Nostradamus Goes Quantum

TECHNISCHE UNIVERSITÄT DARMSTADT

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Technische Universität Darmstadt
www.cryptoplexity.de

The Modern Nostradamus [KK06]

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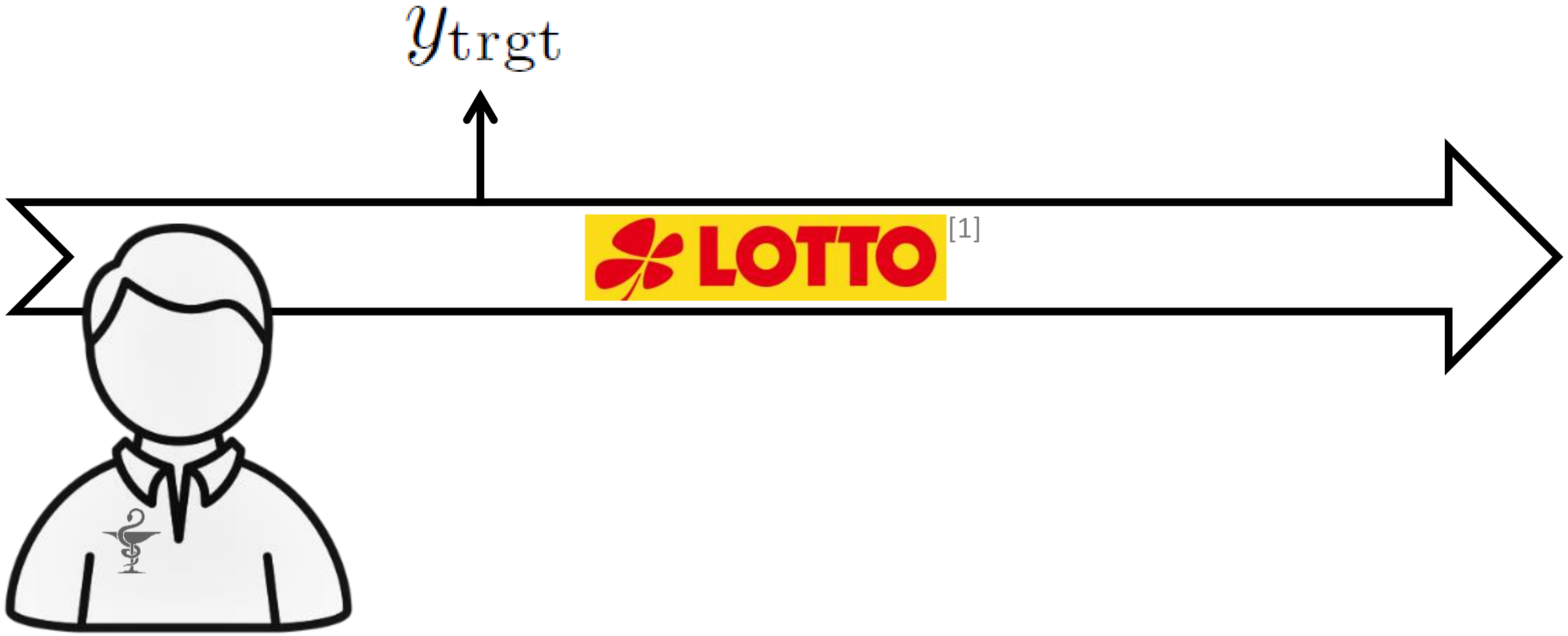

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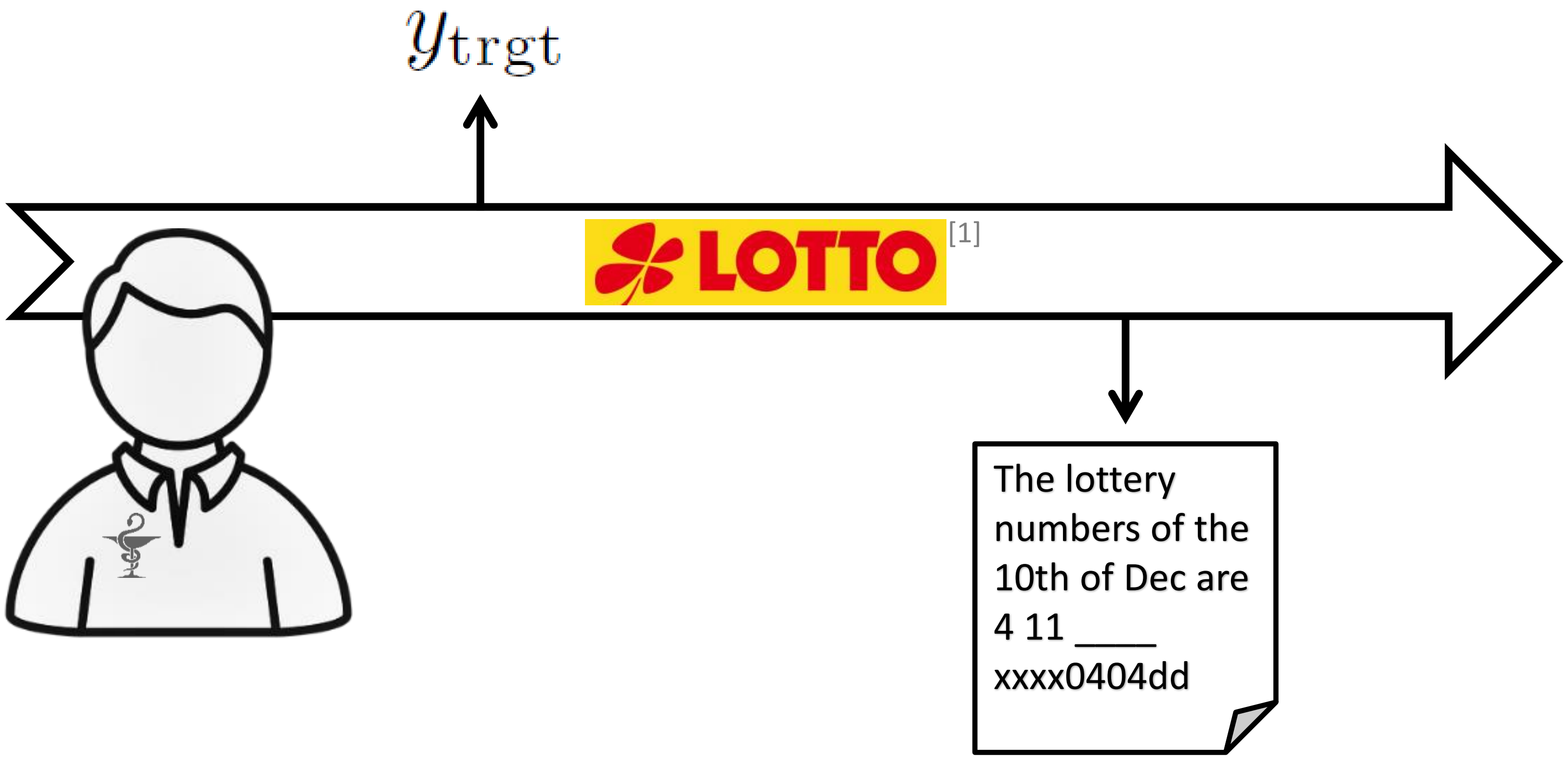

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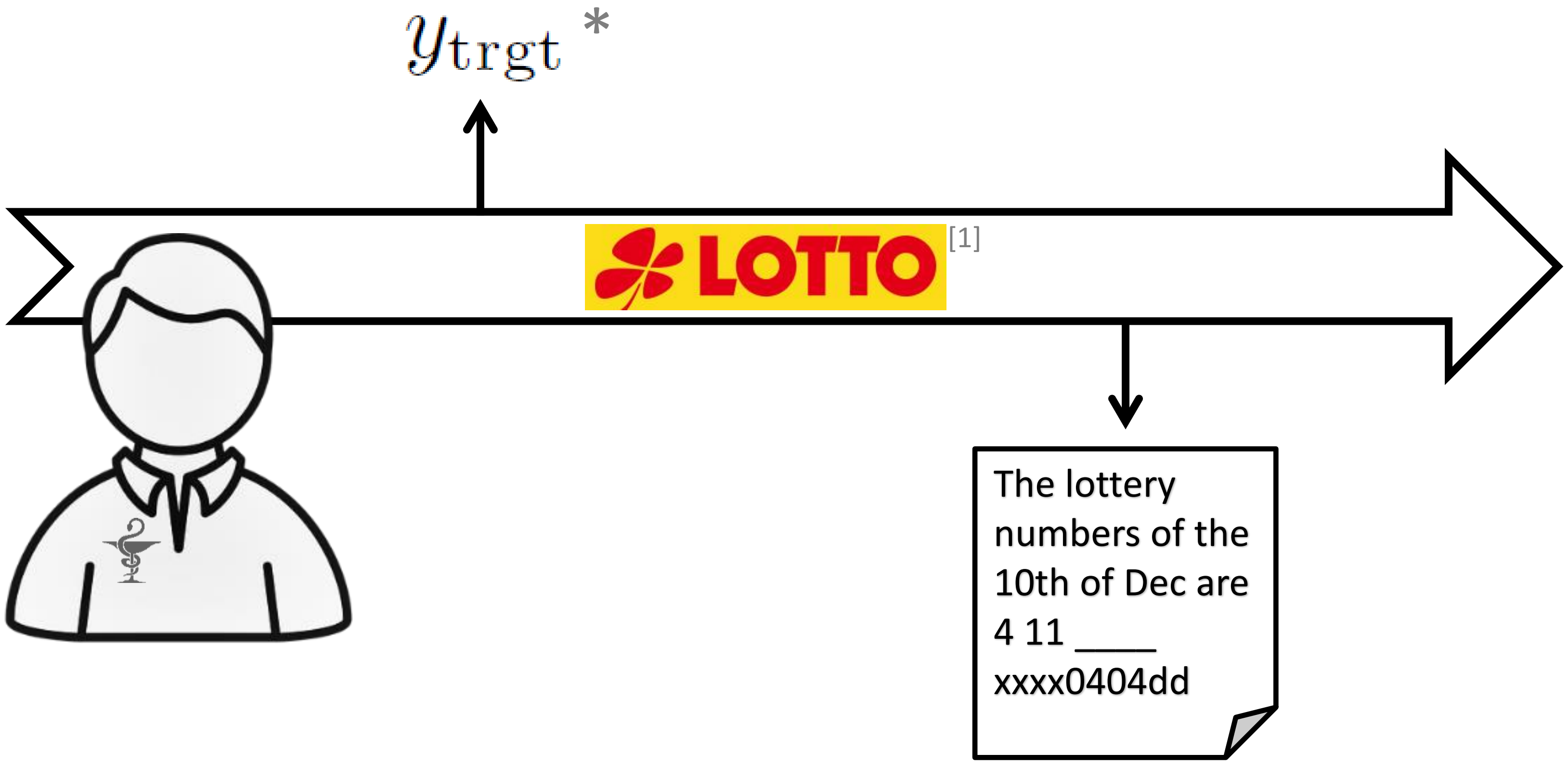

* hash value of an iterated hash function

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History of Nostradamus-Attacker

Legendary Nostradamus

↓

requires magic

Classical Nostradamus [KK06, BSU12]

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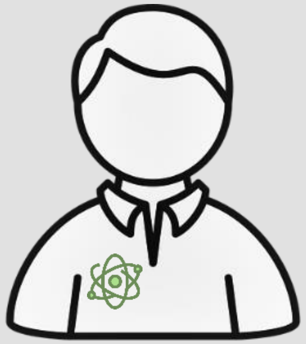

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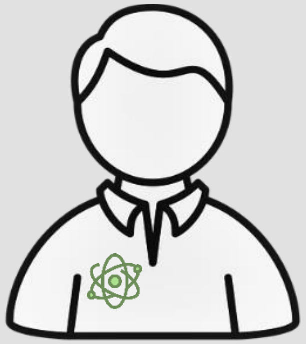

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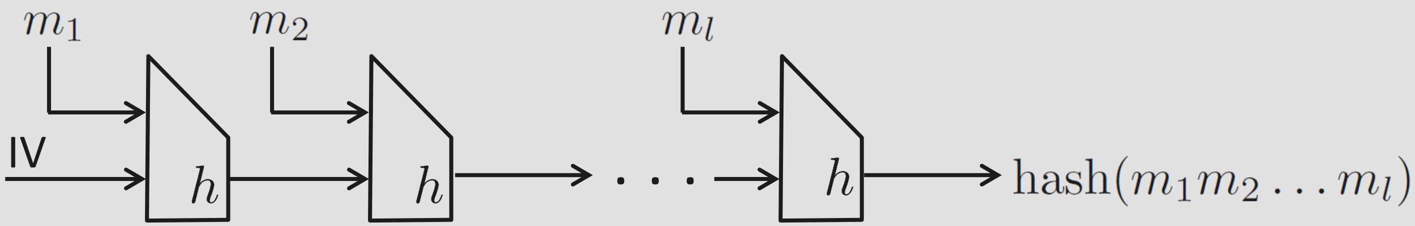
Preliminaries

$\text{hash} : \{0,1\}^* \rightarrow \{0,1\}^n$ **<u>iterated</u>** hash function, e.g.,



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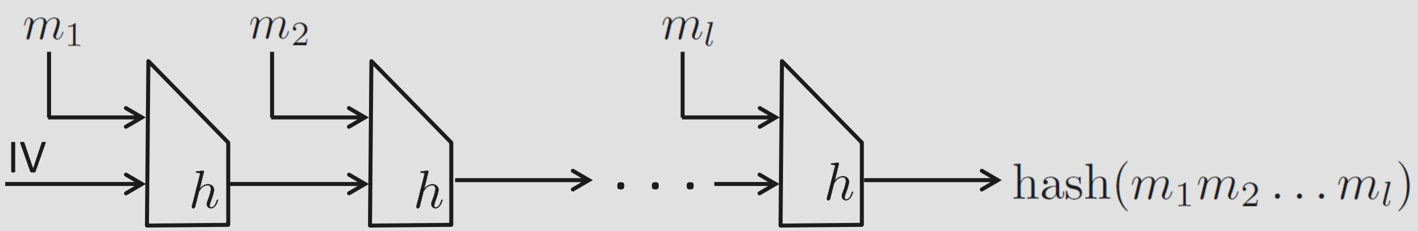
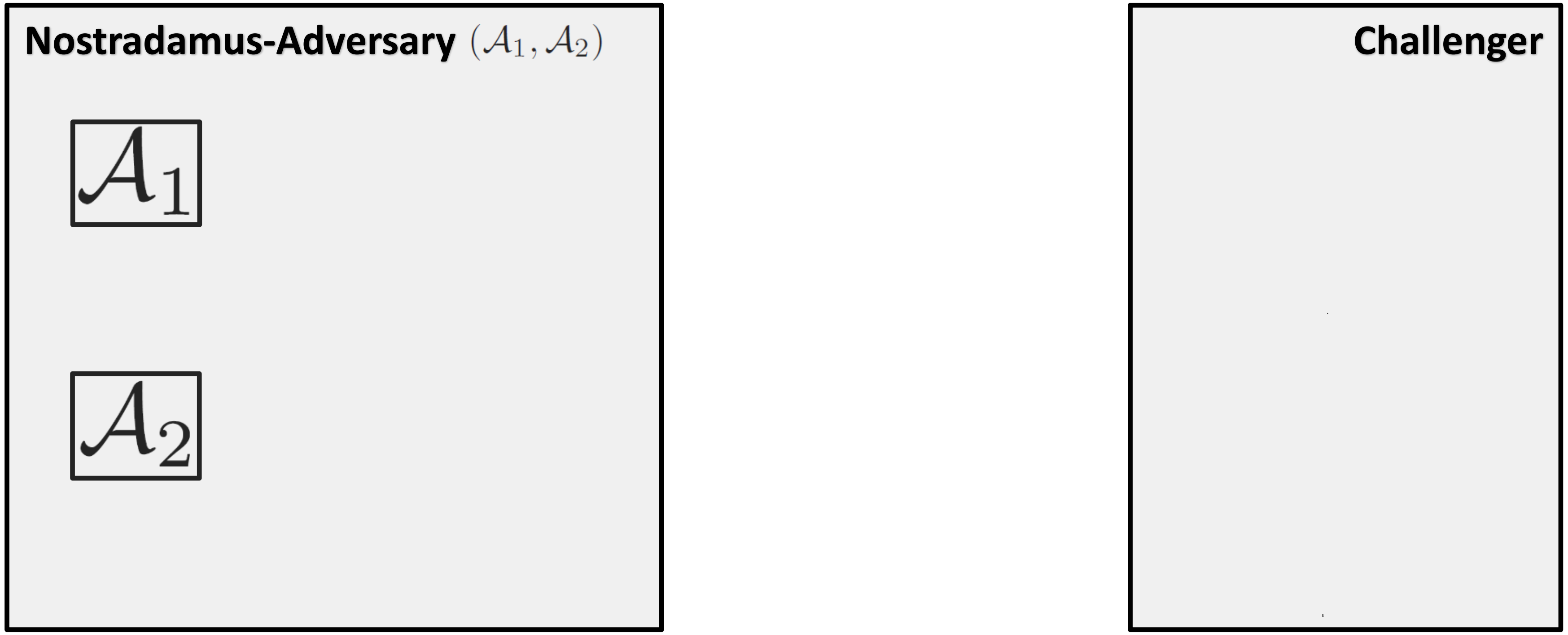


* Results also applicable to SHA-2, SHA-3 or SHAKE.

The Formal Nostradamus-Attack



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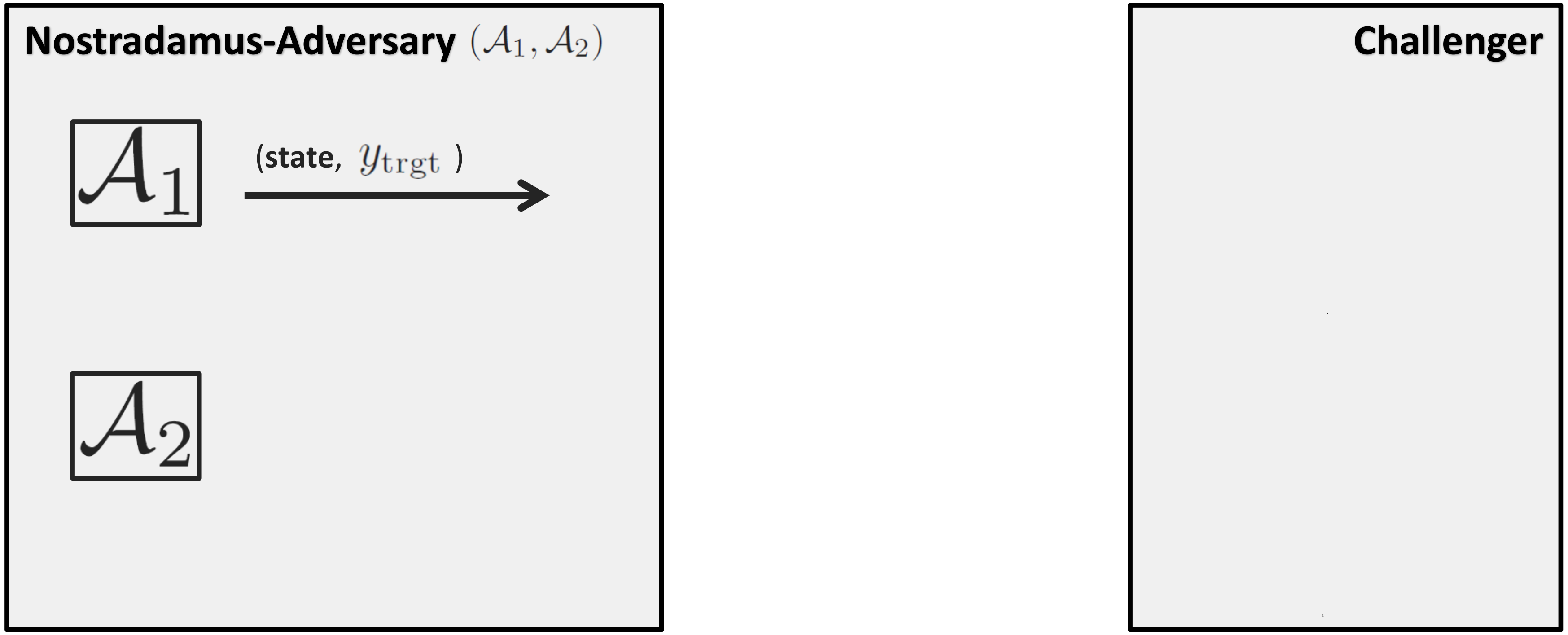

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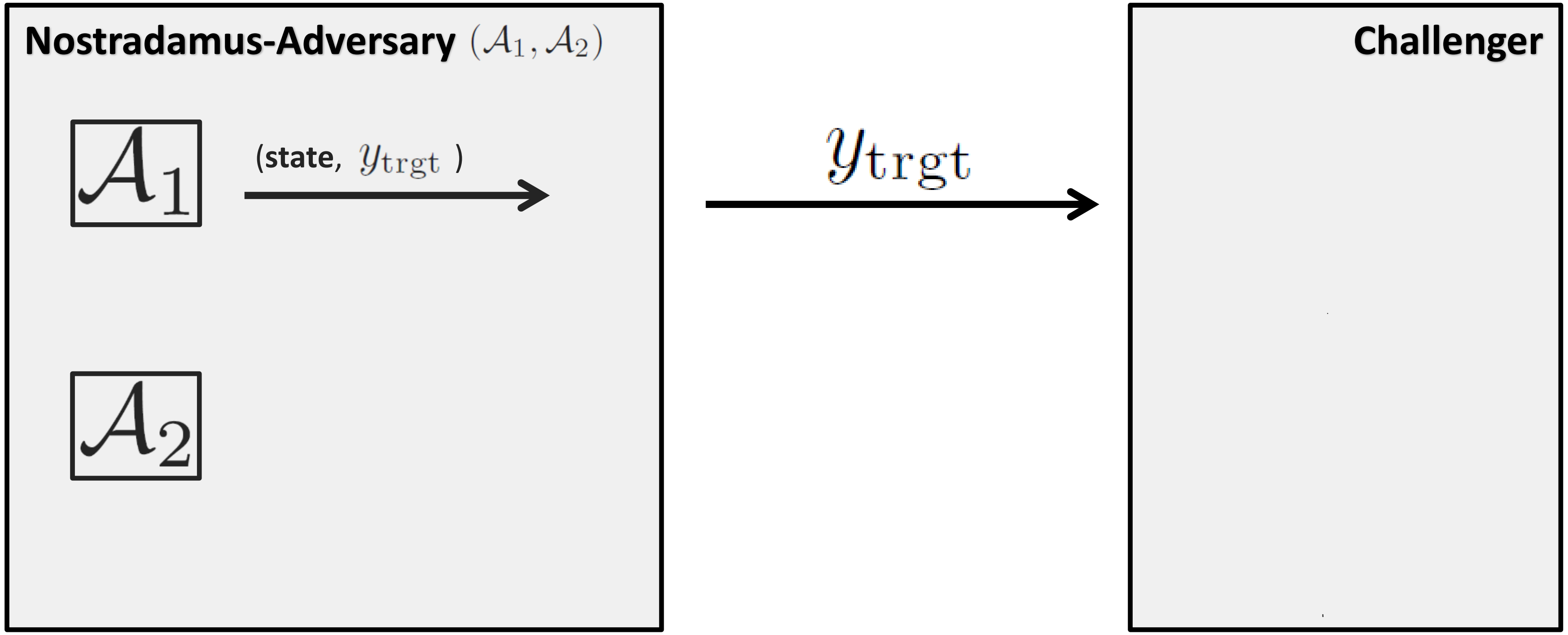

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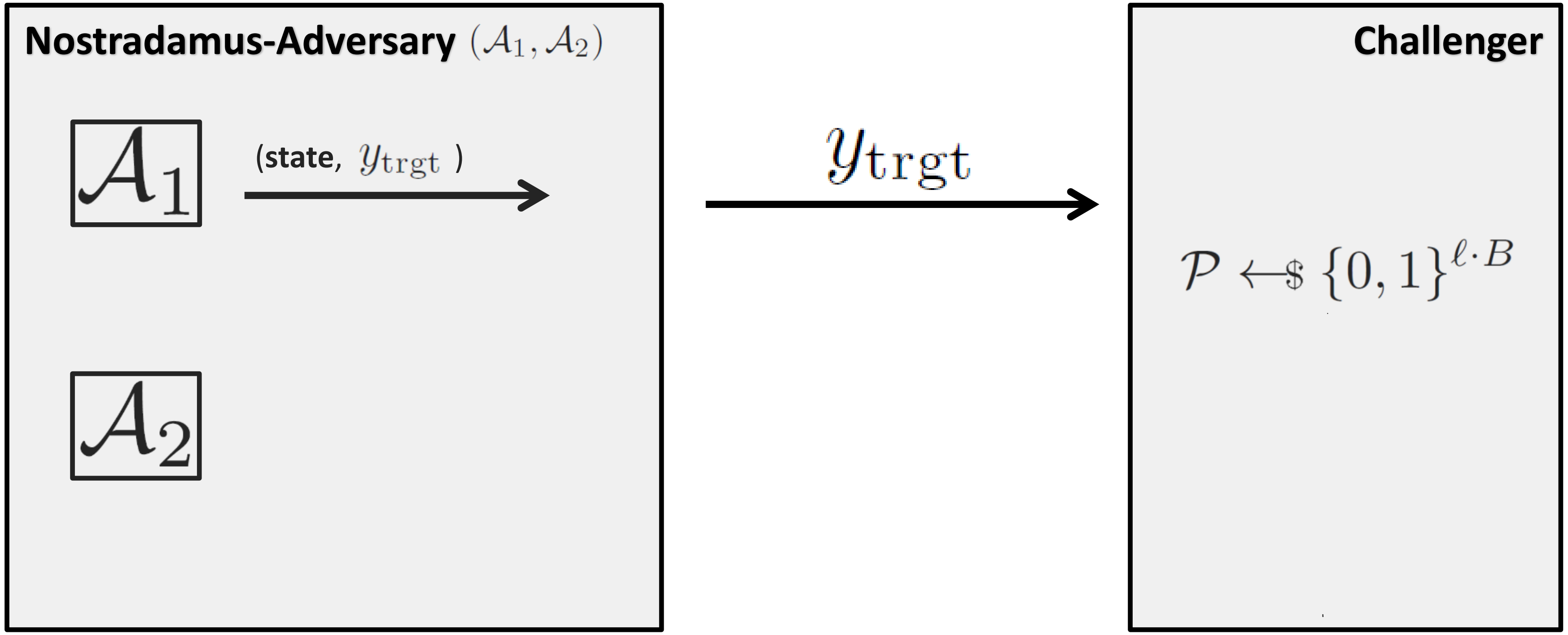

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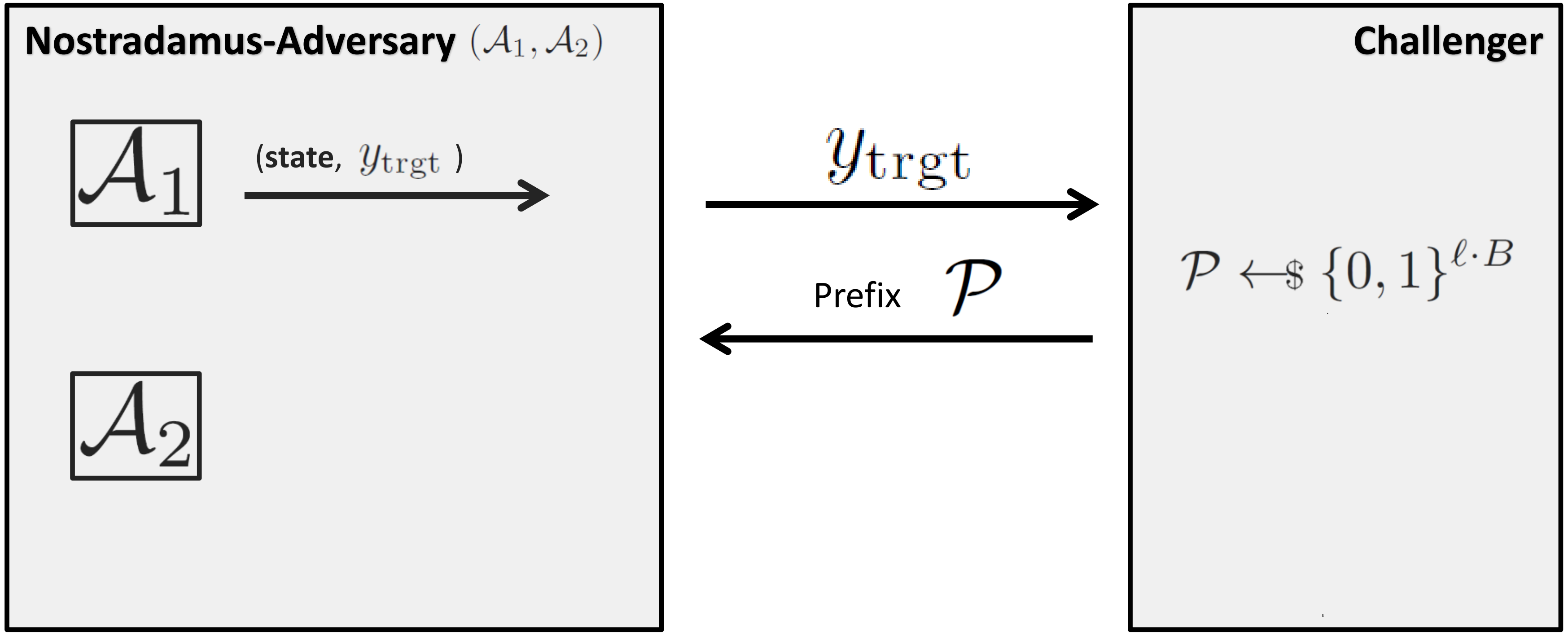

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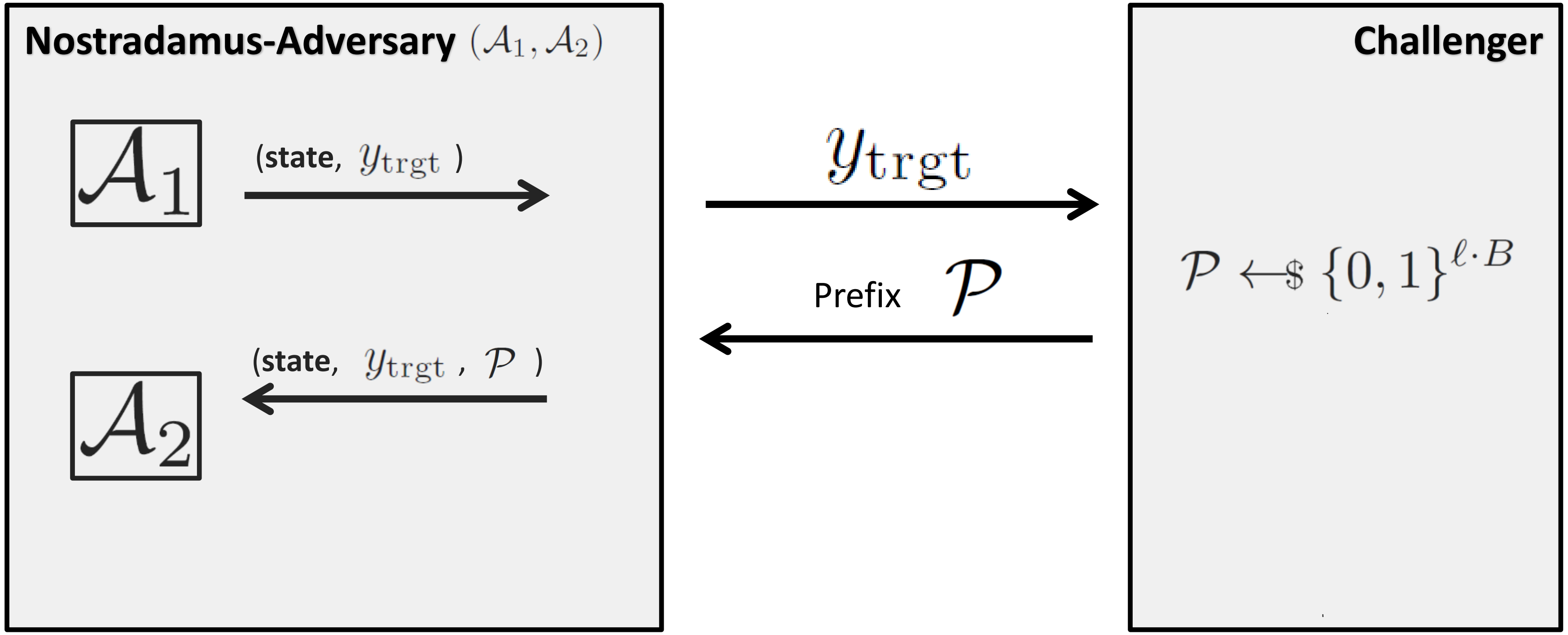

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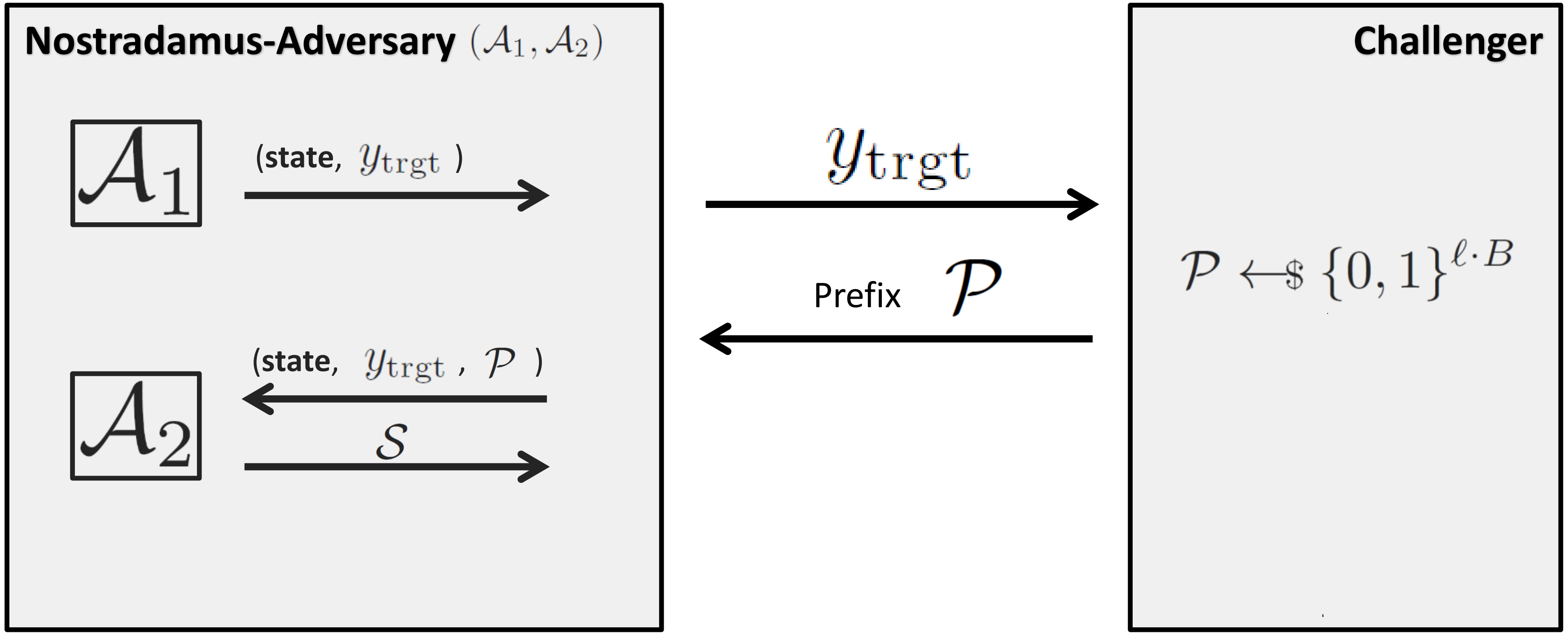

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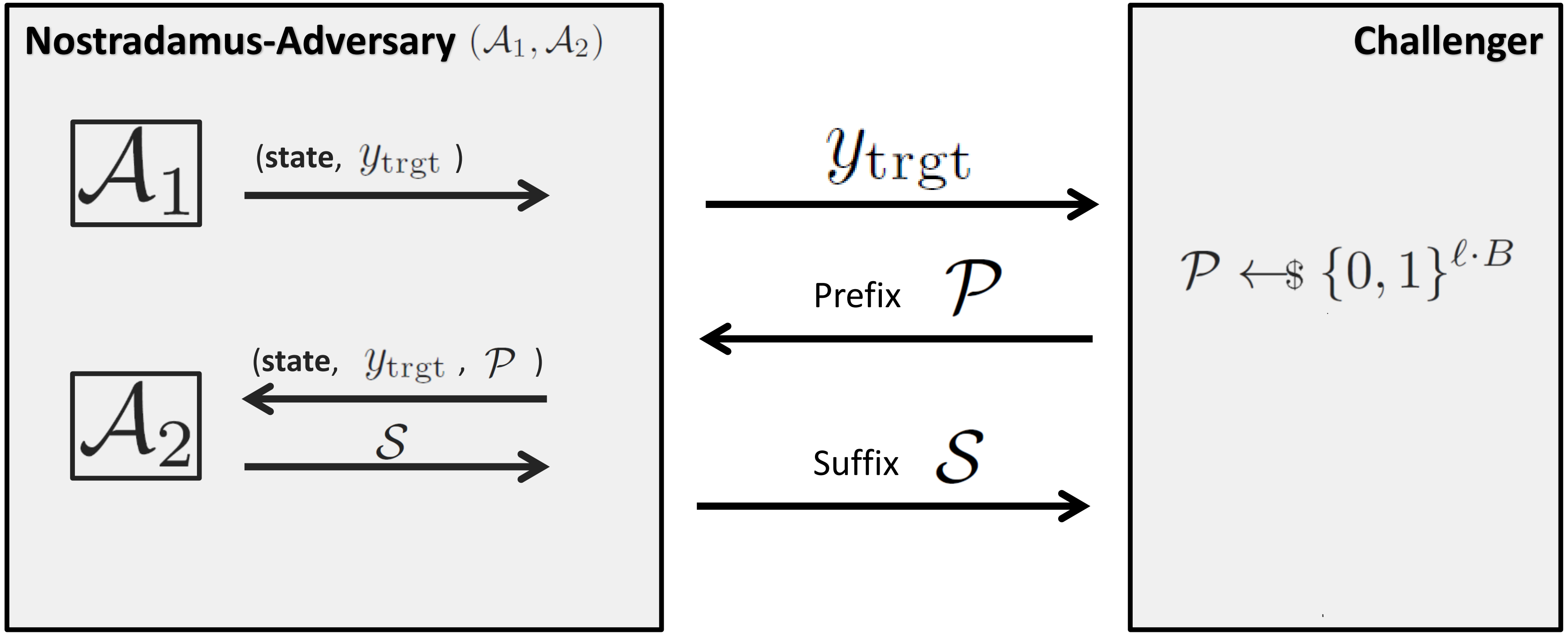

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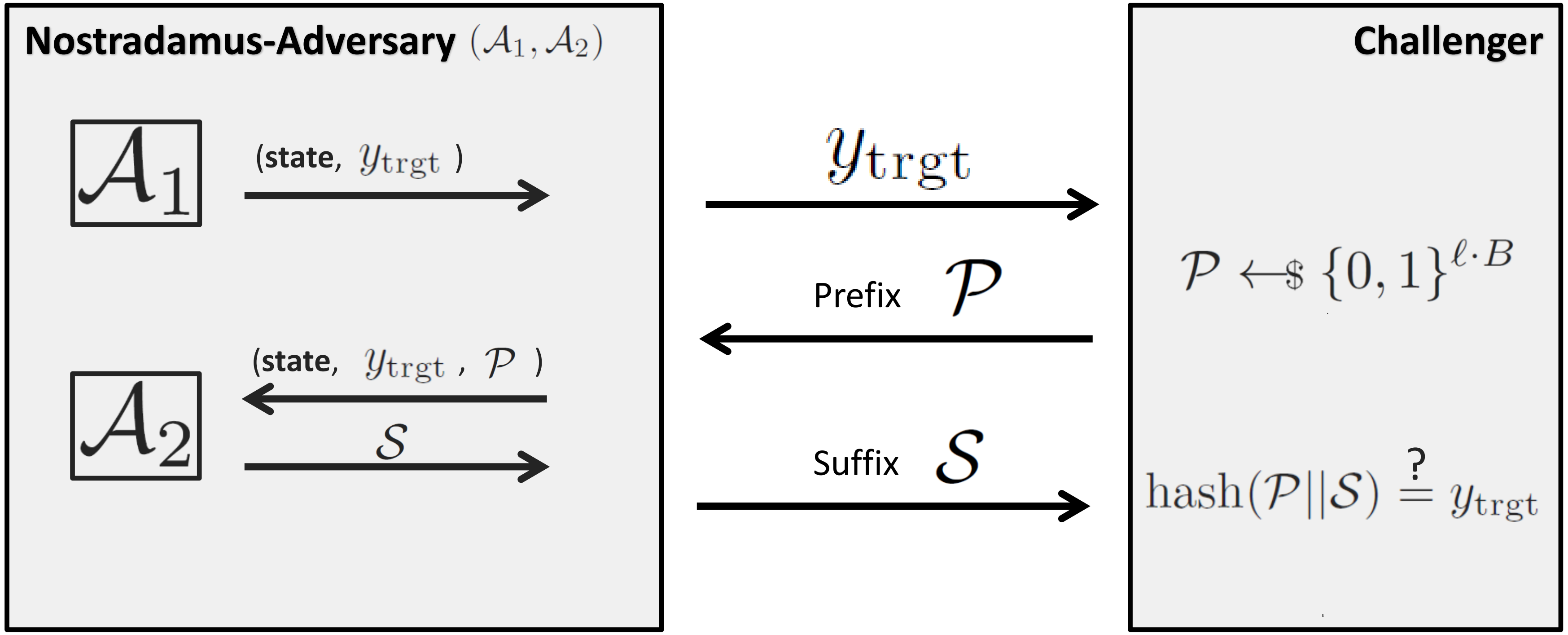

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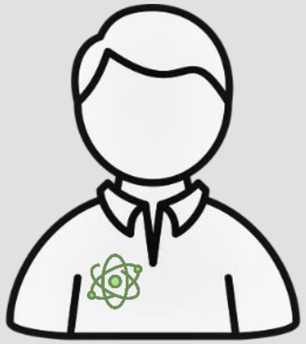

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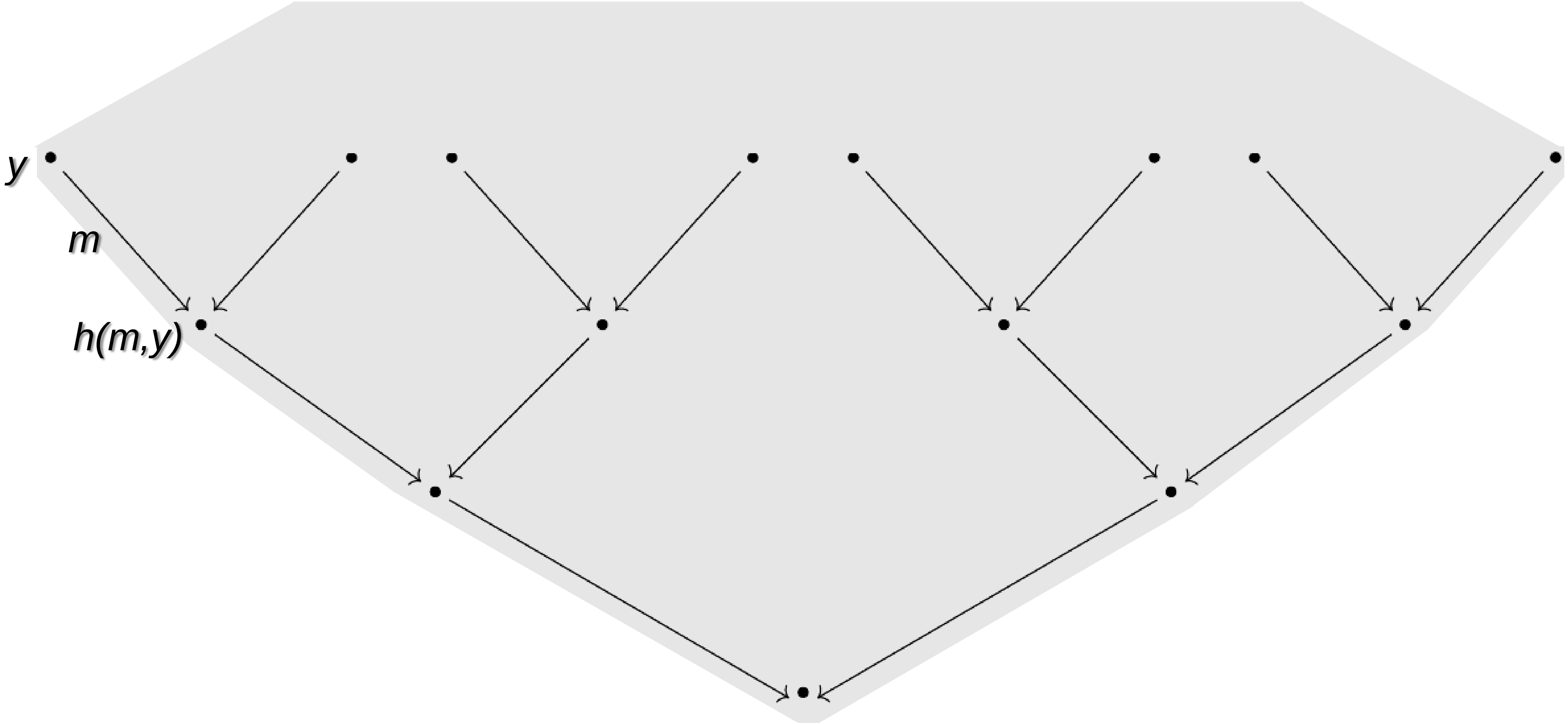

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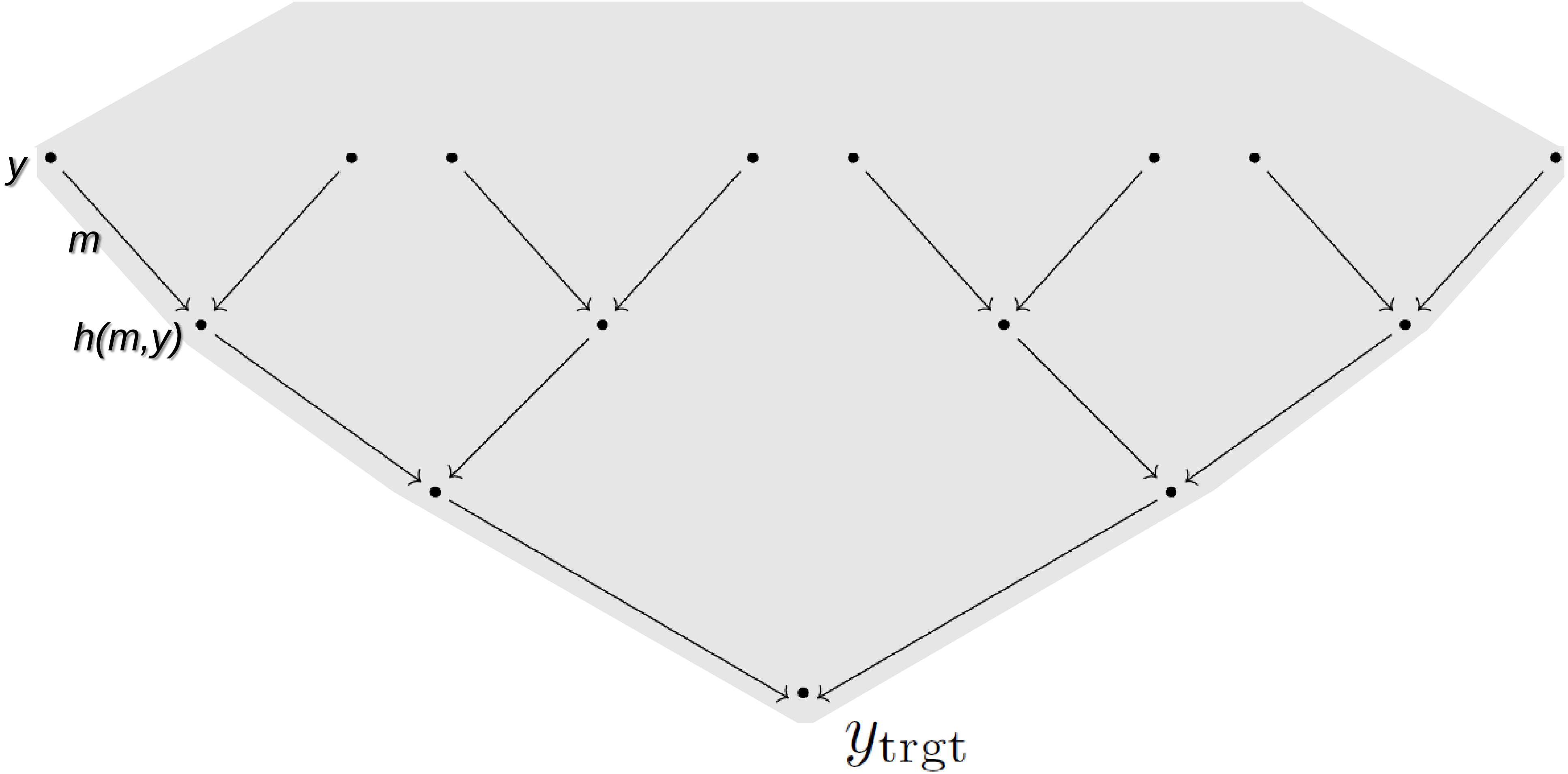

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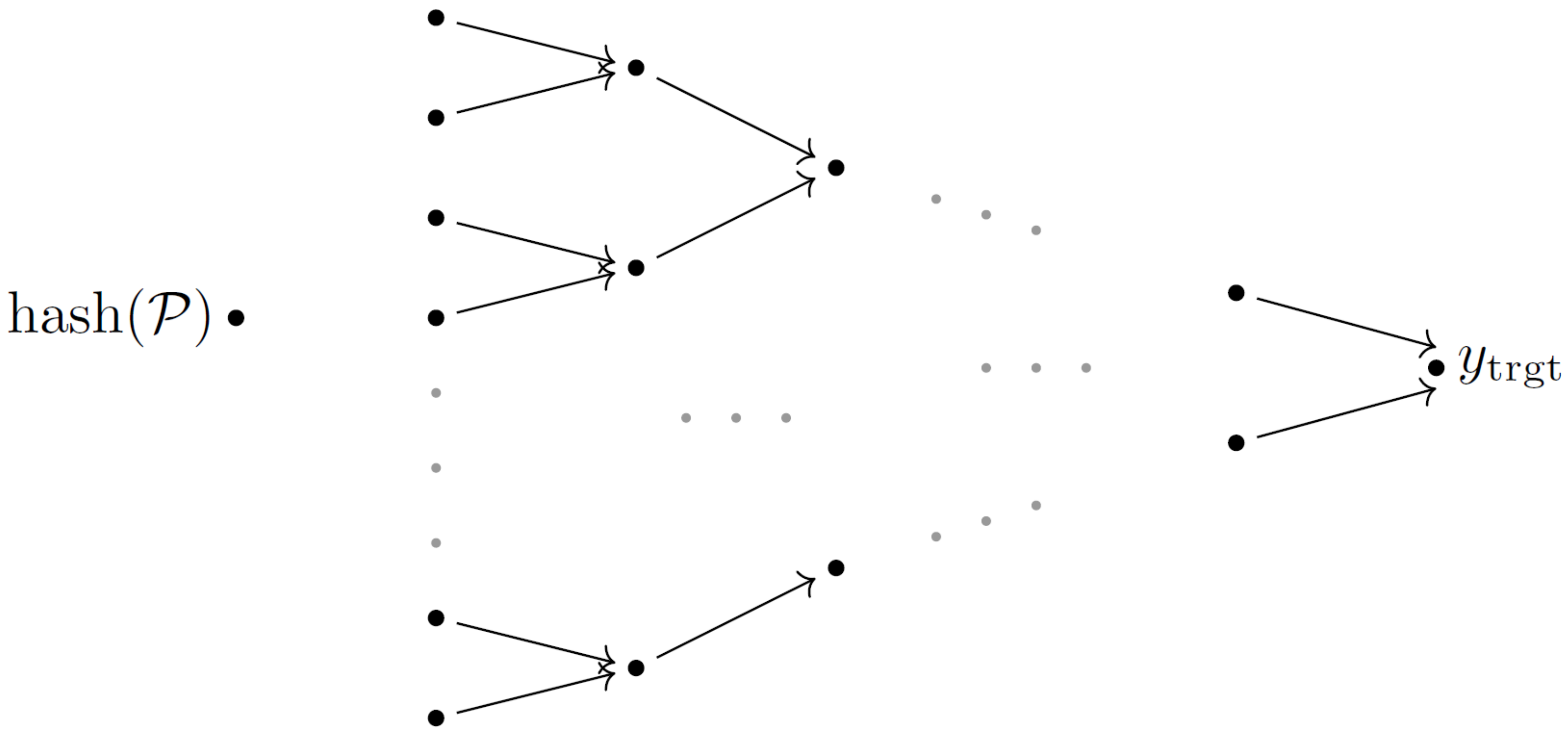

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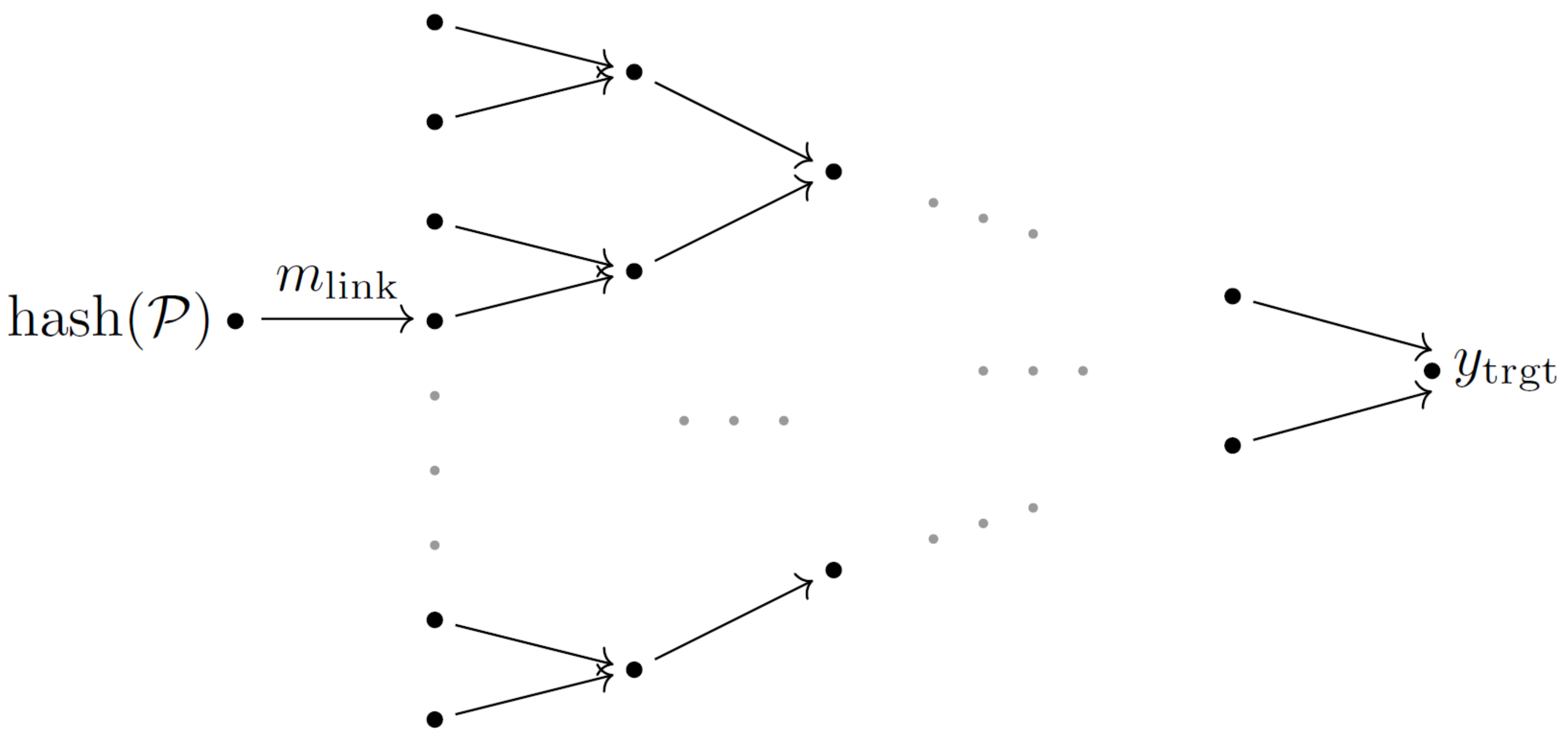

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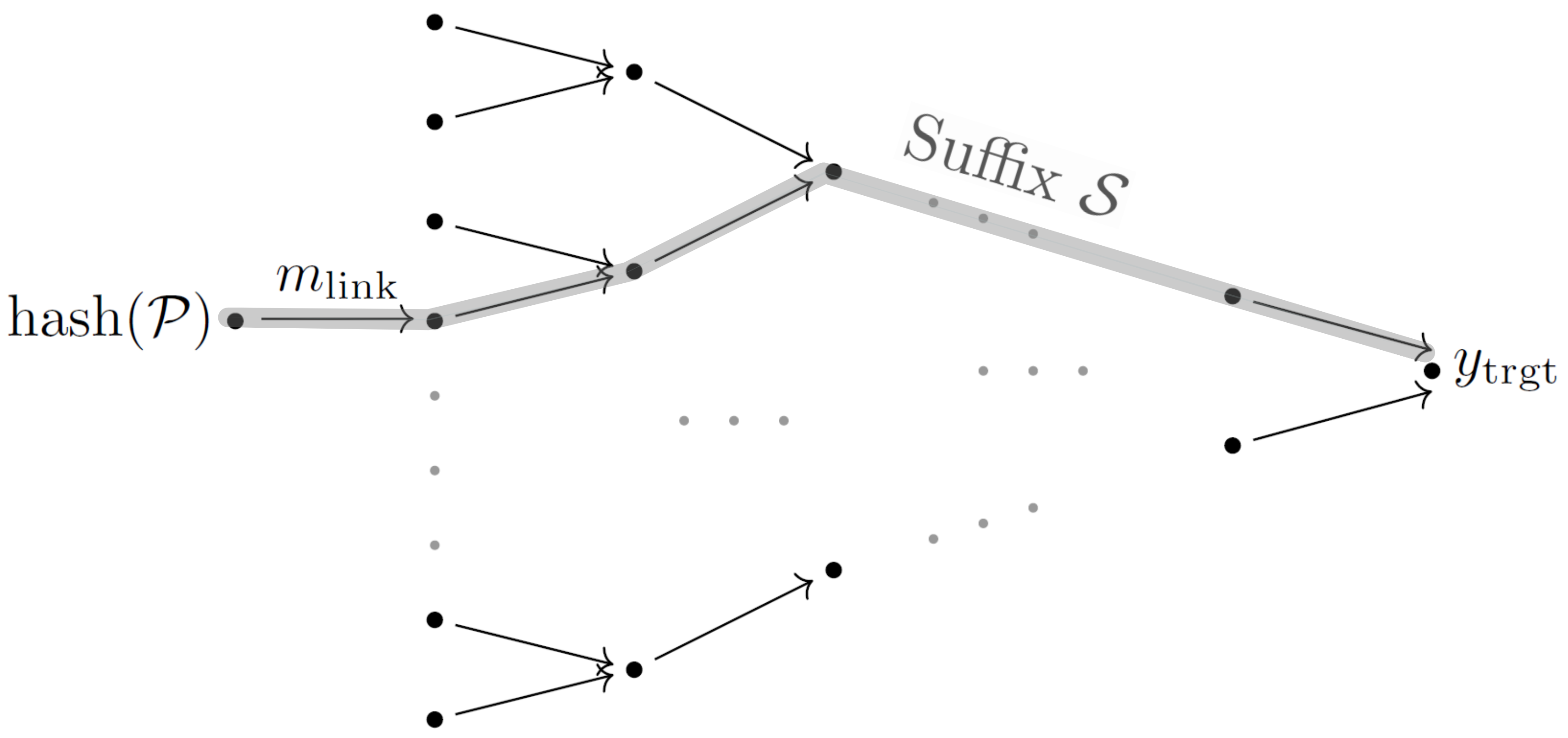

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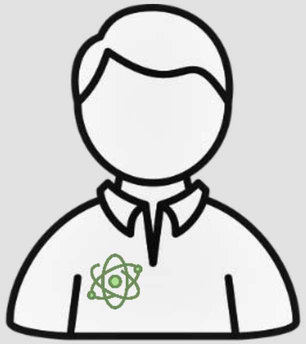

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Grover's Algorithm

Theorem [BBHT98, Gro96]. Let $M^* \subseteq \{0,1\}^B$ be a *non-empty* set of suitable message blocks. Then there is a quantum algorithm, which finds a suitable message block after

$$\mathcal{O}\left(\sqrt{p^{-1}}\right)$$

"valuations" of message blocks, where $p := |M^*| \cdot 2^{-B}$.

[BBHT98] Boyer, Brassard, Høyer and Tapp. Tight bounds on quantum searching. Progress of Physics, 1998.

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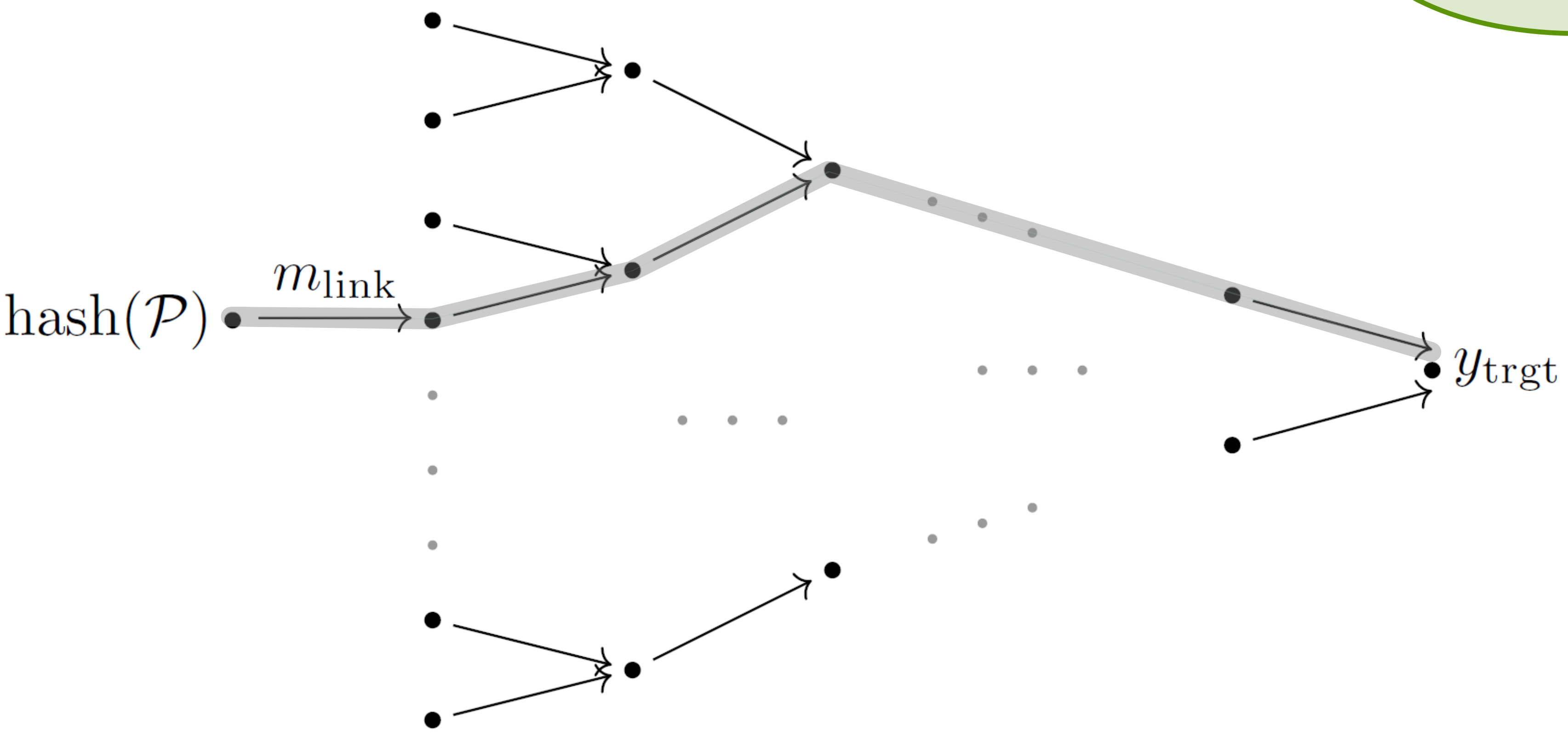
Grover succeeds after $\mathcal{O}\left(\sqrt{p^{-1}}\right)$ evaluations of h.



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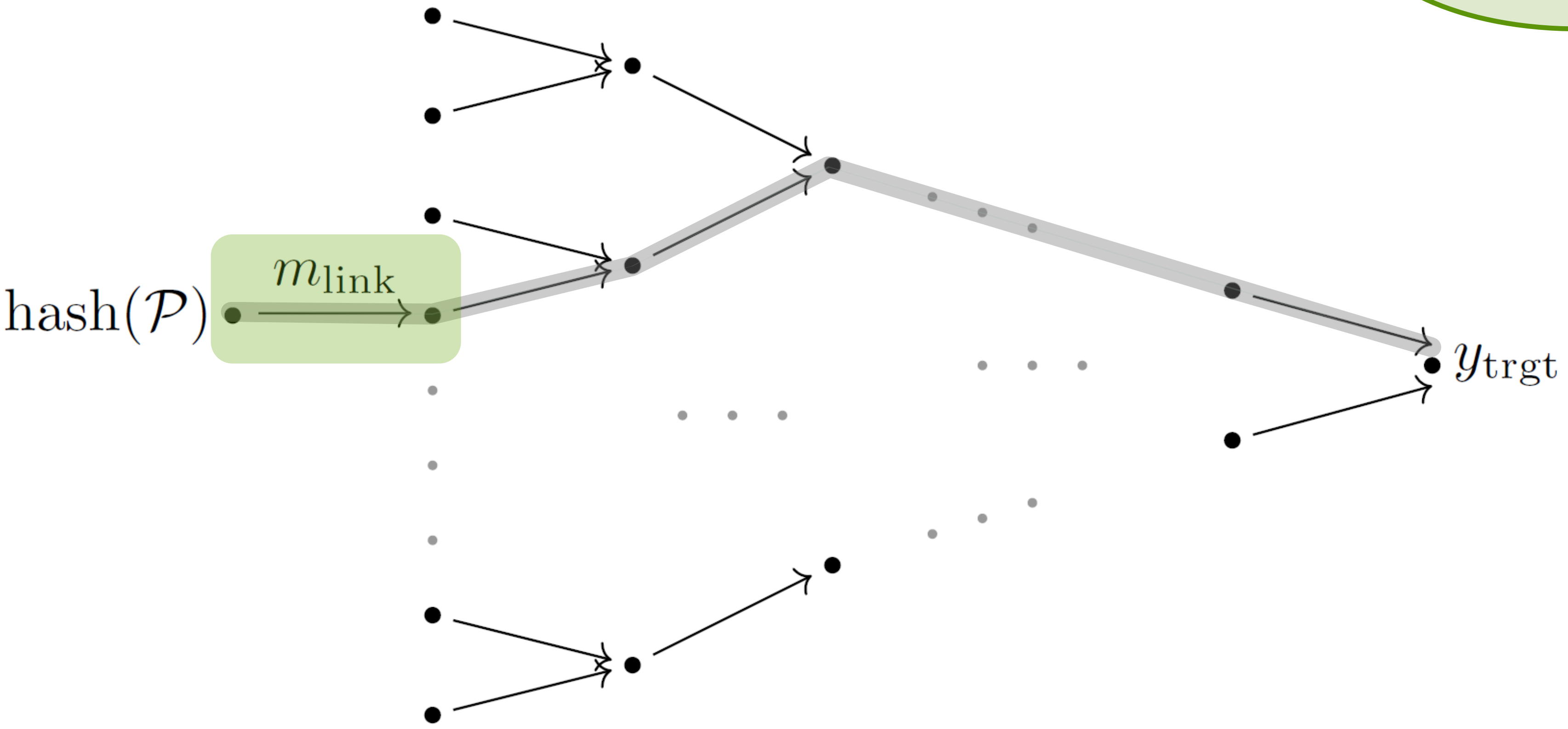

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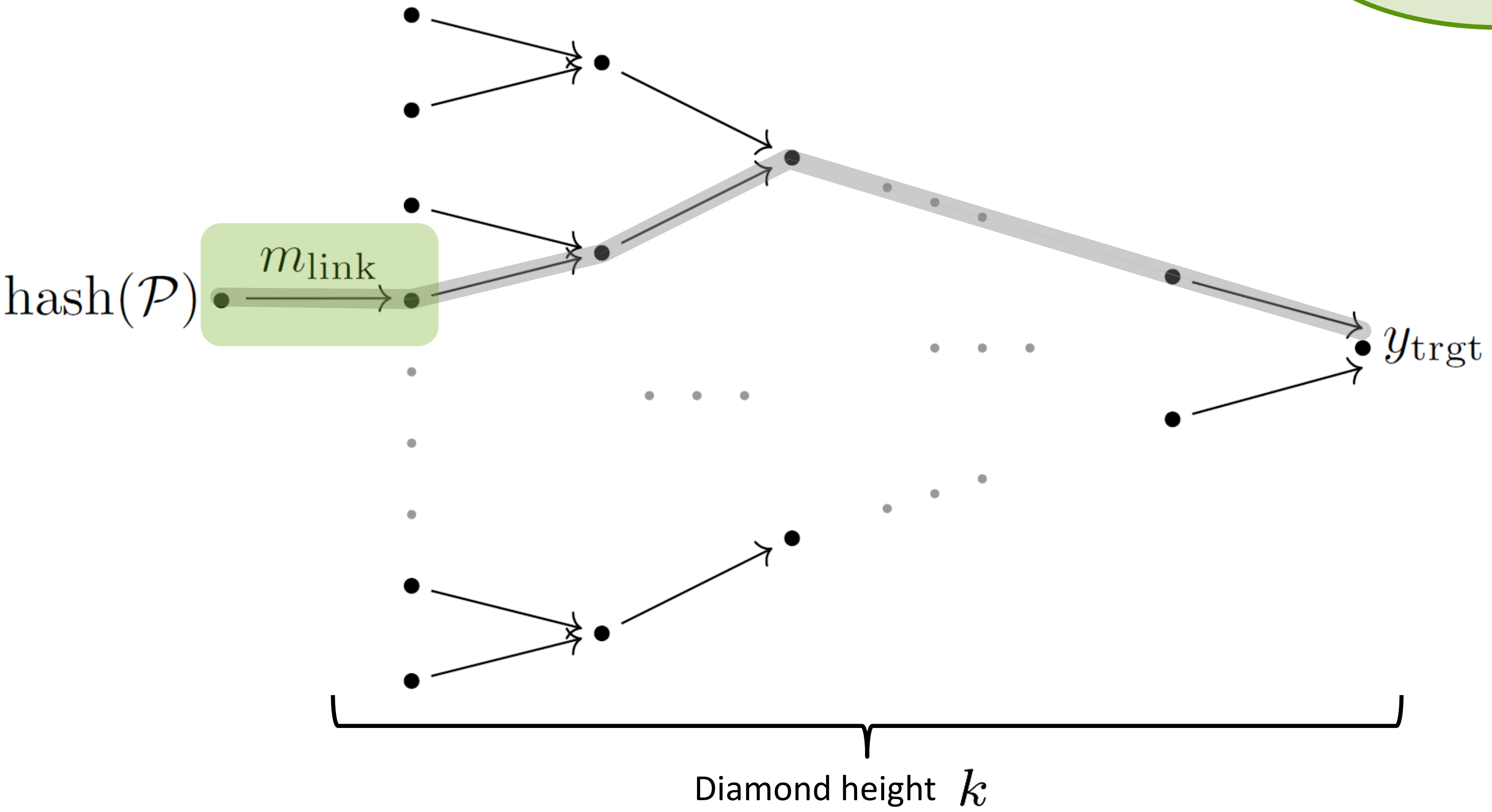

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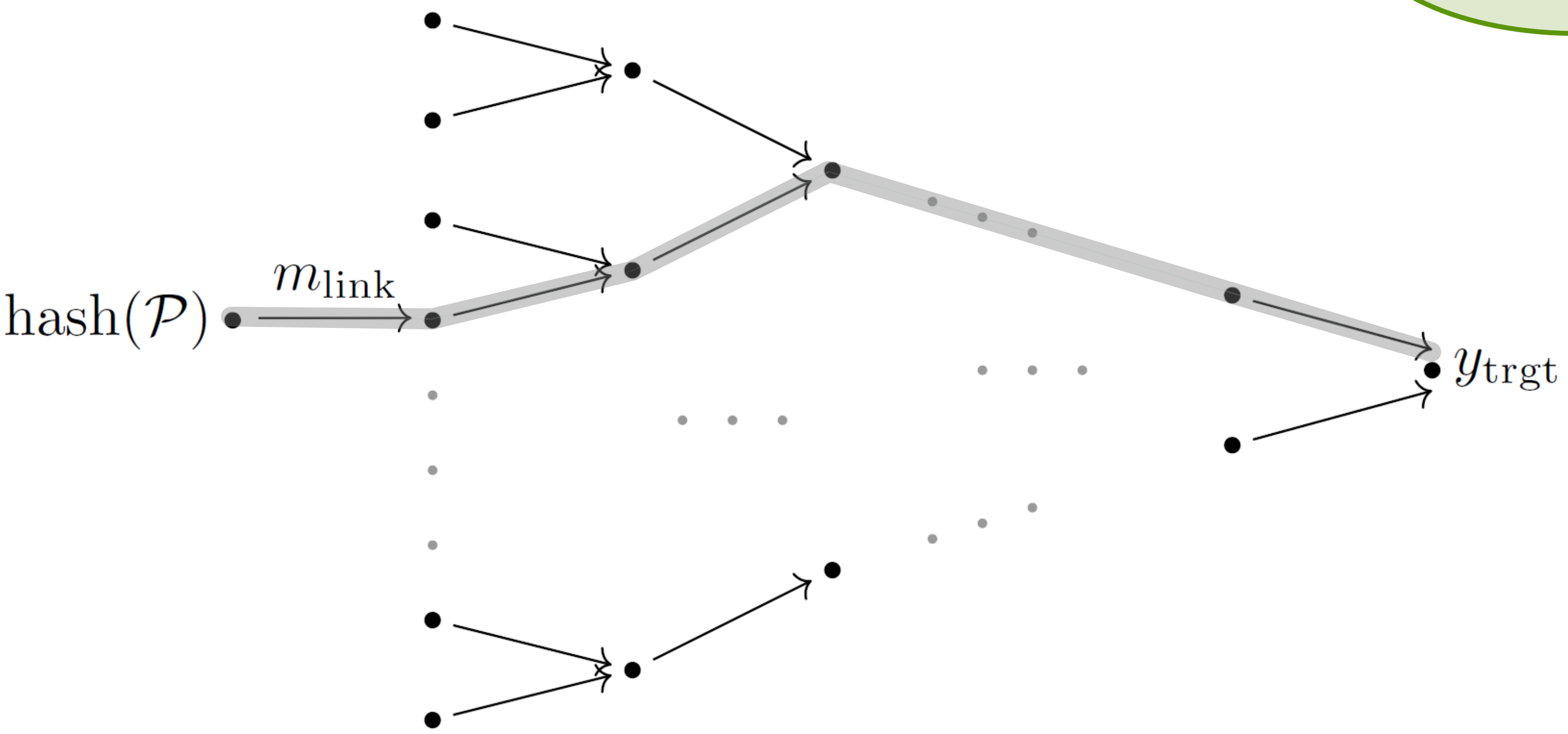

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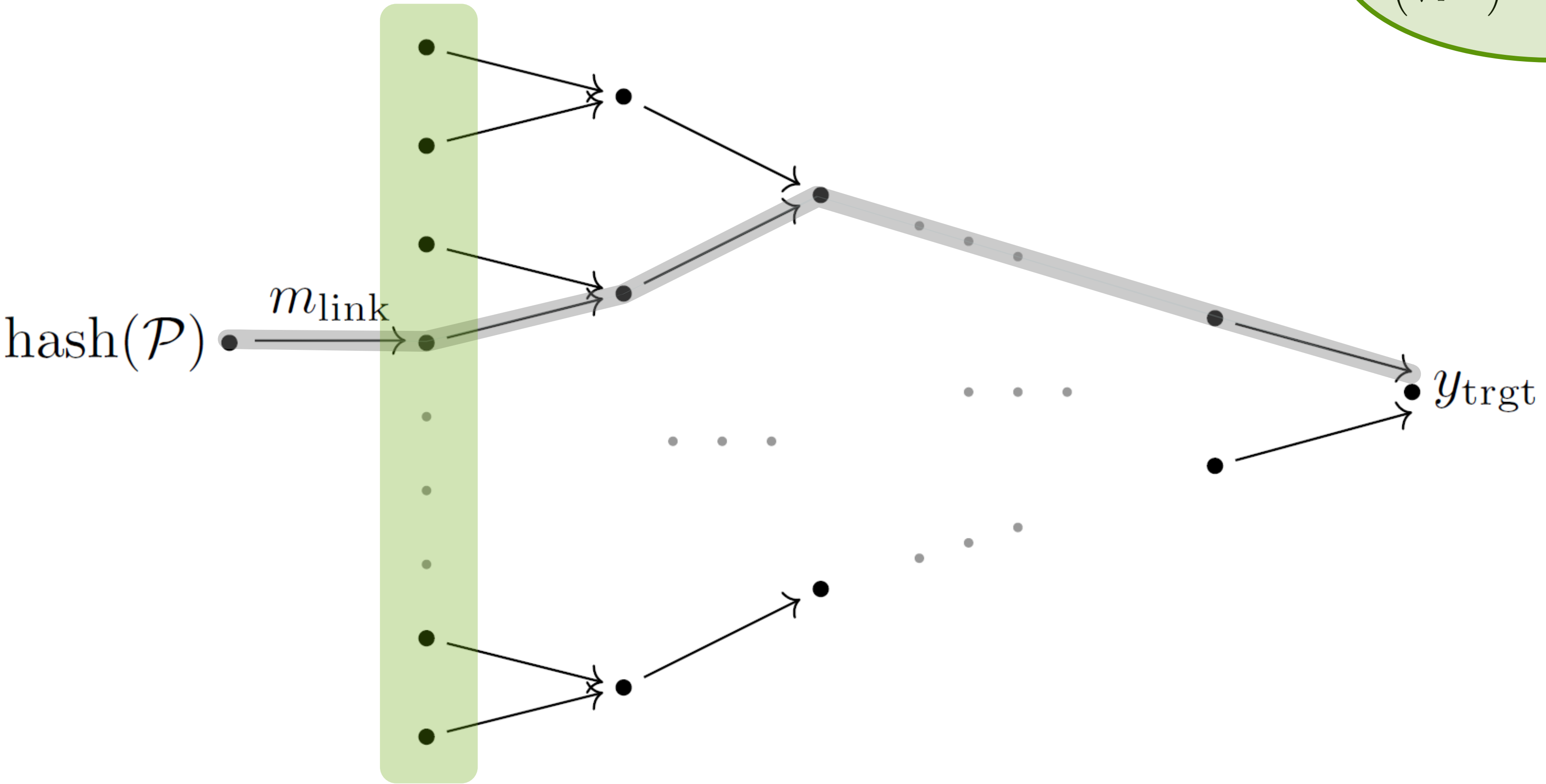

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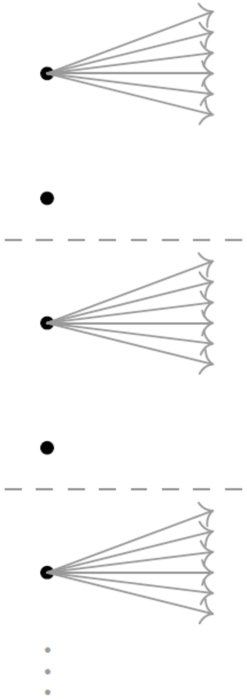

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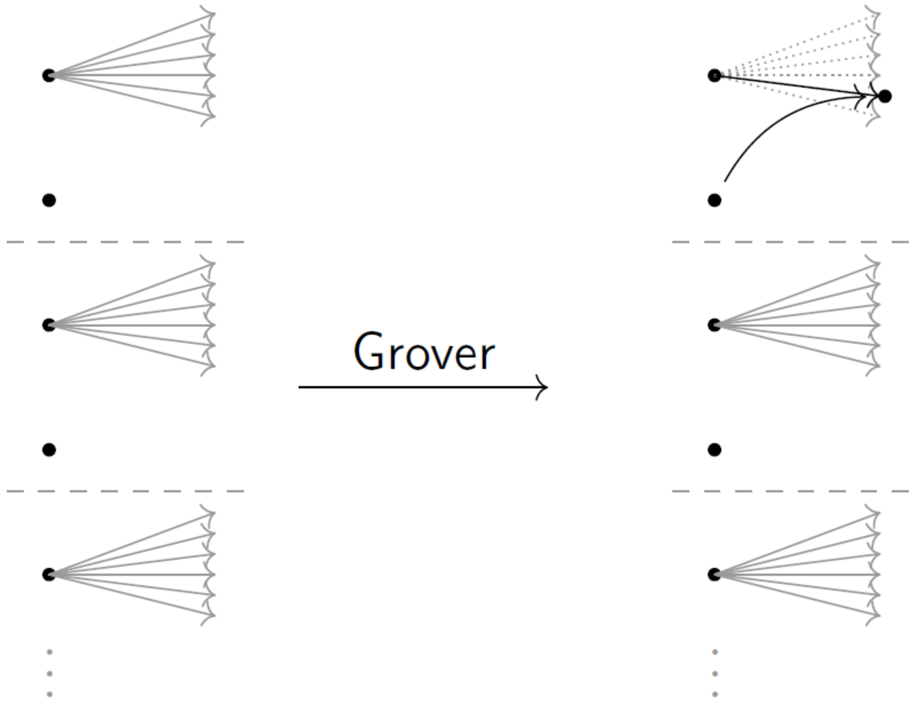

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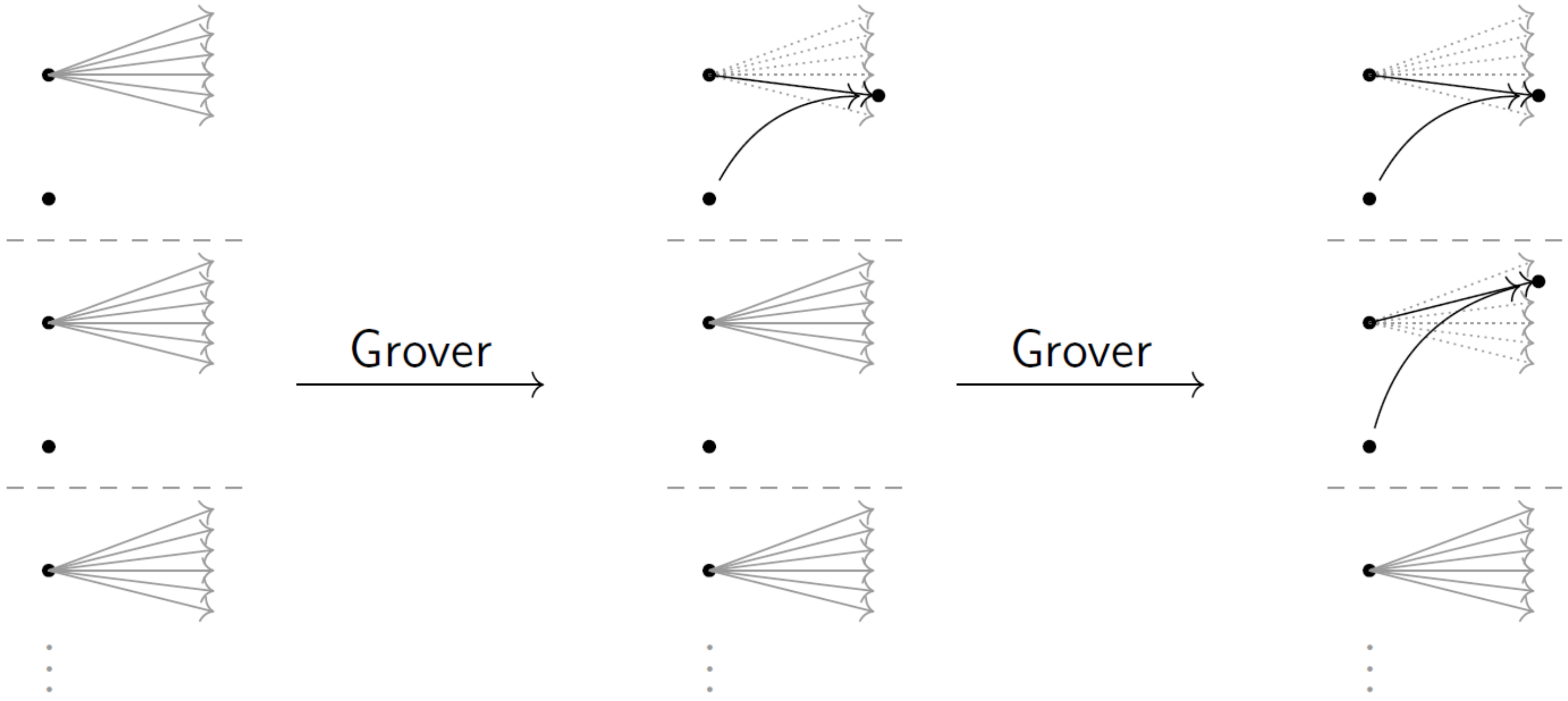

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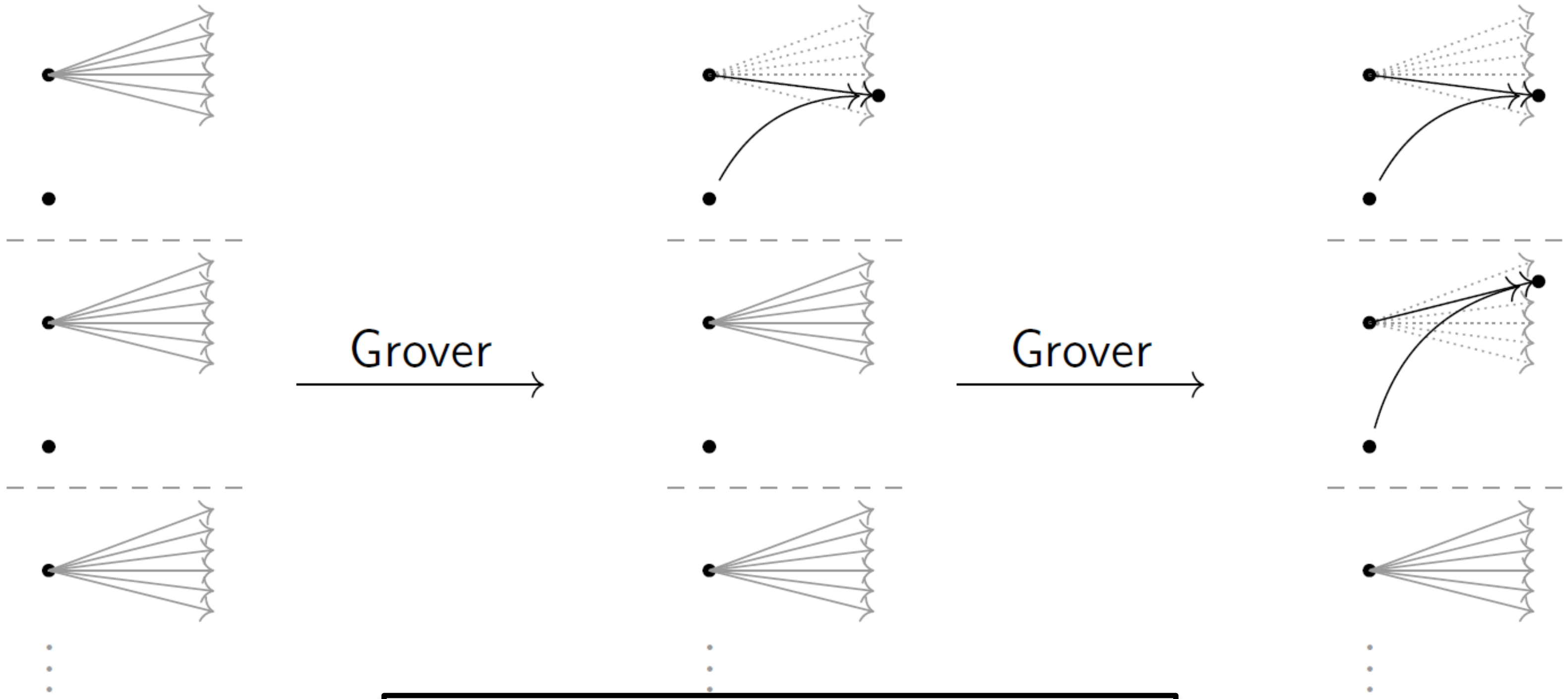

Problem: Needs a lot of evaluations for each pair!

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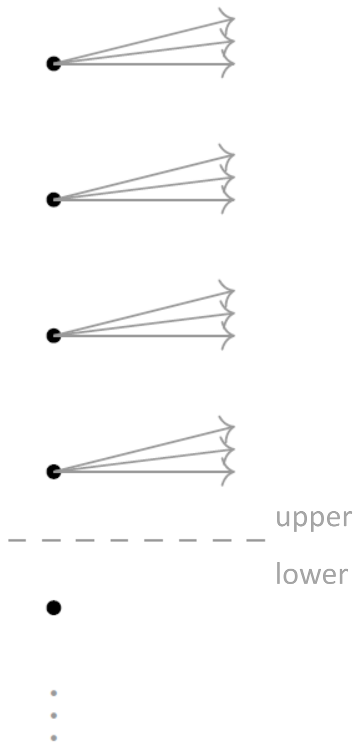

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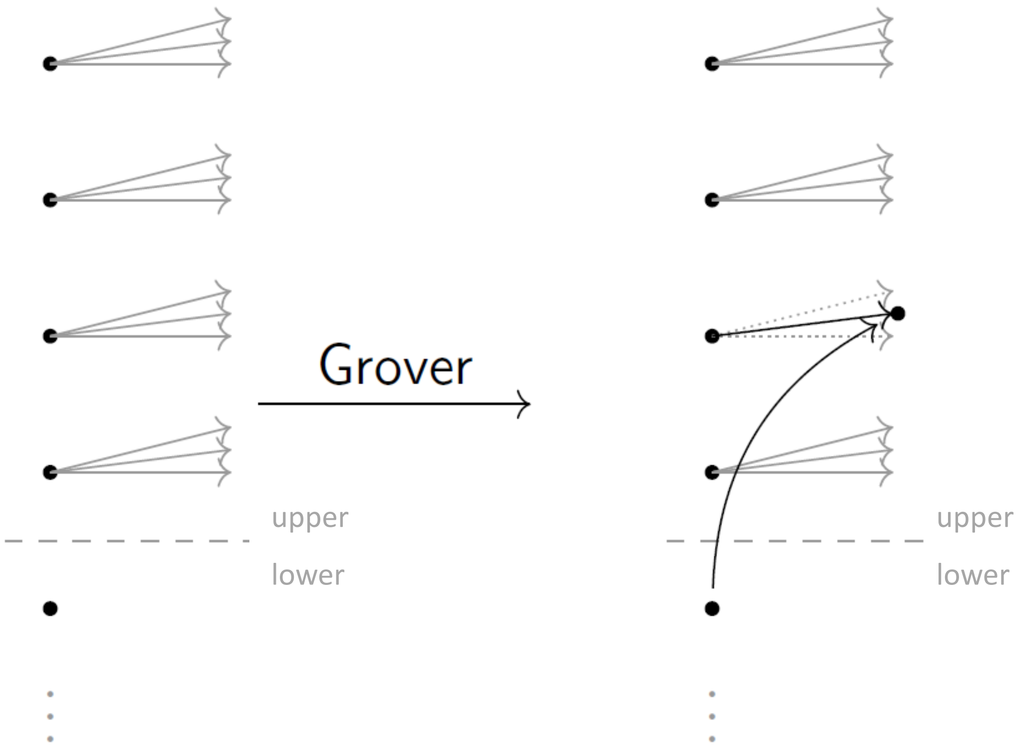

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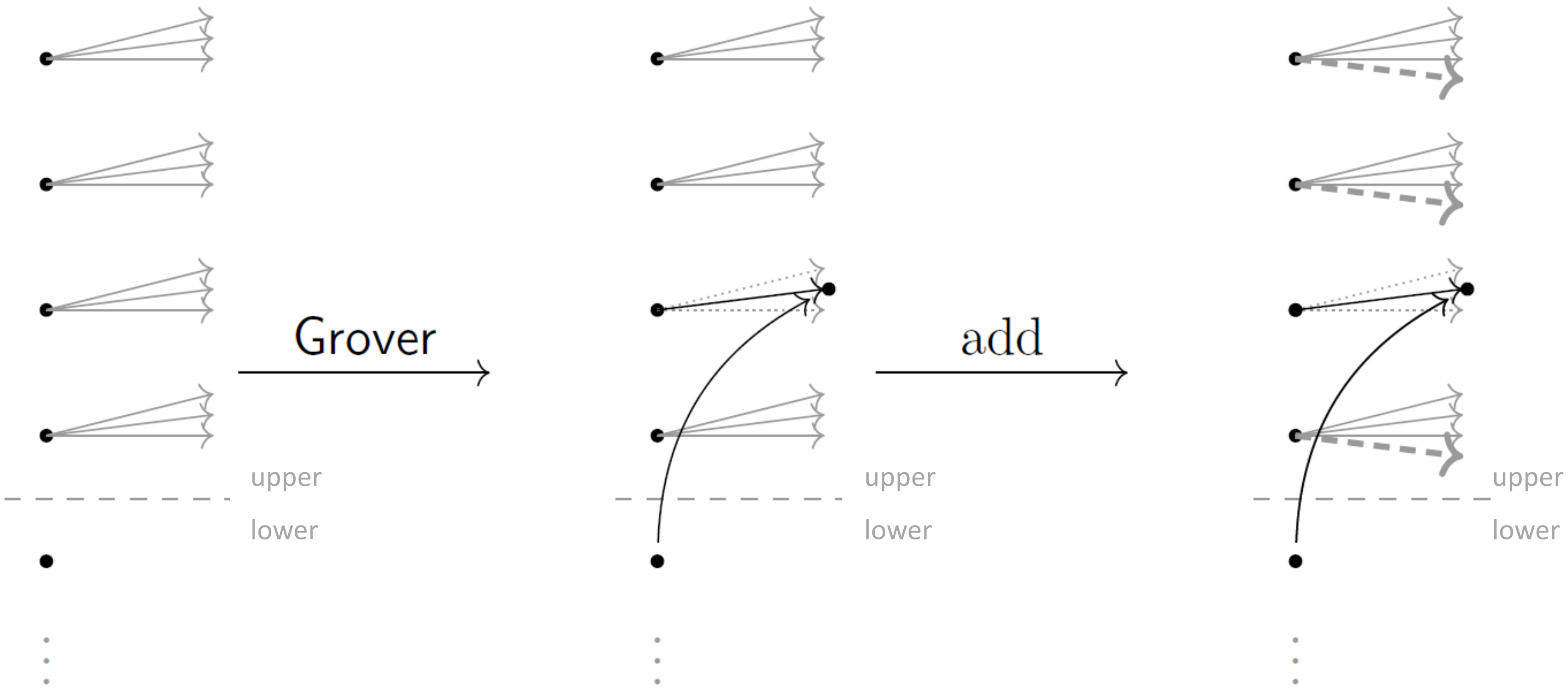

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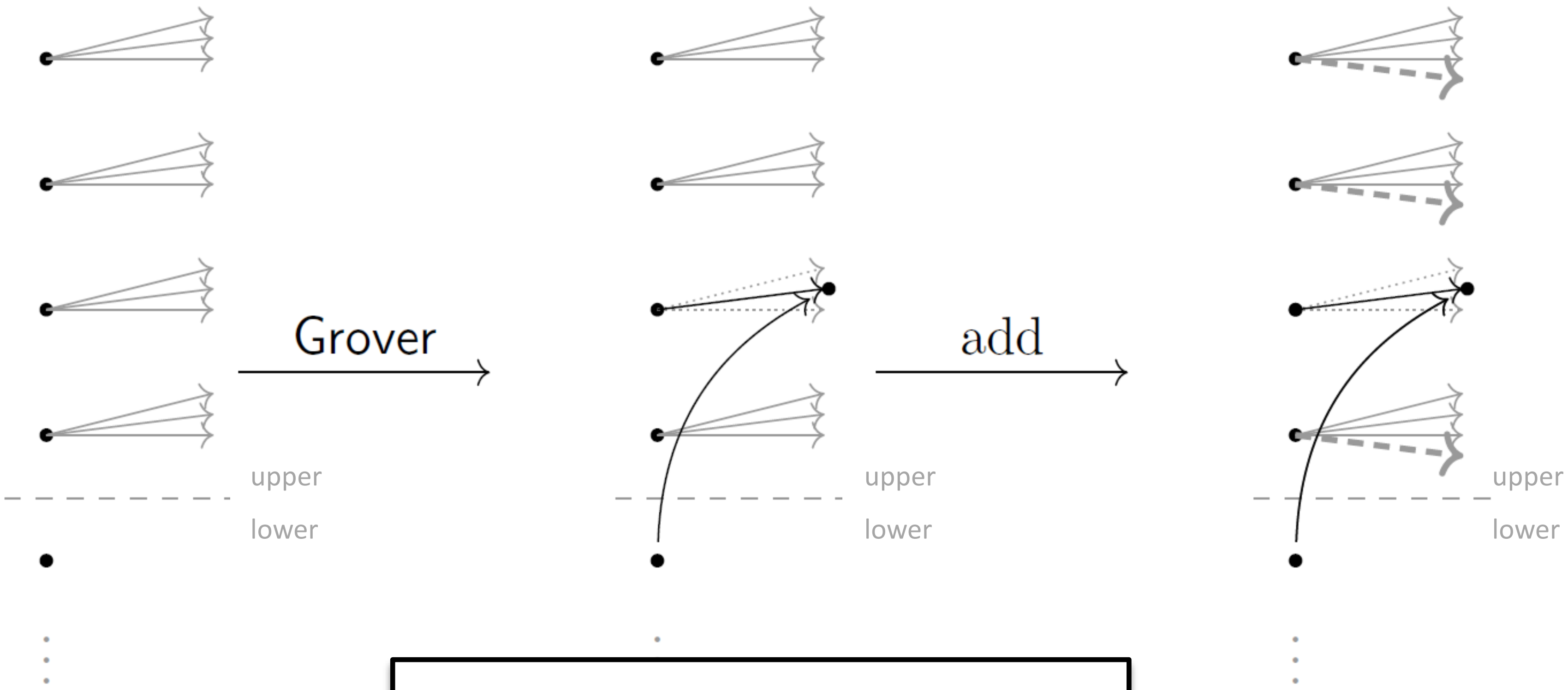

Advantage: Needs fewer evaluations on average for each pair!

	Classical [KK06]	Naïve **[our work]**	Enhanced **[our work]**
$\mathcal{A}_1$	$\mathcal{O}\left(\sqrt{k} \cdot 2^{\frac{n+k}{2}}\right)$	$\mathcal{O}(2^{\frac{n}{3}+k})$	$\mathcal{O}\left(\sqrt[3]{k} \cdot 2^{\frac{n+2k}{3}}\right)$
$\mathcal{A}_2$	$\mathcal{O}(2^{n-k})$	$\mathcal{O}\left(2^{\frac{n-k}{2}}\right)$	$\mathcal{O}\left(2^{\frac{n-k}{2}}\right)$
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About Lower Bounds

Lower Bound of c-Collision Finder [LZ19]

Theorem. Given a *random* function $f : X \to Y$ any quantum algorithm needs at least

$$\Omega\left(2^{\left(1-\frac{1}{2^c-1}\right)\cdot\frac{n}{2}}\right)$$

evaluations of function f to find a **c-collision**, i.e., c different elements $x_1, x_2, \ldots, x_c \in X$ such that $f(x_1) = f(x_2) = \cdots = f(x_c)$ with constant probability.

[LZ19] Liu and Zhandry. On Finding Quantum Multi-Collisions. Advances in Cryptology – EUROCRYPT 2019.

How to derive a Lower Bound for $(\mathcal{A}_1, \mathcal{A}_2)$?

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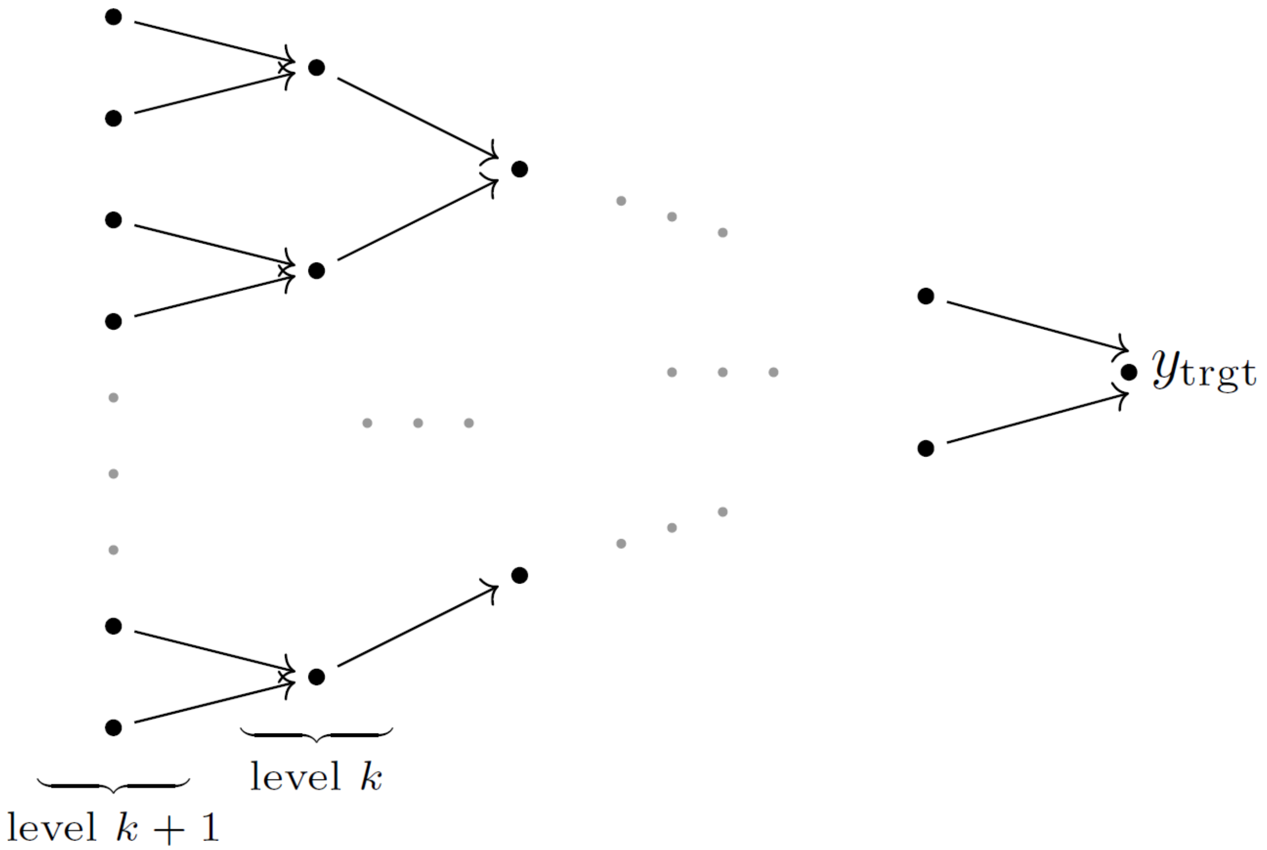

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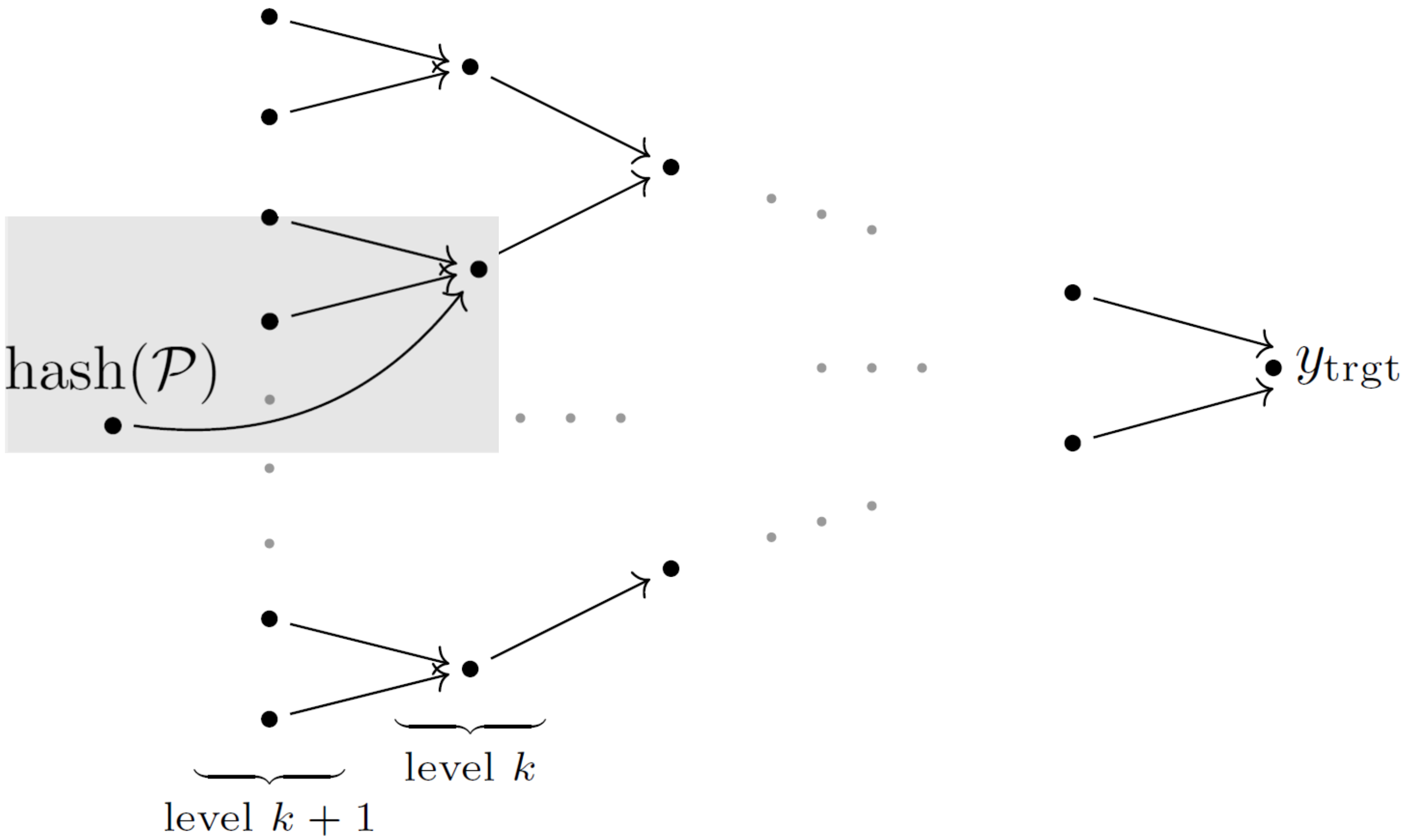

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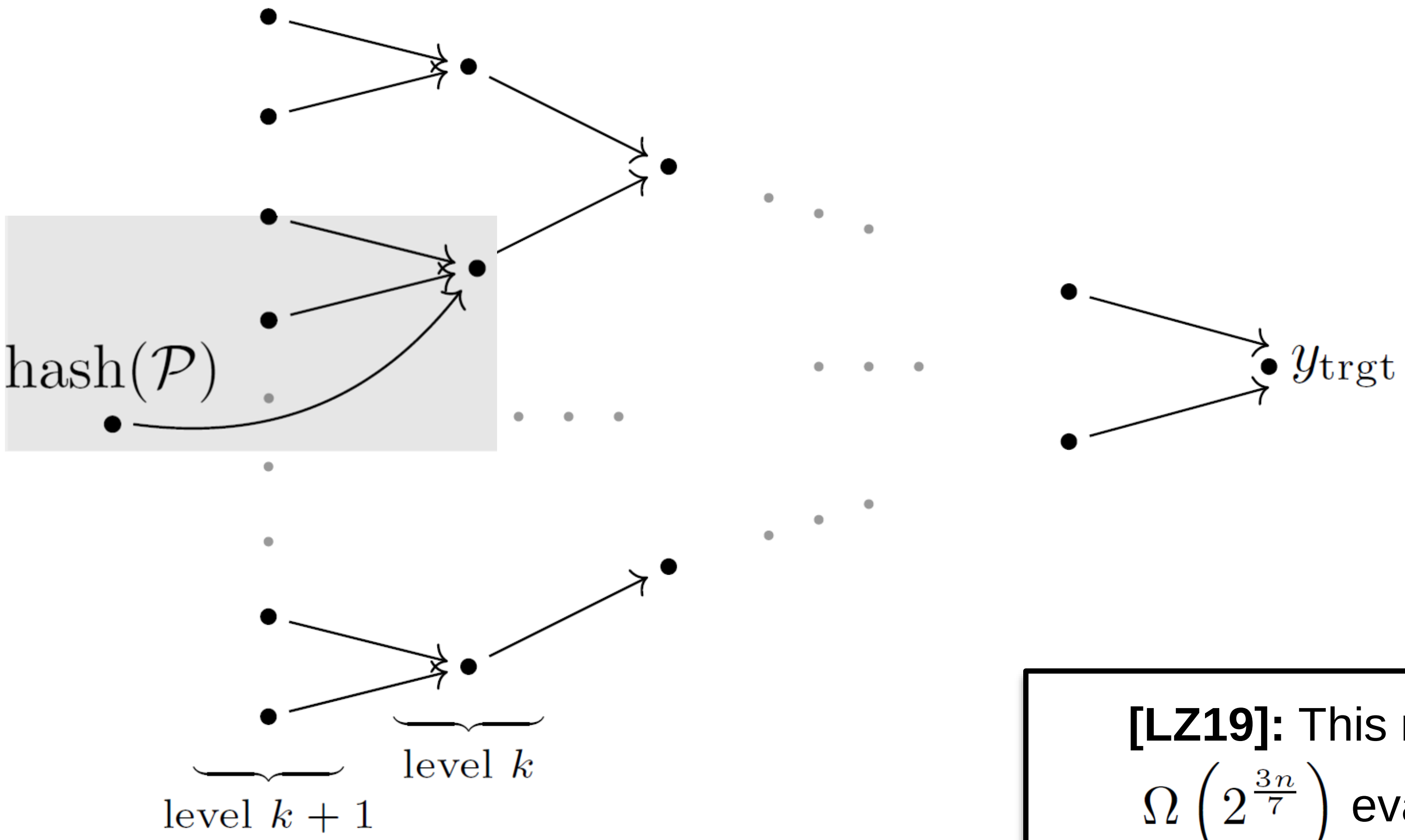

[LZ19]: This needs at least $\Omega\left(2^{\frac{3n}{7}}\right)$ evaluations of h.

[LZ19] Liu and Zhandry. On Finding Quantum Multi-Collisions. Advances in Cryptology – EUROCRYPT 2019.

General Lower Bound

General Lower Bound

Theorem [our work]. Any Nostradamus Attacker $\mathcal{A}$ needs at least

$$\Omega\left(2^{\frac{3n}{7}-s}\right)$$

evaluations of h, where s is the maximal block length of suffix $\mathcal{S}$.

To Sum Up:

- *Quantum Nostradamus Attack*, which needs $\mathcal{O}\left(\sqrt[3]{n} \cdot 2^{\frac{3n}{7}}\right)$ h-evaluations
- There are *Lower Bounds*:
  - $\Omega\left(2^{\frac{3n}{7}}\right)$ with Diamond Structure
  - $\Omega\left(2^{\frac{3n}{7}-s}\right)$ in General

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- *Quantum Nostradamus Attack*, which needs $\mathcal{O}\left(\sqrt[3]{n} \cdot 2^{\frac{3n}{7}}\right)$ h-evaluations
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Thanks for the Attention!

Our full version: eprint.iacr.org/2022/1213
Our Qiskit-Experiments: git.rwth-aachen.de/marc.fischlin/quantum-nostradamus