Nonmalleable Digital Lockers and Robust Fuzzy Extractors in the Plain Model

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In this work, we isolate and realize oracle hashing and nonmalleability.

Point Functions:

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- Hide everything about I_{val} except input/output behavior

$$\text{Obfuscate}: \ \mathcal{O}(\mathsf{val}) = \widetilde{O}$$
$$\text{On use:} \ \widetilde{O}(x) \equiv I_{\mathsf{val}}(x)$$

- **VBB obfuscation**: Ensure the following is negligible

$$|\Pr[\mathcal{A}(\widetilde{O}) = \mathcal{P}(\mathsf{val}) | \widetilde{O} \leftarrow \mathcal{O}(I_{\mathsf{val}})] - \Pr[\mathcal{S}^{I_{\mathsf{val}}(\cdot)}(1^{\lambda}) = \mathcal{P}(\mathsf{val})]|$$

Issue: May be easy to take $\mathcal{O}(\mathsf{val})$ and **obliviously** tamper to some "related" point $\mathsf{val}' = f(\mathsf{val})$. "Preventing this" is called **nonmalleability**.

Nonmalleability and Composition

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Note

"Preventing" mauling to "related" points makes a lot of sense with trusted setup or ROs, but tricky in plain model. More on this later.

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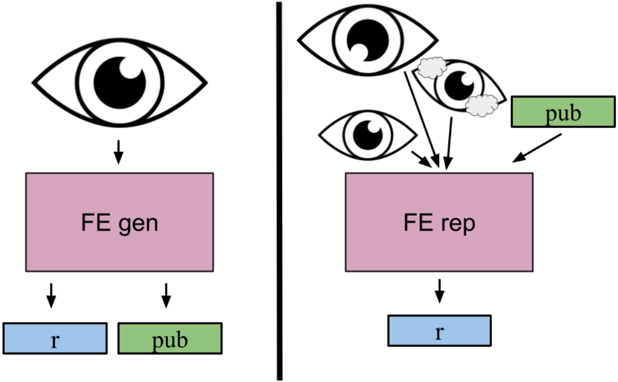
- Maybe nonmalleability over both inputs here

Called a nonmalleable point obfuscation with multibit output...or a **digital locker**

Motivation — (Robust) Fuzzy Extractors

Fuzzy Extractors: retrieve stable random strings from lower entropy and noisy inputs.



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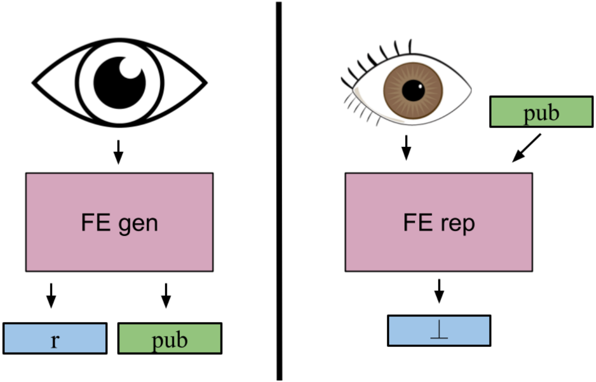

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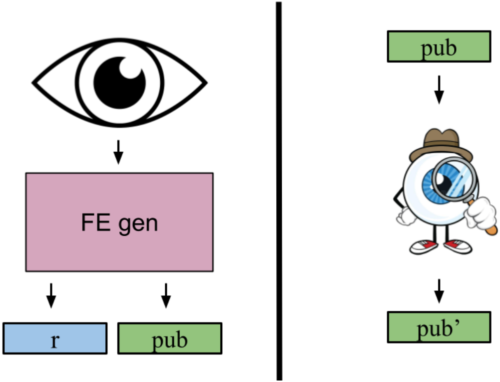
Robustness [Boy04, BDK+05]: Should be hard for adversary with existing *pub* to output *pub*$'$ which reproduces to different $r' \neq r$



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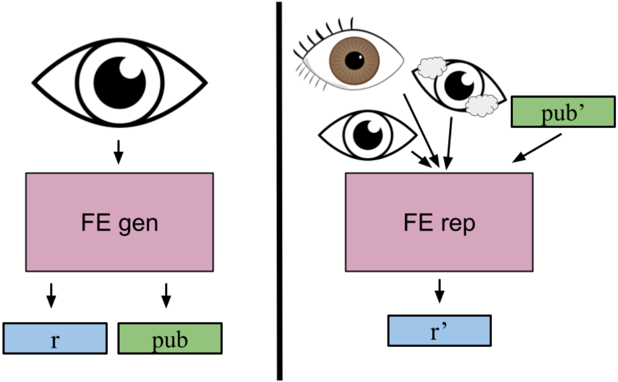

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Robust Fuzzy Extractors over low-entropy inputs are known in ROM and Common Reference String (CRS) model. Meanwhile inputs with entropy less than half their length have been a long-standing barrier in the plain model.

Scheme	Model	Security	SS errors	$H_\infty < 1/2?$
[Boy04],[Boy07]	RO	IT	t	✓
[DKK+12]	Plain	IT	t	X
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[WL18]	CRS	Comp.	$2t$	✓
[FT21]	CRS*	Comp.	$2t$	✓

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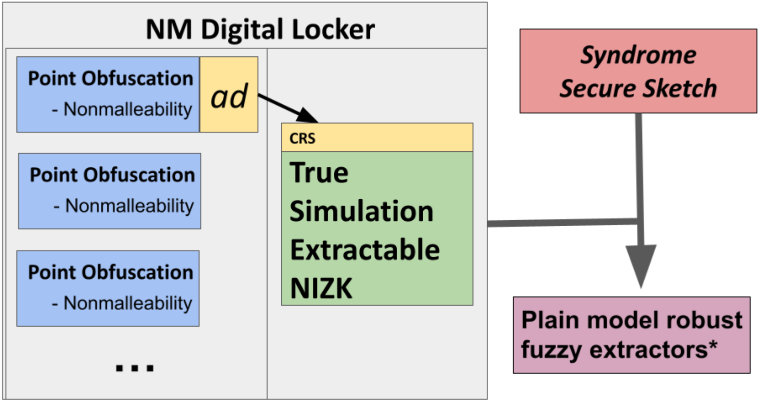
The Plan



NM Digital Locker
Point Obfuscation
- Nonmalleability
ad
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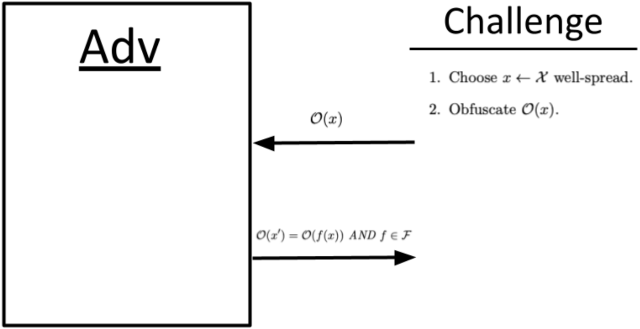
...
CRS
True Simulation Extractable NIZK
Syndrome Secure Sketch
Plain model robust fuzzy extractors*

Nonmalleable Digital Lockers

Komargodski and Yogev [KY18]:

- Nonmalleability defined as adversary outputting tampering function from $\mathcal{F}$



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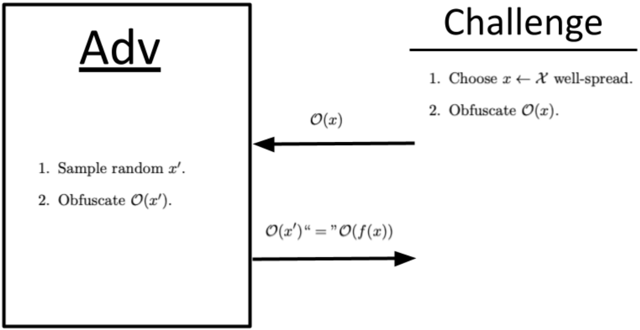

Background — Plain Model NM Point Obfuscation

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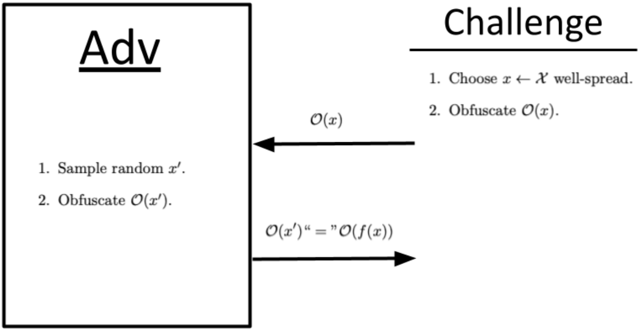

From PO to DL

To get to multibit output, most common method is *Real-or-Random* composition of point obfuscations.

- For each bit of the output key, append $\mathcal{O}(\text{val})$ if the bit is 1 and $\mathcal{O}(r)$ for some random value r if the bit is 0.
- DL functions by reconstructing key bit-by-bit.

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- DL functions by reconstructing key bit-by-bit.

HOWEVER, this requires...

- Point obfuscations composability
- Some way to protect key.

Previous work [FF20] required a CRS to achieve key nonmalleability.

GOAL: Remove CRS to bring NMDLs into plain model!

Nonmalleable Point Obfuscations with Associated Data

Definition

Let $\rho \in \mathbb{N}, \mathcal{X}$ be a family of distributions, and $\mathcal{F}$ be a family of functions. Then, a $(\mathcal{F}, \mathcal{X}, \rho)$-**Nonmalleable Point Obfuscation with Associated Data** is defined as

$$\mathsf{lockPoint}(x; ad) := (ad; \mathsf{unlockPoint}(x; ad)),$$

where $x \leftarrow \mathcal{X}, ad \in \{0,1\}^{\rho}$, and unlockPoint satisfies *completeness*, *VBB security*, and *nonmalleability*.

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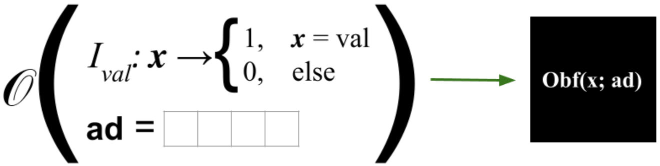

Nonmalleable Point Obfuscations with Associated Data

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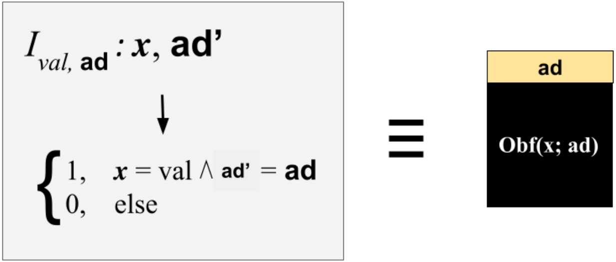

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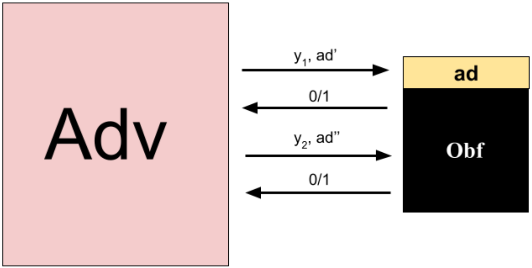

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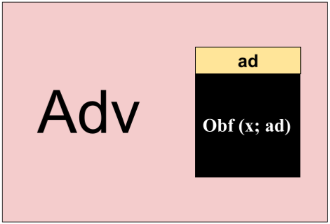

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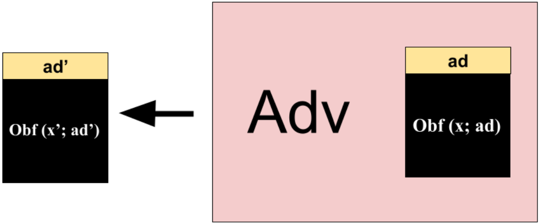

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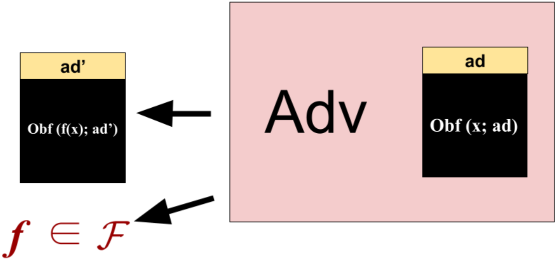

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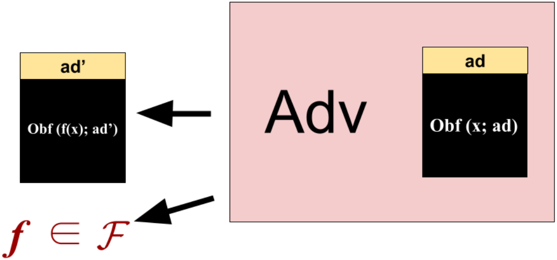

Note

The adversary succeeds if they tamper the ad or underlying point function (or both).

Assumptions

Bartusek, Ma, and Zhandry [BMZ19] studied fixed generator assumptions (toward point obfuscation!) in the GGM, showed following holds there:

Assumption

For $x \leftarrow \mathcal{X}$ well-spread and random r the following is $\mathsf{negl}(\lambda)$ *for all PPT A:*

$$|\Pr[\mathcal{A}(\{k_i, g^{k_i x + x^i}\}_{i \in [2,\tau]}) = 1] - \Pr[\mathcal{A}(\{k_i, g^{k_i r + r^i}\}_{i \in [2,\tau]}) = 1]|.$$

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$$p_{1,ad,c_1}(\mathsf{val}) = c_1\mathsf{val} + \sum_{i=1}^{\rho} ad_i \mathsf{val}^{i+1} + \sum_{i=\rho+2}^{\rho+6} \mathsf{val}^i,$$

$$p_{2,c_2}(\mathsf{val}) = c_2\mathsf{val} + \mathsf{val}^{\rho+7},$$

$$p_{3,c_3}(\mathsf{val}) = c_3\mathsf{val} + \mathsf{val}^{\rho+8},$$

$$p_{4,c_4}(\mathsf{val}) = c_4\mathsf{val} + \mathsf{val}^{\rho+9},$$

$$p_{5,c_5}(\mathsf{val}) = c_5\mathsf{val} + \mathsf{val}^{\rho+10}.$$

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3. Define

$$\mathsf{lockPoint}(\mathsf{val}, ad; c_1, c_2, c_3, c_4, c_5) \stackrel{def}{=} \begin{pmatrix} c_1, & [p_{1,ad,c_1}(\mathsf{val})]_g \\ c_2, & [p_{2,c_2}(\mathsf{val})]_g \\ c_3, & [p_{3,c_3}(\mathsf{val})]_g \\ c_4, & [p_{4,c_4}(\mathsf{val})]_g \\ c_5, & [p_{5,c_5}(\mathsf{val})]_g \end{pmatrix}$$

Note

Reminder: Require nonmalleability for adversaries **outputting** f *and either (1) mauling* x *or (2) mauling* ad *and letting* $f = id$.

Proof route:

Lemma (Lemma 4.3)

Given any degree-ρ *polynomial* P*, no adversary can maul*

$$\mathcal{O}_P(x) = (c_1, [c_1 x + xP(x) + \sum_{i=\rho+2}^{\rho+6} x^i]_g)$$

to any $\mathcal{O}_{P'}(f(x))$ *for any degree-*ρ *polynomial* P' *and* $f \in \mathcal{F}$ *(with non-negligible probability).*

Nonmalleability Proof Roadmap

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Proof route:

Lemma (Lemma 4.5)

Given that x *is not tampered, then for any* $ad \in \{0,1\}^\rho$, *no adversary can maul*

$$\mathcal{O}_{ad}(x) = (c_1, [c_1 x + \sum_{i=1}^{\rho} ad_i x^{i+1} + \sum_{i=\rho+2}^{\rho+6} x^i]_g)$$

to $\mathcal{O}_{ad'}(x)$ *for any* $ad' \neq ad$ *(with non-negligible probability).*

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Proof route:

- **Lemma 4.3** ensures that any non-identity shifts of x are hard to reach
  - Namely, any $\mathcal{O}(f(x))$ is outside the span of elements in $\mathcal{O}(x)$.
- **Lemma 4.5** ensures any maulings of ad when $f = id$ are hard to reach.

Lemma 4.5

We have $f = \mathrm{id}$ and $ad' \neq ad$. So, adversary is given

$$\mathcal{O}_{ad}(x) = (c_1, [c_1 x + \sum_{i=1}^{\rho} ad_i x^{i+1} + \sum_{i=\rho+2}^{\rho+6} x^i]_g)$$

and must construct

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If $ad_i' = 1$ and $ad_i = 0$, then adversary's linear term (c_1') must coincide with term from assumption

$$k_i, g^{k_i x + x^i}.$$

However, never given any input related to k_i

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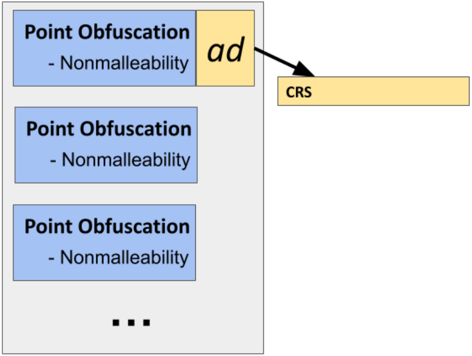
$\implies$ In either case, their success probability is small.

Constructing Nonmalleable Digital Lockers



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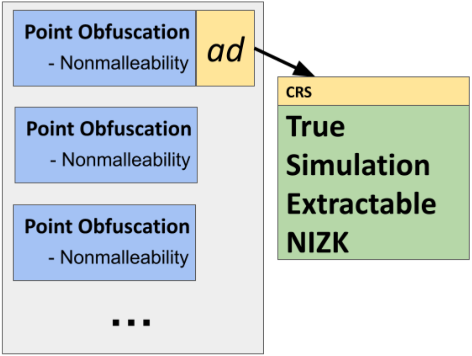

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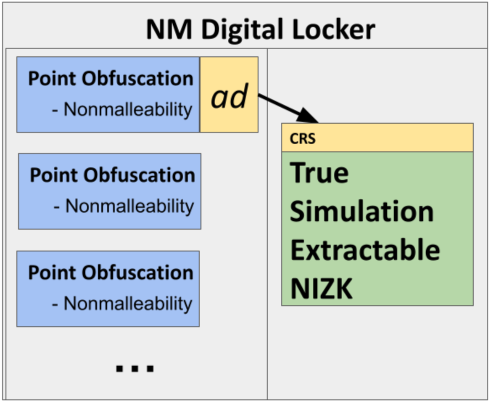
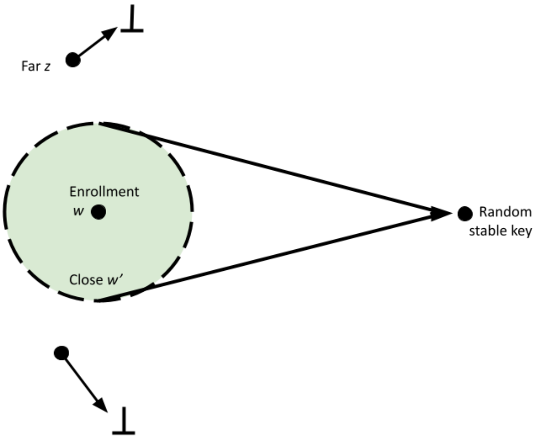


Robust Fuzzy Extractors in the Plain Model

What is a Fuzzy Extractor?



Robust Fuzzy Extractors

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A **Secure Sketch** instead may be thought as recovering w from *pub* and close w':

- $(key, pub) \leftarrow \mathrm{Gen}_{\mathrm{SS}}(w)$.
- $w'' \leftarrow \mathrm{Rep}_{\mathrm{SS}}(pub, w')$

Syndromes and ECCs

Definition

A matrix $\mathrm{Syn} : \mathbb{F}_q^n \to \mathbb{F}_q^{n-k}$ with two properties:

1. $\forall x$ where $|x| \leq t$, $\mathrm{Syn}(x)$ is unique and can be inverted.
2. $\forall s, s'$ where $|s|, |s'|, |s' - s| \leq t$,

$$\begin{aligned}\mathrm{Invert}(\mathrm{Syn}(s' - s)) &= \mathrm{Invert}(\mathrm{Syn}(s') - \mathrm{Syn}(s)) \\ &= \mathrm{Invert}(\mathrm{Syn}(s')) - \mathrm{Invert}(\mathrm{Syn}(s)) \\ &= s' - s\end{aligned}$$

Definition (Syndrome Secure Sketch)

Define $\mathrm{SS}(w) = \mathrm{Syn}(w)$ and

$$\begin{aligned}\mathrm{Rec}(w', s) &= w' - \mathrm{Invert}(\mathrm{Syn}(w') - s) \\ &= w' - \mathrm{Invert}(\mathrm{Syn}(w' - w)) = w.\end{aligned}$$

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- **Essential idea:** We can find the small shift in w' as a unique syndrome!

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$$\begin{aligned}\mathsf{Rec}(w', s) &= w' - \mathsf{Invert}(\mathsf{Syn}(w') - s)\\ &= w' - \mathsf{Invert}(\mathsf{Syn}(w' - w)) = w.\end{aligned}$$

Then, $(\mathsf{SS}, \mathsf{Rec})$ is a Syndrome Secure Sketch.

- **Essential idea:** We can find the small shift in w' as a unique syndrome!
- In particular, can extract the difference in secure sketches by the difference in the Invert of their difference!

Syndrome Secure Sketches

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- **Yields robustness!**

Conclusion

Conclusions and Future Directions

Our Results:

- **Defined** a new primitive, nonmalleable point obfuscations with associated data
- **Constructed** the above and the first nonmalleable digital lockers in the plain model
- **Pulled** robust fuzzy extractors with low input entropy into the plain model

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Future Directions:

- Plain model nonmalleable obfuscation of other evasive functions such as wildcards, conjunctions, hyperplanes
- Achieving more broad notions of composability/composability of digital lockers
- Constructing reusable plain model fuzzy extractors, other desirable properties
- Other applications of nonmalleable point obfuscation with associated data

Thank you!
Any Questions?