Towards Tight Security Bounds for OMAC, XCBC and TMAC

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- MACs and their security

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Towards Tight Security Bounds for OMAC, XCBC and TMAC

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- CBC-MACs: XCBC, TMAC, OMAC
- Observations on their security
- A step towards tight security

Message Authentication Codes (MAC)

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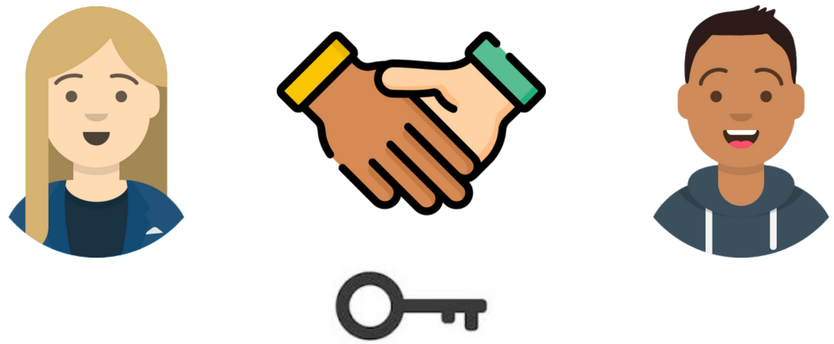

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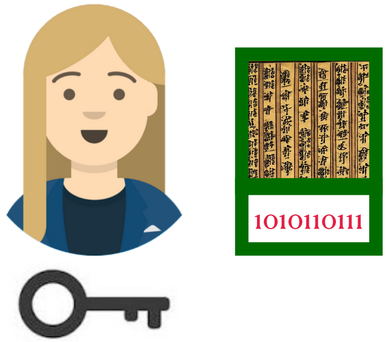

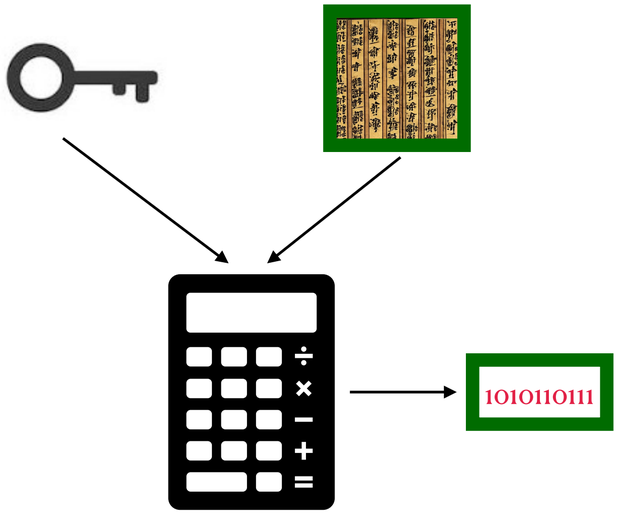

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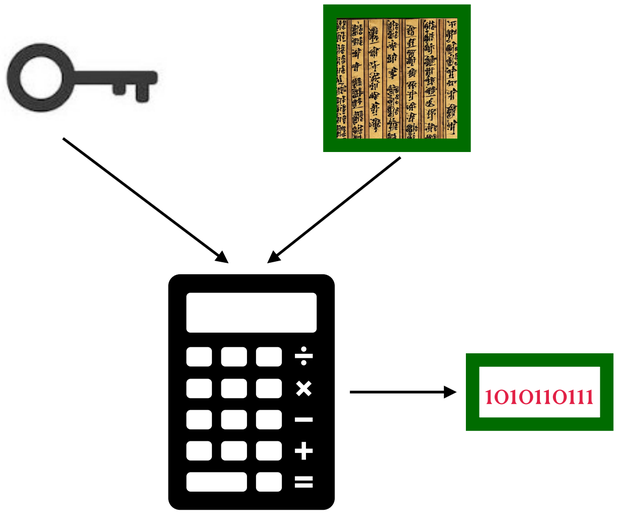

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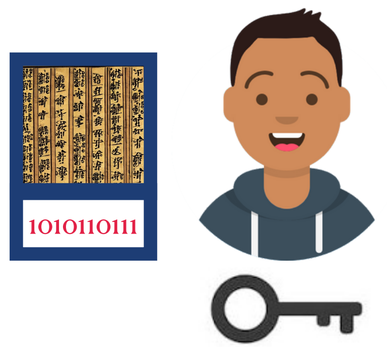
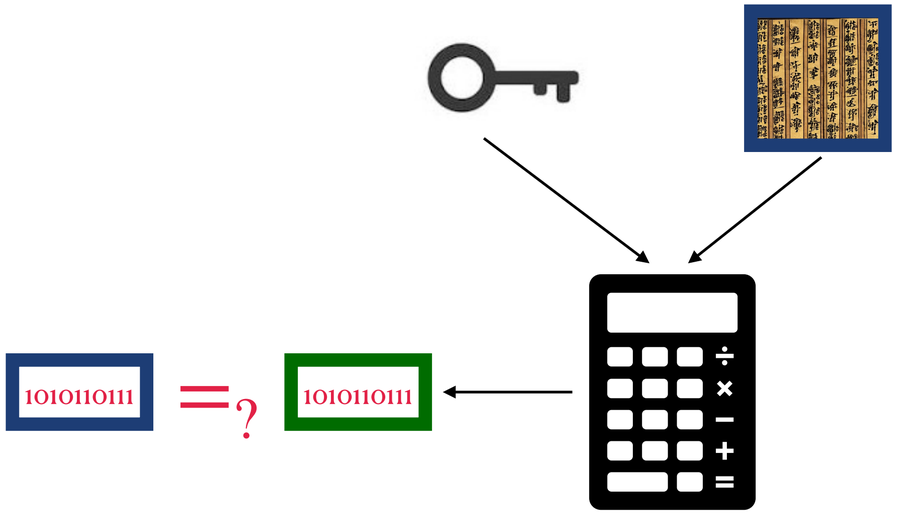
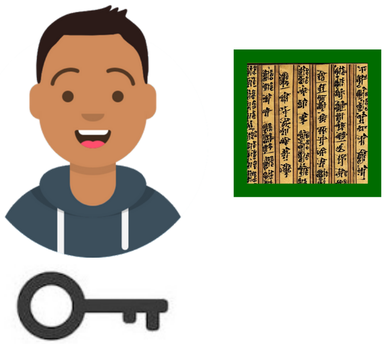
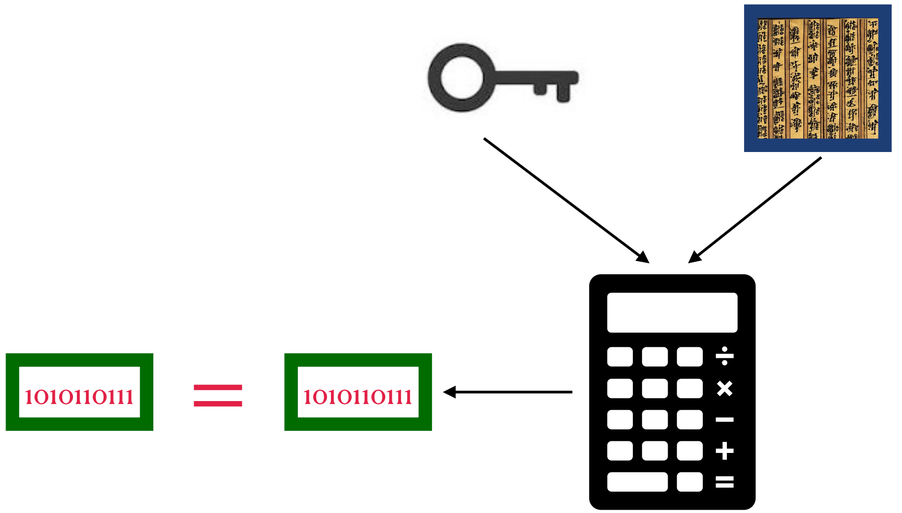




Message Authentication Codes (MAC)

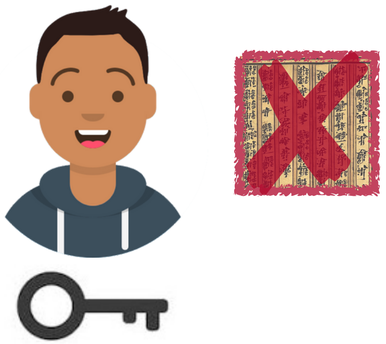
Authentication Succeeds

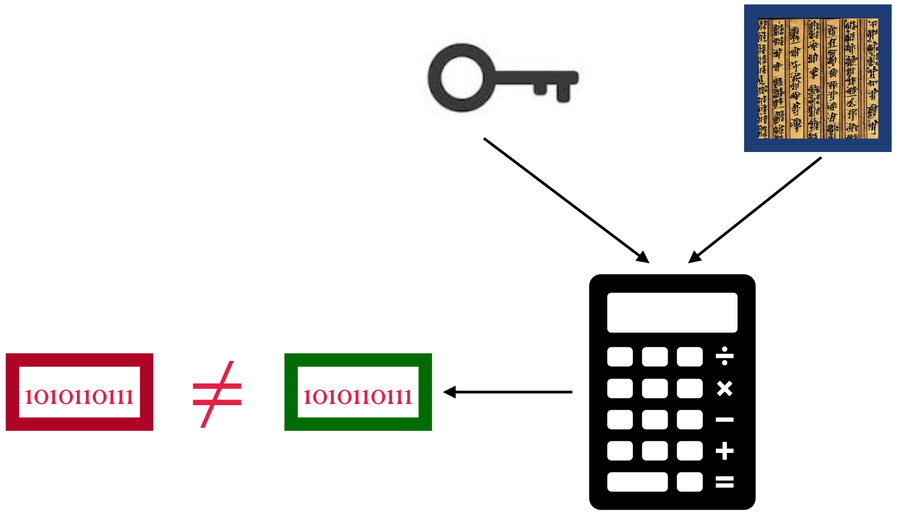




Message Authentication Codes (MAC)

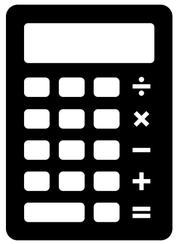
Authentication Fails





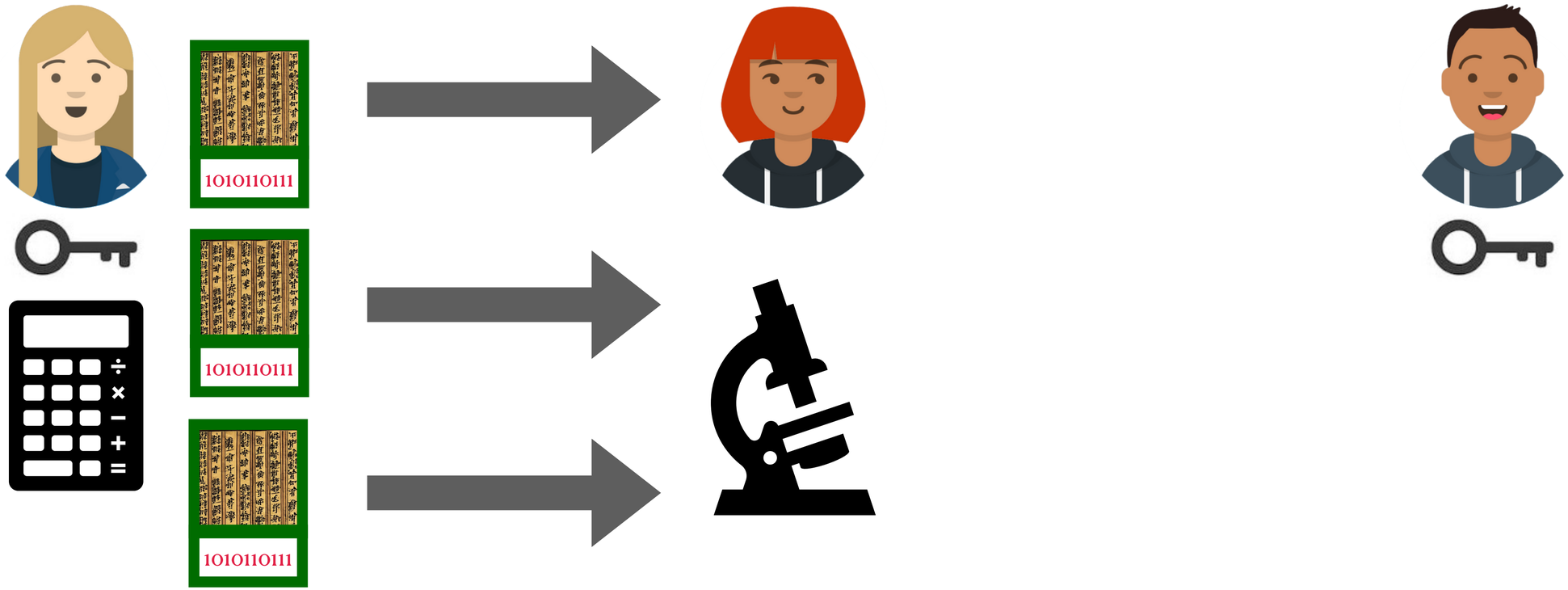
MAC Security

Unforgeability Game



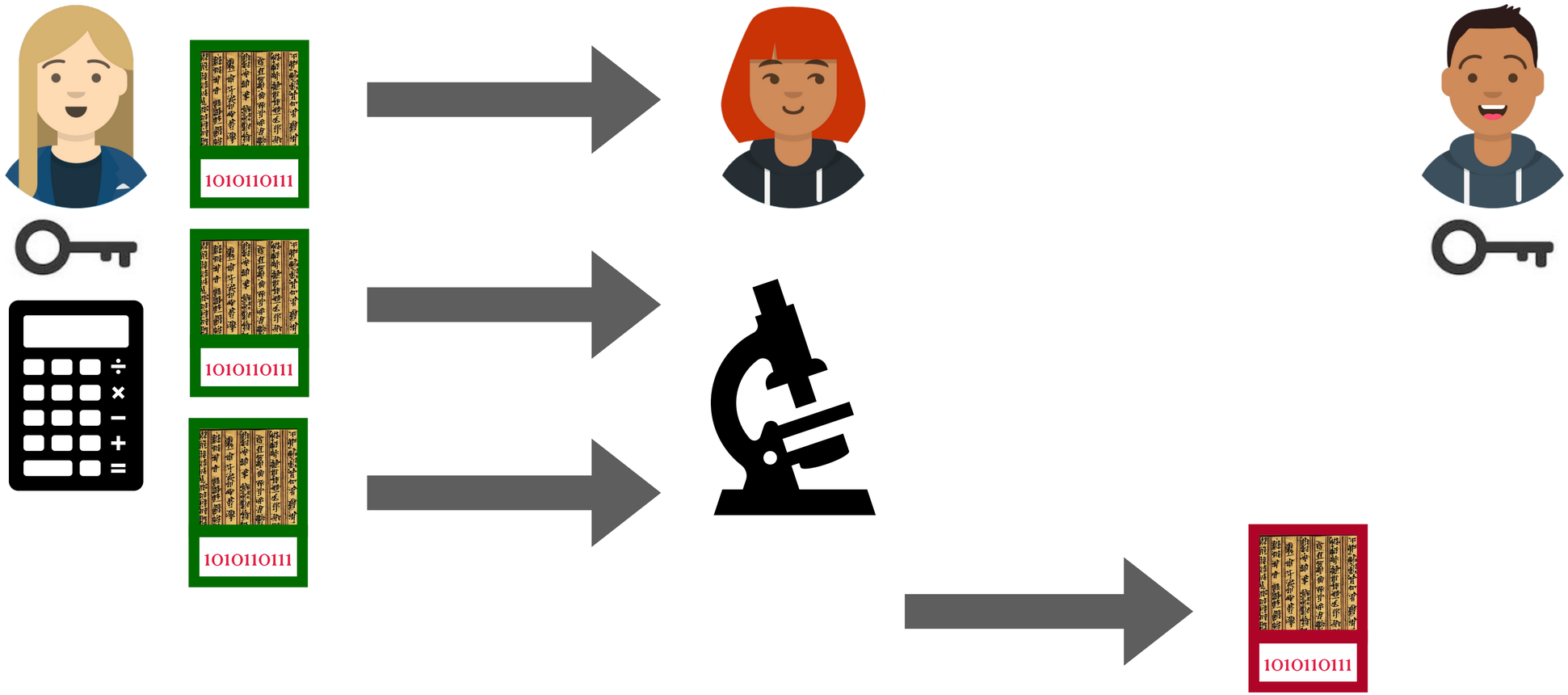
MAC Security

Unforgeability Game: Passive Adversary



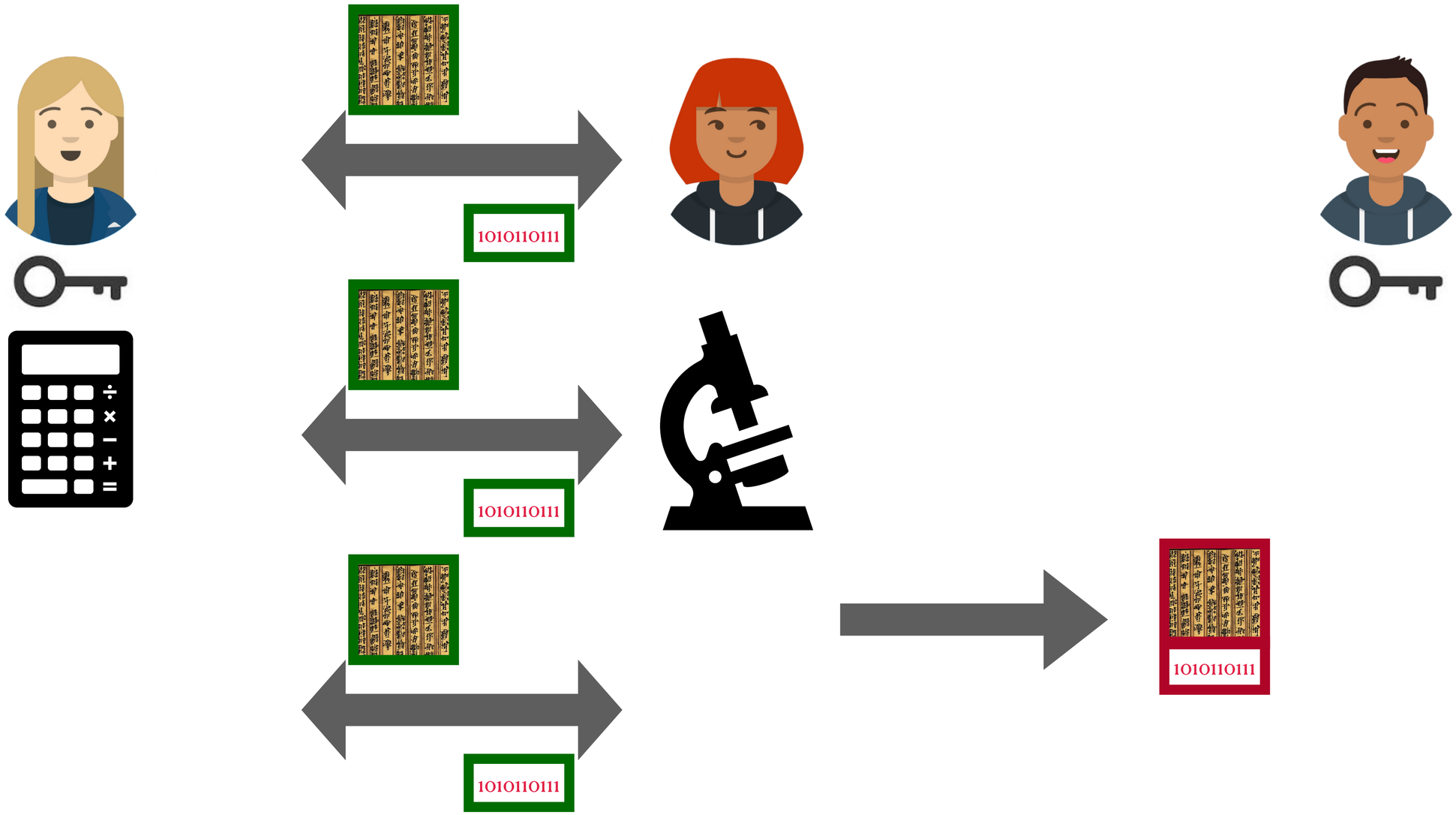
MAC Security

Unforgeability Game: Passive Adversary



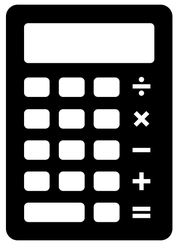
MAC Security

Unforgeability Game: Active Adversary



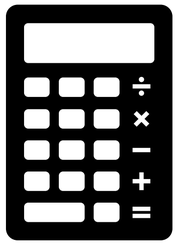
MAC Security

Unforgeability Game: Forgery Succeeds



MAC Security

Unforgeability Game: Forgery Succeeds



$$\text{MAC Security} \triangleq \Pr(\texttt{Forgery Succeeds})$$

MAC Security

Random Functions are Good MACs

MAC Security

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$$\Pr\left(\texttt{Forgery Succeeds}\right) = \Pr\left(\texttt{Guessing}\right)$$

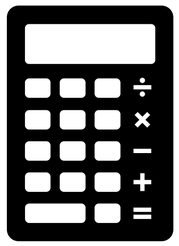
Pseudorandom Function Security

Ideal World (o)

Pseudorandom Function Security

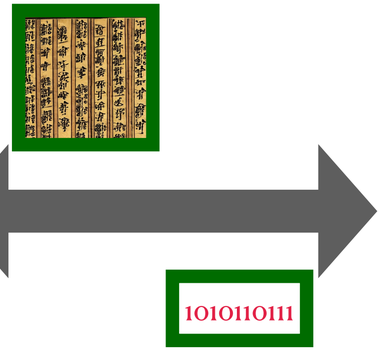
Ideal World (0)

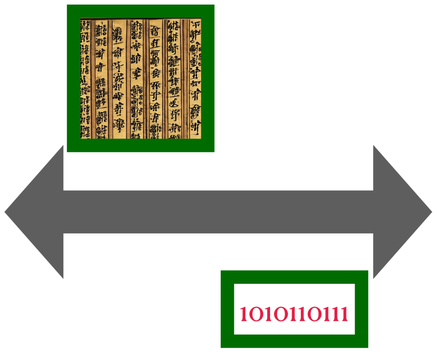
Real World (1)



Pseudorandom Function Security

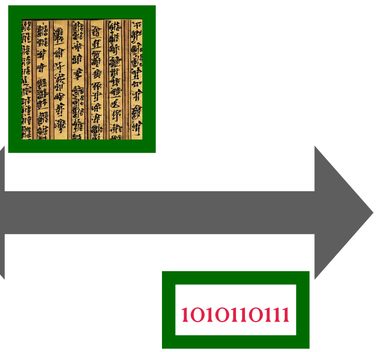
Ideal World (0)

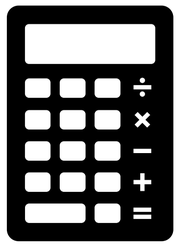


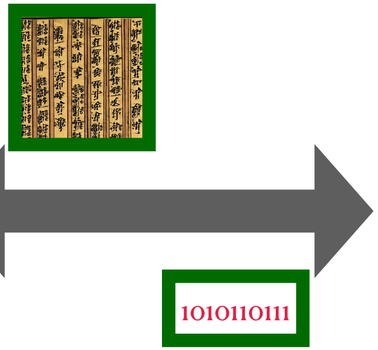


b

Real World (1)







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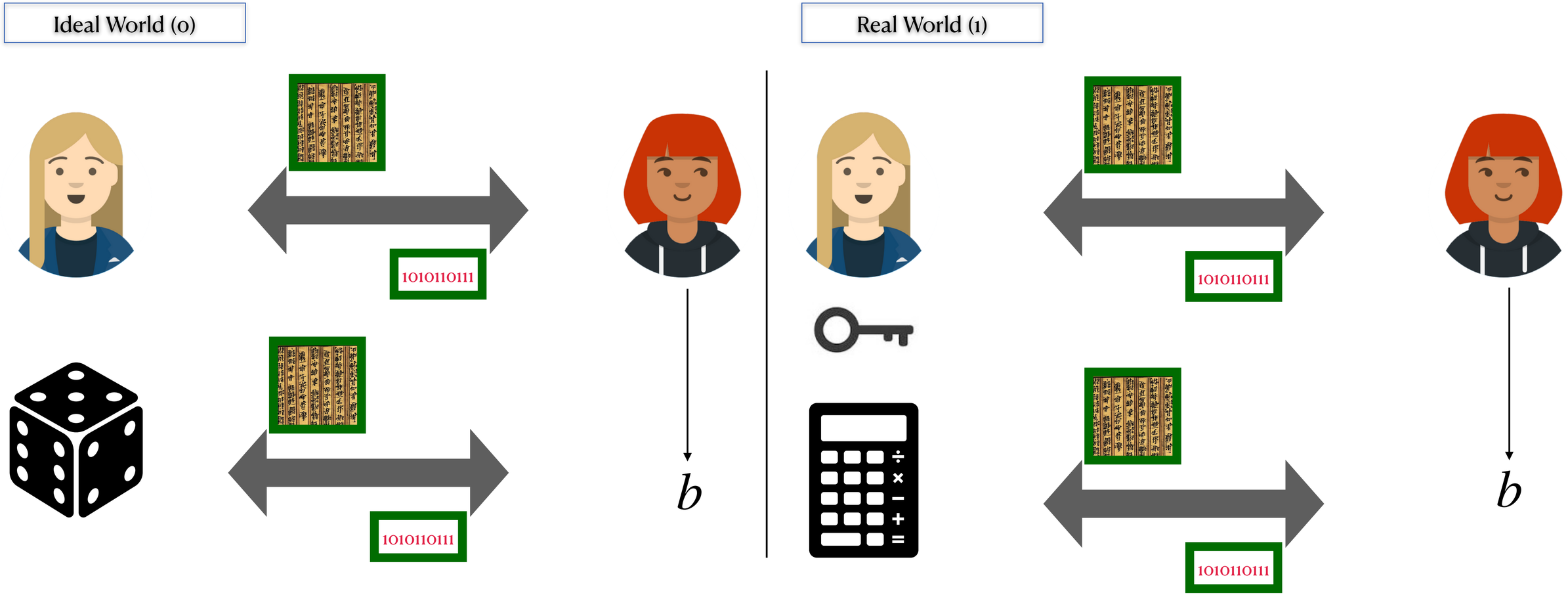
Pseudorandom Function Security



$$\mathbf{Adv}^{\mathrm{prf}}_{\mathbf{Alice}}(\mathbf{Eve}) := \left|\Pr\left(b = 1 \text{ in the real world}\right) - \Pr\left(b = 1 \text{ in the ideal world}\right)\right|$$

CBC-MAC

Ehrsam et al., US Patent 4074066

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$$\left(M[1], \ldots, M[\ell]\right) \xleftarrow{n} M \in \left(\{0,1\}^n\right)^+$$

n: block size

CBC-MAC

Ehrsam et al., US Patent 4074066

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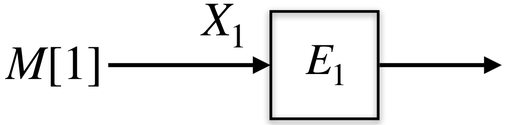

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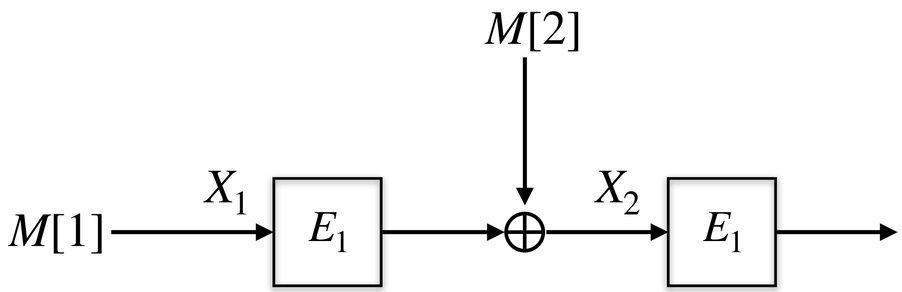

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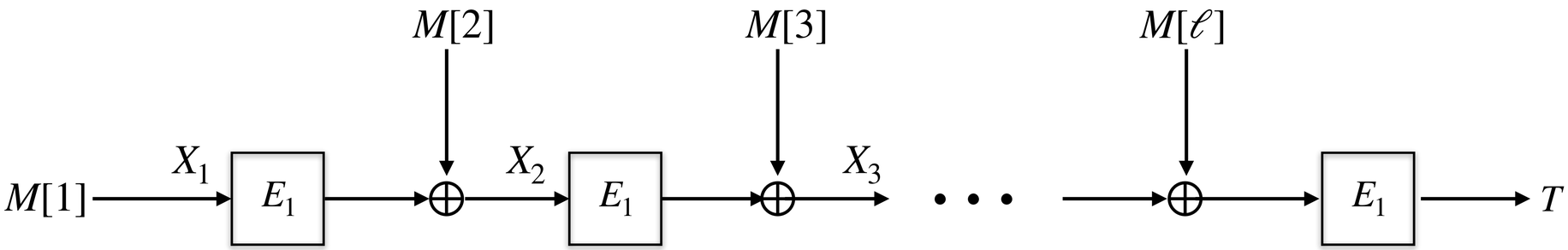

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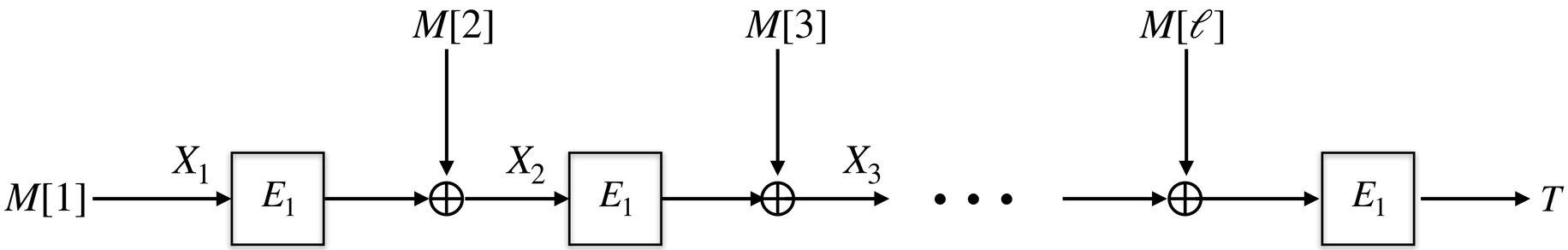


- Susceptible to length extension* attacks.
- Input length must be a non-zero multiple of n.

n: block size

* $\text{CBC-MAC}\left(M \| X\right) = \text{CBC-MAC}\left(\text{CBC-MAC}\left(M\right) \| X\right)$

OMAC, XCBC, and TMAC

OMAC, XCBC, and TMAC

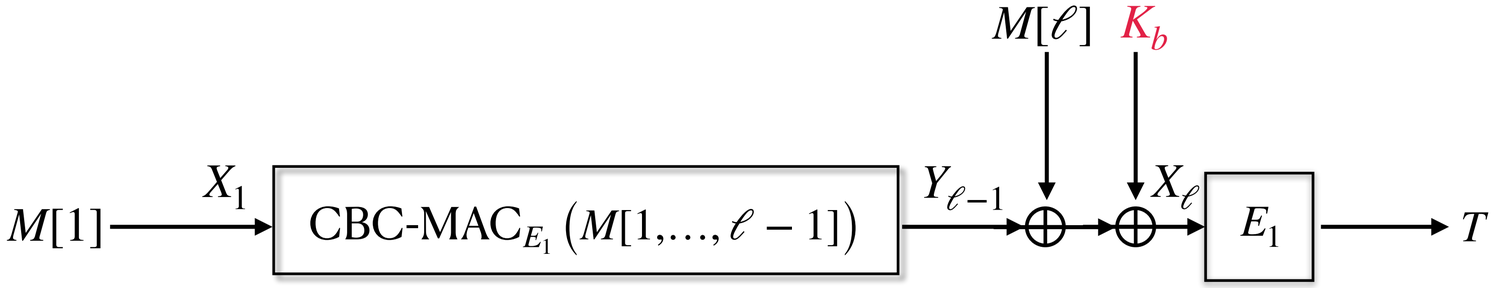
$\left(M[1], \ldots, M[\ell]\right) \xleftarrow{n} M \xleftarrow{\text{pad}} M' \in \{0,1\}^*$

OMAC, XCBC, and TMAC

$$\left(M[1], \ldots, M[\ell]\right) \xleftarrow{n} M \xleftarrow{\text{pad}} M' \in \{0,1\}^*$$

$$\text{XCBC}_{E_1}(M') :=$$

Black and Rogaway, IACR CRYPTO 2000



$b := (M =_? M')\ ?\ 0 : 1$

OMAC, XCBC, and TMAC

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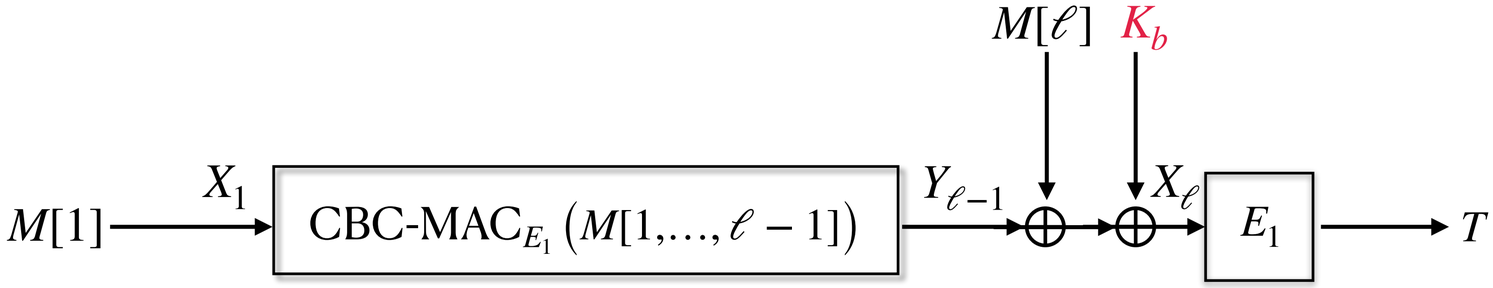
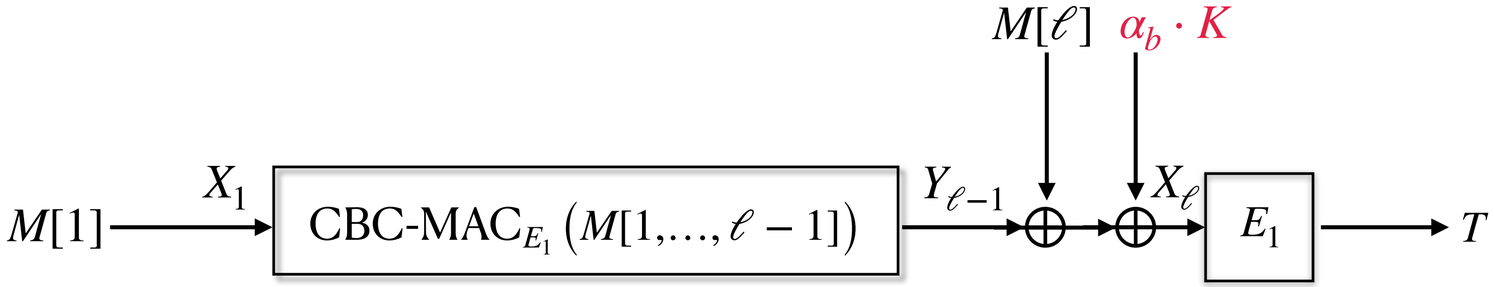


$\text{TMAC}_{E_1}(M') :=$

Kurosawa and Iwata, CT-RSA 2003



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OMAC, XCBC, and TMAC

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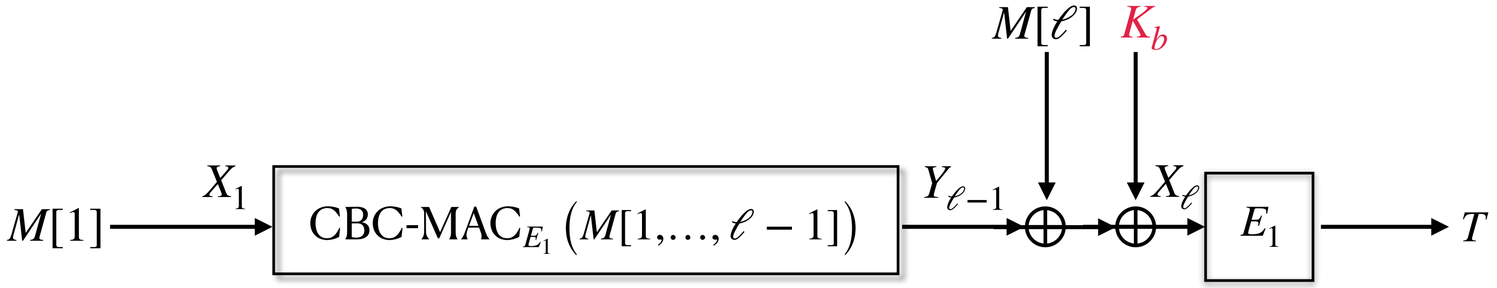
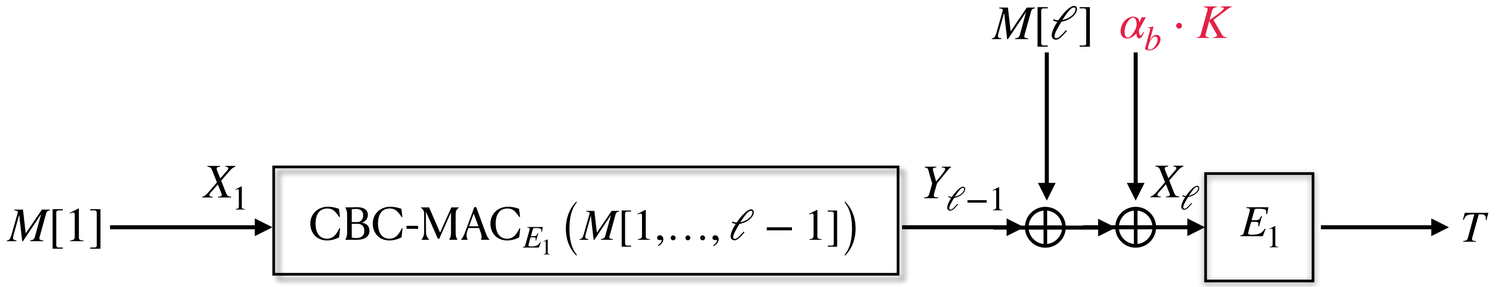
Black and Rogaway, IACR CRYPTO 2000



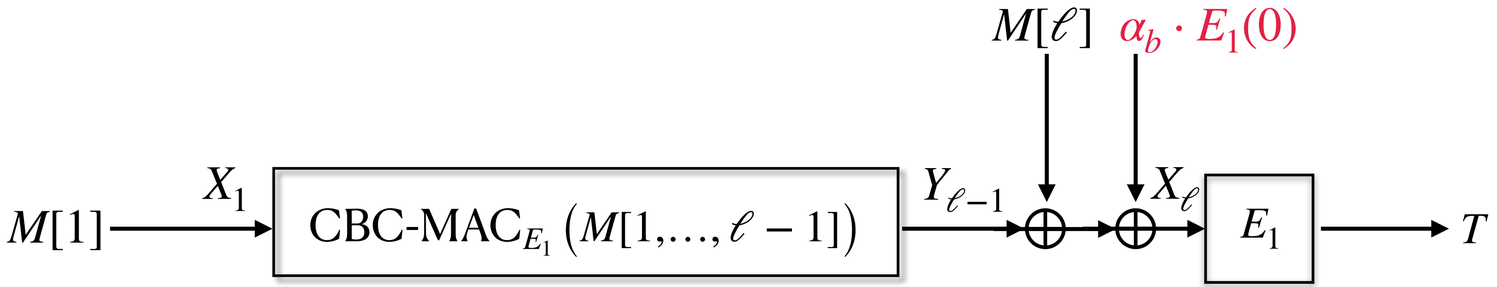
$\text{TMAC}_{E_1}(M') :=$

Kurosawa and Iwata, CT-RSA 2003



$\text{OMAC}_{E_1}(M') :=$

Iwata and Kurosawa, IACR FSE 2003;
NIST FIPS 800-38B;
ISO/IEC 29167-10:2017



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OMAC, XCBC, and TMAC

$\left(M[1], \ldots, M[\ell]\right) \xleftarrow{n} M \xleftarrow{\text{pad}} M' \in \{0,1\}^*$

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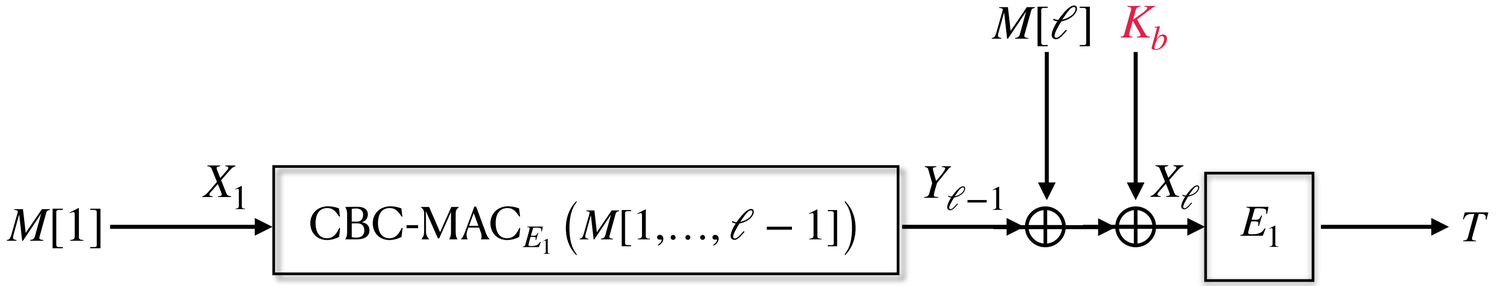
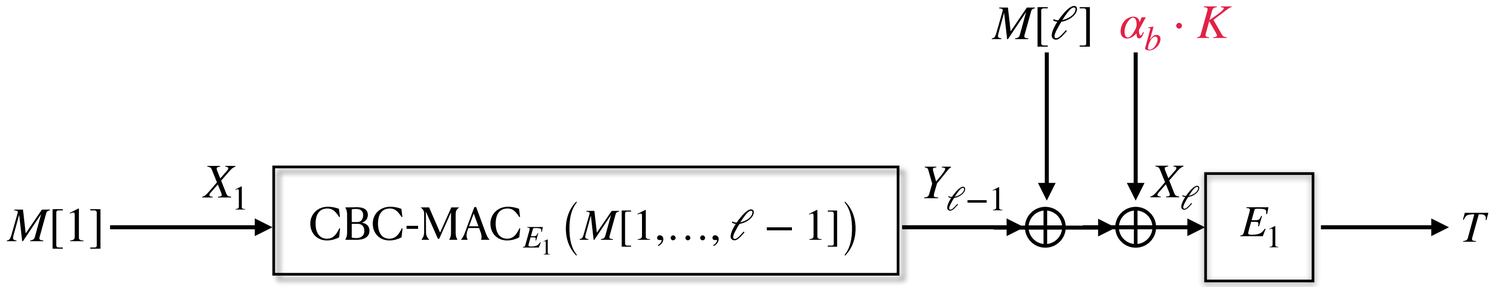


$\text{TMAC}_{E_1}(M') :=$

Kurosawa and Iwata, CT-RSA 2003



$\text{OMAC}_{E_1}(M') :=$

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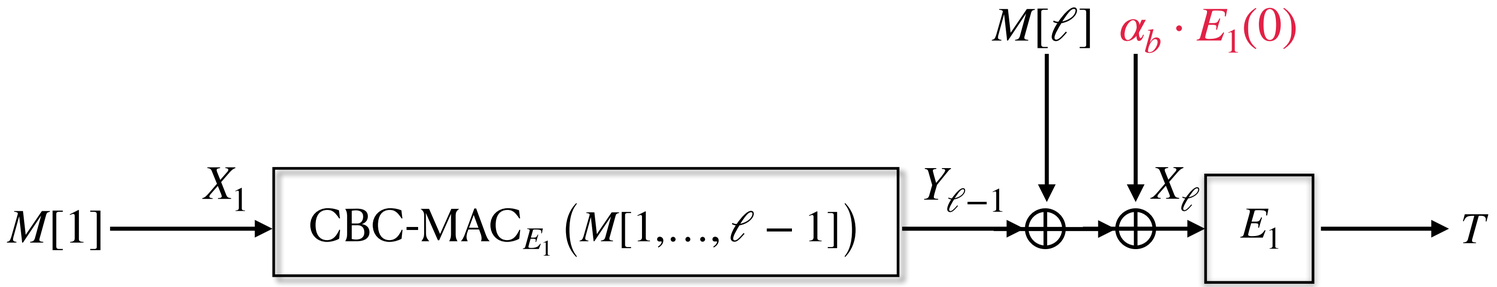

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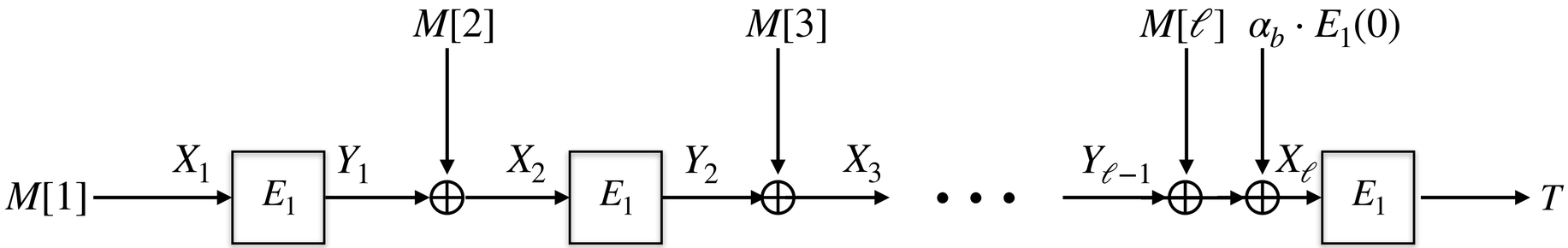
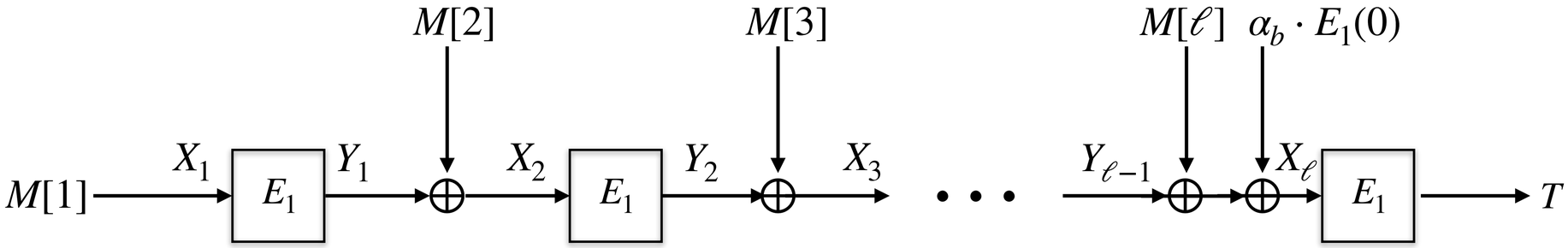
PRF Security of OMAC



- Best security bound:

$$\mathbf{Adv}^{\text{prf}}_{\text{CBC-MAC}}(\mathbf{Eve}) = O\left(\frac{q^2\ell}{2^n}\right)$$

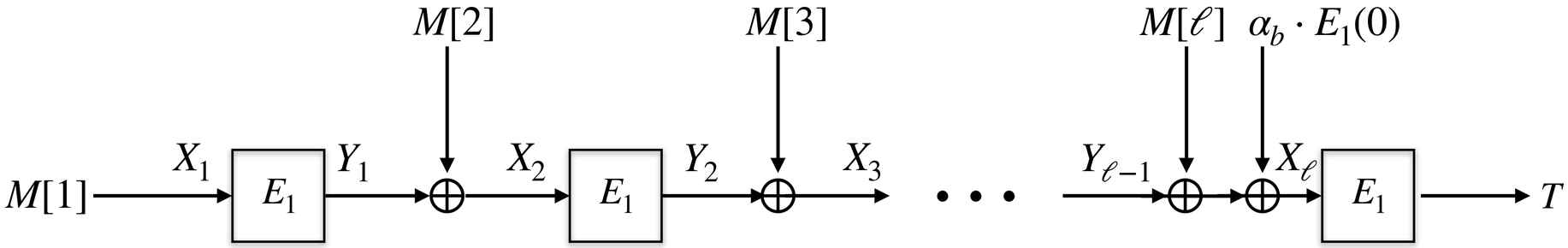
PRF Security of OMAC



- Best security bound:

$$\mathbf{Adv}^{\text{prf}}_{\text{CBC-MAC}}(\mathbf{Eve}) = O\left(\frac{q^2\ell}{2^n}\right) \qquad \text{i.e.} \qquad \mathbf{Eve} \text{ needs roughly } \sqrt{2^n/\ell} \text{ queries.}$$

PRF Security of OMAC



- Best security bound:

$$\mathbf{Adv}^{\text{prf}}_{\text{CBC-MAC}}(\mathbf{Eve}) = O\left(\frac{q^2\ell}{2^n}\right) \quad \text{i.e.} \quad \mathbf{Eve} \text{ needs roughly } \sqrt{2^n/\ell} \text{ queries.}$$

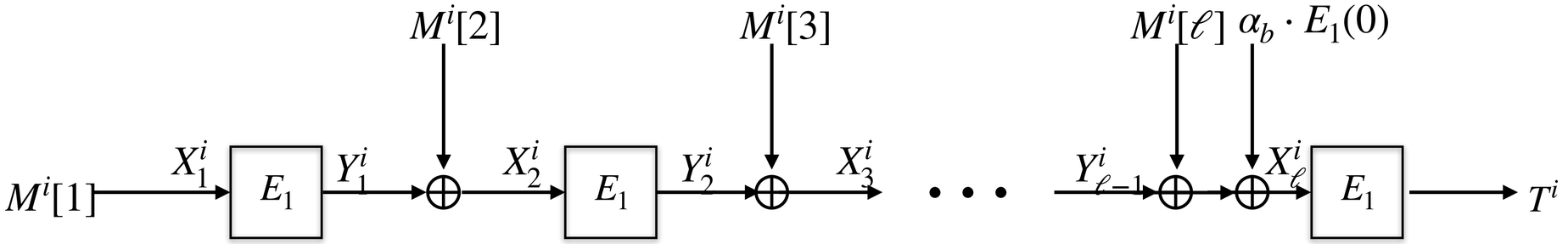
- No matching attack!
  - Folklore collision finding attack needs roughly $\sqrt{2^n}$ queries.

PRF Security of OMAC

Full Collision Event

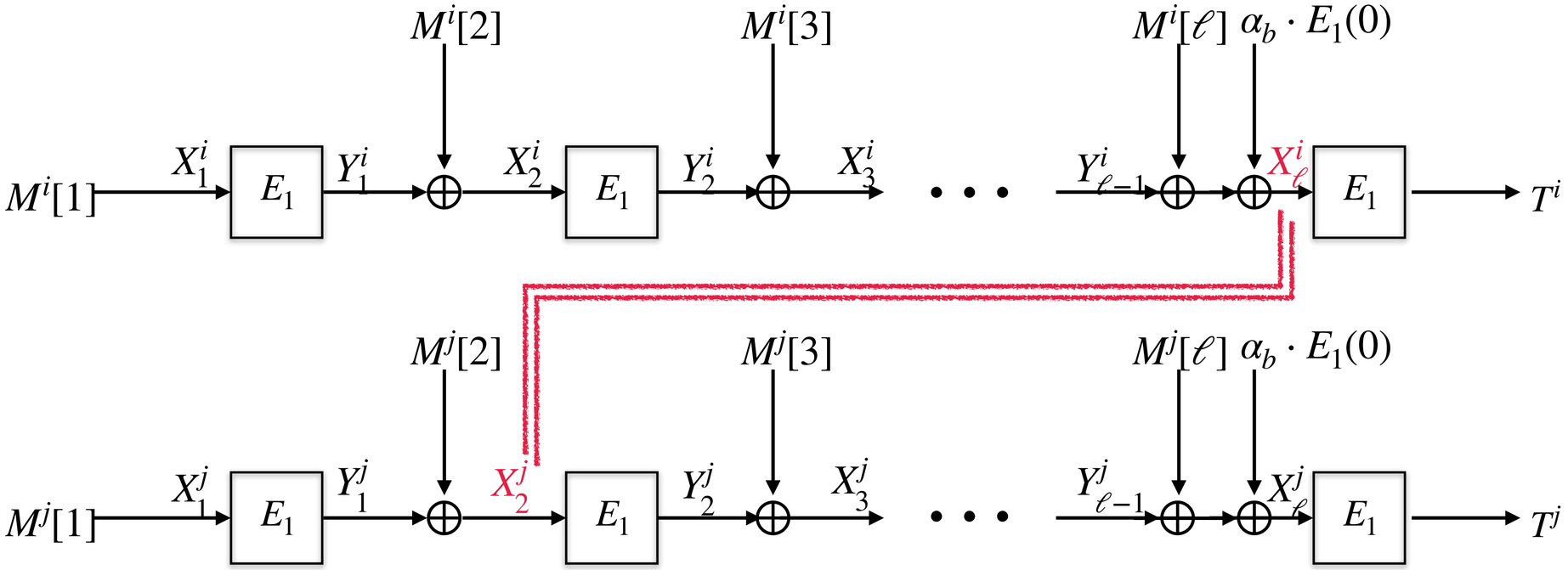
PRF Security of OMAC

Full Collision Event



PRF Security of OMAC

Full Collision Event



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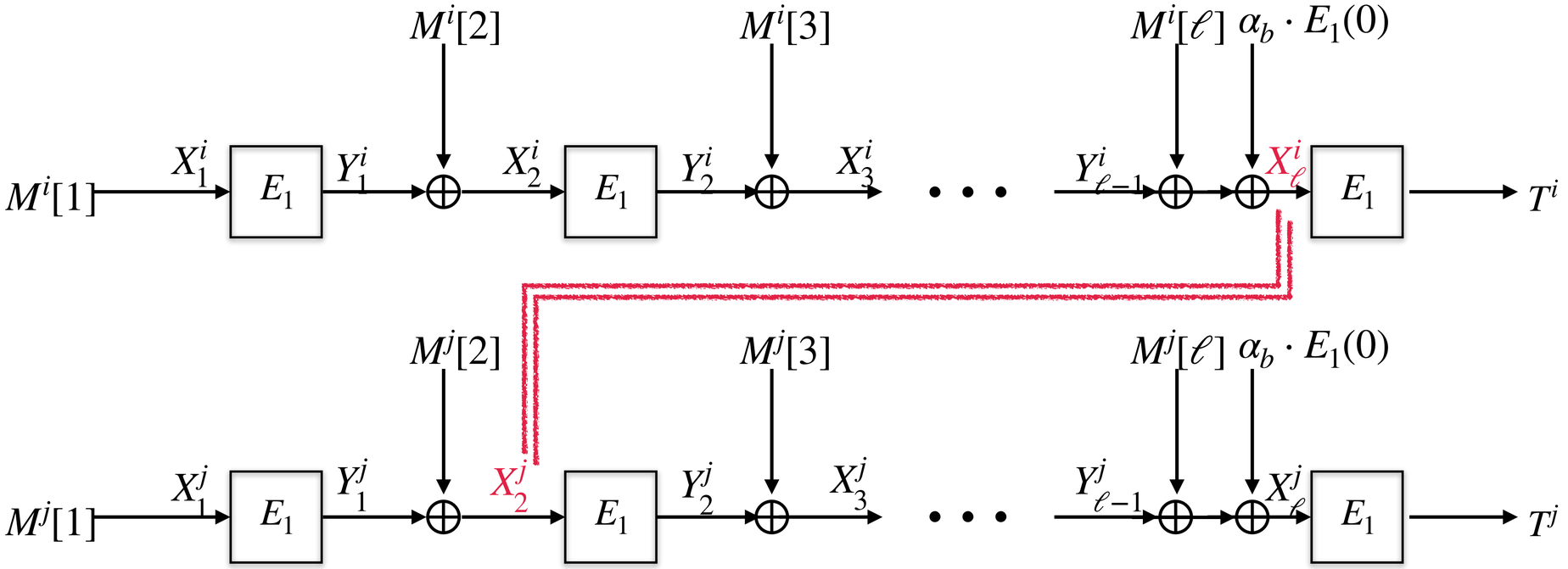

Full Collision : $\exists i, j, a$ such that $X_a^j = X_\ell^i$

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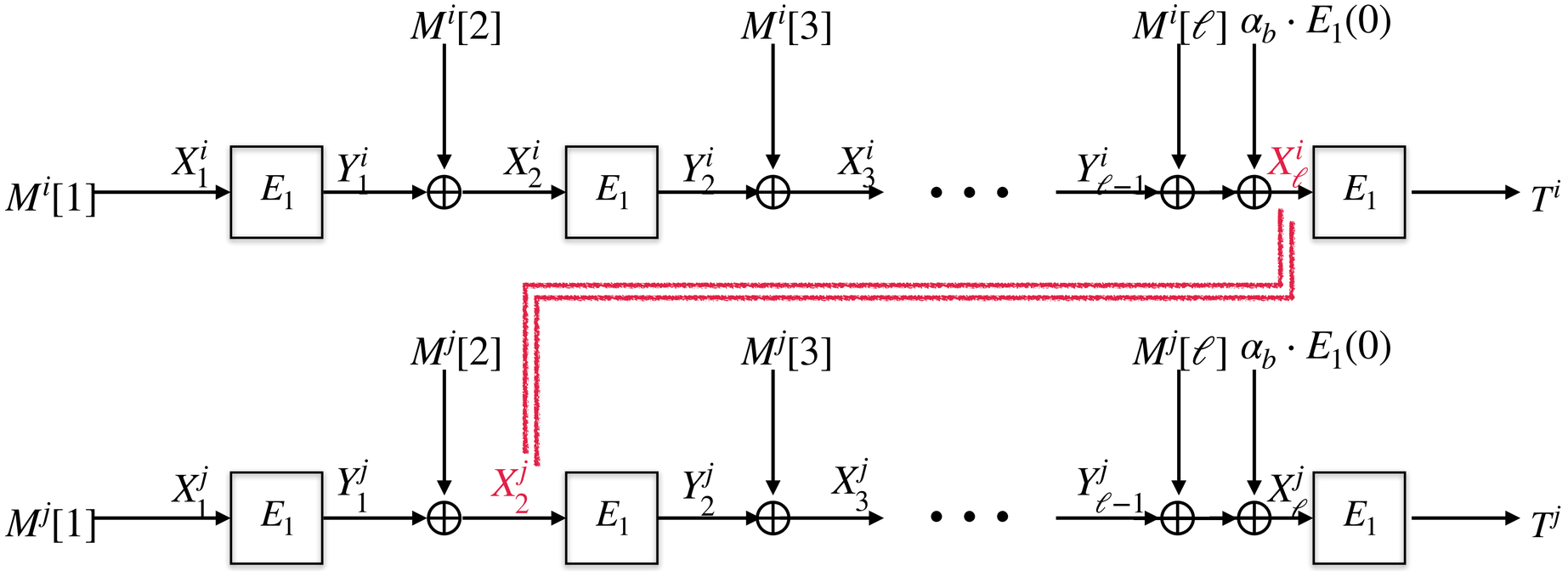


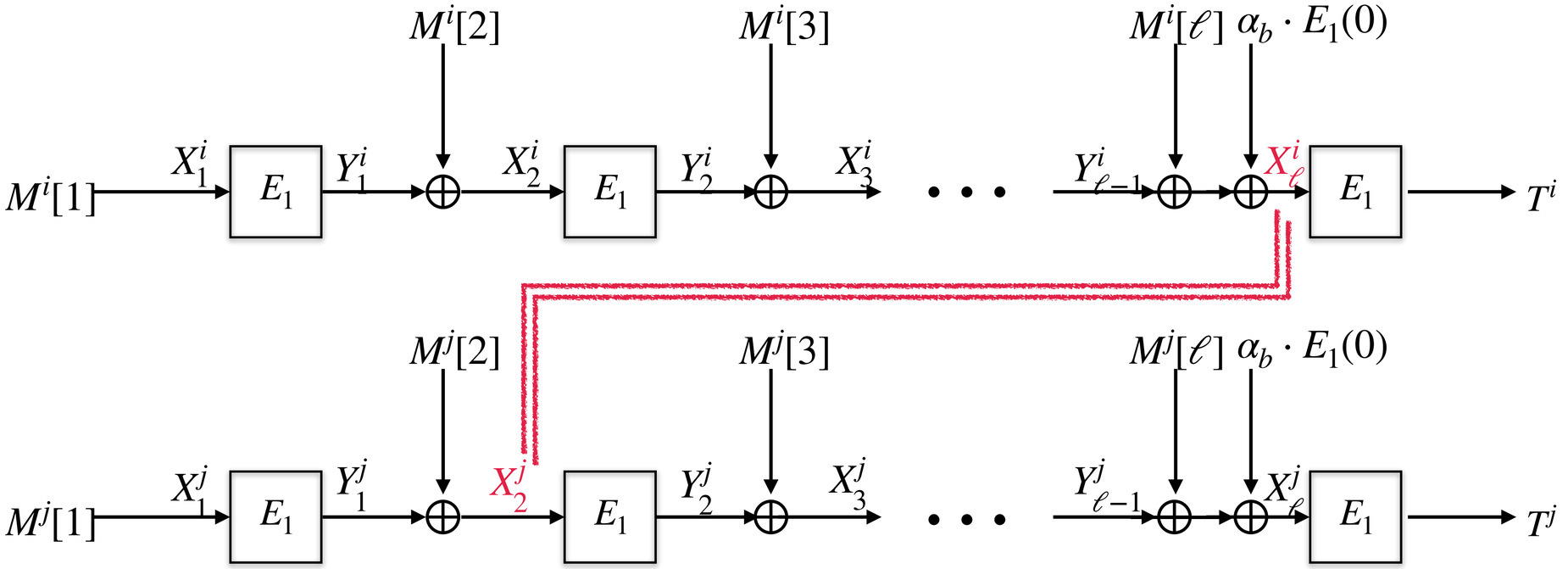
$$\Pr(\texttt{Full Collision}) = O\left(\frac{q^2\ell}{2^n}\right)$$

PRF Security of OMAC

Reset-Sampling Approach (CJN, IACR ASIACRYPT 2021)

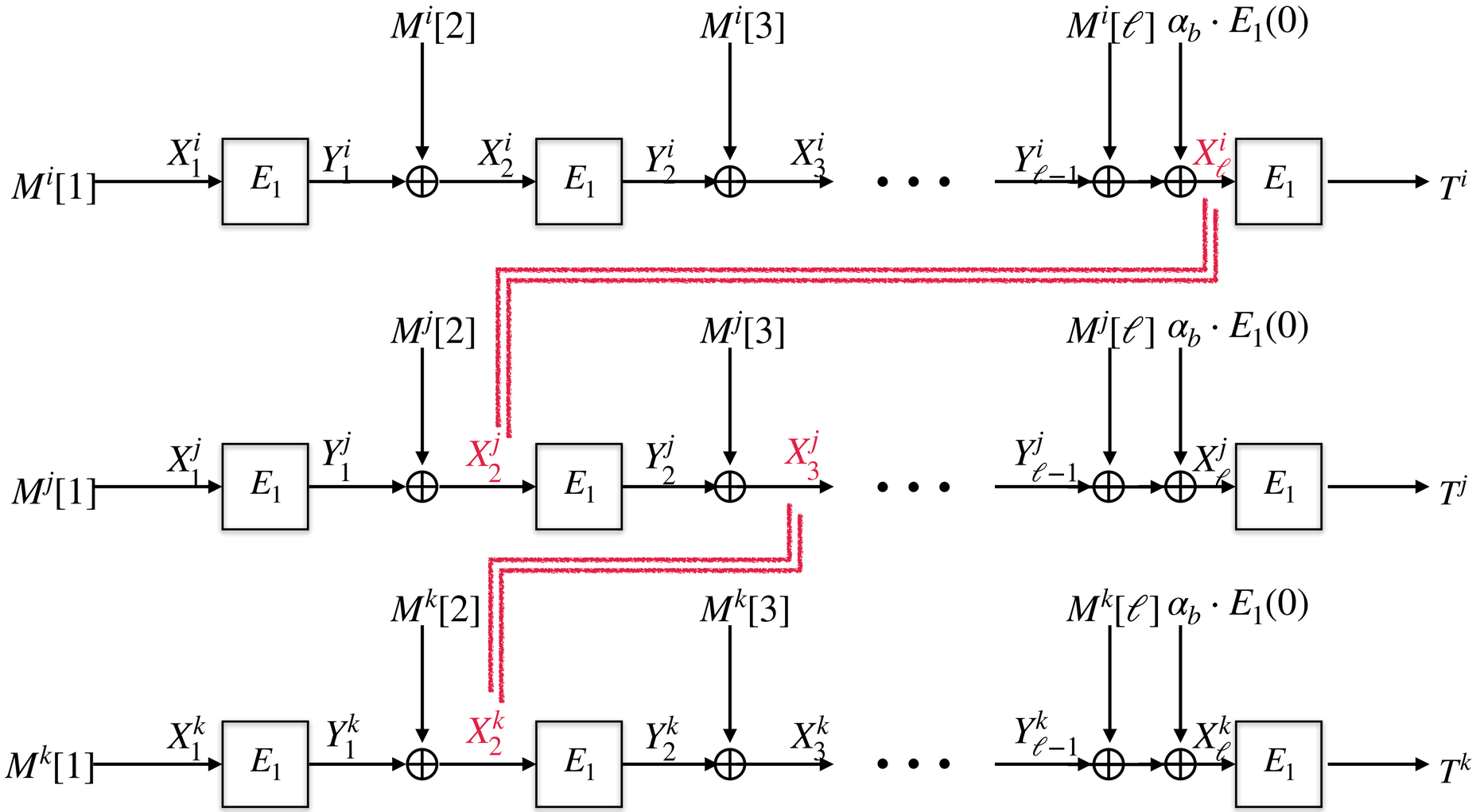
PRF Security of OMAC

Reset-Sampling Approach (CJN, IACR ASIACRYPT 2021)



PRF Security of OMAC

Reset-Sampling Approach (CJN, IACR ASIACRYPT 2021)



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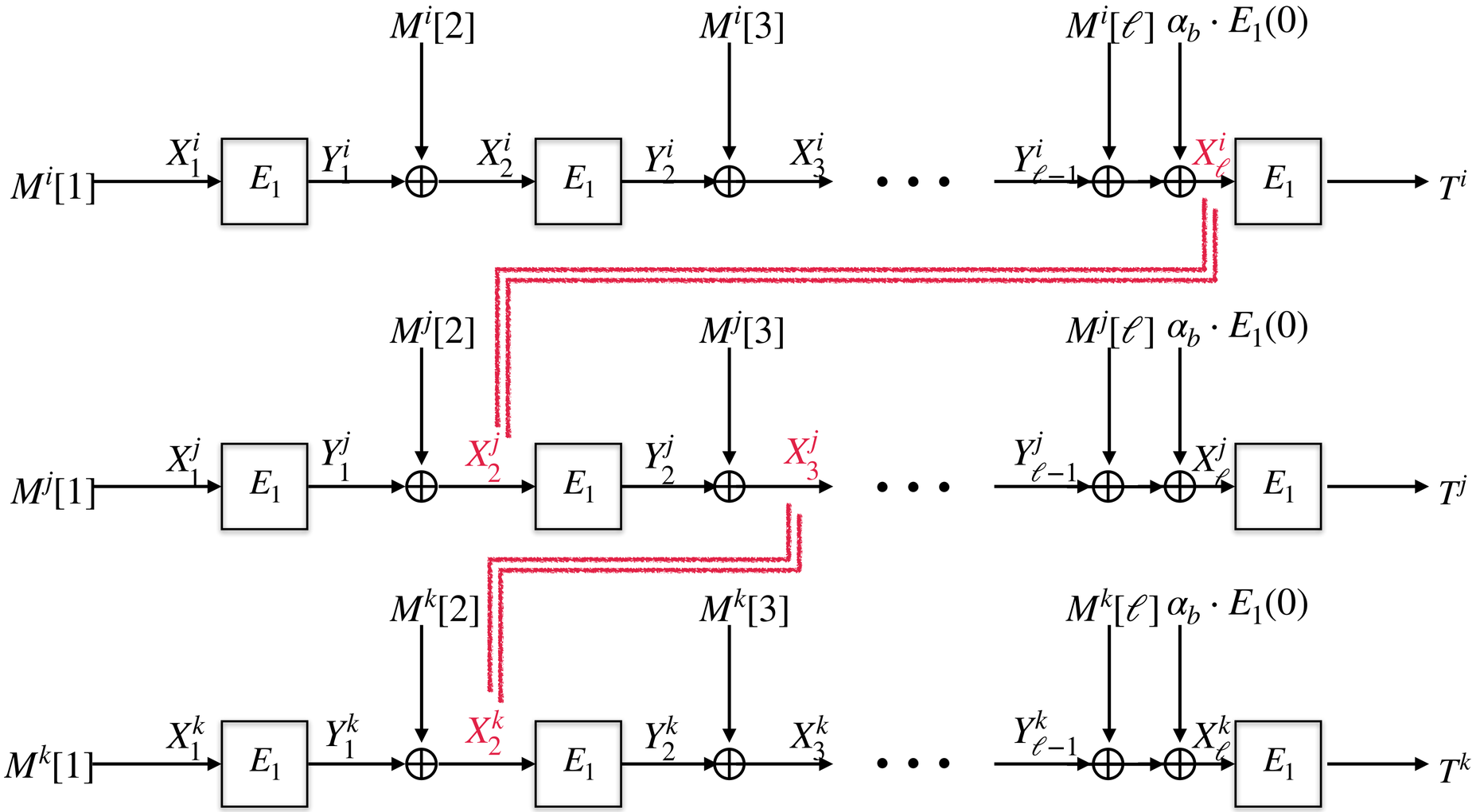

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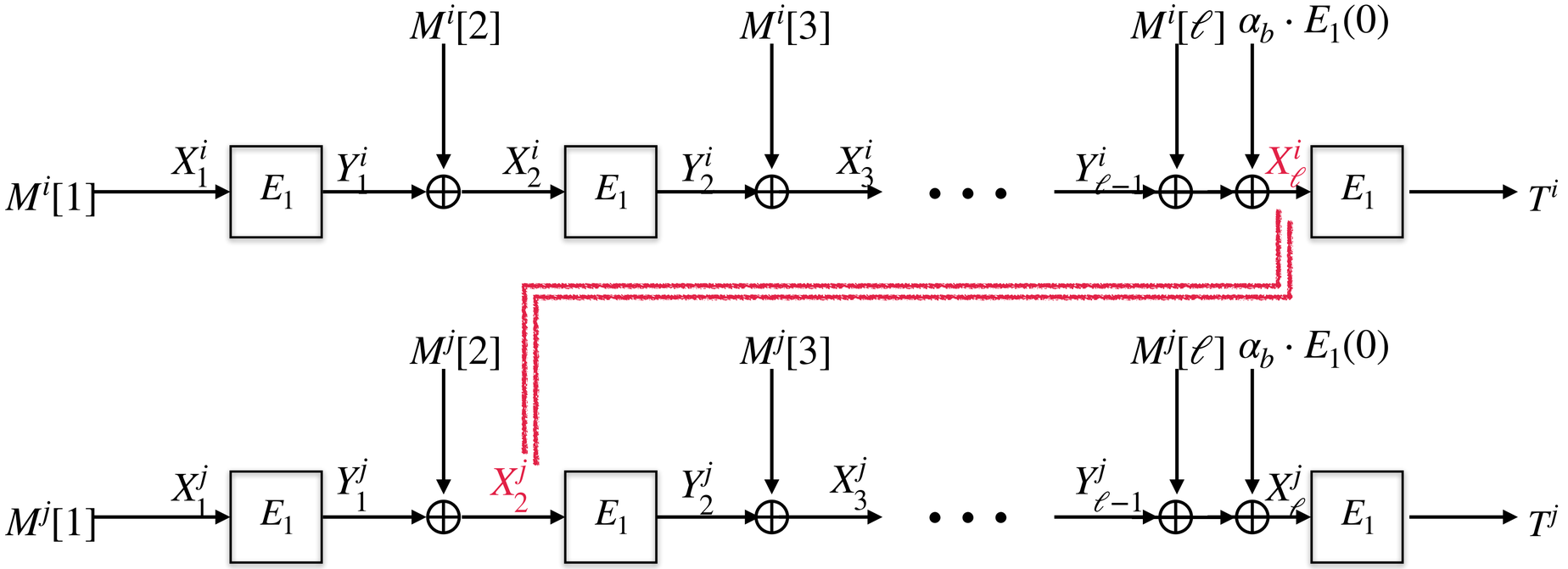

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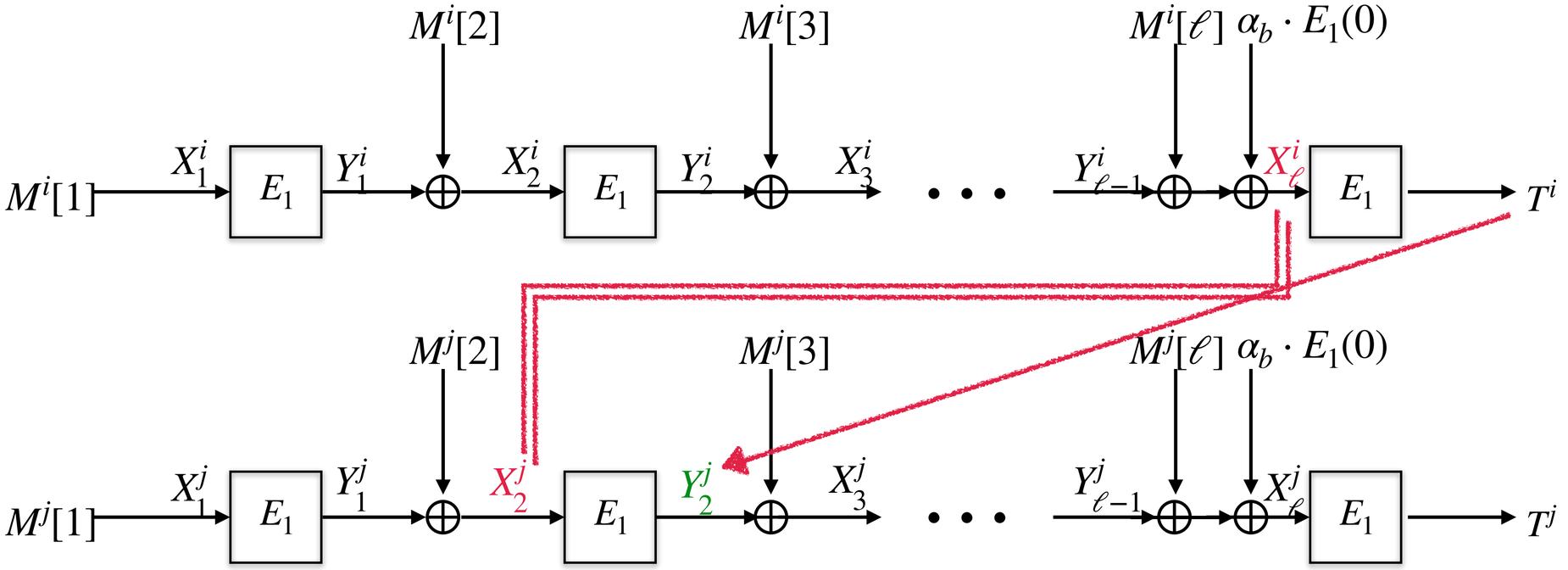
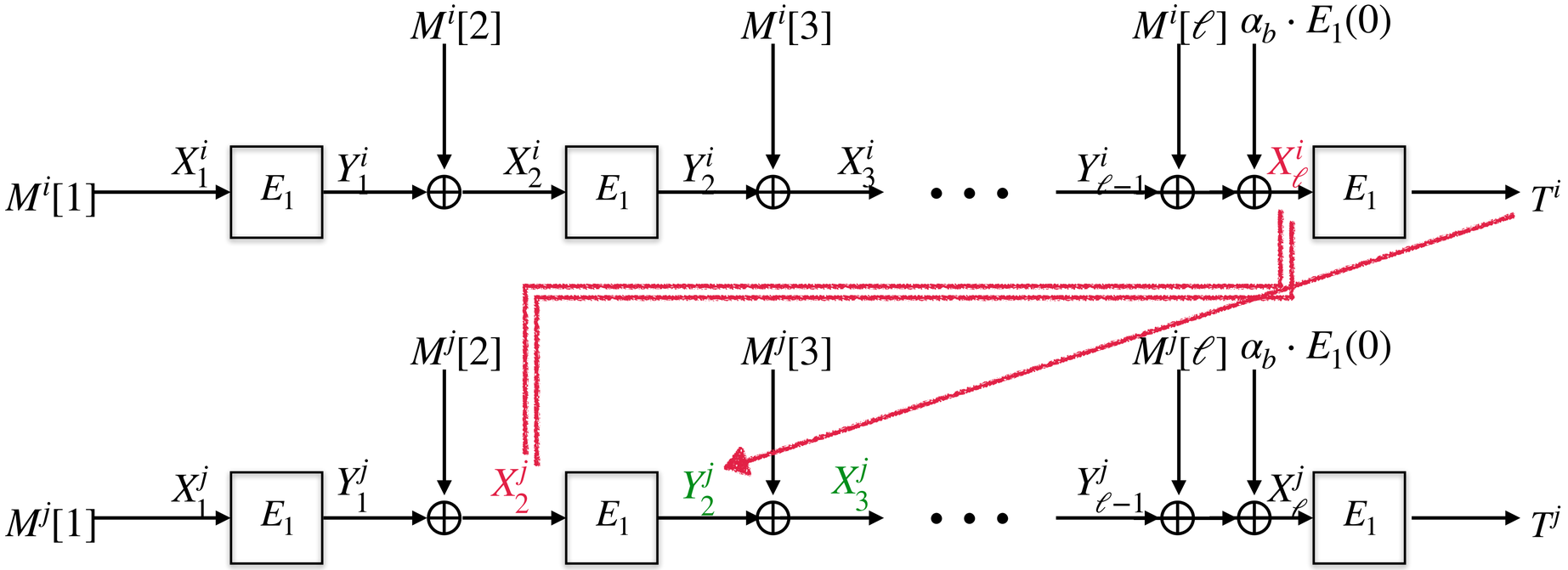


PRF Security of OMAC

Initial Resetting



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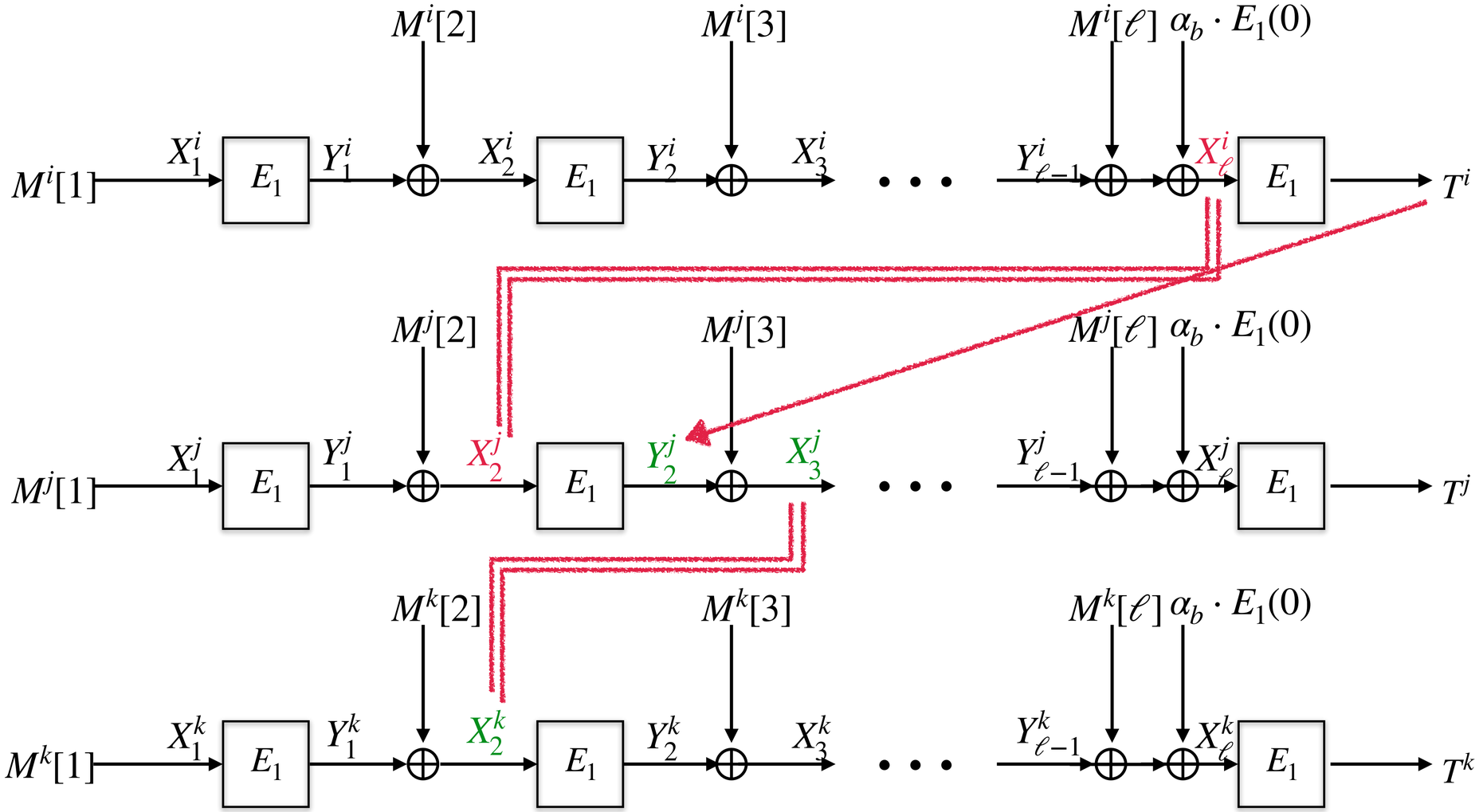

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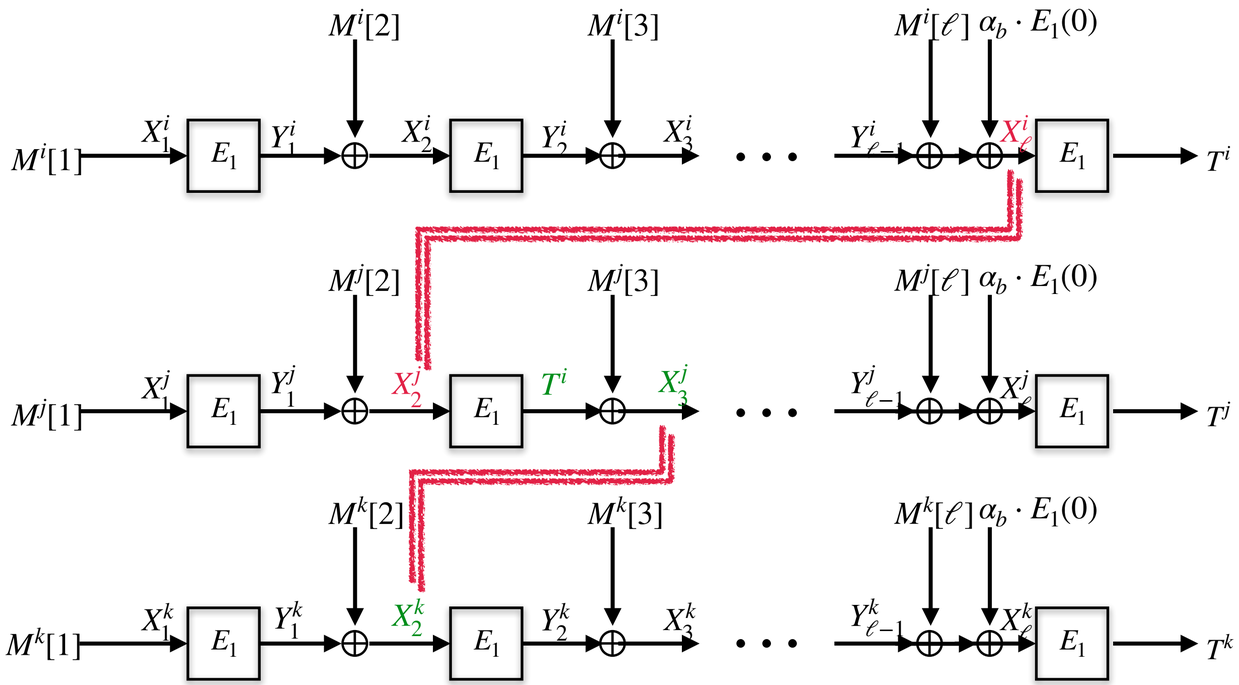
Induced Resetting



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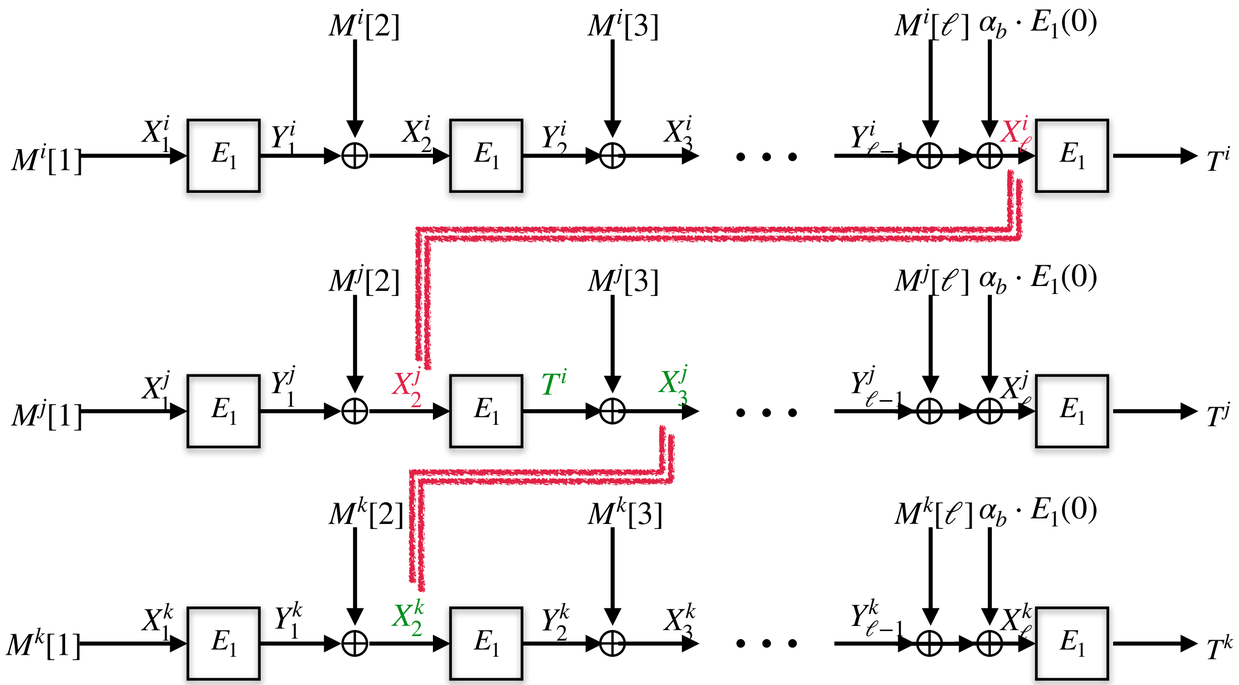

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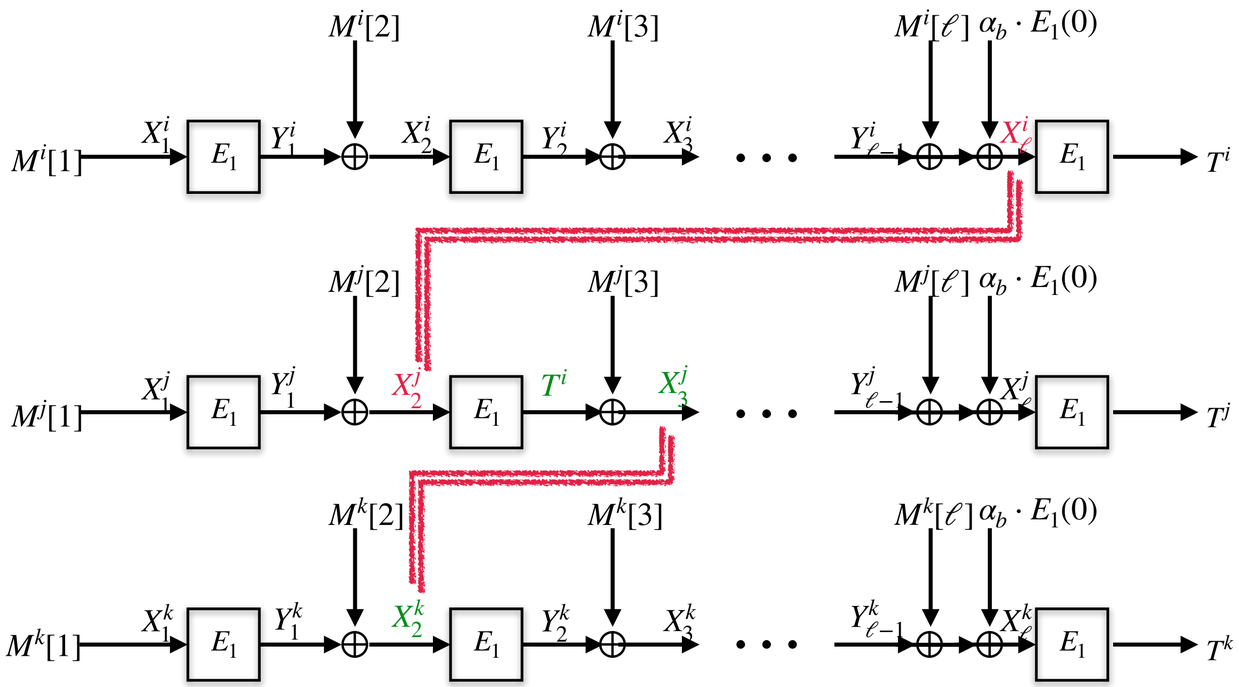

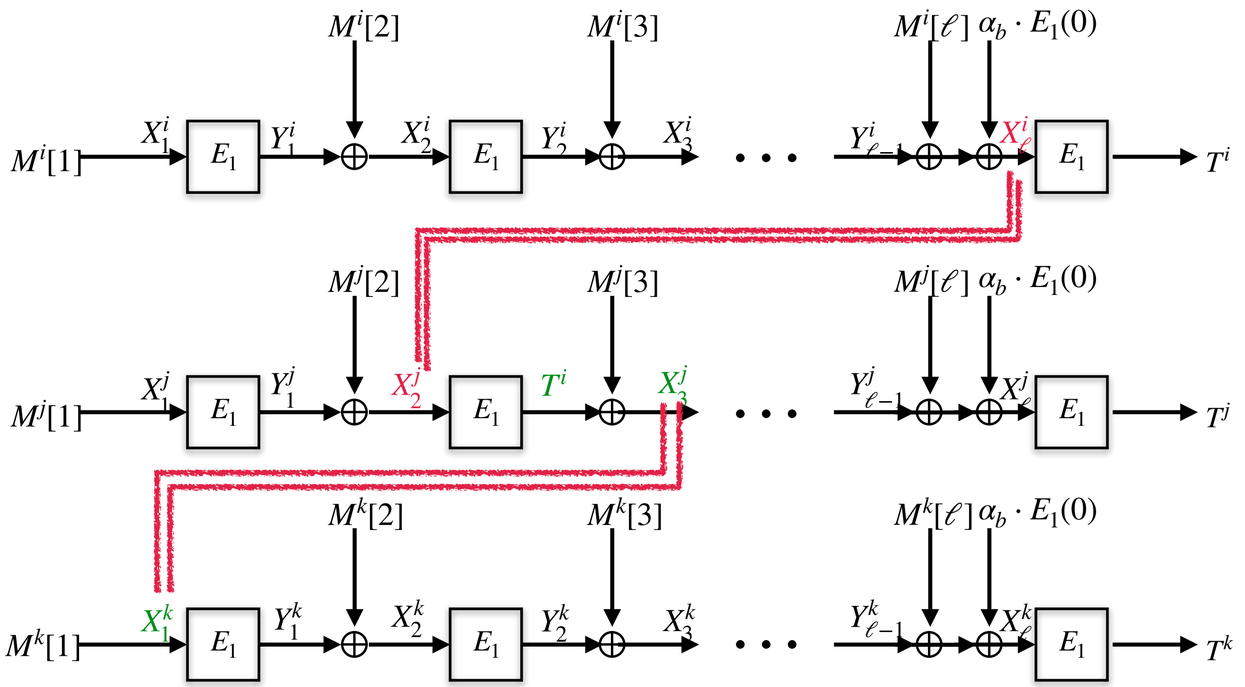

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$$M^k[1] = T^i \oplus M^j[3]$$

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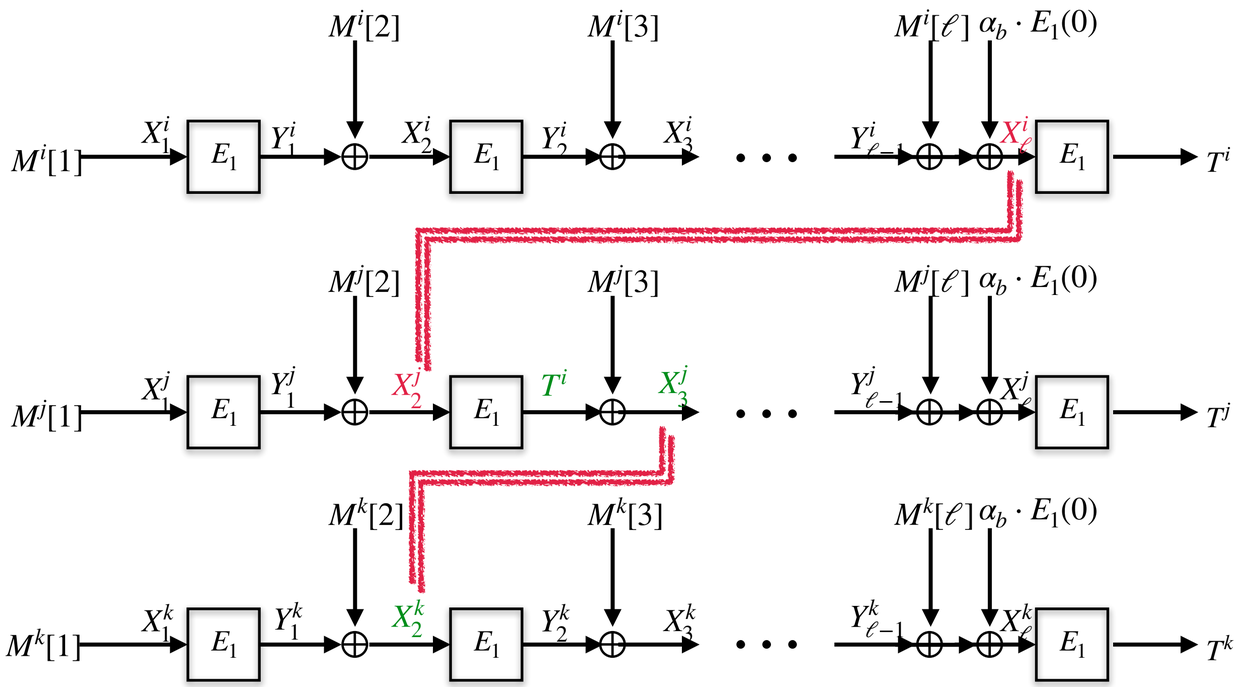

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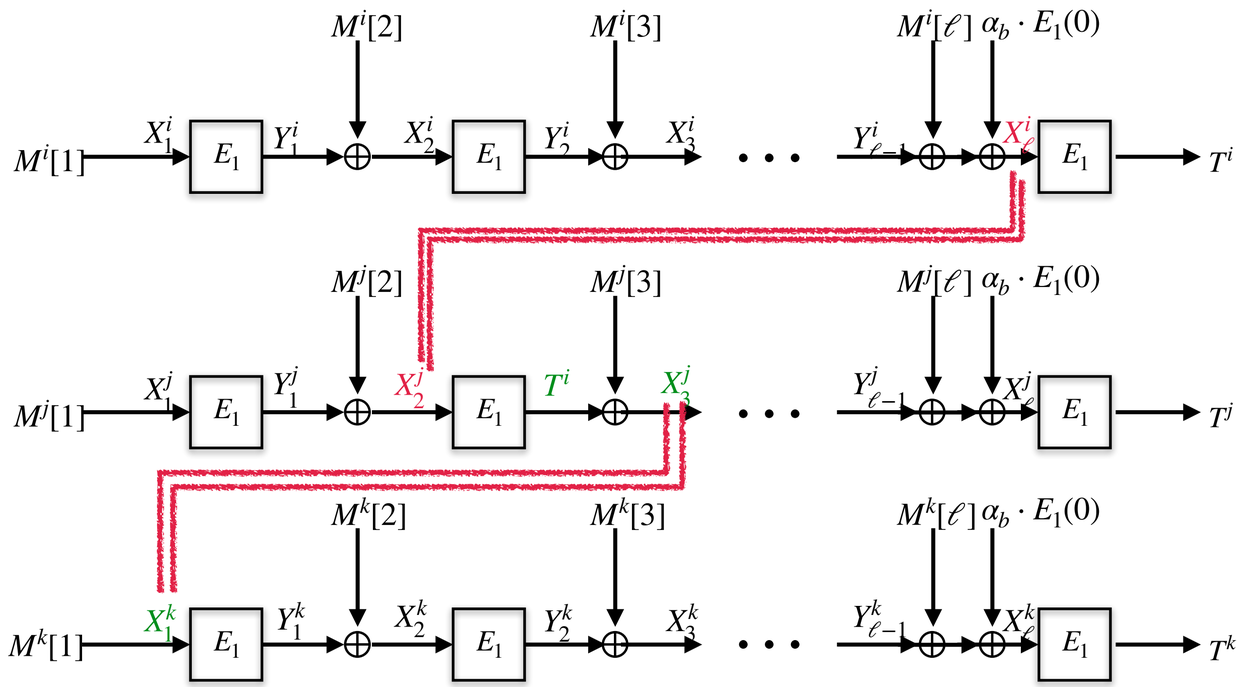

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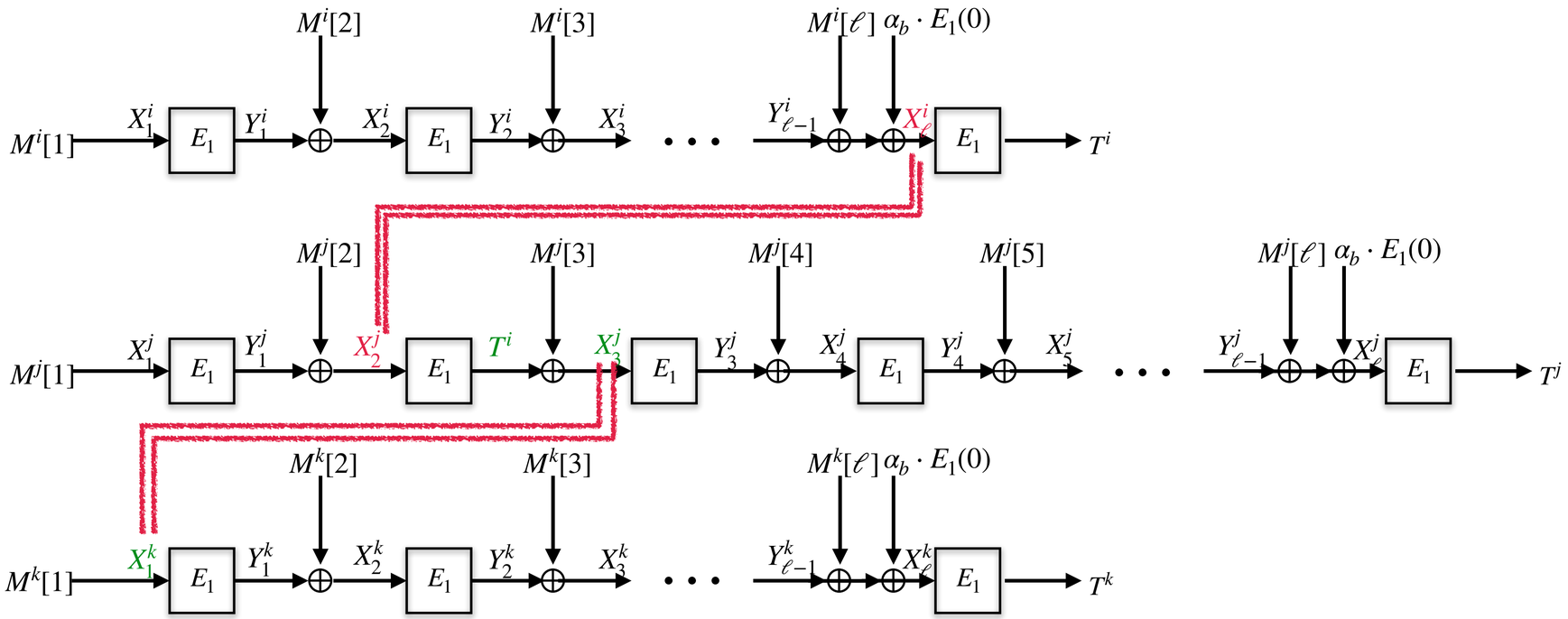

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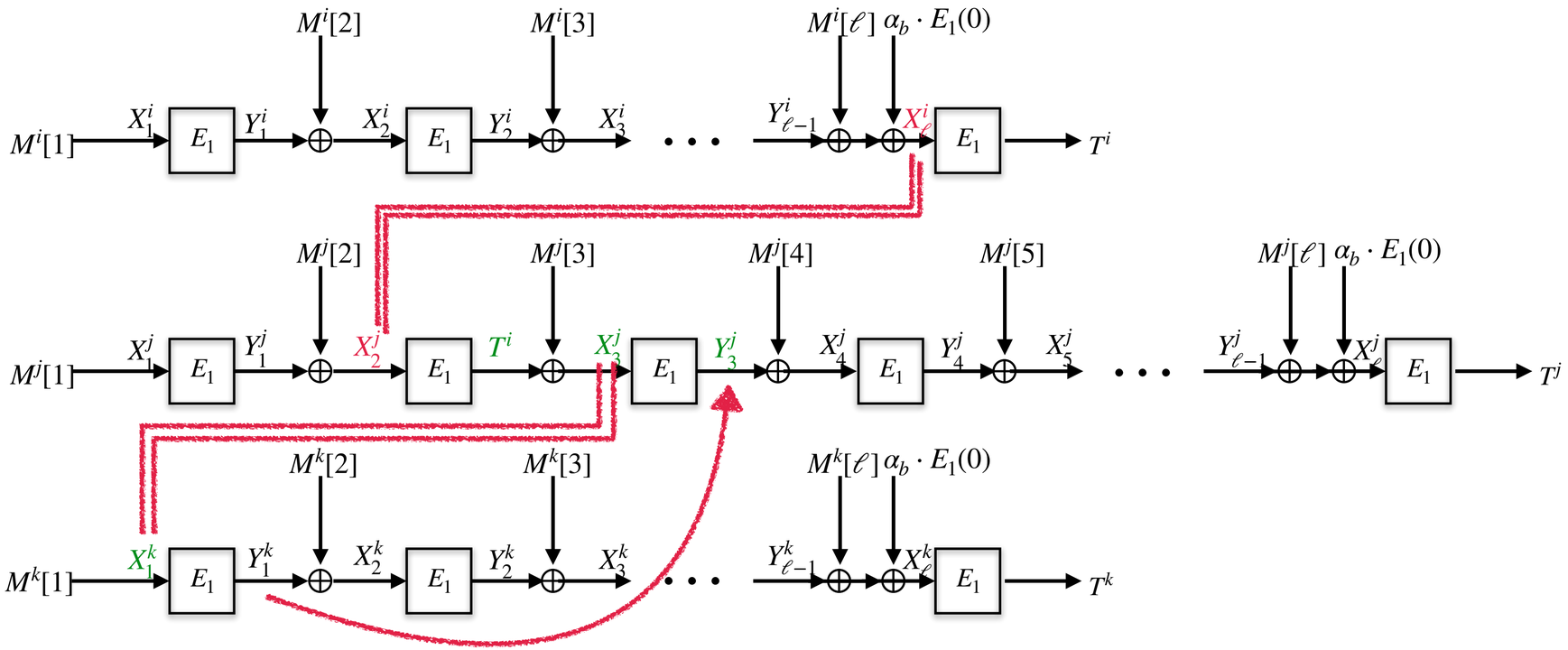

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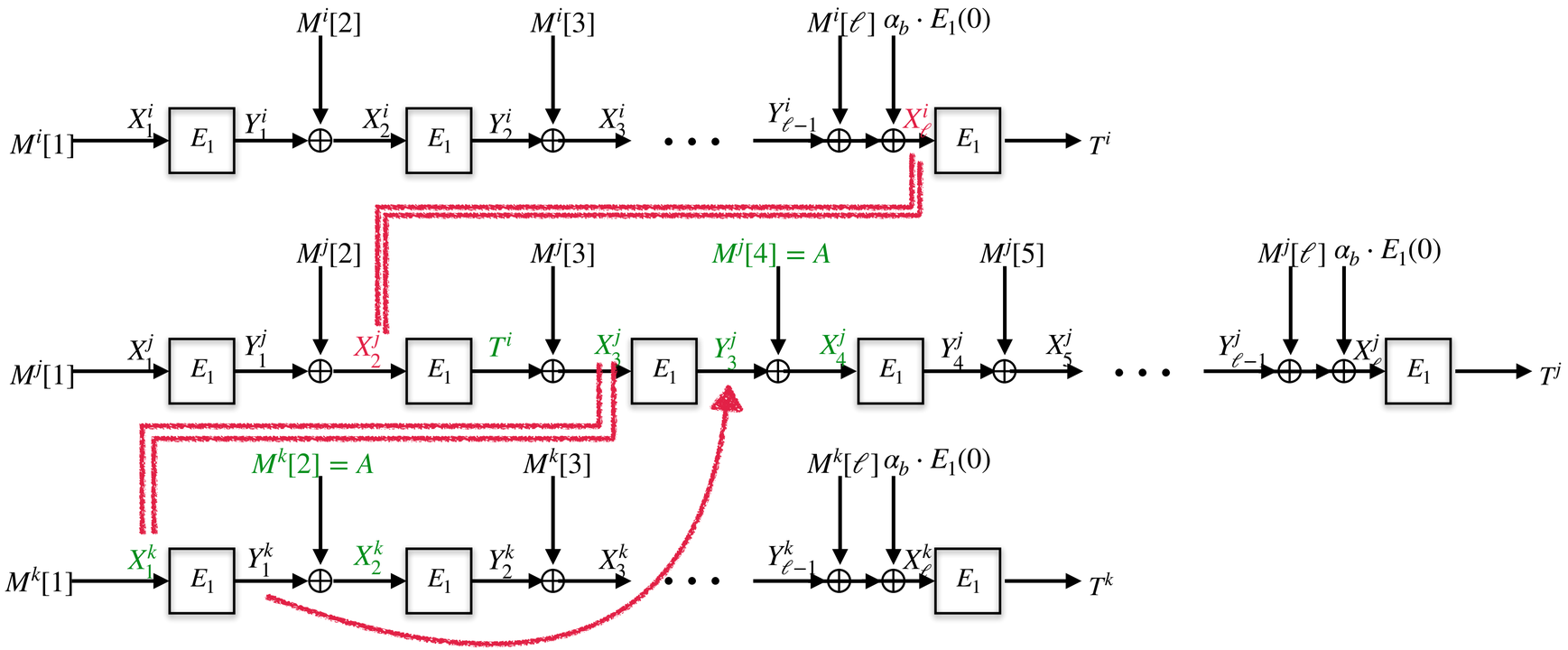

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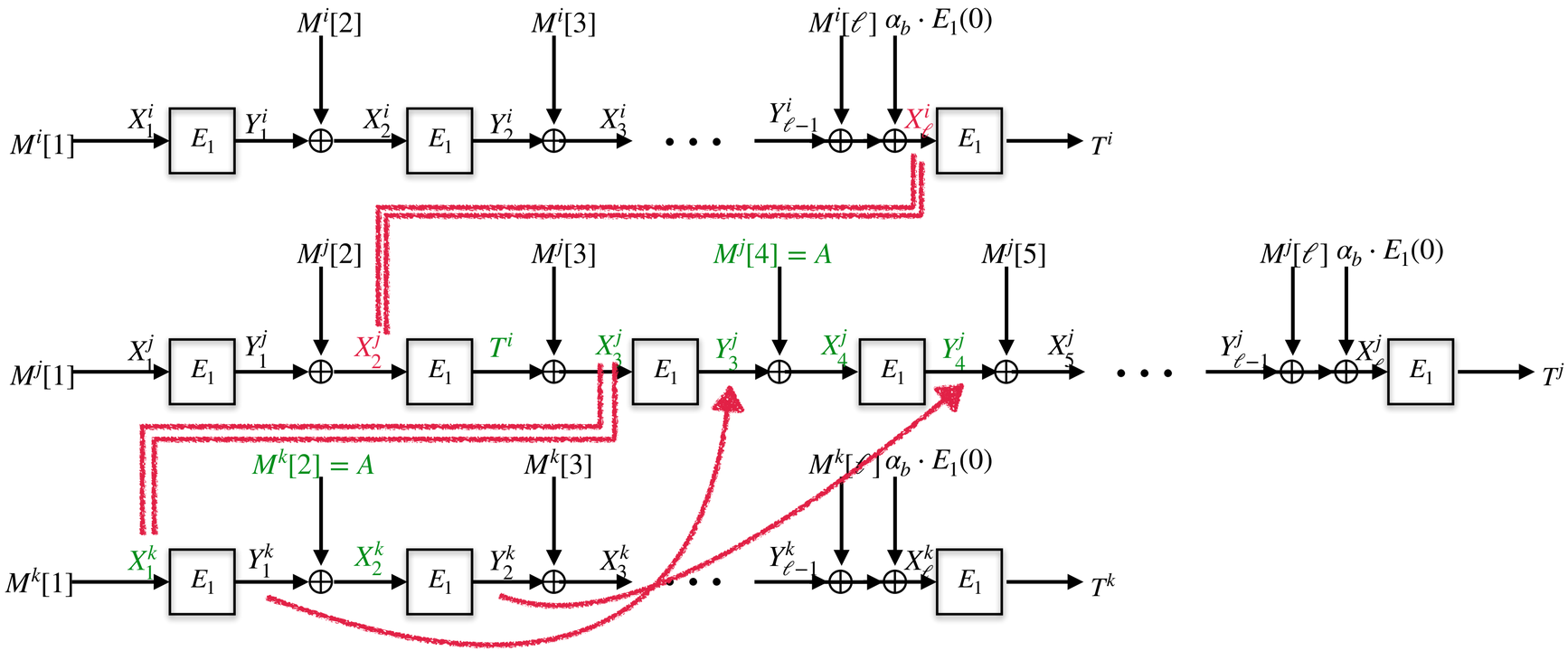

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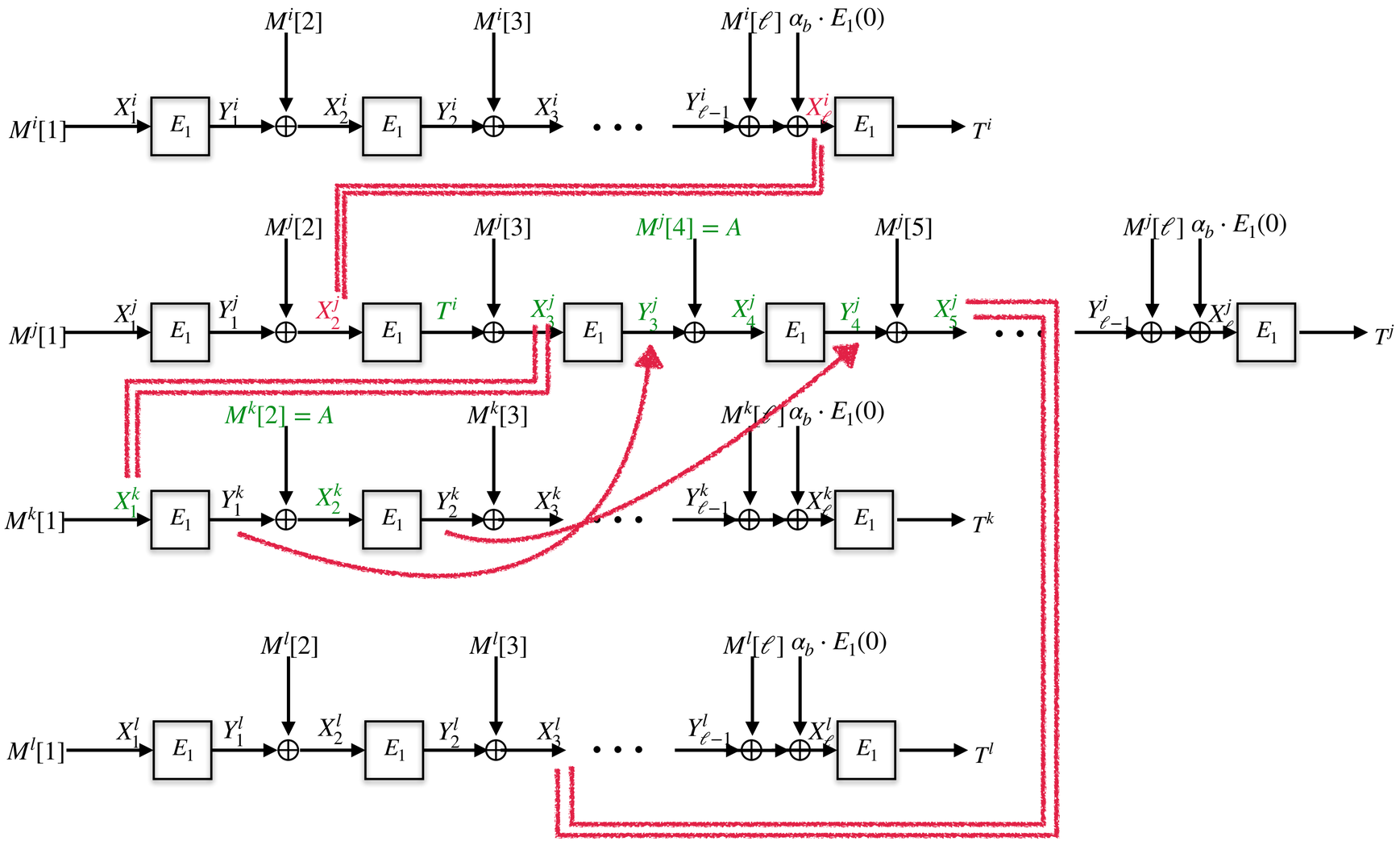

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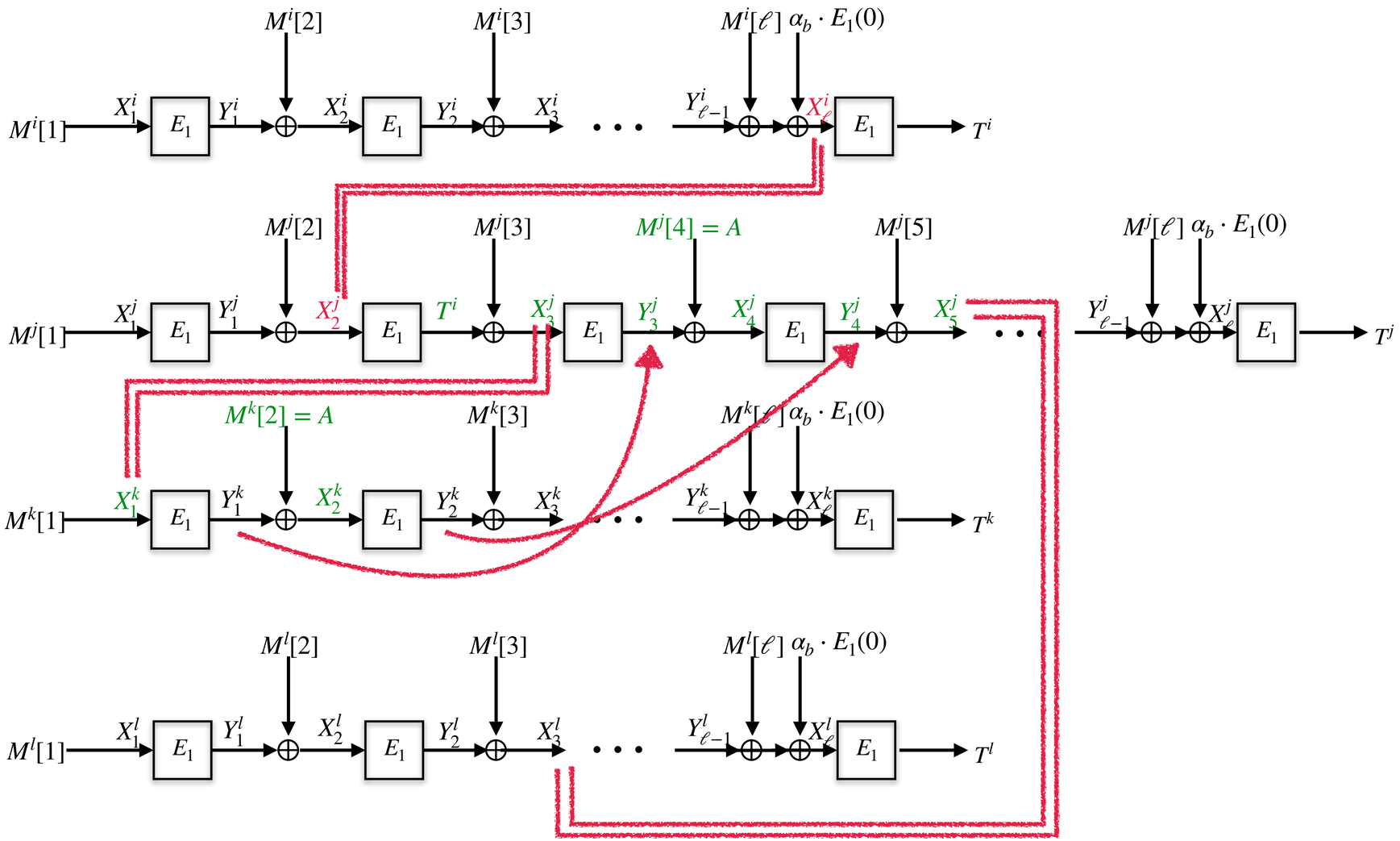

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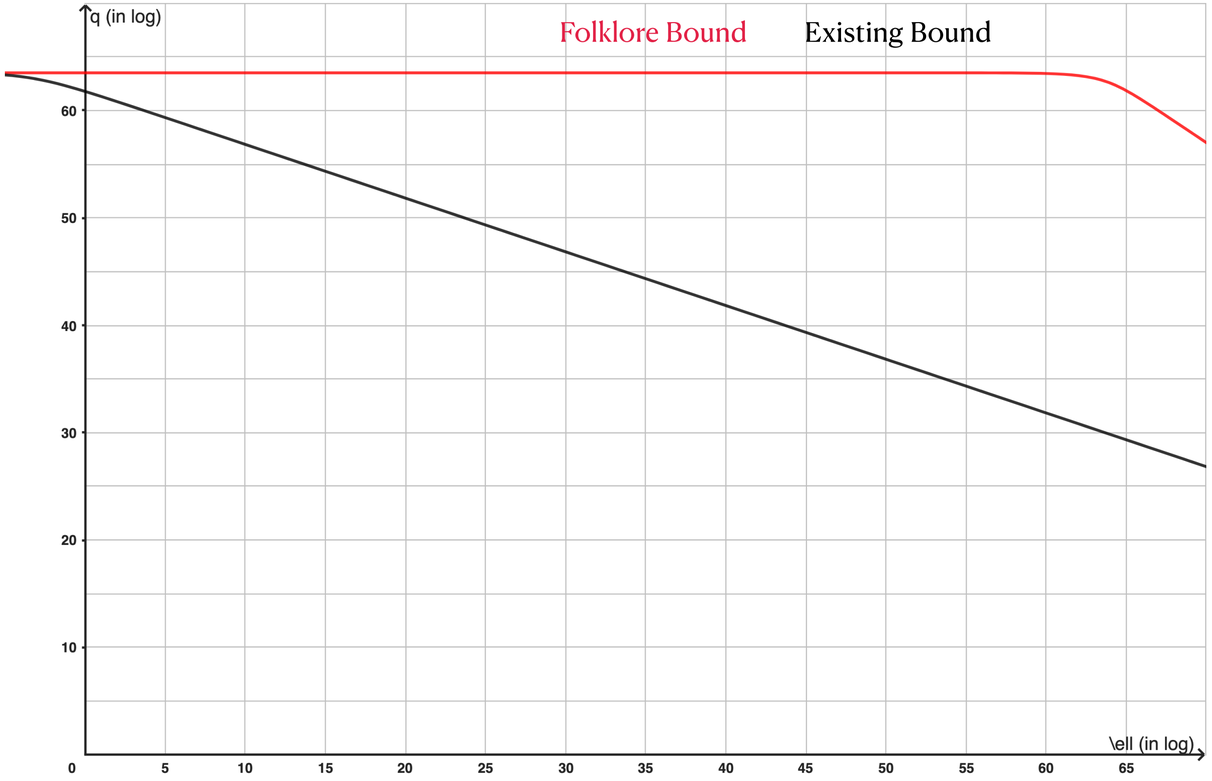

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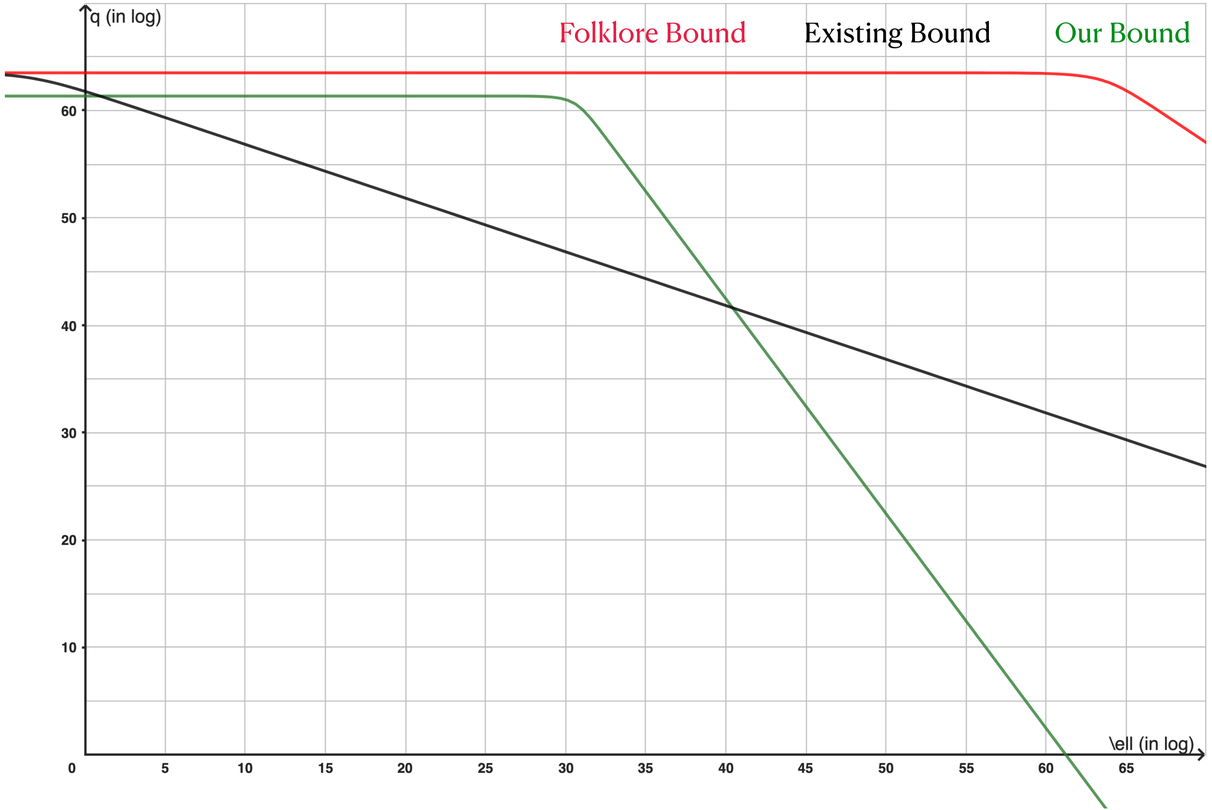

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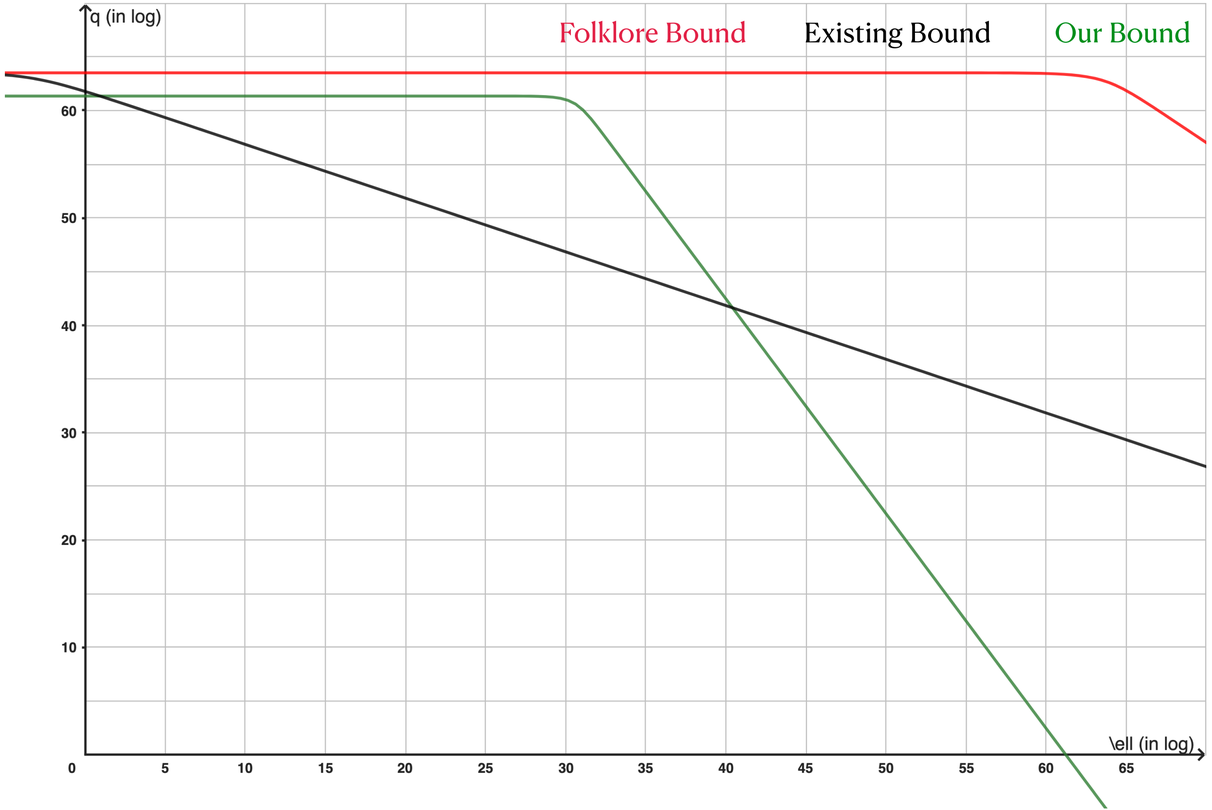

OMAC* is (almost) birthday bound secure.

* see full paper for similar results on XCBC and TMAC

Future Directions

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Thank you