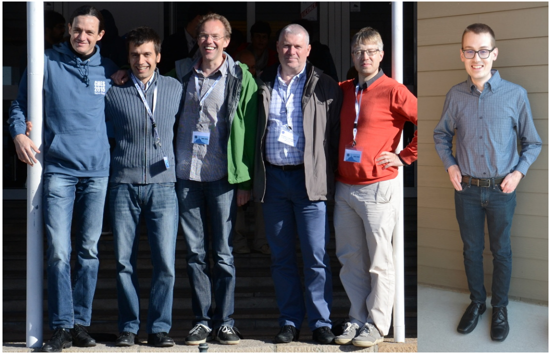
A Modular Approach to the Security Analysis of Two-Permutation Constructions

Yu Long Chen

December 7, 2022

SHA-3 and Permutations



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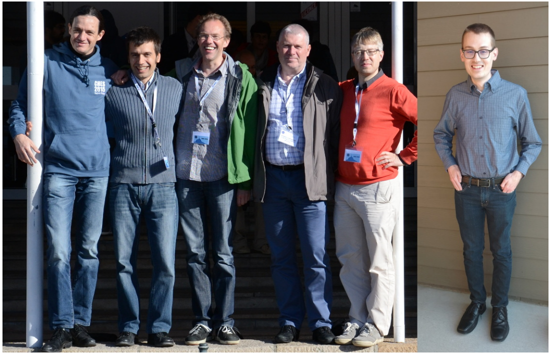

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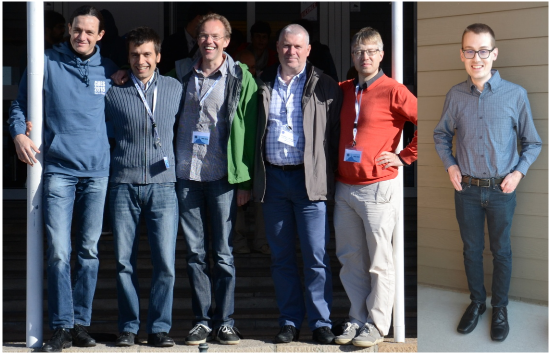

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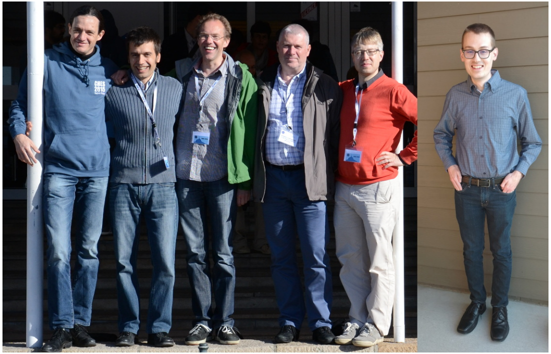

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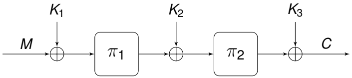
- Keccak is permutation based hash function
- Popularization of public permutation based constructions

Two-Calls Permutation-Based Constructions



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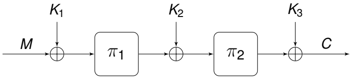

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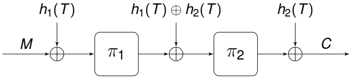

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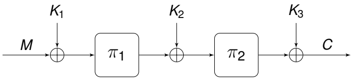

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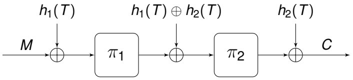

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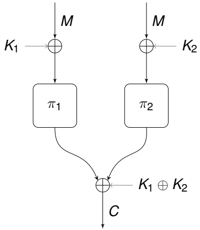

PRF

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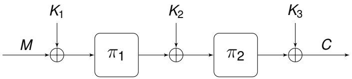

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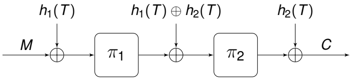

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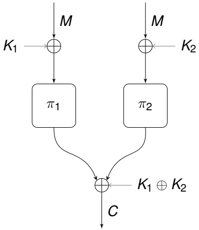

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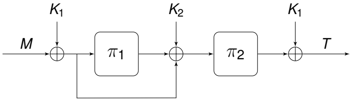

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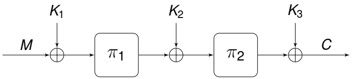

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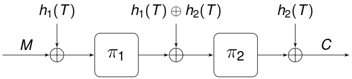

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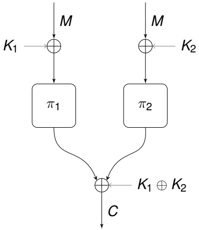

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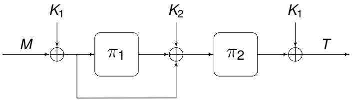

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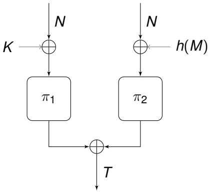

MAC algorithm

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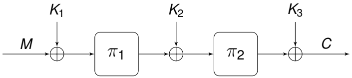

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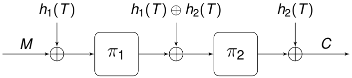

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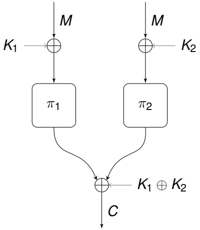

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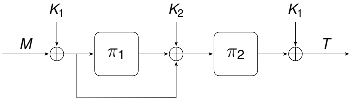

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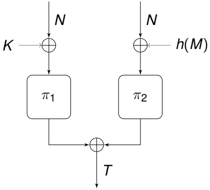

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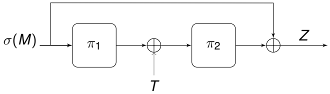


tccr hash function

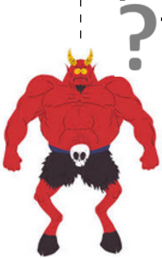
Generic Single-User Security

Our construction $\mathcal{O}$

$\pi_1, \ldots, \pi_r$

Ideal random construction $\mathcal{P}$

$\pi_1, \ldots, \pi_r$



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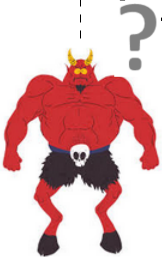

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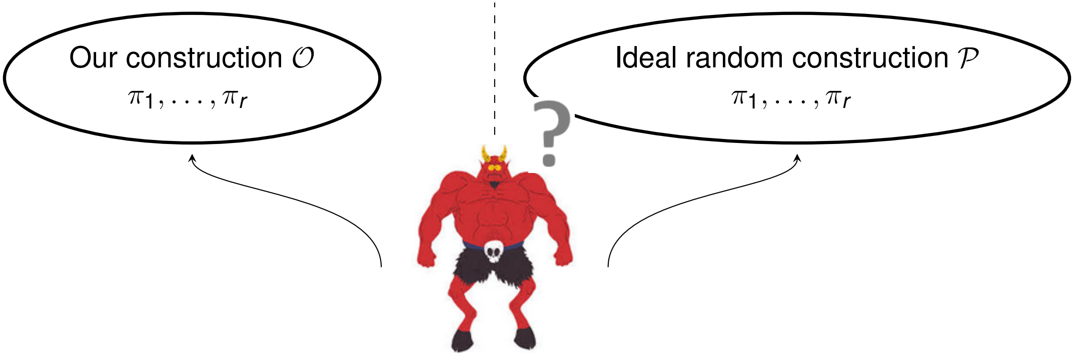

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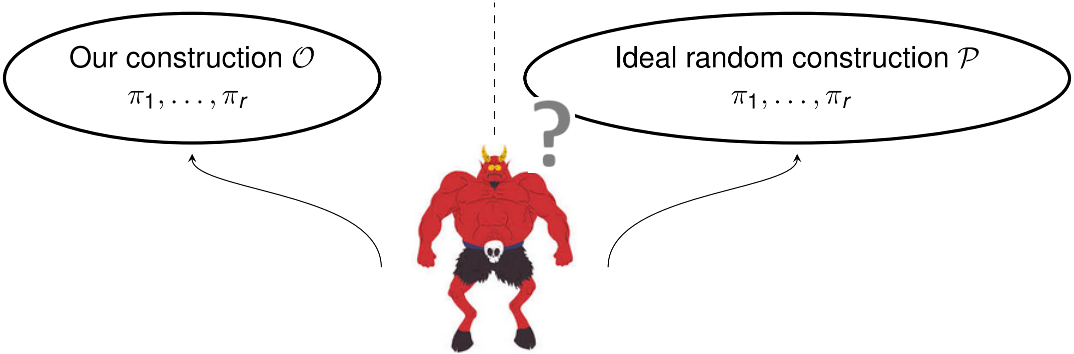

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- $\mathcal{O}$ is secure $\Longleftrightarrow$ $\mathbf{Adv}^{\mathrm{su}}_{\mathcal{O}}(\mathcal{A})$ is negligible

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$\mathcal{O}_{K_1}, \ldots, \mathcal{O}_{K_u}$
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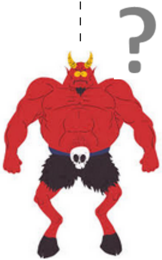

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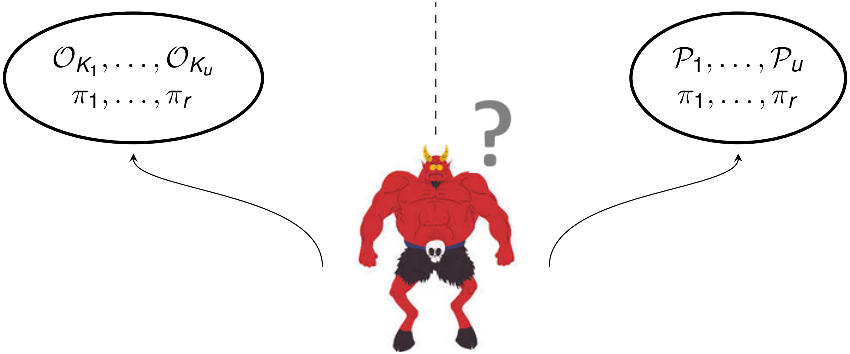

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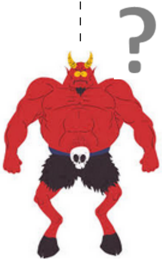

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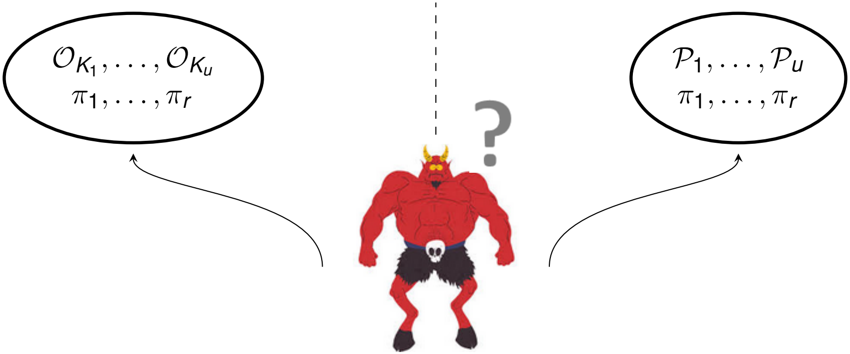

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- Naive hybrid argument $\mathbf{Adv}^{\mathrm{mu}}_{\mathcal{O}}(\mathcal{A}) = u \cdot \mathbf{Adv}^{\mathrm{su}}_{\mathcal{O}}(\mathcal{A})$

Patarin's H-coefficient Technique

$$\frac{\Pr(X_{\mathcal{O}} = \tau)}{\Pr(X_{\mathcal{P}} = \tau)} \geq 1 - \epsilon$$

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Modular approach for Item 1 and Item 3?

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with $\lambda_1, \ldots, \lambda_{qm}$ and $\lambda'_1, \ldots, \lambda'_{qa}$ knowns values

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- Our goal is to give a lower bound on the number of solutions of these systems

Patarin's Mirror Theory

- Represents the system of equations and non-equations by a graph

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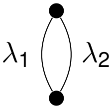

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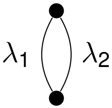

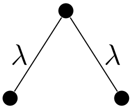

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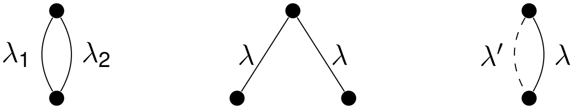

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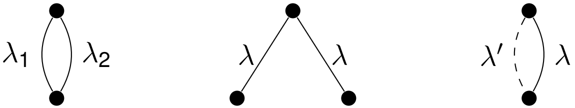
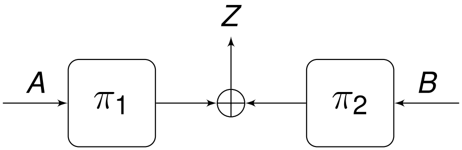
  - these properties define the bad transcripts



Two-Permutation Calls Constructions

Focus on all constructions that can be viewed as:



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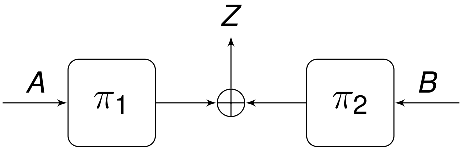

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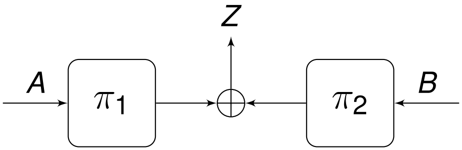

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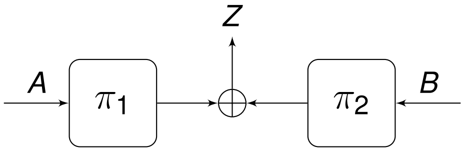
- Security analysis in ideal permutation model
- Query access to the underlying primitives (modeled as random)

Two-Permutation Calls Constructions

Focus on all constructions that can be viewed as:



A, B, and Z are functions of the secret key, the inputs, and the outputs

- Security analysis in ideal permutation model
- Query access to the underlying primitives (modeled as random)
- Primitive queries in the form $\pi_1(u) = v$ and $\pi_2(x) = y$

Include Primitive Queries in The System

$$\mathcal{E}_m^p = \begin{cases} v_{I_1} \oplus y_{I_1} = \lambda_1, \\ \vdots \\ v_{I_{q_m}} \oplus y_{I_{q_m}} = \lambda_{q_m}, \\ v_{I_{q_m+1}} = \lambda_{q_m+1}, \\ \vdots \\ v_{I_{q_m+p}} = \lambda_{q_m+p}, \\ y_{I_{q_m+1}} = \lambda_{q_m+p+1}, \\ \vdots \\ y_{I_{q_m+p}} = \lambda_{q_m+2p}. \end{cases} \qquad \mathcal{E}_a = \begin{cases} v'_{J_1} \oplus y'_{J_1} \neq \lambda'_1, \\ \vdots \\ v'_{J_{q_a}} \oplus y'_{J_{q_a}} \neq \lambda'_{q_a}, \end{cases}$$

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- Two surjective index mappings:

$$\varphi_V^p \colon \{I_1, \ldots, I_{qm+p}, J_1, \ldots, J_{qa}\} \to \{1, \ldots, q_V\},$$
$$\varphi_Y^p \colon \{I_1, \ldots, I_{qm+p}, J_1, \ldots, J_{qa}\} \to \{1, \ldots, q_Y\},$$

Patarin's Mirror Theory For Permutation-Based Construction

- Represents the system of equations and non-equations by a graph
  - A distinct unknown $\rightarrow$ a vertex with unknown value (black)
  - An equation $\rightarrow$ a λ-labeled edge (normal)
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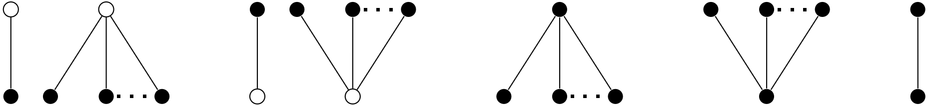
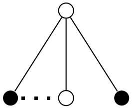
- Simplified the analysis by avoiding components with path of length 3 or higher



Inconsistency in Transcript Graph

- Each component contains at most one known vertex



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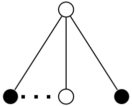
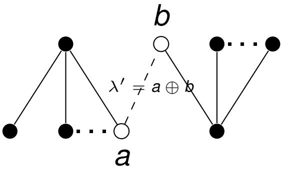
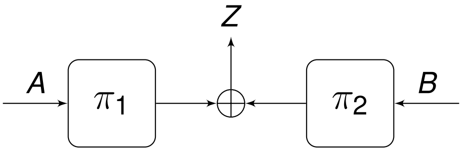


- No λ'-labeled edges that connect two colliding components such that the distance between the two connected vertices is λ'



Framework For Use



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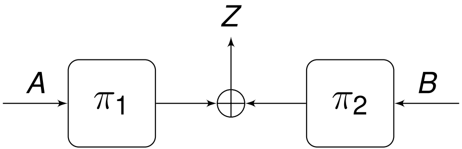

- Query transcript $\tau = \{(A_1, B_1, Z_1), \ldots, (A_{q_m}, B_{q_m}, Z_{q_m}), \tau_{\pi_1}, \tau_{\pi_2}, K_1, \ldots, K_u\}$

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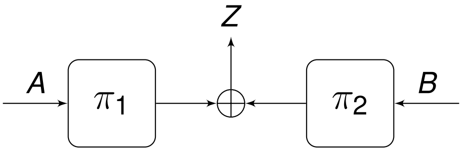

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- Each such algorithm consists of an evaluation of π_1 and an evaluation of π_2

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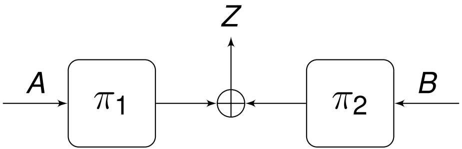

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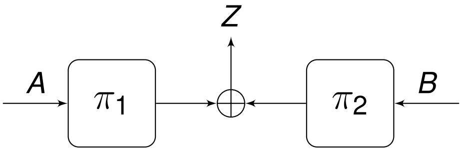

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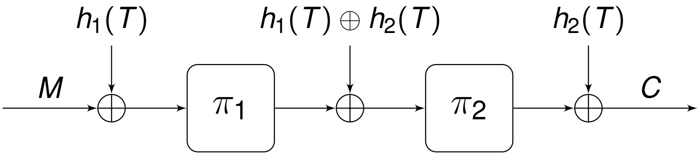
- Define $\mathcal{T}_{\mathrm{bad}}$ such that the graph is consistent
- Obtain ϵ using permutation-based mirror theory

Application on Multi-User Security of TEM



Cogliati et al. 2015

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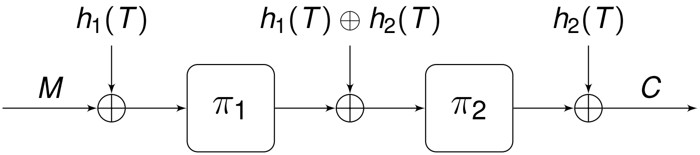

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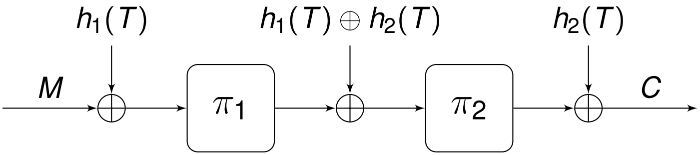

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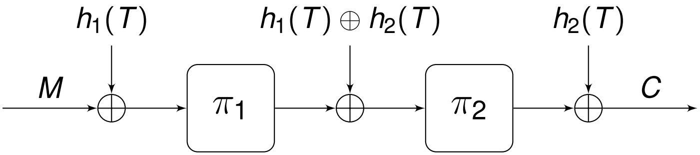

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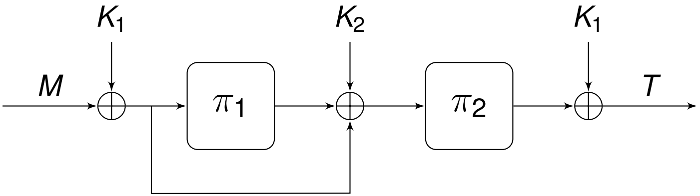

Dutta et al. 2021

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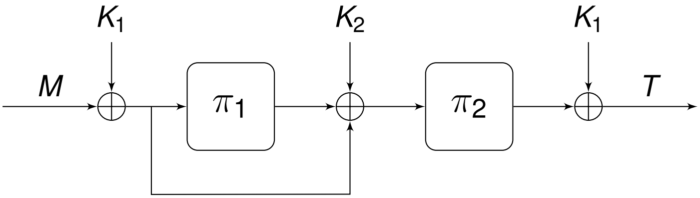

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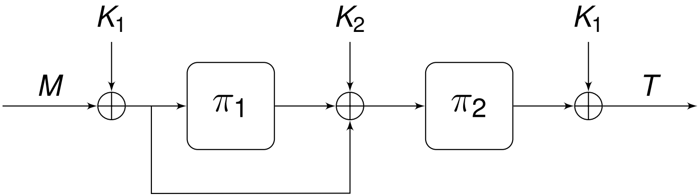

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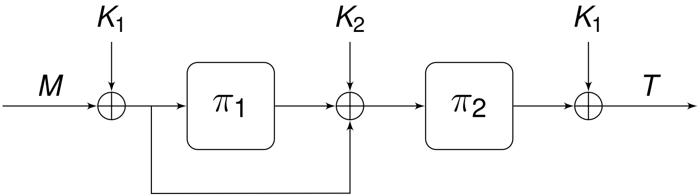

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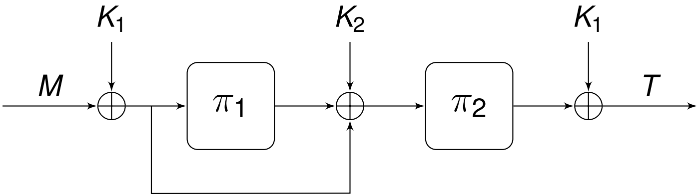

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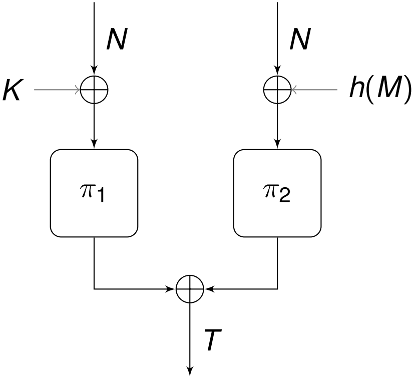

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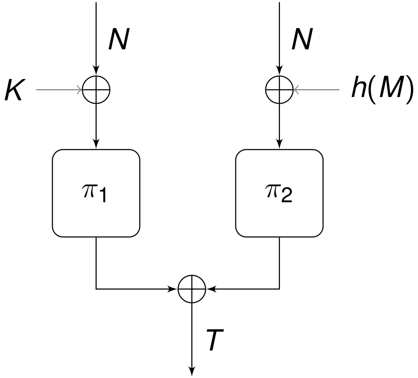

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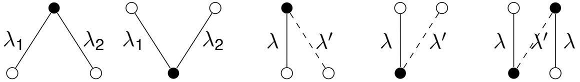

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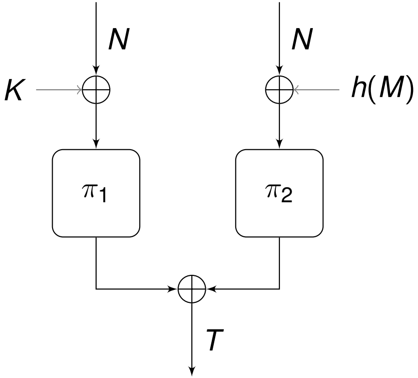

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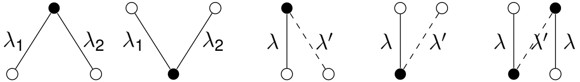

- Good transcript ratio analysis is also incomplete

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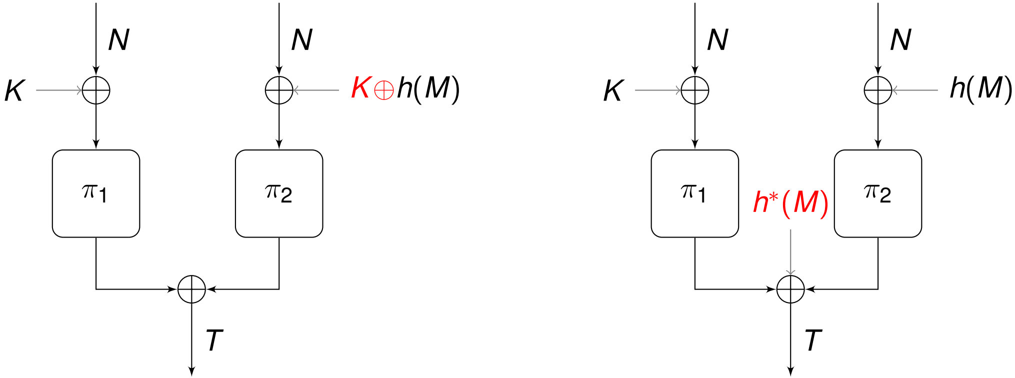

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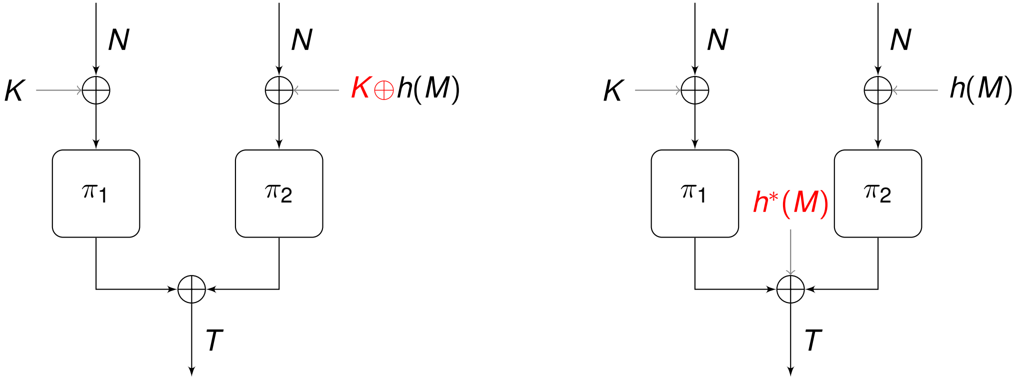

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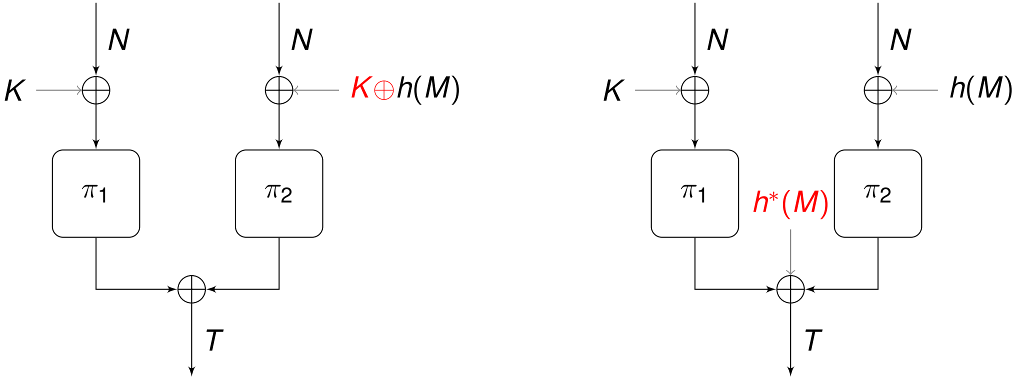

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Conclusion

New results

- Modular proof technique for permutation-based constructions based on mirror theory
- Framework to use this new technique
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Thank you for your attention!