

Efficient Adaptively-Secure Byzantine Agreement for Long Messages

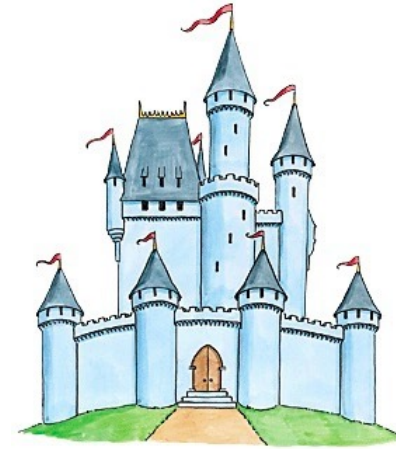
Kartik Nayak

with Amey Bhangale, Chen-Da Liu-Zhang, Julian Loss



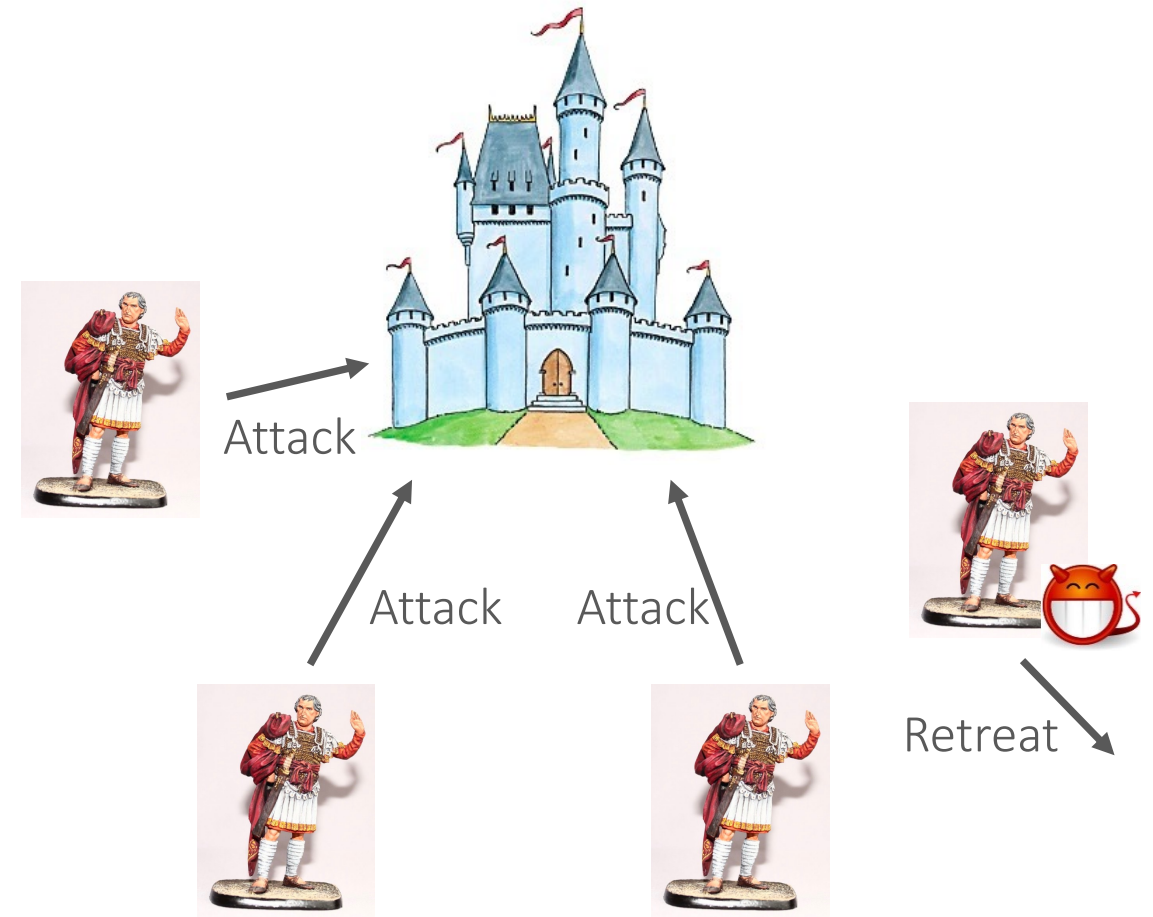
Byzantine Agreement

n generals ($\leq t$ Byzantine) need to agree on a battle plan



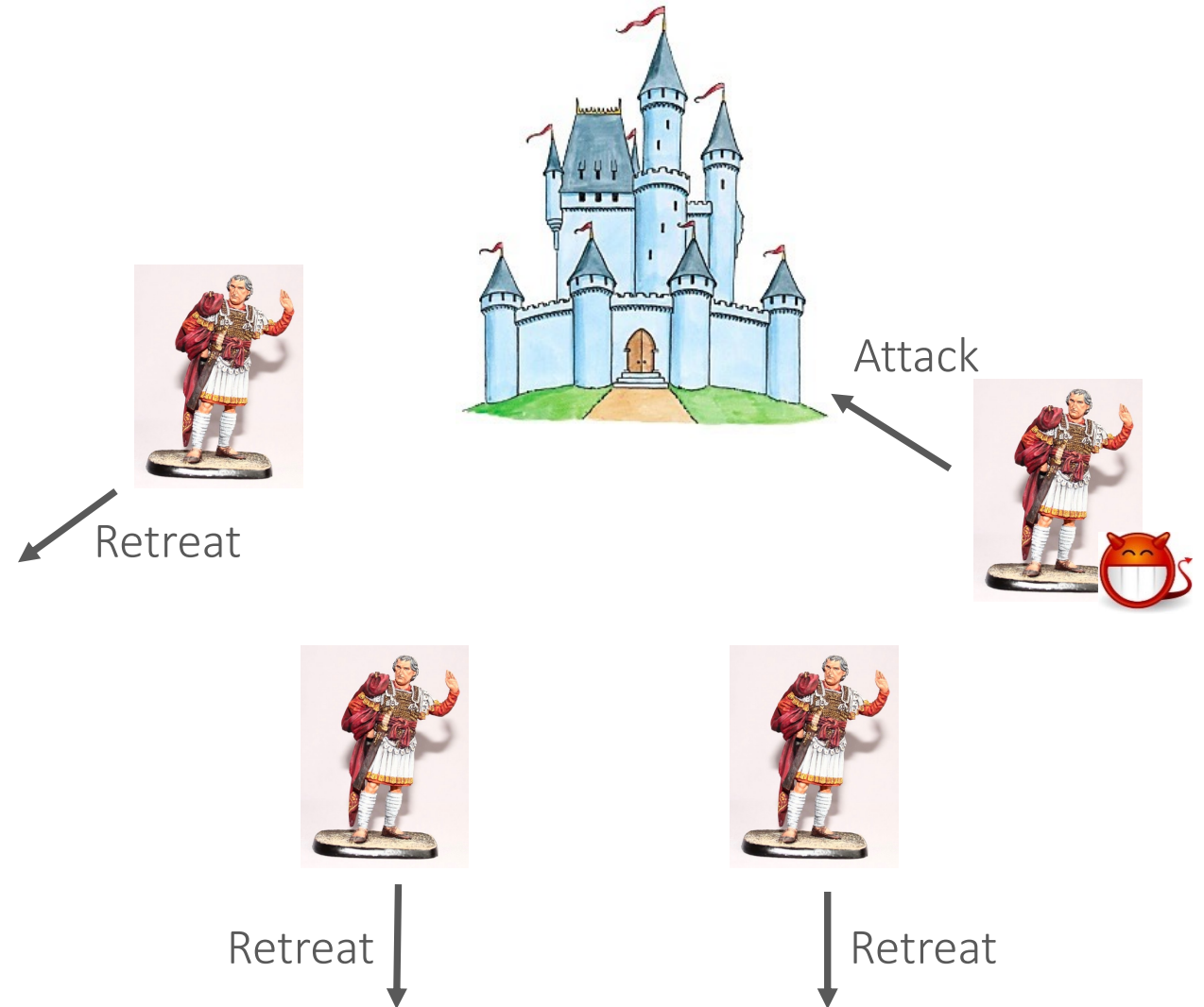
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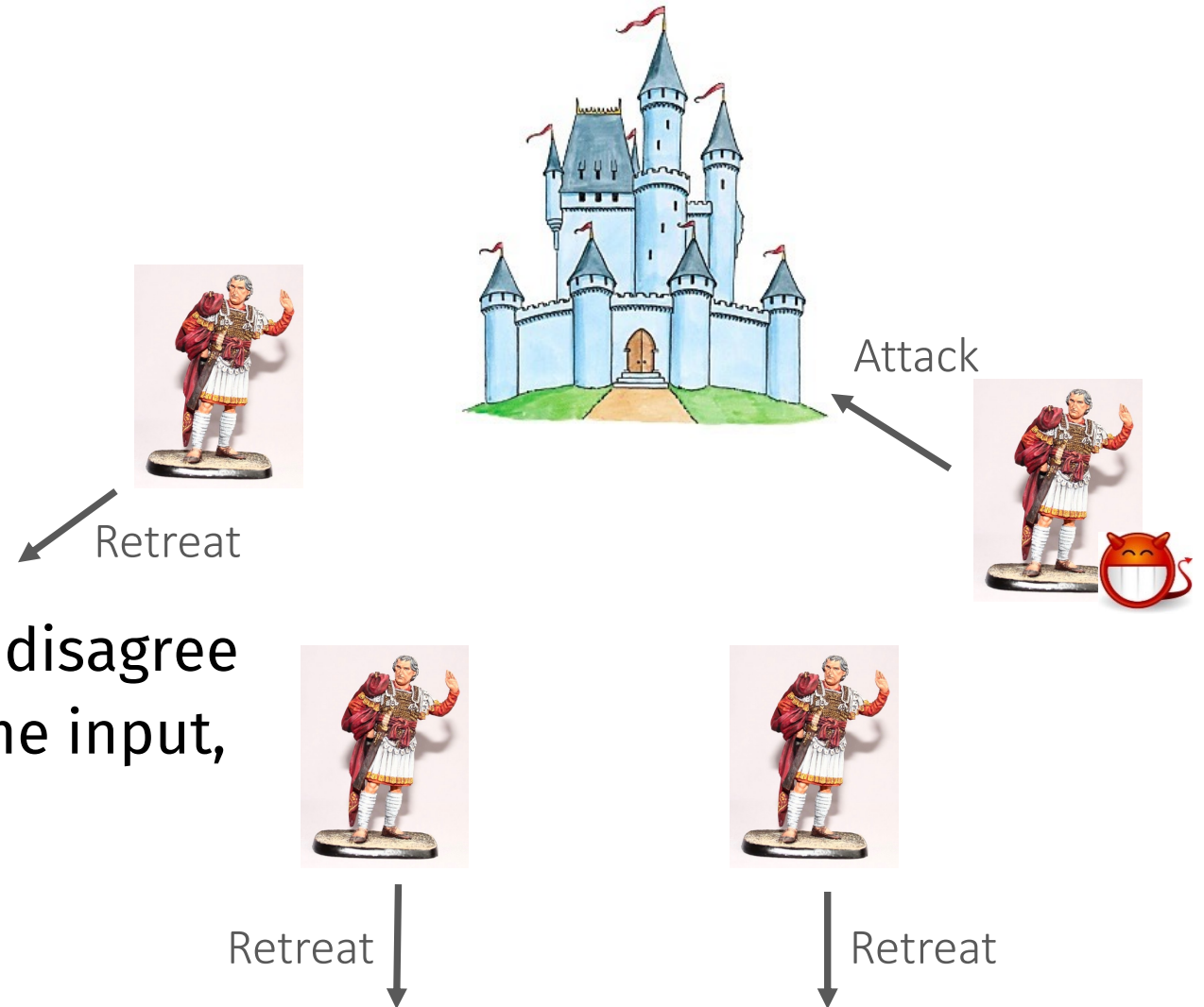


Byzantine Agreement

n generals ($\leq t$ Byzantine) need to agree on a battle plan

Requirements:

- Agreement: no two honest generals disagree
- Validity: if all generals start with same input, they commit that input
- Termination



Some Key Properties For BA Protocols

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1. Communication complexity
2. Security under adaptive adversaries

Goal: Can we achieve a BA protocol with “low communication complexity” while being secure under an adaptive adversary?

Bound on Communication Complexity [DR'82]

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Dolev-Reischuk bound: Any deterministic BA protocol needs honest parties to send $\Omega(t^2)$ messages

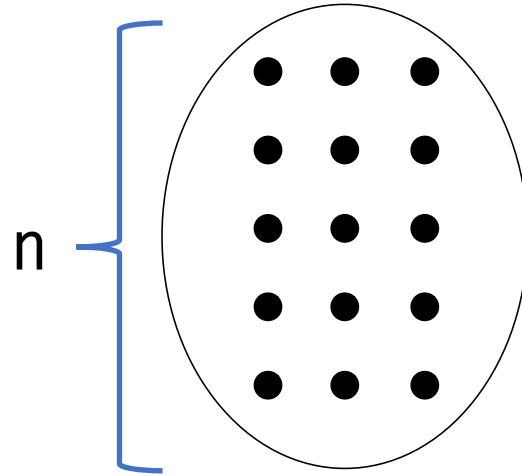
- Typically $t = O(n)$, so $\Omega(n^2)$ messages

Can we achieve BA with $o(n^2)$ messages?

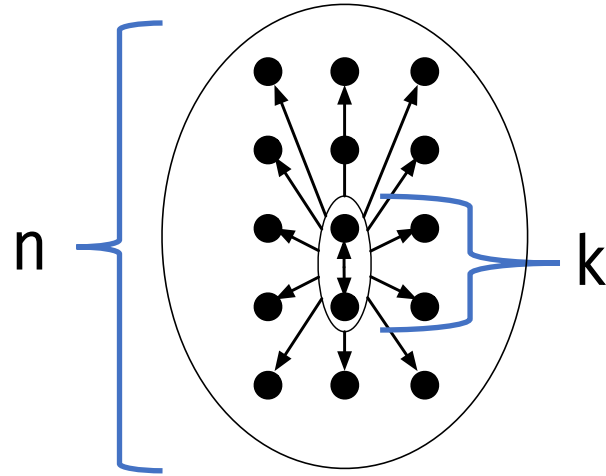
Yes, use randomization!

A Protocol with Sub-Quadratic Messages

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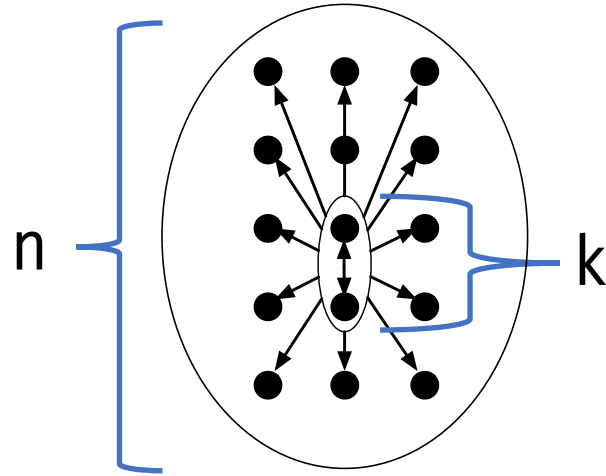


A Protocol with Sub-Quadratic Messages



A Protocol with Sub-Quadratic Messages

Idea: randomly elect a small committee of size k



Only the committee members send messages to all parties;
thus, communication = $O(\text{poly}(k).n)$

Tolerating Adaptive Adversary using Player-Replaceability [CM'16]

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Concern: an adaptive adversary can corrupt the committee

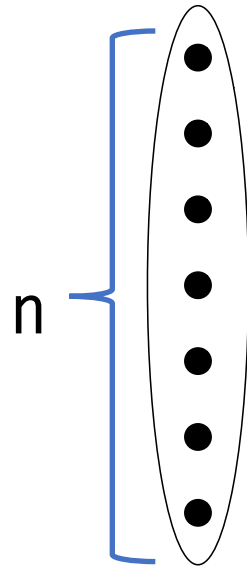
Solution: Player-replaceability, i.e., keep changing the committee after every round

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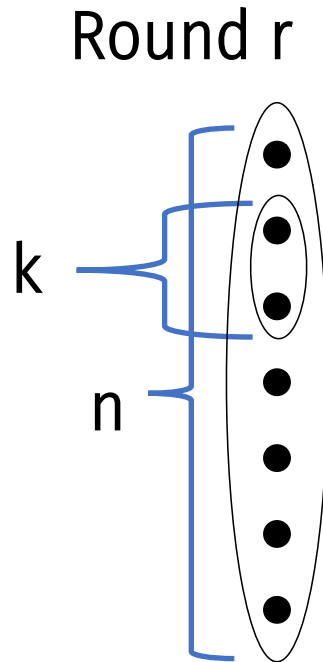
Round r



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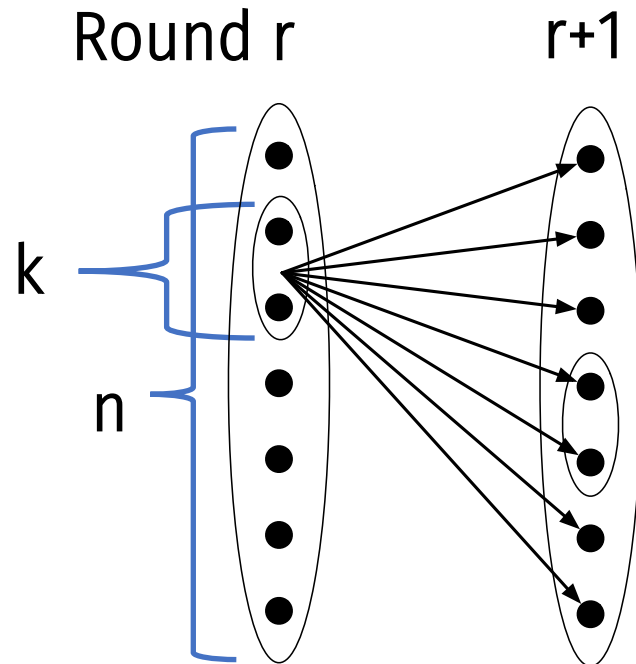
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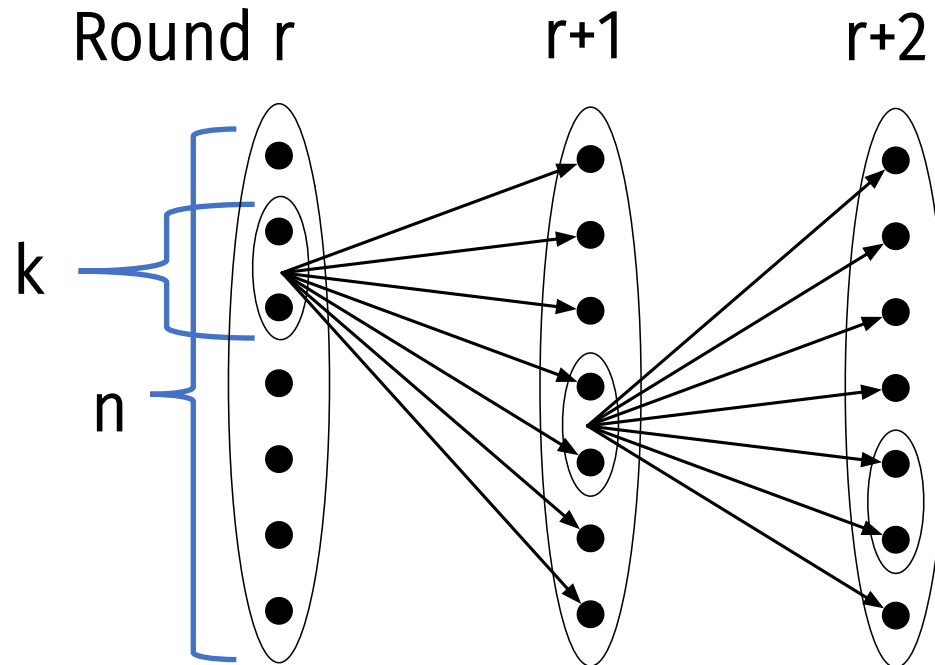
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Communication Complexity of BA

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Thus, we have a BA protocol with $O(\text{poly}(k).n)$ messages. Are we done?

If we have an l -bit value, communication complexity is $O(\text{poly}(k).nl)$ bits

What happens if l is large?

- e.g., $l = \Omega(n^2)$
- e.g., $l = 10 \text{ MB}$ sized block

BA Extension Protocols for l -bit Inputs [[NRSVX'20](#)]

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Intuition: Break down the problem into two steps

- Agree on a k -bit accumulator value corresponding to one of the inputs, requires $O(kn^2)$ communication
- Share the l -bit input using erasure coding techniques

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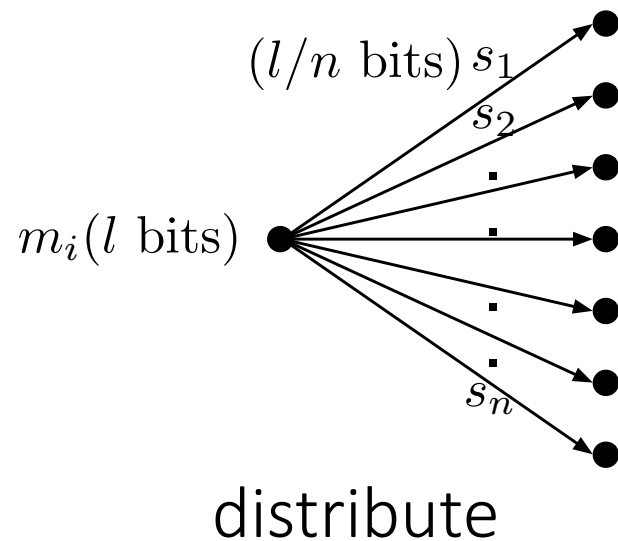
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$m_i(l \text{ bits})$ ●

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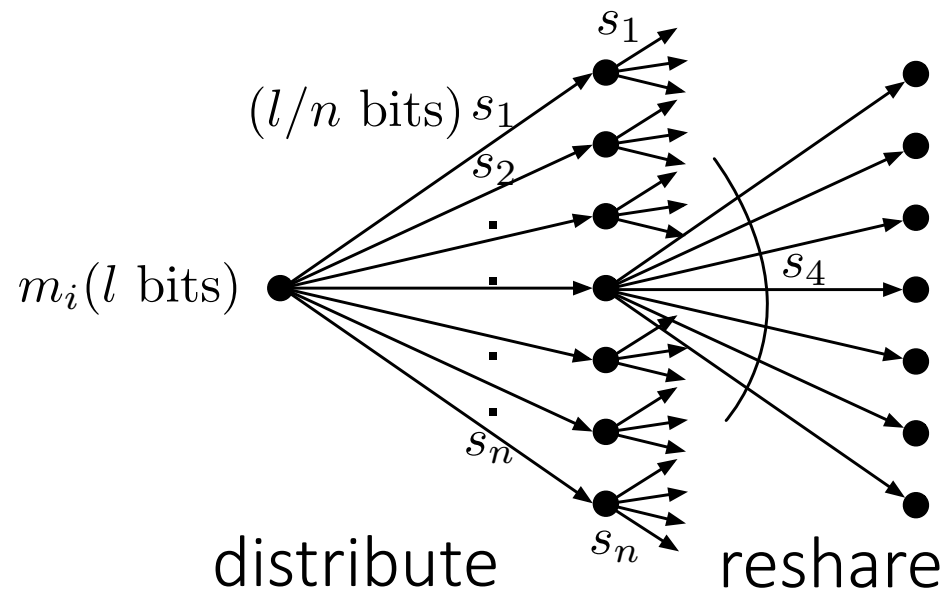
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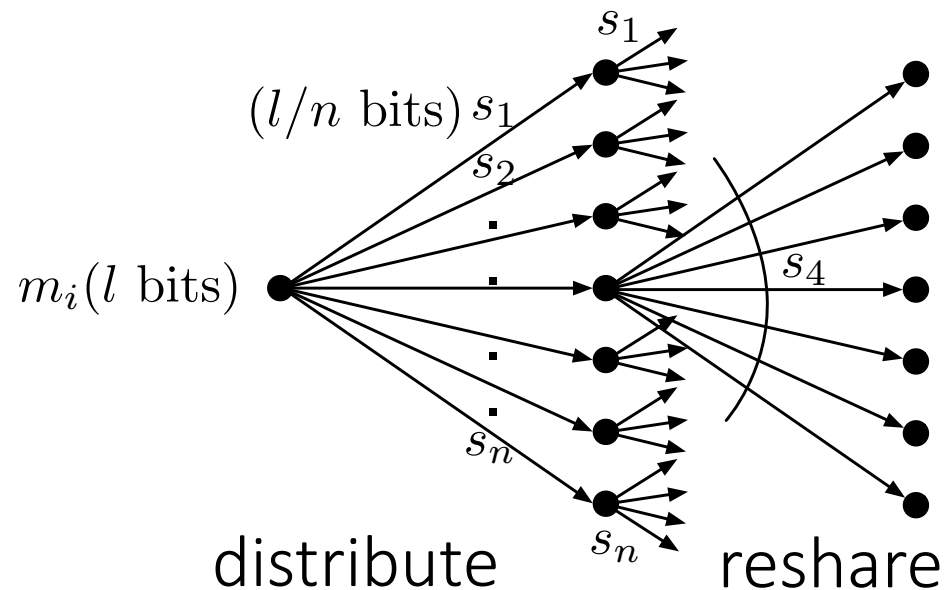
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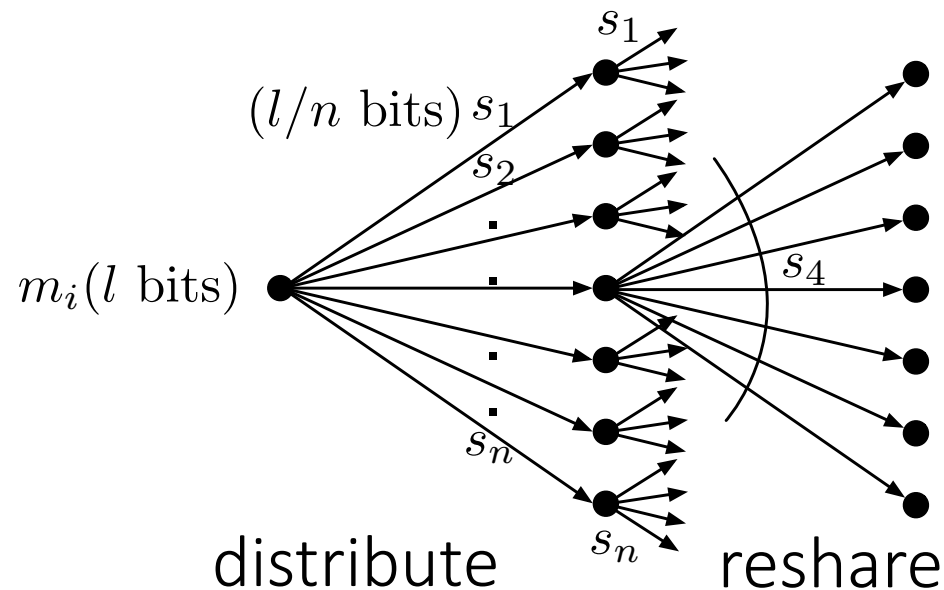
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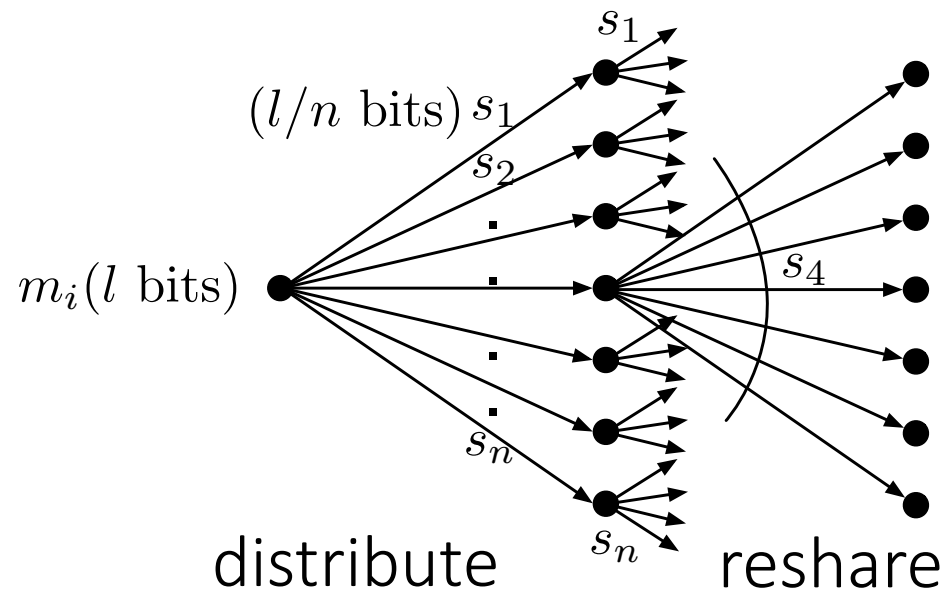
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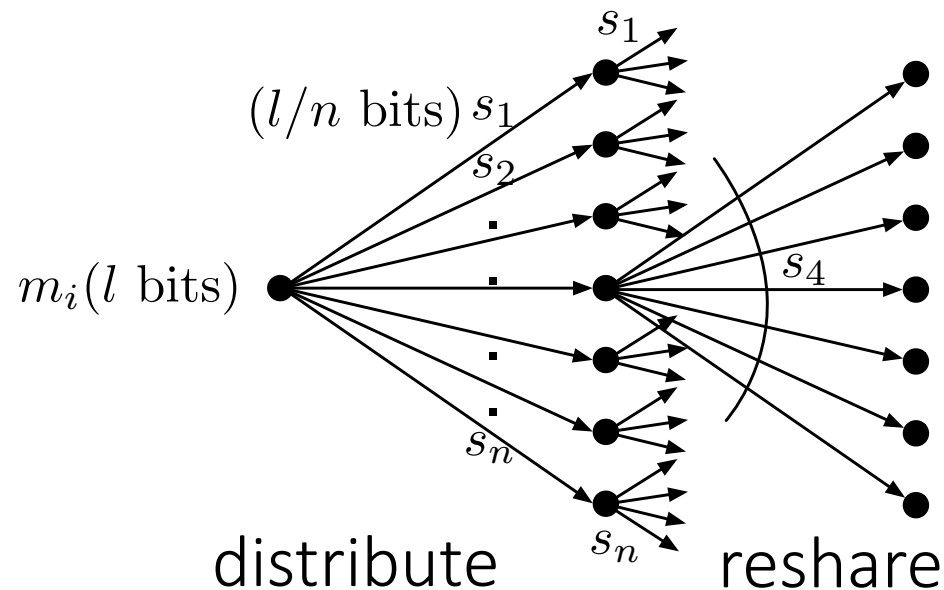
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Total communication: $O(\ln + kn^2)$ bits

State of the Art

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Sub-quadratic communication complexity against an adaptive adversary: $O(\text{poly}(k).nl)$ bits

- Not optimal when l is large

BA Extension protocol for long messages: $O(l + kn^2)$ bits

- Not optimal when $l < kn$

Can we get the best of both worlds? i.e.,

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Can we obtain a communication complexity of $O(\ln + \text{poly}(k).n)$ bits
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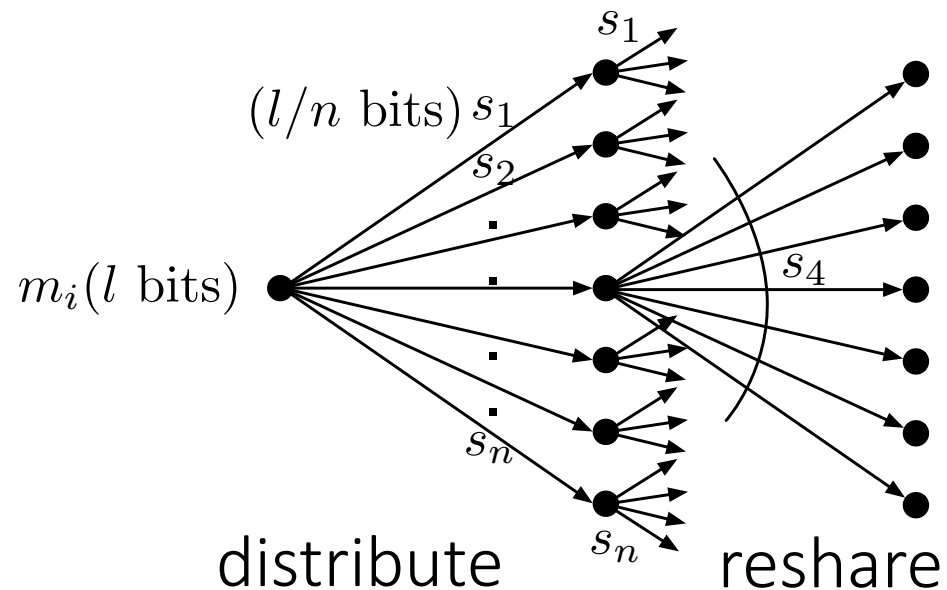
Intuition: Break down the problem into two steps

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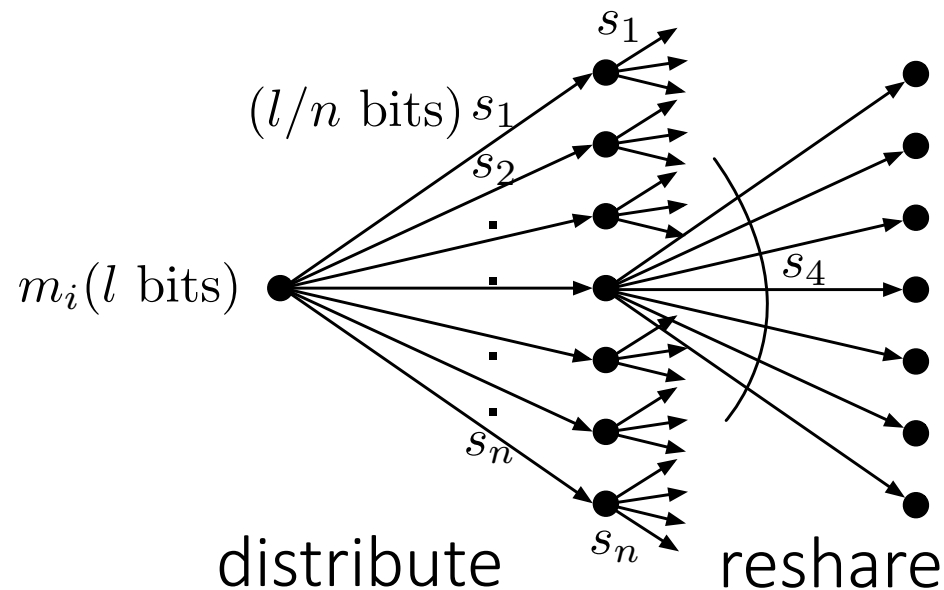
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Concern: Even if each party shares 1-bit value in the reshare phase, communication is $\Omega(n^2)$ bits

Attempt 2: Use Multiple k -sized Committees

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Requirement: Split the message into k shares and each share of the message should be shared by some honest party

Approach: Use an $O(k)$ -sized committee for resharing each share

Two drawbacks/challenges:

- (i) Communication complexity for resharing each share: $\Omega(nk \cdot (l/k))$; for k shares, it is $\Omega(nkl)$
- (ii) Adaptivity: How do we distribute these shares with k different committees?

Our Solution: Approach

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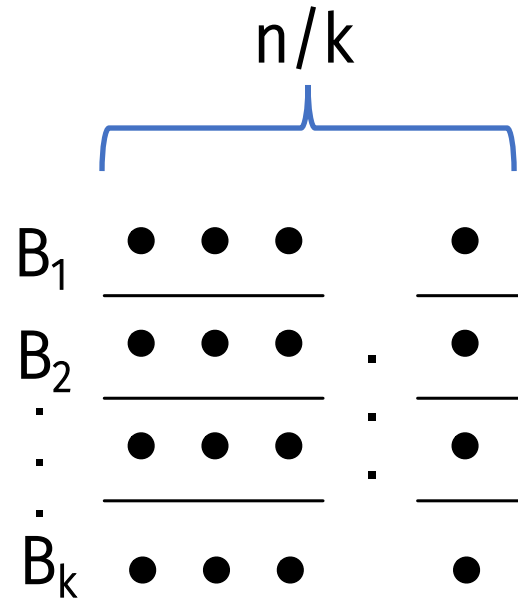
Approach:

- Publicly split the parties into k buckets of size n/k
- Distribute: Share i is shared with parties in bucket i
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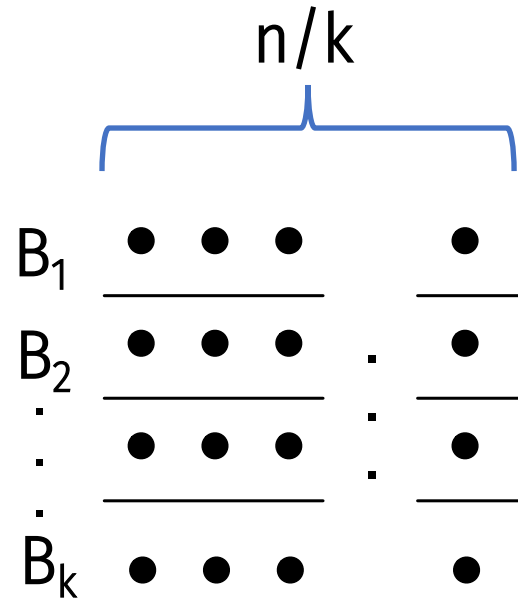
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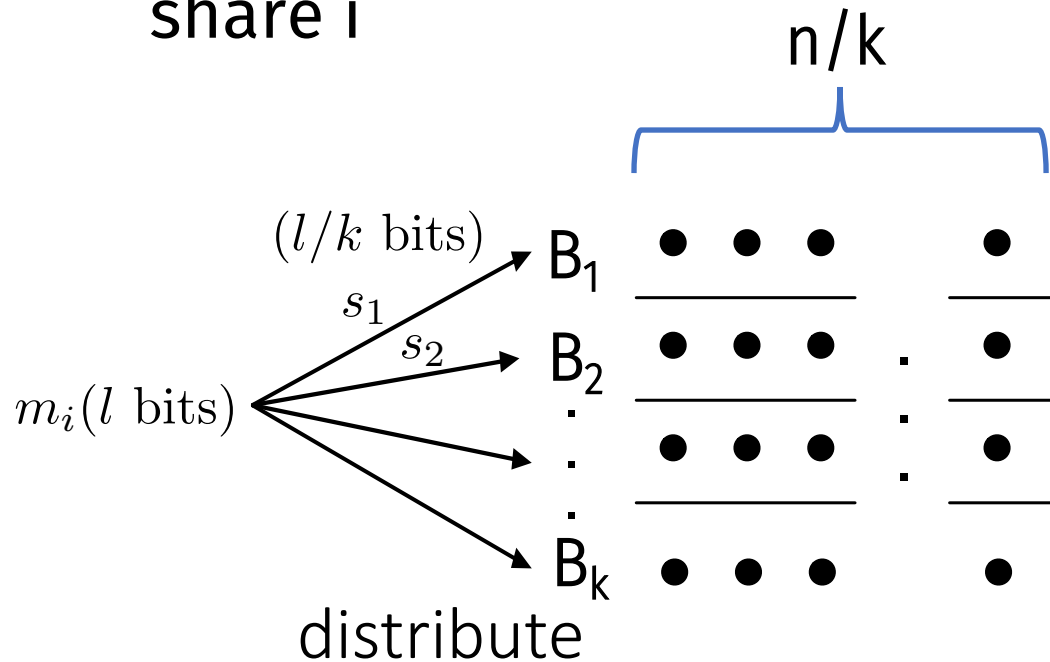
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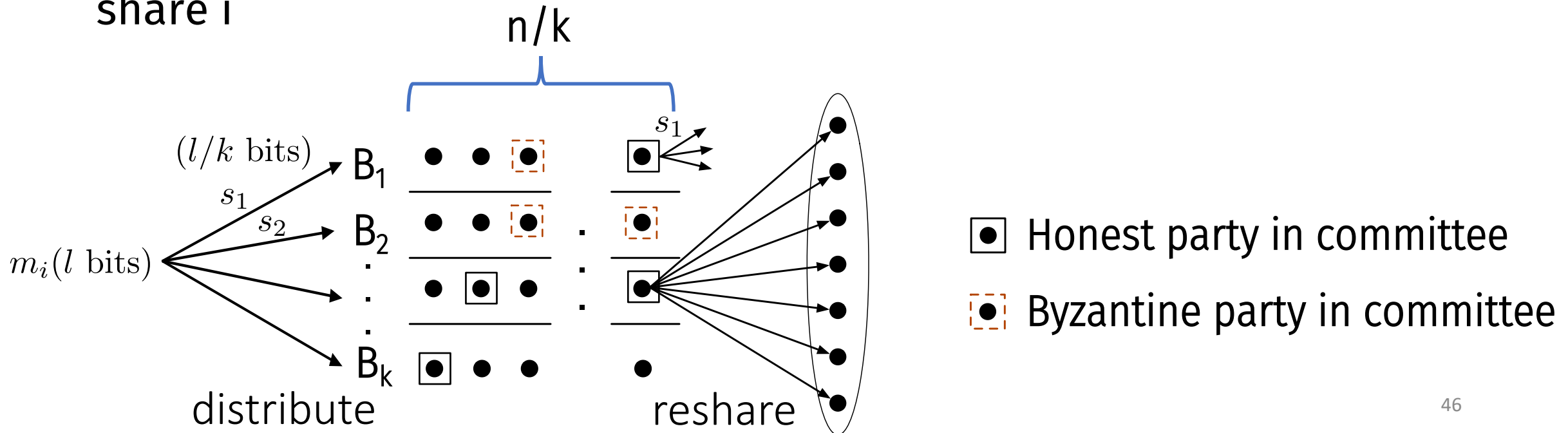
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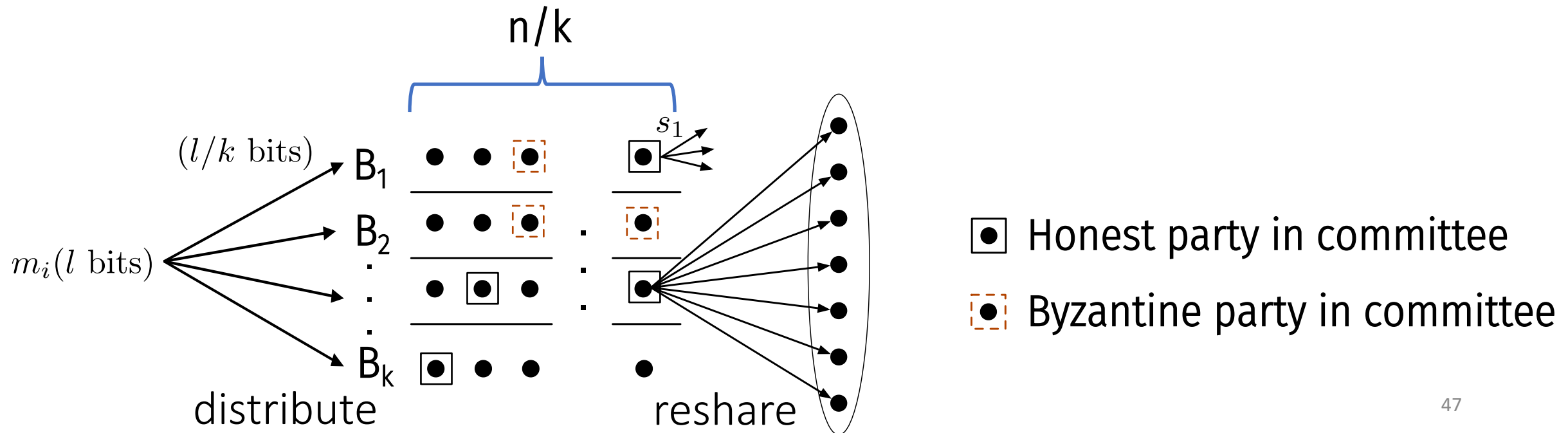
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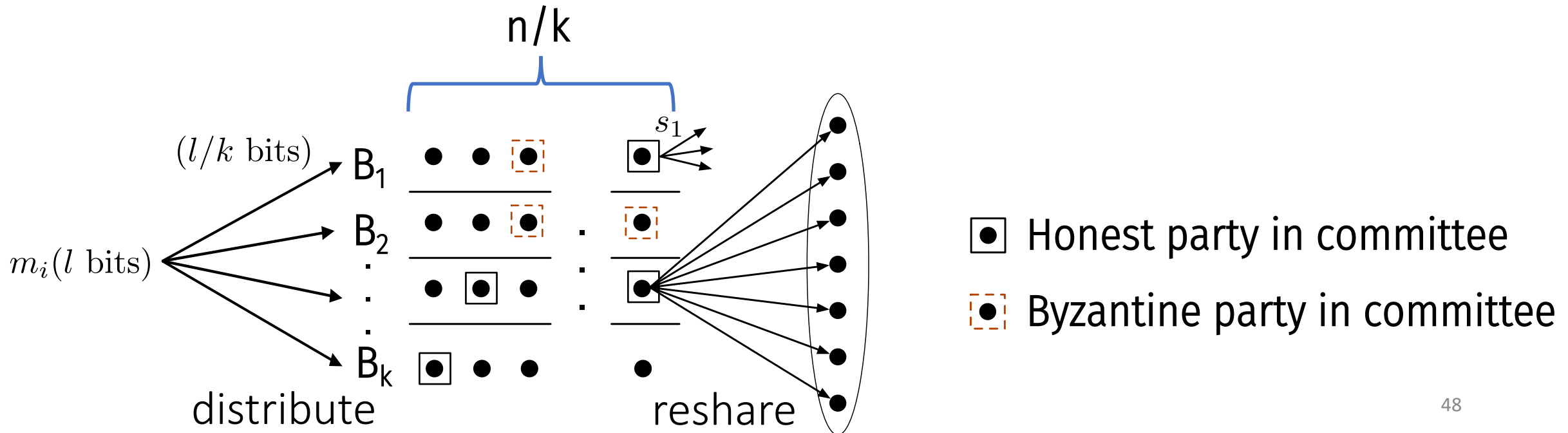


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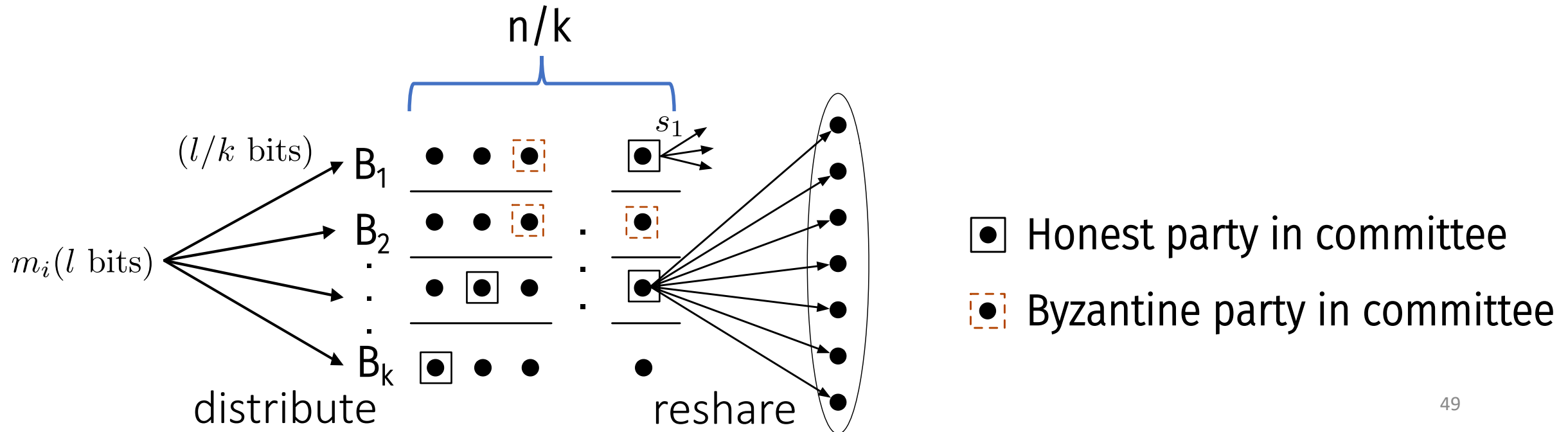


Our Solution: Communication Complexity

- Distribute: $O(k)$ parties sharing l/k -sized shares to n/k parties = $O(\ln/k)$ bits per share
- Reshare: $O(k)$ parties sharing l/k -sized shares to n parties = $O(\ln)$ bits

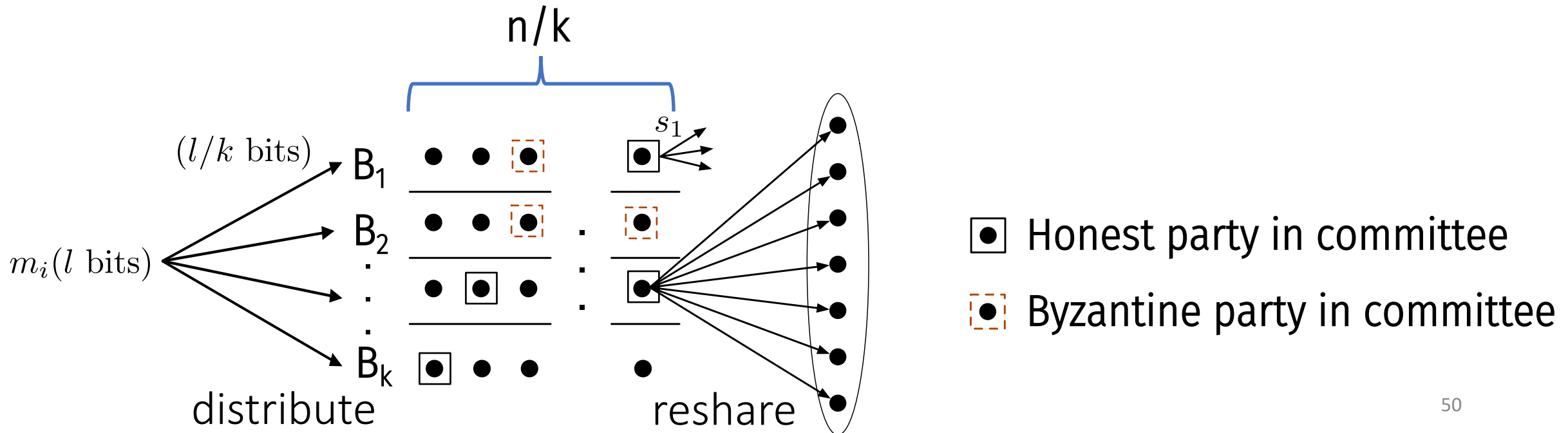


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- (i) Are enough shares reshared? Each bucket i has only $O(1)$ parties who can reshare share i
- (ii) Adaptivity: The adversary can adaptively corrupt parties in different public buckets so that not enough shares are reshared



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Our Result

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Theorem: For any $\varepsilon > 0$, assuming appropriate cryptographic assumptions, there exists an adaptively secure BA protocol achieving a communication complexity of $O(nl + \text{poly}(k).n)$ for l -bit inputs for

- (i) $t < (1 - \varepsilon) n/2$ Byzantine parties under a synchronous network,
- (ii) $t < (1 - \varepsilon) n/3$ Byzantine parties under an asynchronous network

Thank you!
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<https://eprint.iacr.org/2021/1403.pdf>