Additive-Homomorphic Functional Commitments and Applications to Homomorphic Signatures

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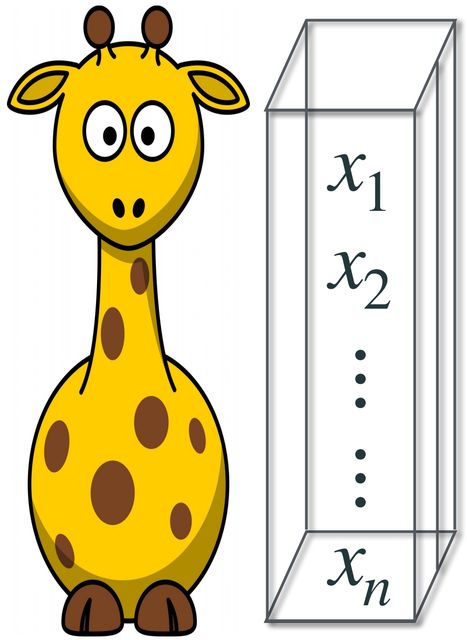
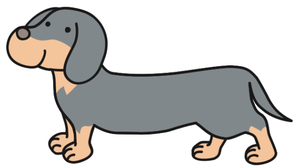
Functional Commitments

[Libert-Ramanna-Yung16]

- Generalization of vector commitments and polynomial commitments





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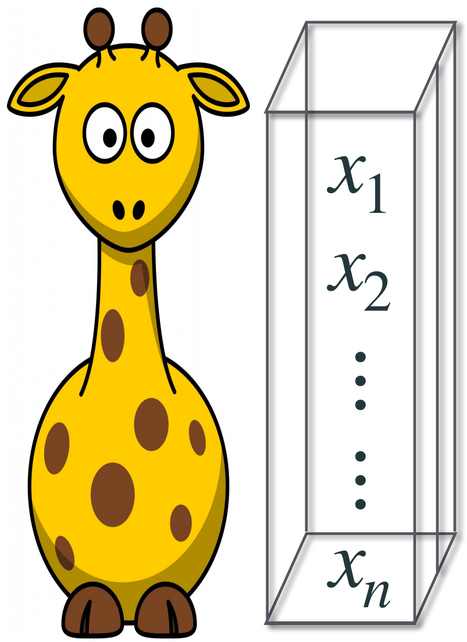
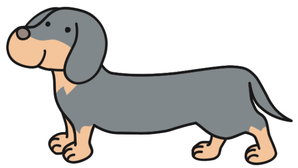


$\mathrm{Com}(\mathbf{x}) \rightarrow$ $C_{\mathbf{x}}$



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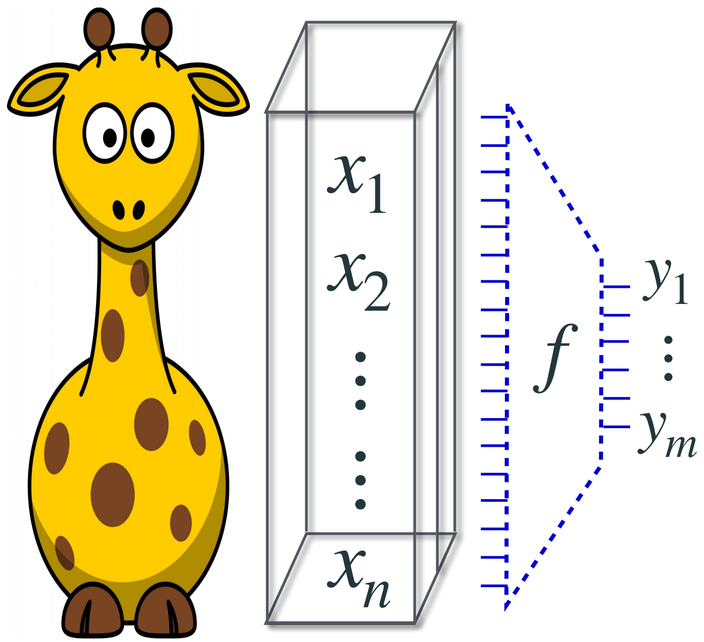

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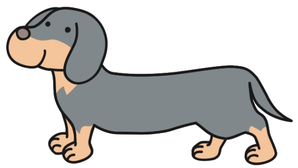

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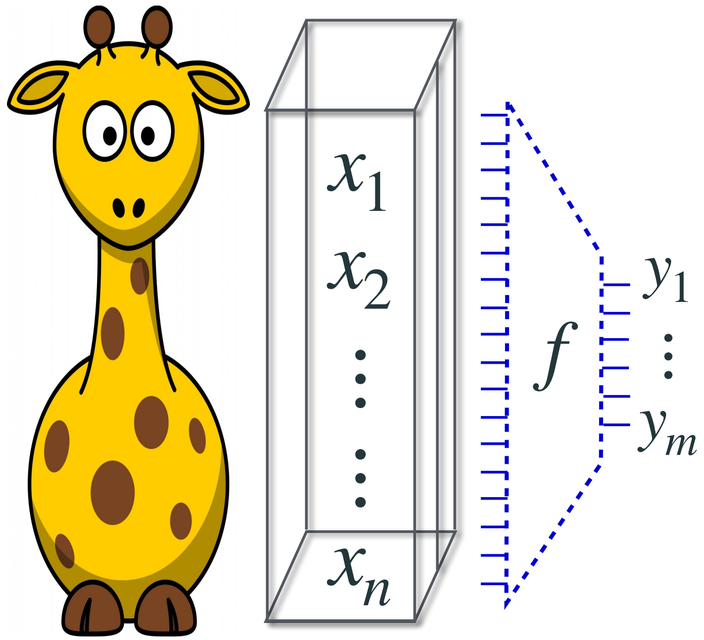

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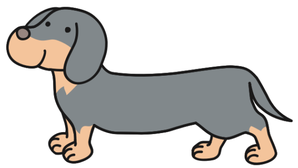

$\mathsf{Open}(f, \mathbf{x}) \rightarrow$ π_f $\mathbf{y} = f(\mathbf{x})$

$\mathsf{Ver}(\mathsf{C}_{\mathbf{x}}, f, \mathbf{y}, \pi_f) \stackrel{?}{=} 1$

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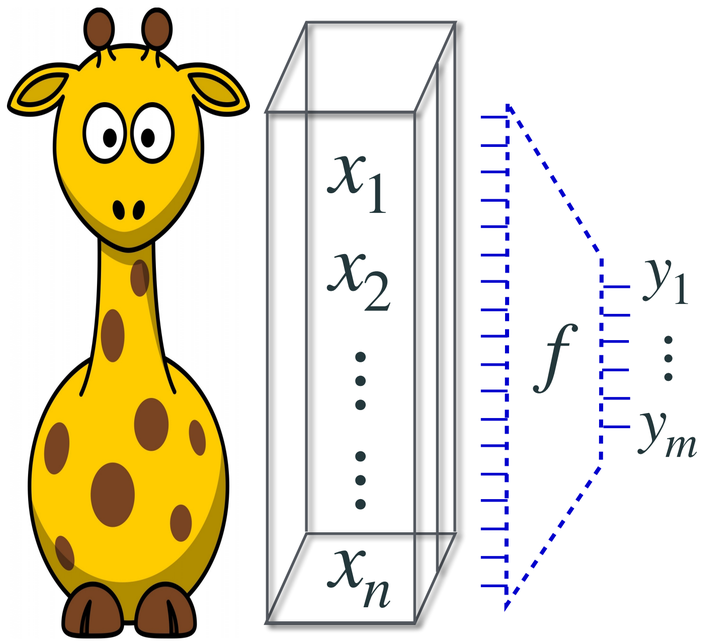

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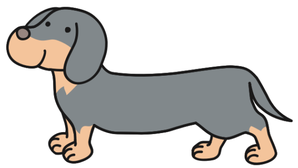

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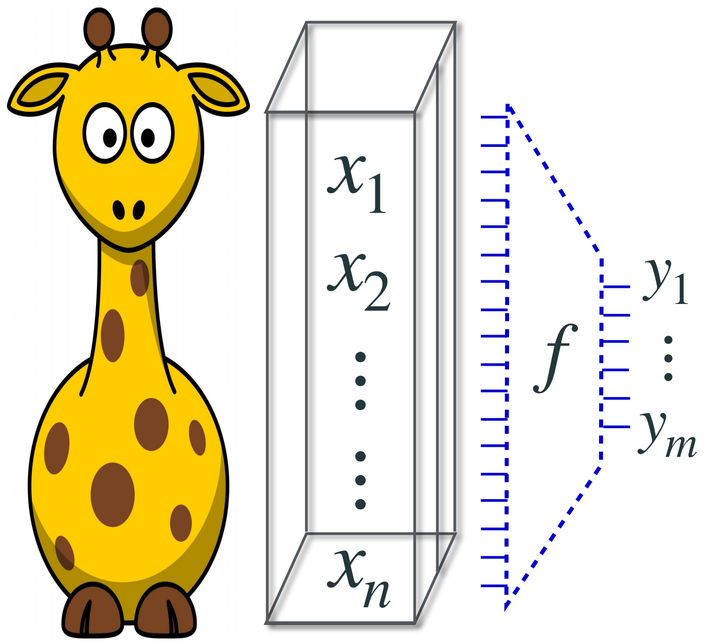

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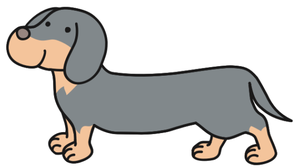

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- **Compactness** [LM19]: *openings "short" w.r.t.* $|\mathbf{x}|$ *and* $|\mathbf{y}|$ $|C_{\mathbf{x}}|, |\pi_f| \leq p(\lambda)$

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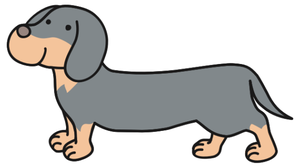
$C_{\mathbf{x}}$ $\quad f, \quad \pi_f, \mathbf{y}, \quad \pi'_f, \mathbf{y}'$



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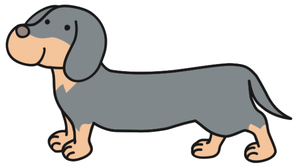

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- **FCs** can be seen as **weaker version of (commit-and-prove) SNARGs** (Eval. binding vs. Soundness)

  Potentially, a replacement for SNARGs in some applications

- **Other properties: hiding, zero-knowledge openings** (see paper, not focus in this talk)

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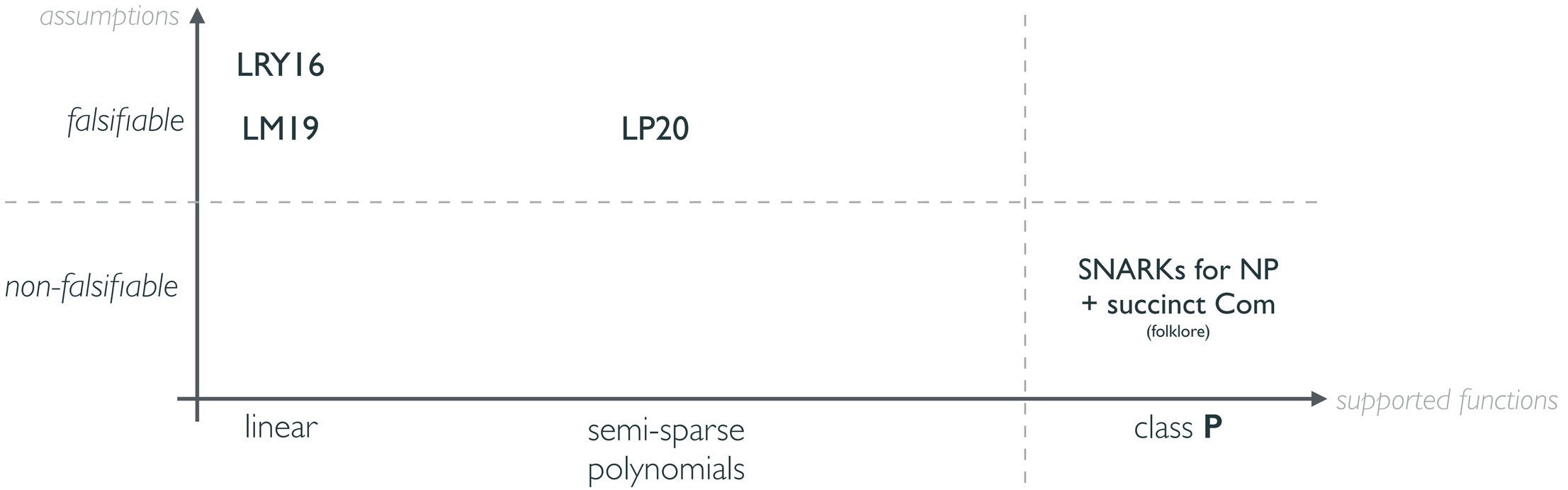

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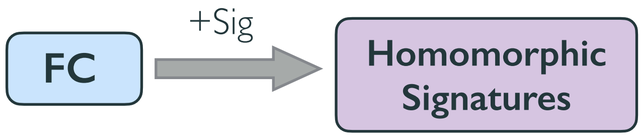

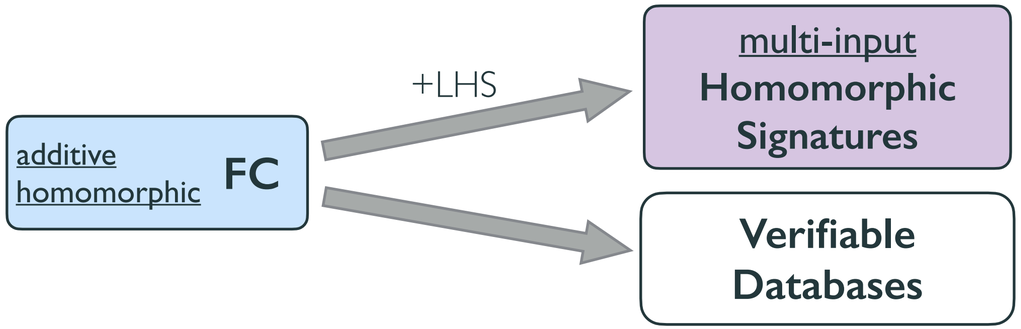

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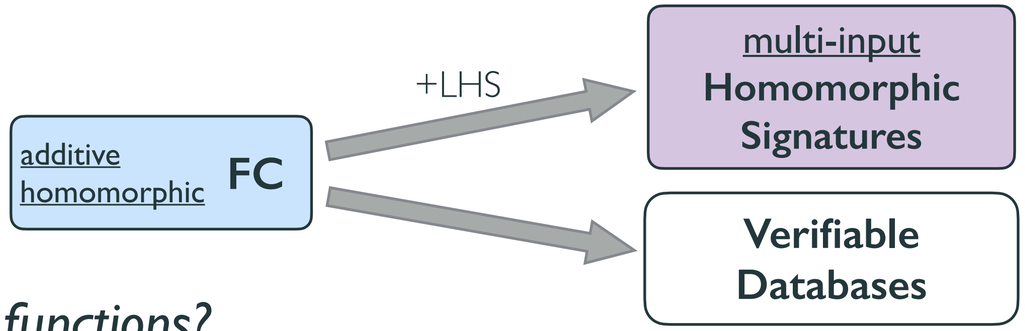
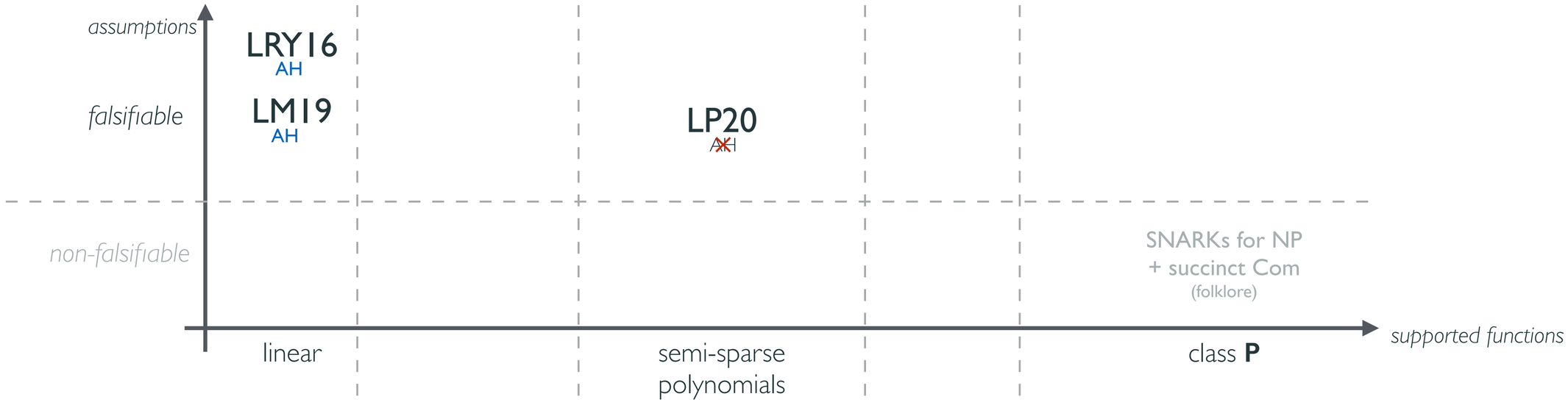
➡ **Motivating Result:** **FC** —+Sig→ **Homomorphic Signatures**



- *Can we construct additive-homomorphic FCs for expressive functions?*



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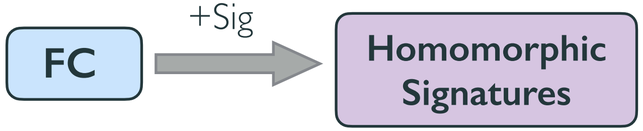

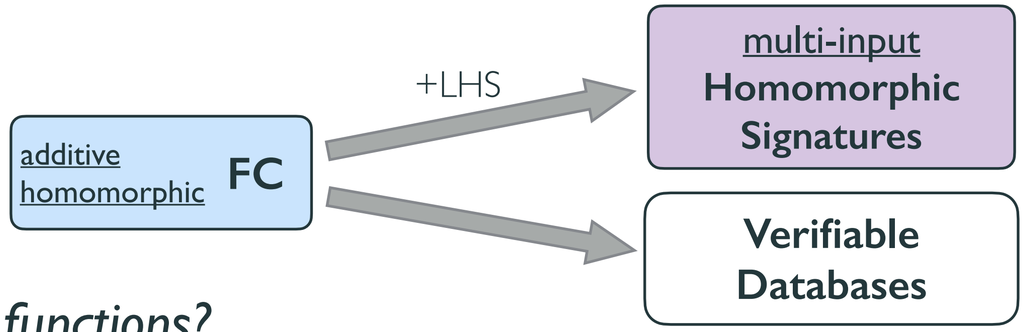

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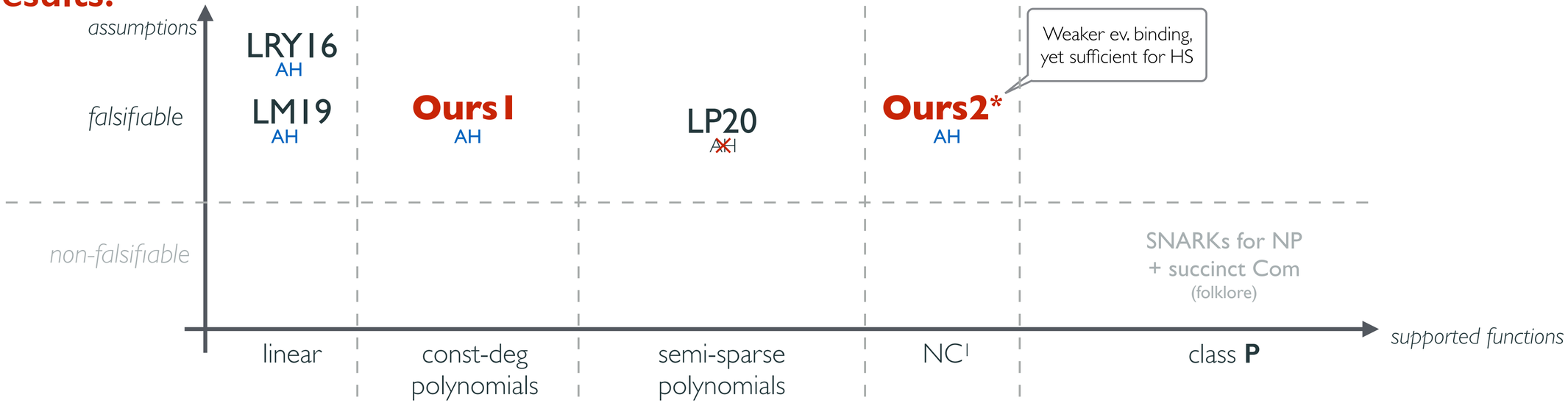

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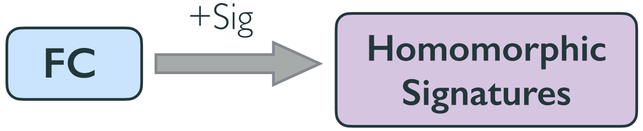

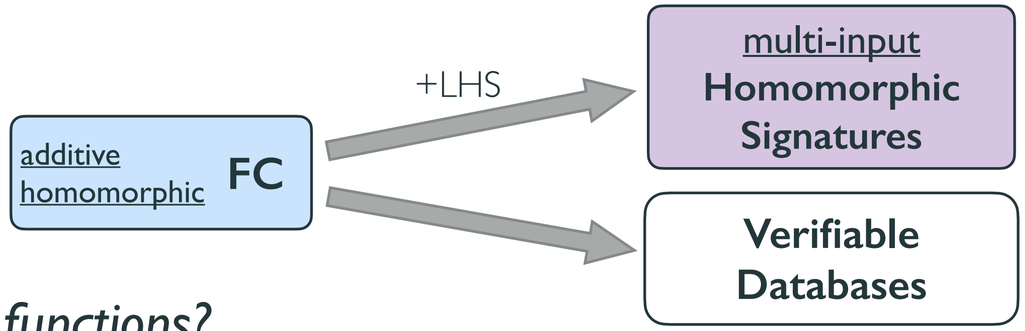

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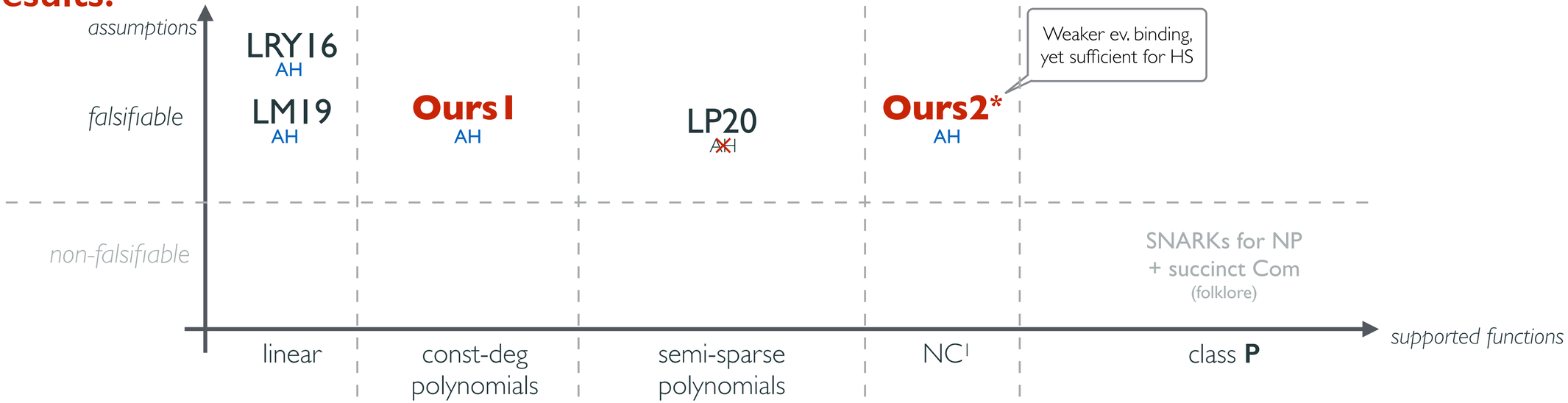
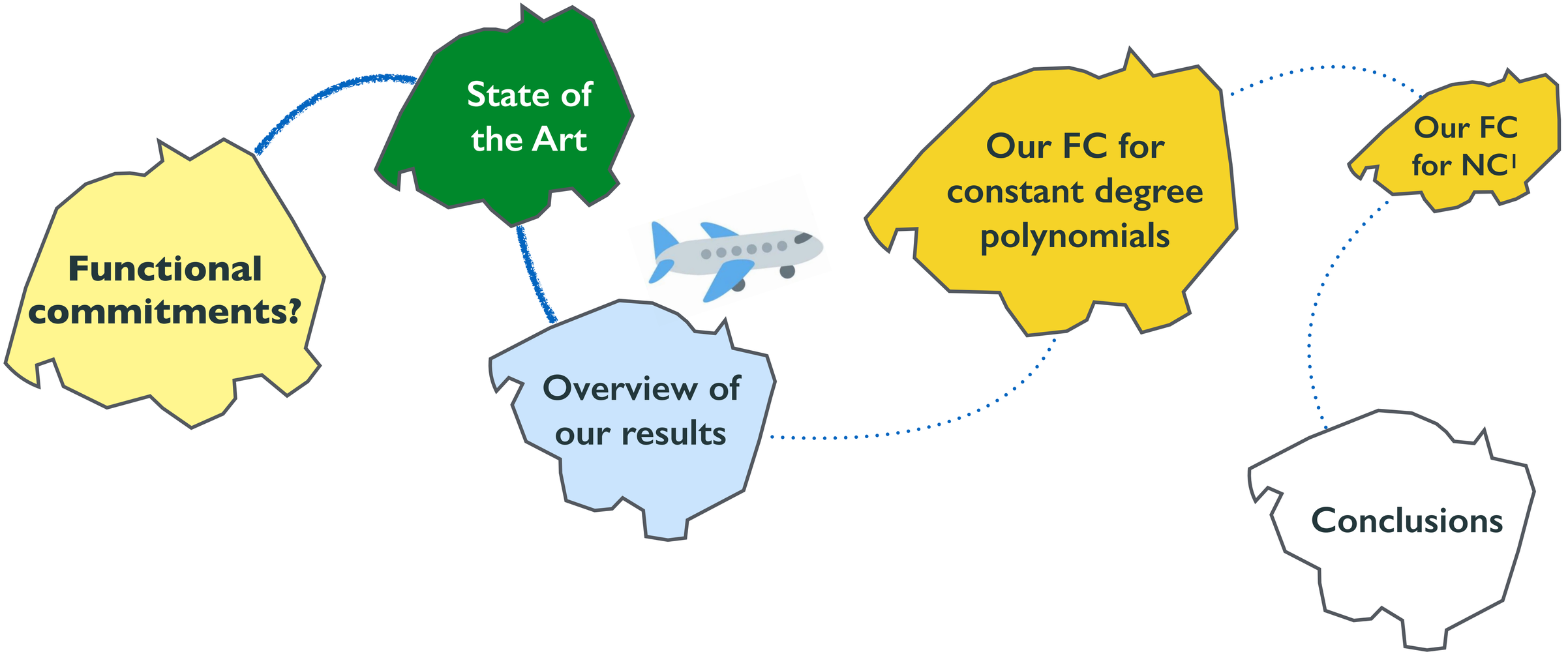


➡ **Implications:** new pairing-based homomorphic signatures

Roadmap of the talk



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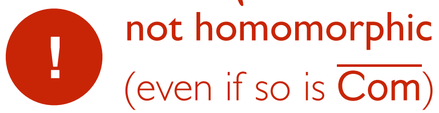

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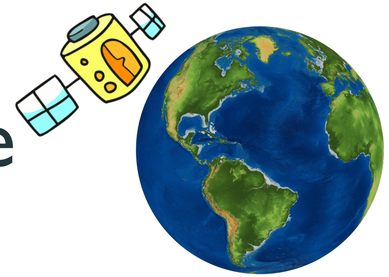
! not homomorphic (even if so is $\overline{\text{Com}}$) e.g., $\overline{\text{Com}}(\cdots, x_i \cdot x_j, x_i \cdot x_{j+1}, \cdots)$

Our FC for polynomials …seen from the space



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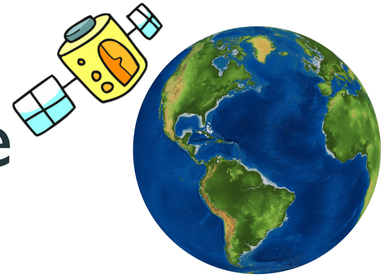

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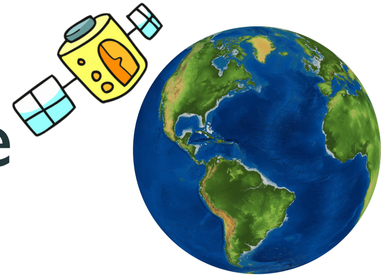

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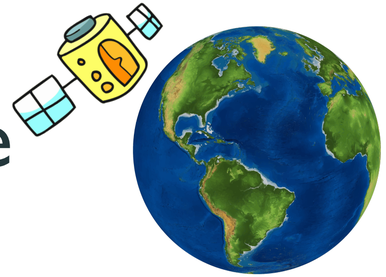

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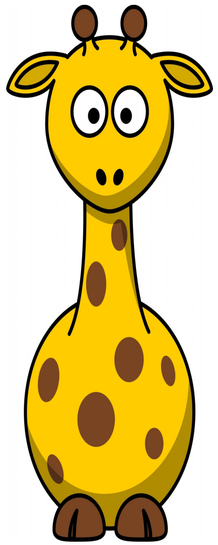
- **Our construction:** *we "open the box" of the FC for linear maps of* [Lai-Malavolta19]

Our FC for polynomials

…from 10Km

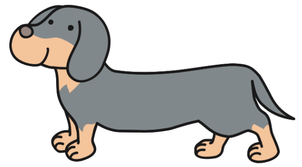
Let's start with f of degree 2

- Bilinear pairings $e : \mathbb{G}_1 \times \mathbb{G}_2 \to \mathbb{G}_T$, Notation: $[\alpha]_i = g_1^{\alpha} \in \mathbb{G}_i$



$\mathsf{Setup}(1^{\lambda}) \to \mathsf{ck} = [\alpha^1, \ldots, \alpha^{n^d}]_{1,2}$ …*and more elements*

Verification:



Our FC for polynomials …from 10Km

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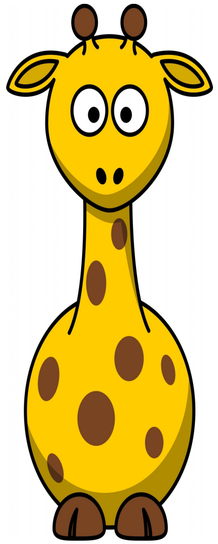

$\text{Com}(\mathbf{x}) \to \text{C}_{\mathbf{x}} =$

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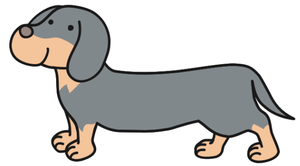

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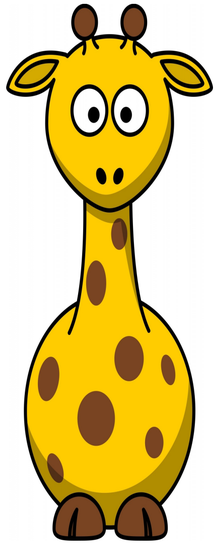

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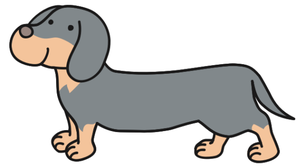

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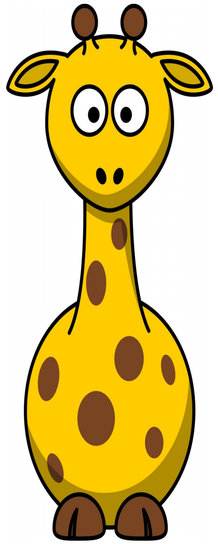

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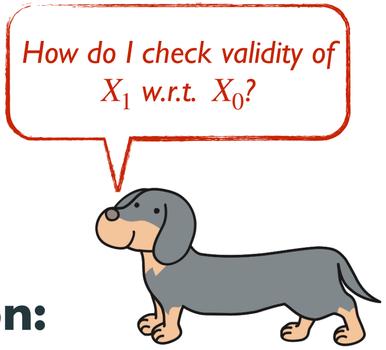

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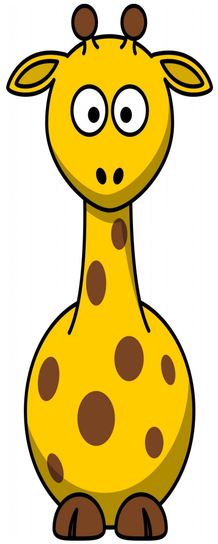

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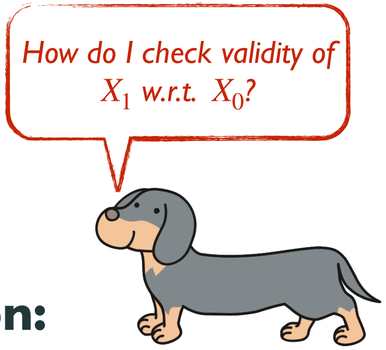

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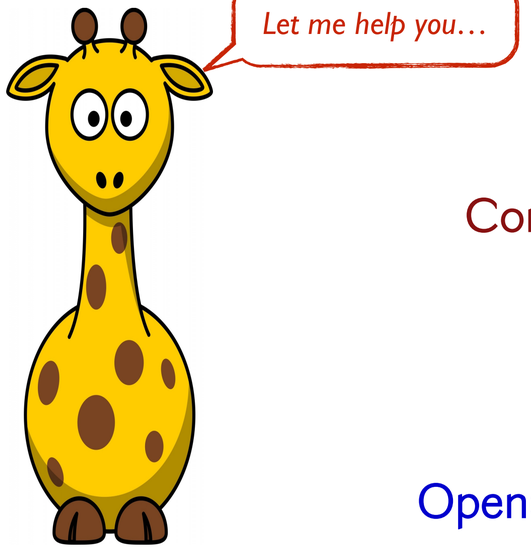

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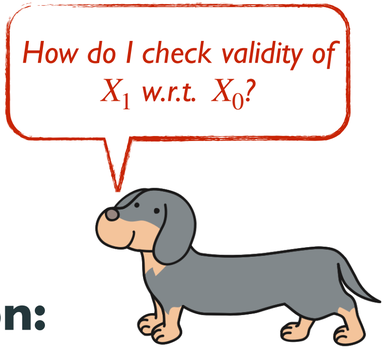

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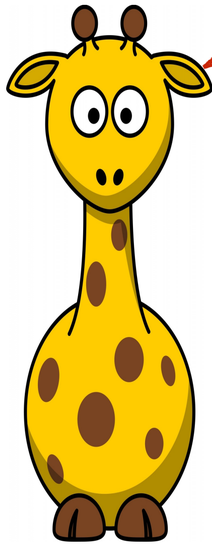

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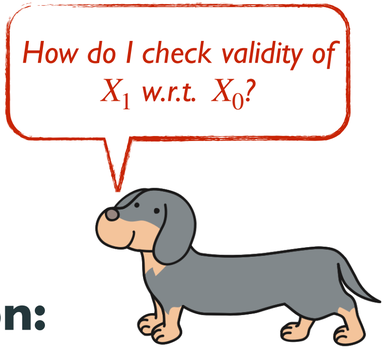

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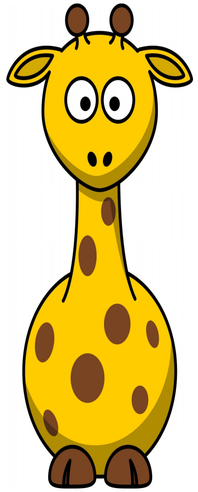
Our FC for polynomials ..almost down to earth



f of degree $d = 2^\delta$

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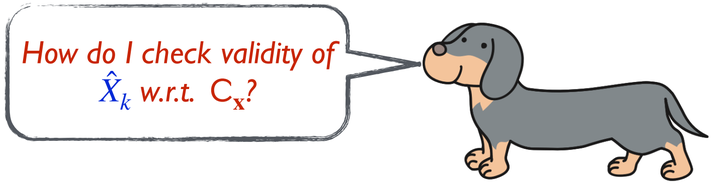

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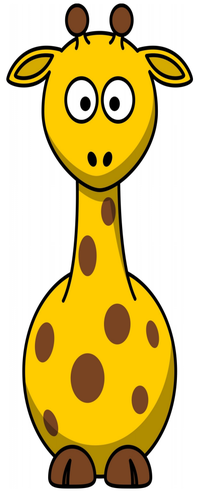

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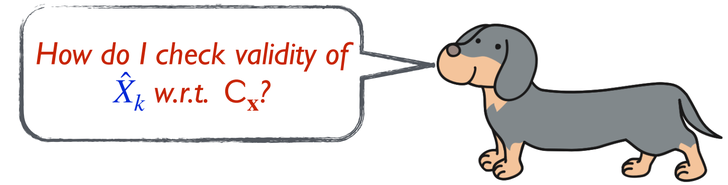

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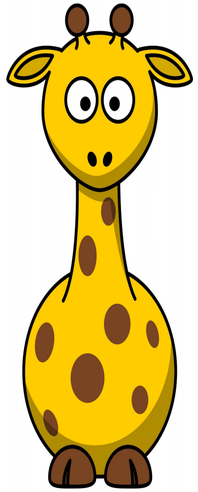

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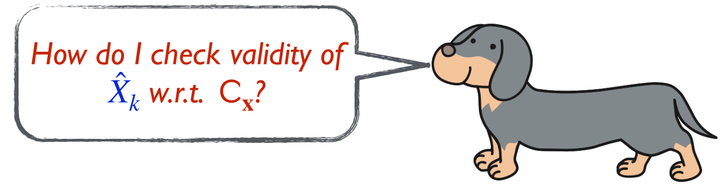

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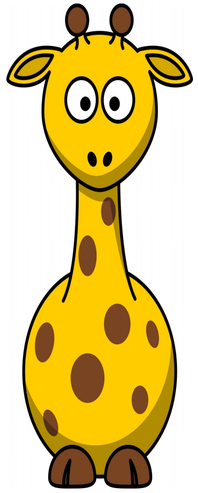

f of degree $d = 2^\delta$

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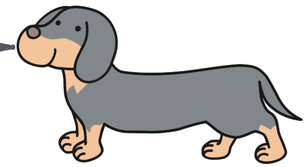
$$\left. \Phi_\ell = [\sum_{j=1}^{n} x_j \cdot \alpha^{n^\ell (j-1)})]_b = [\phi_{\mathbf{x}}^{(\ell)}(\alpha)]_b, l = 2, \ldots, d-1 \right]$$

How do I check validity of $\hat{X}_k$ *w.r.t.* $C_{\mathbf{x}}$*?*



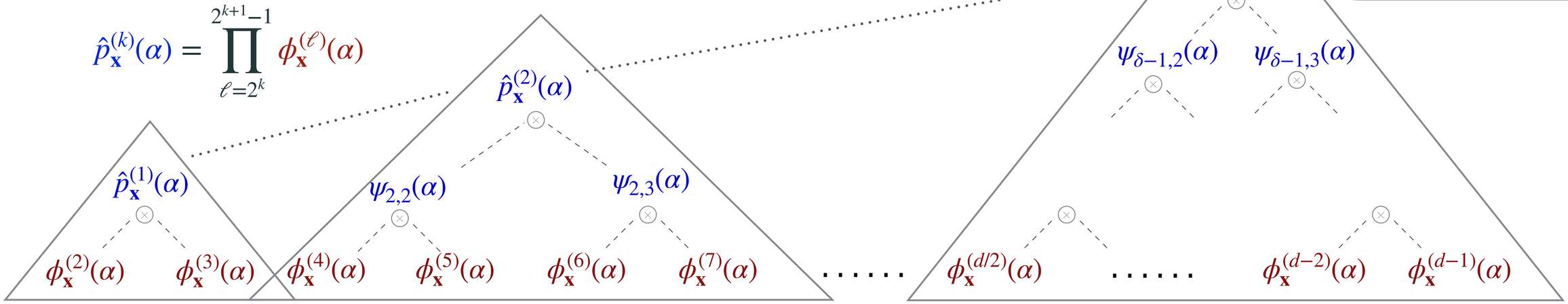
$$\text{Open}(f, \mathbf{x}) \to \pi_f = \left[X_\delta = [p_{\mathbf{x}}^{(\delta)}(\alpha)]_1 \quad \pi_{\hat{\mathbf{f}}} = \overline{\text{Open}}(\hat{\mathbf{f}}, \mathbf{x}^{(\delta)}) \text{ w.r.t. } X_\delta \right.$$
$$\left. \{ X_k = [p_{\mathbf{x}}^{(k)}(\alpha)]_1, \quad \hat{X}_k = [\hat{p}_{\mathbf{x}}^{(k)}(\alpha)]_2 \}_{k=1}^{\delta-1} \quad \{\Psi_{k,\mu} = [\psi_{k,\mu}(\alpha)]\}_{\mu=2^k}^{2^{k+1}-1} \right]$$

Main ideas

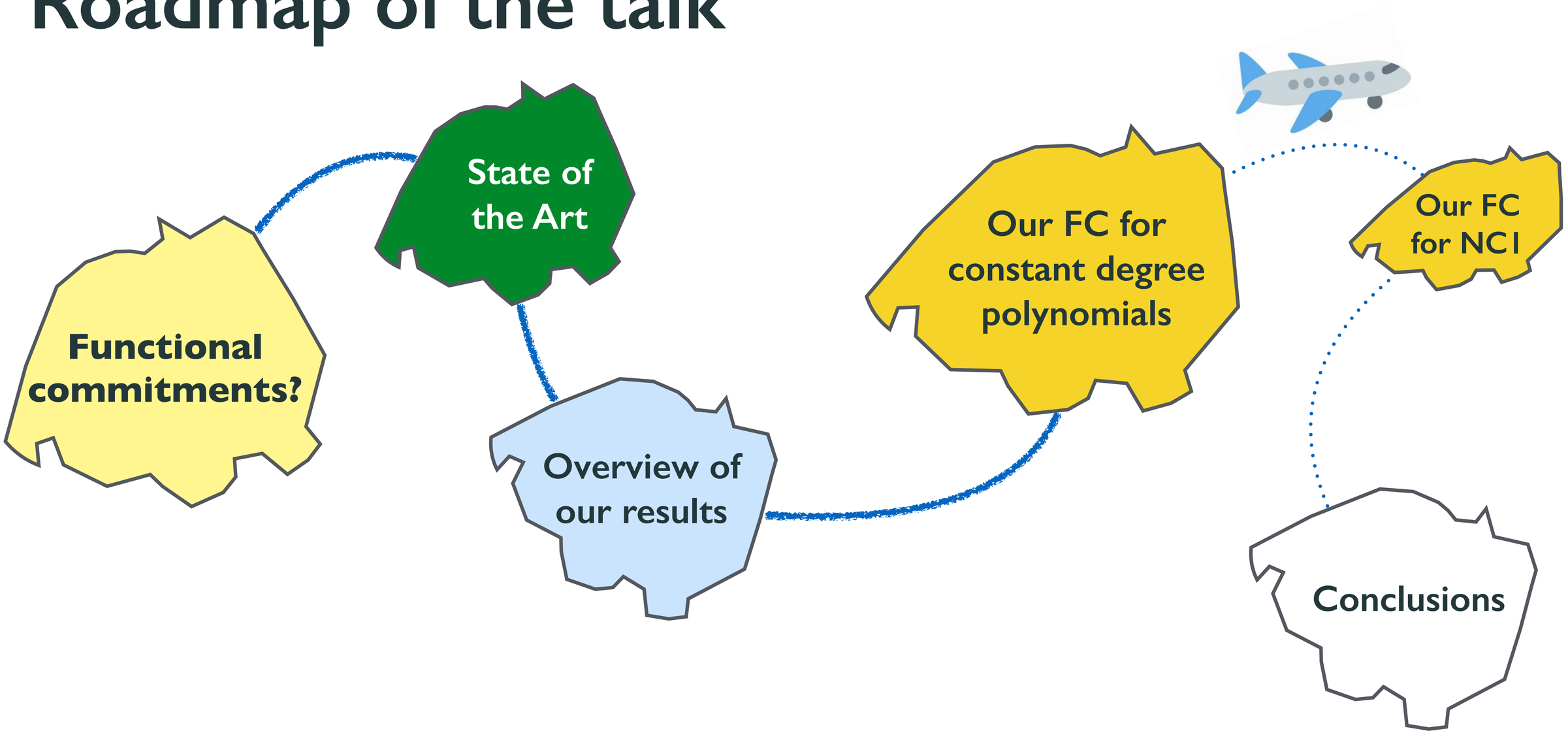
- *Prover gives all the internal nodes of the tree*
- *Verifier checks their consistency bottom-up via quadratic checks (using a pairing)*
  - *Cost* $O(2^\delta) = O(d) = O(1)$

Generalization: $\hat{p}_{\mathbf{x}}^{(k)}(\alpha)$ *factors in* 2^k *polynomials linear in* $\mathbf{x}$*:*

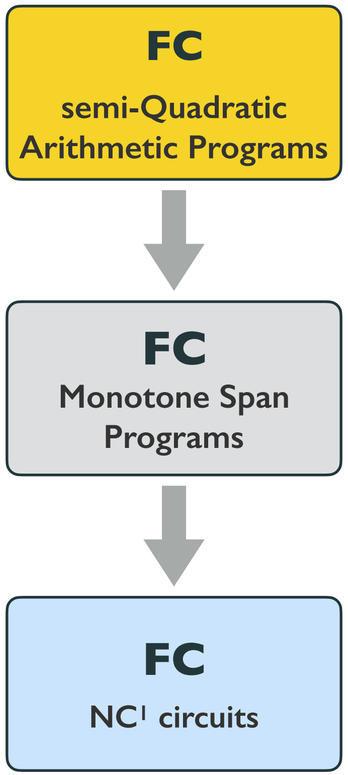
$$\hat{p}_{\mathbf{x}}^{(k)}(\alpha) = \prod_{\ell=2^k}^{2^{k+1}-1} \phi_{\mathbf{x}}^{(\ell)}(\alpha)$$



Roadmap of the talk



Our FC for NC^1



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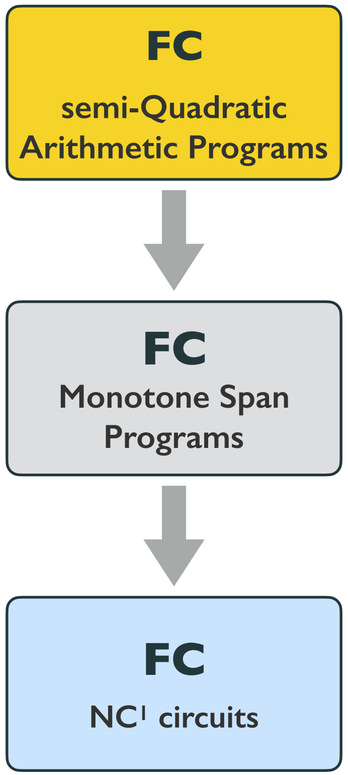

sQAP: semi-Quadratic Arithmetic Programs

$$f : \mathbb{F}^n \times \mathbb{F}^m \to \{\text{true}, \text{false}\} \text{ defined by matrix } \mathbf{F} \in \mathbb{F}^{n \times m}$$

$$f(\mathbf{z}, \mathbf{y}) = \text{true} \iff \exists \mathbf{w} : \mathbf{F} \cdot (\mathbf{w} \circ \mathbf{z}) = \mathbf{y}$$

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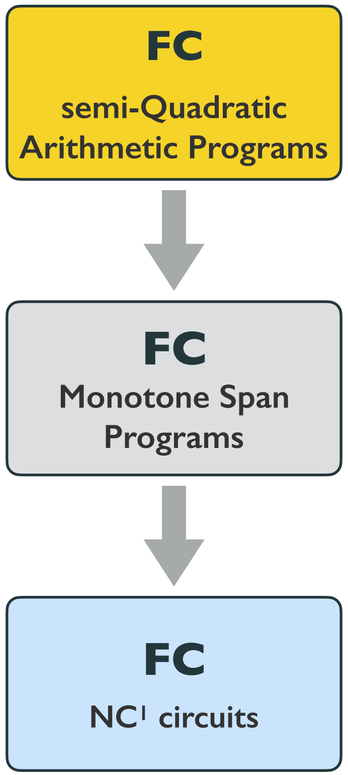

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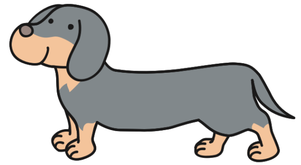
High-level ideas of our approach

(1) linearize the system of equations $\mathbf{F} \cdot (\mathbf{w} \circ \mathbf{z}) = \underbrace{\left(\mathbf{F} \circ \begin{bmatrix} \mathbf{z}^\top \\ \vdots \\ \mathbf{z}^\top \end{bmatrix} \right)}_{\mathbf{F}_\mathbf{z}} \cdot \mathbf{w} = \mathbf{y}$



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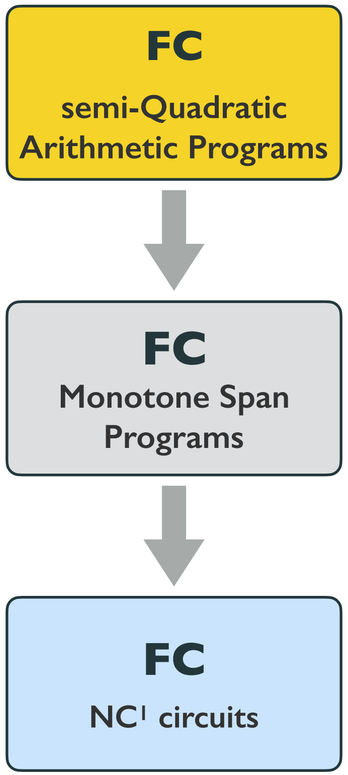

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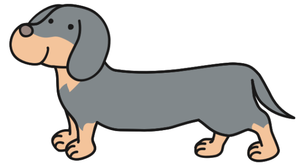

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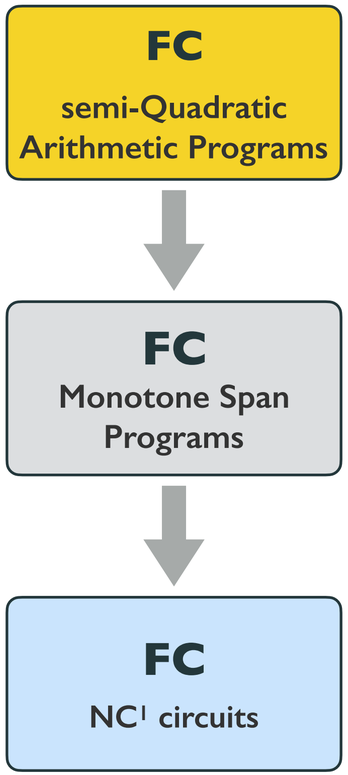
Our FC for NC^1



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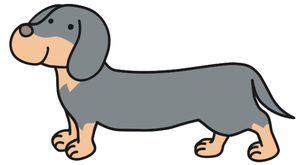

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(*) Prove *strong evaluation binding* of LM19 from a falsifiable assumption (LM19 proved it in the GGM); this implies a falsifiable SNARG for linear systems (of independent interest).

Summary of our results

FC scheme	Functions $f : \mathbb{Z}_q^n \to \mathbb{Z}_q^m$	|pp|	|C|	|π|	Add Hom	Assumption
[LM19]	linear maps	$O(mn)$	$1\ \|\mathbb{G}\|$	$1\ \|\mathbb{G}\|$	yes	n-DHE
[LP20]	semi-sparse polynomials	$O(\mu)$	$O(m)$	$O(1)$	no	
Ours (FC_{poly})	polynomials of deg d=O(1)	$O(mn^d)$	$d/2\|\mathbb{G}_1\|+d/2\|\mathbb{G}_2\|$	$d/2\|\mathbb{G}_1\|+d/2\|\mathbb{G}_2\|$	yes	n^d-DHE
Ours (FC_{sQAP})	sQAP	$O(mn^2)$	$2\ \|\mathbb{G}_1\|$	$2\|\mathbb{G}_1\|+ 1\|\mathbb{G}_2\|$	yes	(n,m)-DP-BDHE

Weak ev. binding, yet sufficient to build HS

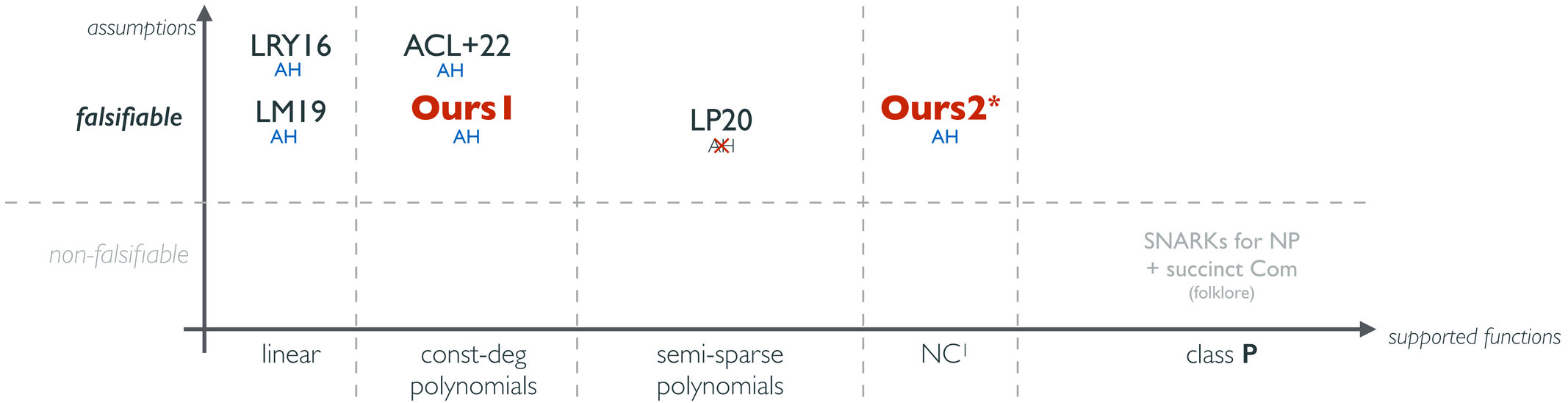
Homomorphic Signatures highlights:

- First HS with *compact* signatures (size constant in number of outputs)
- First *multi-input* HS for NC^1 based on pairings (prior HS for NC1 [KNYY19] only single-input)

Conclusions

Main results: First additive-homomorphic FCs for functions beyond linear

New pairing-based homomorphic signatures



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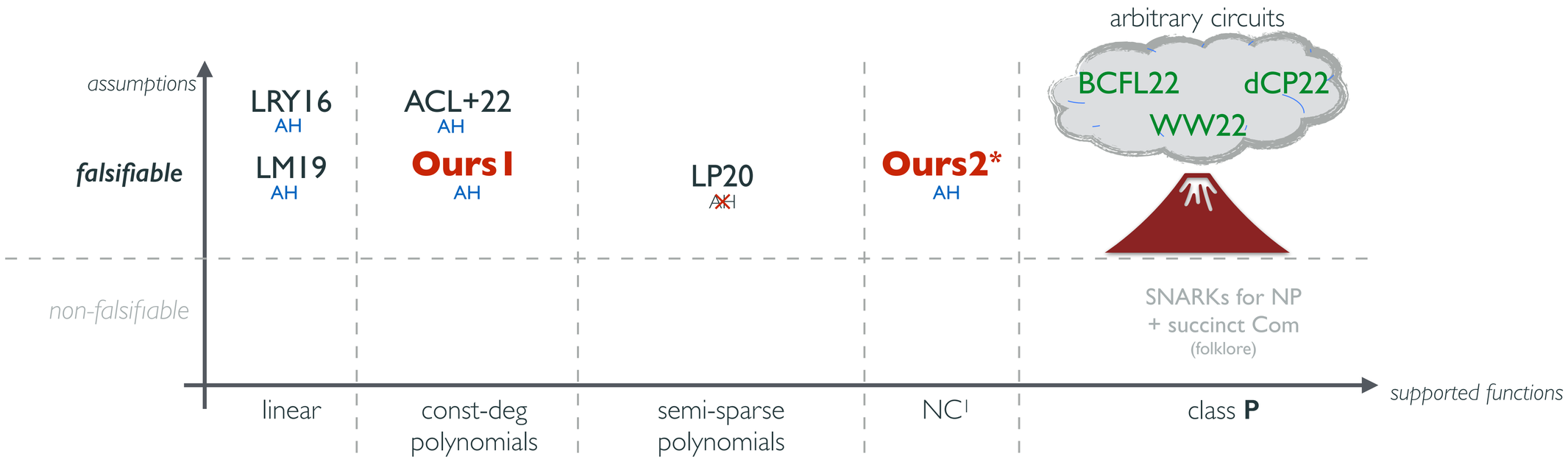

Conclusions

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Open problems: Compact&constant-size FC for NC^1? FC for circuits with shorter openings?

More applications?