Revisiting Related-Key Boomerang attacks on AES using computer-aided tool

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December, 2022

Updated results on AES-192

Key Size	Rounds	Time	Data	Memory	Type	Ref
192 bits	8/12	2^{172}	2^{107}	2^{96}	MITM	[Derbez et al., 2013]
	9/12	$2^{182:5}$	2^{117}	$2^{165:5}$		[Li et al., 2014]
	10/12	2^{183}	2^{124}	N/A	Related-key Rectangle	[Kim et al., 2007]
		2^{156}	2^{156}	2^{65}	Related-key Di erential	[Gerault et al., 2018]
	12/12	$2^{190:16}$	2^{80}	2^{8}	Biclique	[Bogdanov et al., 2011]
		$2^{190:83}$	2	2^{60}		[Bogdanov et al., 2014]
		$2^{189:76}$	2^{48}	2^{60}		[Tao and Wu, 2015]
		2^{176}	2^{123}	2^{152}	Related-key Boomerang	[Biryukov and Khovratovich, 2009]

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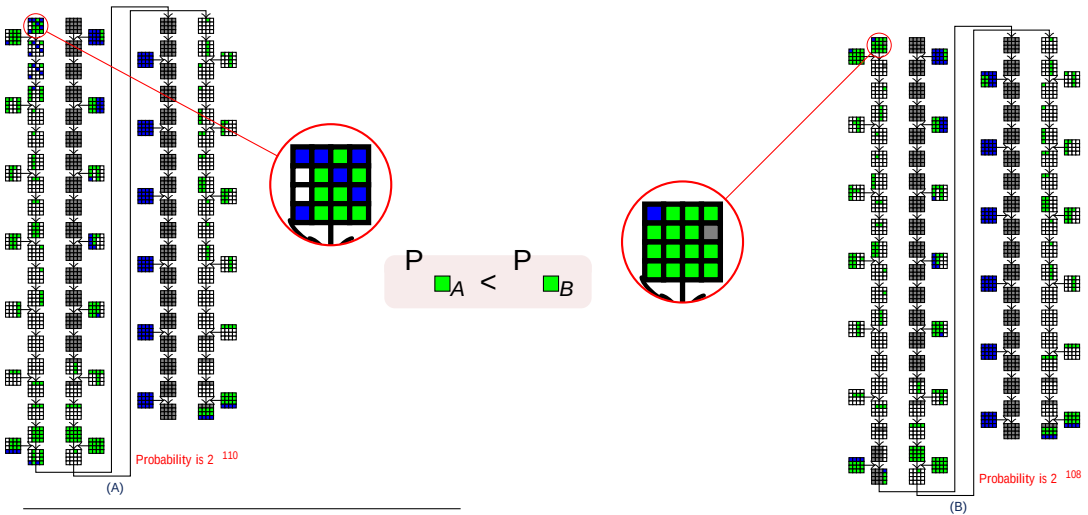
		2^{124}	2^{124}	$2^{79:8}$	Related-key Boomerang	This work

Its time complexity is 2^{52} times lower than the best-known attack!

The best-known attack (A) vs Our attack (B)



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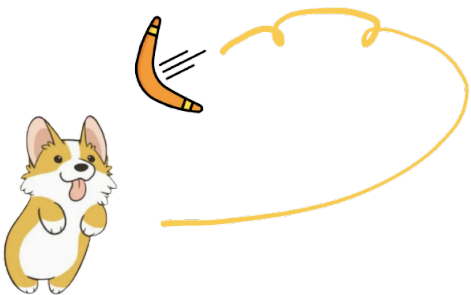
Overview

1. **The boomerang attack**
2. **Previous works**
3. **Application to AES**
4. **Results**

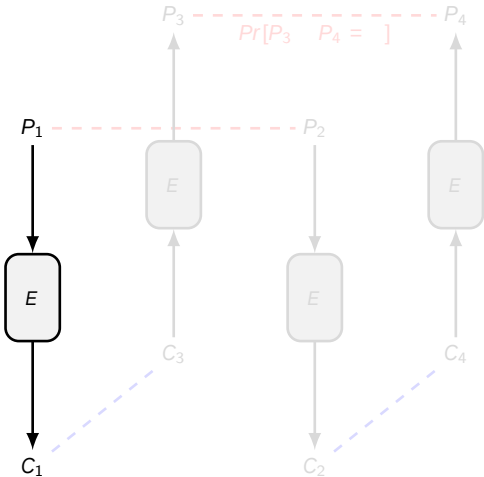
Boomerang Distinguisher 101



The Boomerang attack [Wagner, 1999]

- *When you send it properly, it always comes back to you*

Boomerang Distinguisher 101



1. Pick P_1, ask for $C_1 = E(P_1)$
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3. $C_3 = C_1$, $C_4 = C_2$
4. Ask for $P_3 = E^{-1}(C_3)$, $P_4 = E^{-1}(C_4)$
5. Check if $P_3 \quad P_4 =$

A distinguisher if comes back more often than a *random permutation*!

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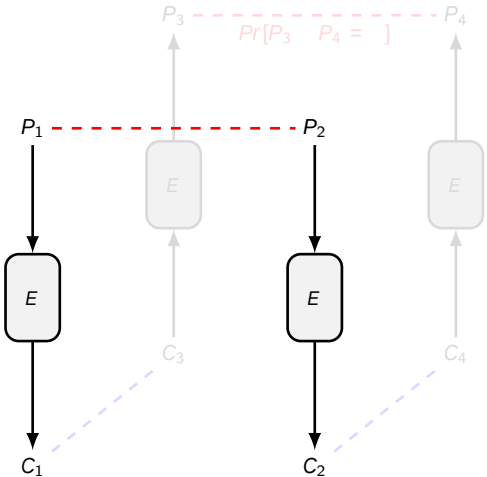

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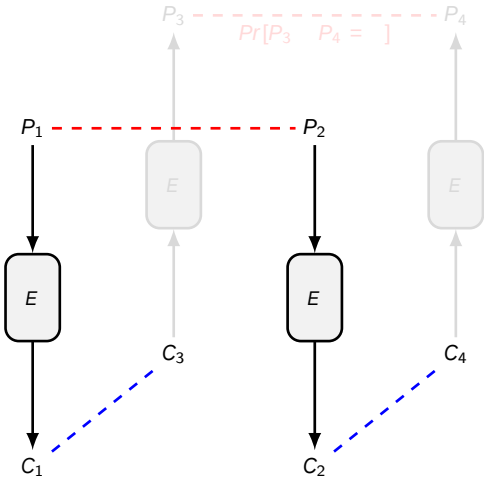

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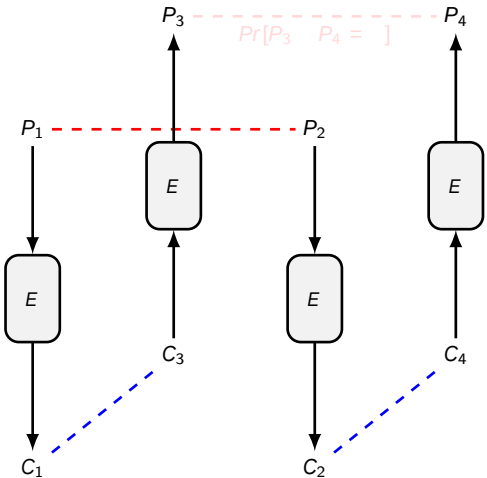

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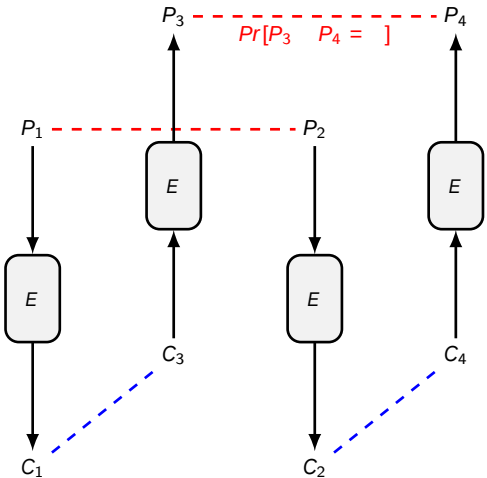

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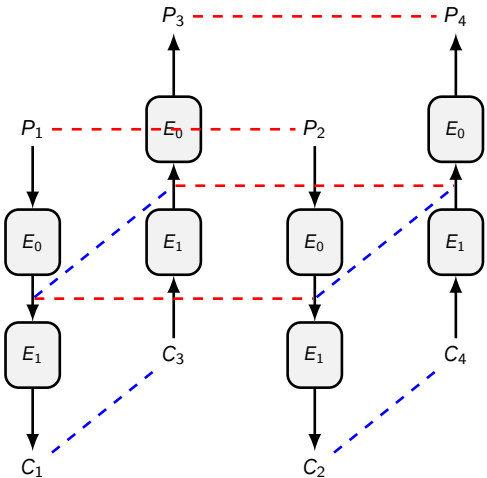


Rewrite $E = E_1 \quad E_0$

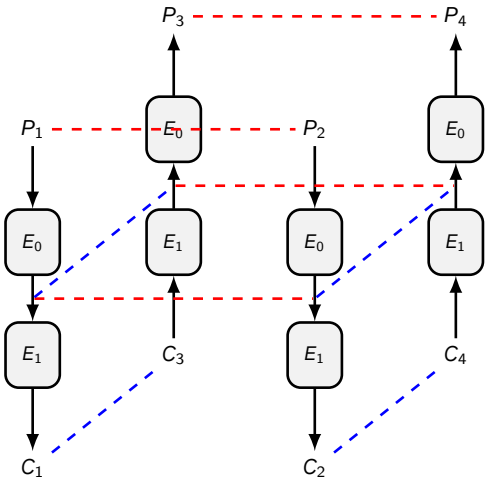
$E_0 : Pr[\quad ! \quad] = p$

$E_1 : Pr[\quad ! \quad] = q$

Expected probability: p^2q^2

Assumed two trails are independent

Boomerang Distinguisher 101



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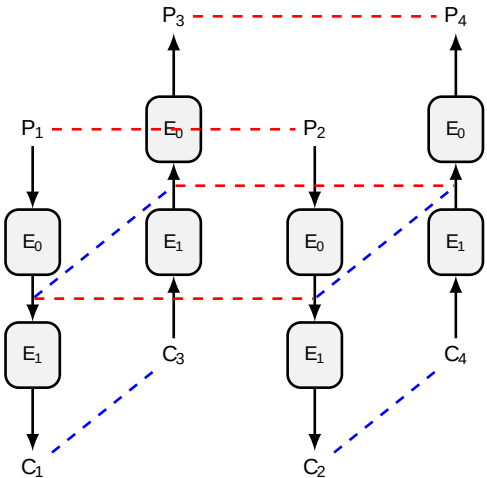
Assumption does NOT hold in practice!

Boomerang Distinguisher 101



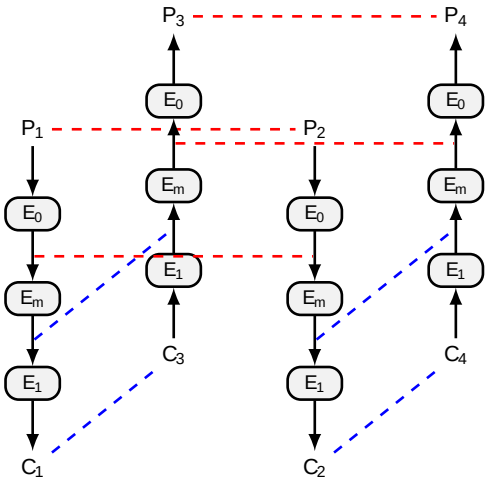
Several examples of non-returning boomerangs [Murphy, 2011]

At the junction of the two trails, dependency may exist

Some attempts to refine the probability: sandwich, ladder switch, ...

Assumption does NOT hold in practice!

Boomerang Distinguisher 101



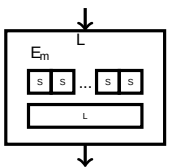
Sandwich attack [Dunkelman et al., 201

Decompose $E = E_1 \quad E_m \quad E_0$

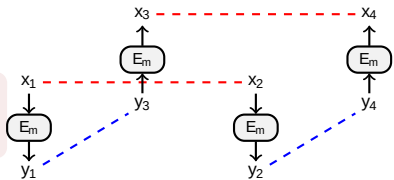
E_m handles the dependency, with probability r

Expected probability: $p^2 q^2 r$

Boomerang Distinguisher 101



How to compute
the probability r of E_m?



$$P(E_m^{-1}(E_m(X) \quad) \quad E_m^{-1}(E_m(X \quad) \quad) = \quad)$$

Boomerang Distinguisher 101

Boomerang Connectivity Table [Cid et al., 2018]

$$\mathrm{BCT}(\ ;\ ) = \#\ \{x \in \mathbb{F}_2^n \mid S^{-1}(S(x) \quad) \quad S^{-1}(S(x \quad) \quad) = \quad \}$$

Boomerang Distinguisher 101

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BCT Framework [Song et al., 2019]

- Determined the boundaries of E_m
- Calculated r of E_m in the sandwich attack

Boomerang Distinguisher 101

Û Automatic Search Boomerangs [Cid et al., 2017]

- Used a MILP model to study the ladder switch
- Improved attacks on Deoxys and Deoxys-BC

ø Automated Related-Key Boomerang [Liu and Sasaki, 2019]

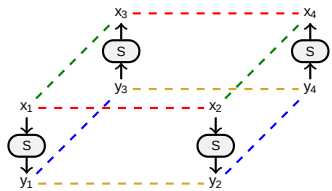
- MILP model to directly search for the best boomerang distinguishers for GIFT
- E_m is restricted to one single round

? Catching the Fastest Boomerangs [Delaune et al., 2020]

- Introduced a set of tables to calculate the probability
- New MILP/CP/ad-hoc approach to search for boomerang distinguishers for SKINNY
- Automatically handle the middle round

Di erential Tables [Delaune et al., 2020]

BCT is only a particular case



UBCT(; ;) = # x 2 F_2^n S(x) S(x) =
S ^{1}(S(x)) S ^{1}(S(x)) =

LBCT(; ;) = # x 2 F_2^n S(x) S(x) =
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EBCT(; ; ;) = # $\begin{cases} \\ \\ \end{cases}$ x 2 F_2^n S(x) S(x) = S(x) S(x) = S ^{1}(S(x)) S ^{1}(S(x)) = $\begin{cases} \\ \\ \end{cases}$;

Computing Probabilities [Delaune et al., 2020]

Given a boomerang characteristic, how to compute the probability for the boomerang to return?

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Multiply the probability of transition for each Sbox separately!

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The di erential tables are used!

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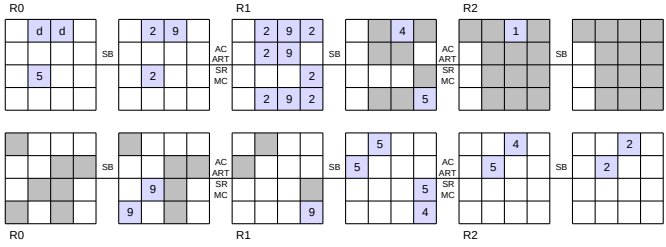

Figure: An example of boomerang characteristic on 3 rounds with $_e$ = [0 ; d ; d ; 0; 0; 0; 0; 0; 0; 5; 0; 0; 0; 0; 0; 0] and r $_e$ = [0 ; 0; 2; 0; 0; 2; 0; 0; 0; 0; 0; 0; 0; 0; 0; 0]

Upper In	Upper Out	Lower In	Lower Out	Proba
*	*			P_{DDT}
		*	*	
*	*			$P_{DDT}(\,;\,)^2$
		*	*	
*			*	P_{BCT}
*	*		*	P_{UBCT}
*		*	*	P_{LBCT}
*	*	*	*	P_{EBCT}

Table: Summary of the used tables to compute probability

$$P_{e}^{E}\, r_{e} = P_{DDT}(d;2)\; P_{DDT}(d;9)\; P_{UBCT}(5;2;9)\; P_{DDT}(9;4)\; P_{BCT}(2;5)^2\; P_{EBCT}(2;5;9;4)\; P_{LBCT}(1;4;2)\; P_{DDT}(5;2)^2$$

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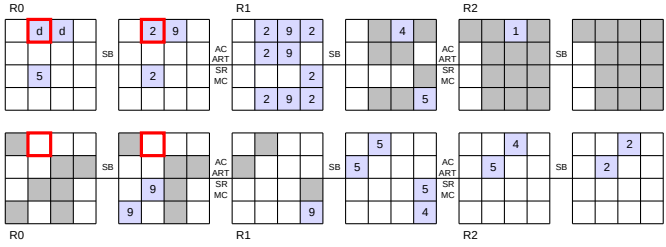
Computing Probabilities [Delaune et al., 2020]



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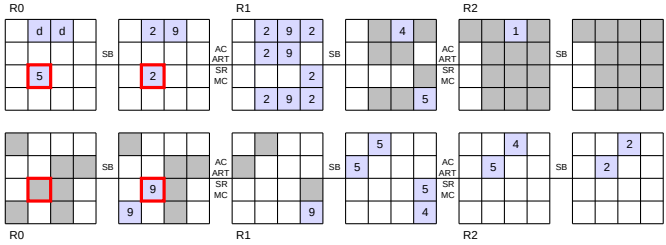

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Truncated Boomerang Characteristics

Idea: convert a MILP model to search for truncated differential characteristics into a MILP model to search for truncated boomerang characteristics

MILP model

Write twice the MILP model for truncated differential, once for the upper characteristic and once for the lower one

Each difference can be either active (non-zero) or inactive (zero)

Each difference can be either controlled (known) or free (unknown)

Objective: an upper bound on the probability (somehow similar to the number of active Sboxes)

MILP model [Delaune et al., 2020]

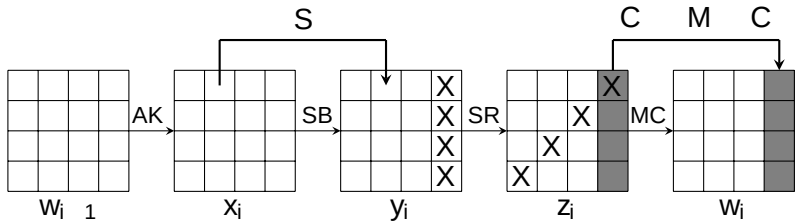
New constraints

- Constraints related to controlled/free variables
  - e.g. propagation of free variables
- Constraints related to controlled/free and active/inactive variables
  - e.g. if x is inactive thenx is controlled
- Constraints related to tables
  - for each Sbox we need to know which table is involved (e.g. DDT, BCT, EBCT, ...)

Objective: weighted sum over all the Sboxes and over all the tables
- weighted by the maximum probability exponent

No "middle round" de ned in the model!

Applications to AES



Advanced Encryption Standard (AES)

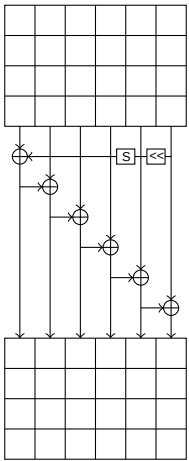
Standardized in 2001

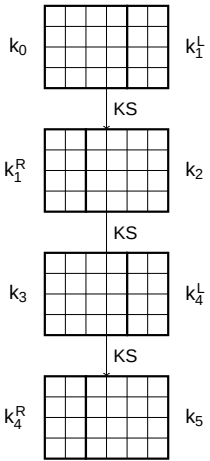
Block size: 4 × 4 bytes (128 bits)

r_i = MC ∘ SR ∘ SB ∘ AK (except the last round)

AES-128 (r = 10), AES-192 (r = 12), AES-256 (r = 14)

AES-192 Key-schedule





AES-192 Key schedule round

$K_{i;0} \quad S(K_{i+1;5}) \quad K_{i;0} \quad C_r ; \qquad 0 \quad i \quad 3$

$K_{i;j} \quad S(K_{i;j \ 1}) \quad K_{i;j} ; \qquad 0 \quad i \quad 3; 1 \quad j \quad 5$

Expand the master key K into $r + 1$ round keys

The key schedule is non linear, the di erence may be unpredictable.

=)

Need a new model!

New MILP Model

- Boomerang on the Related-Keys
  - Handle the non-linear key schedule
- Directly search for attacks, not only for distinguishers
  - Best distinguishers do not always lead to the best attacks!

Boomerang on the Related-Keys

Related-key: All keys $K_1; :::; K_4$ are secret, but relation $_{i;j} = K_i \quad K_j$ are known.

Control di erences at both the input and output of an Sbdx zero di erence

Or consider weak-keys distinguishers

Keys generated by a boomerang with probability 1!

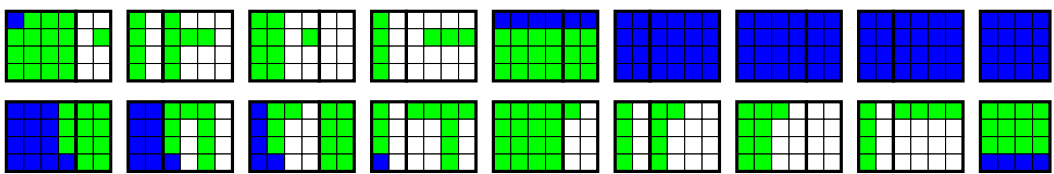


Figure: Key schedule for this attack. The subkeys for the upper trail are represented above the ones of the lower trail

= A known di erence; □ = Zero di erence; = xed but unknown di erence

Searching for Attacks

New variables

$a^d = 1$: if the variable belongs to the distinguisher

$a^z = 1$: if the di erence is *zero*

$a^k = 1$: if the di erence is known

$a^s = 1$: if the di erence is set to a speci c value

New propagation rules:

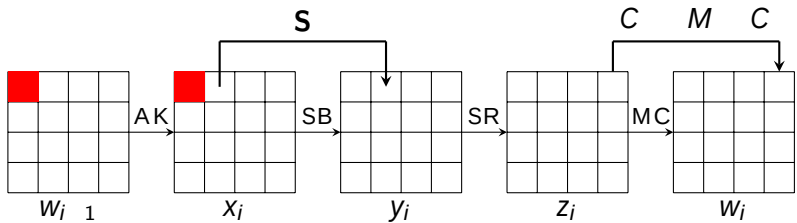
Speci c rules for both *d* and *s*

Each equation $\sum_i x_i =$ implies the constraints

$$x_1^u + \cdots + x_n^u \leq n - 1$$

Use callback and lazy constraints to ensure validity of solutions

Rules (Upper Trail)



$d = 1$: in the distinguisher

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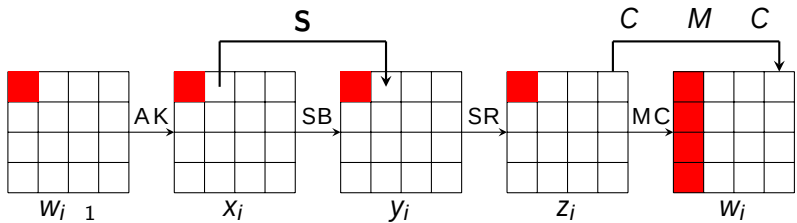

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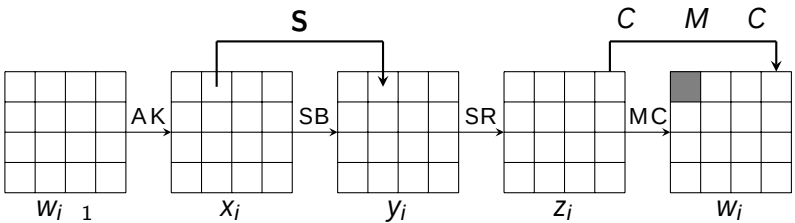

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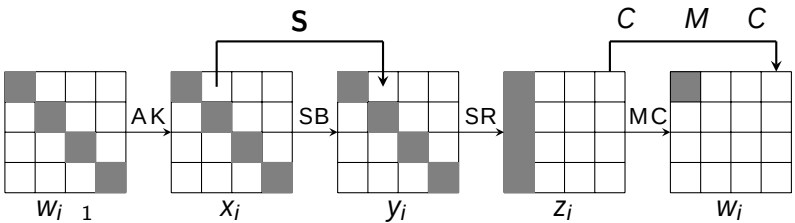


$d = 0$: not in the distinguisher

Computing Probabilities

Proba

$a^{z;up}$, $a^{k;up}$, $a^{s;up}$,
$b^{z;up}$, $b^{k;up}$, $b^{s;up}$,
$a^{z;low}$, $a^{k;low}$, $a^{s;low}$,
$b^{z;low}$, $b^{k;low}$, $b^{s;low}$

$=)$

59 possible cases in practice

compute ! the associated proba

11 possibles values:
2^{0}, $2^{5:4}$, 2^{6}, 2^{8},
2^{12}, $2^{13:4}$, 2^{14},
2^{16}, 2^{20}, $2^{21:4}$,
2^{24}

Extra constraints

Require 5 extra binary variables and 33 inequalities per S-box

The probability of the distinguisher is greater than 2^{127}

Note that: b = S(a)

Objective Function

- **Ideally:** optimize on the complexity of the attack ...
- ... but quite hard to compute (depends on the dimension of several vector spaces)

Idea: Use an approximation

The smaller the vector spaces of plaintexts and ciphertexts, the better the attack

The higher the probability of the distinguisher, the better the attack

Objective function

$$\texttt{Maximize}\quad 2^{\left(\sum_{i=0}^{15} p[i]^{k;up} + c[i]^{k;lo}\right) + 6\left(\sum_{i=0}^{15} p[i]^{s;up} + c[i]^{s;lo}\right)} \cdot p_{dist}$$

Note that: p_{dist} is the $-\log_2$ of the probability

Results

Model is very slow ! impossible to search for the best attacks

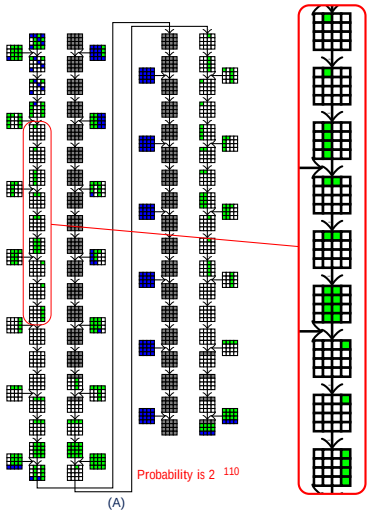
Run the model on a restricted subspace

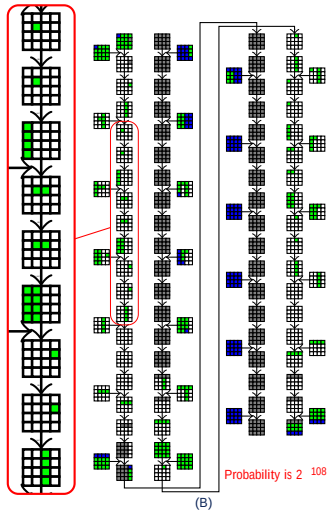
Retrieved the attack against AES-256

Found a better attack on AES-192

Key Size	Rounds	Time	Data	Memory	Type	Ref
192 bits	8/12	2^{172}	2^{107}	2^{96}	MITM	[Derbez et al., 2013]
	9/12	$2^{182.5}$	2^{117}	$2^{165.5}$		[Li et al., 2014]
	10/12	2^{183}	2^{124}	N/A	Related-key Rectangle	[Kim et al., 2007]
		2^{156}	2^{156}	2^{65}	Related-key Di erential	[Gerault et al., 2018]
	12/12	$2^{190.16}$	2^{80}	2^{8}	Biclique	[Bogdanov et al., 2011]
		$2^{190.83}$	2	2^{60}		[Bogdanov et al., 2014]
		$2^{189.76}$	2^{48}	2^{60}		[Tao and Wu, 2015]
		2^{176}	2^{123}	2^{152}	Related-key Boomerang	[Biryukov and Khovratovich, 2009]
		2^{124}	2^{124}	$2^{79.8}$	Related-key Boomerang	This work

The best-known attack (A) vs Our attack (B)





■ = A known di erence; □ = Zero di erence; ■ = A xed di erence

Conclusion

Summary

- Proposed a **new** MILP model to deal with **non-linear** key schedule
- Found a **new** related-keys attack against full AES-192
  - 2^{52} times lower complexity than the [Biryukov and Khovratovich, 2009] attack
- Recovered the attack on AES-256 by [Biryukov and Khovratovich, 2009]

Note

- For more details: ia.cr/2022/725
- Code available at: https://gitlab.inria.fr/pderbez/asia-2022-aes.git

SCAN ME

Thanks for your attention!
Any questions?

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