Revisiting Related-Key Boomerang attacks on AES using computer-aided tool

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Updated results on AES-192

Key Size	Rounds	Time	Data	Memory	Type	Ref
192 bits	8/12	2^{172}	2^{107}	2^{96}	MITM	[Derbez et al., 2013]
	9/12	$2^{182.5}$	2^{117}	$2^{165.5}$		[Li et al., 2014]
	10/12	2^{183}	2^{124}	N/A	Related-key Rectangle	[Kim et al., 2007]
		2^{156}	2^{156}	2^{65}	Related-key Differential	[Gérault et al., 2018]
	12/12	$2^{190.16}$	2^{80}	2^{8}	Biclique	[Bogdanov et al., 2011]
		$2^{190.83}$	2	2^{60}		[Bogdanov et al., 2014]
		$2^{189.76}$	2^{48}	2^{60}		[Tao and Wu, 2015]
		2^{176}	2^{123}	2^{152}	Related-key Boomerang	[Biryukov and Khovratovich, 2009]

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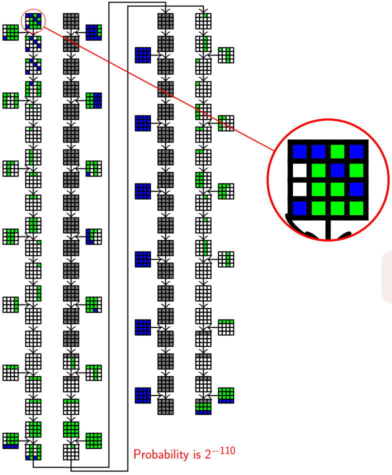
		2^{176}	2^{123}	2^{152}	Related-key Boomerang	[Biryukov and Khovratovich, 2009]
		2^{124}	2^{124}	$2^{79.8}$	Related-key Boomerang	This work

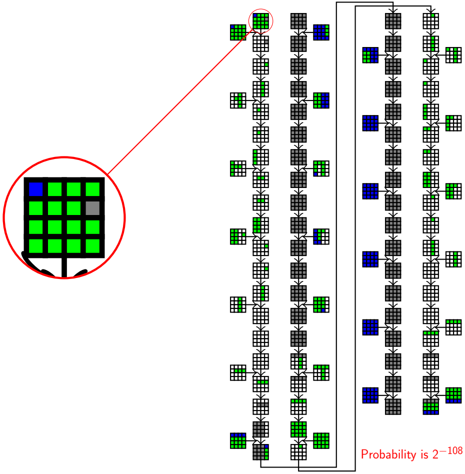
Its time complexity is 2^{52} times lower than the best-known attack!

The best-known attack (A) vs Our attack (B)



(A)

$$\sum \square_A < \sum \square_B$$



(B)

$\square$ = A known difference; $\square$ = Zero difference; $\blacksquare$ = A fixed difference

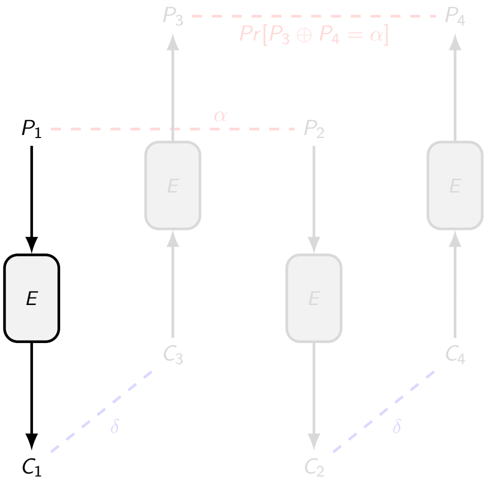
Overview

1. The boomerang attack
2. Previous works
3. Application to AES
4. Results

Boomerang Distinguisher 101

- The Boomerang attack [Wagner, 1999]
  - *When you send it properly,*
    it always comes back to you

Boomerang Distinguisher 101



1. Pick P_1, ask for $C_1 = E(P_1)$
2. $P_2 = P_1 \oplus \alpha$, ask for C_2
3. $C_3 = C_1 \oplus \delta$, $C_4 = C_2 \oplus \delta$
4. Ask for $P_3 = E^{-1}(C_3)$, $P_4 = E^{-1}(C_4)$
5. Check if $P_3 \oplus P_4 = \alpha$

A distinguisher if α comes back
more often than a *random permutation*!

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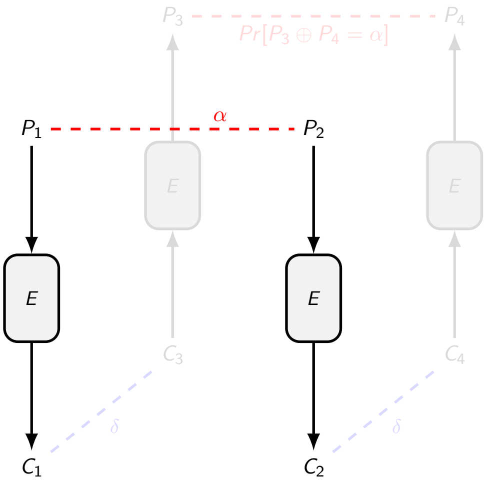

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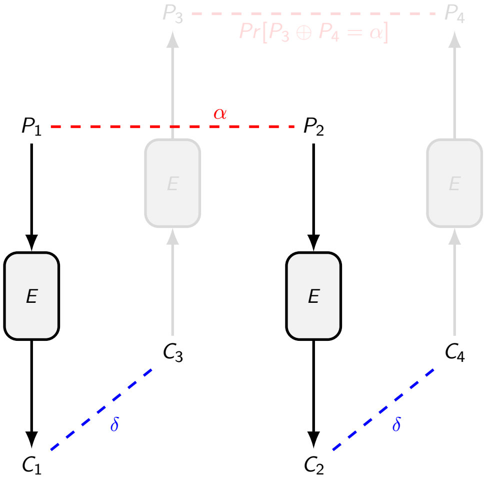

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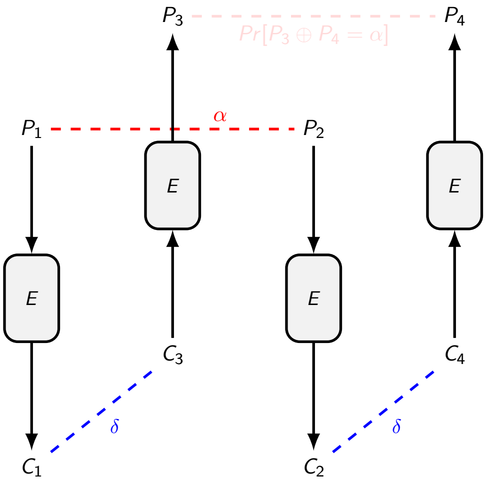

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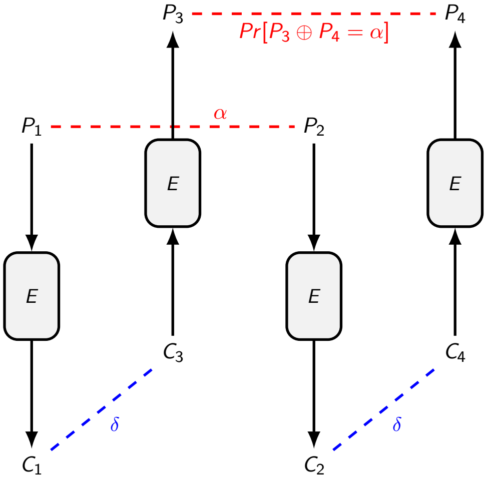

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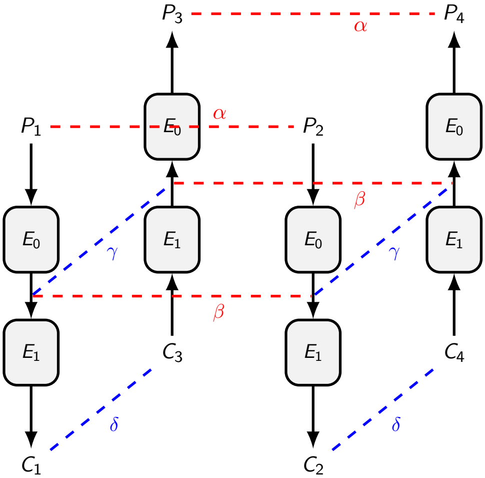
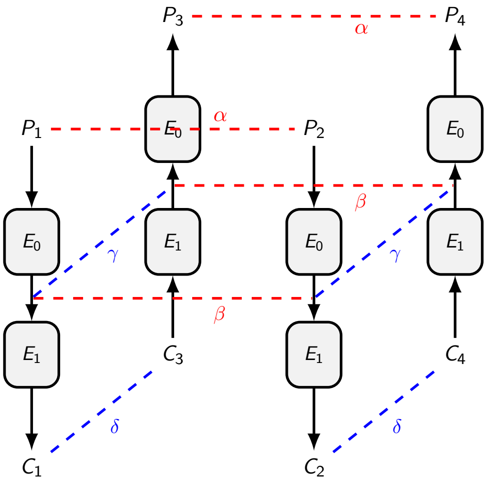


- Rewrite $E = E_1 \circ E_0$
  - $E_0 : Pr[\alpha \rightarrow \beta] = p$
  - $E_1 : Pr[\gamma \rightarrow \delta] = q$
- Expected probability: $p^2 q^2$
- Assumed two trails are independent

Boomerang Distinguisher 101



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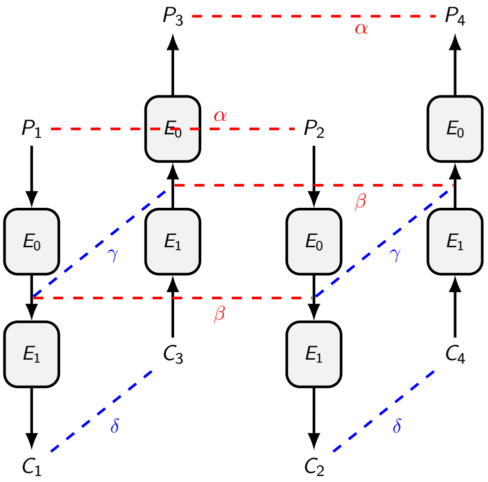
Assumption does NOT hold in practice!

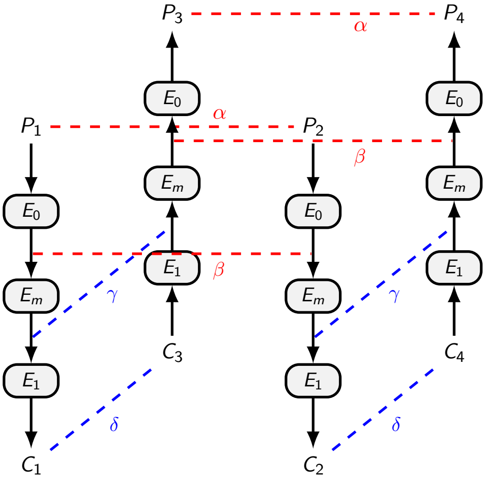
Boomerang Distinguisher 101



- Several examples of non-returning boomerangs [Murphy, 2011]
- At the junction of the two trails, dependency may exist
- Some attempts to refine the probability: sandwich, ladder switch, . . .

Assumption does NOT hold in practice!

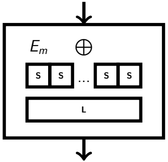
Boomerang Distinguisher 101



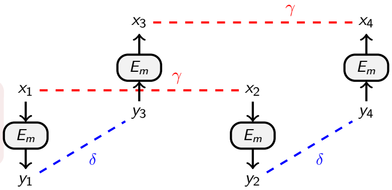
Sandwich attack [Dunkelman et al., 2010]

- Decompose $E = E_1 \circ E_m \circ E_0$
- E_m handles the dependency, with probability r
- Expected probability: $\tilde{p}^2 \tilde{q}^2 r$

Boomerang Distinguisher 101

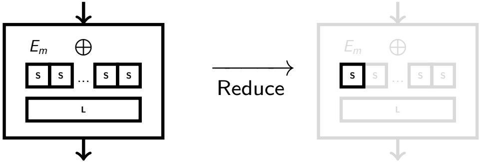


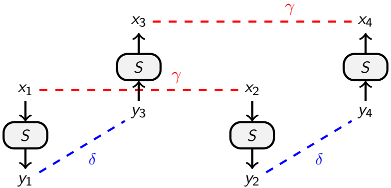
How to compute
the probability r of E_m?



$$\mathbb{P}(E_m^{-1}(E_m(X) \oplus \delta) \oplus E_m^{-1}(E_m(X \oplus \gamma) \oplus \delta) = \gamma)$$

Boomerang Distinguisher 101

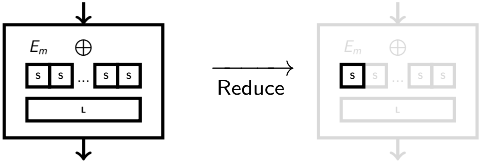


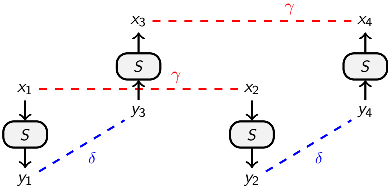


Boomerang Connectivity Table [Cid et al., 2018]

$$\mathrm{BCT}(\gamma, \delta) = \#\{x \in \mathbb{F}_2^n \mid S^{-1}(S(x) \oplus \delta) \oplus S^{-1}(S(x \oplus \gamma) \oplus \delta) = \gamma\}$$

Boomerang Distinguisher 101





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BCT Framework [Song et al., 2019]

- Determined the boundaries of E_m
- Calculated r of E_m in the sandwich attack

Boomerang Distinguisher 101

Automatic Search Boomerangs [Cid et al., 2017]

- Used a MILP model to study the ladder switch
- Improved attacks on `Deoxys` and `Deoxys-BC`

Automated Related-Key Boomerang [Liu and Sasaki, 2019]

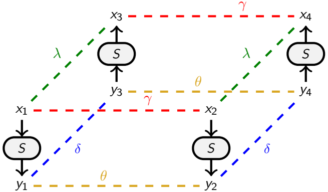
- MILP model to directly search for the best boomerang distinguisher on `GIFT`
- E_m is restricted to one single round

Catching the Fastest Boomerangs [Delaune et al., 2020]

- Introduced a set of tables to calculate the probability
- New MILP/CP/ad-hoc approach to search for boomerang distinguishers on `SKINNY`
- Automatically handle the middle round

Differential Tables [Delaune et al., 2020]

BCT is only a particular case



- $\mathrm{UBCT}(\gamma, \theta, \delta) = \#\left\{ x \in \mathbb{F}_2^n \;\middle|\; \begin{array}{l} S(x) \oplus S(x \oplus \gamma) = \theta \\ S^{-1}(S(x) \oplus \delta) \oplus S^{-1}(S(x \oplus \gamma) \oplus \delta) = \gamma \end{array} \right\}$
- $\mathrm{LBCT}(\gamma, \lambda, \delta) = \#\left\{ x \in \mathbb{F}_2^n \;\middle|\; \begin{array}{l} S(x) \oplus S(x \oplus \lambda) = \delta \\ S^{-1}(S(x) \oplus \delta) \oplus S^{-1}(S(x \oplus \gamma) \oplus \delta) = \gamma \end{array} \right\}$
- $\mathrm{EBCT}(\gamma, \theta, \lambda, \delta) = \#\left\{ x \in \mathbb{F}_2^n \;\middle|\; \begin{array}{l} S(x) \oplus S(x \oplus \gamma) = \theta \\ S(x) \oplus S(x \oplus \lambda) = \delta \\ S^{-1}(S(x) \oplus \delta) \oplus S^{-1}(S(x \oplus \gamma) \oplus \delta) = \gamma \end{array} \right\}$

Computing Probabilities [Delaune et al., 2020]

Given a boomerang characteristic, how to compute the probability for the boomerang to return?

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Multiply the probability of transition for each Sbox separately!

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Computing Probabilities [Delaune et al., 2020]

Given a boomerang characteristic, how to compute the probability for the boomerang to return?

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How to compute the probability of transition for one particular Sbox?

The differential tables are used!

Computing Probabilities [Delaune et al., 2020]

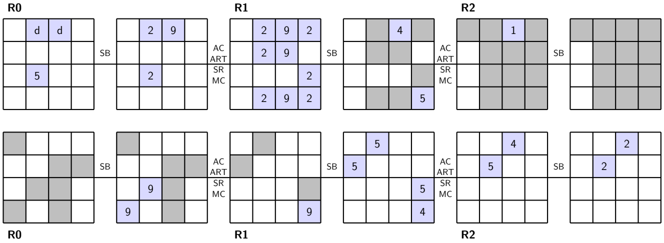


Figure: An example of **boomerang characteristic** on 3 rounds with
$\Delta_e = [0, d, d, 0, 0, 0, 0, 0, 0, 5, 0, 0, 0, 0, 0, 0]$ and
$\nabla_e = [0, 0, 2, 0, 0, 2, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0]$

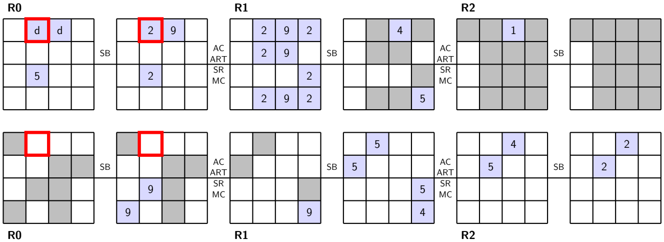
Upper In	Upper Out	Lower In	Lower Out	Proba
*	*			$\mathbb{P}_{DDT}$
		*	*	
*	*			$\mathbb{P}_{DDT}(\cdot,\cdot)^2$
		*	*	
*			*	$\mathbb{P}_{BCT}$
*	*		*	$\mathbb{P}_{UBCT}$
*		*	*	$\mathbb{P}_{LBCT}$
*	*	*	*	$\mathbb{P}_{EBCT}$

Table: Summary of the used tables to compute probability

$$\mathbb{P}\left(\Delta_e \overset{E}{\rightleftarrows} \nabla_e\right) = \begin{array}{l} \mathbb{P}_{\mathrm{DDT}}(\mathrm{d},2)\cdot\mathbb{P}_{\mathrm{DDT}}(\mathrm{d},9)\cdot\mathbb{P}_{\mathrm{UBCT}}(5,2,9)\cdot\mathbb{P}_{\mathrm{DDT}}(9,4)\cdot \\ \mathbb{P}_{\mathrm{BCT}}(2,5)^2\cdot\mathbb{P}_{\mathrm{EBCT}}(2,5,9,4)\cdot\mathbb{P}_{\mathrm{LBCT}}(1,4,2)\cdot\mathbb{P}_{\mathrm{DDT}}(5,2)^2 \end{array}$$

□ = Zero; ■ = Free; □ = Specified

Computing Probabilities [Delaune et al., 2020]



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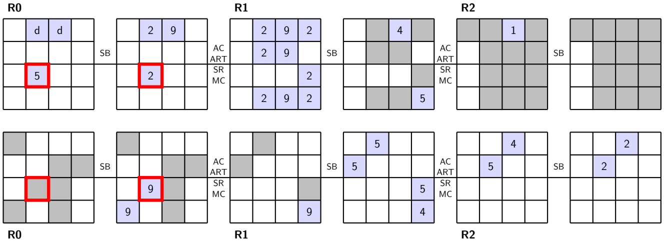

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Truncated Boomerang Characteristics

Idea: convert a MILP model to search for truncated differential characteristics into a MILP model to search for truncated boomerang characteristics

MILP model

- Write twice the MILP model for truncated differential, once for the upper characteristic and once for the lower one
- Each difference can be either active (non-zero) or inactive (zero)
- Each difference can be either controlled (known) or free (unknown)
- **Objective:** an upper bound on the probability (somehow similar to *the number of active Sboxes*)

MILP model [Delaune et al., 2020]

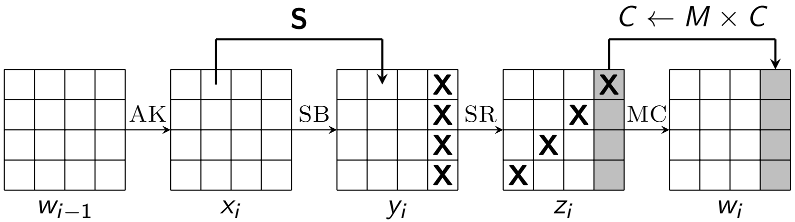
New constraints

- Constraints related to controlled/free variables
  - e.g. propagation of free variables
- Constraints related to controlled/free and active/inactive variables
  - e.g. if x is inactive then x is controlled
- Constraints related to tables
  - for each Sbox we need to know which table is involved (e.g. DDT, BCT, EBCT, ...)

- **Objective:** weighted sum over all the Sboxes and over all the tables
  - weighted by the maximum probability exponent

No "middle round" defined in the model!

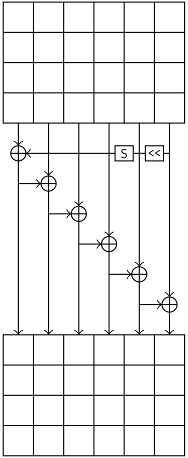
Applications to AES

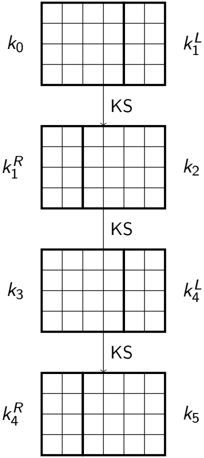


Advanced Encryption Standard (AES)

- Standardized in 2001
- Block size: 4×4 bytes (128 bits)
- $r_i = \mathrm{MC} \circ \mathrm{SR} \circ \mathrm{SB} \circ \mathrm{AK}$ (except the last round)
- AES-128 ($r = 10$), AES-192 ($r = 12$), AES-256 ($r = 14$)

AES-192 Key-schedule





AES-192 Key schedule round

$$
\begin{aligned}
K_{i,0} &\longleftarrow S(K_{i+1,5}) \oplus K_{i,0} \oplus C_r, && 0 \leq i \leq 3 \\
K_{i,j} &\longleftarrow S(K_{i,j-1}) \oplus K_{i,j}, && 0 \leq i \leq 3, 1 \leq j \leq 5
\end{aligned}
$$

Expand the master key K into $r+1$ round keys

The key schedule is
non linear,
the difference may be
unpredictable.

$\Longrightarrow$

Need
a new model!

New MILP Model

- Boomerang on the Related-Keys
  - Handle the non-linear key schedule
- Directly search for attacks, not only for distinguishers
  - Best distinguishers do not always lead to the best attacks!

Boomerang on the Related-Keys

Related-key: All keys $K_1, \ldots, K_4$ are secret, but relation $\Delta_{i,j} = K_i \oplus K_j$ are known.

- Control differences at both the input and output of an Sbox $\rightarrow$ zero difference
  - Or consider weak-keys distinguishers
- Keys generated by a boomerang with probability 1!

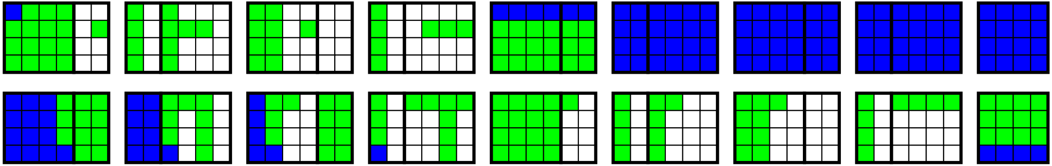


Figure: Key schedule for this attack. The subkeys for the upper trail are represented above the ones of the lower trail

= A known difference; = Zero difference; = fixed but unknown difference

Searching for Attacks

New variables

- $a^d = 1$: if the variable belongs to the distinguisher
- $a^z = 1$: if the difference is *zero*
- $a^k = 1$: if the difference is known
- $a^s = 1$: if the difference is set to a specific value

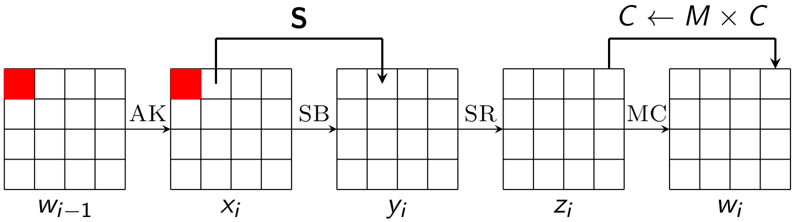
New propagation rules:

- Specific rules for both d and s
- Each equation $\bigoplus \alpha_i x_i = \beta$ implies the constraints

$$x_1^u + \ldots + x_n^u \neq n - 1$$

- Use callback and lazy constraints to ensure validity of solutions

Rules (Upper Trail)



- $d = 1$: in the distinguisher

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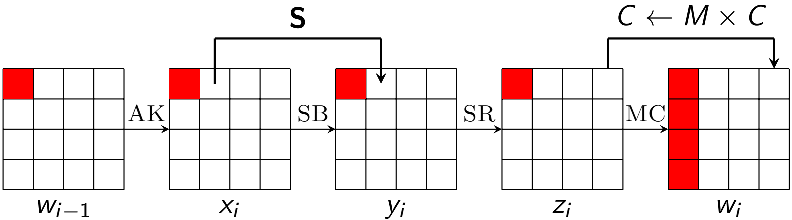
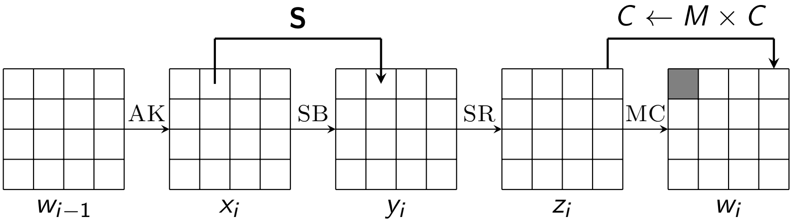


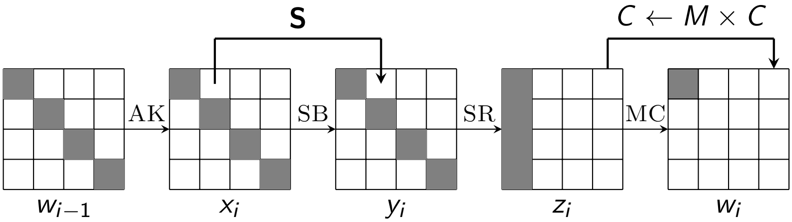
- $d = 1$: in the distinguisher

Rules (Upper Trail)



- $d = 0$: not in the distinguisher

Rules (Upper Trail)



- $d = 0$: not in the distinguisher

Computing Probabilities

Proba

$a^{z,up}$, $a^{k,up}$, $a^{s,up}$, $b^{z,up}$, $b^{k,up}$, $b^{s,up}$, $a^{z,low}$, $a^{k,low}$, $a^{s,low}$, $b^{z,low}$, $b^{k,low}$, $b^{s,low}$

$\Longrightarrow$ 59 possible cases in practice $\xrightarrow[\text{the associated proba}]{\textit{compute}}$

11 possibles values: 2^{0}, $2^{-5.4}$, 2^{-6}, 2^{-8}, 2^{-12}, $2^{-13.4}$, 2^{-14}, 2^{-16}, 2^{-20}, $2^{-21.4}$, 2^{-24}

Extra constraints

- Require 5 extra binary variables and 33 inequalities per S-box
- The probability of the distinguisher is greater than 2^{-127}

Note that: $\mathrm{b} = \mathrm{S}(\mathrm{a})$

Objective Function

- **Ideally:** optimize on the complexity of the attack ...
- ... but quite hard to compute (depends on the dimension of several vector spaces)

Idea: Use an approximation

- The smaller the vector spaces of plaintexts and ciphertexts, the better the attack
- The higher the probability of the distinguisher, the better the attack

Objective function

$$\texttt{Maximize}\left(2\times\left(\sum_{i=0}^{15} p[i]^{k,up}+c[i]^{k,lo}\right)+6\times\left(\sum_{i=0}^{15} p[i]^{s,up}+c[i]^{s,lo}\right)-p_{dist}\right)$$

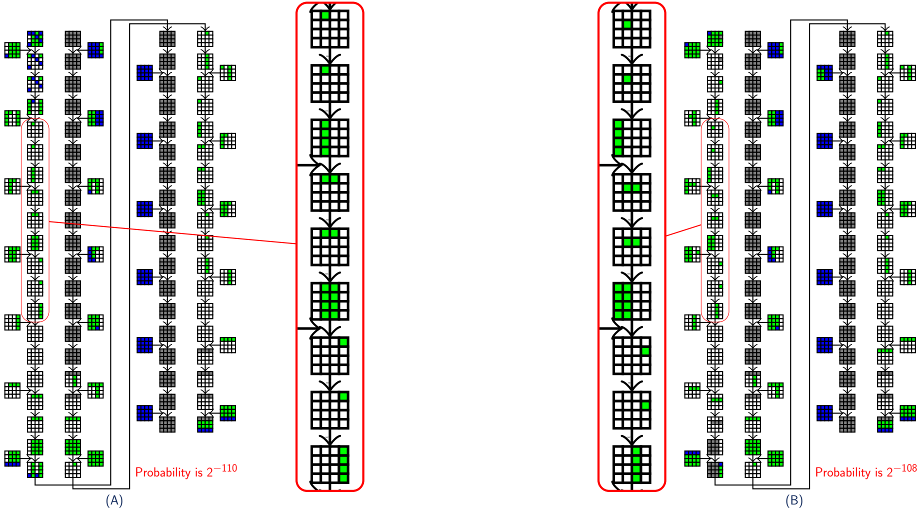
Note that: p_{dist} is the $-\log_2$ of the probability

Results

- Model is very slow $\rightarrow$ impossible to search for the best attacks
- Run the model on a restricted subspace
- Retrieved the attack against AES-256
- Found a better attack on AES-192

Key Size	Rounds	Time	Data	Memory	Type	Ref
192 bits	8/12	2^{172}	2^{107}	2^{96}	MITM	[Derbez et al., 2013]
	9/12	$2^{182.5}$	2^{117}	$2^{165.5}$		[Li et al., 2014]
	10/12	2^{183}	2^{124}	N/A	Related-key Rectangle	[Kim et al., 2007]
		2^{156}	2^{156}	2^{65}	Related-key Differential	[Gérault et al., 2018]
	12/12	$2^{190.16}$	2^{80}	2^{8}	Biclique	[Bogdanov et al., 2011]
		$2^{190.83}$	2	2^{60}		[Bogdanov et al., 2014]
		$2^{189.76}$	2^{48}	2^{60}		[Tao and Wu, 2015]
		2^{176}	2^{123}	2^{152}	Related-key Boomerang	[Biryukov and Khovratovich, 2009]
		2^{124}	2^{124}	$2^{79.8}$	Related-key Boomerang	This work

The best-known attack (A) vs Our attack (B)



■ = A known difference; □ = Zero difference; ■ = A fixed difference

Conclusion

Summary

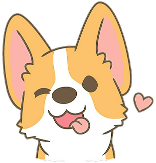
- Proposed a **new** MILP model to deal with **non-linear** key schedule
- Found a **new** related-keys attack against full `AES-192`
  - 2^{52} times lower complexity than the [Biryukov and Khovratovich, 2009] attack
- Recovered the attack on `AES-256` by [Biryukov and Khovratovich, 2009]

Note

- For more details: ia.cr/2022/725
- Code available at: `https://gitlab.inria.fr/pderbez/asia-2022-aes.git`

SCAN ME

Thanks for your attention!
Any questions?



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