Memory-Tight Multi-Challenge Security of Public-Key Encryption

Joseph Jaeger and **Akshaya Kumar**

Georgia Tech

Asymptotic Security v/s Concrete Security

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$\text{Adv}(A) \in [0,1]$

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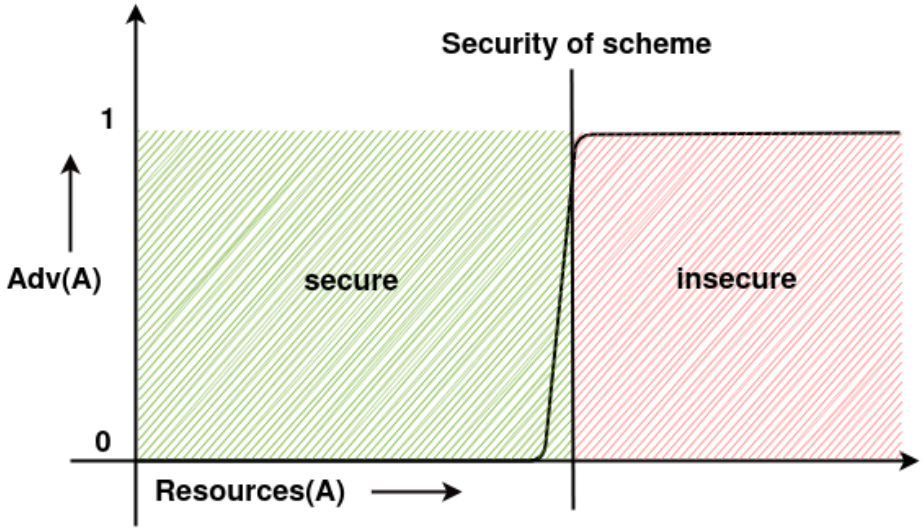

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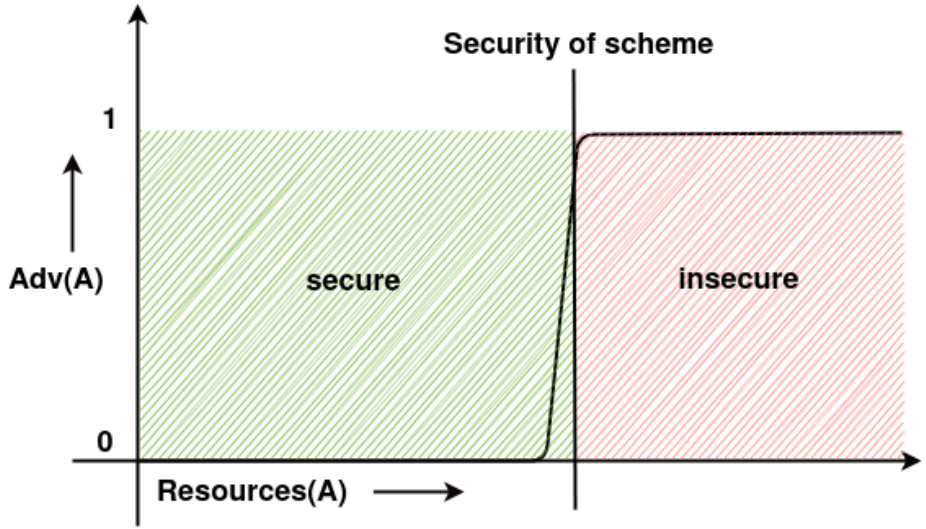

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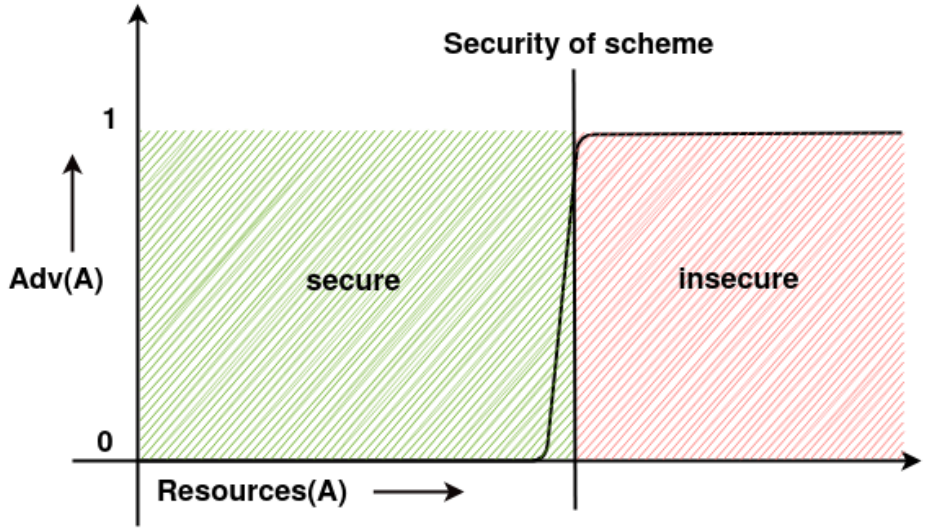

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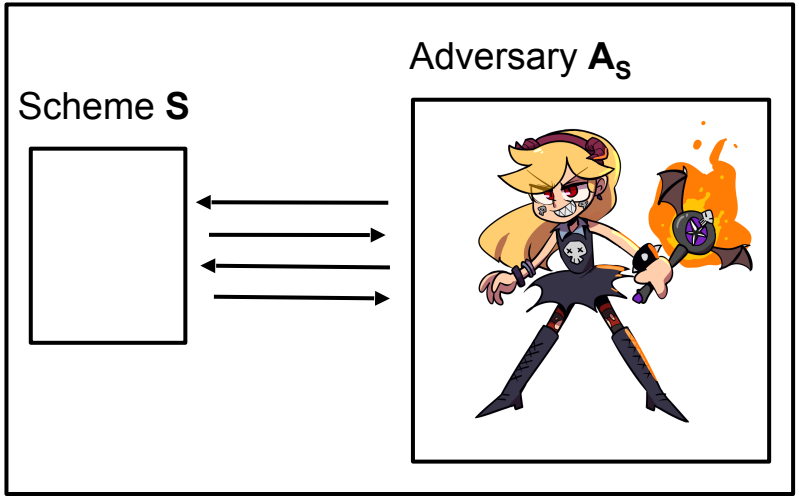
Cryptographic Reductions



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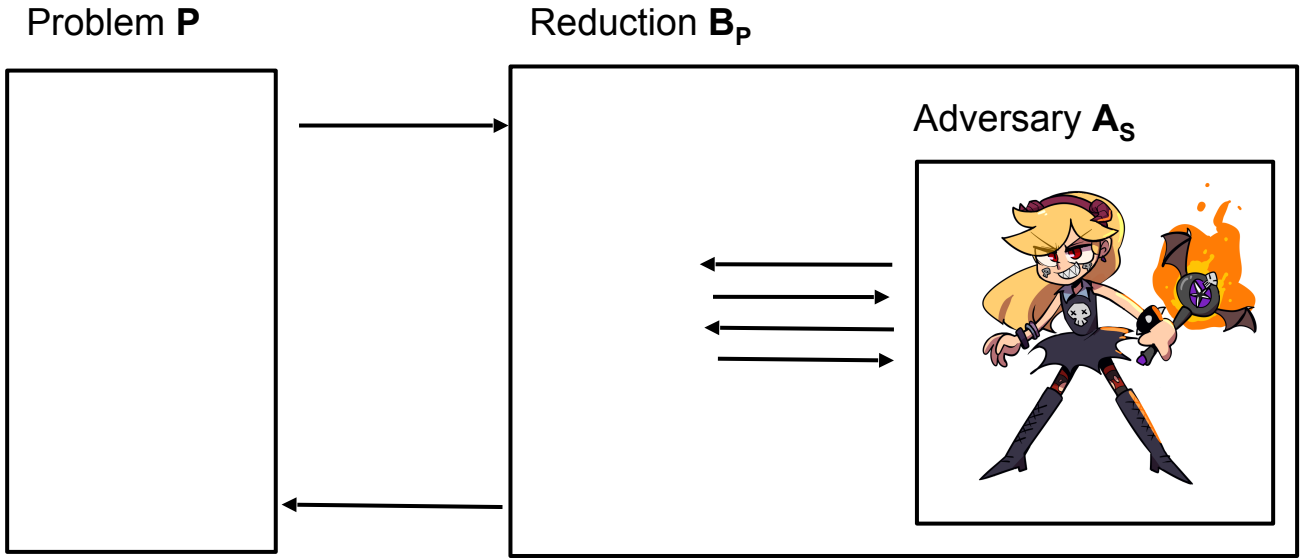

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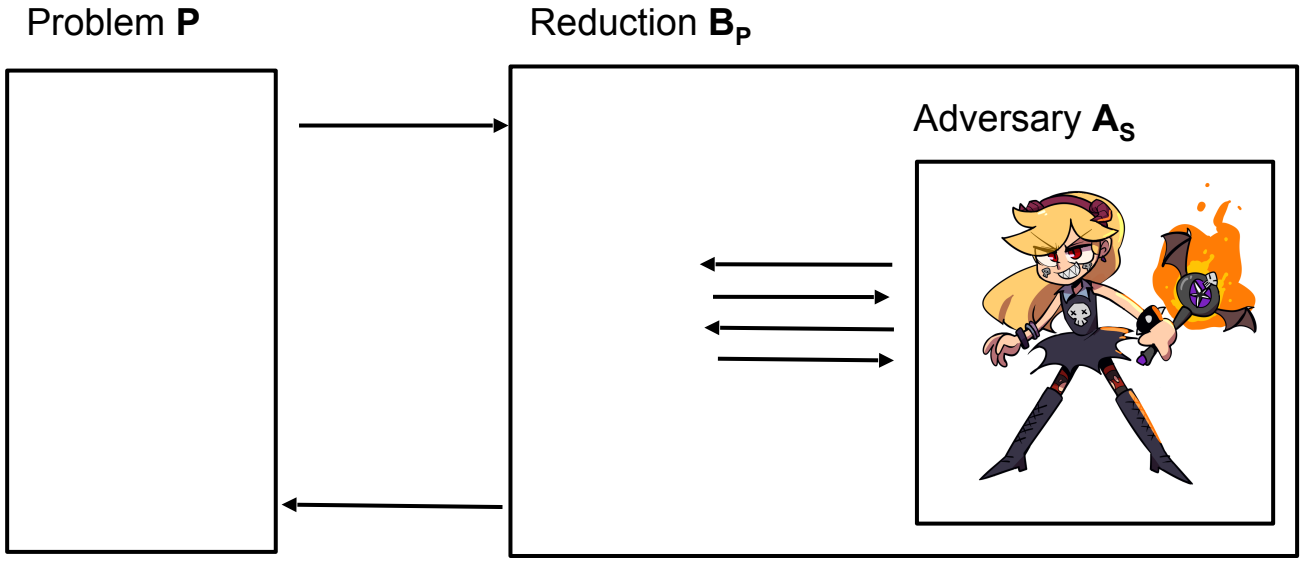

$$\mathbf{Adv}(\mathbf{A_S}) \leq \varepsilon(\mathbf{Adv}(\mathbf{B_P})),$$
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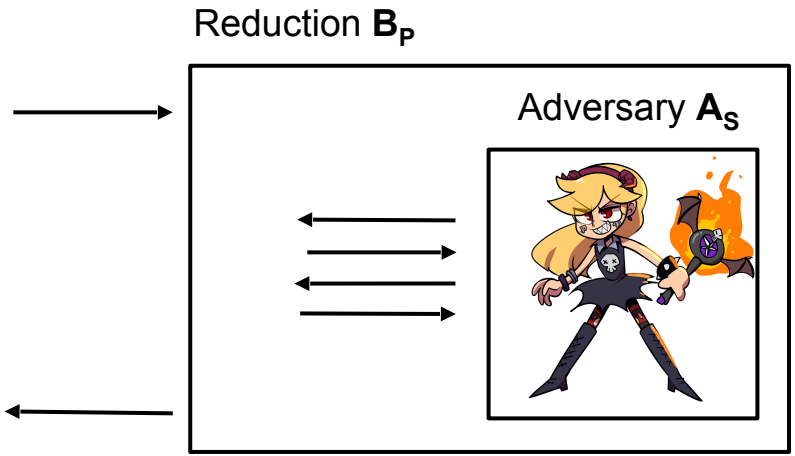
Tightness and Concrete Security



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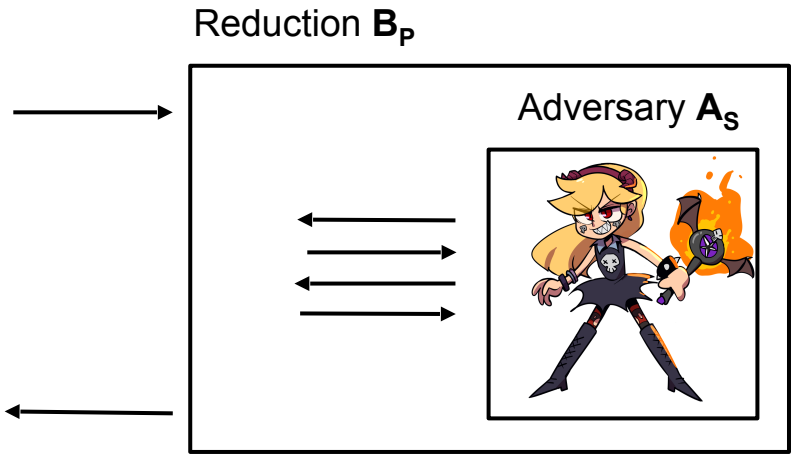

The reduction $\mathbf{B_P}$ is “tight” if

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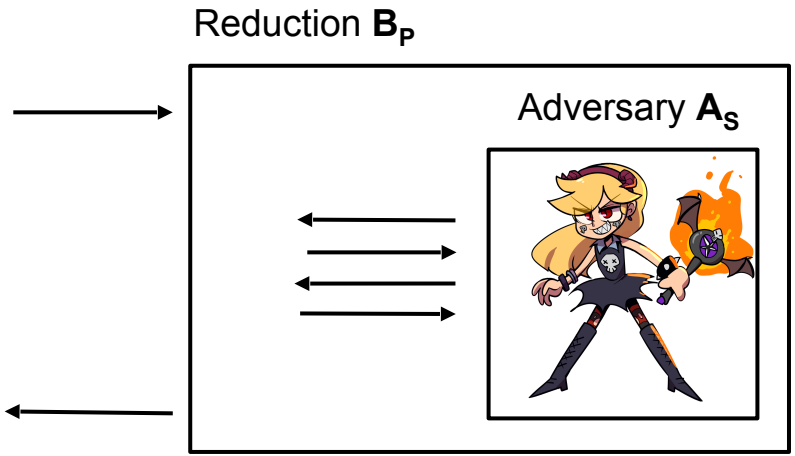

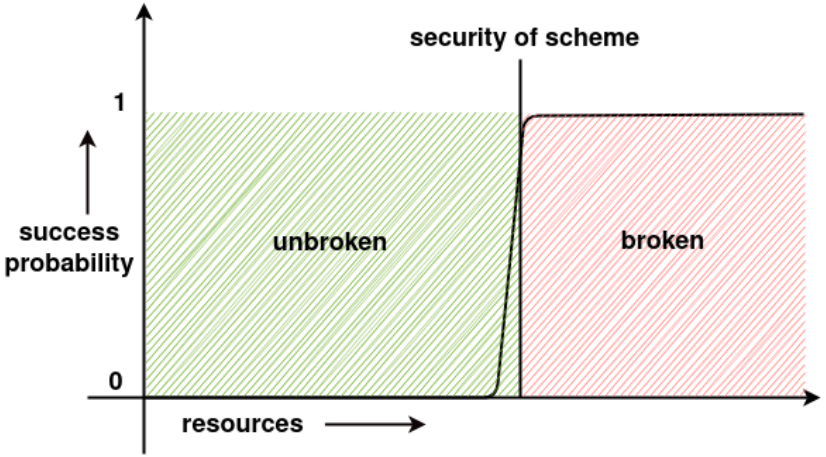

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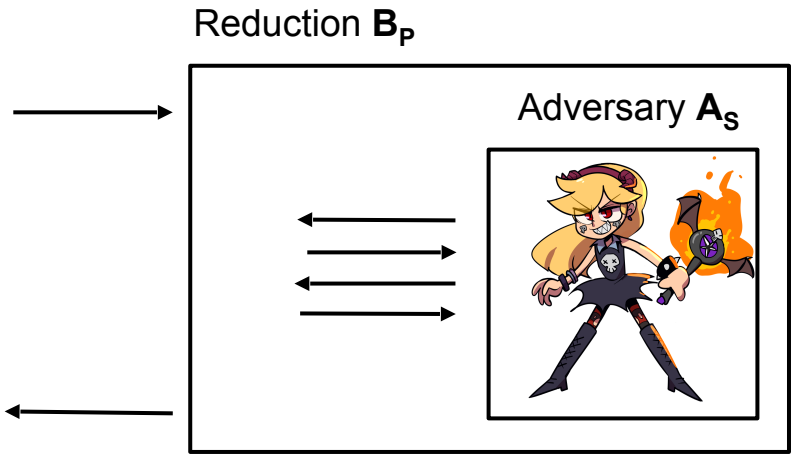

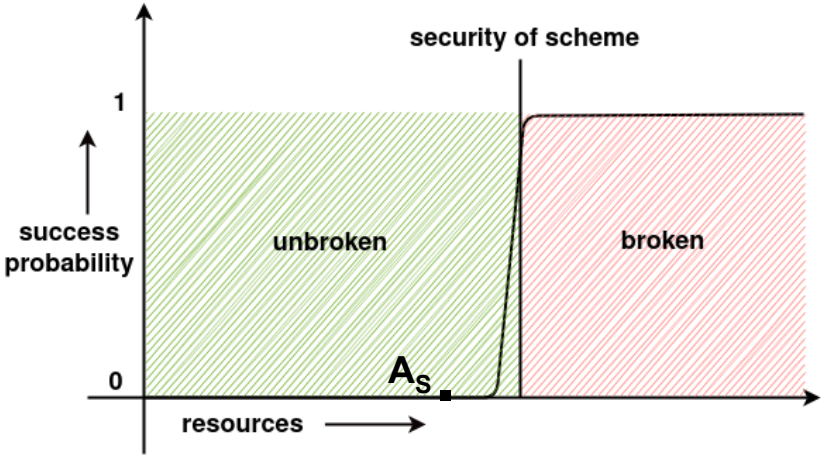

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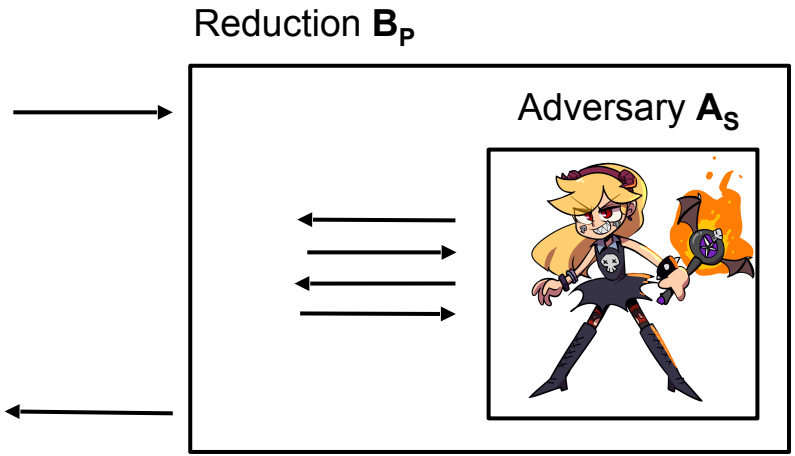

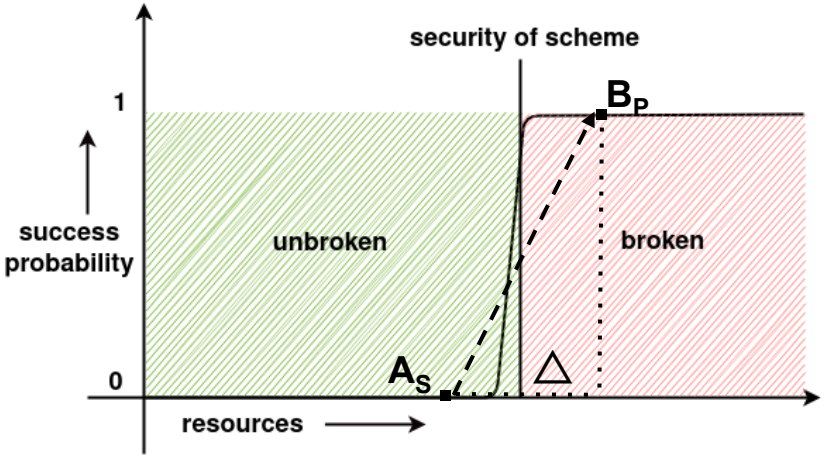

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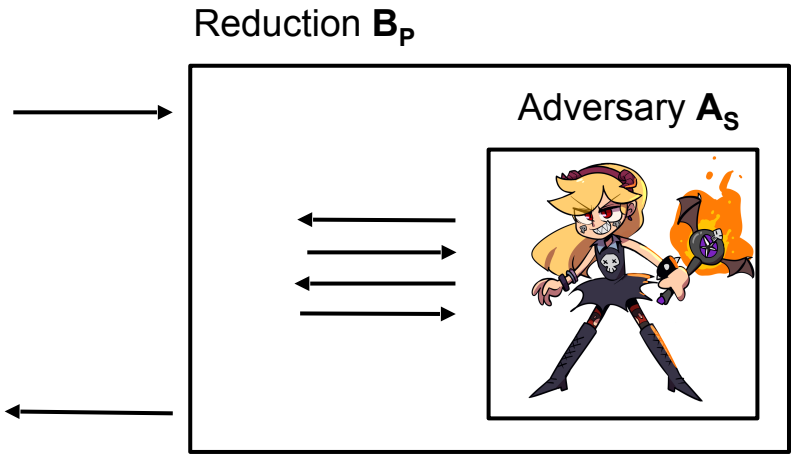

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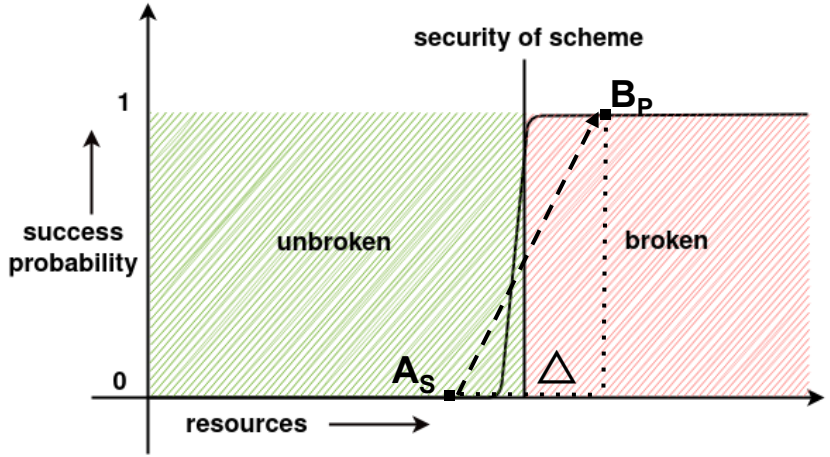

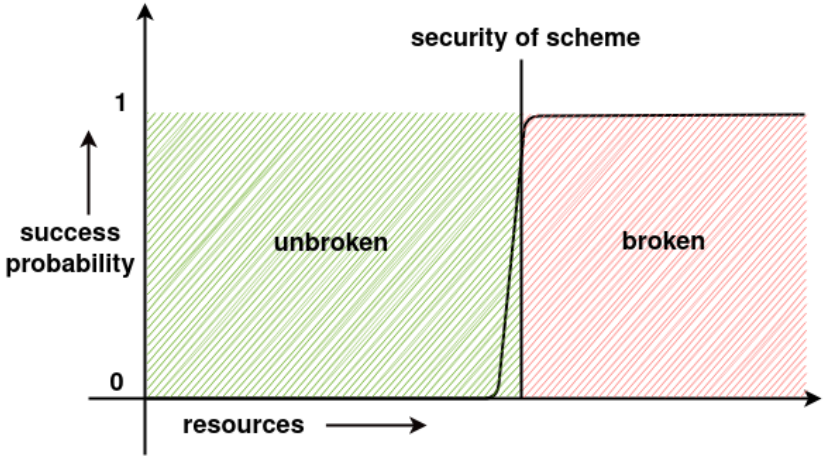

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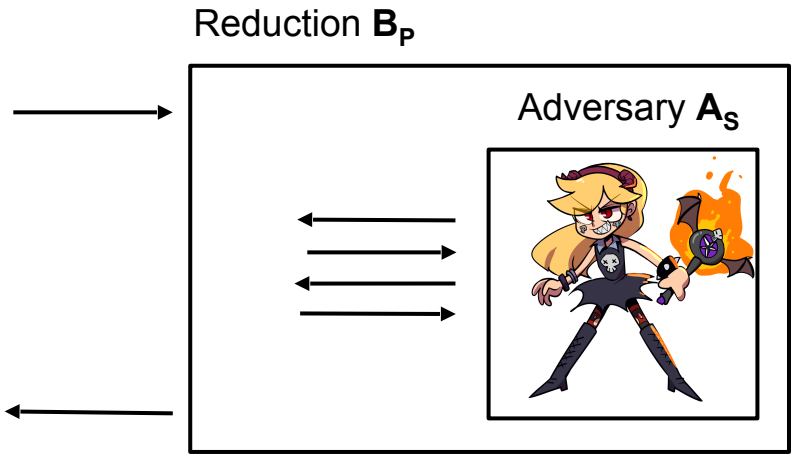

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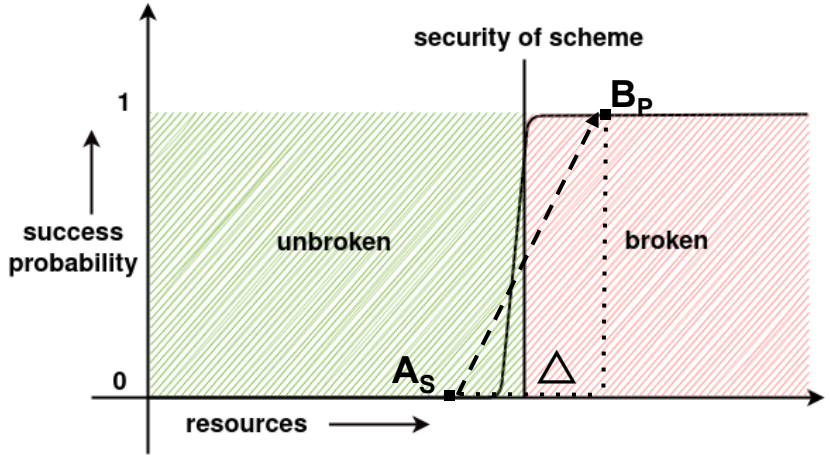

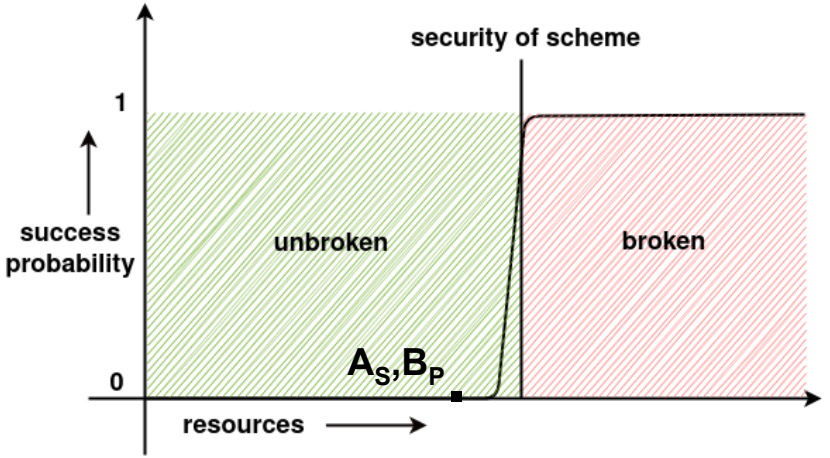

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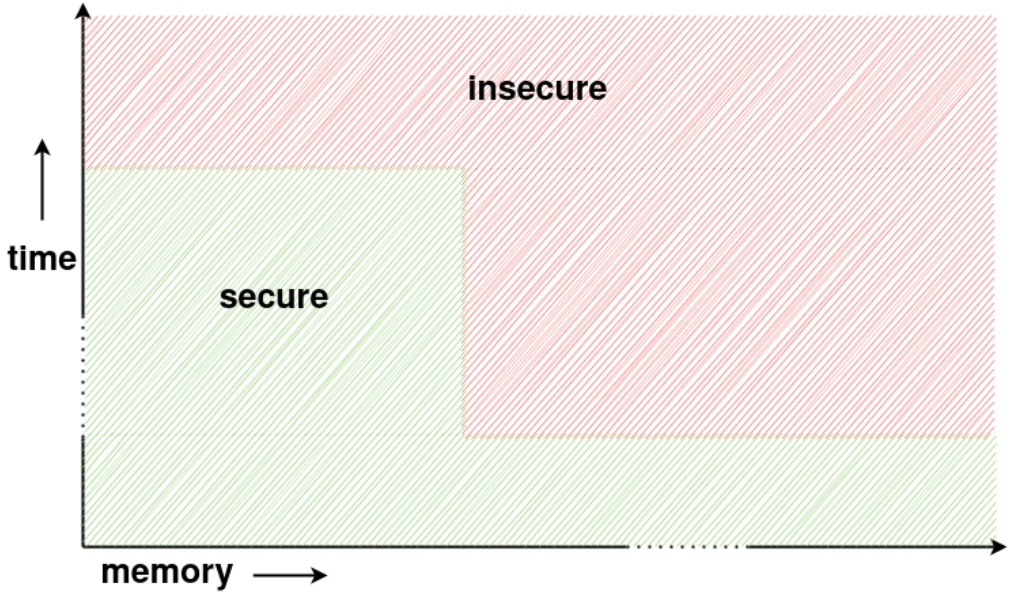

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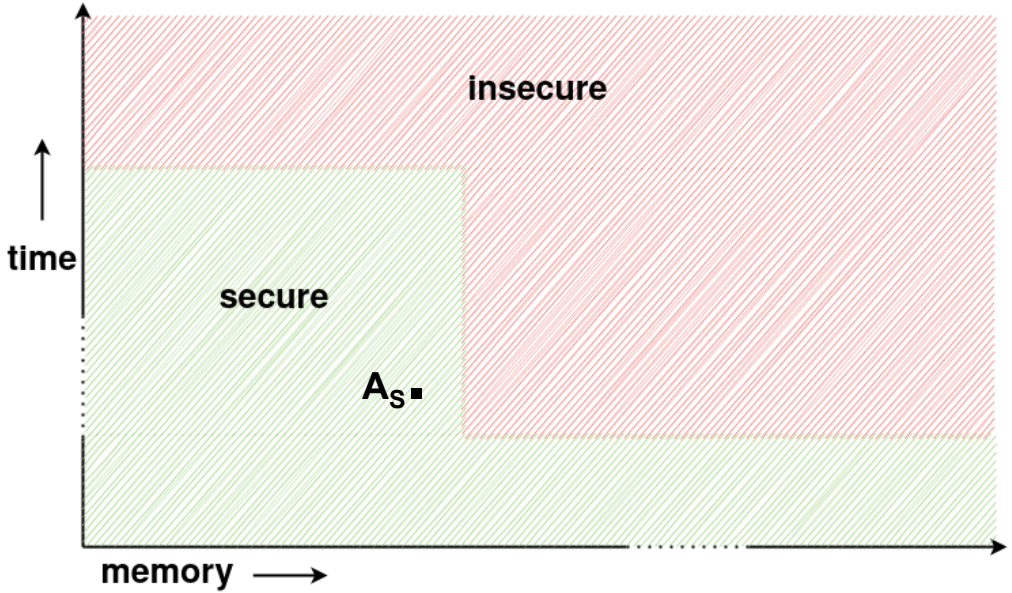

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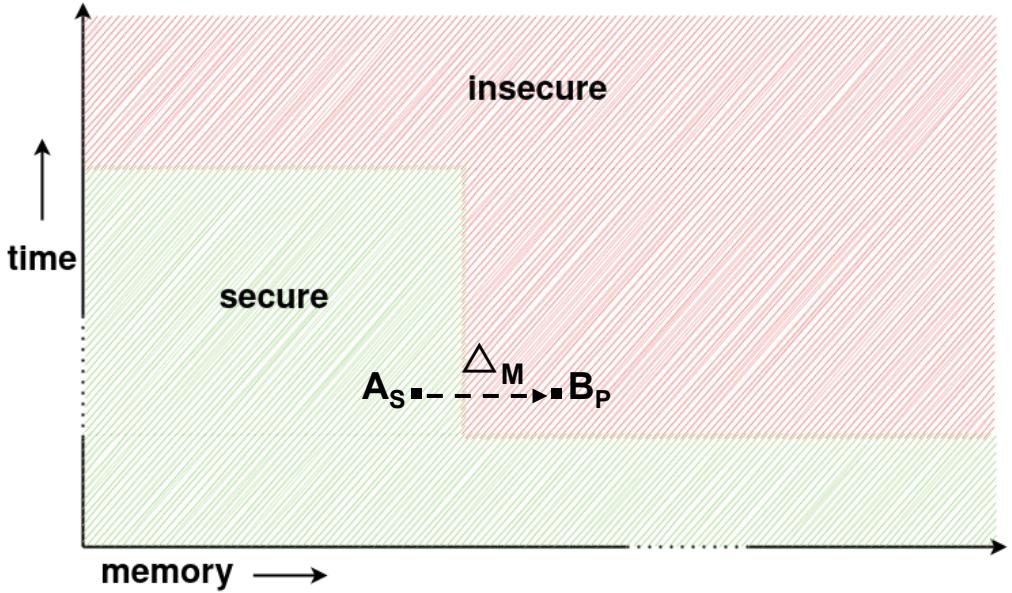

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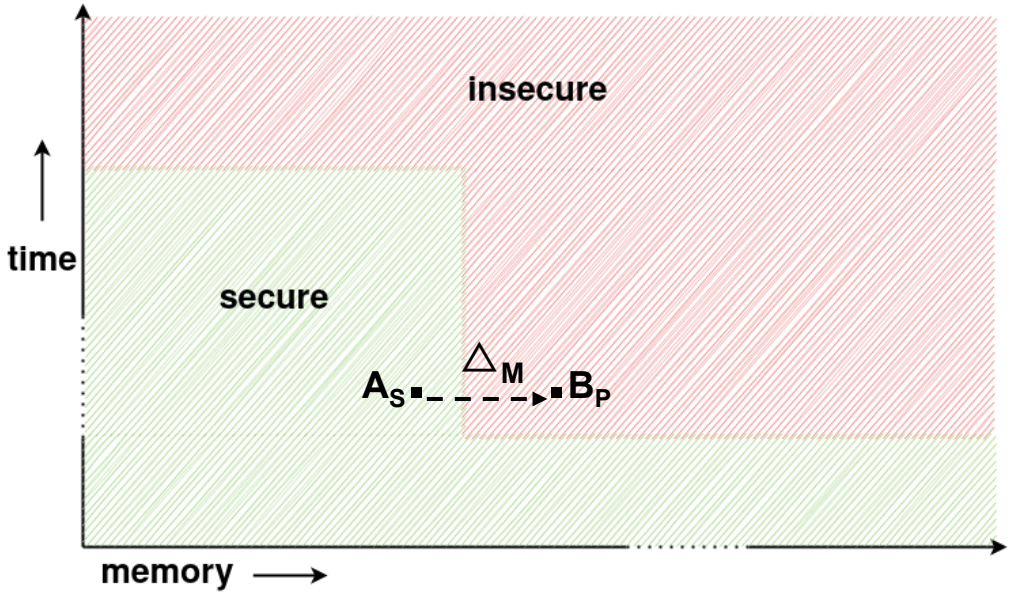

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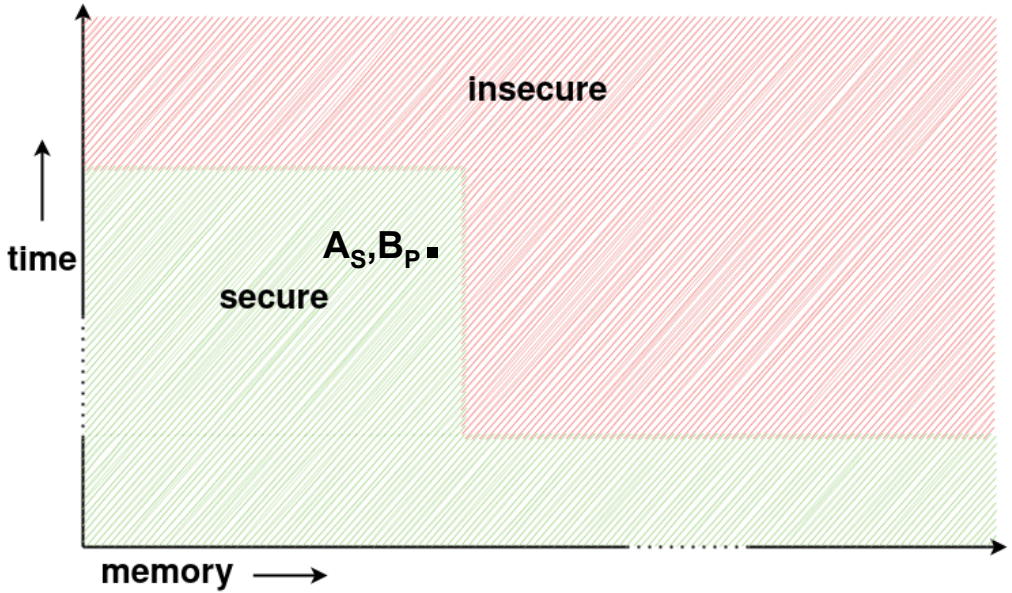

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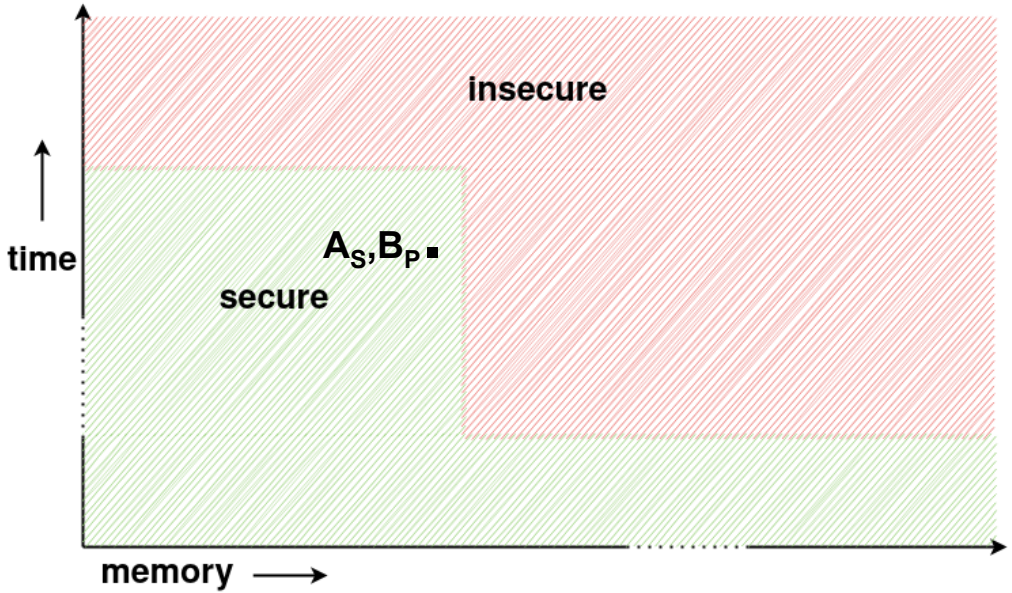

TAM-tight

Our Focus - TAM tightness of PKE

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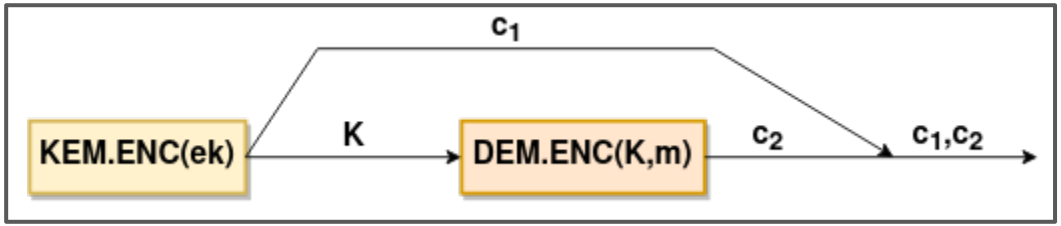

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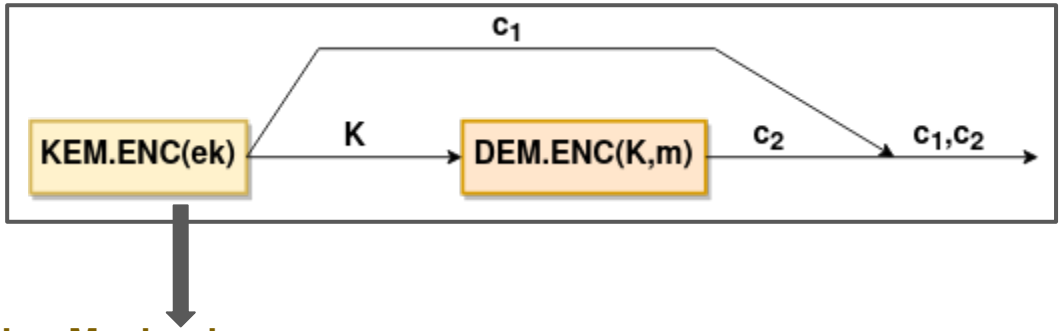

Key-Encapsulation Mechanism

[Bhattacharyya20]

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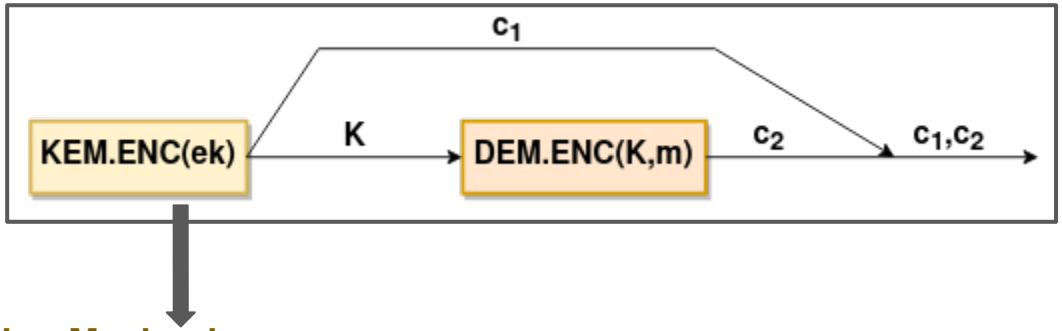

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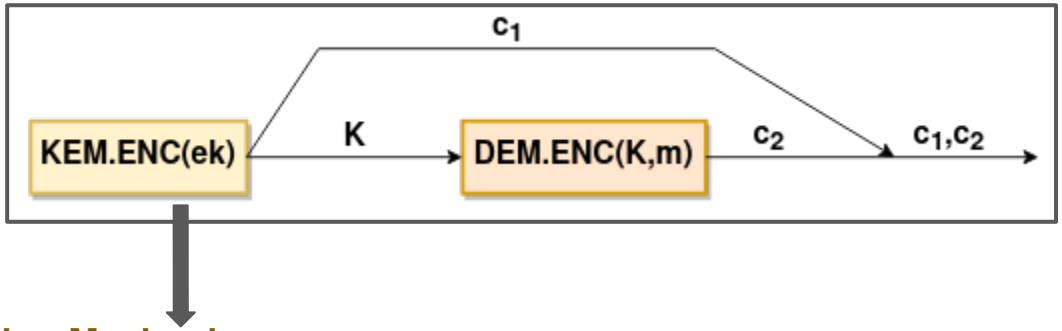

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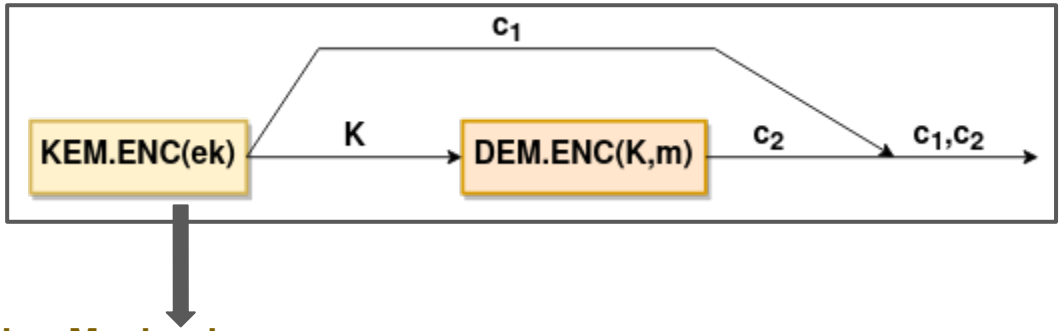

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- ➔ schemes get deployed across multiple users
- ➔ users use scheme multiple times
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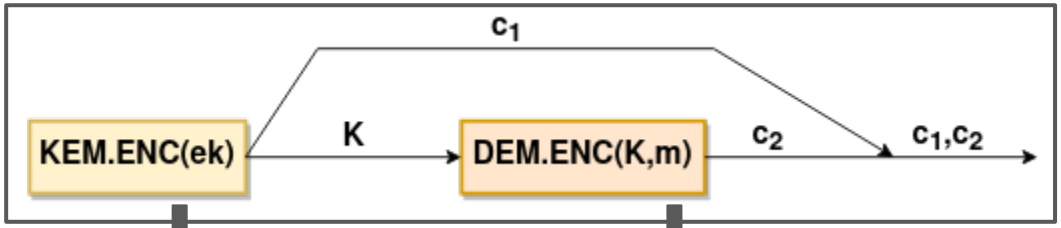

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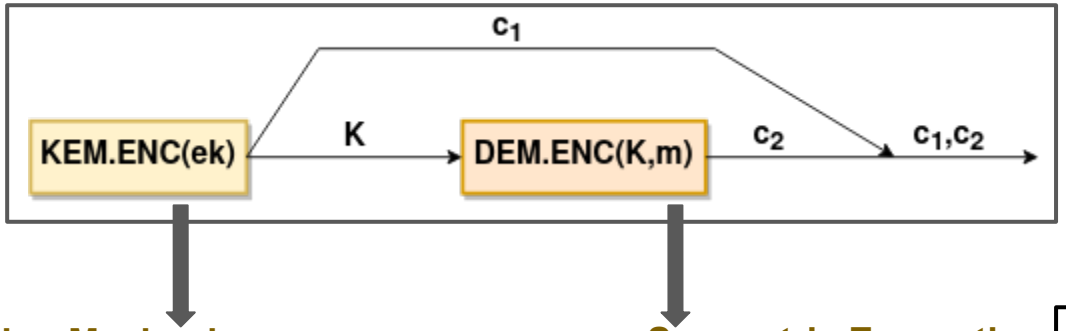

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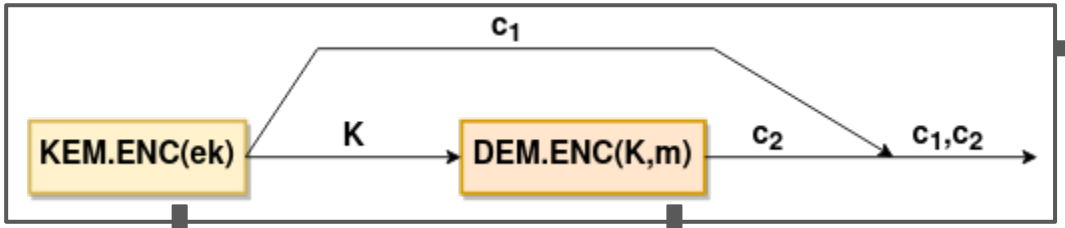

Composition

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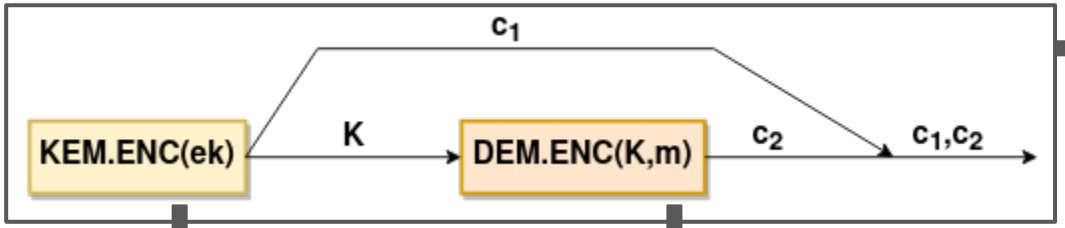

Composition [GGJT22]

Message Encoding

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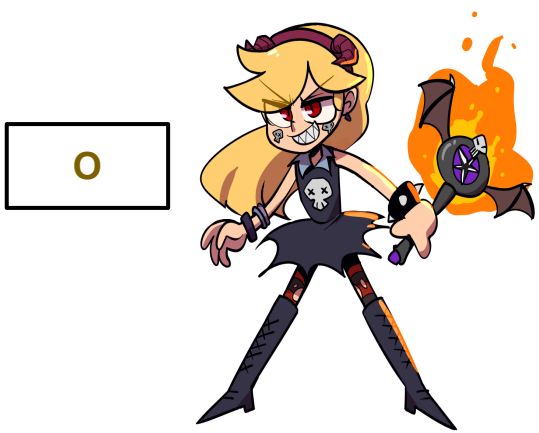

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c, k* ⟵ KEM.E(ek)	**c*, k*** ⟵ $

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Multiple **c*, k*** pairs
- ➔ Identify challenge ciphertexts in **SIMDEC**
- ➔ Identify challenge secrets in **SIMO**
- ➔ Consistent outputs from **SIMDEC**, **SIMO**

Our Contributions

Our Contributions

1. Multi-challenge/user TAM-tightness of “augmented” KEMs

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1. Multi-challenge/user TAM-tightness of “augmented” KEMs

Scheme S	**Problem P**
aECIES [AbdBelRog98]	Pair CDH
aCramer-Shoup [CraSha01b]	Strong CDH
aTwin ElGamal [CasKilSho09]	CDH
aFujisaki-Okamoto [FujOka13]	IND-CPA

Our Contributions

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Hashed ElGamal

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NEW!

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1. Multi-challenge/user TAM-tightness of “augmented” KEMs

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NEW!

2. Multi-challenge/user TAM-tightness of KEM/DEM

Hashed ElGamal KEMs

Hashed ElGamal KEMs

$\underline{\mathsf{KEM.K}}$
$x \leftarrow_{\$} \mathbb{Z}_p^*$
$ek \leftarrow \mathbf{g}^x$
$dk \leftarrow x$
Return (ek, dk)

ECIES	Cramer-Shoup
$\underline{\mathsf{KEM.E}^{\mathcal{H}}(ek)}$ $y \leftarrow_{\$} \mathbb{Z}_p^*$ $Y \leftarrow \mathbf{g}^y$ $Z \leftarrow ek^y$ $K \leftarrow \mathcal{H}(Z)$ Return (Y, K)	$\underline{\mathsf{KEM.E}^{\mathcal{H}}(ek)}$ $y \leftarrow_{\$} \mathbb{Z}_p^*$ $Y \leftarrow \mathbf{g}^y$ $Z \leftarrow ek^y$ $K \leftarrow \mathcal{H}(Y, Z)$ Return (Y, K)
$\underline{\mathsf{KEM.D}^{\mathcal{H}}(dk, Y)}$ $Z \leftarrow Y^{dk}$ $K \leftarrow \mathcal{H}(Z)$ Return K	$\underline{\mathsf{KEM.D}^{\mathcal{H}}(dk, Y)}$ $Z \leftarrow Y^{dk}$ $K \leftarrow \mathcal{H}(Y, Z)$ Return K

Hashed ElGamal KEMs

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The CDH game

Single Challenge

Given X, Y, find $\mathbf{g}^{xy}$ where $x = \mathrm{dlog}(X), y = \mathrm{dlog}(Y)$.

Hashed ElGamal KEMs

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$x \leftarrow_{\$} \mathbb{Z}_p^*$
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The CDH game

Single Challenge

Given X, Y, find $\mathbf{g}^{xy}$ where $x = \mathrm{dlog}(X), y = \mathrm{dlog}(Y)$.

Given $X, Y_1, Y_2, Y_3, ..., Y_n$, find $\mathbf{g}^{xy_i}$ for some $i \in [1, n]$ where $x = \mathrm{dlog}(X), y_i = \mathrm{dlog}(Y_i)$.

Multi-Challenge

Hashed ElGamal KEMs

KEM.K

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$ek \leftarrow \mathbf{g}^x$

$dk \leftarrow x$

Return (ek, dk)

ECIES	Cramer-Shoup
KEM.E$^{\mathcal{H}}(ek)$ $y \leftarrow_{\$} \mathbb{Z}_p^*$ $Y \leftarrow \mathbf{g}^y$ $Z \leftarrow ek^y$ $K \leftarrow \mathcal{H}(Z)$ Return (Y, K)	KEM.E$^{\mathcal{H}}(ek)$ $y \leftarrow_{\$} \mathbb{Z}_p^*$ $Y \leftarrow \mathbf{g}^y$ $Z \leftarrow ek^y$ $K \leftarrow \mathcal{H}(Y, Z)$ Return (Y, K)
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The CDH game

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Multi-Challenge

Single Challenge ⇒ Multi-Challenge (TAM tight from DH rerandomization!)

ECIES

ECIES

ECIES

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ENC

DEC

H

ECIES

ECIES

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$Z \leftarrow Y^{dk}$
$K \leftarrow \mathcal{H}(Z)$
Return K

Y_i, $H(Y_i^x)$

ENC

DEC

H

ECIES

ECIES

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ENC

Y

DEC

H

ECIES

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ENC

Y

$H(Y^x)$

DEC

H

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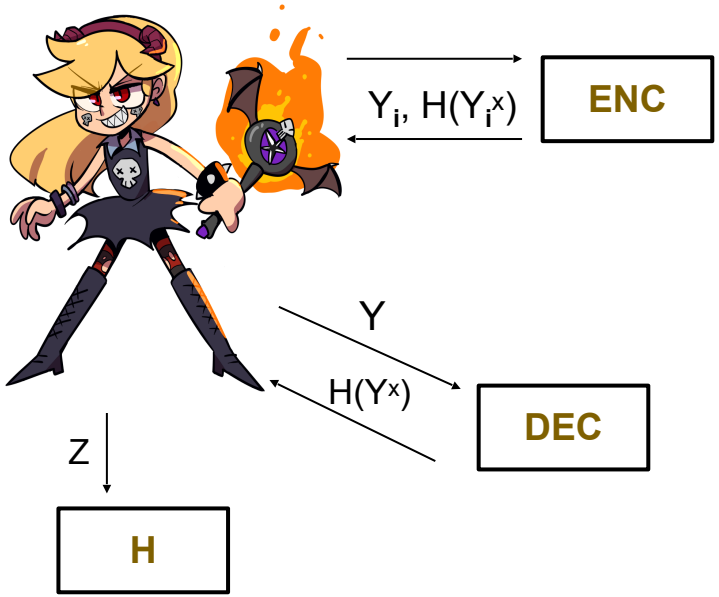

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$K \leftarrow \mathcal{H}(Z)$
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Y_i, $H(Y_i^x)$

ENC

Y

$H(Y^x)$

DEC

Z

H(Z)

H

$X, Y_1, Y_2, \ldots, Y_n$

REDUCTION R

Z

ECIES

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$Z \leftarrow Y^{dk}$
$K \leftarrow \mathcal{H}(Z)$
Return K

Y_i, $H(Y_i^x)$

ENC

SIMENC

Return Y_i, $H'(Y_i)$

Y

$H(Y^x)$

DEC

Z

H(Z)

H

$X, Y_1, Y_2, \ldots, Y_n$

REDUCTION R

Z

ECIES

ECIES

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$\underline{\mathsf{KEM.D}^{\mathcal{H}}(dk, Y)}$
$Z \leftarrow Y^{dk}$
$K \leftarrow \mathcal{H}(Z)$
Return K

Y_i, $H(Y_i^x)$

ENC

SIMENC

Return Y_i, $H'(Y_i)$

Y

$H(Y^x)$

DEC

Z

H(Z)

H

SIMDEC(Y)

Return H'(Y)

$X, Y_1, Y_2, \ldots, Y_n$

REDUCTION R

Z

ECIES

ECIES

$\underline{\mathrm{KEM.E}^{\mathcal{H}}(ek)}$
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Y_i, $H(Y_i^x)$

ENC

SIMENC

Return Y_i, $H'(Y_i)$

Y

$H(Y^x)$

DEC

Z

H(Z)

H

SIMH(Z)

Return H(Z)

SIMDEC(Y)

Return H'(Y)

$X, Y_1, Y_2, \ldots, Y_n$

REDUCTION R

Z

ECIES

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$\underline{\mathsf{KEM.E}^{\mathcal{H}}(ek)}$
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Y_i, $H(Y_i^x)$

ENC

SIMENC

Return Y_i, $H'(Y_i)$

Y

$H(Y^x)$

DEC

Z

H(Z)

H

SIMH(Z)

Return H(Z)

SIMDEC(Y)

Return H'(Y)

$X, Y_1, Y_2, \ldots, Y_n$

REDUCTION R

Z

Reduction Goals

- ➔ ~~Identify challenge ciphertexts in **SIMDEC**~~
- ➔ Identify challenge secrets in **SIMH**
- ➔ Consistent outputs from **SIMDEC**, **SIMH**

ECIES - Standard Proof - Not memory/time-tight

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SIMENC
Return Y_i, $H'(Y_i)$

SIMDEC(Y)
Return $H'(Y)$

SIMH(Z)
Return $H(Z)$

ECIES - Standard Proof - Not memory/time-tight

SIMENC
Return Y_i, $H'(Y_i)$

SIMDEC(Y)
Return H'(Y)

SIMH(Z)
Return H(Z)

$\underline{\textsc{Strong}(Y, Z)}$
$y \leftarrow \mathrm{dlog}(A)$
Return $(Z = X^y)$

ECIES - Standard Proof - Not memory/time-tight

SIMENC

Return Y_i, $H'(Y_i)$

SIMDEC(Y)

Return $H'(Y)$

SIMH(Z)

Return $H(Z)$

SIMH(Z)

If ∃ Y_i s.t. STRONG(Y_i, Z):
    Win with Z
If ∃ SIMDEC(Y) s.t. STRONG(Y, Z):
    Return $H'(Y)$
Return $H(Z)$

$\underline{\textsc{Strong}(Y, Z)}$
$y \leftarrow \text{dlog}(A)$
Return $(Z = X^y)$

ECIES - Standard Proof - Not memory/time-tight

SIMENC

Return Y_i, $H'(Y_i)$

SIMDEC(Y)

Return $H'(Y)$

SIMH(Z)

Return $H(Z)$

SIMH(Z)

If ∃ Y_i s.t. STRONG(Y_i, Z):
    Win with Z
If ∃ SIMDEC(Y) s.t. STRONG(Y, Z):
    Return H'(Y)
Return H(Z)

$\underline{\text{STRONG}(Y, Z)}$
$y \leftarrow \text{dlog}(A)$
Return $(Z = X^y)$

ECIES - Standard Proof - Not memory/time-tight

SIMENC

Return Y_i, $H'(Y_i)$

SIMDEC(Y)

Return $H'(Y)$

SIMH(Z)

Return $H(Z)$

SIMDEC(Y)

If ∃ HASH(Z) s.t. STRONG(Y, Z):
    Return H(Z)
Return $H'(Y)$

SIMH(Z)

If ∃ Y_i s.t. STRONG(Y_i, Z):
    Win with Z
If ∃ SIMDEC(Y) s.t. STRONG(Y, Z):
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Return H(Z)

$\underline{\text{STRONG}(Y, Z)}$
$y \leftarrow \text{dlog}(A)$
Return $(Z = X^y)$

ECIES - Standard Proof - Not memory/time-tight

SIMENC

Return Y_i, H'(Y_i)

SIMDEC(Y)

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SIMH(Z)

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$y \leftarrow \mathrm{dlog}(A)$
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SIMENC

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SIMDEC(Y)

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If ∃ SIMDEC(Y) s.t. STRONG(Y, Z):
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Return H(Z)

$\underline{\text{STRONG}(Y, Z)}$
$y \leftarrow \text{dlog}(A)$
Return $(Z = X^y)$

ECIES - Standard Proof - Not memory/time-tight

SIMENC

Return Y_i, H'(Y_i)

SIMDEC(Y)

Return H'(Y)

SIMH(Z)

Return H(Z)

SIMENC

Return Y_i, H'(Y_i)

SIMDEC(Y)

If ∃ HASH(Z) s.t. STRONG(Y, Z):
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Return H'(Y)

SIMH(Z)

If ∃ Y_i s.t. STRONG(Y_i, Z):
    Win with Z
If ∃ SIMDEC(Y) s.t. STRONG(Y, Z):
    Return H'(Y)
Return H(Z)

$\underline{\textsc{Strong}(Y, Z)}$
$y \leftarrow \text{dlog}(A)$
Return $(Z = X^y)$

ECIES - Standard Proof - Not memory/time-tight

SIMENC

Return Y_i, H'(Y_i)

SIMDEC(Y)

Return H'(Y)

SIMH(Z)

Return H(Z)

SIMENC

Return Y_i, H'(Y_i)

SIMDEC(Y)

If ∃ HASH(Z) s.t. STRONG(Y, Z):
    Return H(Z)
Return H'(Y)

SIMH(Z)

If ∃ Y_i s.t. STRONG(Y_i, Z):
    Win with Z
If ∃ SIMDEC(Y) s.t. STRONG(Y, Z):
    Return H'(Y)
Return H(Z)

$\underline{\text{STRONG}(Y, Z)}$
$y \leftarrow \text{dlog}(A)$
Return $(Z = X^y)$

Reduction Goals

- ➔ ~~Identify challenge ciphertexts in **SIMDEC**~~
- ➔ ~~Identify challenge secrets in **SIMH**~~
- ➔ ~~Consistent outputs from **SIMDEC**, **SIMH**~~

ECIES - Standard Proof - Not memory/time-tight

SIMENC

Return Y_i, $H'(Y_i)$

SIMDEC(Y)

Return $H'(Y)$

SIMH(Z)

Return $H(Z)$

SIMENC

Return Y_i, $H'(Y_i)$

SIMDEC(Y)

If ∃ HASH(Z) s.t. STRONG(Y, Z):
    Return H(Z)
Return H'(Y)

SIMH(Z)

If ∃ Y_i s.t. STRONG(Y_i, Z):
    Win with Z
If ∃ SIMDEC(Y) s.t. STRONG(Y, Z):
    Return H'(Y)
Return H(Z)

Not time/memory tight 😕

$\underline{\text{STRONG}(Y, Z)}$
$y \leftarrow \text{dlog}(A)$
Return $(Z = X^y)$

Reduction Goals

- ➔ ~~Identify challenge ciphertexts in **SIMDEC**~~
- ➔ ~~Identify challenge secrets in **SIMH**~~
- ➔ ~~Consistent outputs from **SIMDEC**, **SIMH**~~

Improvement - map-then-hash - [Bhattacharya20]

SIMENC

Return Y_i, $H'(Y_i)$

SIMDEC(Y)

If ∃ HASH(Z) s.t. STRONG(Y, Z):
    Return H(Z)
Return H'(Y)

SIMH(Z)

If ∃ Y_i s.t. STRONG(Y_i, Z):
    Win with Z
If ∃ SIMDEC(Y) s.t. STRONG(Y, Z):
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Improvement - map-then-hash - [Bhattacharya20]

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If ∃ Y_i s.t. STRONG(Y_i, Z):
    Win with Z
If ∃ SIMDEC(Y) s.t. STRONG(Y, Z):
    Return H'(Y)
Return H(Z)

Assumes a pairing friendly group

Improvement - map-then-hash - [Bhattacharya20]

SIMENC

Return Y_i, $H'(Y_i)$

SIMDEC(Y)

If ∃ HASH(Z) s.t. STRONG(Y, Z):
    Return H(Z)
Return H'(Y)

SIMH(Z)

If ∃ Y_i s.t. STRONG(Y_i, Z):
    Win with Z
If ∃ SIMDEC(Y) s.t. STRONG(Y, Z):
    Return H'(Y)
Return H(Z)

SIMH(Z)

If ∃ Y_i s.t. pair(X,Y_i) = pair(g, Z):
    Win with Z
Return H(pair(g,Z))

Assumes a pairing friendly group

Improvement - map-then-hash - [Bhattacharya20]

SIMENC

Return Y_i, $H'(Y_i)$

SIMDEC(Y)

If ∃ HASH(Z) s.t. STRONG(Y, Z):
    Return H(Z)
Return H'(Y)

SIMH(Z)

If ∃ Y_i s.t. STRONG(Y_i, Z):
    Win with Z
If ∃ SIMDEC(Y) s.t. STRONG(Y, Z):
    Return H'(Y)
Return H(Z)

SIMH(Z)

If ∃ Y_i s.t. pair(X,Y_i) = pair(g, Z):
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Improvement - map-then-hash - [Bhattacharya20]

SIMENC

Return Y_i, $H'(Y_i)$

SIMDEC(Y)

If ∃ HASH(Z) s.t. STRONG(Y, Z):
    Return H(Z)
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SIMH(Z)

If ∃ Y_i s.t. STRONG(Y_i, Z):
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Return H(Z)

SIMH(Z)

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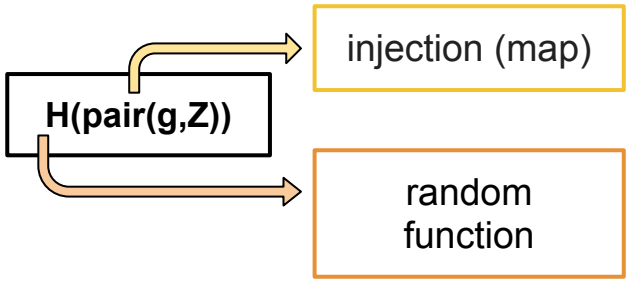
Assumes a pairing friendly group



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SIMH(Z)

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    Win with Z
If ∃ SIMDEC(Y) s.t. STRONG(Y, Z):
    Return H'(Y)
Return H(Z)

SIMENC

Return Y_i, $H'(Y_i)=H(pair(X,Y_i))$

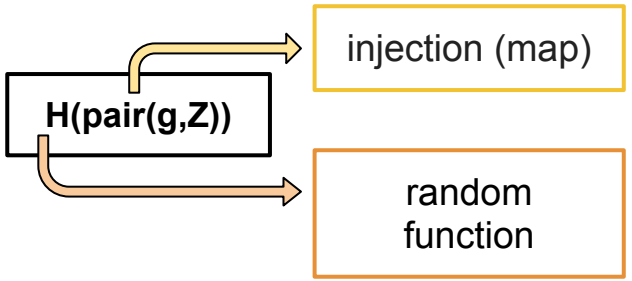
SIMDEC(Y)

Return $H'(Y)=H(pair(X,Y))$

SIMH(Z)

If ∃ Y_i s.t. $pair(X,Y_i) = pair(g, Z)$:
    Win with Z
Return H(pair(g,Z))

Assumes a pairing friendly group



Improvement - map-then-hash - [Bhattacharya20]

SIMENC

Return Y_i, $H'(Y_i)$

SIMDEC(Y)

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SIMENC

Return Y_i, $H'(Y_i)=H(pair(X,Y_i))$

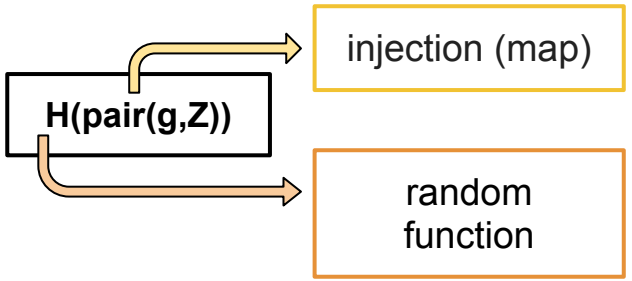
SIMDEC(Y)

Return $H'(Y)=H(pair(X,Y))$

SIMH(Z)

If ∃ Y_i s.t. $pair(X,Y_i) = pair(g, Z)$:
    Win with Z
Return H(pair(g,Z))

Assumes a pairing friendly group



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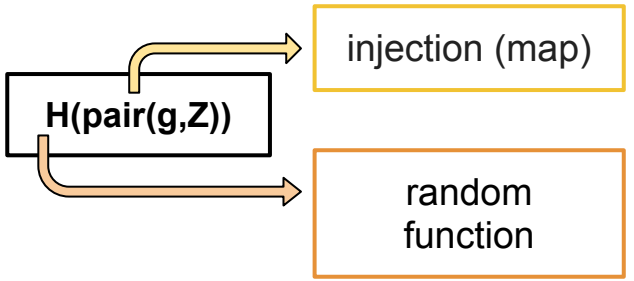

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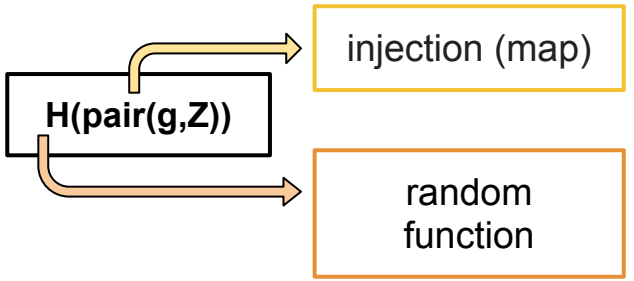

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Reduction checks ALL challenge ciphertexts for SIMH 😕

H(pair(g,Z))

injection (map)

random function

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Augmented Construction

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aECIES

aECIES.K

$x \leftarrow_{\$} \mathbb{Z}_p^*$
$ek \leftarrow \mathbf{g}^x$
$dk \leftarrow x$
Return (ek, dk)

aECIES.E$^{\mathcal{H}}(ek)$

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random string a included in ciphertexts and H queries

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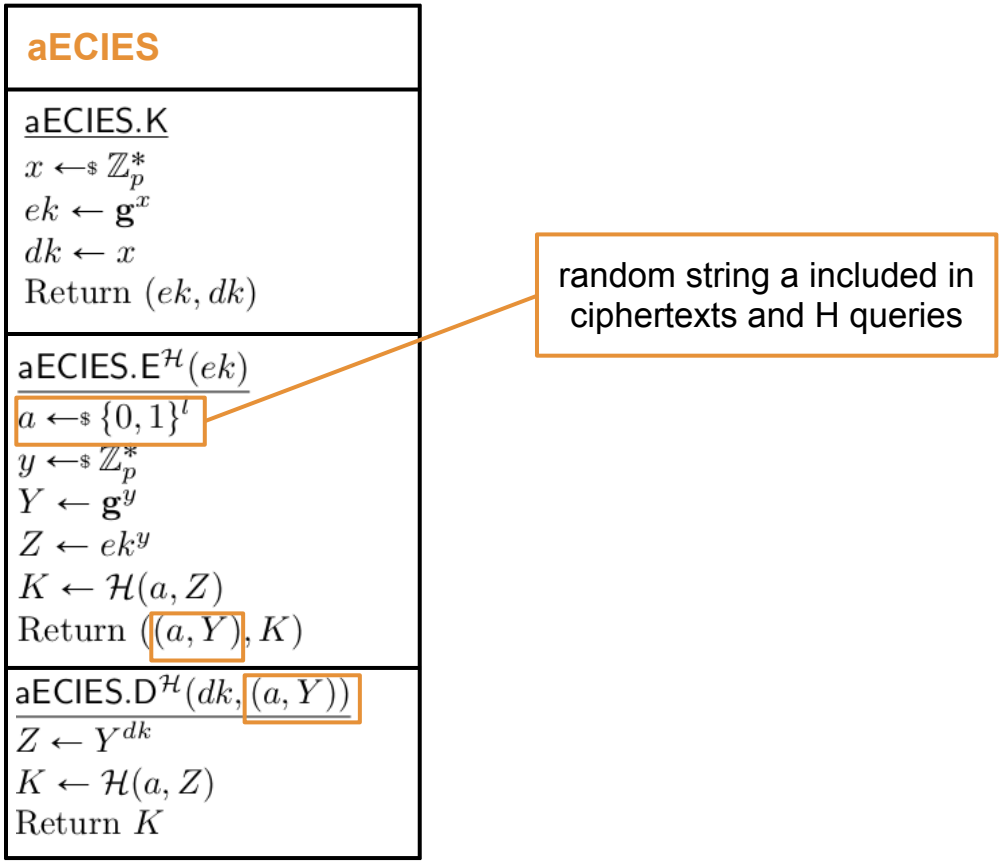

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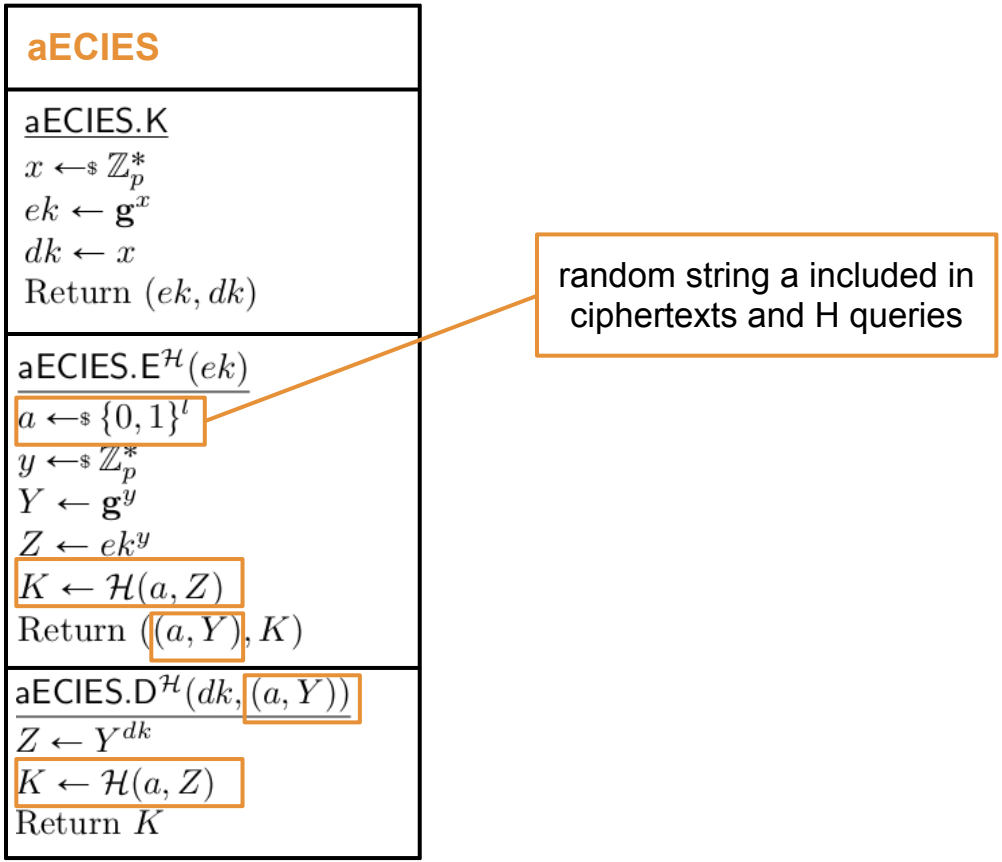

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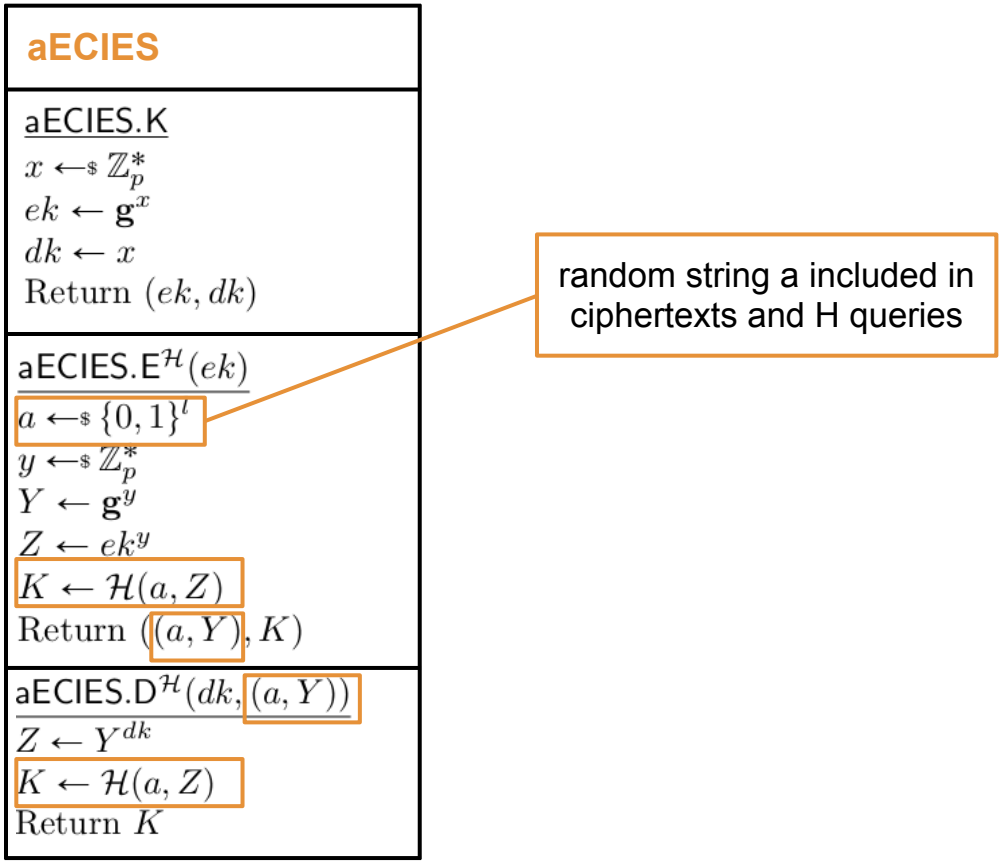

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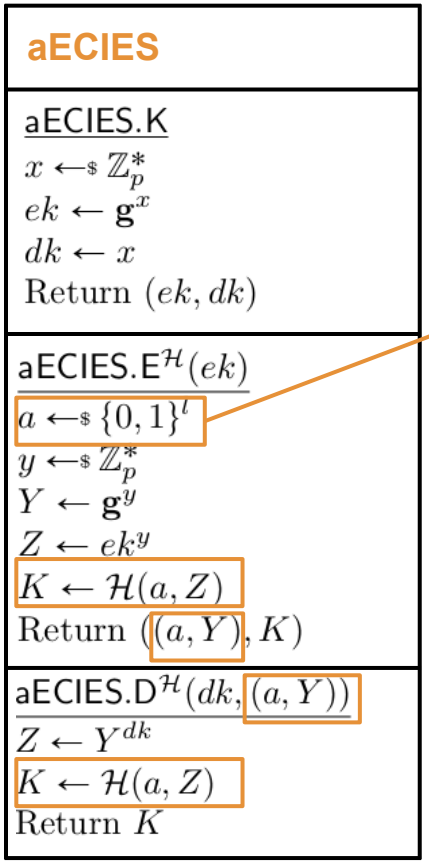

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Check one ciphertext in SIMH 🙂

TAM-Tight ✅

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Extensions

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➔ **Groups without pairings - The Pair CDH Assumption**

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Given $X, Y_1, Y_2, Y_3, ..., Y_n$, find $\mathbf{g}^{xy_i}$ for some $i \in [1, n]$
where $x = \text{dlog}(X), y_i = \text{dlog}(Y_i)$.

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$\underline{\text{PAIR}(A, B)}$
$a \leftarrow \text{dlog}(A);\ b \leftarrow \text{dlog}(B)$
Return $f(ab)$

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- ★ Attacker is given oracle access to a pairing to a “random group”
- ★ Heuristic justification: Showed hard in Algebraic / Generic Group model

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➔ Multi-user

a_i = encode(i) → a_i = encode(u,i)

Cramer-Shoup

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$\underline{\mathsf{KEM.E}^{\mathcal{H}}(ek)}$
$y \leftarrow_{\$} \mathbb{Z}_p^*$
$Y \leftarrow \mathbf{g}^y$
$Z \leftarrow ek^y$
$K \leftarrow \mathcal{H}(Y, Z)$
Return (Y, K)

$\underline{\mathsf{KEM.D}^{\mathcal{H}}(dk, Y)}$
$Z \leftarrow Y^{dk}$
$K \leftarrow \mathcal{H}(Y, Z)$
Return K

Y_i, $H(Y_i, Y_i^x)$

ENC

SIMENC

Return Y_i, $H'(Y_i)$

Y

$H(Y,Y^x)$

DEC

Y,Z

H(Y,Z)

H

SIMH(Y,Z)

Return H(Y,Z)

SIMDEC(Y)

Return H'(Y)

$X, Y_1, Y_2, \ldots, Y_n$

REDUCTION R

Z

Reduction Goals

- ➔ ~~Identify challenge ciphertexts in **SIMDEC**~~
- ➔ Identify challenge secrets in **SIMH**
- ➔ Consistent outputs from **SIMDEC**, **SIMH**

CS Standard Proof - Not memory/time-tight

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SIMDEC(Y)
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SIMH(Y,Z)
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$$\frac{\textsc{Strong}(Y, Z)}{y \leftarrow \text{dlog}(A)}$$
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Return Y_i, H'(Y_i)=H(Y_i,★)

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Reduction checks ALL challenge ciphertexts for SIMH 😕

Solution - Augment the scheme!
Check one ciphertext in SIMH 🙂
TAM-Tight ✅

Extensions

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➔ **Multi-user**

Extensions

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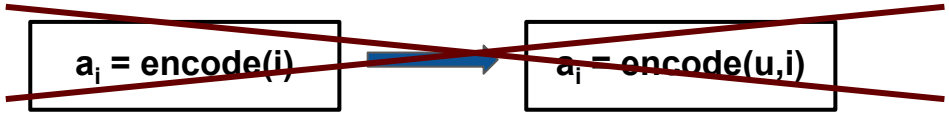

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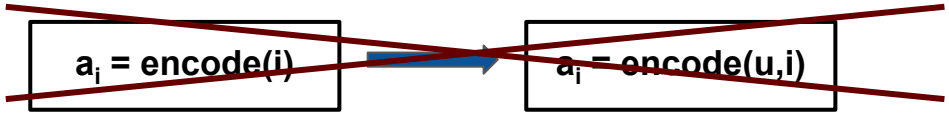

★ Also augment encapsulation key with a random string α!

Extensions

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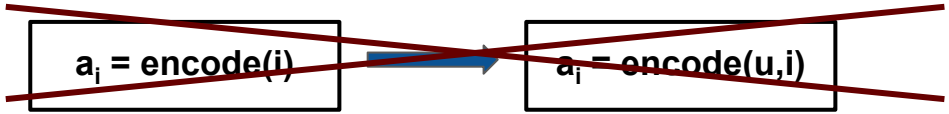

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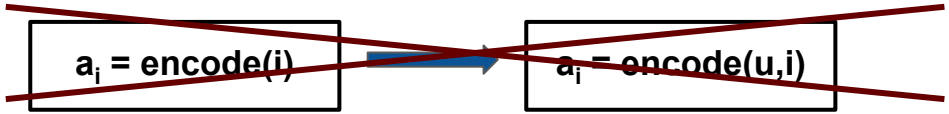
$\underline{\mathsf{aCS.K}}$	$\underline{\mathsf{aCS.E}^{\mathcal{H}}((\alpha, X))}$	$\underline{\mathsf{aCS.D}^{\mathcal{H}}((\alpha, x), (a, Y))}$
$\alpha \leftarrow\!\!\$\ \{0,1\}^{l_1}$	$a \leftarrow\!\!\$\ \{0,1\}^{l_2}$	$Z \leftarrow Y^x$
$x \leftarrow\!\!\$\ \mathbb{Z}_p^*$	$y \leftarrow\!\!\$\ \mathbb{Z}_p^*$	$K \leftarrow \mathcal{H}(\alpha, a, Y, Z)$
$ek \leftarrow (\alpha, \mathbf{g}^x)$	$Y \leftarrow \mathbf{g}^y$	Return K
$dk \leftarrow (\alpha, x)$	$Z \leftarrow X^y$	
Return (ek, dk)	$K \leftarrow \mathcal{H}(\alpha, a, Y, Z)$	
	Return $((a, Y), K)$	

Extensions

➔ **Multi-user**



★ Also augment encapsulation key with a random string α!

$\underline{\mathsf{aCS.K}}$	$\underline{\mathsf{aCS.E}^{\mathcal{H}}((\alpha, X))}$	$\underline{\mathsf{aCS.D}^{\mathcal{H}}((\alpha, x), (a, Y))}$
$\alpha \leftarrow\!\!\$\ \{0,1\}^{l_1}$	$a \leftarrow\!\!\$\ \{0,1\}^{l_2}$	$Z \leftarrow Y^x$
$x \leftarrow\!\!\$\ \mathbb{Z}_p^*$	$y \leftarrow\!\!\$\ \mathbb{Z}_p^*$	$K \leftarrow \mathcal{H}(\alpha, a, Y, Z)$
$ek \leftarrow (\alpha, \mathbf{g}^x)$	$Y \leftarrow \mathbf{g}^y$	Return K
$dk \leftarrow (\alpha, x)$	$Z \leftarrow X^y$	
Return (ek, dk)	$K \leftarrow \mathcal{H}(\alpha, a, Y, Z)$	
	Return $((a, Y), K)$	

random string **α** included in ciphertexts and H queries

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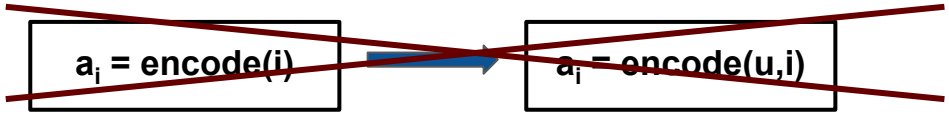

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In proof **α = encode'(u), $a_{u,i}$ = encode_u(i)**

Summary

Hashed ElGamal

Fujisaki-Okamoto

Scheme S	Problem P
aECIES [AbdBelRog98] ✅	Pair CDH
aCramer-Shoup [CraSha01b] ✅	Strong CDH
aTwin ElGamal [CasKilSho09]	CDH
aFujisaki-Okamoto [FujOka13]	IND-CPA
PKE = KD[KEM,SKE]	IND-CCA(KEM), IND-CCA(SKE)

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Thank You!
Questions?