PointProofs, Revisited

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Definition. Commit to a vector, and selectively open single entries

Position Binding

Security.

 \mathbf{X} Open to two different values at the same position \mathbf{X}



PointProofs

Gorbunov, Reyzin, Wee, Zhang, 2020 (CCS'20)

PointProofs:

- Vector commitment supporting non-interactive aggregations of proofs
- Supports commitment updates
- Very efficient
 - Applications: Blockchain smart contracts (reducing storage and bandwidth 60%)
- Security: Perfectly hiding & Computationally binding

Under n-wBDHE in AGM+ ROM

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Under n-wBDHE in AGM+ ROM

• Security: Perfectly hiding & Computationally binding

PointProofs:

- Vector commitment supporting non-interactive aggregations of proofs
- Supports commitment updates
- Very efficient
 - Applications: Blockchain smart contracts (reducing storage and bandwidth 60%) Ο
- Security: Perfectly hiding & Computationally binding

Our contribution. Under n-DHE in AGMI+ ROM

Removing AGM from the proof without changing the scheme (& from a weaker assumption)

PointProofs:

- Vector commitment supporting non-interactive aggregations of proofs
 - Same-commitment aggregations

Aggregating single-position proofs for the same commitment

→ Proofs for sub-vectors

• Cross-commitment aggregations

Aggregating proofs across different commitments

PointProofs:

- Vector commitment supporting non-interactive aggregations of proofs
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Cross-commitment aggregations

Aggregating proofs across different commitments









Security. Batch Binding

X Open to two different batches with different values at their intersection X



Security. Batch Binding

X Open to two different batches with different values at their intersection X



 $\exists i^\star \, \in S \cap S' \; ext{ s.t. } m[i^\star]
eq m'[i^\star] \; ig |$

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 $\operatorname{Verify}(C,S',m'\lceil S'
ceil,\pi')=1$

Security. Batch Binding



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Security. Batch Binding

X Open to two different batches with different values at their intersection X



Security. Batch Binding

X Open to two different batches with different values at their intersection X

5/17



Our contribution.

A new security proof for the PointProofs in the ROM without AGM, from a weaker assumption.

+ perfectly-hiding polynomial commitment with constant size batch openings inspired by PointProofs techniques (full version).

Our contribution.

A new security proof for the PointProofs in the ROM without AGM, from a weaker assumption.

Generic Group Model (GGM) [Sho97]

- Only *generic* adversaries are considered.

X Only giving access to a random representation of the group.

Computing group operations via oracle queries.

Black-Box Access To The Group

Algebraic Group Model (AGM) [FKL18]

- Only *algebraic* adversaries are considered.
- ✓ No restriction on the access to the group representation

$$\bigstar \quad \text{If} \; \mathcal{A} \to z \, \in G, \; \text{then} \;$$

$$\mathcal{A} o (a_1,\ldots,a_t) \,\in\, \mathbb{Z}_p^t \, ext{ s.t. } z = \prod_{i=1}^t L_i^{a_i}$$

for known (L_1, \ldots, L_t) .

Our contribution.

A new security proof for the PointProofs in the ROM without AGM, from a weaker assumption.



PointProofs [GRWZ'20]

Improved Proof of Same-Commitment Aggregation

PointProofs [GRWZ'20]

Construction

• Public Parameters

$$\mathbb{G}=\langle g
angle ext{ of prime order } p, \left(g,g^lpha,g^{lpha^2},\ldots,g^{lpha^n},oldsymbol{X},g^{lpha^{n+2}},\ldots,g^{lpha^{2n}}
ight)$$

$$\begin{array}{c} \bullet \quad \text{Commit to } \vec{m} = (m_1, \ldots, m_n) \\ & 1 \quad \alpha^1 \quad \alpha^i \quad \alpha^n \quad \alpha^{n+1} \quad \alpha^{2n} \\ \hline \gamma \quad m_1 \quad \ldots \quad m_i \quad \ldots \quad m_n \quad 0 \quad \cdots \quad 0 \\ \end{array} \\ C = g^\gamma \cdot \prod_{j=1}^n g^{\alpha^j \cdot m_j} = g \end{array}$$

• Public Parameters

$$\mathbb{G}=\langle g
angle ext{ of prime order } p, \left(g,g^lpha,g^{lpha^2},\ldots,g^{lpha^n},oldsymbol{X},g^{lpha^{n+2}},\ldots,g^{lpha^{2n}}
ight)$$

• Commit to
$$\vec{m} = (m_1, \dots, m_n)$$

1 α^1 α^i α^n α^{n+1} α^{2n}
 γ m_1 \dots m_i \dots m_n 0 \dots 0
 $C = g^{\gamma} \cdot \prod_{j=1}^n g^{\alpha^j \cdot m_j} = g$
• Open the position i
 $\pi_i = \left(C/g^{\alpha^i \cdot m_i}\right)^{\alpha^{n+1-i}} = g$
Remove and shift
 $\pi_i = g$

$$egin{aligned} m{C} &= g^{\gamma} \cdot \prod_{j=1}^n g^{lpha^j \cdot m_j} \ &= g^{rac{1-lpha^1-lpha^1-lpha^i-lpha^n-lpha^{n+1}-lpha^{2n}}{\gamma - m_1 \cdots - m_i \cdots - m_n - 0} \end{aligned}$$

• Open the position *i*

$$\pi_i = \left(C/g^{lpha^i \cdot m_i}
ight)^{lpha^{n+1-i}} = g^{rac{1 \ lpha^1 \ lpha^1 \ lpha^{n+1-i} \ lpha^n \ lpha^{n+1-i} \ lpha^n \ lpha^{n+1-i} \ lpha^{2n+1-i} \ lpha^{2n}} = g$$

$$egin{aligned} m{C} &= g^{\gamma} \cdot \prod_{j=1}^n g^{lpha^j \cdot m_j} \ &= g^{rac{1-lpha^1-lpha^1-lpha^i-lpha^n-lpha^{n+1}-lpha^{2n}}{\gamma - m_1 \cdots - m_i \cdots - m_n - 0} \end{aligned}$$

• Verify
$$(C, i, \pi_i, m_i)$$

$$e\Big(oldsymbol{C}, {\hat{g}}^{lpha^{n+1-i}}\Big) \,=\, e(\pi_{oldsymbol{i}}, {\hat{g}}) \cdot g_T^{m_{oldsymbol{i}} \cdot lpha^{n+1}}$$

• Open the position *i*

$$egin{aligned} \pi_{i} &= \left(C/g^{lpha^{i}\cdot m_{i}}
ight)^{lpha^{n+1-i}} & \ &= g \end{aligned} = g \end{array}$$









• Public Parameters

$$\mathbb{G} = \langle g
angle ext{ of prime order } p, \left(g, g^{lpha}, g^{lpha^2}, \dots, g^{lpha^n}, oldsymbol{X}, g^{lpha^{n+2}}, \dots, g^{lpha^{2n}}
ight) \ \left(\hat{g}, \hat{g}^{lpha}, \dots, \hat{g}^{lpha^n}
ight), \quad \hat{\mathbb{G}} = \langle \hat{g}
angle, \quad e: \mathbb{G} imes \hat{\mathbb{G}} o \mathbb{G}_T, \quad g_T = e(g, \hat{g})$$

• Commit to $ec{m}=(m_1,\ldots,m_n)$

• Verify
$$(C, i, \pi_i, m_i)$$

$$C=g^{\gamma}\cdot\prod_{j=1}^n g^{lpha^j\cdot m_j}$$

$$e\Big(C, {\hat g}^{lpha^{n+1-i}}\Big) \,=\, e(\pi_i, {\hat g}) \cdot g_T^{m_i \cdot lpha^{n+1}}$$

• Open the position *i*

$$\pi_{i}=\left(C/g^{lpha^{i}\cdot m_{i}}
ight) ^{lpha^{n+1-i}}$$

• Public Parameters

$$\mathbb{G} = \langle g
angle ext{ of prime order } p, \left(g, g^{lpha}, g^{lpha^2}, \dots, g^{lpha^n}, \mathbf{X}, g^{lpha^{n+2}}, \dots, g^{lpha^{2n}}
ight)$$
 it is hard to compute $g^{lpha^{n+1}}$
 $\left(\hat{g}, \hat{g}^{lpha}, \dots, \hat{g}^{lpha^n}
ight), \quad \hat{\mathbb{G}} = \langle \hat{g}
angle, \quad e: \mathbb{G} imes \hat{\mathbb{G}} o \mathbb{G}_T, \quad g_T = e(g, \hat{g})$

• Commit to $ec{m}=(m_1,\ldots,m_n)$

• Verify
$$(C,i,\pi_i,m_i)$$

$$C=g^{\gamma}\cdot\prod_{j=1}^ng^{lpha^j\cdot m_j}$$

$$e\Big(C, {\hat g}^{lpha^{n+1-i}}\Big) \,=\, e(\pi_i, {\hat g}) \cdot g_T^{m_i \cdot lpha^{n+1}}$$

• Open the position *i*

$$\pi_{i}=\left(C/g^{lpha^{i}\cdot m_{i}}
ight) ^{lpha^{n+1-i}}$$

n-DHE assumption:

• Verify (C,i,π_i,m_i)

 $1 \quad \alpha^1$

0

 g_T

0

$$e\left(C,\hat{g}^{\alpha^{n+1-i}}\right) = e(\pi_{i},\hat{g}) \cdot g_{T}^{m_{i} \cdot \alpha^{n+1}}$$

$$\stackrel{\alpha^{n+1-i} \alpha^{n} \alpha^{n+1} \alpha^{2n+1-i} \alpha^{2n}}{\cdots \gamma \cdots m_{i-1}} \times \cdots \times m_{n} \cdots 0 = \begin{pmatrix} 1 & \alpha^{1} & \alpha^{n+1-i} & \alpha^{n} \alpha^{n+1} & \alpha^{2n+1-i} & \alpha^{2n} \\ 0 & 0 & \cdots & \gamma & \cdots & m_{i-1} \times \cdots & m_{n} & \cdots & 0 \\ & & & & & \\ \hline 0 & 0 & \cdots & 0 & \cdots & 0 & m_{i} & \cdots & 0 & \cdots & 0 \\ & & & & & & \\ g_{T}$$

• Verify (C, i, π_i, m_i)

$$e\left(C,\hat{g}^{\alpha^{n+1-i}}\right) = e(\pi_{i},\hat{g}) \cdot g_{T}^{m_{i} \cdot \alpha^{n+1}}$$

$$\stackrel{\alpha^{n+1-i} \alpha^{n} \alpha^{n+1} \alpha^{2n+1-i} \alpha^{2n}}{\stackrel{\alpha^{n+1-i} \alpha^{n} \alpha^{n+1} \alpha^{2n+1-i} \alpha^{2n}}{g_{T}} = \begin{array}{c} 1 & \alpha^{1} & \alpha^{n+1-i} \alpha^{n} \alpha^{n+1} & \alpha^{2n+1-i} \alpha^{2n} \\ \hline 0 & 0 & \cdots & \gamma & \cdots & m_{i-1} \not X & \cdots & m_{n} & \cdots & 0 \\ \hline 0 & 0 & \cdots & 0 & \cdots & 0 & m_{i} & \cdots & 0 \\ g_{T} & g_{T} & g_{T} \end{array}$$

 • Verify (C,i,π_i,m_i)

1

0

$$e\left(C,\hat{g}^{\alpha^{n+1-i}}\right) = e(\pi_{i},\hat{g}) \cdot g_{T}^{m_{i} \cdot \alpha^{n+1}}$$

$$\stackrel{\alpha^{n+1-i} \alpha^{n} \alpha^{n+1} \alpha^{2n+1-i} \alpha^{2n}}{\stackrel{\alpha^{n+1-i} \alpha^{n} \alpha^{n+1} \alpha^{2n+1-i} \alpha^{2n}}{g_{T}} = \underbrace{\begin{array}{c}1 \alpha^{1} \alpha^{n+1-i} \alpha^{n} \alpha^{n+1} \alpha^{2n+1-i} \alpha^{2n} \\ 0 0 \cdots \gamma \cdots m_{i-1} \mathbf{X} \cdots m_{n} \cdots 0 \\ \mathbf{y}_{T} \cdots \mathbf{y} \cdots \mathbf{y}_{T} \cdots \mathbf{y}_{T} \cdots \mathbf{y}_{T} \\ g_{T} \end{array}}$$

Binding
$$\checkmark$$
 $(C, \pi_i) \mapsto m_i$

PointProofs [GRWZ'20]

Same-Commitment Proof Aggregation

$$C=g^{\gamma}\cdot\prod_{j=1}^n g^{lpha^j\cdot m_j}$$

• Open the position *i*

$$\pi_i = \left(C/g^{lpha^i \cdot m_i}
ight)^{lpha^{n+1-i}}$$

• Aggregate $(\pi_i)_{i \in S}$

$$\pi_S = \prod_{i \in S} {\pi_i}^{h_i}; \hspace{1em} h_i = H(i, C, S, m[S])
onumber \ (H: \mathbb{G}
ightarrow \mathbb{Z}_p)$$

$$C = g^{\gamma} \cdot \prod_{j=1}^{n} g^{\alpha^{j} \cdot m_{j}}$$

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$$\pi_{S} = \prod_{i \in S} \pi_{i}^{h_{i}}; \quad h_{i} = H(i, C, S, m[S])$$

$$(H : \mathbb{G} \to \mathbb{Z}_{p})$$

$$h_{i} \propto \frac{1 \quad \alpha^{1} \qquad \alpha^{n+1-i} \quad \alpha^{n} \quad \alpha^{n+1-i} \quad \alpha^{2n+1-i} \quad \alpha^{2n}}{1 \quad \alpha^{n-1-i}}$$

$$\bullet \text{ h}_{i} \times \frac{1 \quad \alpha^{1} \qquad \alpha^{n+1-i} \quad \alpha^{n} \quad \alpha^{n+1} \quad \alpha^{2n+1-i} \quad \alpha^{2n}}{1 \quad \alpha^{n} \quad \cdots \quad n_{n-1}} \times \cdots \quad m_{n} \quad 0$$

$$\vdots$$

$$h_{j} \times 0 \quad 0 \quad \cdots \quad \gamma \quad \cdots \quad m_{j-1} \times \cdots \quad m_{n} \quad 0$$

$$\vdots$$

$$h_{k} \times 0 \quad 0 \quad \cdots \quad \cdots \quad m_{k-1} \times \cdots \quad m_{n} \quad 0$$

$$g$$

$$Missing Part of \pi_{S} \cdot \sum_{i \in S} m_{i} \cdot h_{i}$$

$$C = g^{\gamma} \cdot \prod_{j=1}^{n} g^{\alpha^{j} \cdot m_{j}}$$

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$$(H : \mathbb{G} \to \mathbb{Z}_{p})$$

$$\bullet \frac{1}{\alpha^{1}} \xrightarrow{\alpha^{n+1-i}} \alpha^{n} \alpha^{n+1-i} \alpha^{2n}}{\alpha^{n+1-i}} \xrightarrow{\alpha^{2n+1-i}} \alpha^{2n}}$$

$$C=g^{\gamma}\cdot\prod_{j=1}^ng^{lpha^j\cdot m_j}$$

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onumber \ (H: \mathbb{G}
ightarrow \mathbb{Z}_p)$$

• Verify $ig(C,S,\{\pi_i\}_{i\,\in S},m[S]ig)$

$$\prod_{i\in S} e\Big(C, \hat{g}^{lpha^{n+1-i}}\Big)^{h_i} = e(\pi_S, \hat{g}) \cdot \prod_{i\in S} g_T^{m_i \cdot h_i \cdot lpha^{n+1}}$$
Shift and sum all of the committed values Adding the missing values to the proofs $\sum_{i\in S} m_i \cdot h_i$

$$C=g^{\gamma}\cdot\prod_{j=1}^ng^{lpha^j\cdot m_j}$$

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Shift and sum all of the committed values Adding the missing values to the proofs $\sum_{i\in S} m_i \cdot h_i$

$$\mathcal{A} o (\emph{C}, S_0, S_1, ec{m_0}[S_0], ec{m_1}[S_1], \pi_0, \pi_1)$$

such that

 $\exists i^{\star} \in S_0 \cap S_1 \;\; s.t. \;\; ec{m_0}[i^{\star}]
eq ec{m_1}[i^{\star}].$

$$\mathcal{A} o (\emph{C}, S_0, S_1, ec{m_0}[S_0], ec{m_1}[S_1], \pi_0, \pi_1)$$

such that

$$\mathrm{Verify}({\color{black} C},S_0,ec{m_0}[S_0],\pi_0)=1$$



 $\mathrm{Verify}({\color{black} C},S_1,ec{m_1}[S_1], \pi_1)=1$

 \wedge

 \wedge

 $\exists i^{\star} \, \in S_0 \cap S_1 \; \; s.t. \; \; ec{m_0}[i^{\star}]
eq ec{m_1}[i^{\star}].$



$$\mathcal{A} o (\emph{C}, S_0, S_1, ec{m_0}[S_0], ec{m_1}[S_1], \pi_0, \pi_1)$$

such that

 $(C,\pi_0) \mapsto \sum m_0[i] \cdot h_i^{(0)}$ $Verify(C, S_0, \vec{m_0}[S_0], \pi_0) = 1$ $i \in S_0$ Λ $(C,\pi_1) \mapsto \sum m_1[i] \cdot h_i^{(1)}$ $Verify(C, S_1, \vec{m_1}[S_1], \pi_1) = 1$ $i{\in}S_1$ Get rid of other elements to find conflicting proofs for the single position i^{\star} $m_1[i^\star]\cdot h$ $\exists i^{\star} \in S_0 \cap S_1 \;\; s.t. \;\; ec{m_0}[i^{\star}]
eq ec{m_1}[i^{\star}].$

 $-m_0[i^\star]\cdot h_i^($

$$\mathcal{A} o (\emph{C}, S_0, S_1, ec{m_0}[S_0], ec{m_1}[S_1], \pi_0, \pi_1)$$

such that



 $_{-}m_0[i^{\star}]\cdot h_i^{(}$

PointProofs [GRWZ'20]

Improved Proof of Same-Commitment Aggregation

Using

Local Forking Lemma

Bellare, Dai, Li (Asiacrypt'19)















• Extracting two conflicting proofs for position i^* from two conflicting batches:

$$egin{aligned} &eigl(C, {\hat g}^{lpha^{n+1-i^\star}}igr) = eiggl(igl(\pi_0/\pi_0'igr)^{1/\Delta h_{i^\star}^{(0)}}, {\hat g}igr) \cdot g_T^{lpha^{n+1}\cdot m_0[i^\star]} \ &eigl(C, {\hat g}^{lpha^{n+1-i^\star}}igr) = eiggl(igl(\pi_1/\pi_1''igr)^{1/\Delta h_{i^\star}^{(1)}}, {\hat g}igr) \cdot g_T^{lpha^{n+1}\cdot m_1[i^\star]} \end{aligned}$$

• Contradicting position binding.

• Solving n-DHE problem. 🗡

• Extracting two conflicting proofs for position i^* from two conflicting batches:

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• Contradicting position binding.

Batch Binding 🗸

• Solving n-DHE problem. 🗡

Conclusion

- A new security proof for the PointProofs in the ROM (without AGM) without changing the scheme.
 - Same-commitment (generalized or local forking lemma)
 - Cross-commitment (local forking lemma)

• Proposing the first perfectly-hiding polynomial commitment with optimal batch openings (1 group element) inspired by PointProofs techniques.

Based on inner-product functional commitment of Libert, Ramanna, and Yung (ICALP'16), under the n-DHE assumption.

Conclusion

- A new security proof for the PointProofs in the ROM (without AGM) without changing the scheme.
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• Proposing the first perfectly-hiding polynomial commitment with optimal batch openings (1 group element) inspired by PointProofs techniques.

Based on inner-product functional commitment of Libert, Ramanna, and Yung (ICALP'16), under the n-DHE assumption.