Linear-Map Vector Commitments

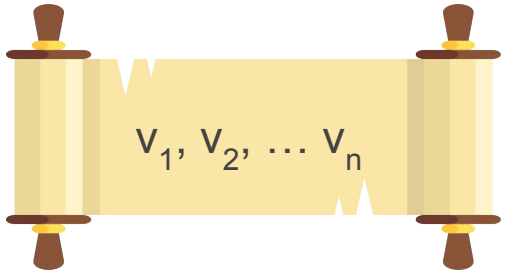
and their Practical Applications

with Matteo Campanelli, Carla Ràfols, Alexandros Zacharakis, Arantxa Zapico

Anca Nitulescu

Protocol Labs

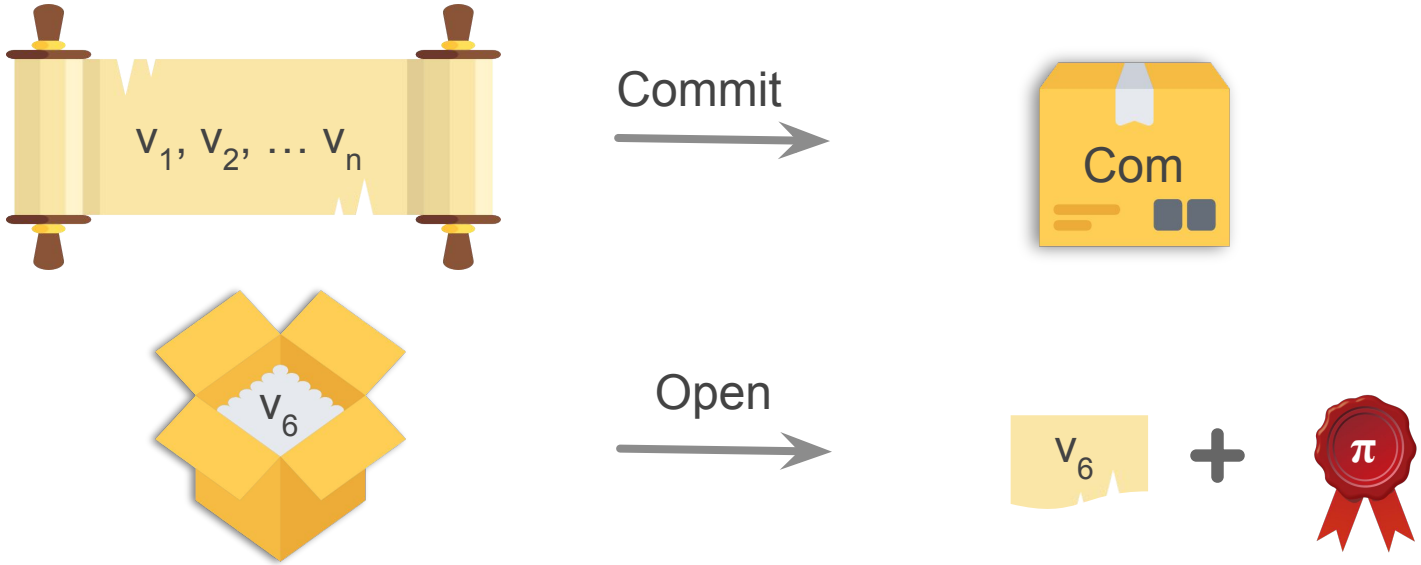
Vector Commitments



Commit

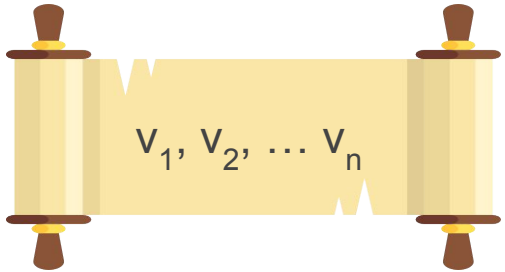
Vector Commitments and their Applications, by Dario Catalano and Dario Fiore

Vector Commitments



Vector Commitments and their Applications, by Dario Catalano and Dario Fiore

Binding



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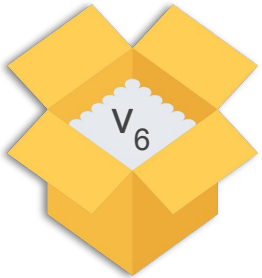
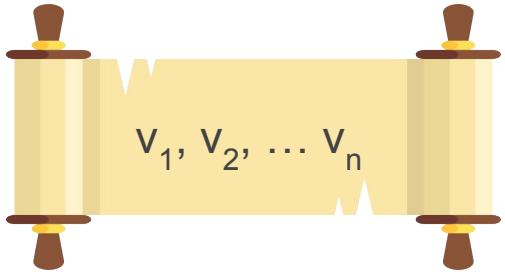


Open

SubVector Commitment



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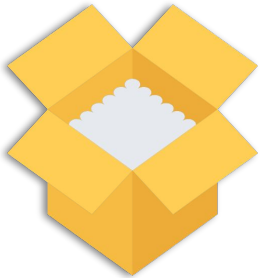
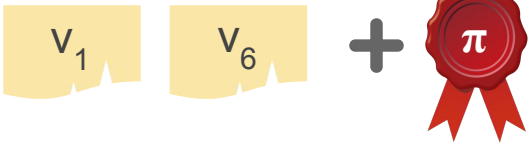
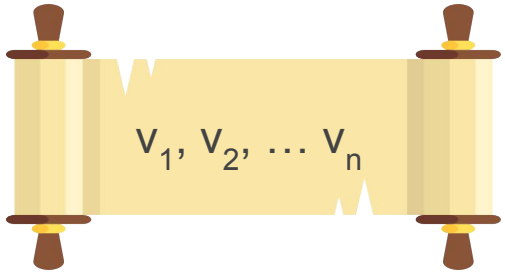


Open

subvector



Functional VC



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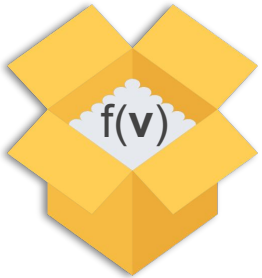
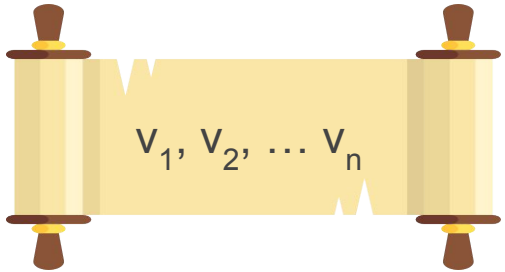


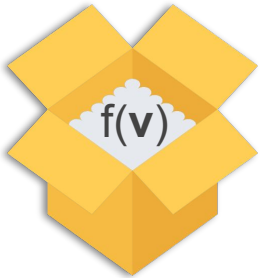
Open

Linear-Map VC [LM19]



Commit

$\mathbf{v} \in \mathbb{F}^n$

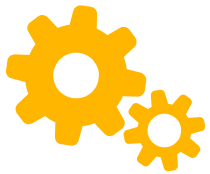


Open

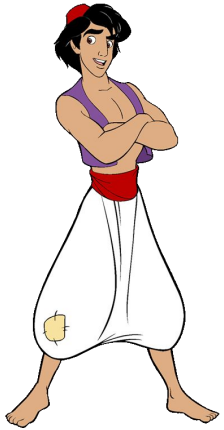
$f : \mathbb{F}^n \to \mathbb{F}^\ell$

Motivation

Storage Delegation



V_1 V_2 V_3 V_4 V_5



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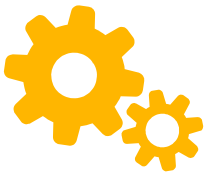
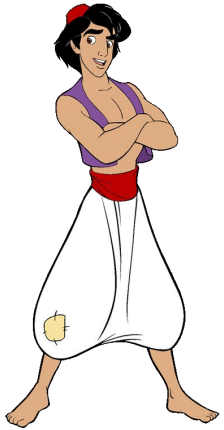


V_1 V_2 V_3 V_4 V_5



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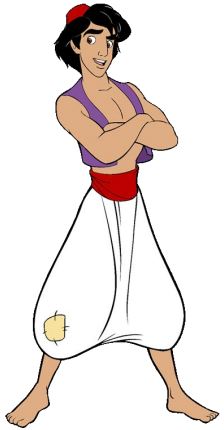
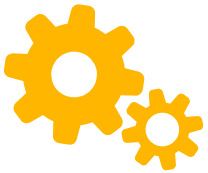
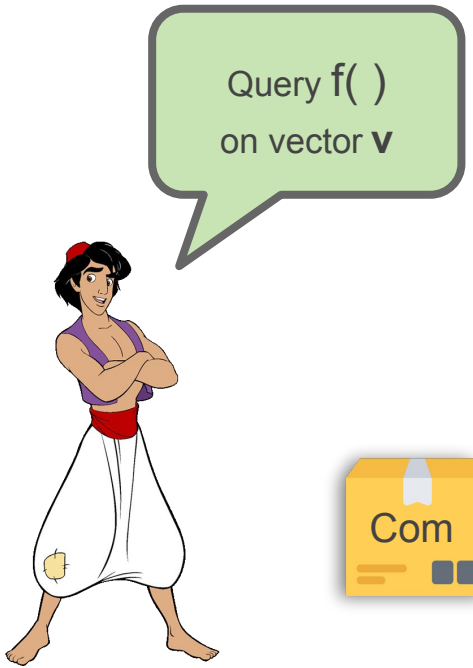


Storage Delegation





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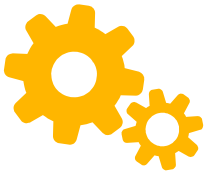

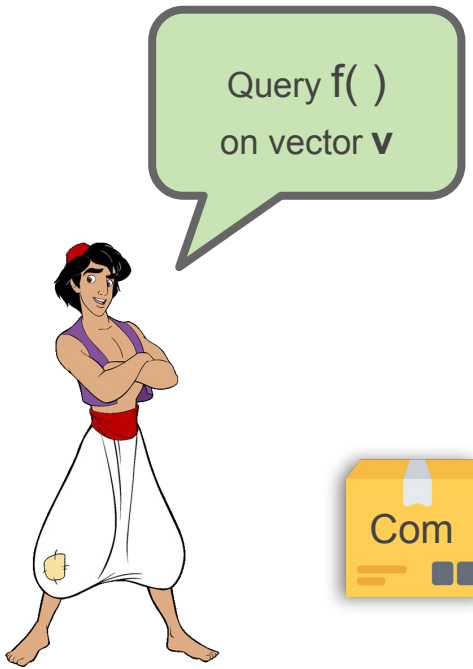

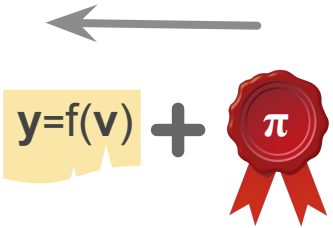

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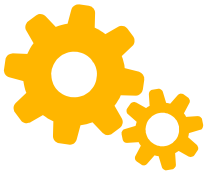
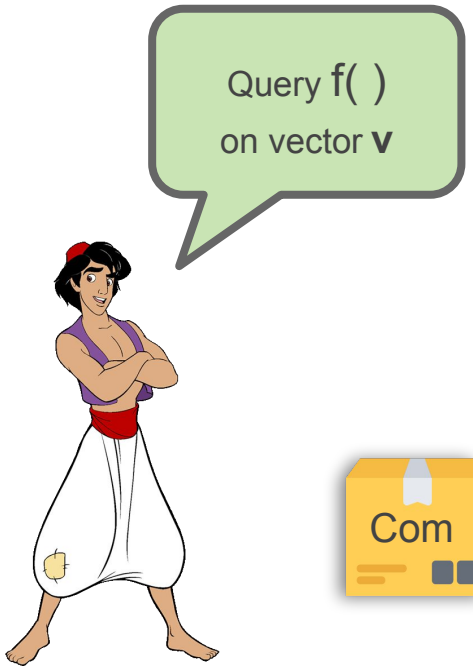
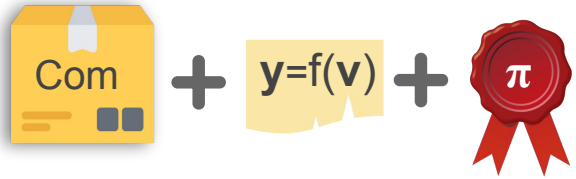


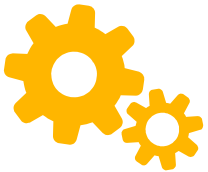


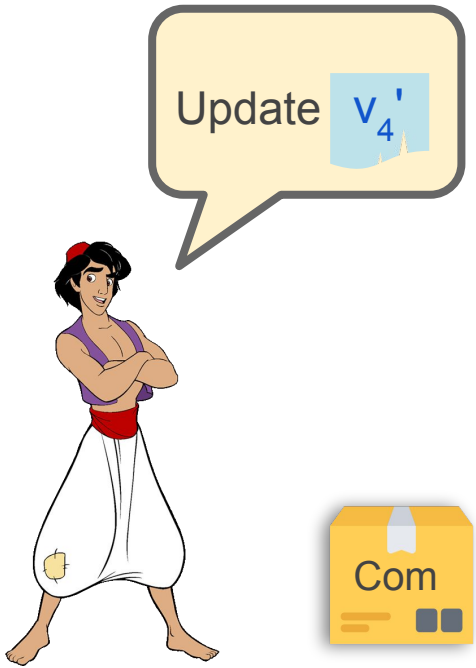


verify

More Properties

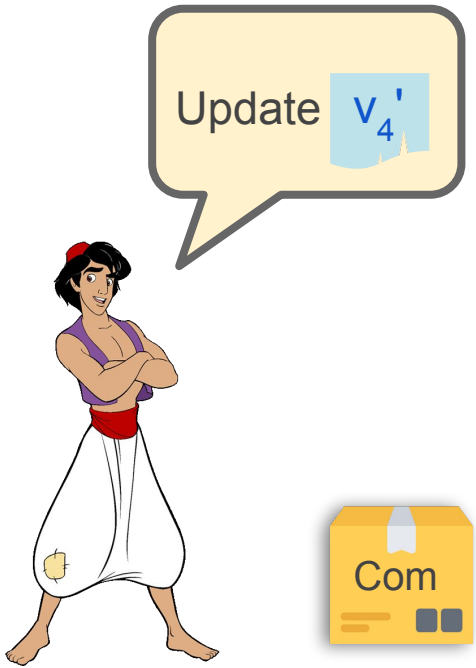
Updates for the vector

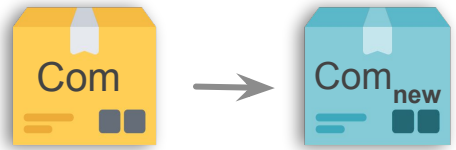




v_1 v_2 v_3 v_4 v_5

Updates for the vector





v_1 v_2 v_3 v_4 v_5

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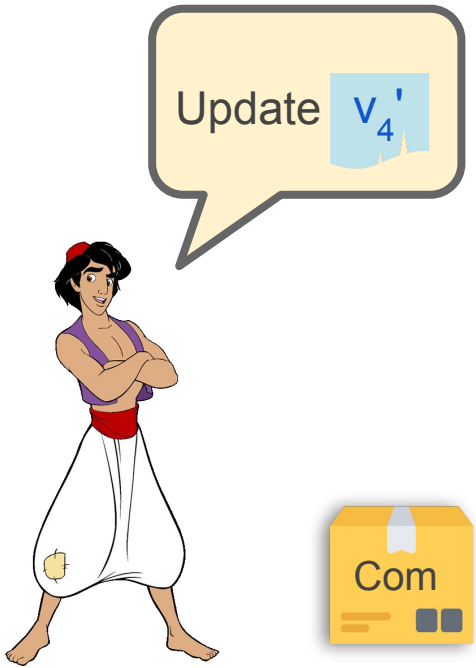

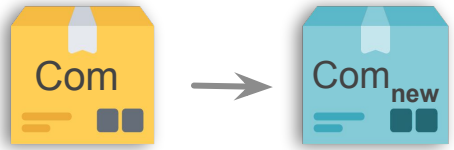

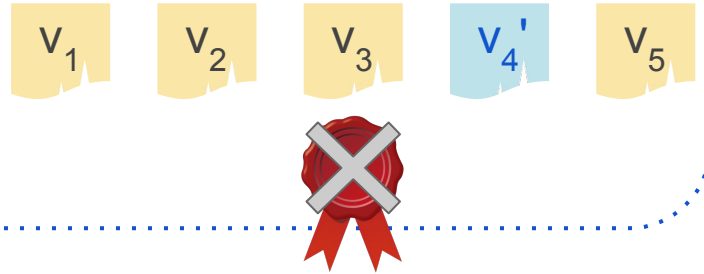

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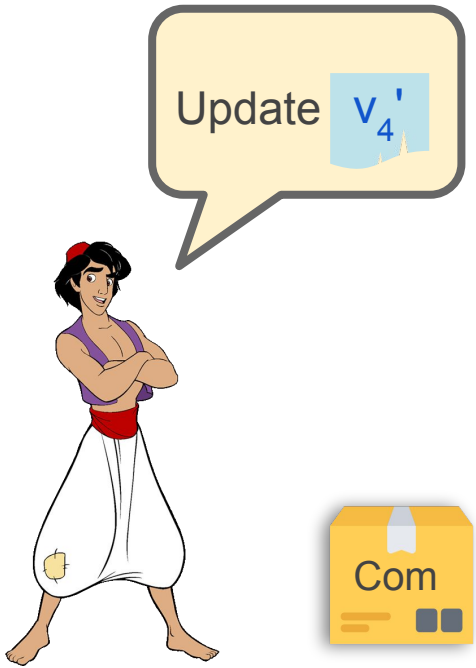

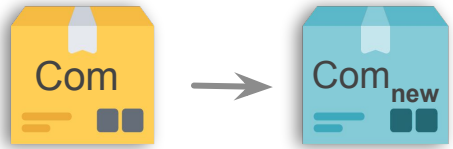

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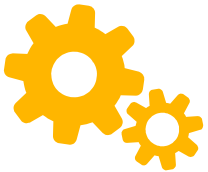
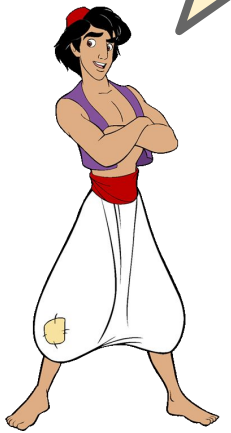
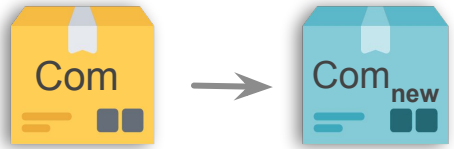


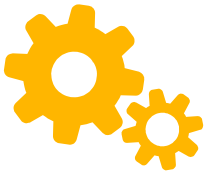


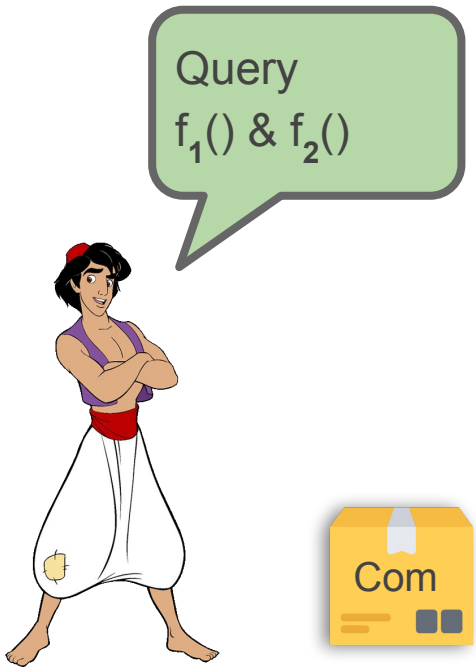
Updates types

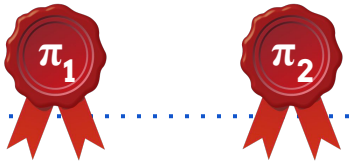
- **hint**: needs opening
- **key**: needs static key



Proof Aggregation







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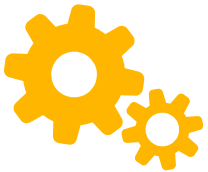

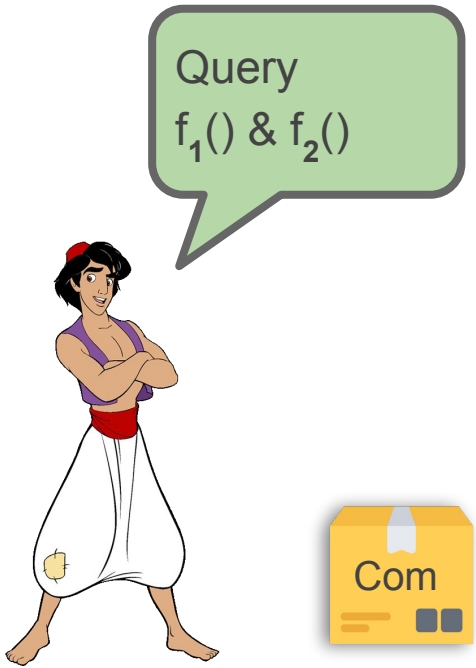

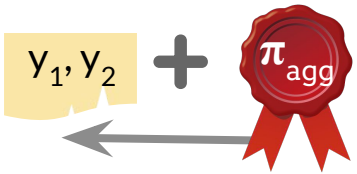

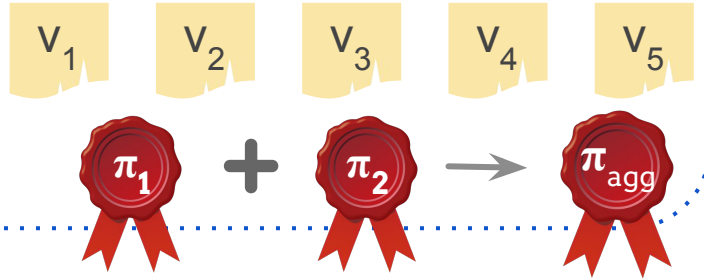

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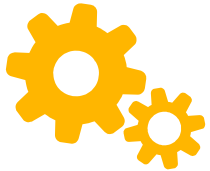
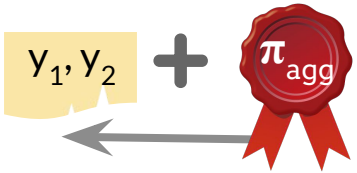
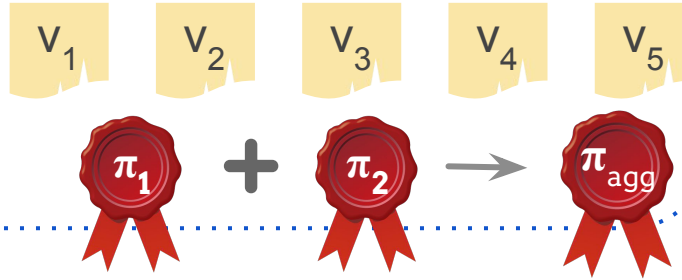


Query
$f_1()$ & $f_2()$

- same-commitment/**cross-commitment**
- one-hop/**unbounded**
- **incremental**: can disaggregate





Homomorphic

Commitments

Homomorphic

Commitments

Homomorphic

Commitments

Openings

$f_1(v)$

$f_2(v)$

Homomorphic

Commitments

Openings

α 

$+ \beta$ 

$=$ 

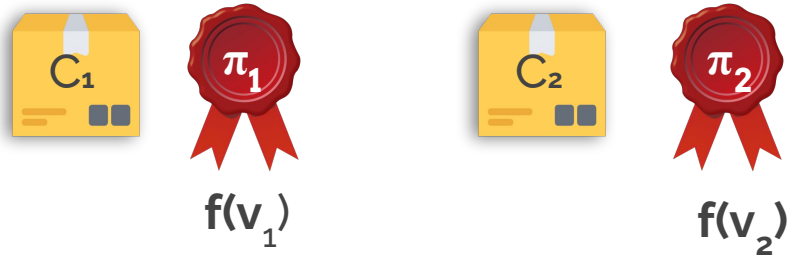
$f_1(v)$ $f_2(v)$ $\alpha f_1(v) + \beta f_2(v)$

Homomorphic

Commitments

Openings

Proofs

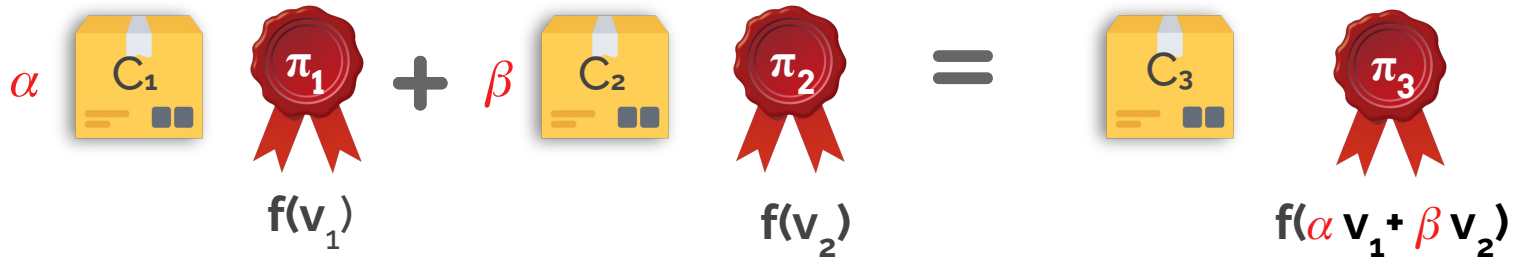


Homomorphic

Commitments

Openings

Proofs

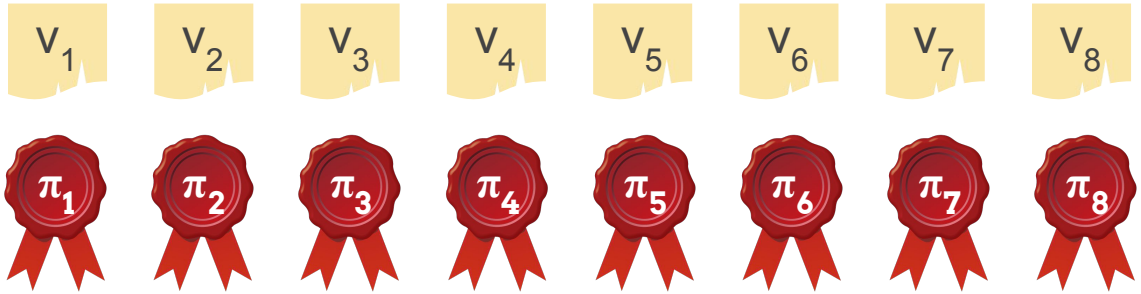


Drawbacks

Properties:

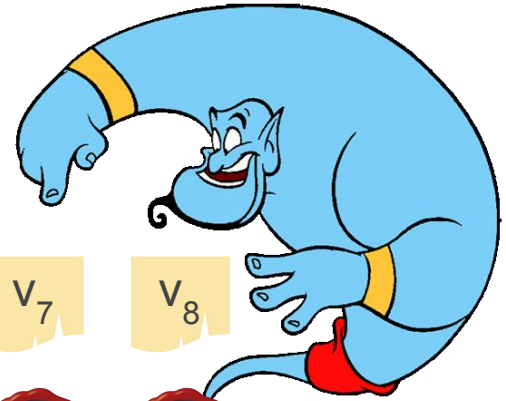
- Short proofs
- Fast verification
- Updates & Aggregation
- Homomorphism
- **Slow Prover**

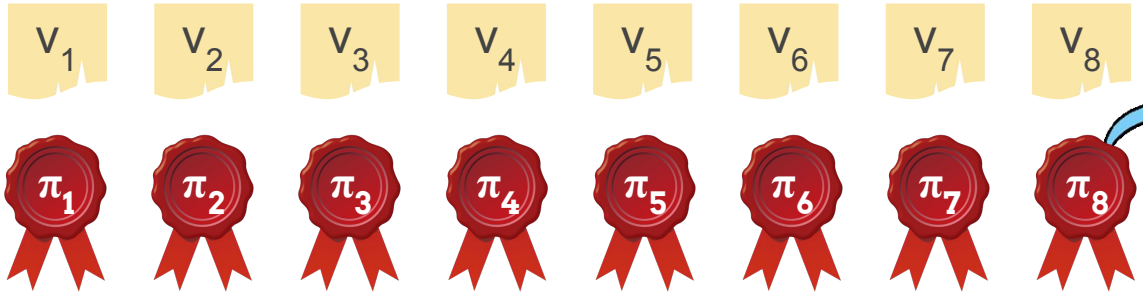
Trade Space for Time



Offline: Pre-compute all proofs

Trade Space

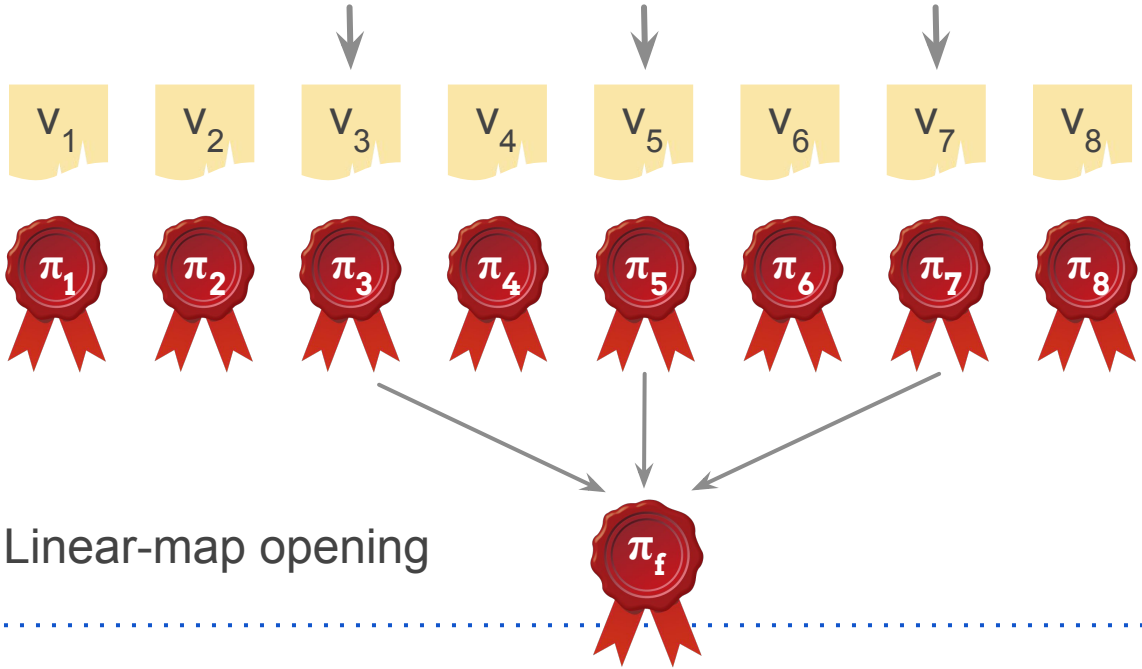




Offline: Pre-compute all proofs (for individual position openings)

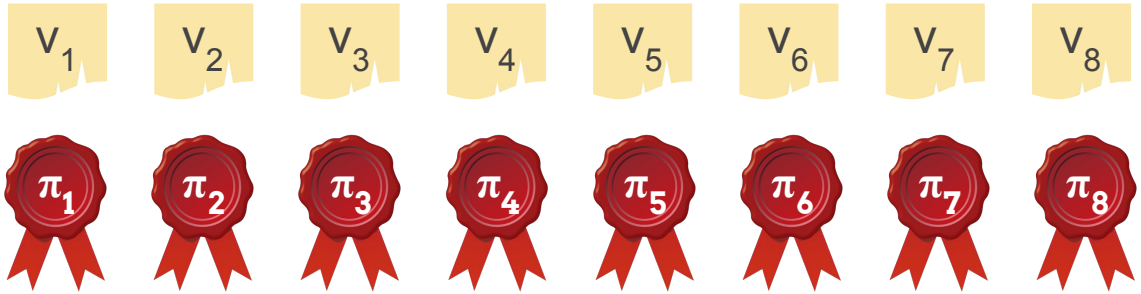
Online: Open by using only the required stored proofs

Trade Space

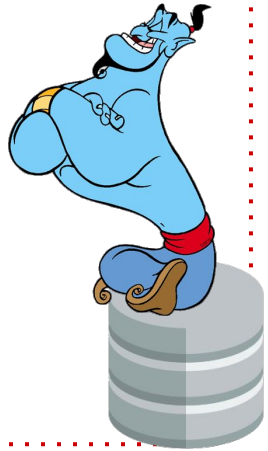


Linear-map opening

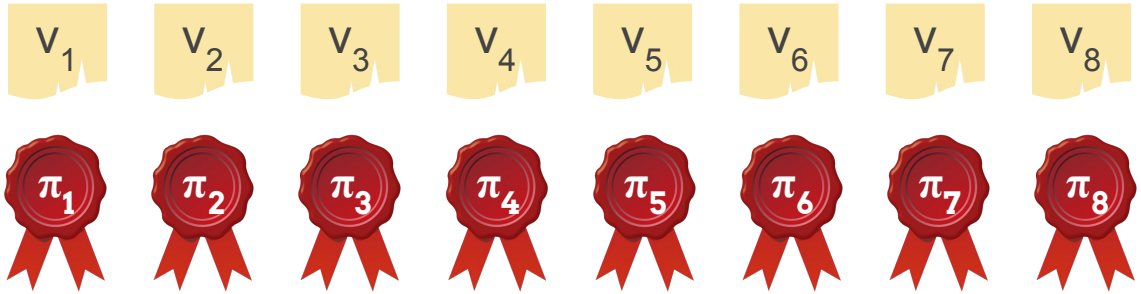
Space = expensive

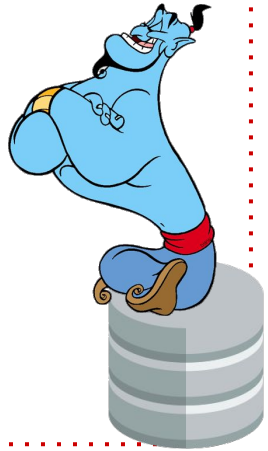


Computation: preprocessing with *small* amortized cost



Space = expensive

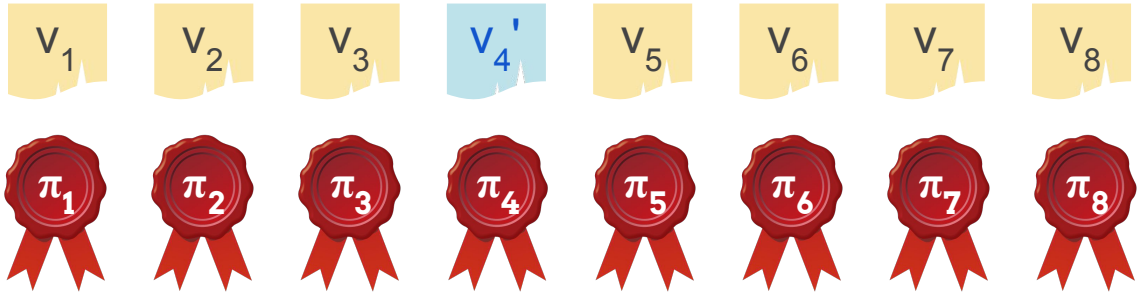


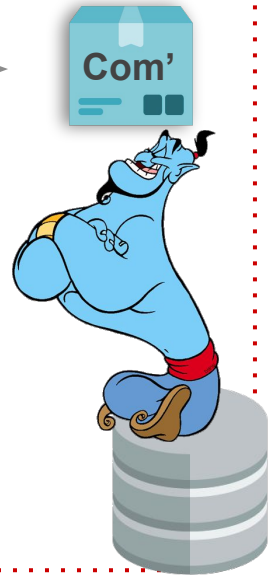


Computation: preprocessing with *small* amortized cost

Storage: doubles **Updates:** recompute all

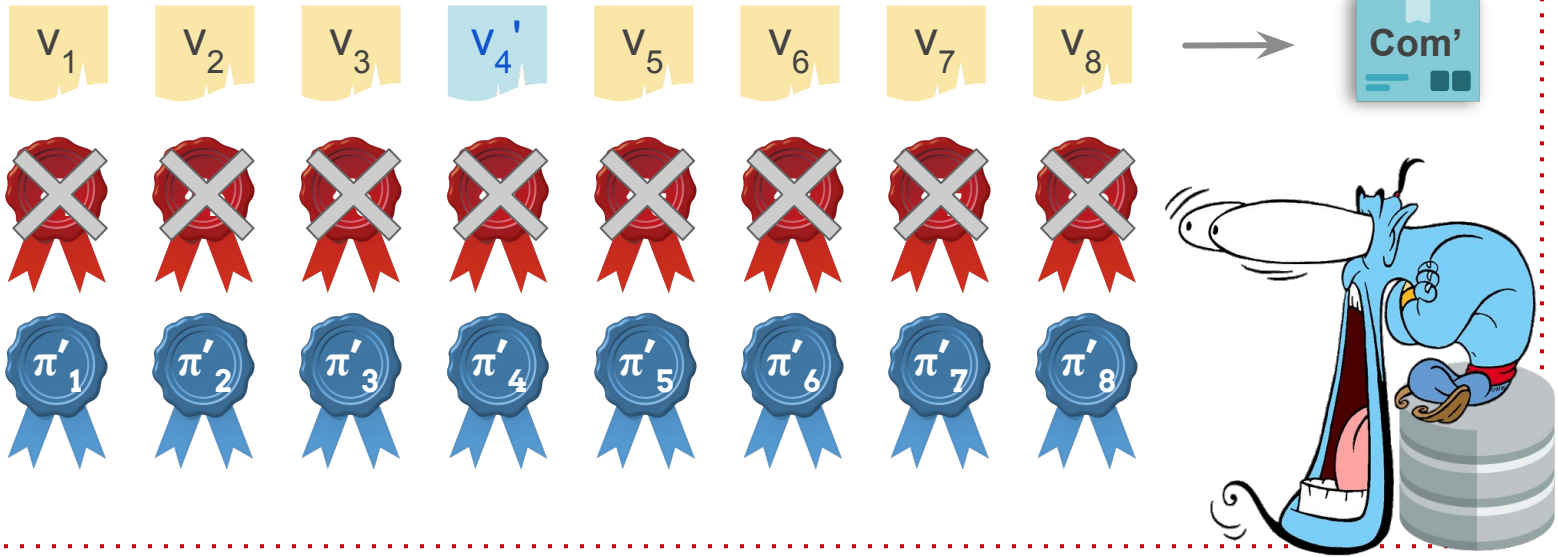
Updates = expensive





Updates = expensive

Updates = expensive



Maintainability

Update all: cost significantly smaller than recomputing all proofs

Space vs. Time

Space

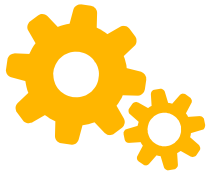
- Sublinear, fast updates
- Ideally: flexibility - tune it

Time

- Sublinear(size of vector)
- Ideally: Sublinear(opening size)

Our Results

Contributions



Inner-Product Vector Commitments: new simple constructions

- Framework to build generic LVC from Inner-Product VC
- Unbounded Aggregation & Updatability with static keys
- Special Subset Openings and Subset opening with Efficient Verifier

Maintainable Tree-based construction from any LVC

- Trade-offs for the prover with flexible Space and Time savings
- Universal Setup (existing ceremonies “powers of tau”)

Inner Product VC

Inner Product of a committed vector with a public vector

$f_b(a) = \langle a, b \rangle$

$a, b \in \mathbb{F}^n$

Inner Product VC

Inner Product of a committed vector with a public vector

$f_b(a) = \langle a, b \rangle$

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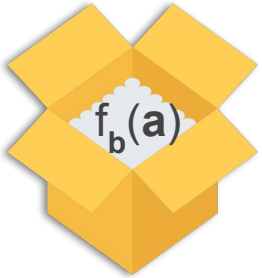
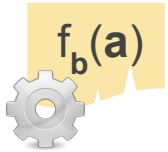


Open



$f : \mathbb{F}^n \to \mathbb{F}$

Inner Product VC

Inner Product of a committed vector with a public vector

$f_b(a) = \langle a, b \rangle$

$a, b \in \mathbb{F}^n$

Com(a)

$a = a_1, a_2, \ldots a_n$

Polynomial A(X)

Monomial basis

$$M_i(X) = X^i$$

Lagrange basis

$$L_i(X) = \prod_{j \neq i} \frac{X - j}{i - j}$$

Monomial basis

$$M_i(X) = X^i$$

$a = a_1, a_2, \ldots a_n$

$A(X) = a_1 + a_2X + a_3X^2 + \ldots a_nX^{n-1}$

Lagrange basis

$$L_i(X) = \prod_{j \neq i} \frac{X - j}{i - j}$$

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$a = a_1, a_2, \ldots a_n$

$A(X) = a_1L_1(X) + a_2L_2(X) + \ldots a_nL_n(X)$

Monomial basis

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$A(X) = a_1L_1(X) + a_2L_2(X) + \ldots a_nL_n(X)$

$A(1) = a_1,\ A(2) = a_2,\ \ldots\ A(n) = a_n$

Background

Bilinear Groups

$$[a]_1 \in \mathbf{G}_1, [b]_2 \in \mathbf{G}_2$$

$$e : \mathbf{G}_1 \times \mathbf{G}_2 \rightarrow \mathbf{G}_T$$

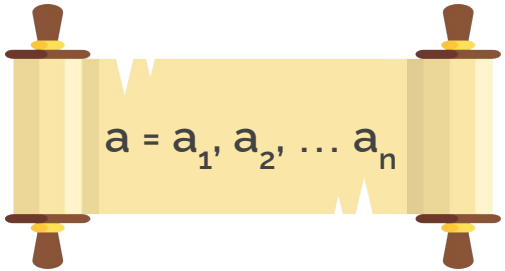
$$e([a]_1, [b]_2) = [ab]_T$$

Inner Product VC

Inner Product of a committed vector with a public vector

$f_b(a) = \langle a, b \rangle$

$a, b \in \mathbb{F}^n$



$A(X) = a_1 + a_2X + a_3X^2 + \ldots a_nX^{n-1}$

Inner Product VC

$f_b(\mathbf{a}) = \langle \mathbf{a}, \mathbf{b} \rangle$

KeyGen(λ, n) → ck: $[1]_1, [\tau]_1, [\tau^2]_1, \ldots [\tau^{n-1}]_1$

Inner Product VC

$f_b(\mathbf{a}) = \langle \mathbf{a}, \mathbf{b} \rangle$

KeyGen(λ, n) $\rightarrow$ ck: $[1]_1, [\tau]_1, [\tau^2]_1, \ldots [\tau^{n-1}]_1$

Commit(ck, **a**) $\rightarrow$ $C = [A(\tau)]_1$

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Commit(ck, **a**) $\rightarrow$ $C = [A(\tau)]_1$

$A(X) = a_1 + a_2X + a_3X^2 + \ldots a_nX^{n-1}$

Open(ck, **a**, **b**, y=<**a**, **b**>) $\rightarrow$ π: $[H(\tau)]_1, [R(\tau)]_1, [\check{R}(\tau)]_1$

Inner Product VC

$f_b(\mathbf{a}) = \langle \mathbf{a}, \mathbf{b} \rangle$

KeyGen(λ, n) $\rightarrow$ ck: $[1]_1, [\tau]_1, [\tau^2]_1, \ldots [\tau^{n-1}]_1$

Commit(ck, **a**) $\rightarrow$ $C = [A(\tau)]_1$

$A(X) = a_1 + a_2X + a_3X^2 + \ldots a_nX^{n-1}$

Open(ck, **a**, **b**, y=<**a**, **b**>) $\rightarrow$ π: $[H(\tau)]_1, [R(\tau)]_1, [\check{R}(\tau)]_1$

$B(X) = b_n + b_{n-1}X + b_{n-2}X^2 + \ldots b_1X^{n-1}$

Inner Product VC

$f_b(\mathbf{a}) = \langle \mathbf{a}, \mathbf{b} \rangle$

KeyGen(λ, n) $\rightarrow$ ck: $[1]_1, [\tau]_1, [\tau^2]_1, \ldots [\tau^{n-1}]_1$

Commit(ck, **a**) $\rightarrow$ $C = [A(\tau)]_1$

Open(ck, **a**, **b**, y=<**a**, **b**>) $\rightarrow$ π: $[H(\tau)]_1, [R(\tau)]_1, [\check{R}(\tau)]_1$

$B(X) = b_n + b_{n-1}X + b_{n-2}X^2 + \ldots b_1X^{n-1}$

$A(X) = a_1 + a_2X + a_3X^2 + \ldots a_nX^{n-1}$

$A(X)\,B(X) =$

Inner Product VC

$f_b(\mathbf{a}) = \langle \mathbf{a}, \mathbf{b} \rangle$

KeyGen(λ, n) $\rightarrow$ ck: $[1]_1, [\tau]_1, [\tau^2]_1, \ldots [\tau^{n-1}]_1$

Commit(ck, **a**) $\rightarrow$ $C = [A(\tau)]_1$

Open(ck, **a**, **b**, y=$\langle \mathbf{a}, \mathbf{b} \rangle$) $\rightarrow$ π: $[H(\tau)]_1, [R(\tau)]_1, [\check{R}(\tau)]_1$

$B(X) = b_n + b_{n-1}X + b_{n-2}X^2 + \ldots b_1X^{n-1}$

$A(X) = a_1 + a_2X + a_3X^2 + \ldots a_nX^{n-1}$

$A(X)\,B(X) = \langle \mathbf{a}, \mathbf{b} \rangle X^{n-1}$

Inner Product VC

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$A(X) = a_1 + a_2X + a_3X^2 + \ldots a_nX^{n-1}$

$A(X)\,B(X) = \langle \mathbf{a}, \mathbf{b} \rangle X^{n-1} + H(X)\,X^n$

Inner Product VC

$f_b(\mathbf{a}) = \langle \mathbf{a}, \mathbf{b} \rangle$

KeyGen(λ, n) $\rightarrow$ ck: $[1]_1, [\tau]_1, [\tau^2]_1, \ldots [\tau^{n-1}]_1$

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$A(X)\,B(X) = \langle \mathbf{a}, \mathbf{b} \rangle X^{n-1} + H(X)\,X^n + R(X)$

Inner Product VC

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$A(X)\,B(X) = \langle \mathbf{a}, \mathbf{b} \rangle X^{n-1} + H(X)\,X^n + R(X)$

$\check{R}(X) = XR(X)$

Inner Product VC

$f_b(\mathbf{a}) = \langle \mathbf{a}, \mathbf{b} \rangle$

KeyGen(λ, n) $\rightarrow$ ck: $[1]_1, [\tau]_1, [\tau^2]_1, \ldots [\tau^{n-1}]_1$

Commit(ck, **a**) $\rightarrow$ $C = [A(\tau)]_1$

Open(ck, **a**, **b**, y=<**a**, **b**>) $\rightarrow$ π : $[H(\tau)]_1, [R(\tau)]_1, [\check{R}(\tau)]_1$

Verify(vk, C, **b**, y, π): $A(X)\,B(X) = y\,X^{n-1} + H(X)\,X^n + R(X)$ $\quad \check{R}(X) = XR(X)$

Inner Product VC

$f_b(\mathbf{a}) = \langle \mathbf{a}, \mathbf{b} \rangle$

KeyGen(λ, n) $\rightarrow$ ck: $[1]_1, [\tau]_1, [\tau^2]_1, \ldots [\tau^{n-1}]_1$

Commit(ck, **a**) $\rightarrow$ $C = [A(\tau)]_1$

Open(ck, **a**, **b**, y=<**a**, **b**>) $\rightarrow$ π : $[H(\tau)]_1, [R(\tau)]_1, [\check{R}(\tau)]_1$

Verify(vk, C, **b**, y, π): $e([A(\tau)]_1, [B(\tau)]_2) = e([y\,\tau^{n-1}]_1, [1]_2)\, e([H(\tau)]_1, [\tau^n]_2)\, e([R(\tau)]_1, [1]_2)$

Inner Product VC

$f_b(\mathbf{a}) = \langle \mathbf{a}, \mathbf{b} \rangle$

KeyGen(λ, n) $\rightarrow$ ck: $[1]_1, [\tau]_1, [\tau^2]_1, \ldots [\tau^{n-1}]_1$ vk: $[1]_2, [\tau]_2, \ldots [\tau^n]_2$

Commit(ck, **a**) $\rightarrow$ $C = [A(\tau)]_1$

Open(ck, **a**, **b**, y=<**a**, **b**>) $\rightarrow$ π : $[H(\tau)]_1, [R(\tau)]_1, [\check{R}(\tau)]_1$

Verify(vk, C, **b**, y, π): $e([A(\tau)]_1, [B(\tau)]_2) = e([y\,\tau^{n-1}]_1, [1]_2)\, e([H(\tau)]_1, [\tau^n]_2)\, e([R(\tau)]_1, [1]_2)$

Inner Product VC

$f_b(\mathbf{a}) = \langle \mathbf{a}, \mathbf{b} \rangle$

KeyGen(λ, n) $\rightarrow$ ck: $[1]_1, [\tau]_1, [\tau^2]_1, \ldots [\tau^{n-1}]_1$ vk: $[1]_2, [\tau]_2, \ldots [\tau^n]_2$

Commit(ck, **a**) $\rightarrow$ $C = [A(\tau)]_1$

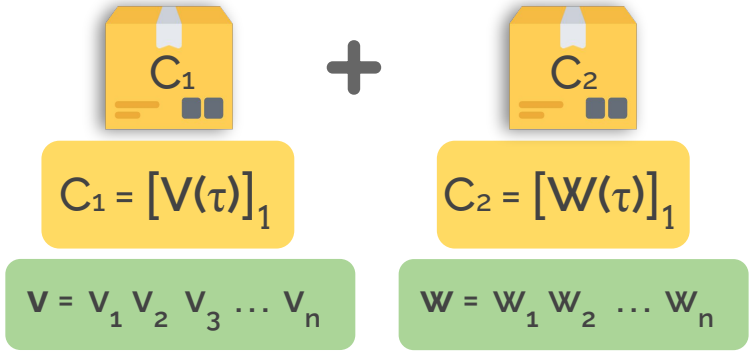
Open(ck, **a**, **b**, y=<**a**, **b**>) $\rightarrow$ π : $[H(\tau)]_1, [R(\tau)]_1, [\check{R}(\tau)]_1$

Verify(vk, C, **b**, y, π): $e([\check{R}(\tau)]_1, [1]_2) = e([R(\tau)]_1, [\tau]_2)$ $\check{R}(X) = XR(X)$

Properties of LVC

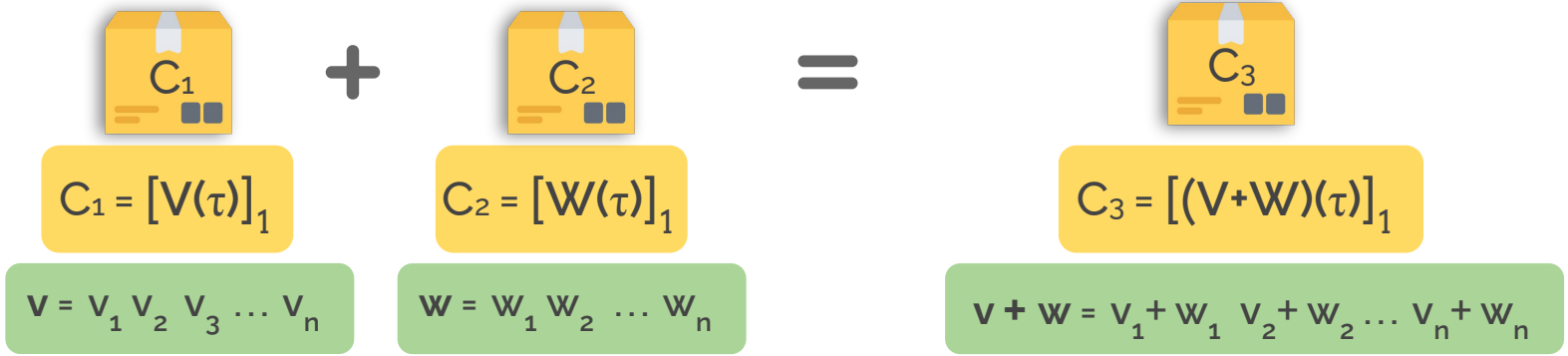
Homomorphism

Commitments



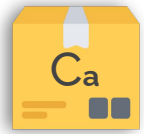
Homomorphism

Commitments



Homomorphism

Openings



+

=

$y_1 = \langle a, b_1 \rangle$ $y_2 = \langle a, b_2 \rangle$ $y_3 = \langle a, b_1 + b_2 \rangle$

Homomorphism

Openings

Proofs

Updatability for Com

$a' = a_1\ a_2 \ldots a_i + \delta \ldots a_n$

upk: $[1]_1, [\tau]_1, [\tau^2]_1, \ldots [\tau^{n-1}]_1$

$C' = [A(\tau)]_1 + [\delta\tau^i]_1$

Updatability for Proof

$\mathbf{a'} = a_1\ a_2 \ldots a_i + \delta \ldots a_n$

upk: $[1]_1, [\tau]_1, [\tau^2]_1, \ldots [\tau^{n-1}]_1$

$C' = [A(\tau)]_1 + [\delta\tau^i]_1$

$y' = \langle \mathbf{a} + \delta \mathbf{e}_i, \mathbf{b} \rangle = y_1 = \langle \mathbf{a}, \mathbf{b} \rangle + \delta \langle \mathbf{e}_i, \mathbf{b} \rangle$

Updatability for Proof

$\mathbf{a'} = a_1\ a_2 \dots a_i + \delta \dots a_n$

upk: $[1]_1, [\tau]_1, [\tau^2]_1, \dots [\tau^{n-1}]_1$

$C' = [A(\tau)]_1 + [\delta\tau^i]_1$

$A(X)\,B(X) = \langle \mathbf{a}, \mathbf{b} \rangle X^{n-1} + H(X)\,X^n + R(X)$

$y' = \langle \mathbf{a} + \delta \mathbf{e}_i, \mathbf{b} \rangle = y_1 = \langle \mathbf{a}, \mathbf{b} \rangle + \delta \langle \mathbf{e}_i, \mathbf{b} \rangle$

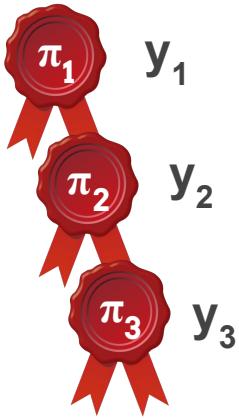
$\pi_{ij}: \langle \mathbf{e}_i, \mathbf{e}_j \rangle = 0$

$H(X)/R(X) = X^{i+n-j}$

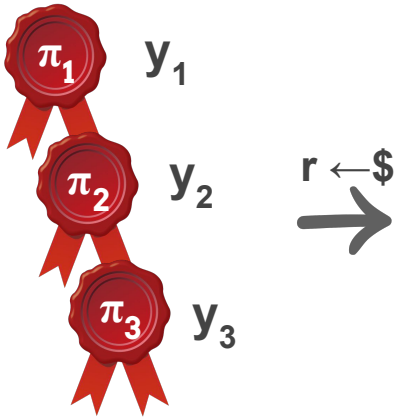
$\pi_{ii}: \langle \mathbf{e}_i, \mathbf{e}_i \rangle = 1$

$H(X)/R(X) = 0$

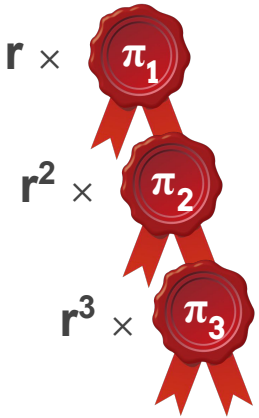
Aggregation

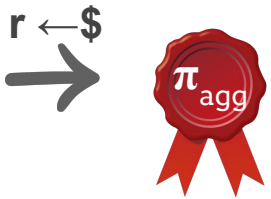


Aggregation



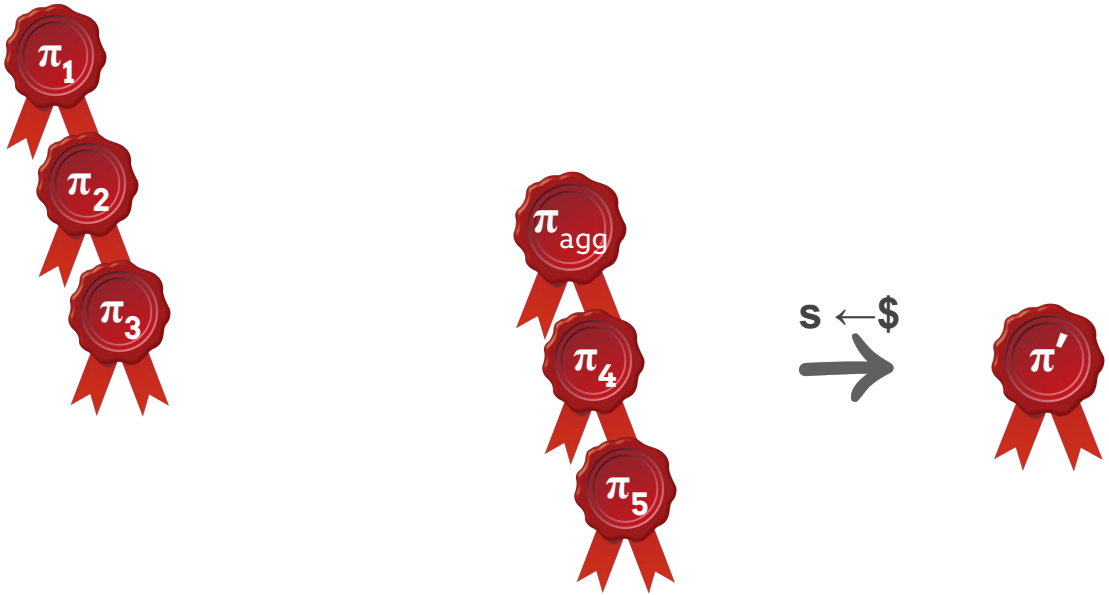
Aggregation



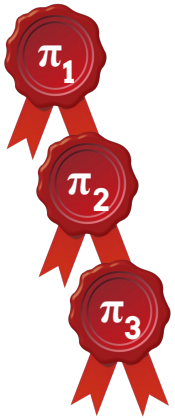


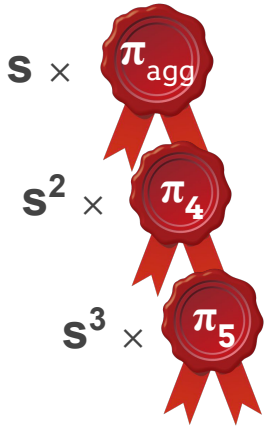
$$\pi_{agg} = \sum r^i \pi_i$$

Aggregation

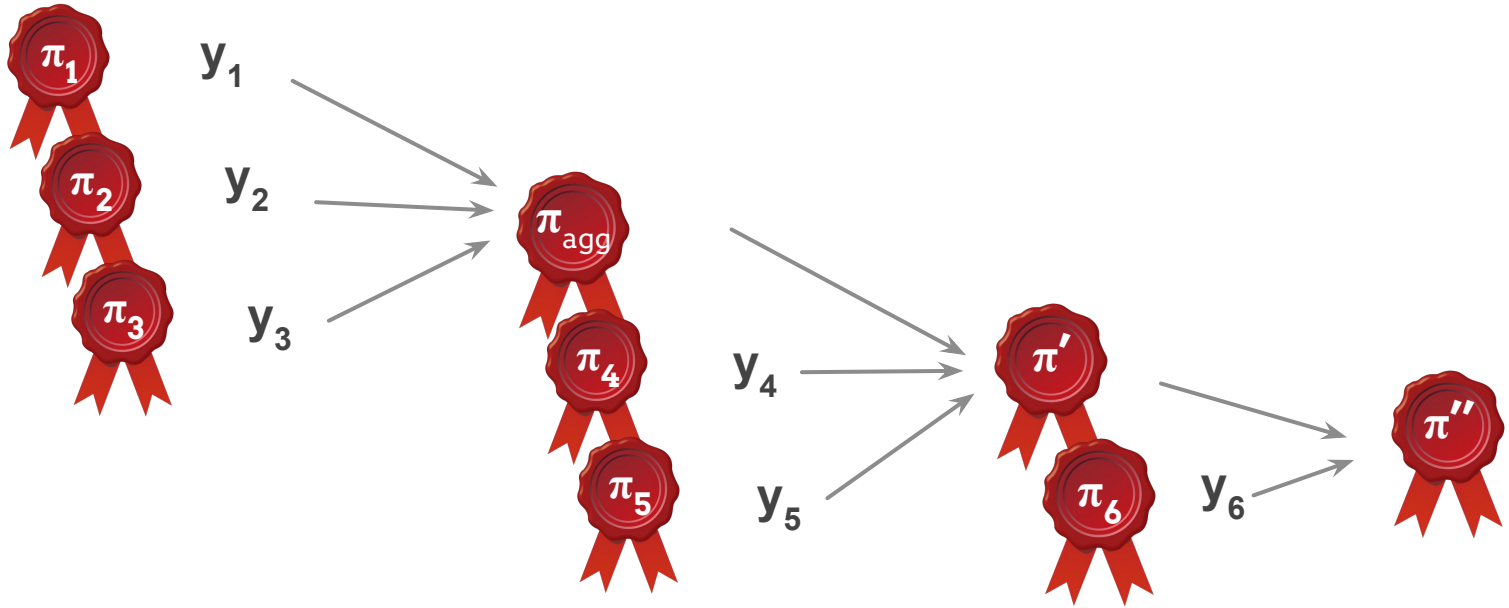


Aggregation





Aggregation



Linear-Map VC from IP

$f : \mathbb{F}^n \to \mathbb{F}^\ell$

KeyGen(λ, n) ⟶ ck: $[1]_1, [\tau]_1, [\tau^2]_1, \ldots [\tau^{n-1}]_1$ vk: $[1]_2, [\tau]_2, \ldots [\tau^n]_2$

Commit(ck, **a**) ⟶ $C = [A(\tau)]_1$

Open(ck, **a**, f: **B**=(**b**$_i$), **y**) ⟶ $\pi_1, \pi_2, \ldots \pi_\ell$ ⟶ π_{agg}

Verify(vk, C, f , **y**, π_{agg})

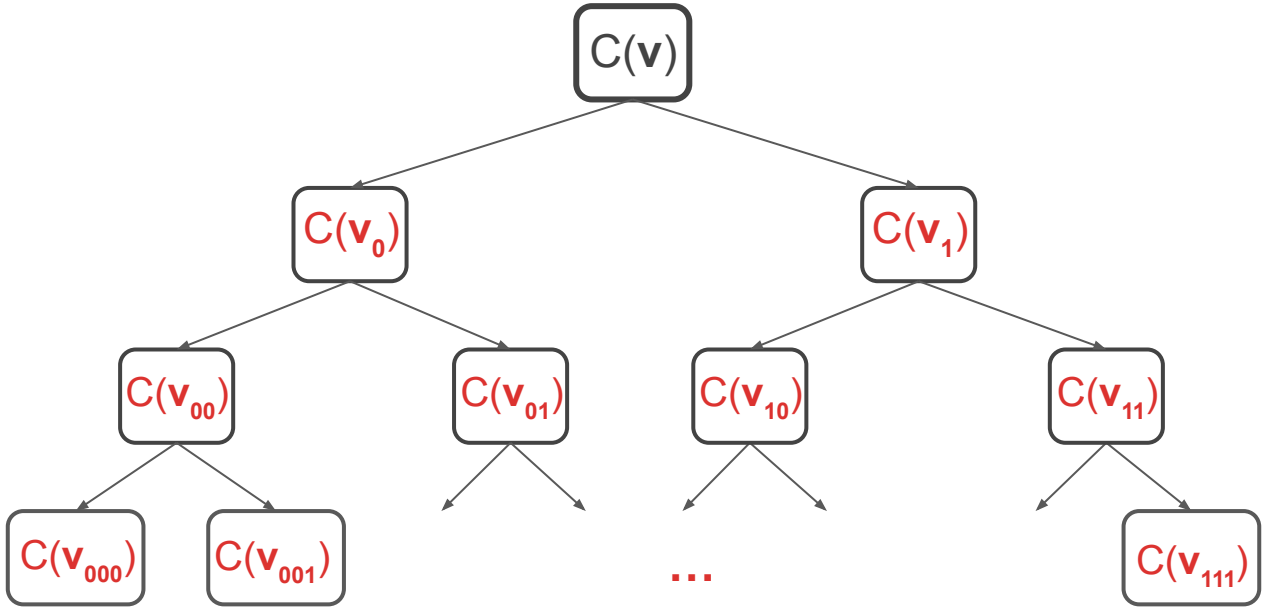
LVC comparison

$f : \mathbb{F}^n \to \mathbb{F}^\ell$

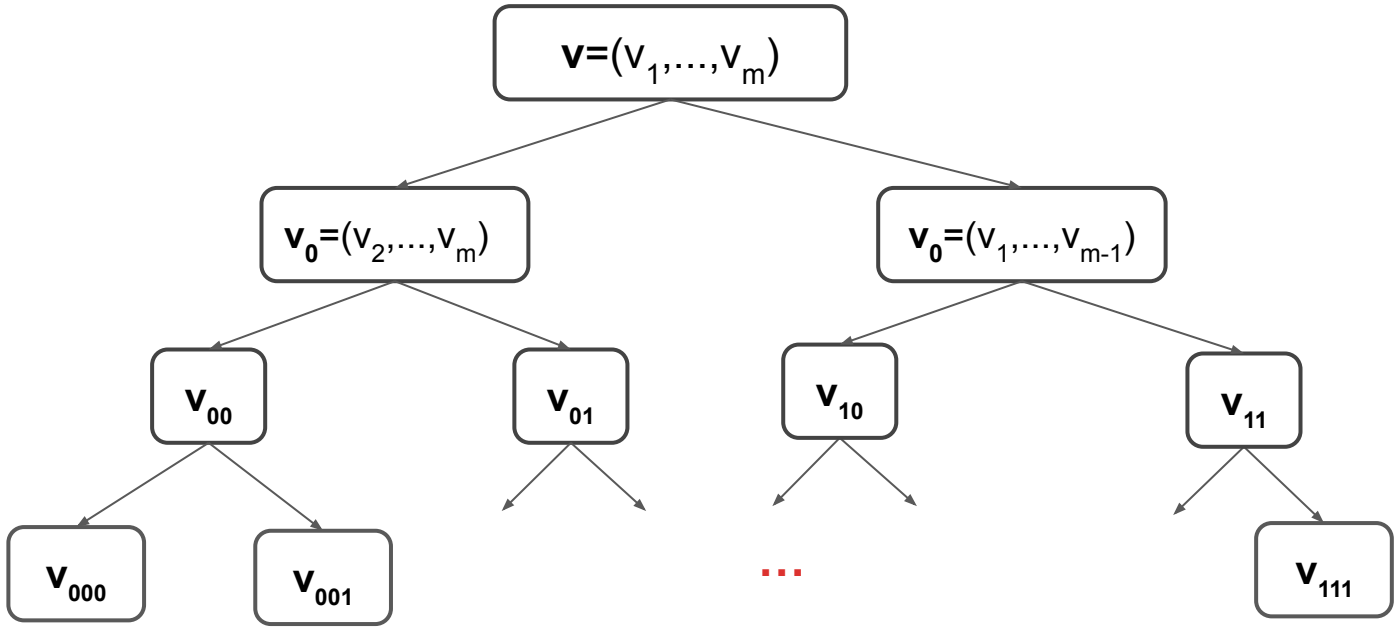
Schemes		|ck|	|vk|	|upk|	Updates	Aggregation	SVC	LVC
[LRY16]		n	n	—	✗	✗	✗	✓
[LM19]	SVC	n^2	n	n	key	✗	✓	✗
	LVC	$n\ell$	n	n	key	✗	✓	✓
Pointproofs		n	n	n	key	cross & one-hop	✓	✗
[CFG+20]		1	1	1	hint	same & incremental	✓	✗
Our Monomial		**n**	**n**	—	**key**	**cross & unbounded**	✓	✓
Our Lagrange		**n**	ℓ	**n**	**key**	**cross & unbounded**	✓	✓

Maintainable Trees

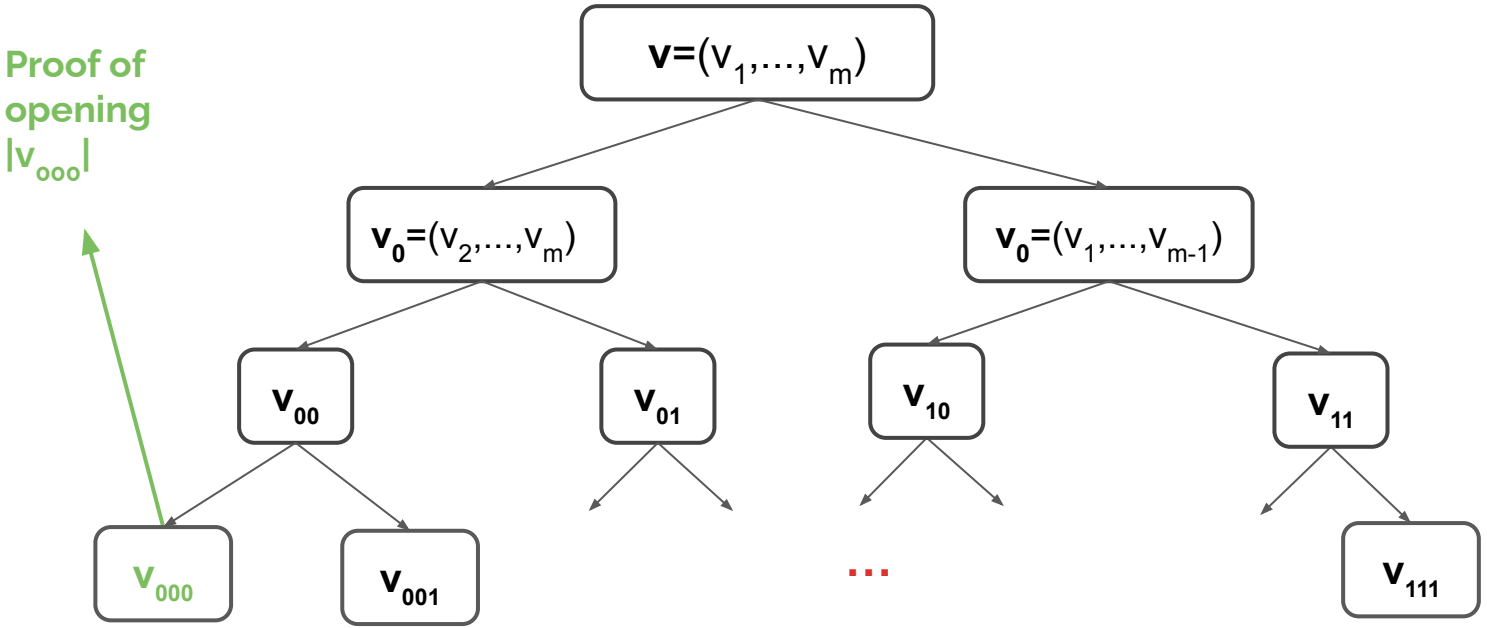
VC Tree



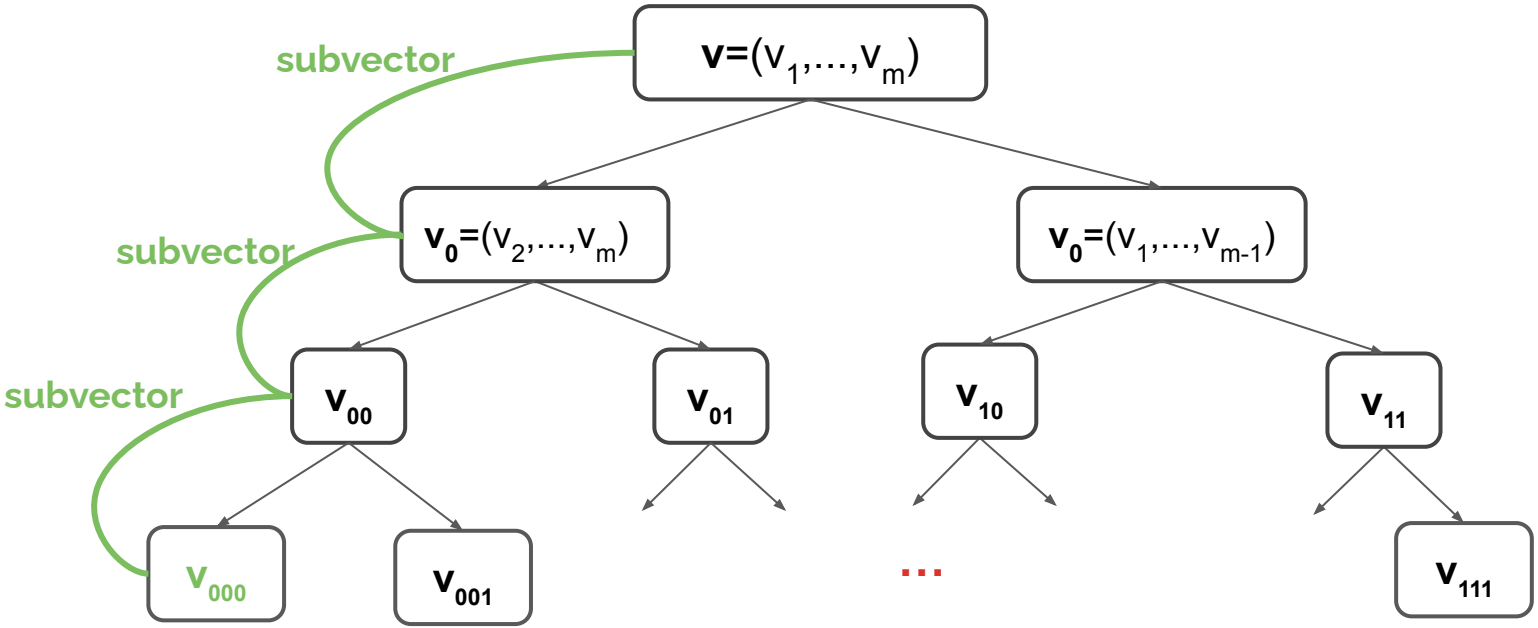
VC Tree



VC Tree - opening proof



VC Tree - opening proof



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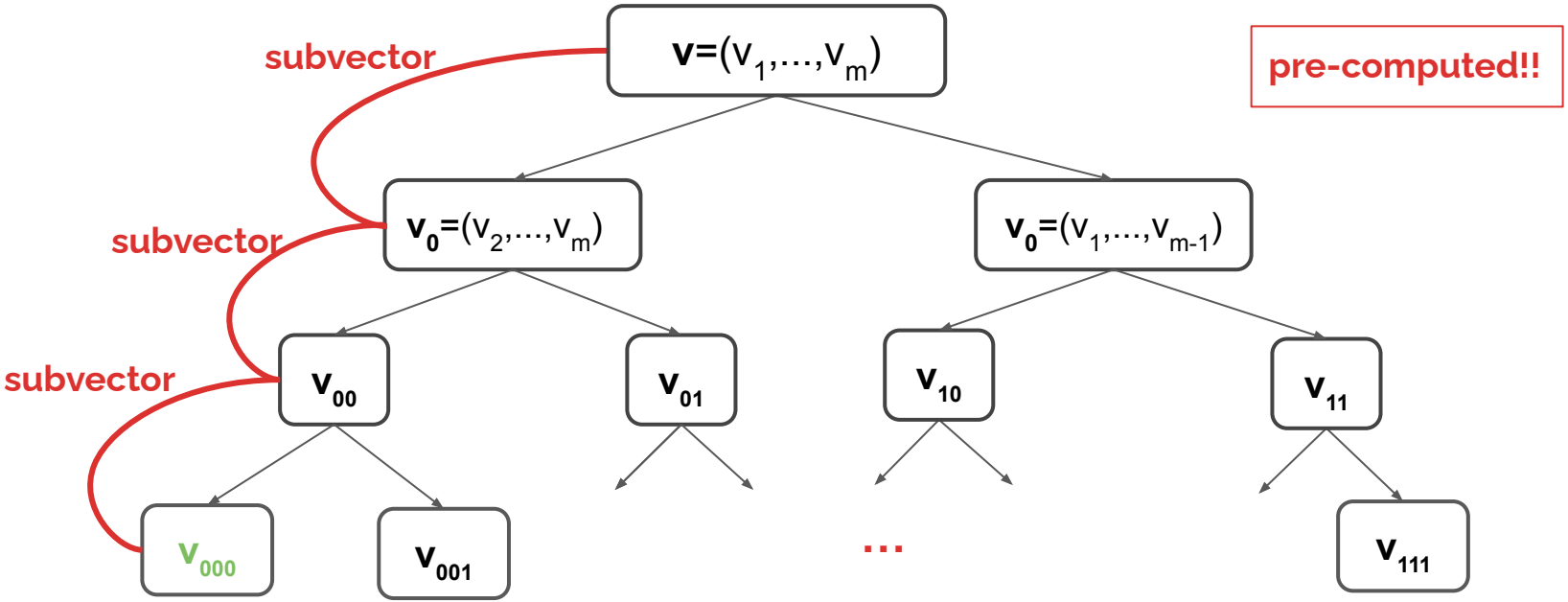

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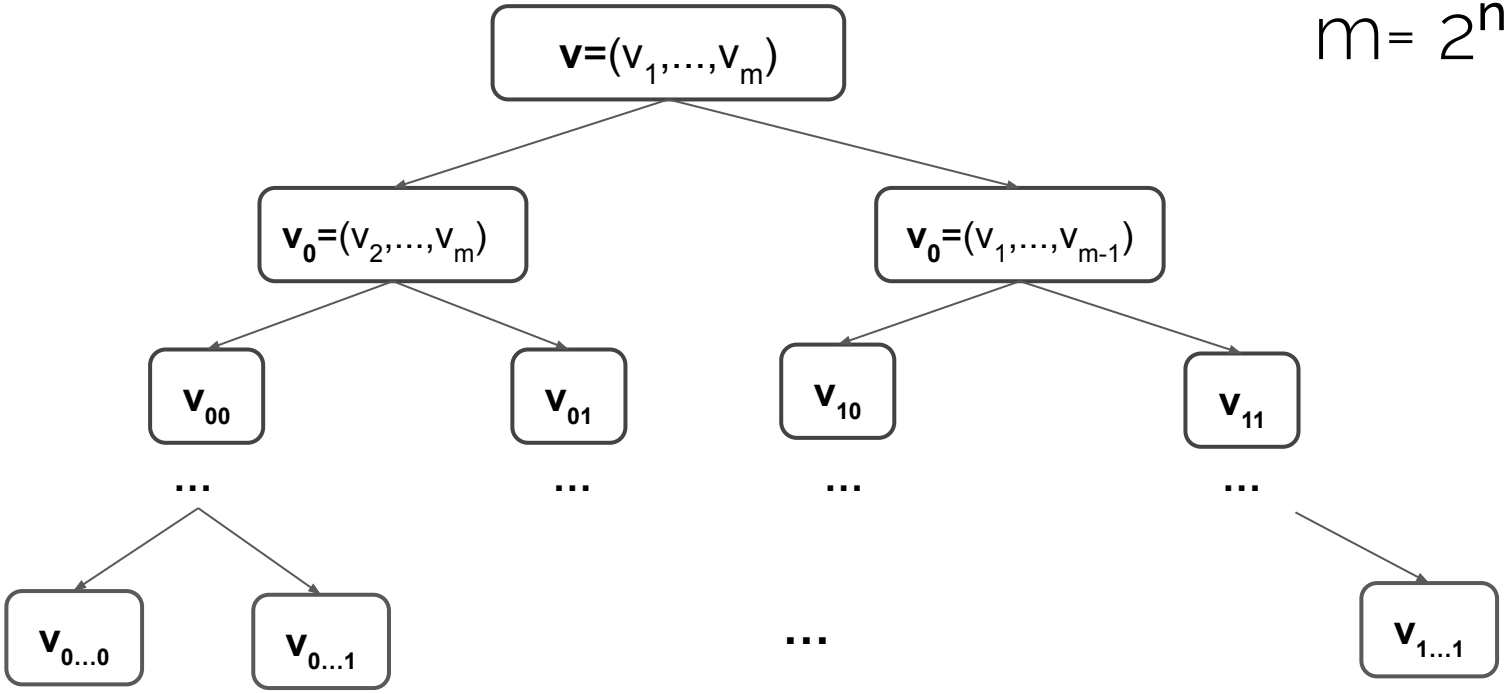
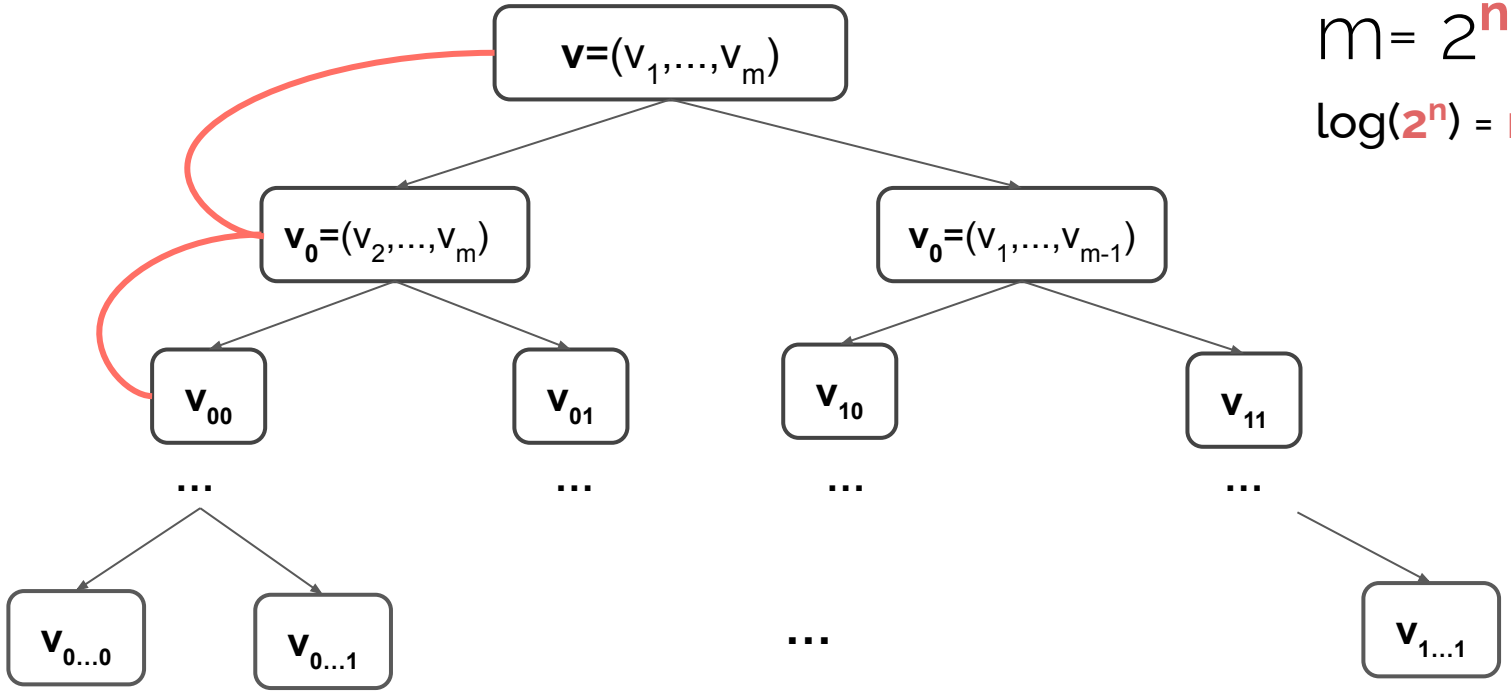


$m = 2^{n+k}$

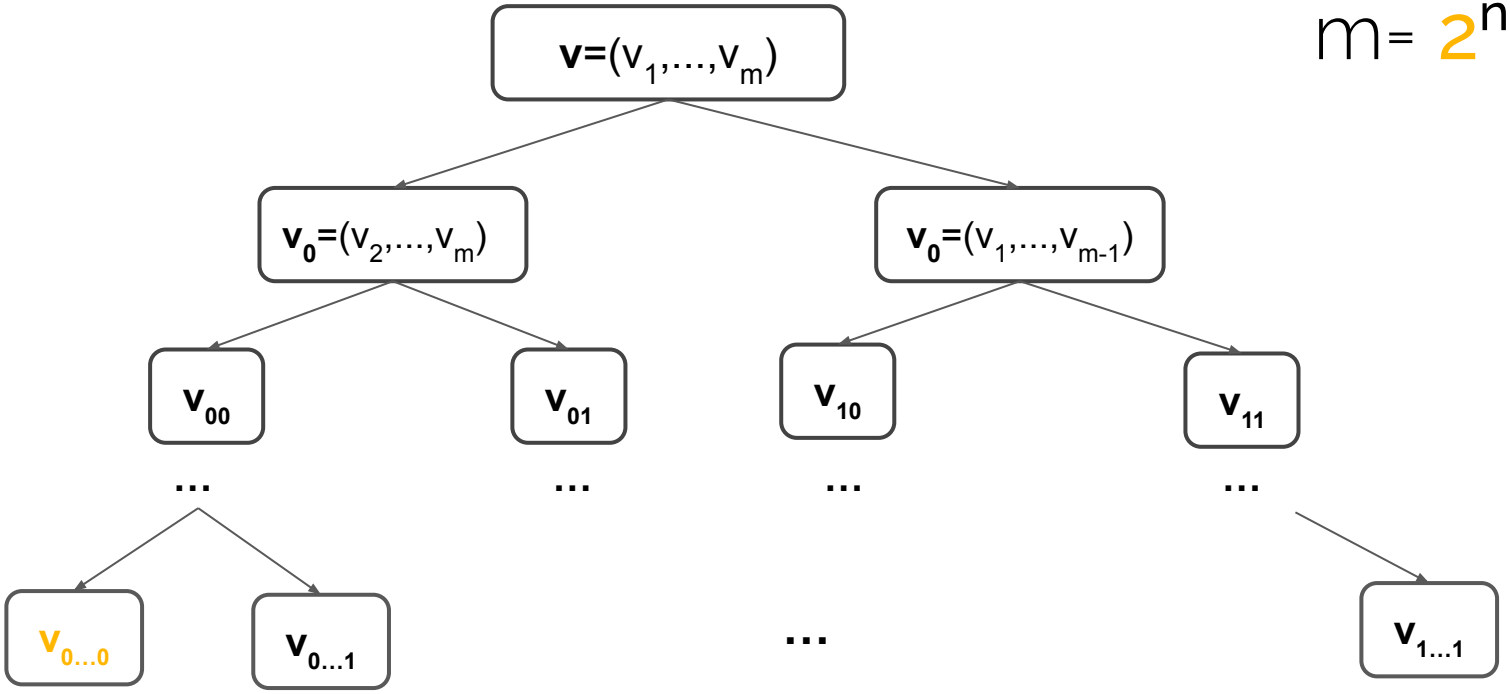
VC Tree



$m = 2^{n+k}$

$\log(2^n) = n$

VC Tree



$m = 2^{n+k}$

Future Directions

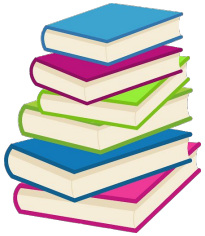
What's next?

- Transparent Setup Vector Commitments
- Functional Vector Commitments for larger classes of functions
- Post-Quantum Vector Commitments with same properties
- More efficient Aggregation for Tree-based VC (not relying on IPA)

Thanks!

https://eprint.iacr.org/2022/705

References



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Credits

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