# Unconditionally Secure NIZK in the Fine-Grained Setting

**Yuyu Wang**[1] and Jiaxin pan[2]

1. University of Electronic Science and Technology of China
2. NTNU - Norwegian University of Science and Technology

# Standard cryptography

Honest party

polynomial-time

Adversary

polynomial-time

Assumption:

- Basic ones (e.g., one-way function)
- More advanced ones (e.g., factoring, discrete logarithm, DDH, LWE)
- Exotic ones (e.g, generic groups, algebraic groups)

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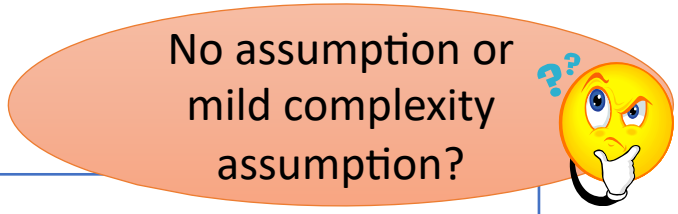


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Existing fine-grained primitives:
NIKE [Mer78], OWF [BC20], PKE [DVV16], verifiable computation [CG18], HPS and trapdoor one-way function [EWT21], ABE [WPC21]

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runs in $NC^1$

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secure against adversaries in $NC^1$ if $NC^1 \neq \oplus L/poly$

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The resources of an adversary can be a-prior bounded.

Unconditionally secure fine-grained NIZK?

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# Our results

A fully fine-grained NIZK for AC0-circuit satisfiability (SAT)

- ❖the CRS generator, prover, verifier, simulator run in AC0
- ❖Perfect soundness and composable zero-knowledge against AC0
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Circuits with constant depth,
polynomial size, and unbounded fan-in
using AND, OR, and NOT gates.

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A statement circuit cannot go beyond AC0. Otherwise, even the honest prover in AC0 cannot decide with the witness whether the statement is true or not.

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If we allow the prover to run in polynomial time as in [BDK20] (Crypto '20), our NIZK supports all statements in NP.

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Extensions in the AC0-fine-grained setting:

1. Unconditionally secure non-interactive zaps
2. Unconditionally secure NIZKs in the URS model

# Applications

Applications:

1. Unconditional security against adversaries with limited parallel running-time.

2. Systems requiring “online security”, where attacks are valid only if they succeed immediately:

   - Our NIZK with composable zero-knowledge can be used to protect secrets only valuable in a short period of time.

   - The dual mode with computational soundness and perfect zero-knowledge perfectly protects secrets and guarantees security in a system requiring users to provide proofs in a short time.

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# Impact of Our Work

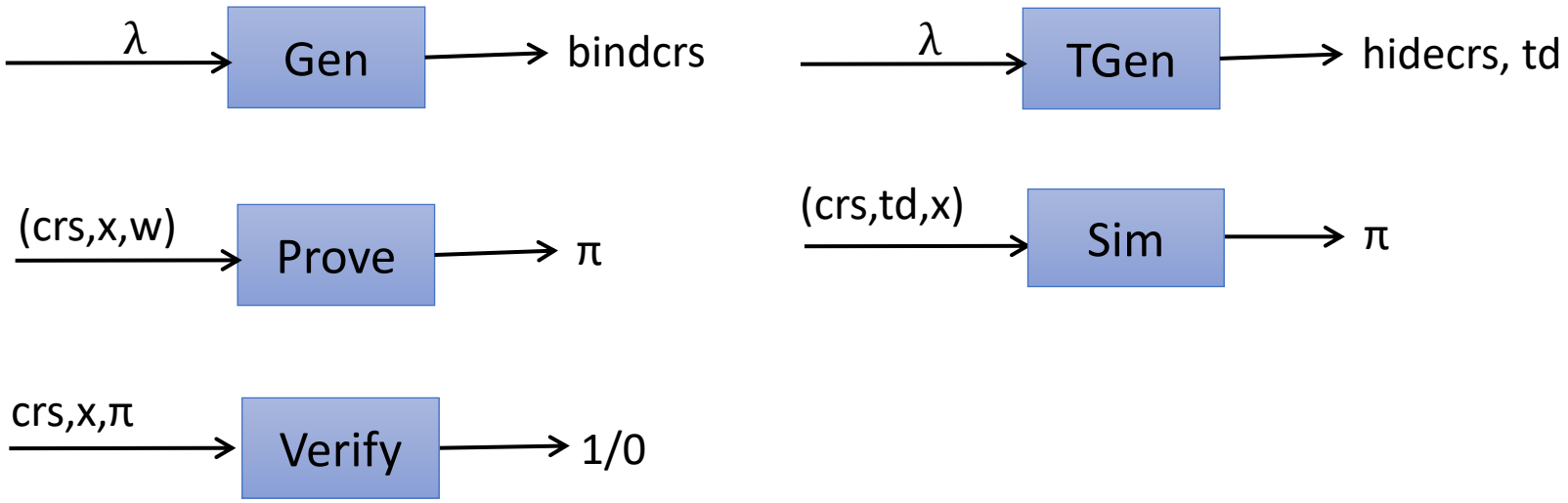
Before our work, it seemed that cryptographic assumptions implying PKE were necessary for NIZK in the standard model. Namely, NIZK seemed to be in Cryptomania.

In the AC0 fine-grained setting, only “minicrypt primitives” (e.g., OWF, SKE, wPRF) are known to exist, while we achieved NIZK.

NIZK is in minicrypt?

# Definition of NIZK

$x \in L$ iff $\exists w$ s. t. $R(x, w) = 1$



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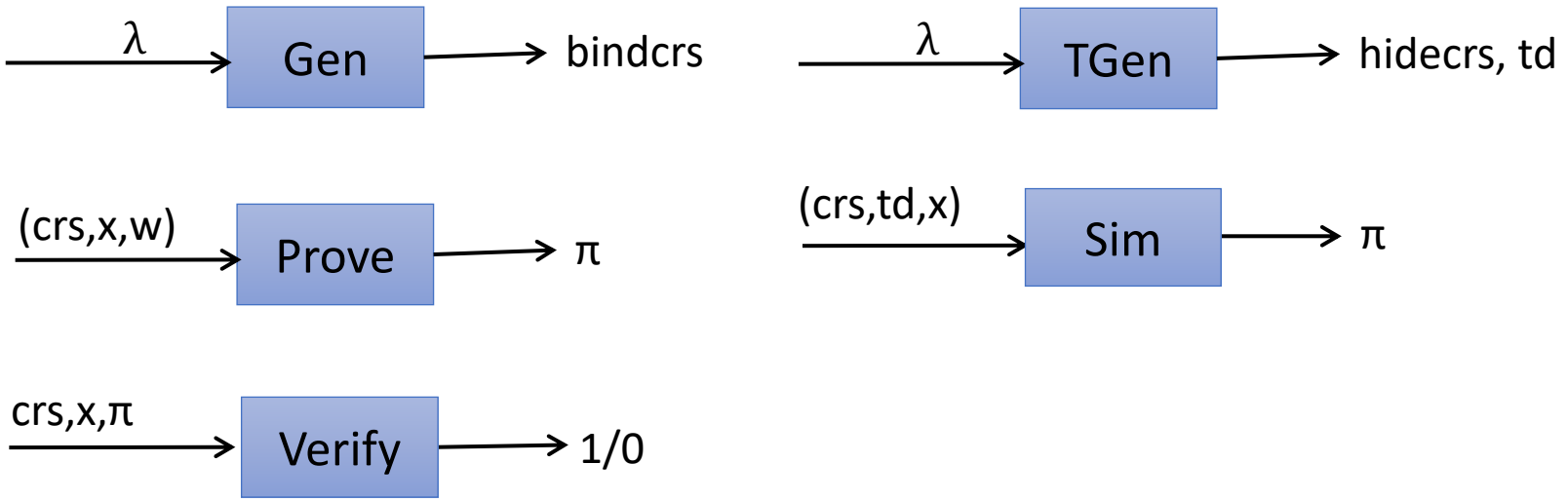
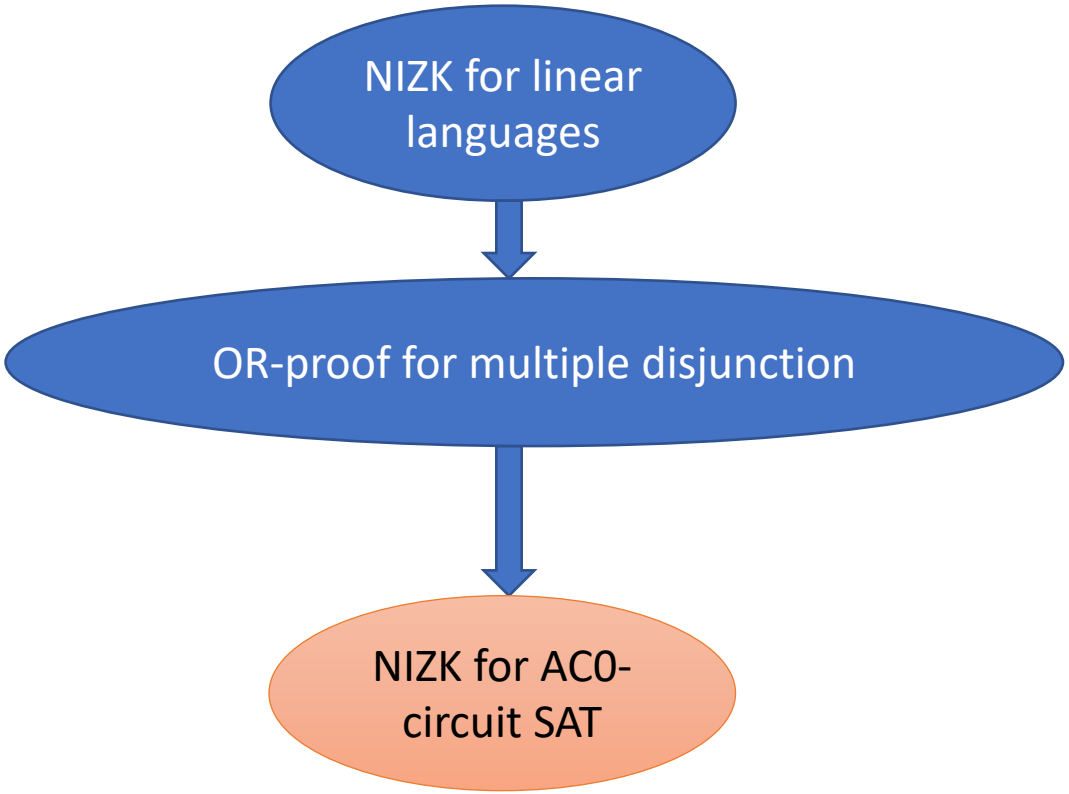



Completeness: honest proofs must pass the verification.

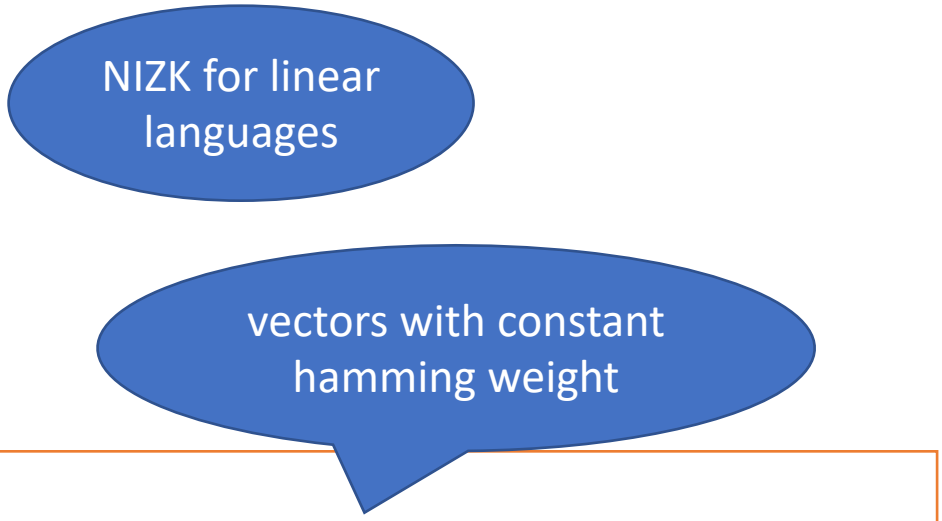
Perfect soundness: when crs is binding, there exists no valid proof for x if $x \notin L$.

(Composable) zero knowledge: bindcrs and hidecrs are indistinguishable, and when crs is hiding, Sim perfectly simultates honest proofs.

# NIZK for AC0-circuit SAT



# NIZK for linear languages




Two facts:

1. Inner product of two sparse vectors is computable in AC0.

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1. Inner product of two sparse vectors is computable in AC0.
2. The parity of a random vector is not computable in AC0 [Hås14,IM11].

# NIZK for linear languages

$$\mathbf{E} = \begin{pmatrix} 1 & & & & \\ 1 & 1 & & & \\ & & \ddots & & \\ & & & 1 & 1 \\ & & & & 1 \end{pmatrix} \in \{0,1\}^{\lambda \times (\lambda - 1)} \qquad \mathbf{e} = \begin{pmatrix} 0 \\ \vdots \\ 0 \\ 1 \end{pmatrix} \in \{0,1\}^{\lambda}$$

# NIZK for linear languages

The parity of a random string is not computatble in AC0 [Hås14,IMP12].

$$D_0=\{Et \mid t \leftarrow \{0,1\}^{\lambda-1}\} \approx_{AC0} D_1=\{Et+e \mid t \leftarrow \{0,1\}^{\lambda-1}\}$$

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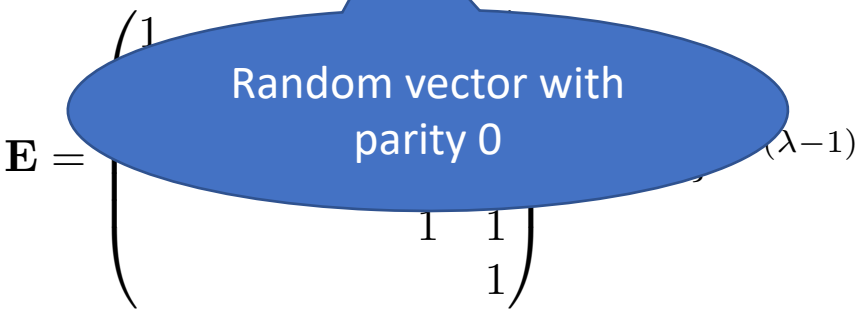
Random vector with parity 0

Random vector with parity 1



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Sampleable in AC0 since the row vectors in E are sparse

# NIZK for linear languages

Column vectors are sparse

$M \in \{0,1\}^{n \times t}$

$\exists\ w\ s.t.\ x=Mw$

$r = Et + e \quad (t \leftarrow \{0,1\}^{\lambda-1})$

bindCRS

# NIZK for linear languages

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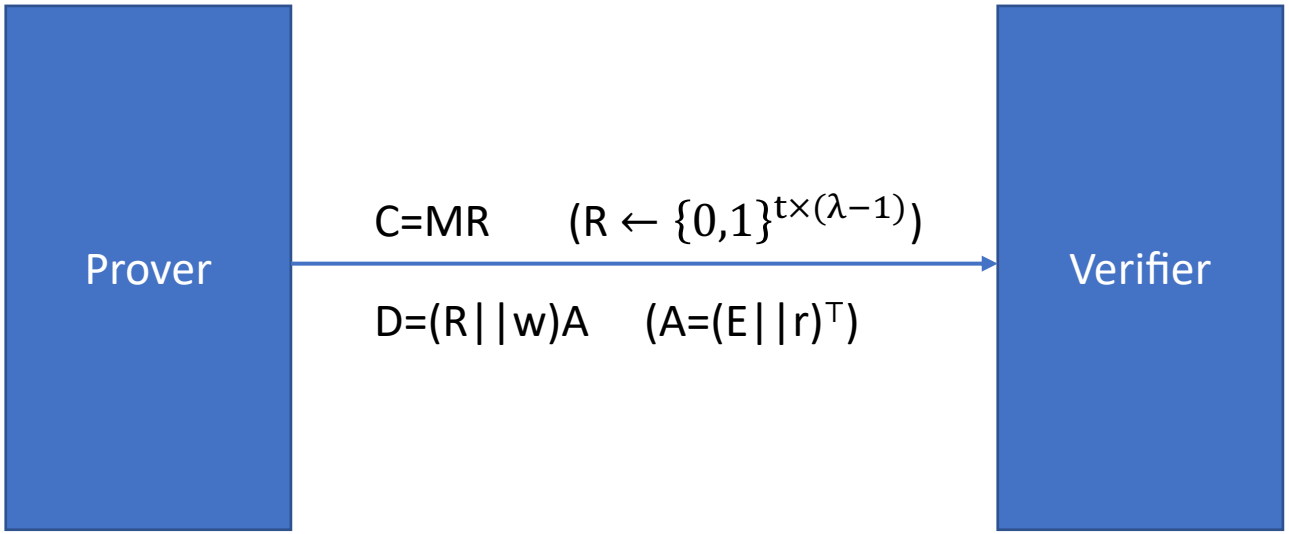


Verify if (C||x)A=MD

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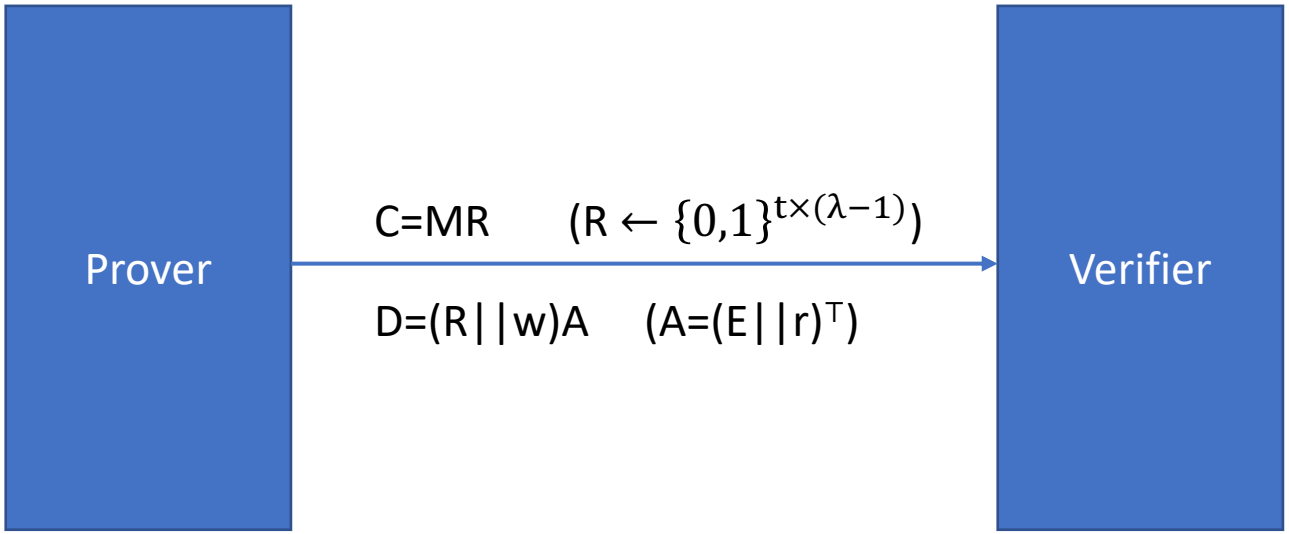
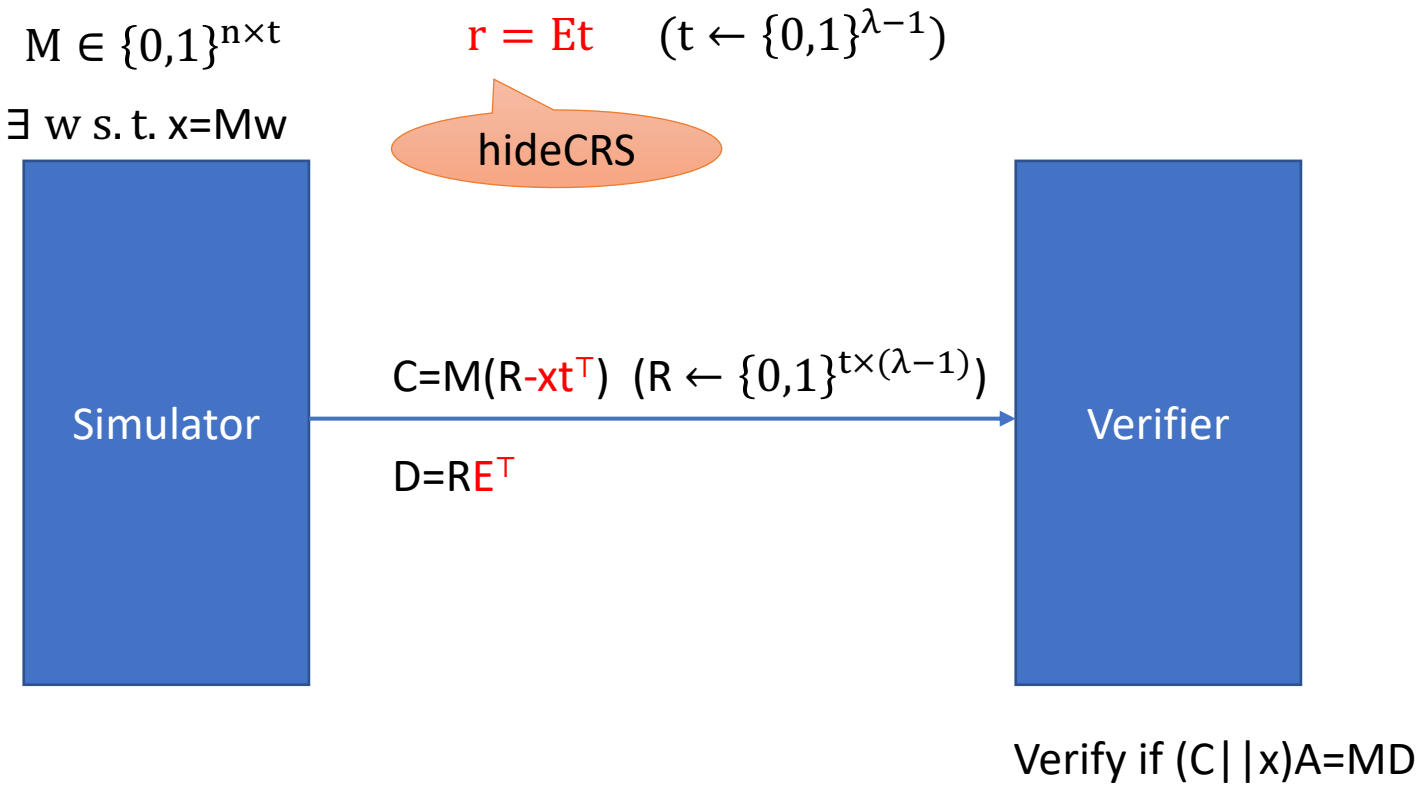



Verify if (C||x)A=MD

Complexity: row vectors of M and column vectors of A are sparse => prover and verifier run in AC0.

Perfect soundness: A is of full rank => C||x=MD$A^{-1}$ => x is in Span(M)

# NIZK for linear languages




Composable zero-knowledge: when switching r to a random vector in Span(E), there exists a simulator in AC0 that perfectly simulates the prover.

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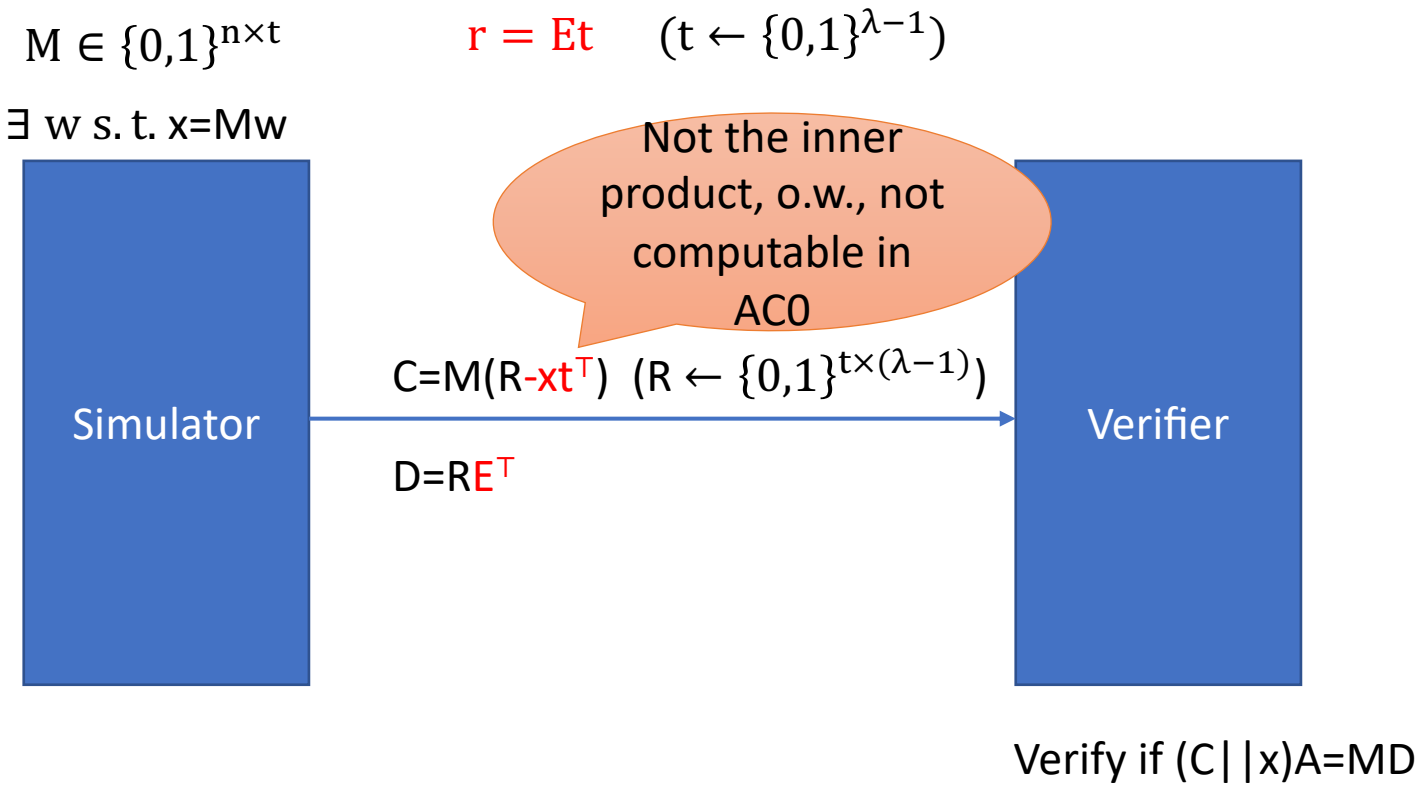


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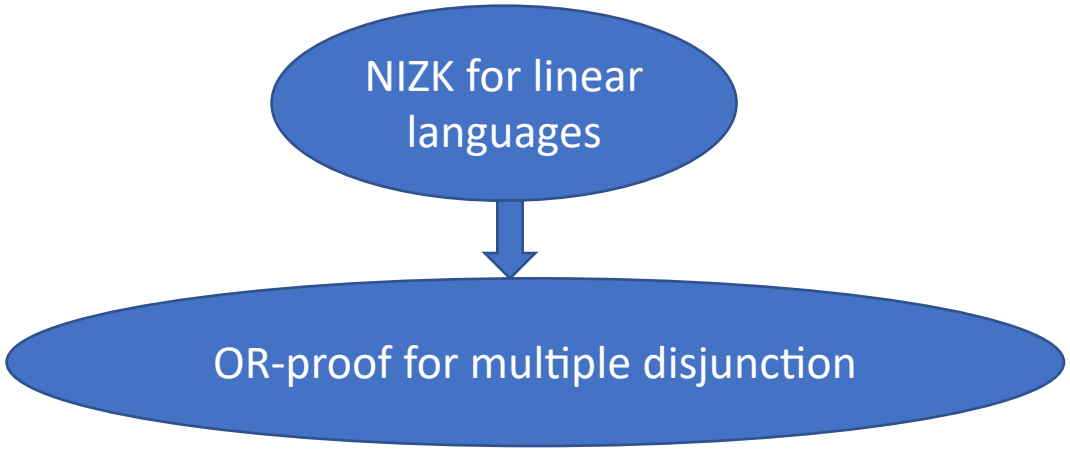
# NIZK for AC0 circuits



# OR-proof for 1-out-of-2 disjunction

$M_0 \in \{0,1\}^{n \times t}$
$M_1 \in \{0,1\}^{n \times t}$
$x_j = M_j w$ for some $j \in \{0,1\}$

$r = Et + e \ \ (t \leftarrow \{0,1\}^{\lambda - 1})$

Prover

Verifier

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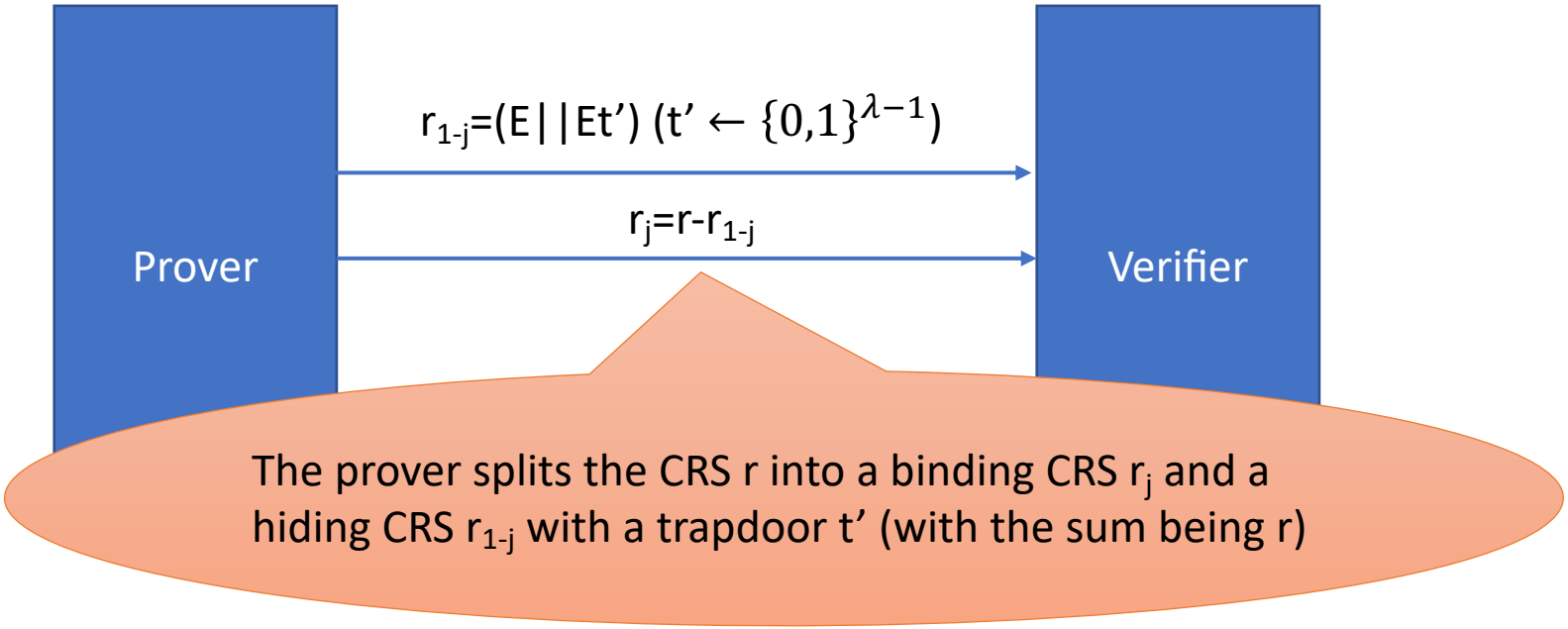

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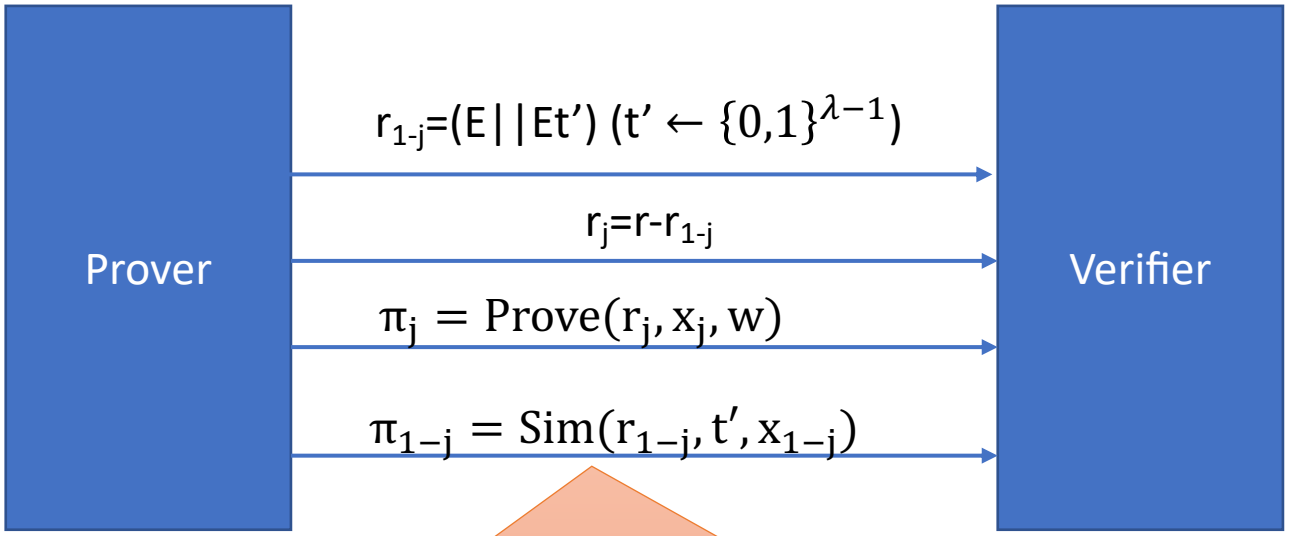


Then it generates proofs for $x_j$ and $x_{1-j}$ with w and t' respectively by making use of the prover and simulator of NIZK for linear languages.

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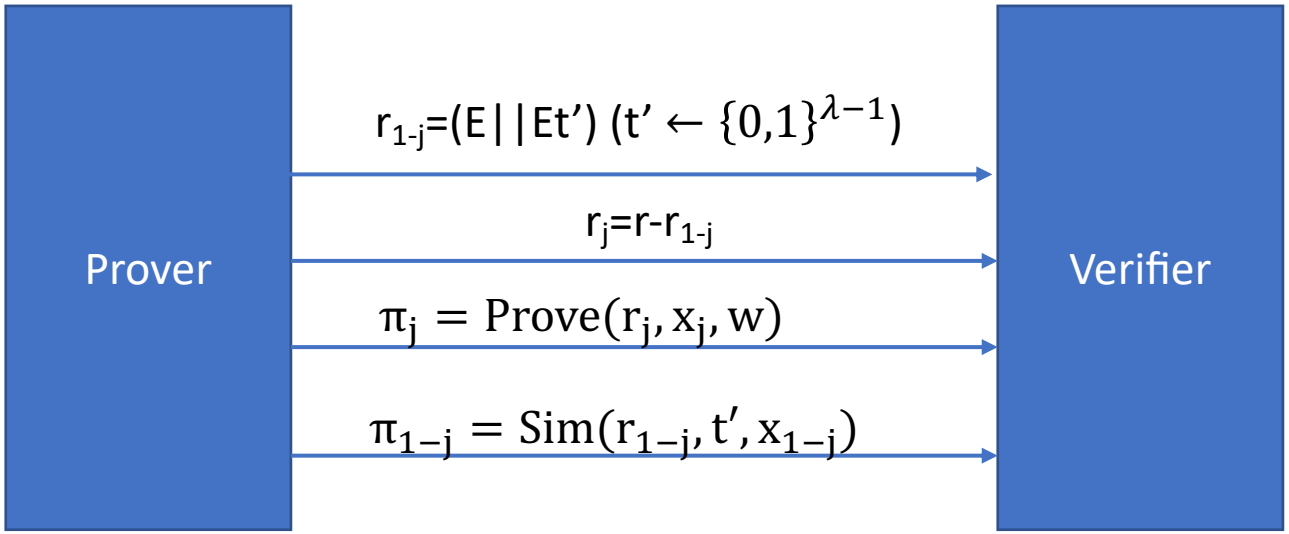


Soundness: when the parity of r is 1, either $r_0$ or $r_1$ has parity 1 (binding).

Zero-knowledge: when the parity of r is 0, both $r_0$ and $r_1$ have parity 0 (hiding).

# OR-proof for 1-out-of-n disjunction

A natural idea:

1. Assuming the jth statement is true, let the prover split the original bindCRS r into $r_1, \cdots, r_n$ s.t. $r = r_1 + r_2 + \cdots + r_n$ and only $r_j$ has parity 1.
2. The verifier checks $r = r_1 + r_2 + \cdots + r_n$.

Neither can be performed in AC0

# OR-proof for 1-out-of-n disjunction

Solution: a double layers sampler to sample n CRSs with the sum being r in AC0.

First layer:
Sample n trapdoors $(t_1, \cdots t_n)$ with the sum being a 0-vector.
- by sampling n-bit strings with parity being 0

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Second layer:
1. Sample n hiding CRSs as $(r_1, \cdots, r_n) = (Et_1, \cdots, Et_n)$.

We now obtain n hiding CRSs with the sum being 0.

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2. Add $r_j$ with r.

We now obtain n random CRSs with the sum being r and the jth CRS being binding.

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This sampler allows a prover to split r into n CRSs in AC0

# OR-proof for 1-out-of-n disjunction

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With the proof, a verifier can check the validity of split CRSs in AC0.

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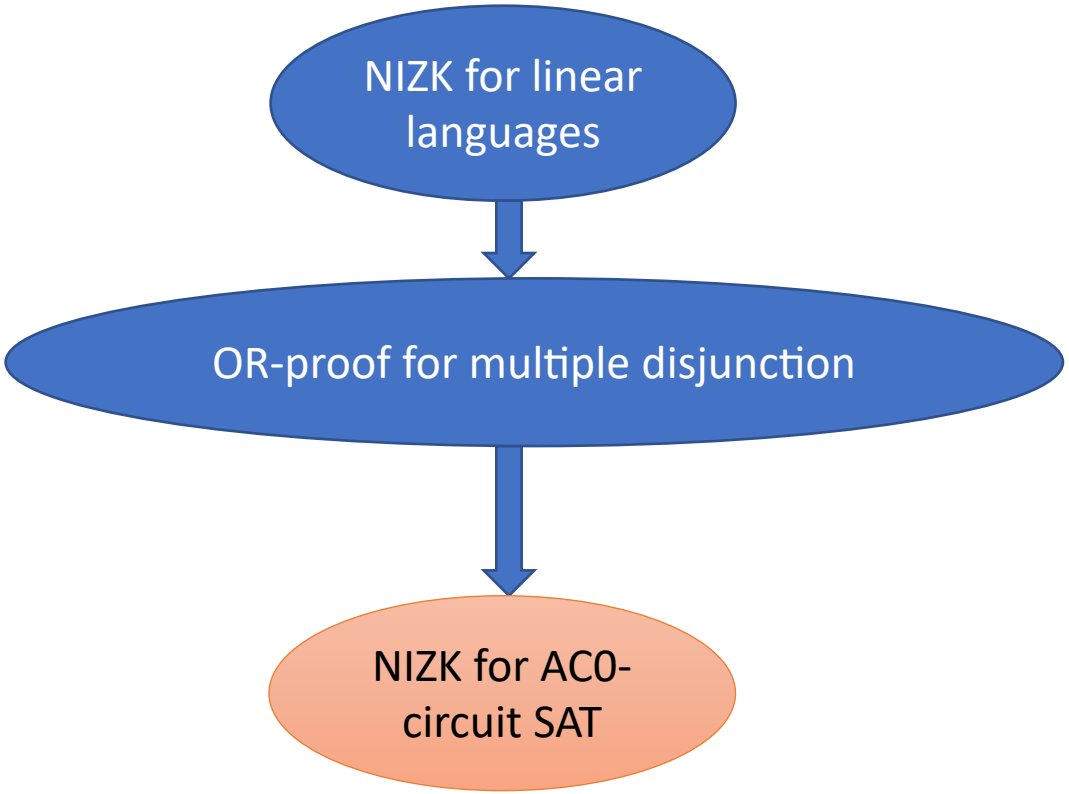
2. Add $r_j$ with r.

Additional step for verification:
Use the NIZK for linear languages to prove that $r_1 + \cdots + r_n = r$
- an AC0 extractor extracting the witness from $(t_1, \cdots t_n)$

# NIZK for AC0-circuit SAT



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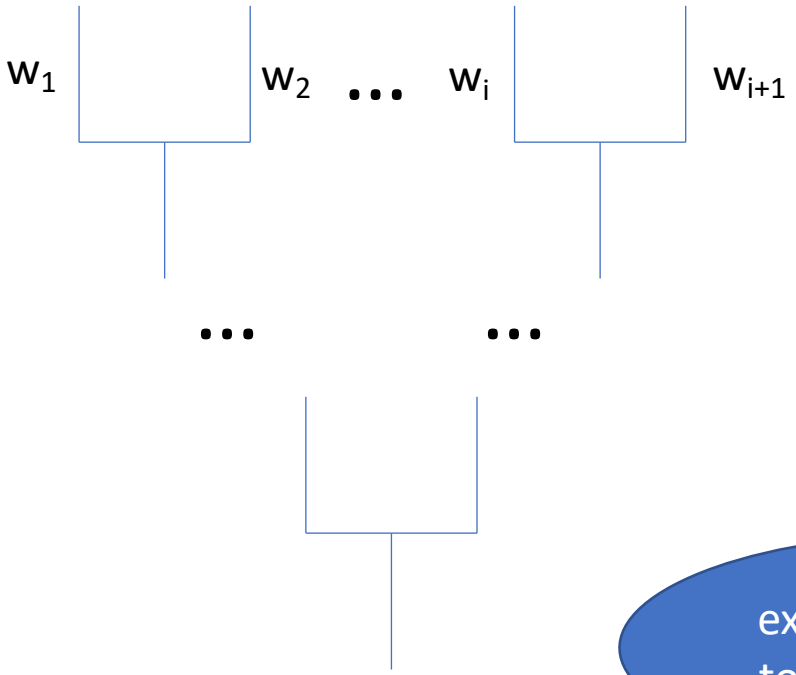
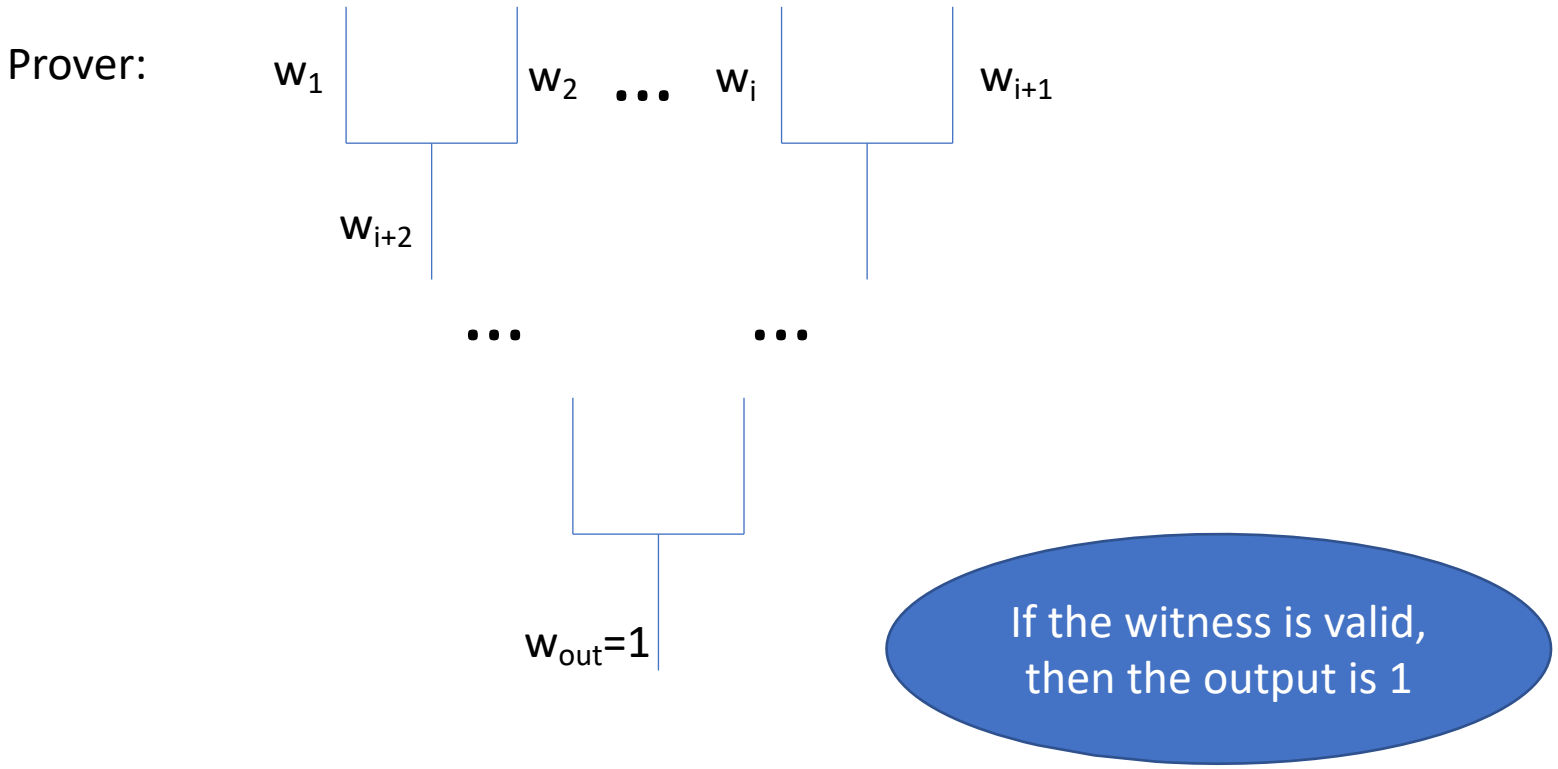
Prover:




The prover first extends the witness to contain bits of all wires

# NIZK for AC0-circuit SAT



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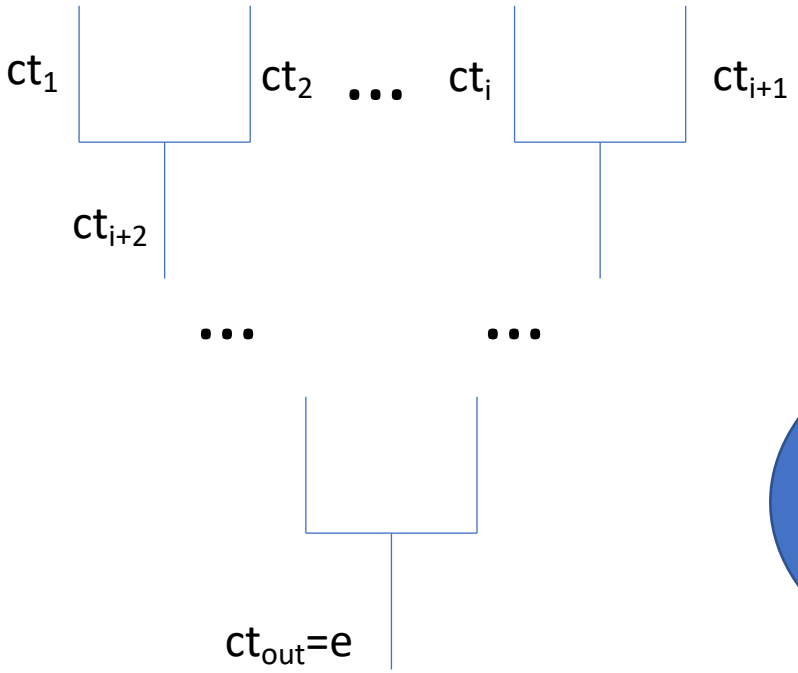


$$ct_i = Er_i + ew_i$$

The prover encodes $w_i$ as a random vector $ct_i$ with the parity being $w_i$

# NIZK for AC0-circuit SAT

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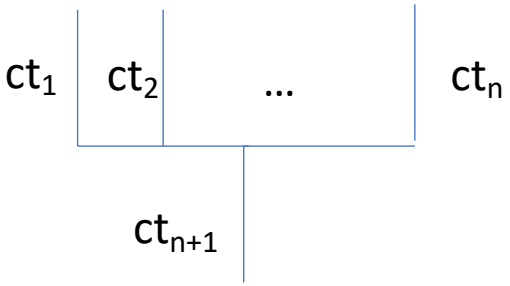


Unbounded fan-in AND gate

For AND-gate, use the OR-proof for multiple disjunction to prove that either $ct_i$-e$\in \mathrm{Span}(\mathrm{E})$ for all i or $ct_j$ ,$ct_{n+1} \in \mathrm{Span}(\mathrm{E})$ for some j.

# NIZK for AC0-circuit SAT

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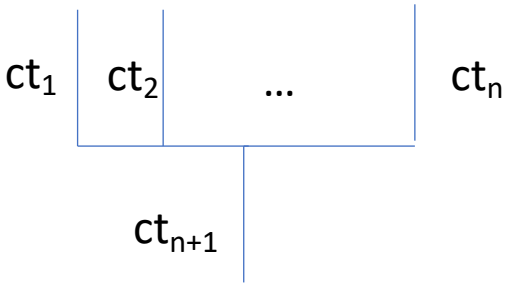


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Enough to ensure consistence of encoded wires

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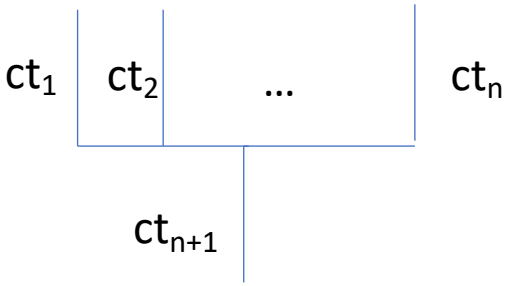


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A challenge: to get the witness, the prover has to decide which wire corresponds to 0 when the output is 0. The prover cannot find such wire by searching each wire in order.

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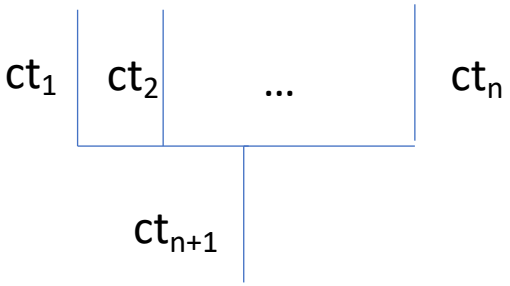


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We prove the existence of an AC0-algorithm that can output the index of the first 0-bit or 1-bit amongst n bits.

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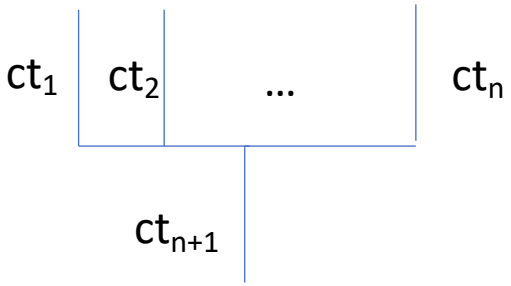


Unbounded fan-in AND gate

For AND-gate, use the OR-proof for multiple disjunction to prove that either $ct_i - e \in \mathrm{Span}(E)$ for all i or $ct_j, ct_{n+1} \in \mathrm{Span}(E)$ for some j.

For OR-gates, we prove the consistence in a similar way.
For NOT-gates, we prove that the sum of e and the input and output encodings are in Span(E).

# Extensions

- Conversion to non-interactive zaps (NIWI in the plain model)

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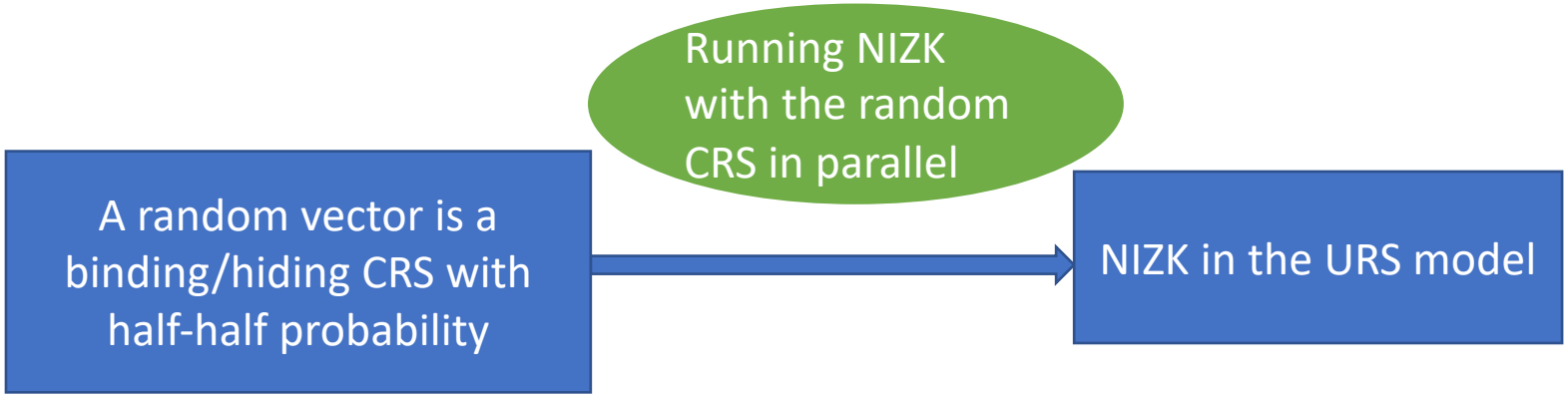

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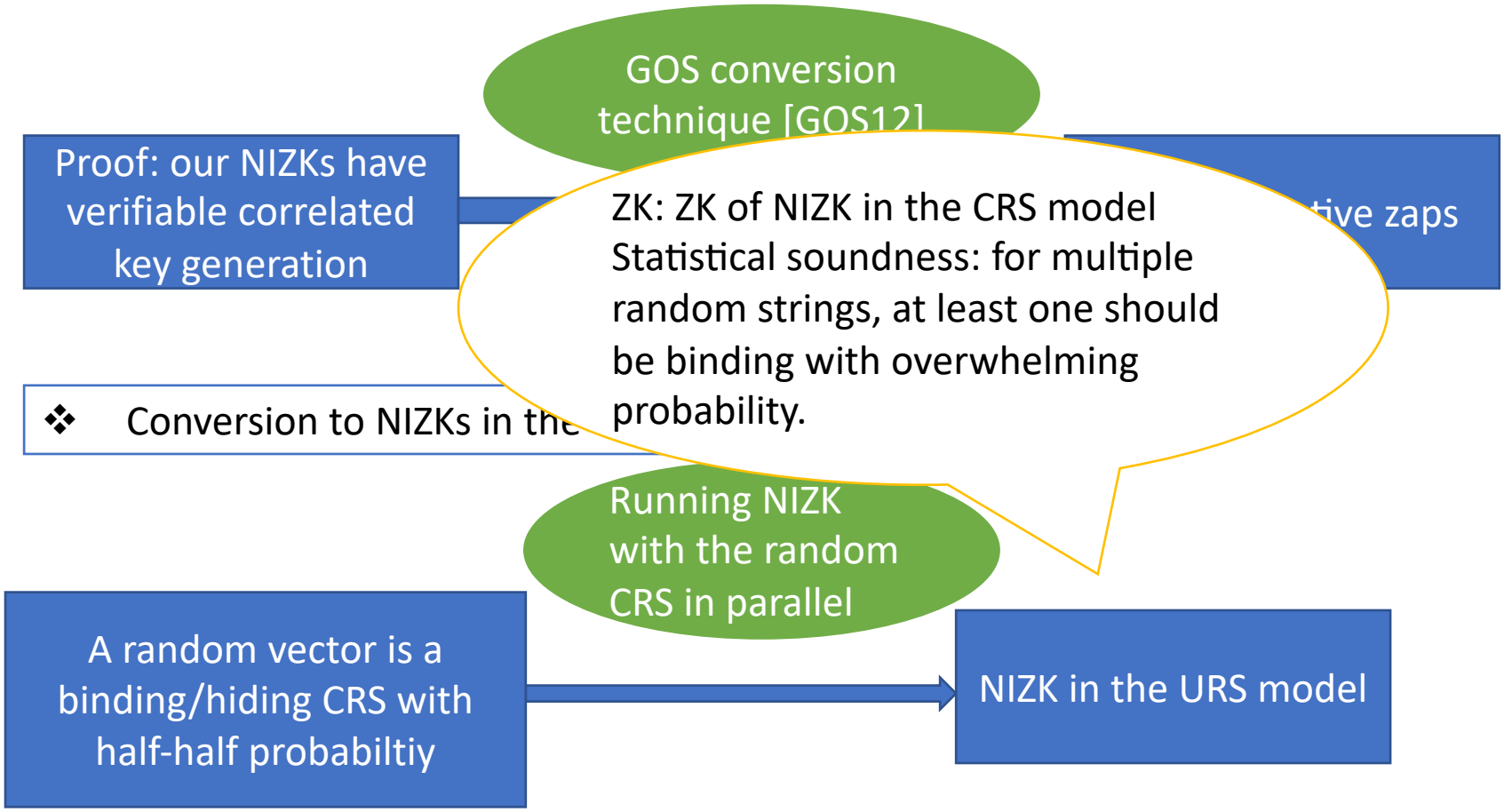


- Conversion to NIZKs in the

# Conclusion

Fine-grained proof systems unconditionally secure against AC0 adversaries

1. NIZK for AC0-circuit SAT
2. Non-interactive zaps
3. NIZKs in the URS model