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Security of Truncated Permutation Without Initial Value

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Motivation and Overview

(1) A truncated permutation applied to the concatenation of an input value with a **fixed** *IV* is *indifferentiable* from a random function up to a certain bound;

(2) We prove that (almost) the same bound holds **if** the fixed value is replaced by a randomized value;

(3) Concrete applications in the context of parallel variable-length digest generation.

Preliminary

About Public Permutations and Functions

- Many symmetric cryptographic primitives are based on public permutations and/or functions:
  - Even-Mansour construction $x \mapsto k + \mathcal{P}(x + k)$ for a public permutation $\mathcal{P}$;
  - Merkle-Damgård construction instantiated by a public compression function $\mathcal{F}$;
  - sponge/duplex construction instantiated either by a public permutation $\mathcal{P}$ or by a public function $\mathcal{F}$.

- We know how to construct a public permutation $\mathcal{P}$ with "good" properties (e.g., via the SPN strategy).

- *What about the case of public functions?*

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- *What about the case of public functions?*

SOP Function

- Sum of permutations:
  - let $\mathcal{P}_0, \mathcal{P}_1$ be two public (random) permutations over $(\{0,1\}^b, +)$;
  - the Sum-Of-Permutations (SOP) function

$$\mathrm{SOP}(x) = \mathcal{P}_0(x) + \mathcal{P}_1(x)$$

    is indifferentiability from a random function up to 2^b queries [BN18];
- Analogous result holds given a single public (random) permutation $\mathcal{P}$:

$$\mathrm{SOP}'(x) = \mathcal{P}(x\|0) + \mathcal{P}(x\|1)\,.$$

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TRUNC Function

- Let $\mathcal{P}$ be a public (random) permutation over $(\{0,1\}^b, +)$;
- Let the truncation function

$$\mathrm{TRUNC} : \{0,1\}^{b-m} \to \{0,1\}^n$$

be defined as

$$\mathrm{TRUNC}(x) = \mathrm{left}_n \mathcal{P}(IV \| x)\,,$$

where

- $IV \in \{0,1\}^m$ is an initial value;
- $\mathrm{left}_n : \{0,1\}^b \to \{0,1\}^n$ returns the n leftmost bits (where $n < b$).

Security of TRUNC

Given

- $m =$ size of IV,
- $n =$ output size of the TRUNC,
- $b =$ size of $\mathcal{P}$,

then

- Dodis et al. [DRR+09] proved that TRUNC is indifferentiable from a random function up to
$$\approx \min\left\{2^{\frac{b-n}{2}}, 2^{\frac{b}{2}}, 2^m\right\} \text{ queries};$$
- Choi et al. [CLL19] proved that it is indifferentiable from a random function up to
$$\approx \min\left\{2^{\frac{2b-n}{3}}, \frac{2^{b-n}}{b-n}, 2^m\right\} \text{ queries}.$$

From a Fixed to a Randomized Initial Value

About the Initial Value

- Initial value is crucial for indifferentiability!
- If *omitted* (hence, $m = 0$), the obtained TRUNC function $\mathcal{F}(x) = \mathrm{left}_n(\mathcal{P}(x))$ can be easily distinguished from a random function:
  - $y \xleftarrow{\$} \{0,1\}^b$, and let $x = \mathcal{P}^{-1}(y)$;
  - if $\mathrm{left}_n(y) \neq \mathcal{O}(x)$, then $\mathcal{O}$ is the *random function* with prob. 1.
- *The initial value is overkill:* hash function "instantiated" by a truncated permutation with $m = 0$ can be still secure!

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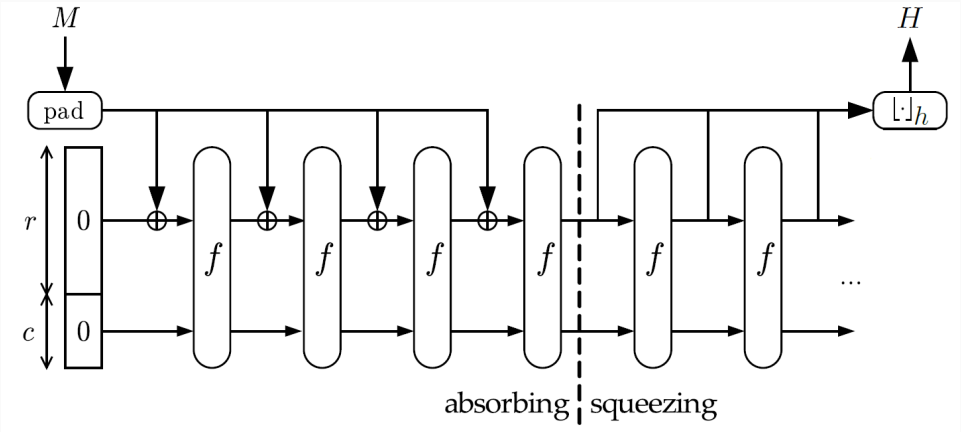

Truncation Without Fixed *IV*

- *Idea:* Replace the *fixed* initial value with a **randomized** one!
- From

$$\mathrm{TRUNC}(x) = \mathrm{left}_n \mathcal{P}(IV \| x)\,,$$

to

$$\mathrm{RTRUNC}(M, x) = \mathrm{left}_n \mathcal{P}(\mathcal{H}(M) \| x)$$

where

- $\mathcal{H} : \{0,1\}^\star \to \{0,1\}^m$ is a hash function;
- $M \in \{0,1\}^\star$ and $x \in \{0,1\}^{b-m}$;
- $\mathcal{P}(\cdot)$, IV, $\mathrm{left}_n(\cdot)$ as before.

Security of RTRUNC

Under the assumption that $\mathcal{H}$ is a random oracle and $\mathcal{P}$ is a random permutation, the RTRUNC construction is indifferentiable from a random oracle up to

$$\approx \min\left\{2^{\frac{2b-n}{3}}, \frac{2^{b-n}}{b-n}, 2^{\frac{m}{2}}\right\}$$

queries, where $m =$ output size of $\mathcal{H}$, and (as before) $n =$ output size of the RTRUNC, $b =$ size of $\mathcal{P}$.

For k bits of security:

$$m \geq 2 \cdot k\,,$$

$$b \geq \max\left\{m, k+n, \frac{3 \cdot k + n}{2}\right\}\,.$$

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Sketch of the Proof (1/2)

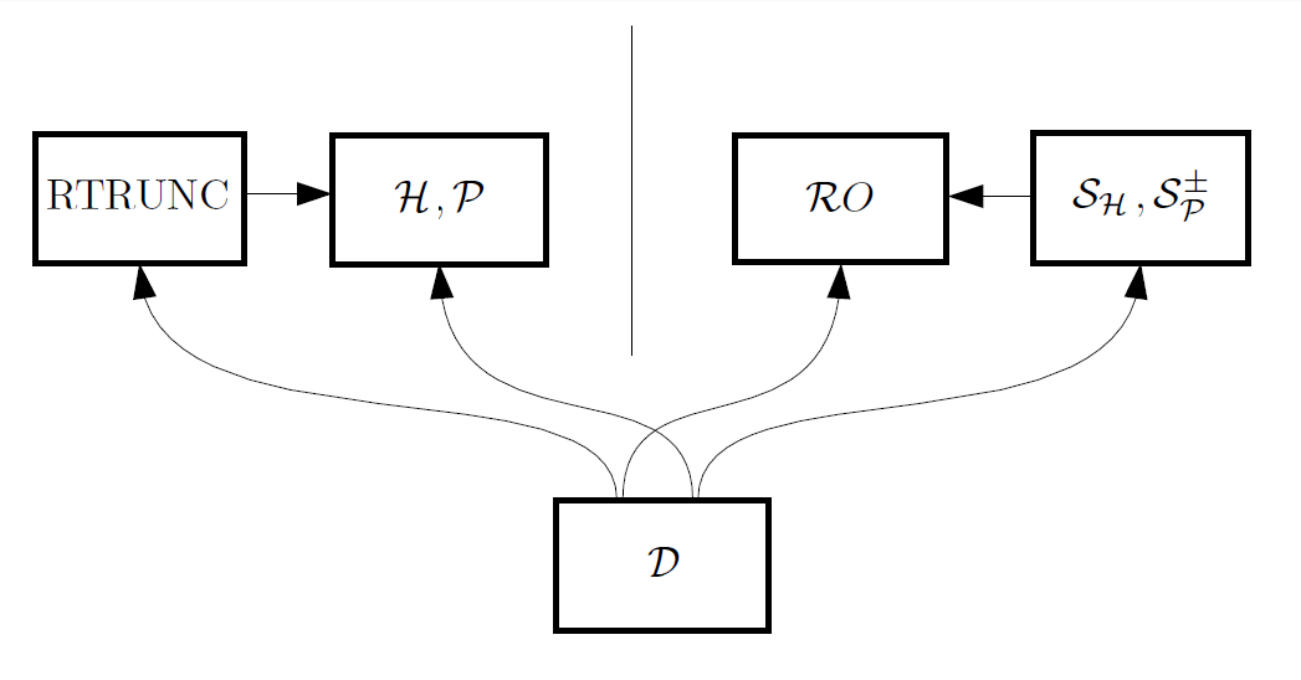

Figure: The indifferentiability model [MRH04, CDM+05].

Sketch of the Proof (2/2)

- Strategy of the proof similar to the one proposed by Choi et al. [CLL19] (but with some crucial differences!): see the paper for details;
- For comparison:
  - TRUNC is indifferentiable from a random oracle up to

$$\approx \min\left\{2^{\frac{2b-n}{3}}, \frac{2^{b-n}}{b-n}, \mathbf{2^m}\right\} \text{ queries;}$$

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  - *Difference:* existence of collisions at the output of $\mathcal{H}$.

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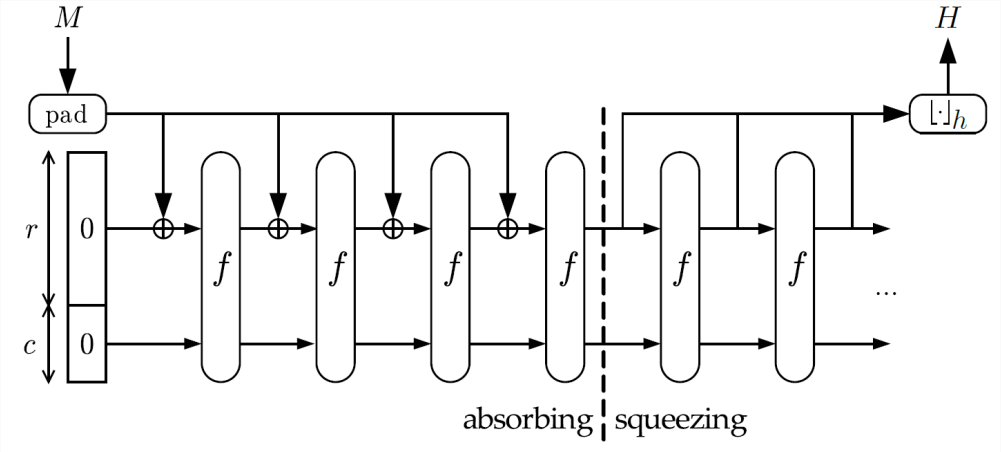
Applications

Parallel Digest Generation (1/2)

- Merkle-Damgård construction [Mer89, Dam89] outputs a *fixed* sized digest:
  - inputs: $M = M_1 \| \cdots \| M_\mu$, and $h_0 = IV$;
  - output: h_μ, where

$$\forall i \in \{0, 1, \ldots, \mu - 1\} : \qquad h_{i+1} = \mathcal{F}(h_i, M_i) ;$$

- sponge construction [BDP+07] does allow for arbitrarily sized outputs, but the *output generation is inherently sequential*:



Parallel Digest Generation (2/2)

- Quest for *parallel digest generation*: evaluating several permutations simultaneously in modern CPUs is faster than evaluating them in sequence;
- In the case of PRFs: see the Farfalle construction proposed by Bertoni et al. [BDH+17] at ToSC 2017;
- In the case of hashing: "Mask Generation Functions" (MGFs);

$\longrightarrow$ we propose a *new MGF construction* – called Cascade-MGF – based on RTRUNC: it extends any hash function $\mathcal{H}$ into a parallel eXtendable Output Function (XOF) that generates arbitrarily sized outputs.

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Cascade-MGF

- Let $\mathcal{H} : \{0,1\}^\star \to \{0,1\}^m$ be a (fixed output length) hash function, and let $\mathcal{P} : \{0,1\}^b \to \{0,1\}^b$ a public (random) permutation;
- Cascade-MGF:

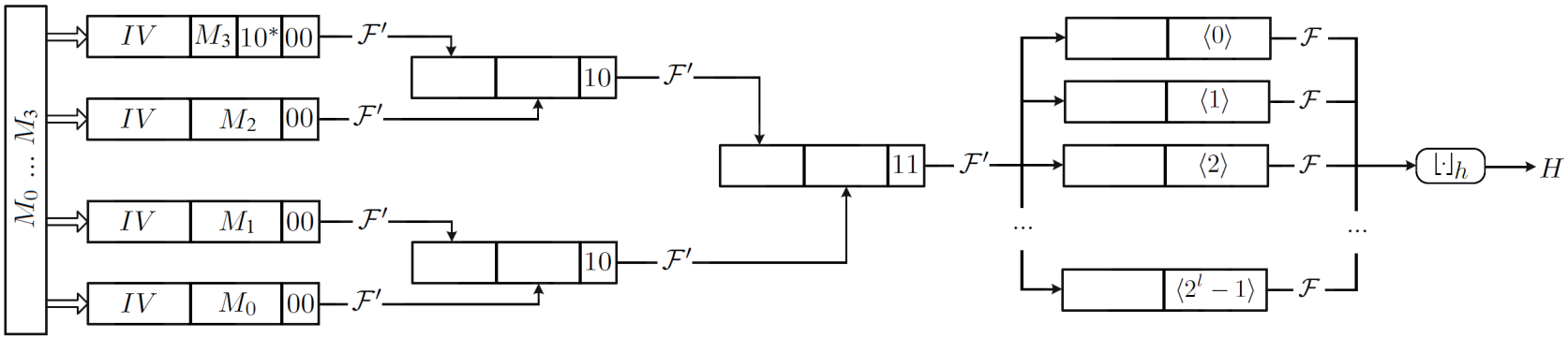
$$\mathrm{RTRUNC}(M, \langle 0 \rangle_l) \| \cdots \| \mathrm{RTRUNC}(M, \langle i \rangle_l) \| \cdots$$

  where

$$\mathrm{RTRUNC}(M, x) := \mathrm{left}_n \mathcal{P}(\mathcal{H}(M), x)\,;$$

- Cascade-MGF behaves as a (variable output length) random oracle if $\mathcal{H}$ behaves as a (fixed output length) random oracle.

Example: Cascade-MGF Construction via Tree Hash



Counter-MGF [KS98]

- Let $\mathcal{H} : \{0,1\}^\star \to \{0,1\}^m$ be a (fixed output length) hash function;
- (Variable output length) Counter-MGF:

$$\mathcal{H}(M\|\langle 0\rangle_l)\|\mathcal{H}(M\|\langle 1\rangle_l)\|\cdots\|\mathcal{H}(M\|\langle i\rangle_l)\|\cdots$$

- Counter-MGF behaves as a (variable output length) random oracle if $\mathcal{H}$ behaves as a (fixed output length) random oracle [SY12].

Chained-MGF [Nist09]

- Let $\mathcal{H} : \{0,1\}^\star \to \{0,1\}^m$ be a (fixed output length) hash function, and let $\mathcal{F} : \{0,1\}^{m+l} \to \{0,1\}^n$ a one way function;
- Chained-MGF:

$$\mathcal{F}(\mathcal{H}(M), \langle 0 \rangle_l) \| \mathcal{F}(\mathcal{H}(M), \langle 1 \rangle_l) \| \cdots \| \mathcal{F}(\mathcal{H}(M), \langle i \rangle_l) \| \cdots$$

- Chained-MGF behaves as a (variable output length) random oracle if $\mathcal{H}$ and $\mathcal{F}$ behave as (fixed output length) random oracles [SY12].

Cascade-MGF versus Chained-MGF with TRUNC (1/2)

Given

- $k =$ be the security level,
- $\mathcal{P} =$ public (random) permutation over $\{0,1\}^b$ for $b \geq 3 \cdot k$,
- $\mathcal{H}(M) \in \{0,1\}^{2 \cdot k}$,

then:

- $l = b - 2 \cdot k$ bits of counter for Cascade-MGF:

$$\mathsf{left}_{2 \cdot k} \mathcal{P}(\underbrace{\mathcal{H}(M)}_{2 \cdot k \text{ bits}} \| \langle i \rangle_l) \, ;$$

- $l = b - 3 \cdot k$ bits of counter for Chained-MGF instantiated by TRUNC:

$$\mathcal{F}(\mathcal{H}(M), \langle i \rangle_l) = \mathsf{left}_{2 \cdot k} \mathcal{P}(\underbrace{IV}_{k \text{ bits}} \| \underbrace{\mathcal{H}(M)}_{2 \cdot k \text{ bits}} \| \langle i \rangle_l) \, .$$

Cascade-MGF versus Chained-MGF with TRUNC (2/2)

Difference of k bits in the counter is crucial for permutations $\mathcal{P}$ over "small" fields:

- GIMLI [BKL+17] and XOODOO [DHV+18] are two permutations over $b = 384$ bits;
- assume $k = 128$ bits;
- Chained-MGF with TRUNC:

$$l = b - 3 \cdot k = 0 \text{ bits}$$

versus Cascade-MGF:

$$l = b - 2 \cdot k = 128 \text{ bits}$$

$\longrightarrow$ 1 digest block versus 2^{128} digest blocks!

Cascade-MGF versus Chained-MGF with SOP

Given $\mathcal{P}_0, \mathcal{P}_1 =$ two public (random) permutations over $\{0,1\}^b$ for $b \geq 2 \cdot k$:

- Chained-MGF instantiated by SOP:

$$\mathcal{F}(\mathcal{H}(M), \langle i \rangle_l) = \mathcal{P}_0(\mathcal{H}(M) \| \langle i \rangle_l) + \mathcal{P}_1(\mathcal{H}(M) \| \langle i \rangle_l)$$

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- Cascade-MGF always improves over Chained-MGF with SOP:

$$\underbrace{b-k}_{\text{size digest block}} \geq \underbrace{b}_{\text{size digest block}} \cdot \underbrace{\frac{1}{2}}_{\text{2 permutation calls}}$$

  that is, $b \geq 2 \cdot k$, which is always satisfied.

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Summary

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- We proved that the truncated permutation construction instantiated by a randomized value is still indifferentiability from a random function (up to a certain limit);
- New Cascade-MGF construction for extending any hash function $\mathcal{H}$ into a parallel XOF;
- Concrete practical advantages of Cascade-MGF with respect to other MGF constructions.

Thanks for your attention!

Questions?

Comments?

Cascade-MGF versus Counter-MGF (with Tree Hash)

- Comparison between Counter-MGF and Cascade-MGF: **not** possible without being specific about the hash function $\mathcal{H}$;
- *If* $\mathcal{H}$ = *tree hash*, then the cost (= number function/permutation calls) for generating an output of arbitrary length:
  - independent of the size of the input message for Cascade-MGF;
  - depends on it for Counter-MGF (minimum if size of input = power of 2):
  - cost for Cascade-MGF $<$ cost for Counter-MGF.

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