Cryptographic Primitives with Hinting Property

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Hints in real life

What is a good password hint?

A hint could be anything from **a single word that reminds you of your password to a full-fledged sentence, a string of characters, or anything else that helps you jump-start your associative memory**. The one thing that should not be in your password hint is your actual password, obviously.

wikiHow to do anything... UPGRADE

Question

If I forget my password, should I make my password hint the same as my password?

Community Answer

No, it's a very bad idea. This will make it very easy for someone to hack into your computer.

Helpful 36 Not Helpful 5

Password hint: “what is the sound of one hand clapping?”

Hints in cryptography

Pseudorandom Generator (PRG)

Assumption: The distinguisher is given no additional information about the seed $\mathbf{s}$

$$\mathbf{s} = (s_1, \ldots, s_n) \leftarrow \{0,1\}^n$$

$$\text{PRG}: \{0,1\}^n \rightarrow \{0,1\}^{n^2}$$

$y_1 \quad y_2 \quad y_3 \quad \cdots \quad y_{n-1} \quad y_n$	n output strings of length n bits each
$\approx_c$	(computationally indistinguishable)
$z_1 \quad z_2 \quad z_3 \quad \cdots \quad z_{n-1} \quad z_n$	n **random** strings of length n bits each

Hints in cryptography

Hinting PRG [KW19]

The distinguisher is given some additional information or **"hint"** about the seed $\mathbf{s}$

$\mathbf{s} = (s_1, \ldots, s_n) \leftarrow \{0,1\}^n$

$\text{HPRG}: \{0,1\}^n \rightarrow \{0,1\}^{n^2}$

$y_1 \quad y_2 \quad y_3 \quad \ldots \quad y_{n-1} \quad y_n$ — n output strings of length n bits each

Swap (y_i, z_i) if $s_i = 1$

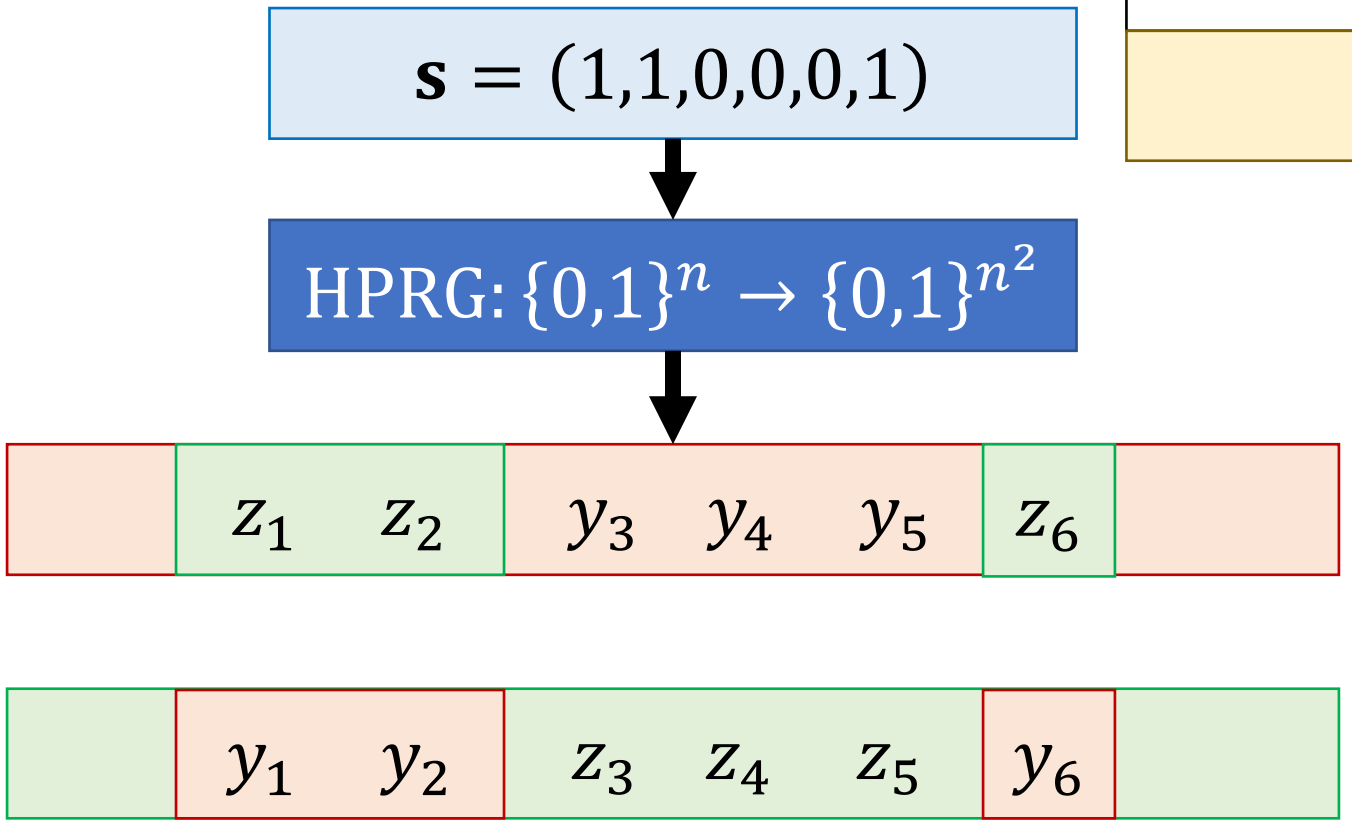
$z_1 \quad z_2 \quad z_3 \quad \ldots \quad z_{n-1} \quad z_n$ — n **random** strings of length n bits each

Hints in cryptography

Hinting PRG [KW19]

The distinguisher is given some additional information or **"hint"** about the seed $\mathbf{s}$

A toy example with $n = 6$



Hints in cryptography

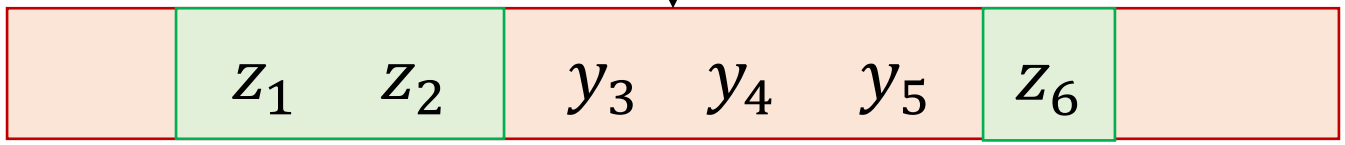
Hinting PRG [KW19]

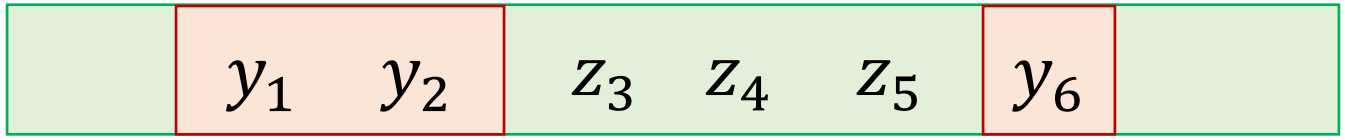
The distinguisher is given some additional information or **"hint"** about the seed **s**

A toy example with $n = 6$

$\mathbf{s} = (1,1,0,0,0,1)$

$\text{HPRG}: \{0,1\}^n \to \{0,1\}^{n^2}$





Embeds information about the seed **s**

But remains pseudorandom
(computationally indistinguishable from a $2 \times n$ matrix of random n-bit strings)

Hints in cryptography

Hinting PRG [KW19]

$\mathbf{s} = (s_1, \ldots, s_n) \leftarrow \{0,1\}^n$

$\mathrm{HPRG}: \{0,1\}^n \to \{0,1\}^{n^2}$

$y_1 \quad y_2 \quad y_3 \quad \cdots \quad y_{n-1} \quad y_n$

$z_1 \quad z_2 \quad z_3 \quad \cdots \quad z_{n-1} \quad z_n$

The Hinting PRG game

Let $\mathrm{HPRG}(\mathbf{s} = (s_1, \ldots, s_n)) = (y_1, \ldots, y_n)$

$\mathbf{Z} = [z_{j,b}]_{b \in \{0,1\}, j \in [n]} \in (\{0,1\}^n)^{2 \times n}$

$z_{j,s_j} = y_j\,, \qquad z_{j,1-s_j} = z_j \leftarrow \{0,1\}^n$

$\mathbf{Z} \approx_c \mathbf{U} \leftarrow (\{0,1\}^n)^{2 \times n}$

$\mathbf{Z}$ is pseudorandom despite encoding additional "hints" about $\mathbf{s}$

Why study Hinting PRGs?

Cryptographic “booster” [KW19]

- Generic transformation from CPA to CCA security for public-key encryption (PKE)
- Transformation also works for attribute-based encryption (ABE) and predicate encryption (PE)

A myriad of other cryptographic applications

Key ingredient in new constructions of:

- Trapdoor one-way functions [KMT19,GHMO21]
- Designated-verifier NIZKs [LQRWW19]
- Black-box non-interactive non-malleable commitments [GKLW21]
- CCA-compatible public-key infrastructure [KW21]

Hinting Property vs Circular Security

Can view hinting property as a “deterministic” form of symmetric-key circular security

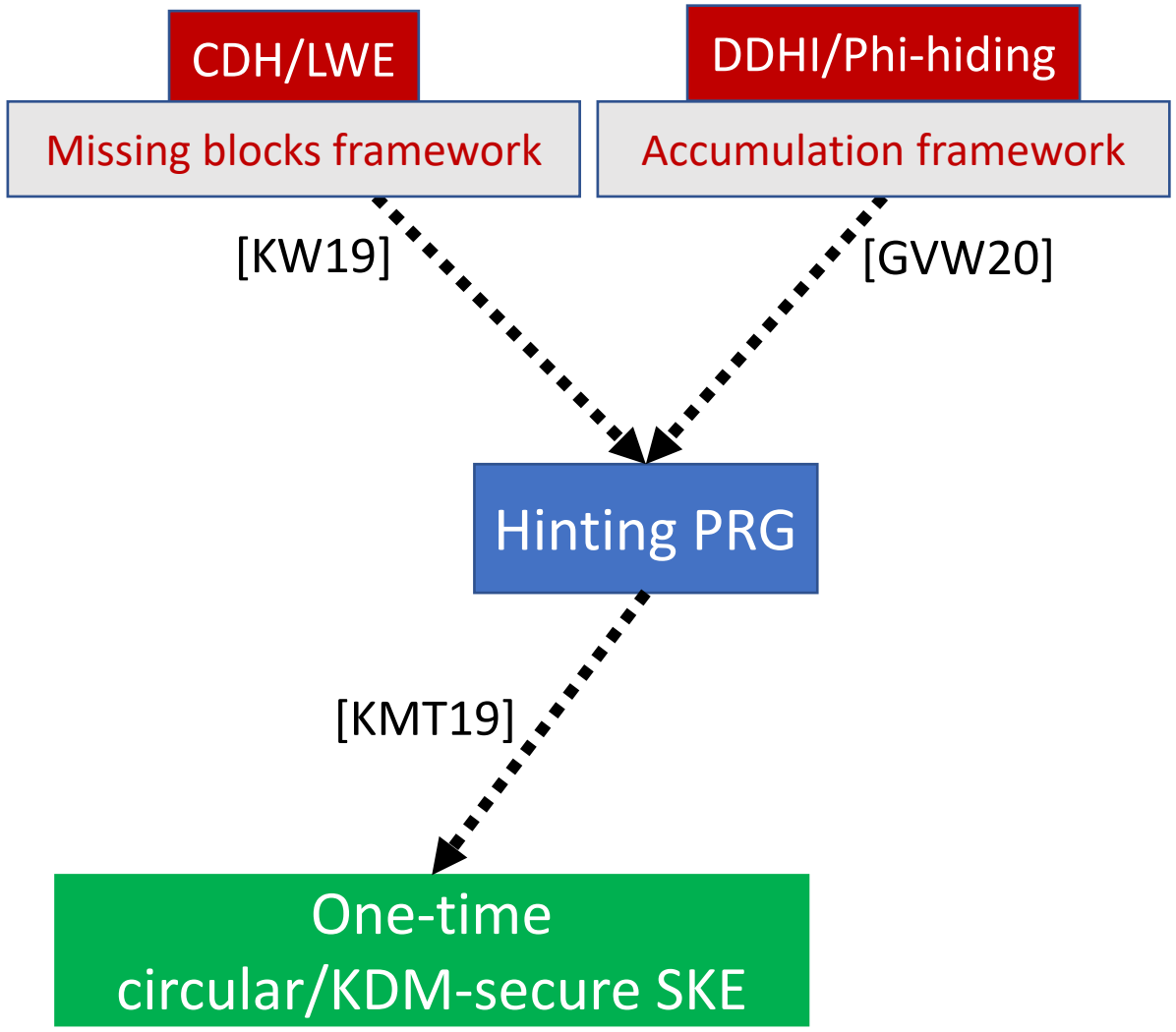
Hinting PRG	• Retains pseudorandomness of (deterministic) output even given some “hinting” information about the secret seed
Circular-Secure Symmetric-Key Encryption	• Retains semantic security of (randomized) encryption even when encrypting the bits of the secret key

Hinting Property vs Circular Security

Can view hinting property as a “deterministic” form of symmetric-key circular security

Hinting PRG	• Retains pseudorandomness of (deterministic) output even given some “hinting” information about the secret seed • Construction and proof techniques are complicated and inherently rooted in algebraic/ “public-key style” mathematical assumptions [KW19, GVW20] • Cryptographic complexity unclear (inherently requires public-key techniques?)
Circular-Secure Symmetric-Key Encryption	• Retains semantic security of (randomized) encryption even when encrypting the bits of the secret key • Simple construction and proof techniques from certain standard decisional assumptions (e.g., DDH [BHHO08]) • Weaker than public-key encryption (realizable from random oracles [BRS03])

Hinting PRGs: Current landscape vs what we ask



- **Simpler constructions and proofs** from standard decisional assumptions (e.g., DDH)?
- Realizations from a **wider class of assumptions** (e.g., isogeny-based assumptions)?

- Can we **extend** the hinting property to **other** symmetric-key primitives?
- What are the applications of such extensions? Can we realize them from standard assumptions?

- **Cryptographic complexity** of hinting PRGs?
- Are mathematically structured “public-key-style” assumptions **necessary**?

Our Contributions

Bold arrows denote results/implications from our paper

- New extensions of the hinting property to **pseudorandom functions (PRFs)**
  - PRF → Hinting PRF
  - weak PRF* → Hinting weak PRF
- New security definitions:
  - (Weak) pseudorandomness in the presence of **hints about the secret key**
  - **Multi-evaluation** security (as opposed to single-evaluation security for hinting PRG)

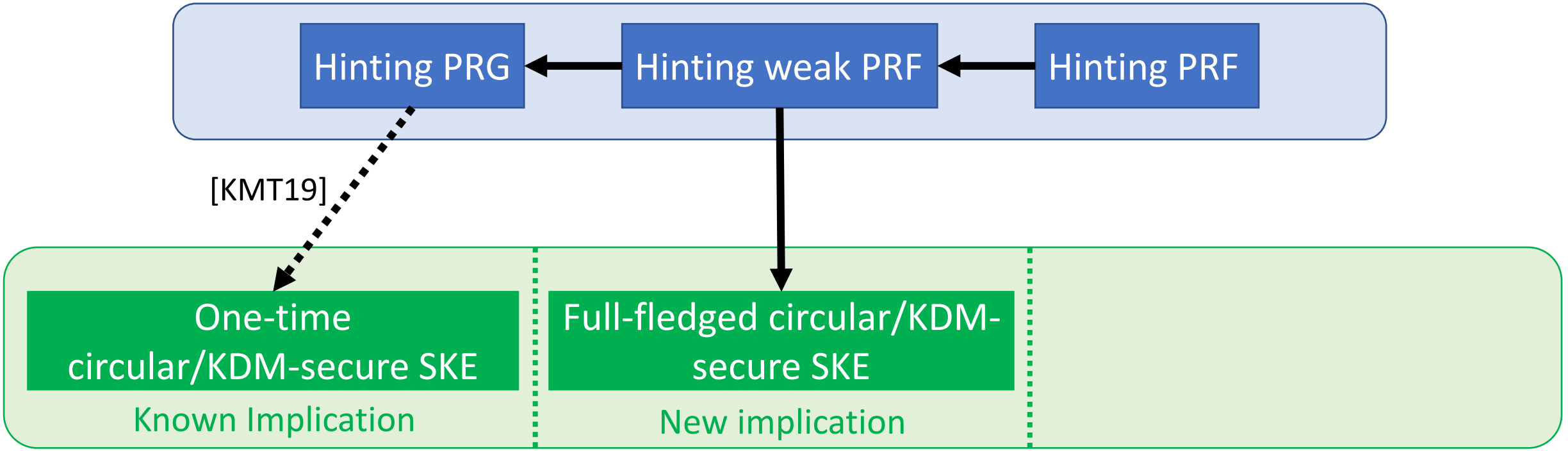
Cryptographic primitives with hinting property

* A weak PRF (informally) guarantees pseudorandomness on uniformly randomly sampled inputs

Our Contributions

Bold arrows denote results/implications from our paper

- New **stronger** implications of the hinting property:
  - A hinting weak PRF implies (in a black-box manner) a full-fledged circular-secure symmetric-key encryption (SKE) scheme
    - Circular security guarantee holds with respect to multiple encryptions of the secret key
  - Non-black-box amplification to full-fledged KDM-secure SKE [App14]



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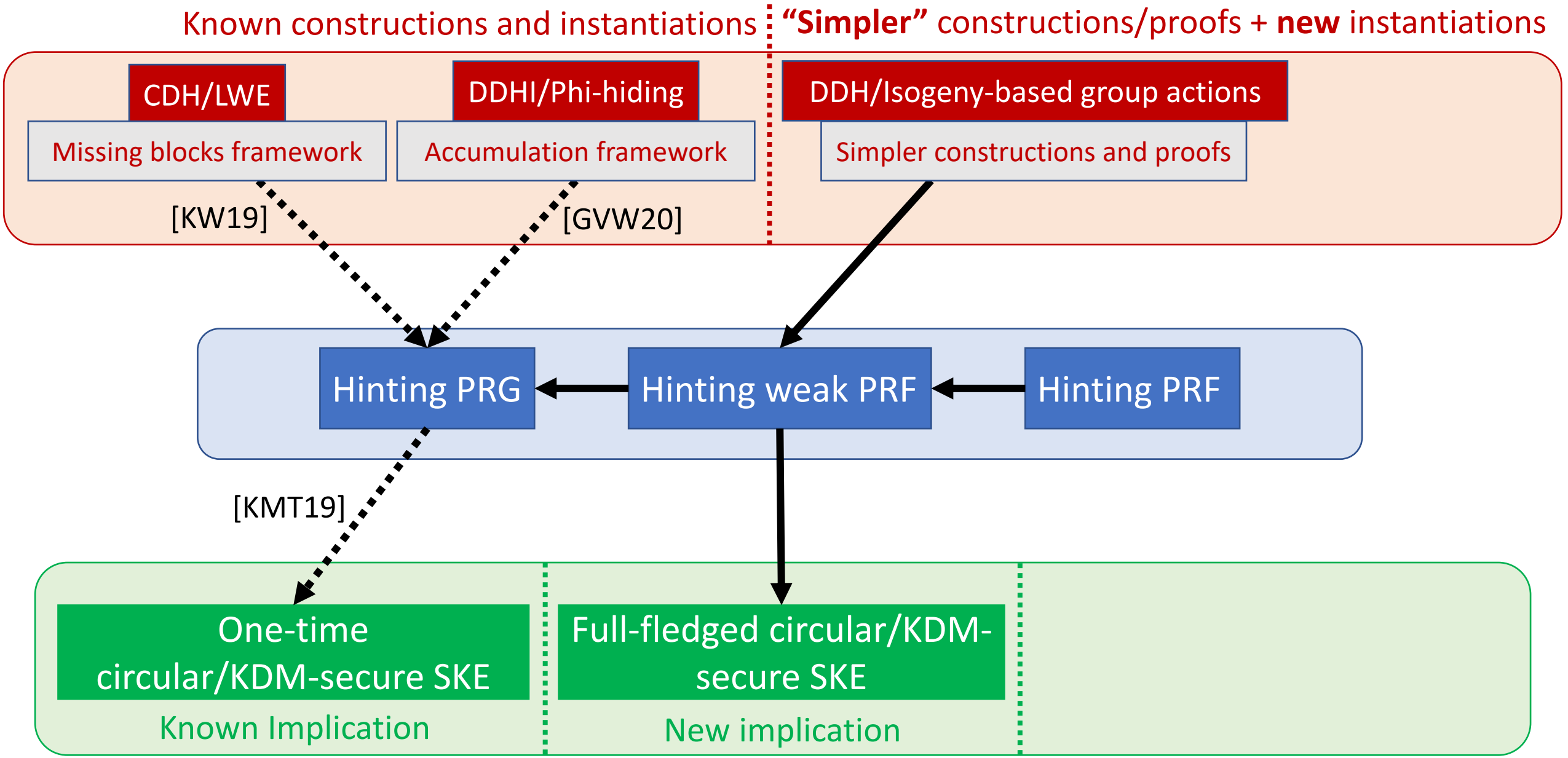

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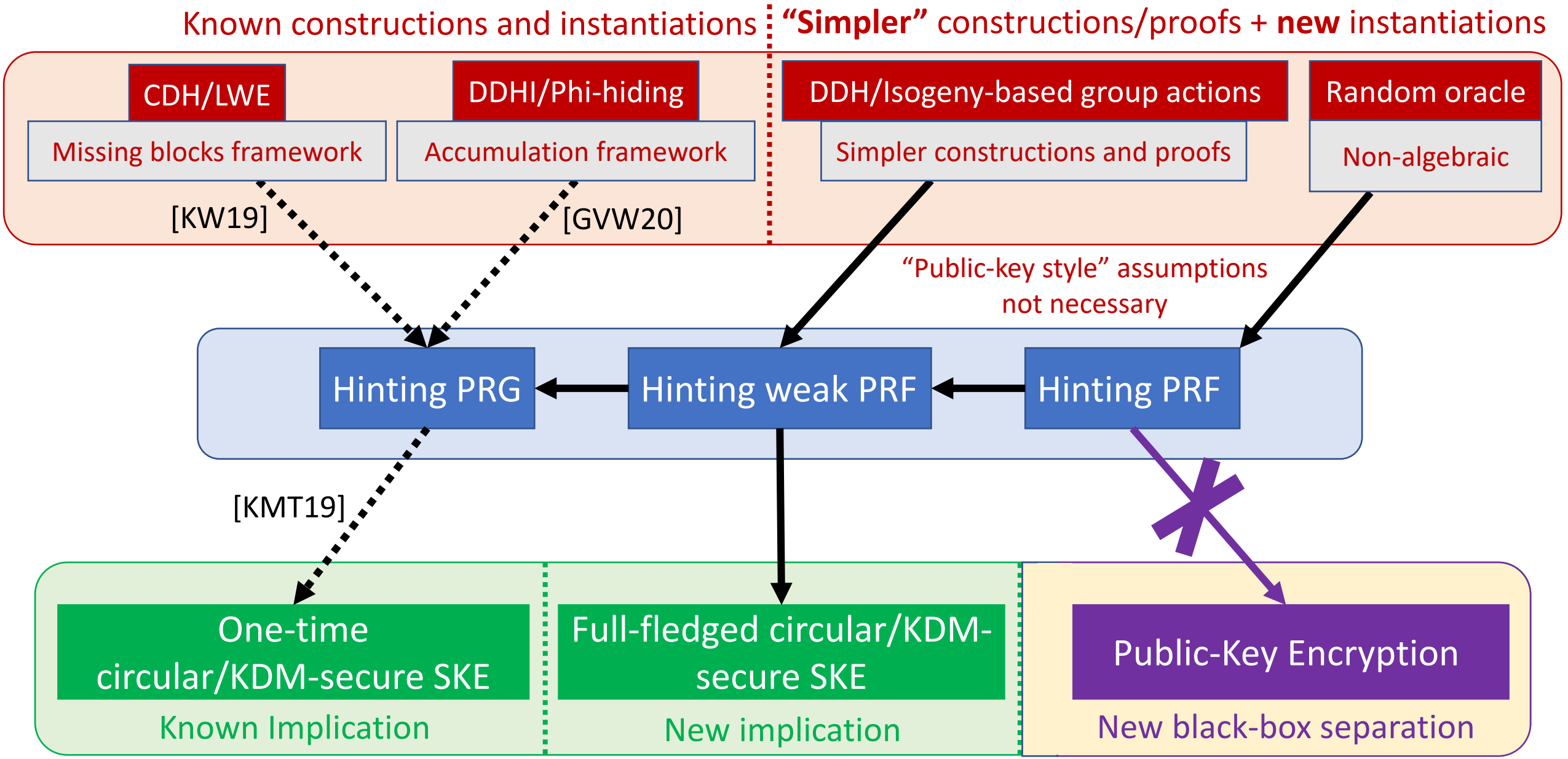
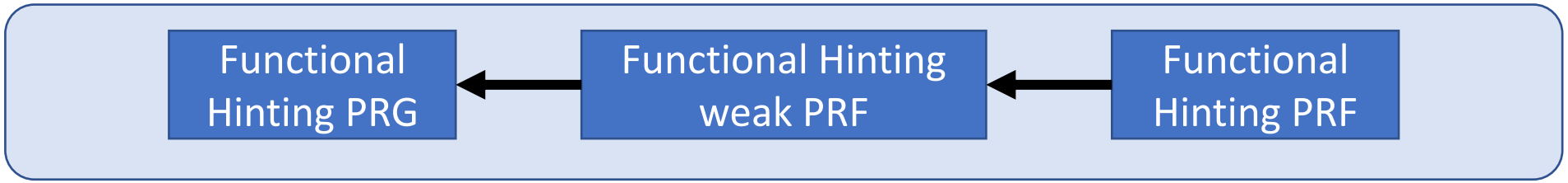


Our Contributions: Functional Hinting Property

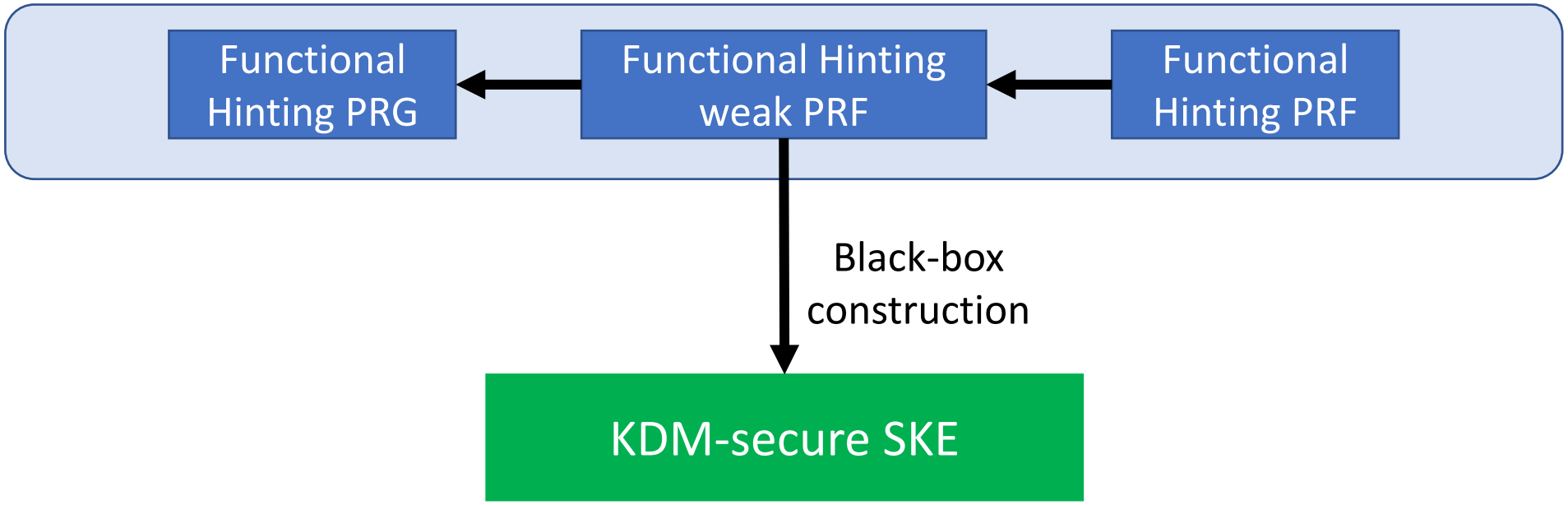
- New strengthening of the hinting property to **functional hinting property**
  - Functional hinting PRG: hints about each bit of **some function** of the seed
  - Functional hinting (weak) PRF: hints about each bit of **some function** of the secret key
  - The function may be chosen **adversarially** from a fixed function family
- Example:
  - Seed $\mathbf{s} = (s_1, \ldots, s_n) \in \{0,1\}^n$
  - Hints about each bit of the **quadratic** function $f(\mathbf{s}) = \{s_i . s_j\}_{i,j\in[n]} \in \{0,1\}^{n^2}$



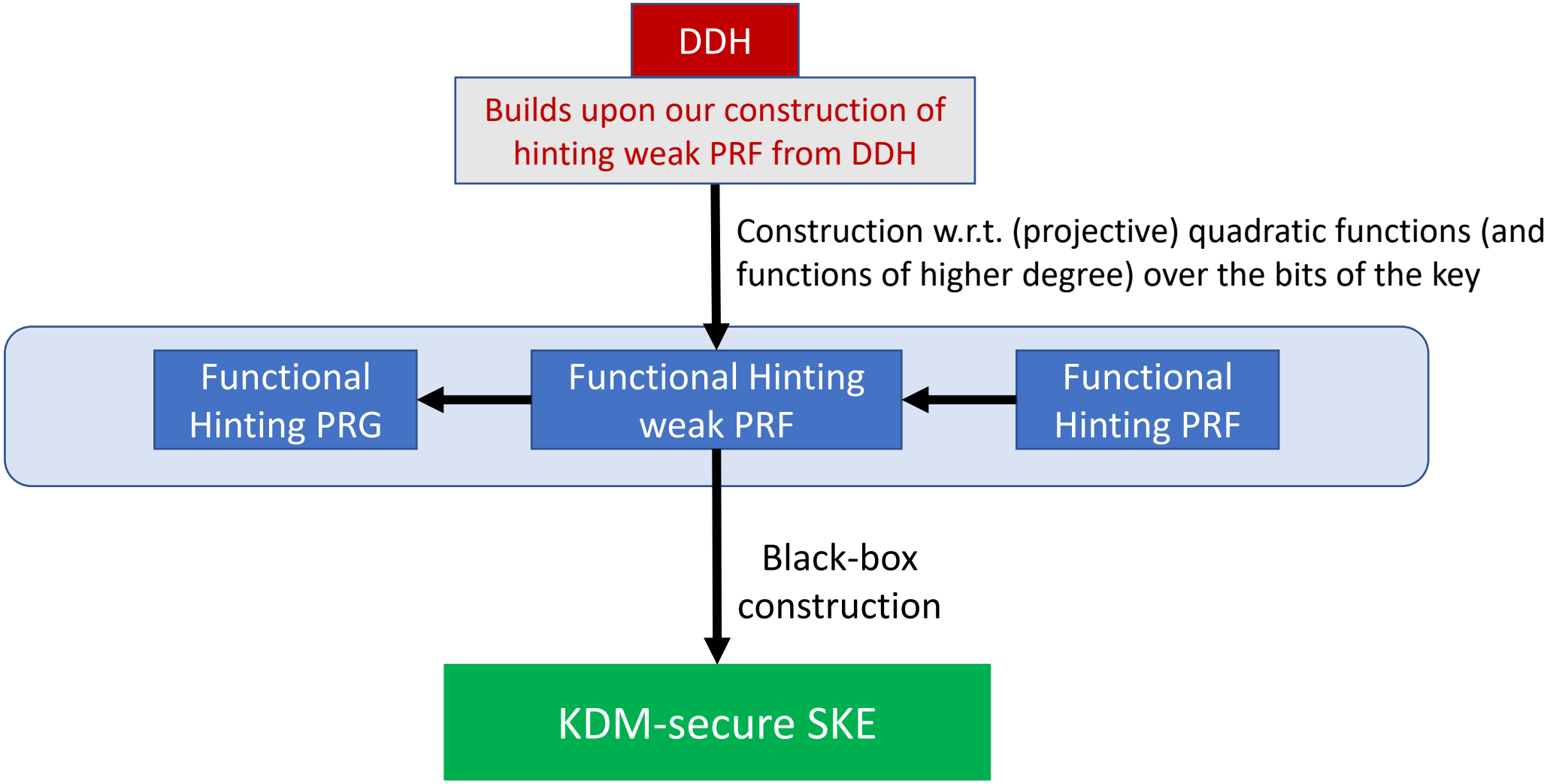
Cryptographic primitives with **functional** hinting property

Our Contributions: Functional Hinting Property

- Even stronger implications of the functional hinting property:
  - A functional hinting weak PRF w.r.t. some function family $\mathcal{F}$ implies (in a black-box manner) a KDM-secure symmetric-key encryption (SKE) w.r.t. the same function family $\mathcal{F}$
  - New approach for achieving KDM-security in a black-box manner w.r.t. Boolean function families
  - Prior approaches [BGK11,KMT19] are designed specifically for arithmetic function families and inherently require some algebraic structure on the key space



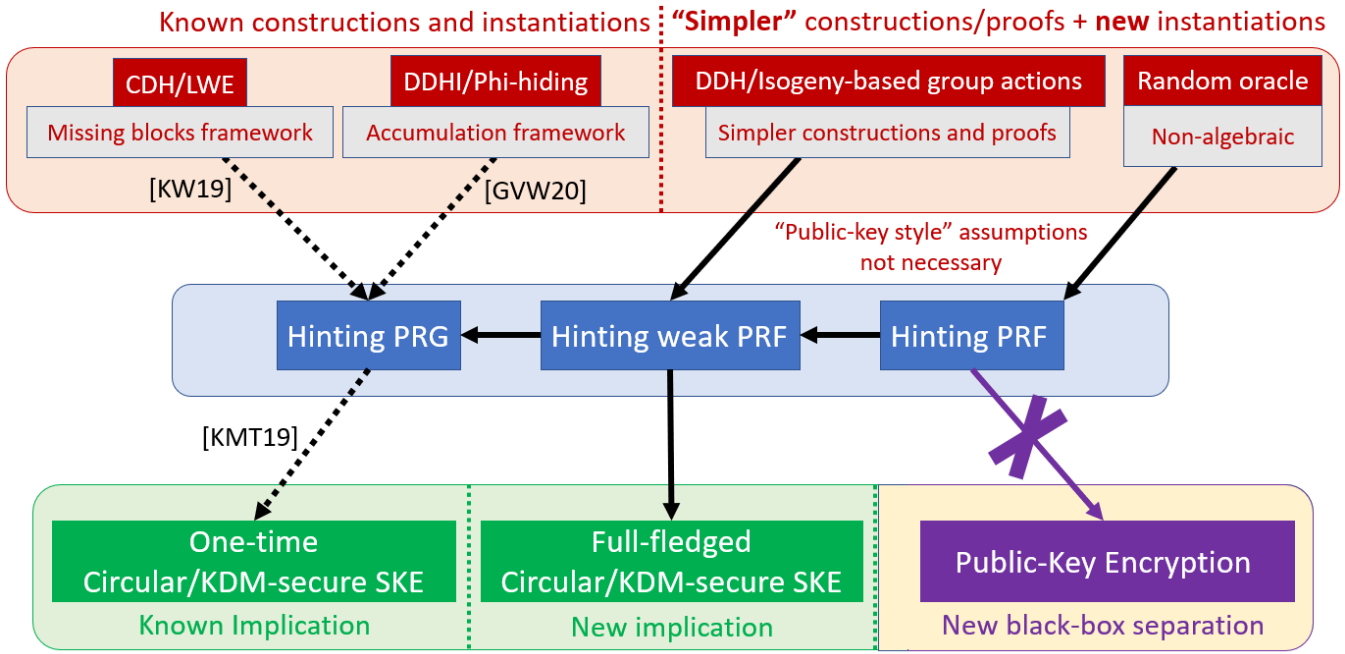
Our Contributions: Functional Hinting Property



Our Contributions: Summary

Bold arrows denote results/implications from our paper

Primitives with hinting property



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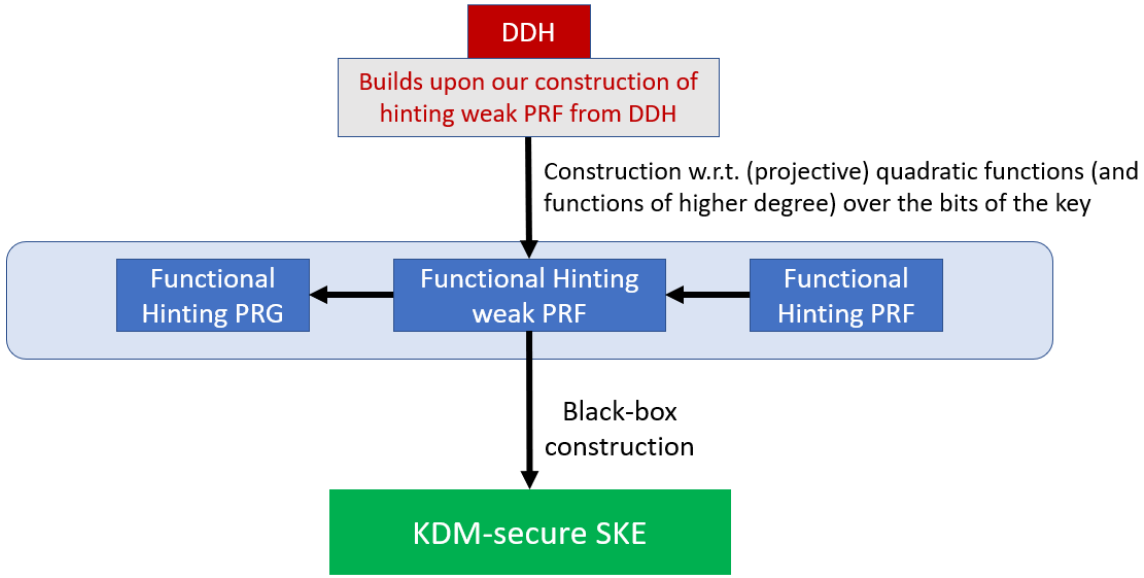

Talk Outline

- Simple Hinting PRG from DDH
- Hinting PRG from cryptographic group actions (brief overview)
- Hinting weak PRF (brief overview)
- Summary and open questions

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Simple Hinting PRG from DDH

- Let G be a DDH-hard group of prime order q with generator g, and let $n > \log|G| + \omega(\log \lambda)$
- Let $\mathrm{M} \leftarrow \mathbb{Z}_q^{n \times n}$ be a matrix with entries $m_{i,j}$ uniformly sampled from $\mathbb{Z}_q$
- Let $[\mathrm{M}]$ denote the matrix of group elements $g^{\mathrm{M}} \in G^{n \times n}$ (exponentiation applied component-wise)
- Define the function $H_{[\mathrm{M}]}: \{0,1\}^n \rightarrow G^n$ as:

$$H_{[\mathrm{M}]}(\mathrm{s} \in \{0,1\}^n) = [\mathrm{Ms}]$$

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[PW08,FGK+10,AMP19]

H is a PRG assuming that G is DDH-hard

(Hint: apply the leftover hash lemma)

Simple Hinting PRG from DDH

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Our Result

H is a **hinting** PRG assuming that G is DDH-hard

(Hint: see the next few slides)

Simple Hinting PRG from DDH

$$H_{[\mathrm{M}]}(\mathrm{s} \in \{0,1\}^n) = [\mathrm{Ms}]$$

Our Result

H is a **hinting** PRG assuming that G is DDH-hard

Lemma

H is a **hinting** PRG assuming that it is a PRG

Simple Hinting PRG from DDH

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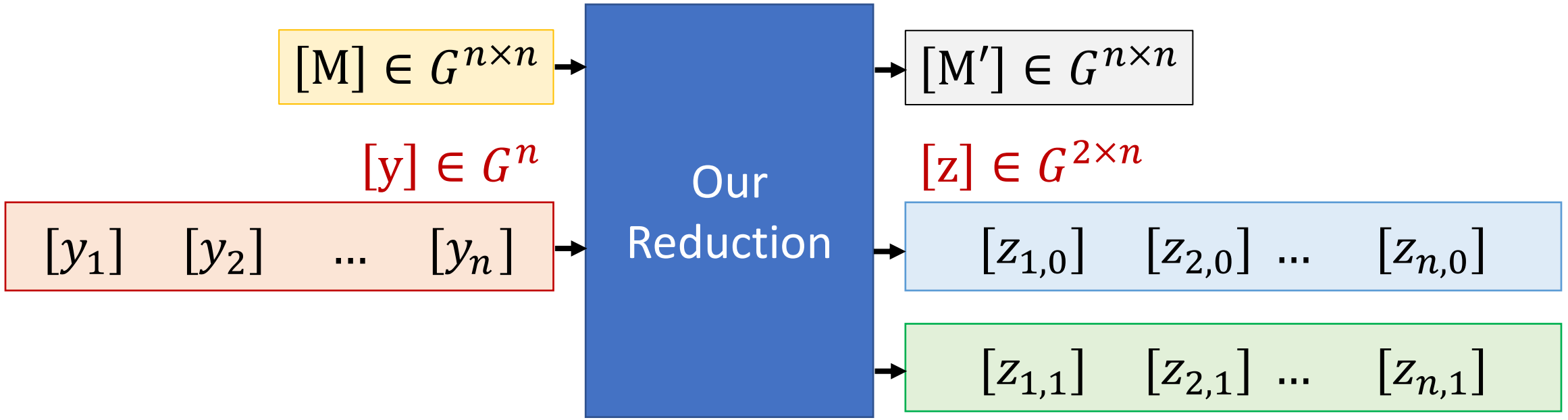

Simple Hinting PRG from DDH

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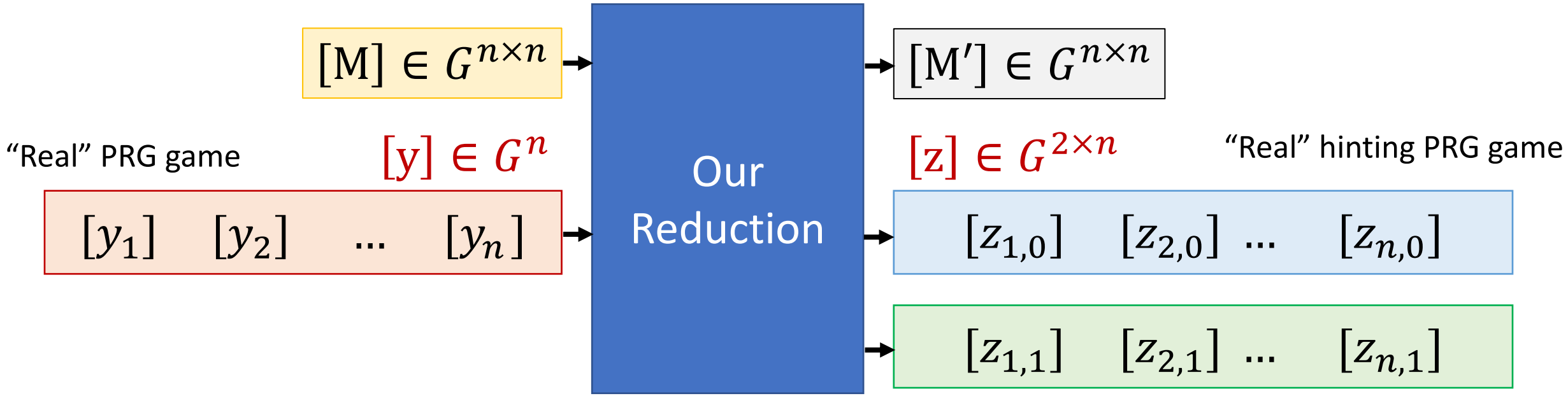

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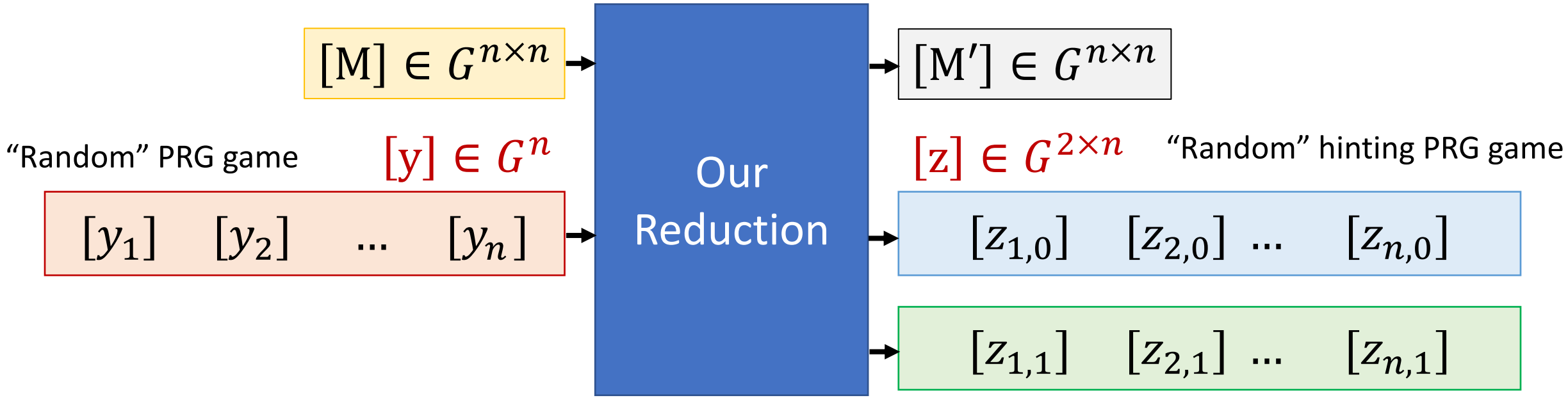

Simple Hinting PRG from DDH

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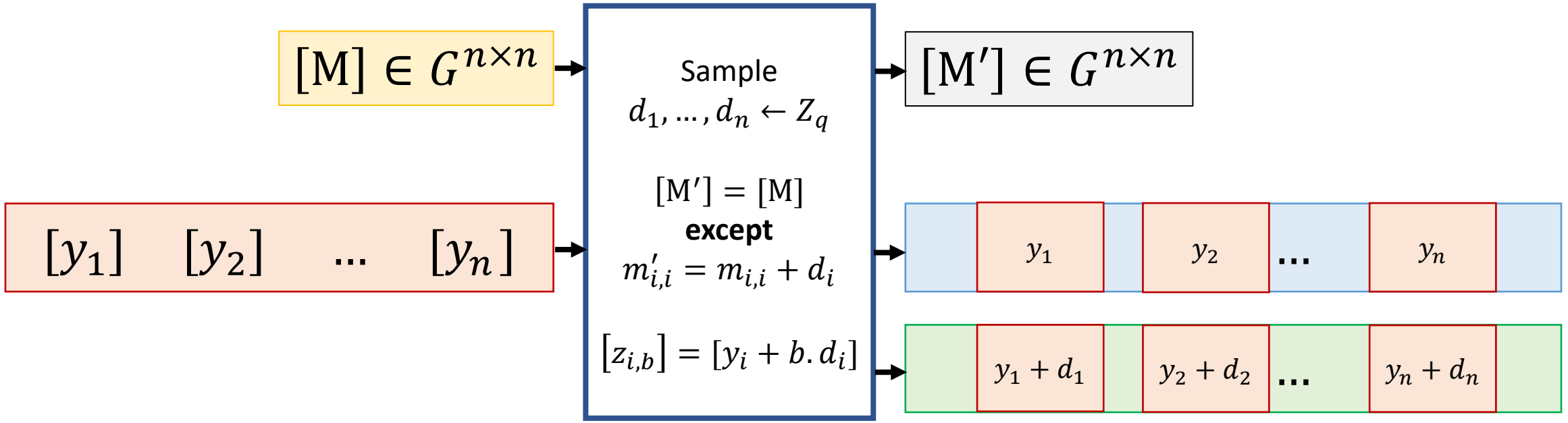
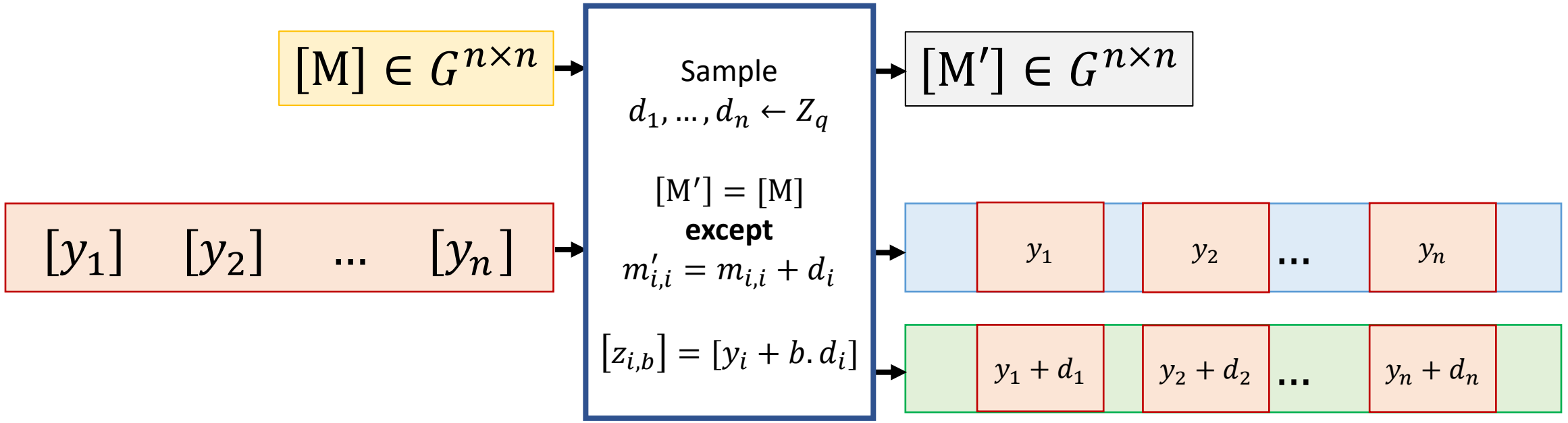


Simple Hinting PRG from DDH

$[y_i]$-s are distributed as in the "real" PRG game w.r.t. the PRG parameter [M]

- $y_i = \sum_{j\in[n]} m_{i,j}s_j, \quad z_{i,0} = y_i, \quad z_{i,1} = y_i + d_i$



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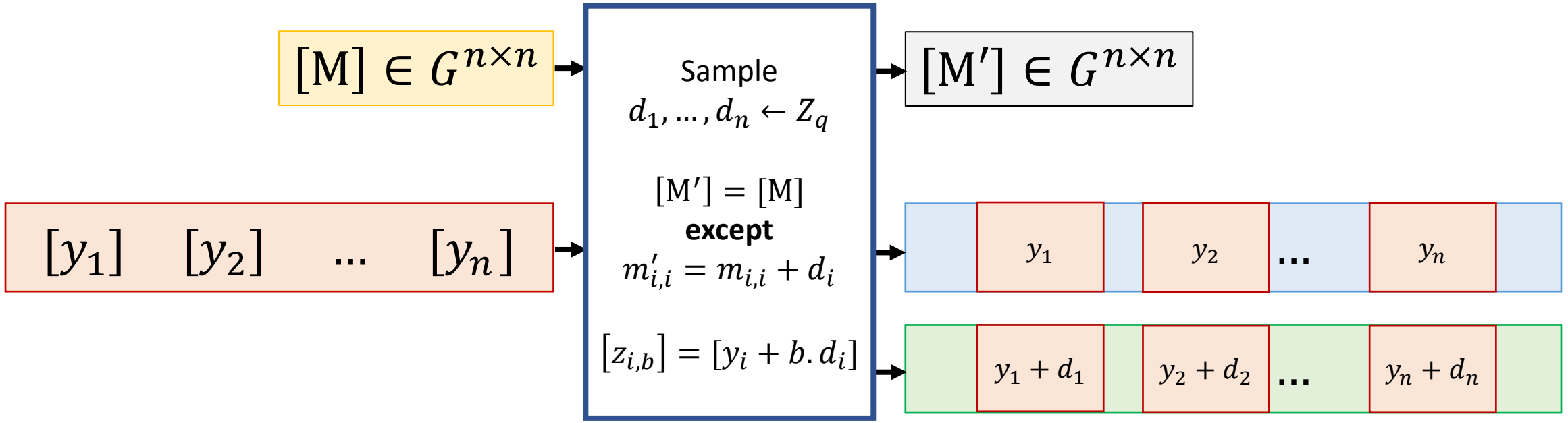
$z_{i,1}$ is distributed uniformly randomly in Z_q



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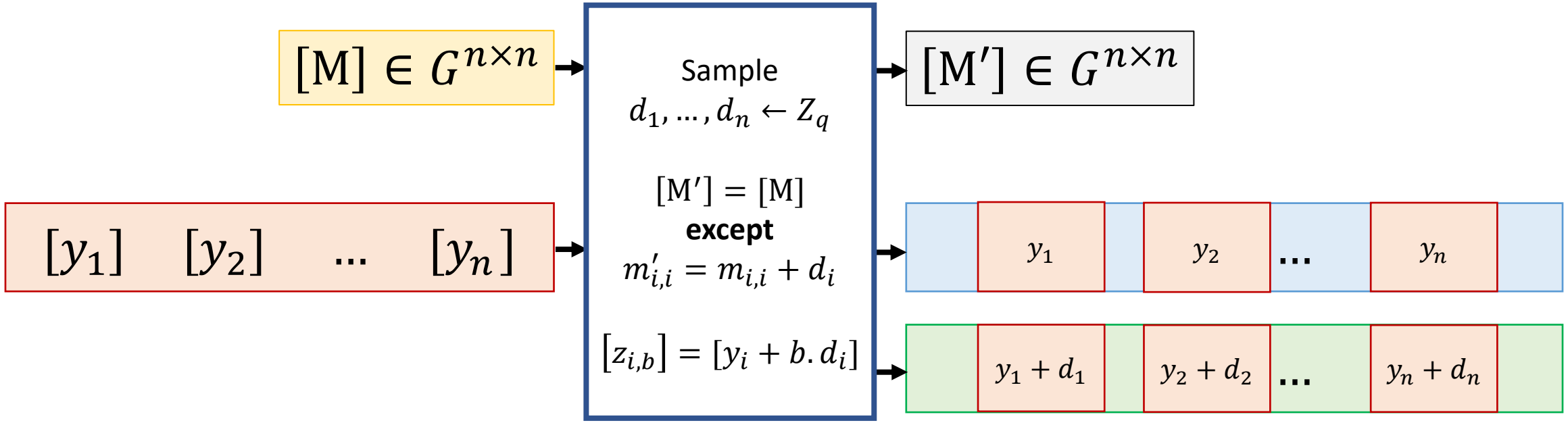
- If $s_i = 1$, $z_{i,1} = \sum_{j\in[n]} m_{i,j}s_j + d_i = \sum_{j\in[n]} m'_{i,j}s_j$, $z_{i,0}$ is distributed uniformly randomly in Z_q



Simple Hinting PRG from DDH

$[y_i]$-s are distributed as in the "real" PRG game w.r.t. the PRG parameter [M] → $[z_{i,b}]$-s are distributed as in the "real" hinting PRG game w.r.t. the PRG parameter [M′]

- $y_i = \sum_{j\in[n]} m_{i,j}s_j, \quad z_{i,0} = y_i, \quad z_{i,1} = y_i + d_i$
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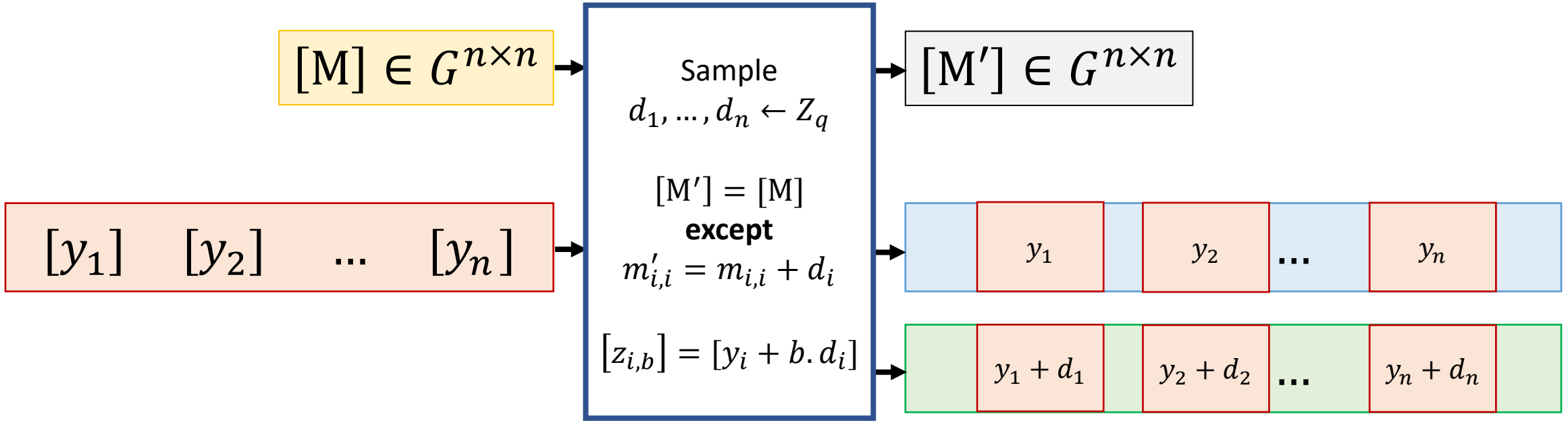
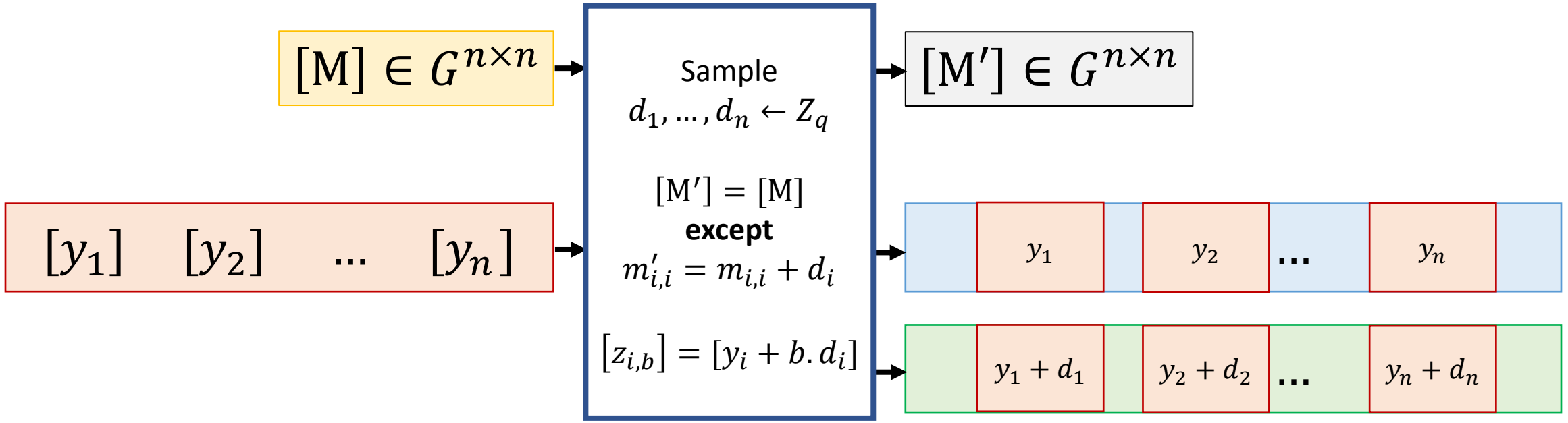


Simple Hinting PRG from DDH

$[y_i]$-s are distributed as in the "random" PRG game w.r.t. the PRG parameter [M] → $[z_{i,b}]$-s are distributed as in the "random" hinting PRG game w.r.t. the PRG parameter [M′]

- y_i is distributed uniformly randomly in Z_q, $z_{i,0} = y_i$, $z_{i,1} = y_i + d_i$
- Both $z_{i,0}$ and $z_{i,1}$ are distributed uniformly randomly in Z_q



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Hinting PRG from cryptographic group actions (brief overview)

Our simple approach for constructing hinting PRGs from DDH readily generalizes to cryptographic group actions (commutative and regular group actions)

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A group action of a group $(G,\cdot)$ on a set $\mathcal{X}$ is a function $\star\colon G \times \mathcal{X} \to \mathcal{X}$ such that:

- Letting e be the identity element in G, for every $x \in \mathcal{X}$ we have $e \star x = x$
- For every $g, h \in G$ and for every $x \in \mathcal{X}$ we have $(g \cdot h) \star x = g \star (h \star x)$
- G is a commutative/abelian group
- For any $x, x' \in \mathcal{X}$, there exists a **unique** $g \in G$ such that $g \star x = x'$

Plausibly **post-quantum secure** instantiations from the CSIDH family of isogenies [CLMPR18] and derivatives of CSIDH with known group structure (such as CSI-Fish [BKV19])

Definitions adopted from [ADMP20]

(Same as definitions in the previous talk)

Hinting PRG from cryptographic group actions (brief overview)

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Construction of hinting PRG from group action (generalizes construction from plain DDH)

- Let $\mathrm{M} = [\mathrm{m}_1\ \mathrm{m}_2\ \dots\ \mathrm{m}_n] \leftarrow G^{n\times n}$ and let $\mathrm{x} = [x_1\ x_2\ \dots\ x_n] \leftarrow \mathcal{X}^n$ for $n > \log|G| + \omega(\log\lambda)$
- Define the function $H_{\mathrm{M},\mathrm{x}}\colon \{0,1\}^n \to \mathcal{X}^n$ as:

$$H_{\mathrm{M},\mathrm{x}}(\mathrm{s} = (s_1, \dots, s_n)) = \mathrm{M}\mathrm{s} \star \mathrm{x} = (< \mathrm{m}_1, \mathrm{s} > \star\, x_1, < \mathrm{m}_2, \mathrm{s} > \star\, x_2, \dots, < \mathrm{m}_n, \mathrm{s} > \star\, x_n)$$

$$\text{where } < \mathrm{m}_\mathrm{i}, \mathrm{s} > = \prod_{j\in[n]} m_{i,j}^{s_j}$$

Hinting PRG from cryptographic group actions (brief overview)

Our simple approach for constructing hinting PRGs from DDH readily generalizes to cryptographic group actions (commutative and regular group actions)

A group action of a group $(G,\cdot)$ on a set $\mathcal{X}$ is a function $\star\colon G \times \mathcal{X} \to \mathcal{X}$ such that:

- Letting e be the identity element in G, for every $x \in \mathcal{X}$ we have $e \star x = x$
- For every $g, h \in G$ and for every $x \in \mathcal{X}$ we have $(g \cdot h) \star x = g \star (h \star x)$
- G is a commutative/abelian group
- For any $x, x' \in \mathcal{X}$, there exists a **unique** $g \in G$ such that $g \star x = x'$

Plausibly **post-quantum secure** instantiations from the CSIDH family of isogenies [CLMPR18] and derivatives of CSIDH with known group structure (such as CSI-Fish [BKV19])

Definitions adopted from [ADM**P**20]

(Same as definitions in the previous talk)

Construction of hinting PRG from group action (generalizes construction from plain DDH)

- Let $\mathrm{M} = [\mathrm{m}_1\ \mathrm{m}_2\ \dots\ \mathrm{m}_n] \leftarrow G^{n\times n}$ and let $\mathrm{x} = [x_1\ x_2\ \dots\ x_n] \leftarrow \mathcal{X}^n$ for $n > \log|G| + \omega(\log\lambda)$
- Define the function $H_{\mathrm{M},\mathrm{x}}\colon \{0,1\}^n \to \mathcal{X}^n$ as:

$$H_{\mathrm{M},\mathrm{x}}(\mathrm{s} = (s_1, \dots, s_n)) = \mathrm{M}\mathrm{s} \star \mathrm{x} = (< \mathrm{m}_1, \mathrm{s} > \star\, x_1, < \mathrm{m}_2, \mathrm{s} > \star\, x_2, \dots, < \mathrm{m}_n, \mathrm{s} > \star\, x_n)$$

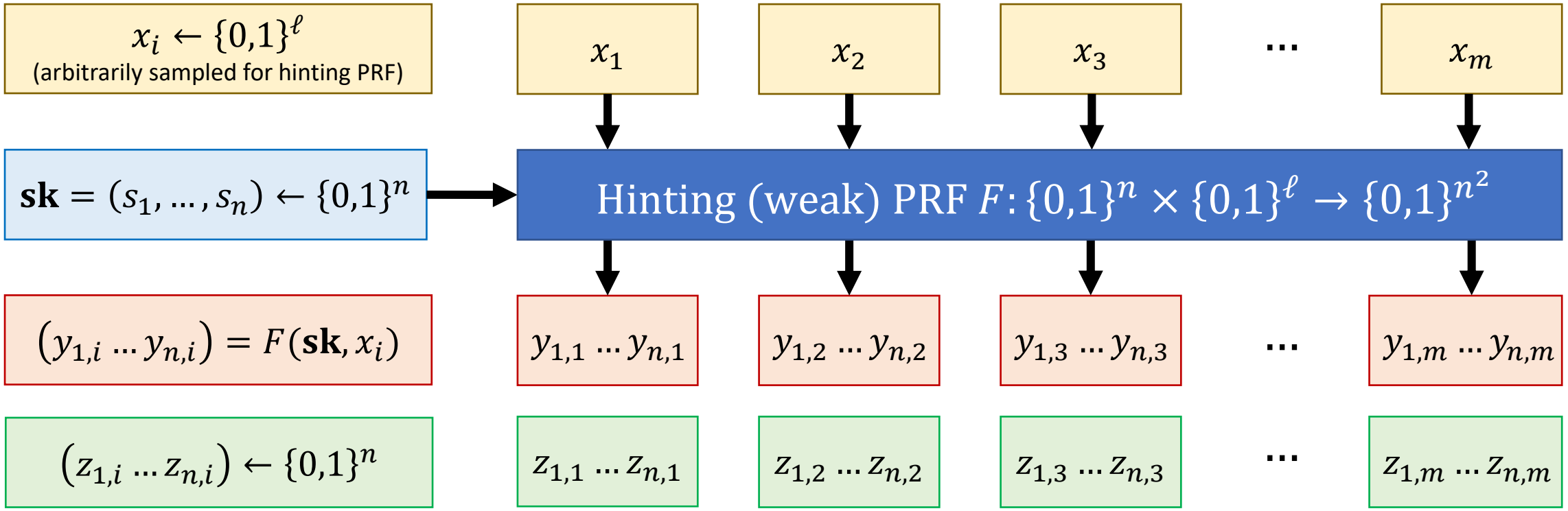
Our result (see paper for details)

H is a hinting PRG if the group action $\star\colon G \times \mathcal{X} \to \mathcal{X}$ satisfies the **linear hidden shift (LHS) assumption** [ADM**P**20]

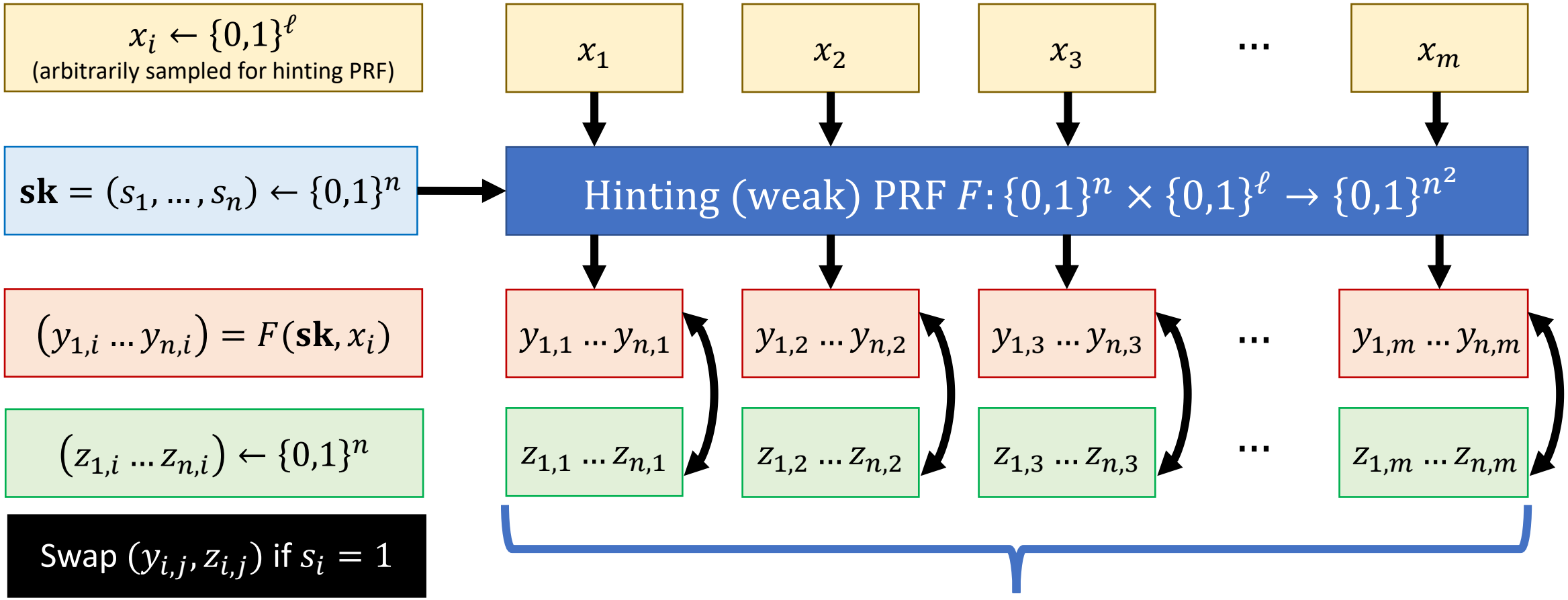
Talk Outline

- Simple Hinting PRG from DDH
- Hinting PRG from cryptographic group actions (brief overview)
- Hinting weak PRF (brief overview)
- Summary and open questions

Hinting (weak) PRF (brief overview)



Hinting (weak) PRF (informal definition)



Multi-evaluation hinting property (each wPRF evaluation leaks info about the secret key **sk**)

Remains pseudorandom across multiple evaluations
(computationally indistinguishable from a set of m matrices of dimension $2 \times n$, where each matrix consisting of random n-bit strings)

Hinting weak PRF (construction outlines)

Our simple approaches for constructing hinting PRGs from DDH/group actions naturally extend to hinting weak PRFs from the same assumptions

Hinting weak PRF from DDH (informal outline)

- Let G be a DDH-hard group of prime order q with generator g, and let $n > \log|G| + \omega(\log\lambda)$
- Define the function $F: \{0,1\}^n \times G^{n\times n} \rightarrow G^n$ as:

$$F(\mathrm{s} \in \{0,1\}^n, [\mathrm{M}]) = [\mathrm{Ms}]$$

Our Result-1

F is a **hinting** weak PRF assuming that G is a DDH-hard group

Hinting weak PRF from Group Actions (informal outline)

- Let $(G, \mathcal{X}, \star)$ be an LHS-hard cryptographic group action, and let $n > \log|G| + \omega(\log\lambda)$, and let $\mathrm{x} = [x_1\ x_2\ \ldots x_n] \leftarrow \mathcal{X}^n$
- Define the function $F'_{\mathrm{x}}: \{0,1\}^n \times G^{n\times n} \rightarrow \mathcal{X}^n$ as:

$$F'_{\mathrm{x}}(\mathrm{s} \in \{0,1\}^n, [\mathrm{M}]) = [\mathrm{Ms}] \star \mathrm{x}$$

Our Result-2

F' is a **hinting** weak PRF assuming that $(G, \mathcal{X}, \star)$ is an LHS-hard group action

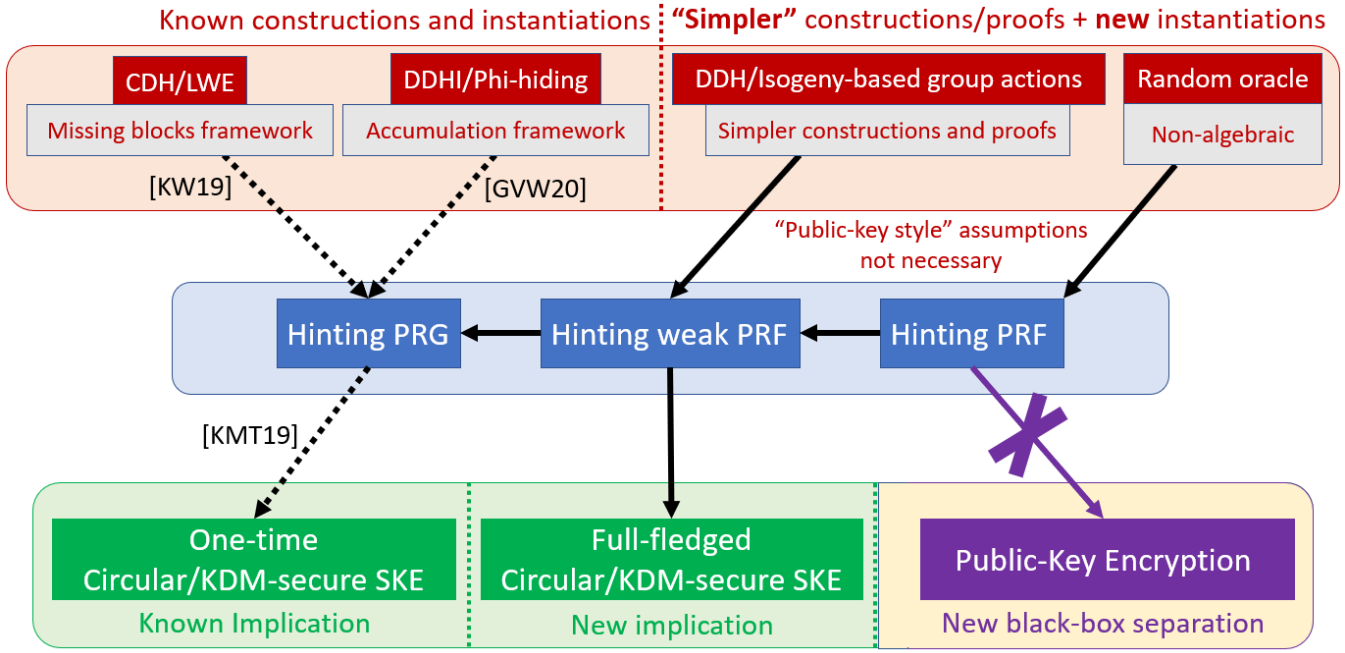
See paper for detailed constructions and proofs

Talk Outline

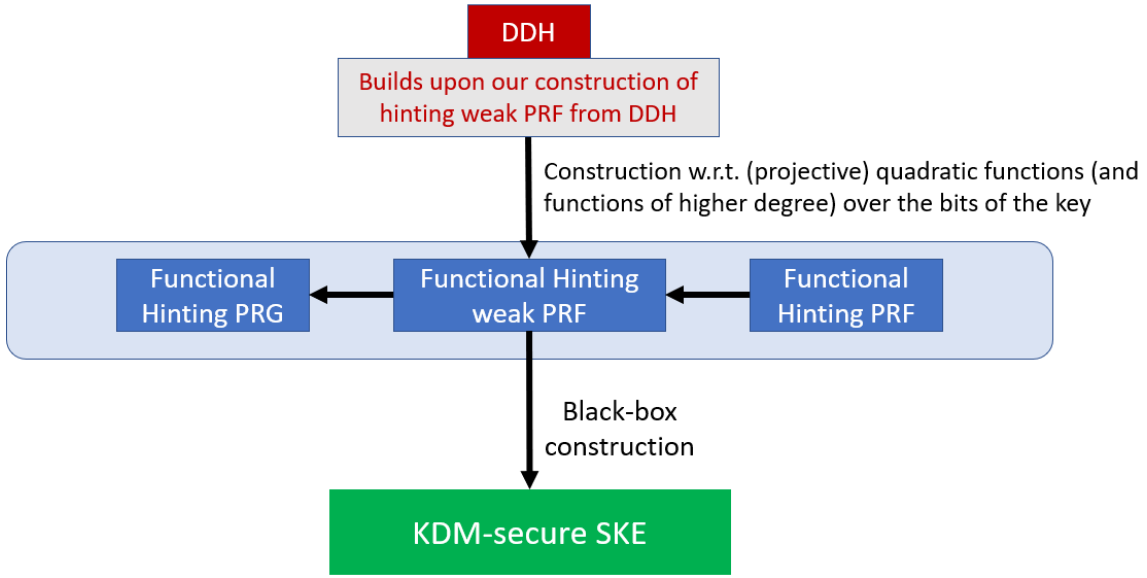
- Simple Hinting PRG from DDH
- Hinting PRG from cryptographic group actions (brief overview)
- Hinting weak PRF (brief overview)
- Summary and open questions

Summary and Open Questions

Primitives with hinting property



Primitives with functional hinting property



- Construct (functional) hinting PRFs from standard mathematical assumptions
- Realize primitives with the functional hinting property from a wider set of (quantum-safe) assumptions
- Investigate additional applications of primitives with the (functional) hinting property

Full version coming on ePrint soon

Thank You! Questions?