Efficient NIZKs and Signatures from Commit-and-Open Protocols in the QROM

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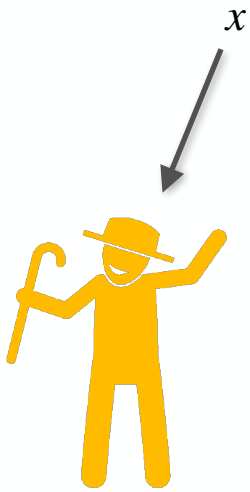
Outline

- The Problem
- Our Results
- Commit-and-Open Protocols
- The QROM extractor
- Main Result
- The CHFL21 Framework
- Improved Unruh Transform

The Problem

Are there Fiat-Shamir NIZKs/DSS with a tight security proof?

Σ-protocols & Fiat-Shamir



$x \in \mathscr{L}$, witness relation R

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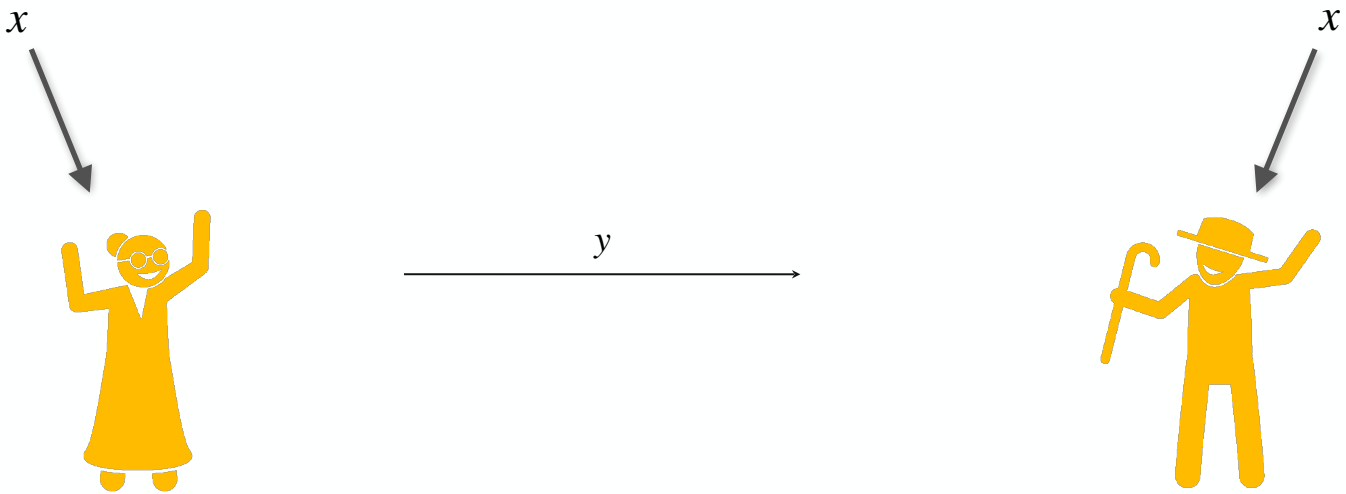

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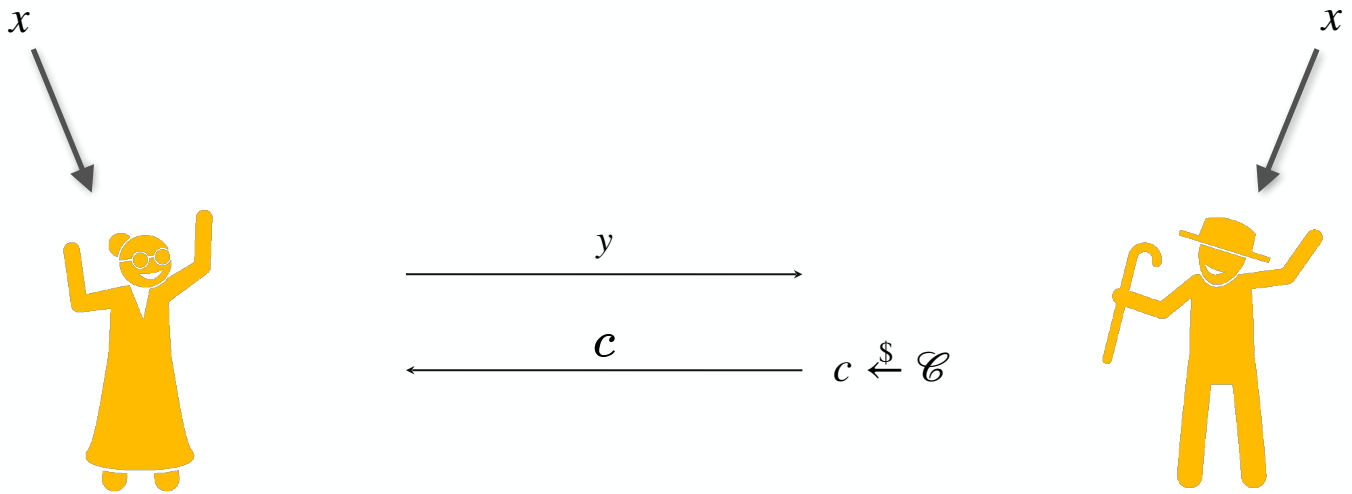

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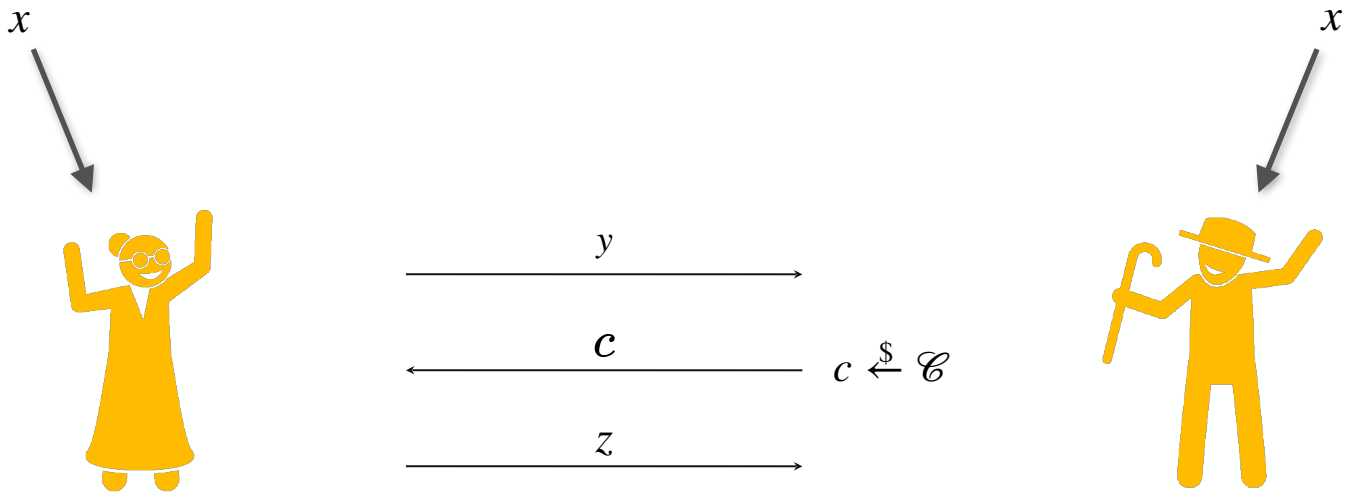

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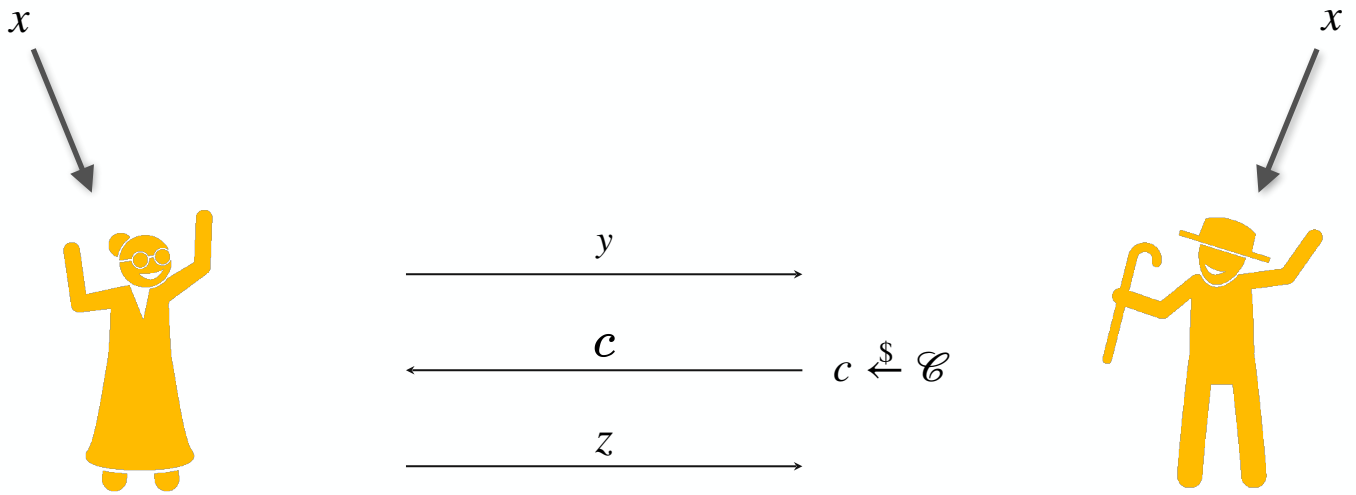

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$V(x, y, c, z) \in \{\text{accept}, \text{reject}\}$

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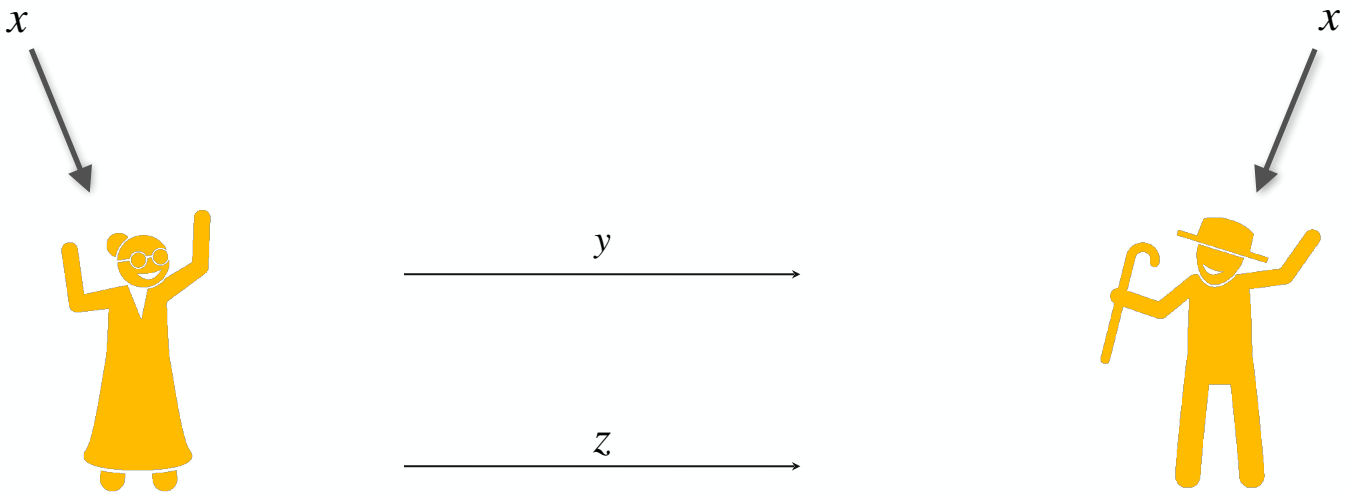

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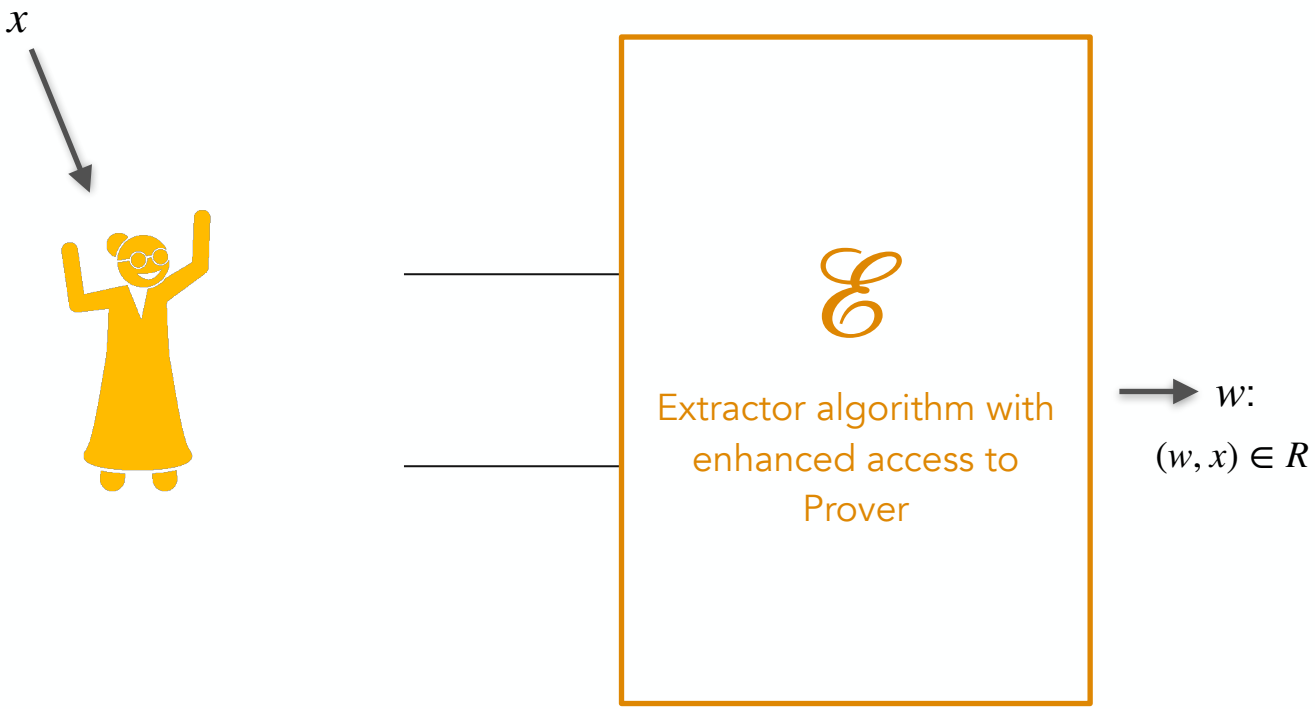
Fiat-Shamir: $c := H(x, y)$

Extractability



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FS of…	Prover success probability	Extractor success probability
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Our Results

Online-extractability for some Fiat-Shamir-NIZKs in the QROM

Results

We…

- construct an online extractor for the Fiat-Shamir transform of commit-and-open protocols in the QROM
- extend the technique to Merkle-tree commitments
- modularize and improve the Unruh transform
- apply our result to the Picnic signature scheme

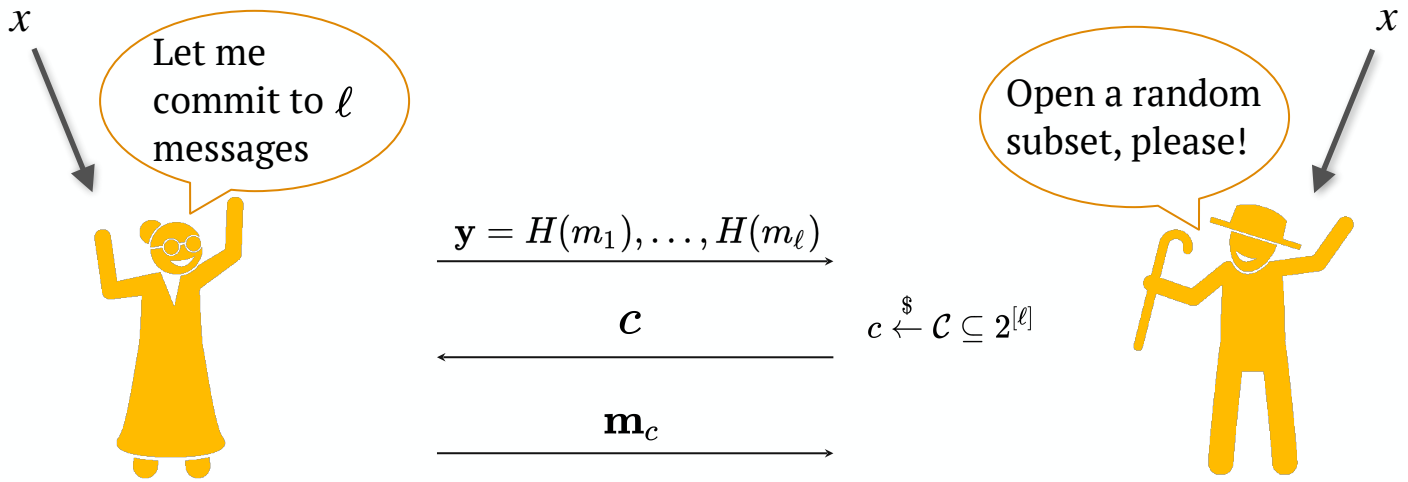
Related work

- Chailloux (eprint ’21): same result, but under scheme-specific assumption
- Chiesa, Manohar, Spooner (TCC ’19): Analysis of a specific SNARG construction.
  - Quite similar to our Merkle tree commitment result
  - Was somehow missed in the analysis of, e.g, Picnic until now

Commit-and-Open Protocols

A special class of Σ-protocols

Commit-and-Open (C&O) Protocols



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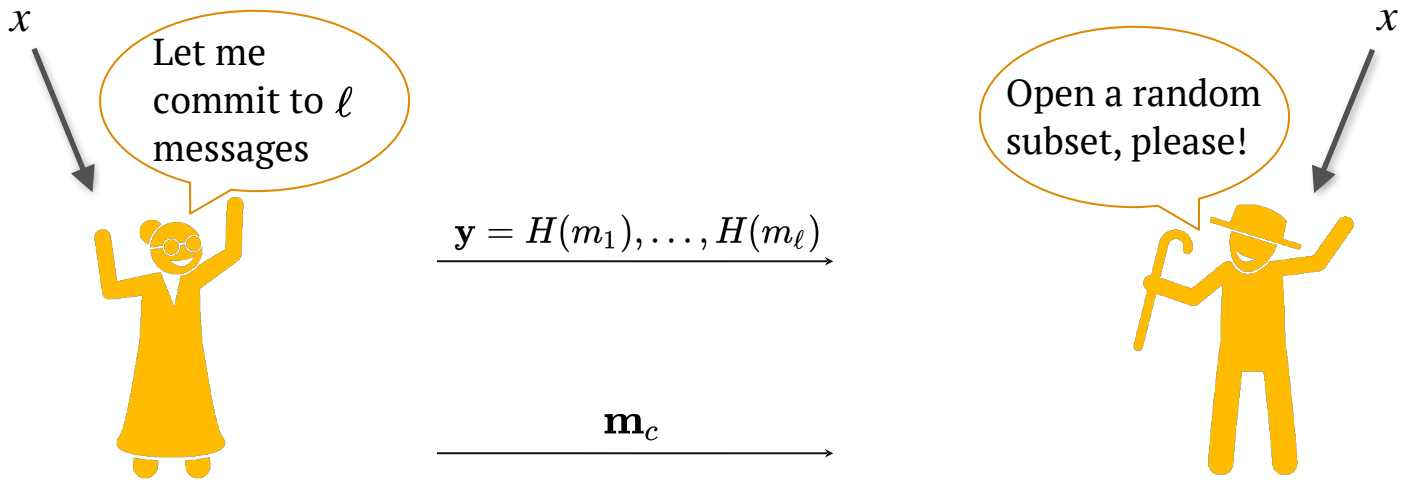

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Merkle tree commitments with octopus opening

- First message in C&O protocols: $H(m_1), \ldots, H(m_\ell)$
- Third message: $(m_i)_{i \in c}$ for $c \subset \{1,...,\ell\}$

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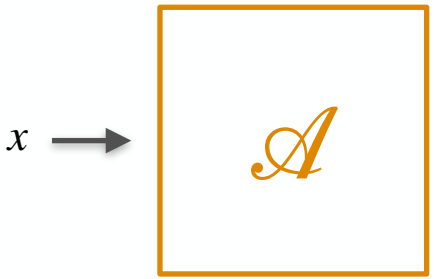
In Picnic: Merkle tree commitment

Online-Extractability in the ROM

$\mathscr{A}$: Adversary against FS transform of special-sound C&O protocol



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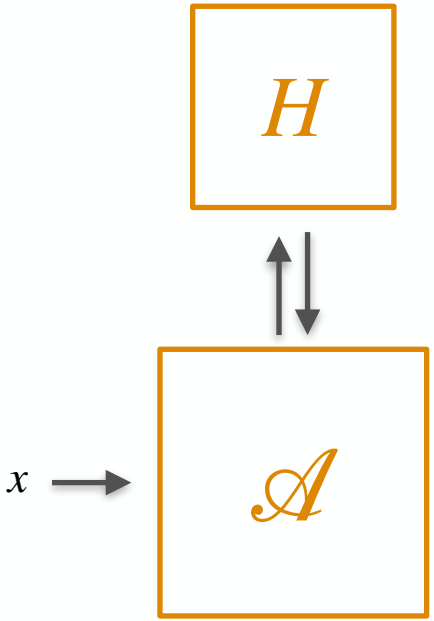

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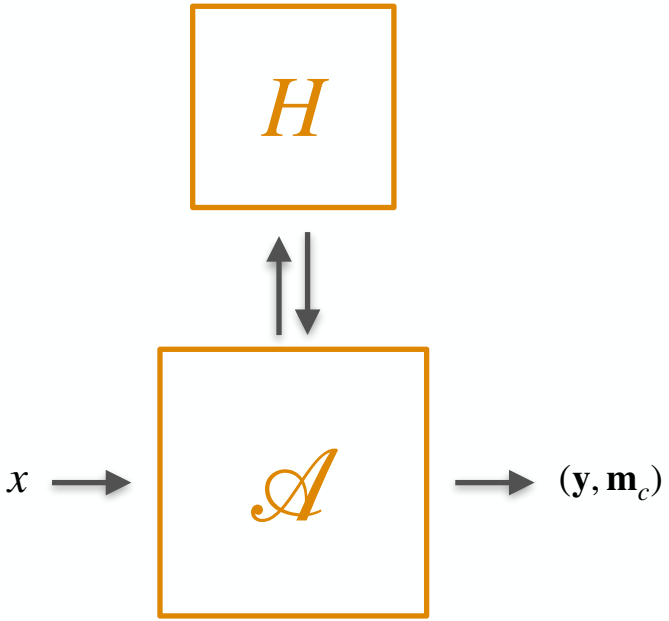

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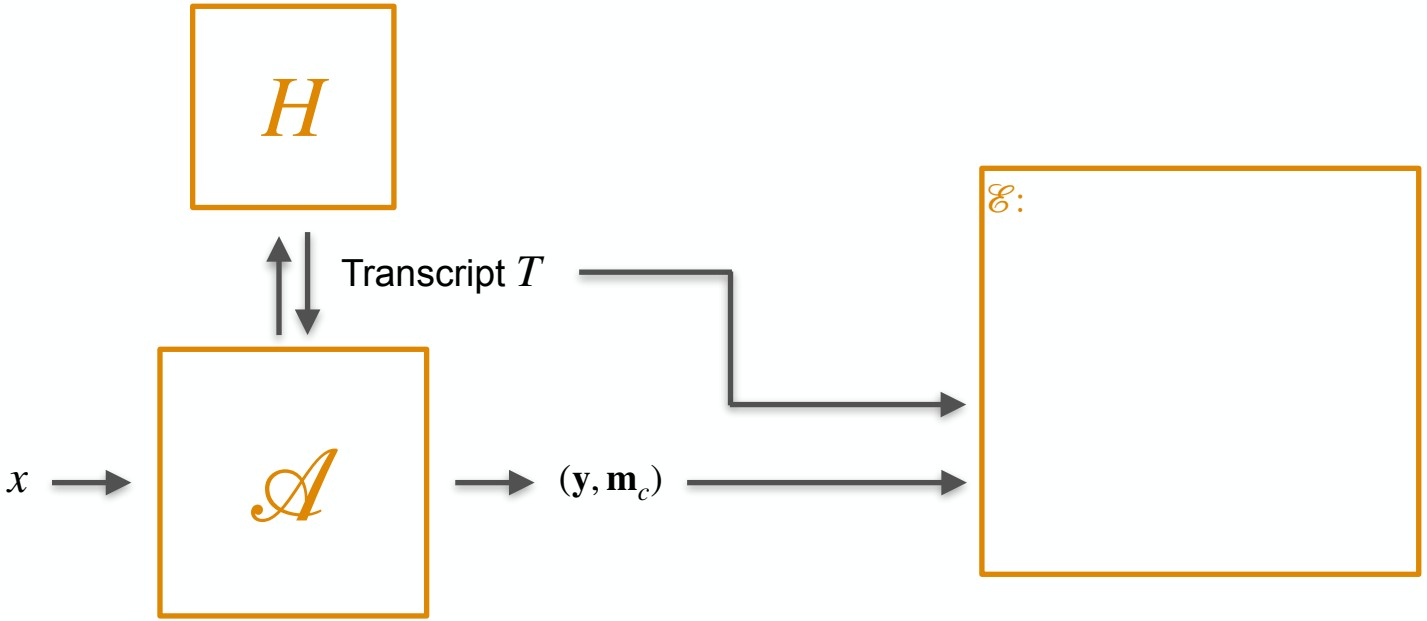

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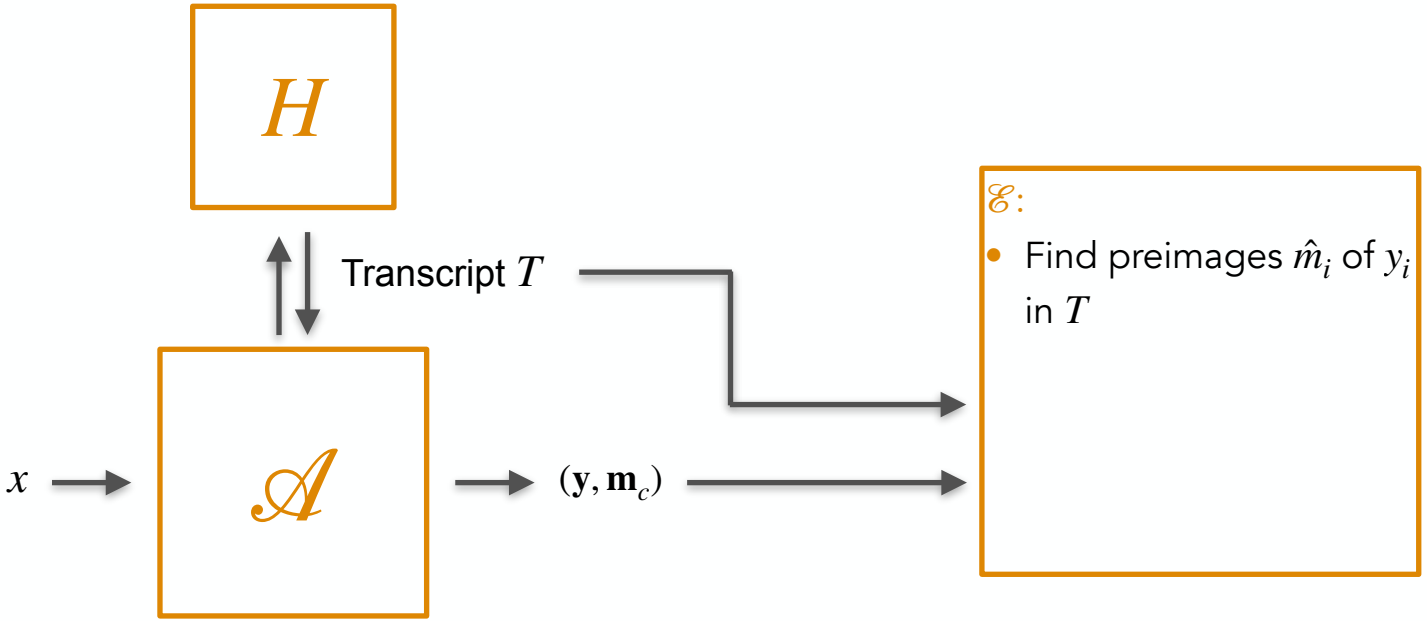

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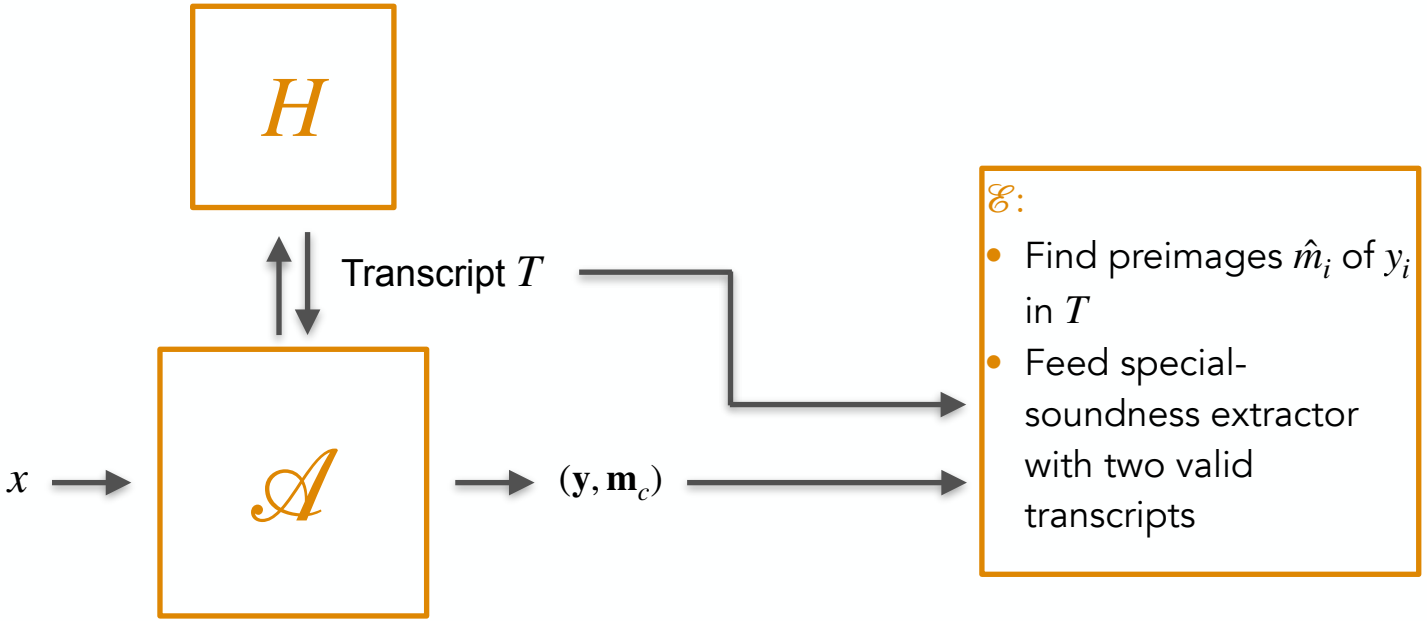

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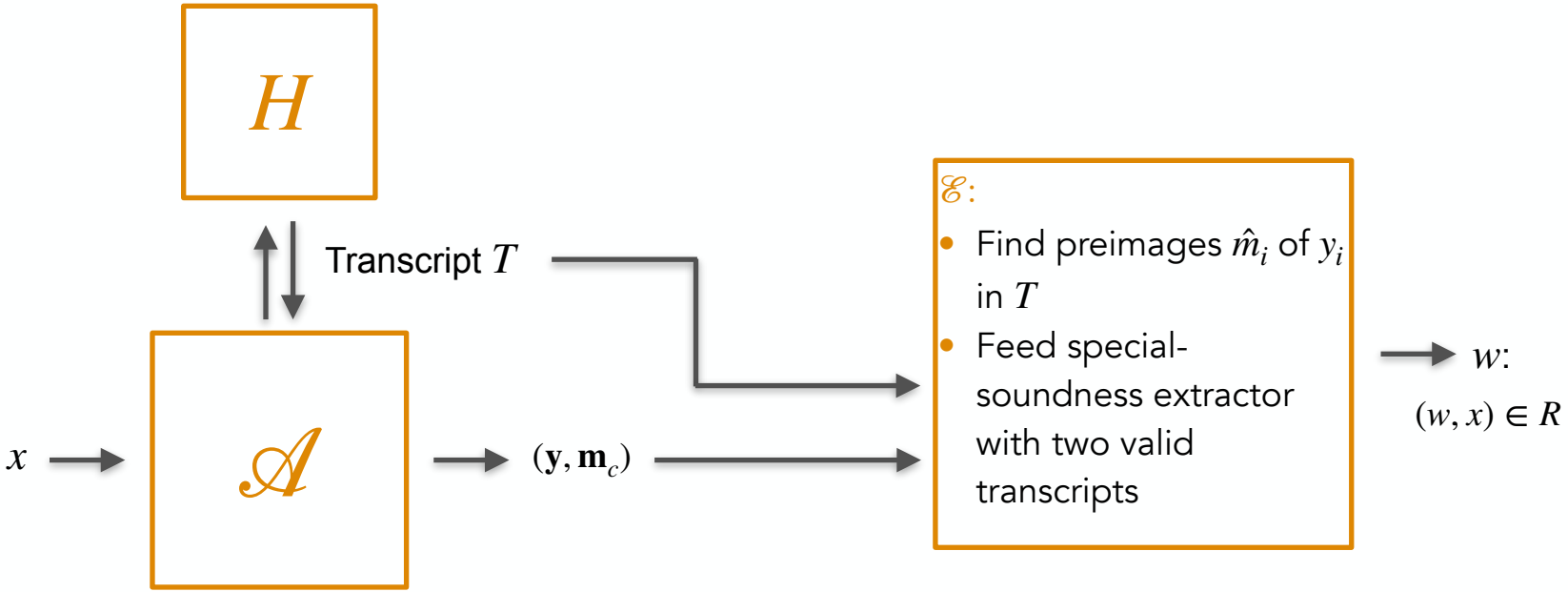

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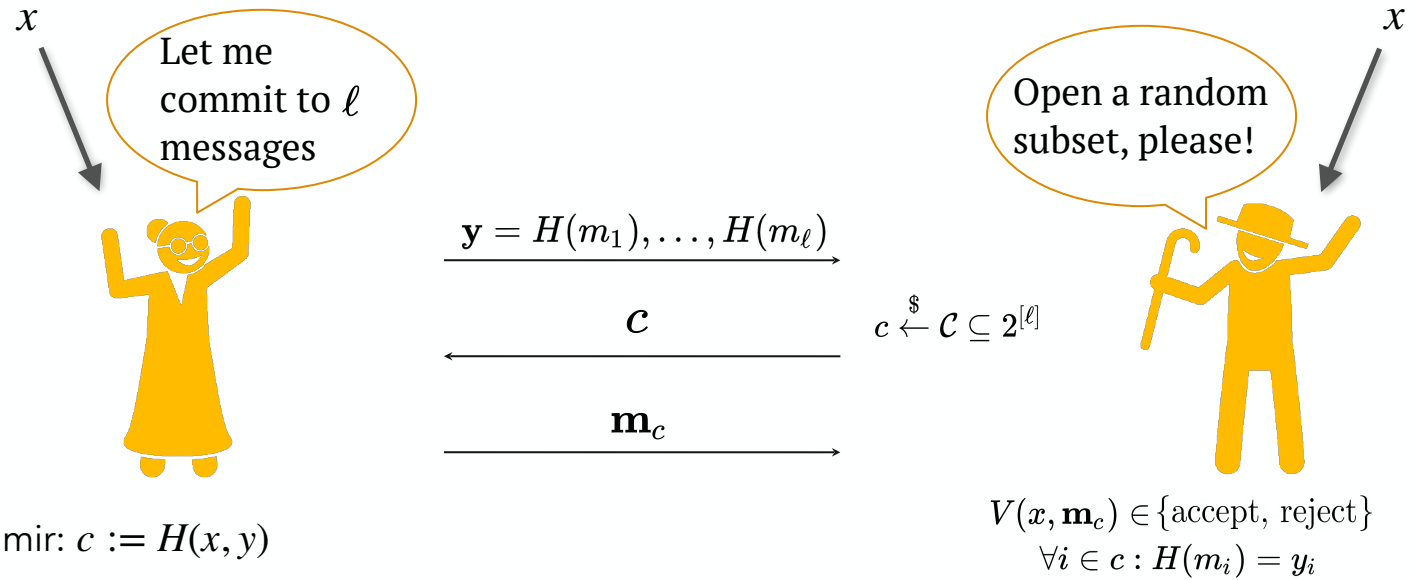

Fiat-Shamir: $c := H(x, y)$

It's (usually) easy to produce $\mathbf{y}$ and a valid $\mathbf{m}_c$ for **one** value of c….

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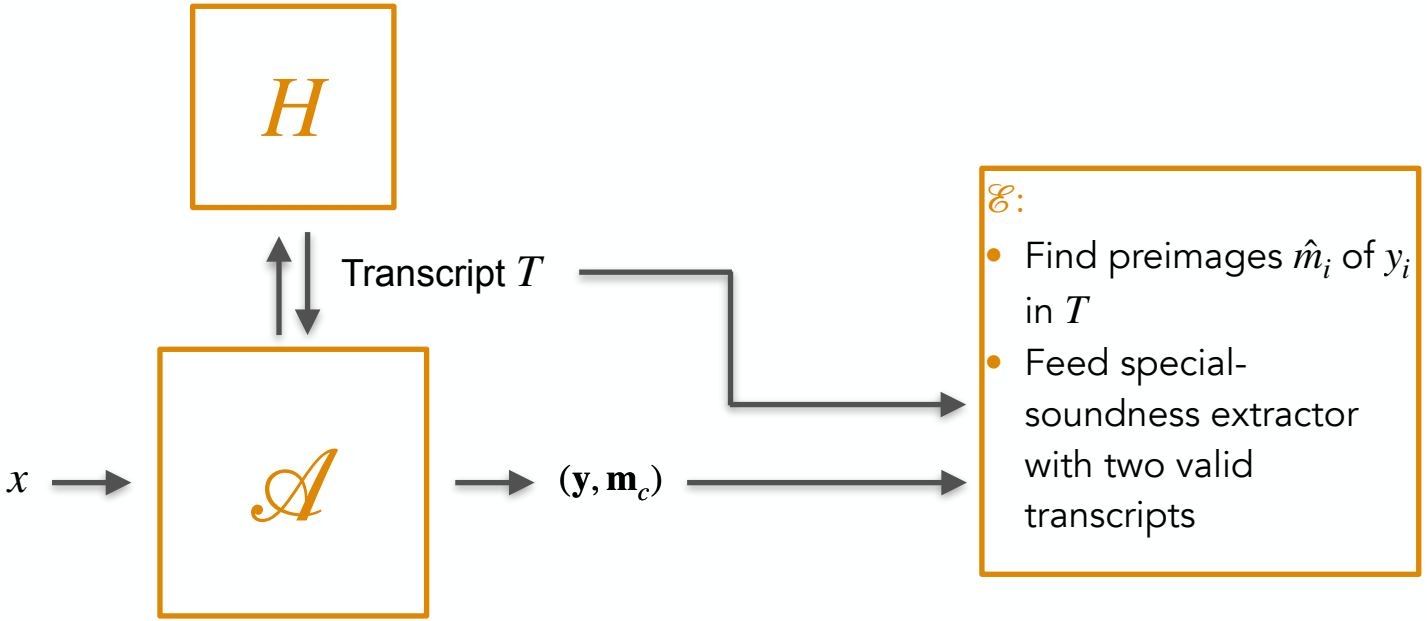

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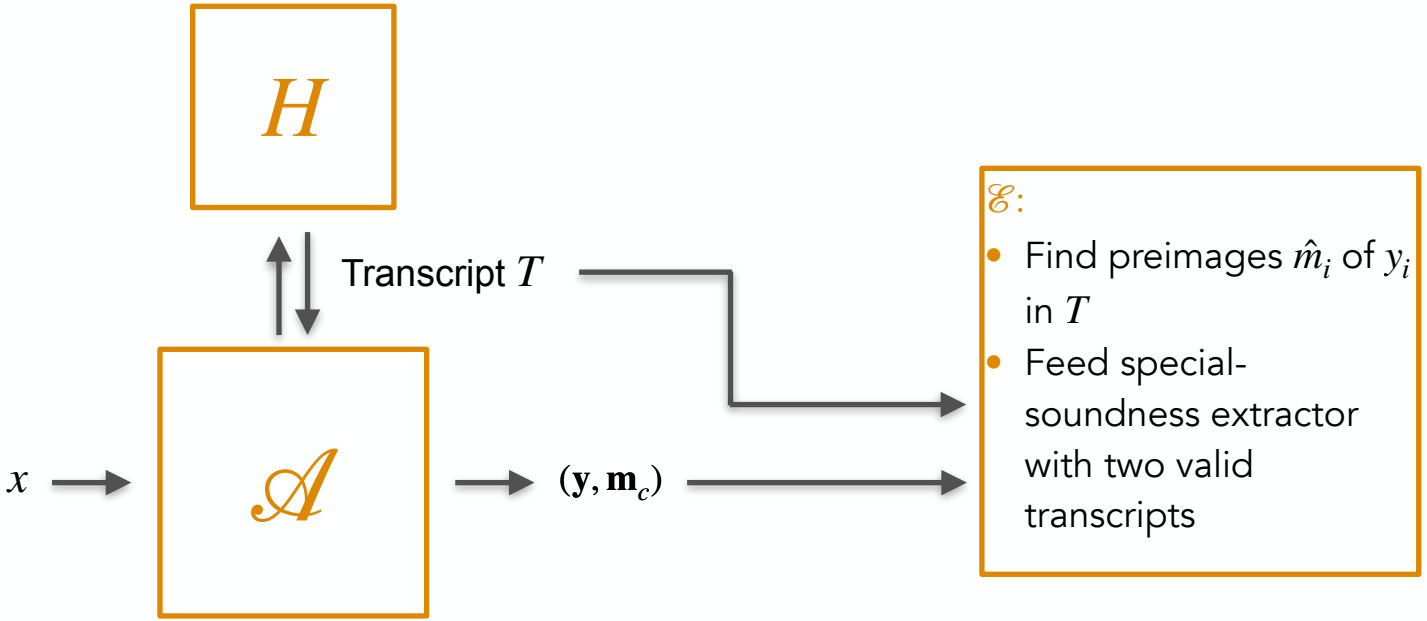

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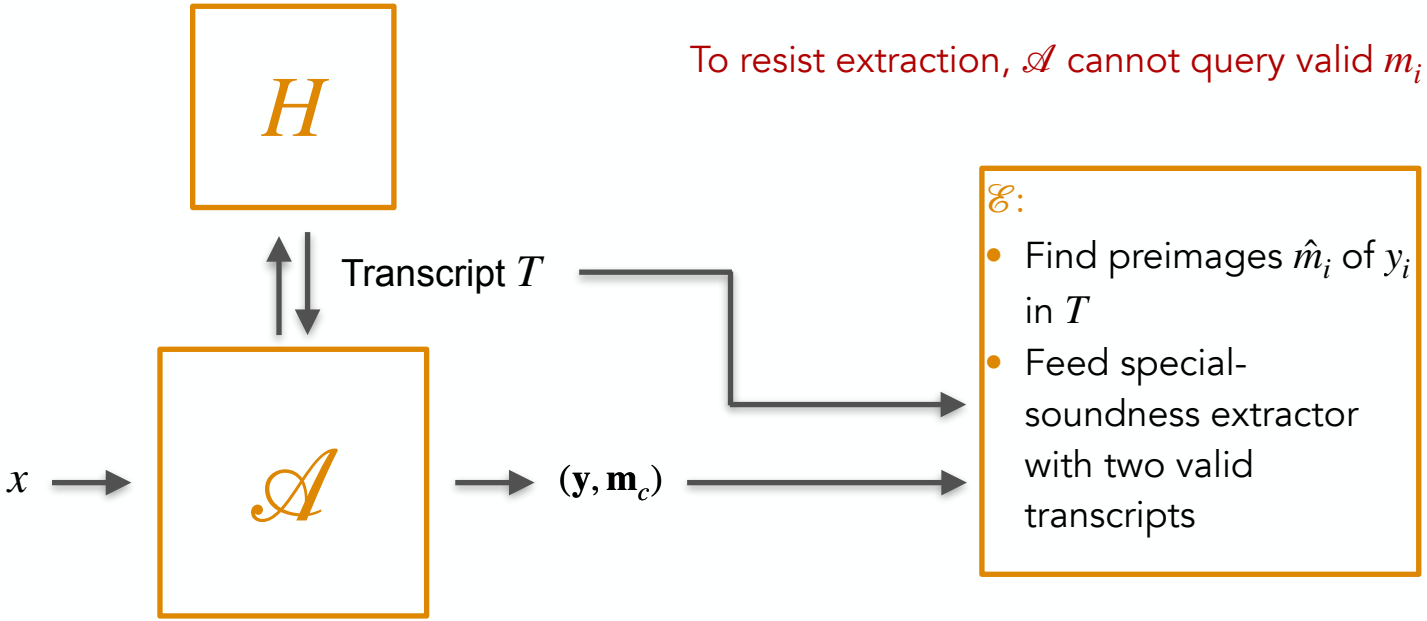

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This is a hard oracle search task!

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Σ-protocols with collapsingness

q=number of (Q)RO queries

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The QROM extractor

“Just use a compressed oracle!”

Online-Extractability in the QROM

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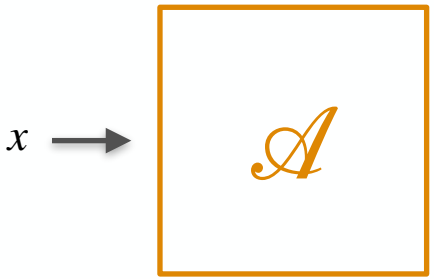

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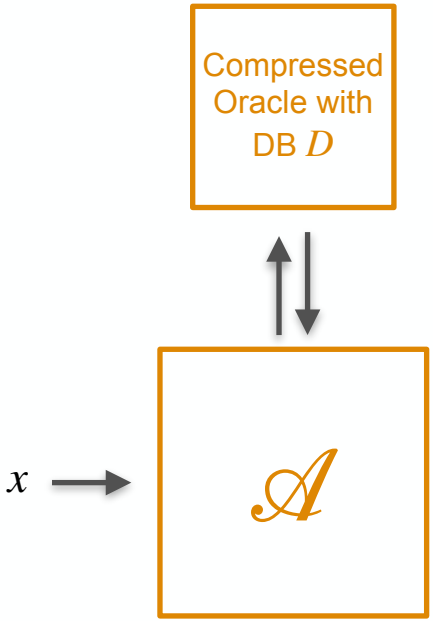

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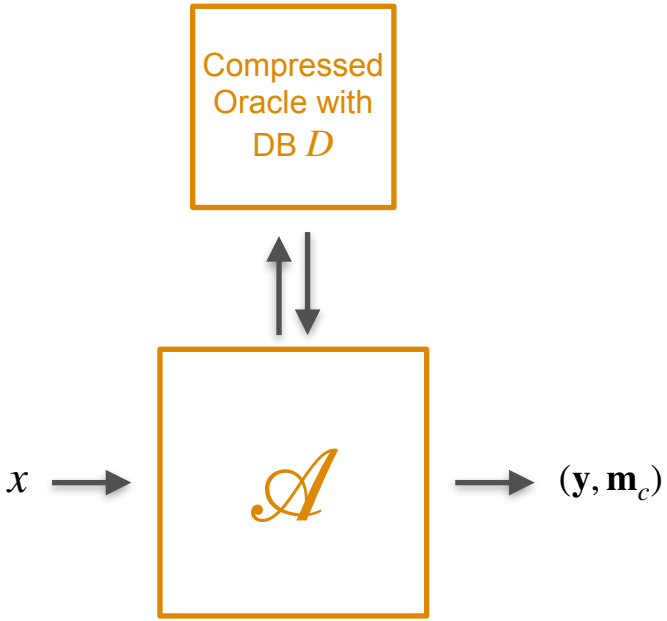

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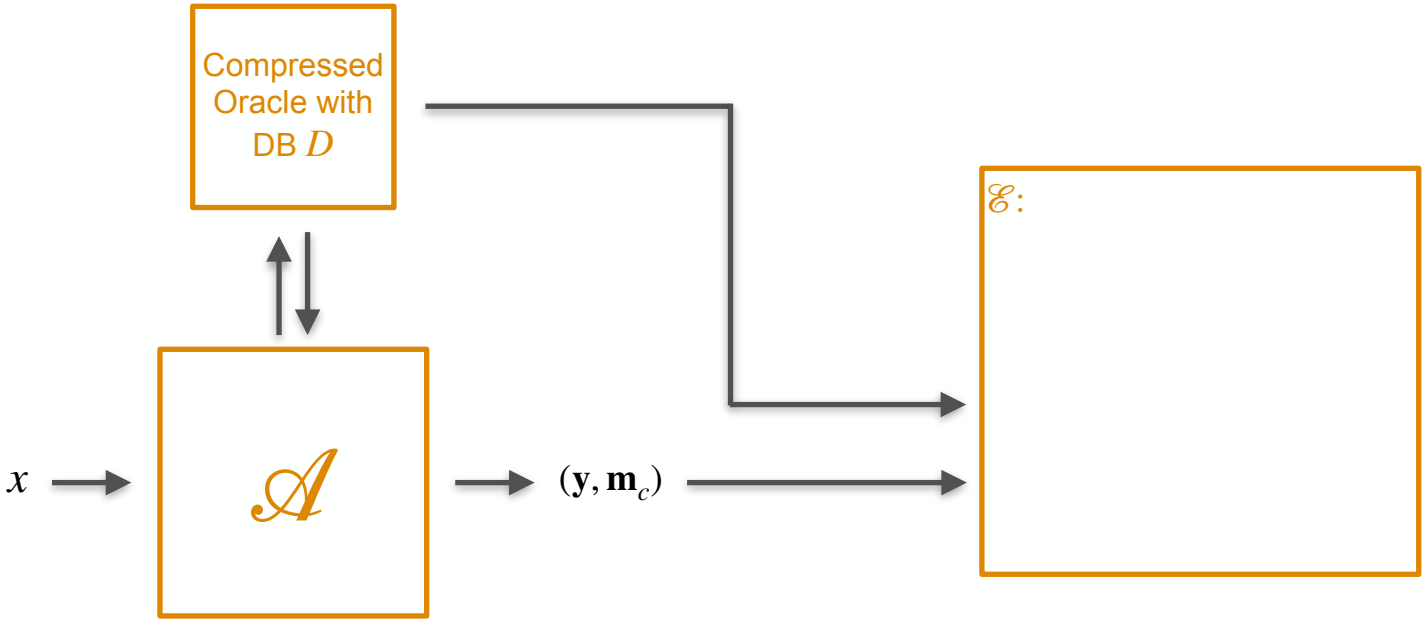

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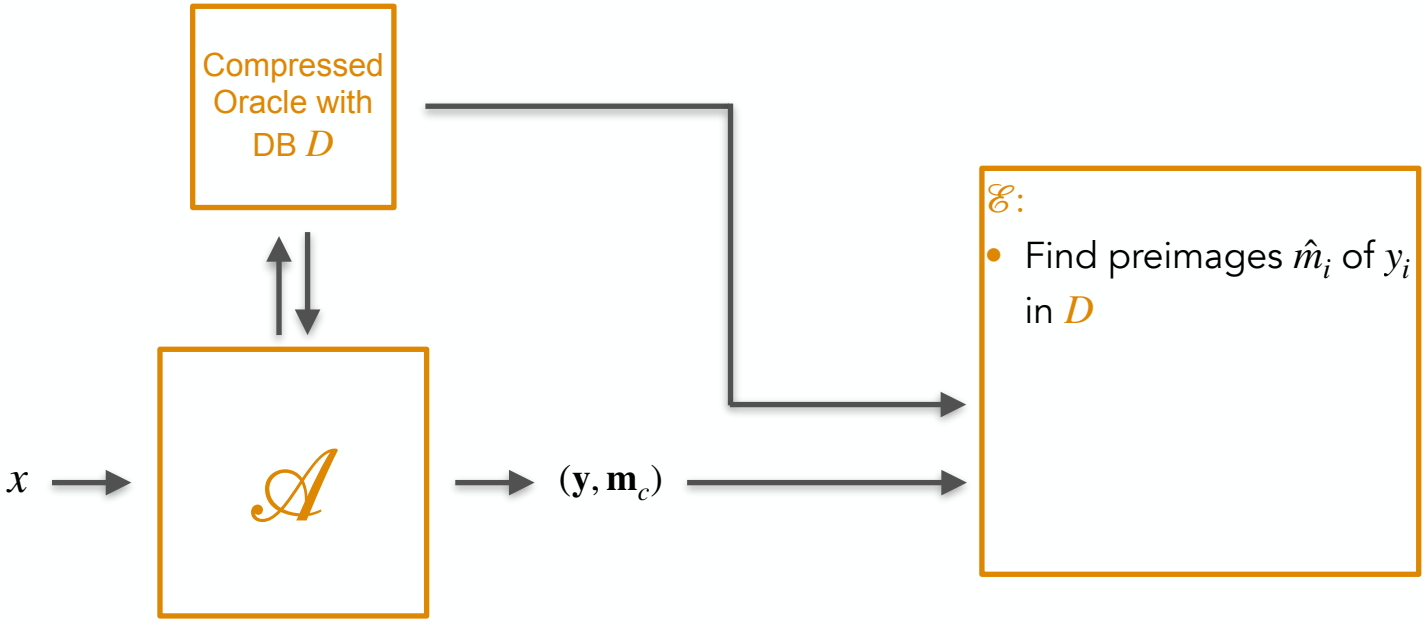

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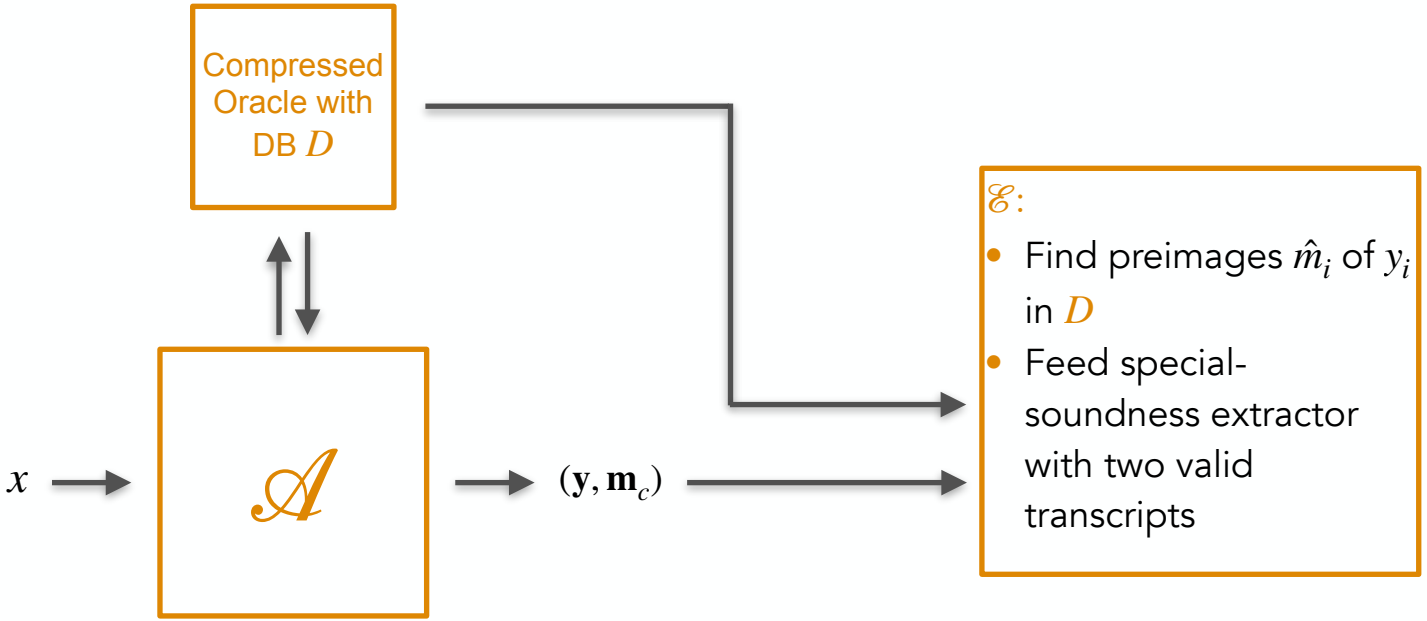

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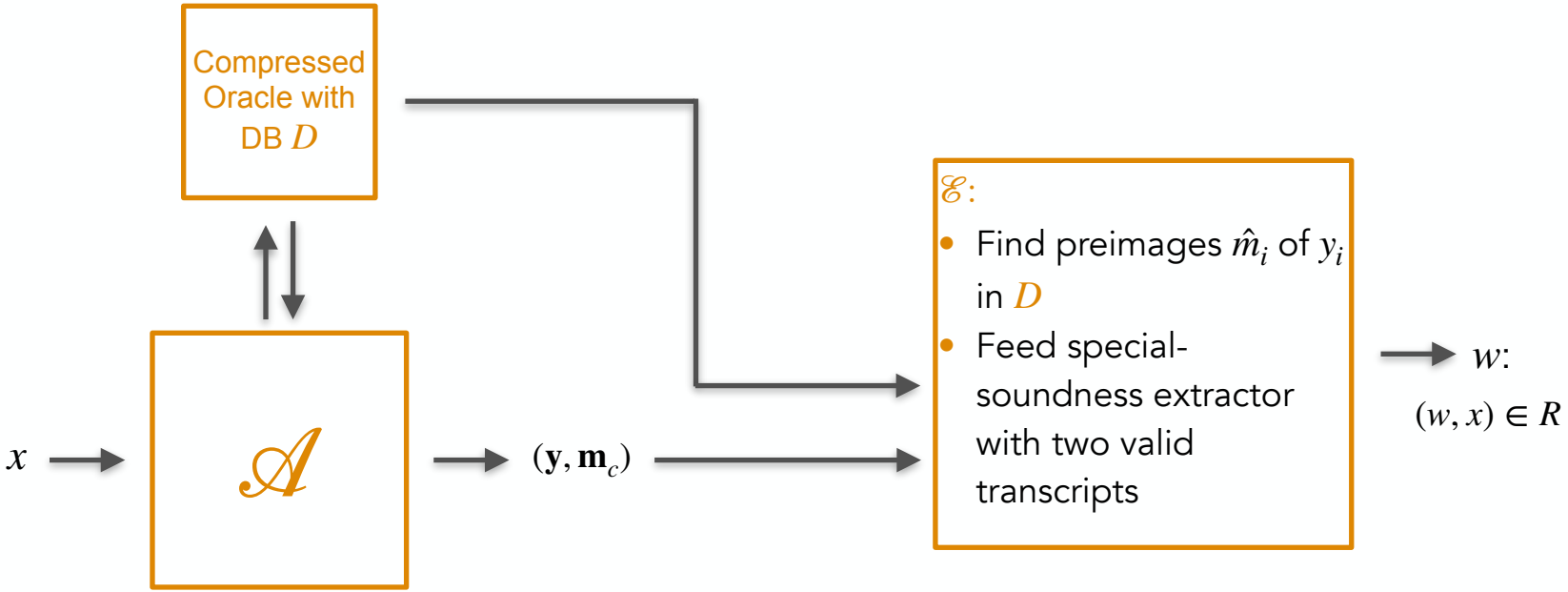

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Query lower bounds for compressed oracles: plenty.

But not if the predicate needs to read the compressed oracle database!

Main Result

Online-extractability in the QROM via artificial compressed-oracle tasks

Main Theorem

Theorem (Don, Fehr, M, Schaffner 22, informal): *For the FS transformation of a C&O protocol with any of a very general class of special-soundness-like properties there exists an online extractor that extracts a witness whenever the prover succeeds, except with negligible error, in the QROM.*

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For the proof, we generalize the compressed-oracle query lower bound framework of Chung, Fehr, Huang, Liao (Eurocrypt ’21).

Result: Online-extractability of NIZK from C&O

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- Provably secure in the QROM, based on security of the underlying block cipher (and a couple more primitives)
- Previously: Very much non-tight security proof with both power loss, q^c loss
- Our result ⇒ multiplicatively tight reduction, additive error terms essentially match collision- and preimage attacks on hash function.

The CFHL21 Framework

A query lower bound framework for oracle search tasks

Query lower bounds for compressed oracles

Lemma (Zhandry '18): *If there's no pair $(x, y) \in R$ in the compressed oracle database, the oracle algorithm can't output such a pair.*

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Generalizes to relations on tuples of input-output pairs. Write $D \in R$

⇒ Bounds for (multi)collision, space-time trade-offs, chain-of-values,…

Query lower bounds from transition capacities (TCs)

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Theorem (Chung, Fehr, Huang, Liao, Eurocrypt '21, very informal): *There's a closed-form quantum query bound for the task of satisfying a relation R with input-output pairs of a QRO in terms of the quantum TC.*

Removing one technical restriction allows application to our artificial search task!

Improved Unruh transform

Any Σ-protocol can become a C&O protocol!

Unruh Transform

- Less efficient than FS, but first NIZK provably secure in the QROM
- Online extractable, can be applied to any sigma-protocol
- Prover commits to a response for every possible challenge
- Requires length preserving hash for the commitment

$$a, y_i := H(z_i||r_i)_{i \in \mathcal{C}}$$

$$c = i \qquad c \xleftarrow{\$} \mathcal{C}$$

$$z_i, r_i$$

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- ~~Requires length preserving hash for the commitment~~
- Can use Merkle tree commitment

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$$\leftarrow c = i \quad c \xleftarrow{\$} \mathcal{C}$$

$$\rightarrow z_i, r_i$$

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Summary

- We give the first tight QROM reduction of Fiat-Shamir NIZKs for the subclass of C&O protocols
- The reduction works on Merkle-C&O
- ⇒Tight QROM reduction for the Picnic signature scheme
- More efficient Unruh transform

The End!

Questions?