Lower Bound on SNARGs in the Random Oracle Model

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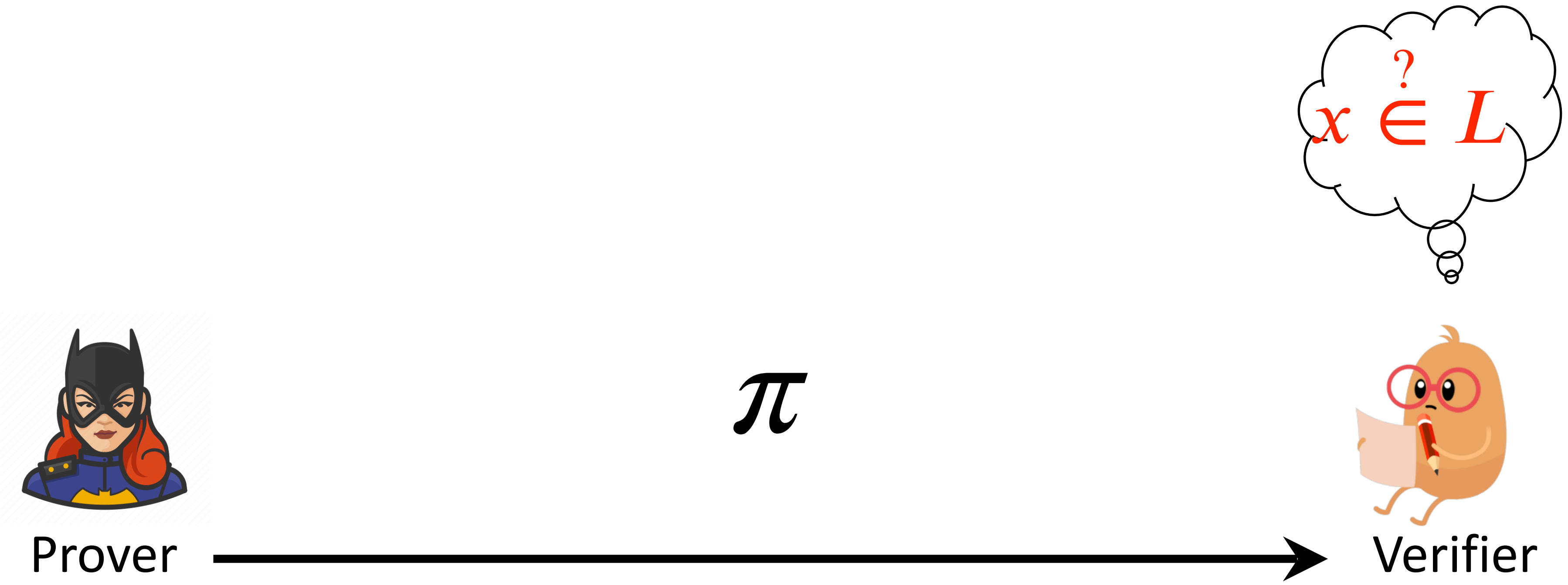
Joint work with

Iftach Haitner & Eylon Yogev

Succinct **N**on-Interactive **Arg**uments
SNARGS

SNARGs



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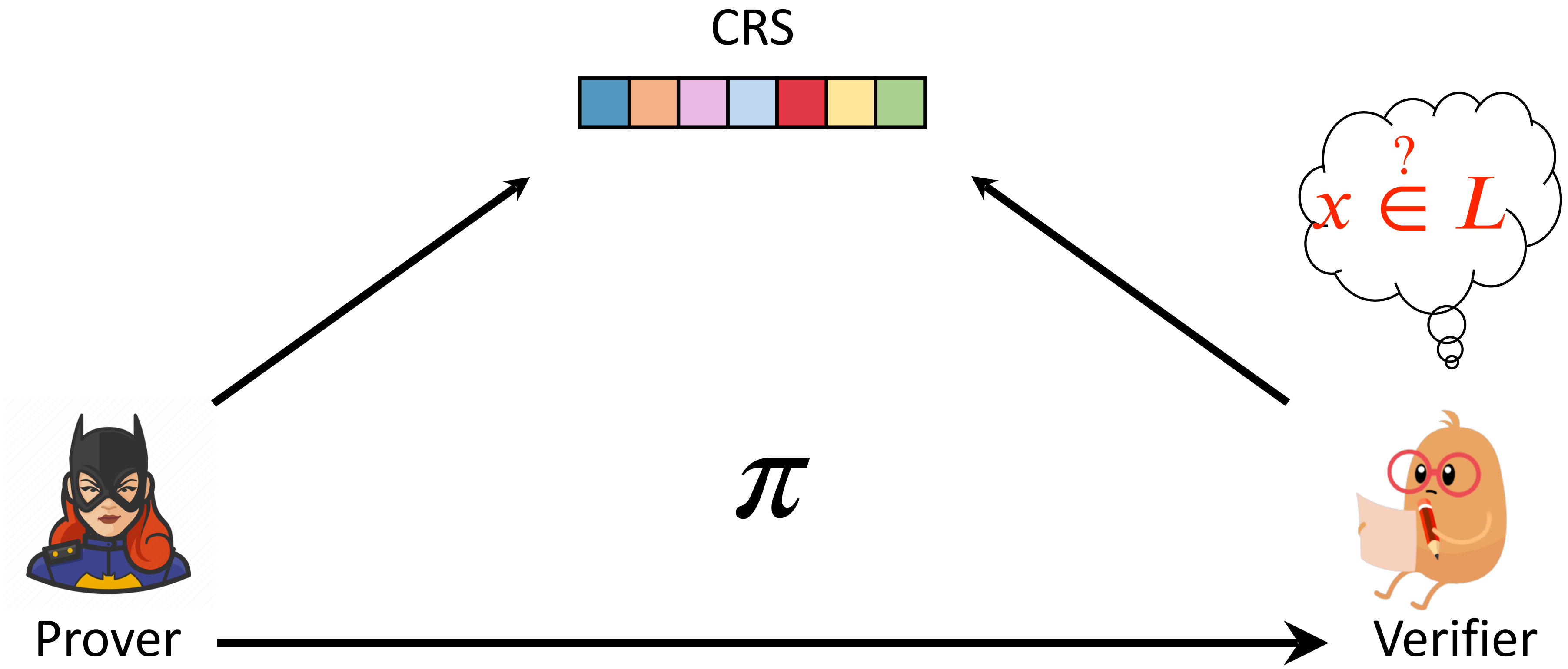
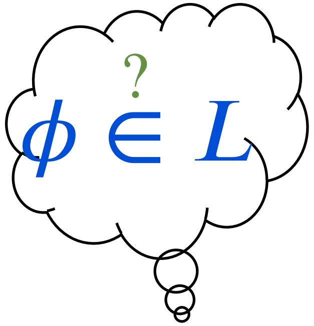


SNARGs in the ROM

SNARG: **S**uccinct **N**on-interactive **A**rgument

ROM: **R**andom **O**racle **M**odel



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π

Verifier

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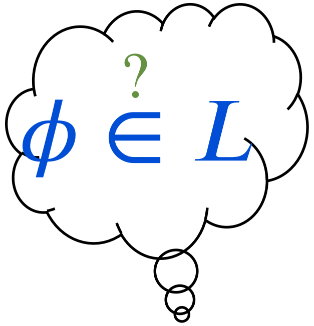

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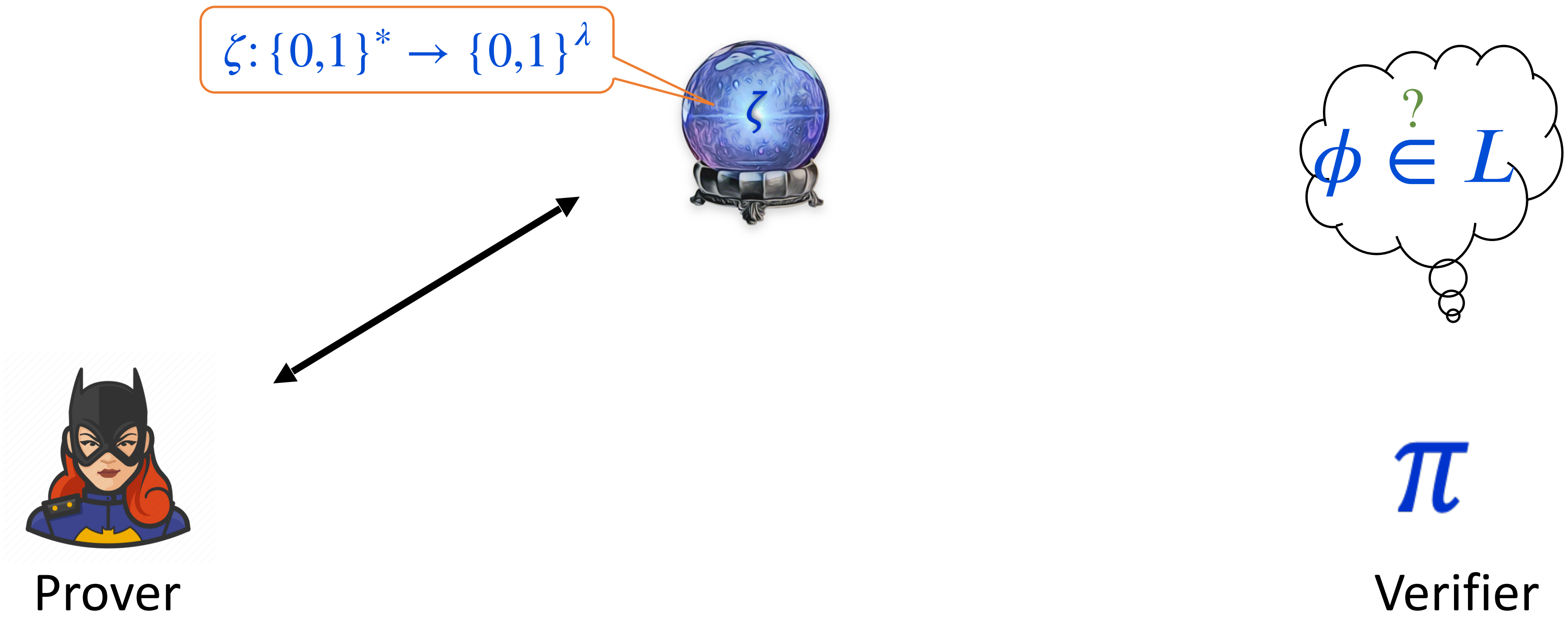

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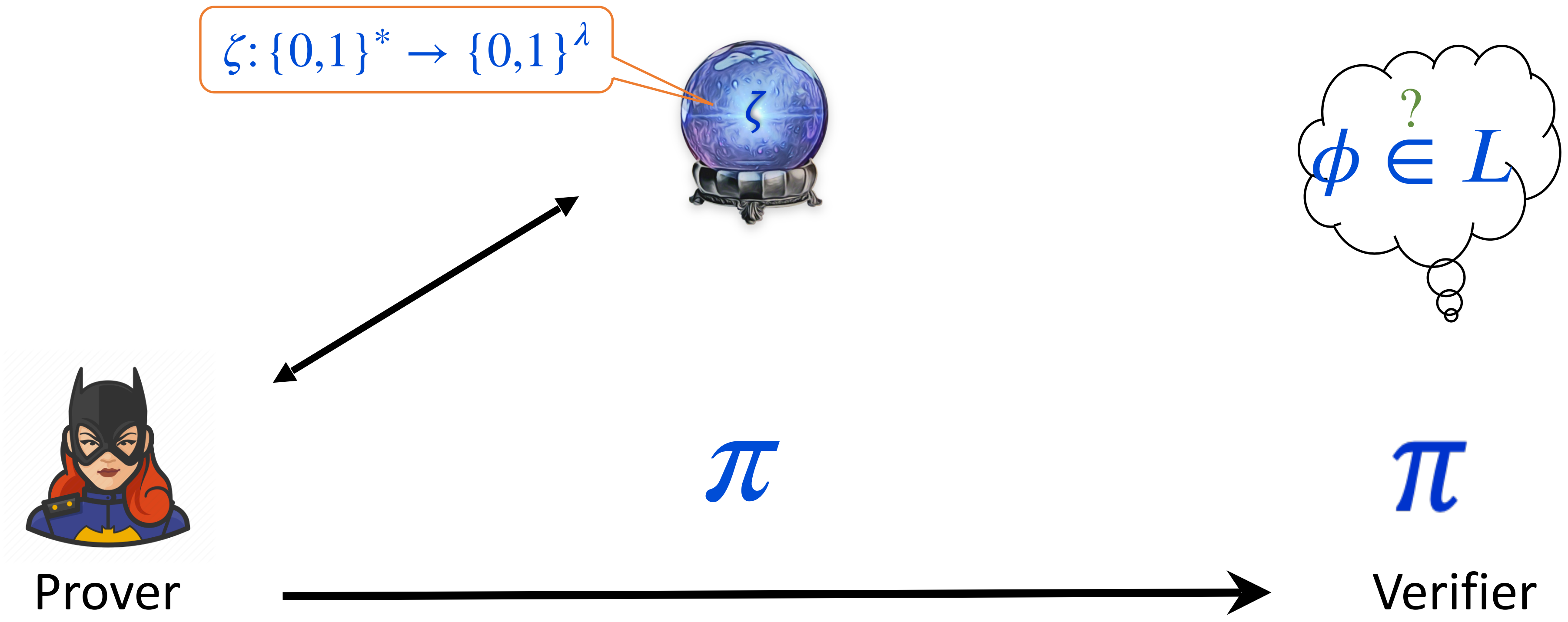

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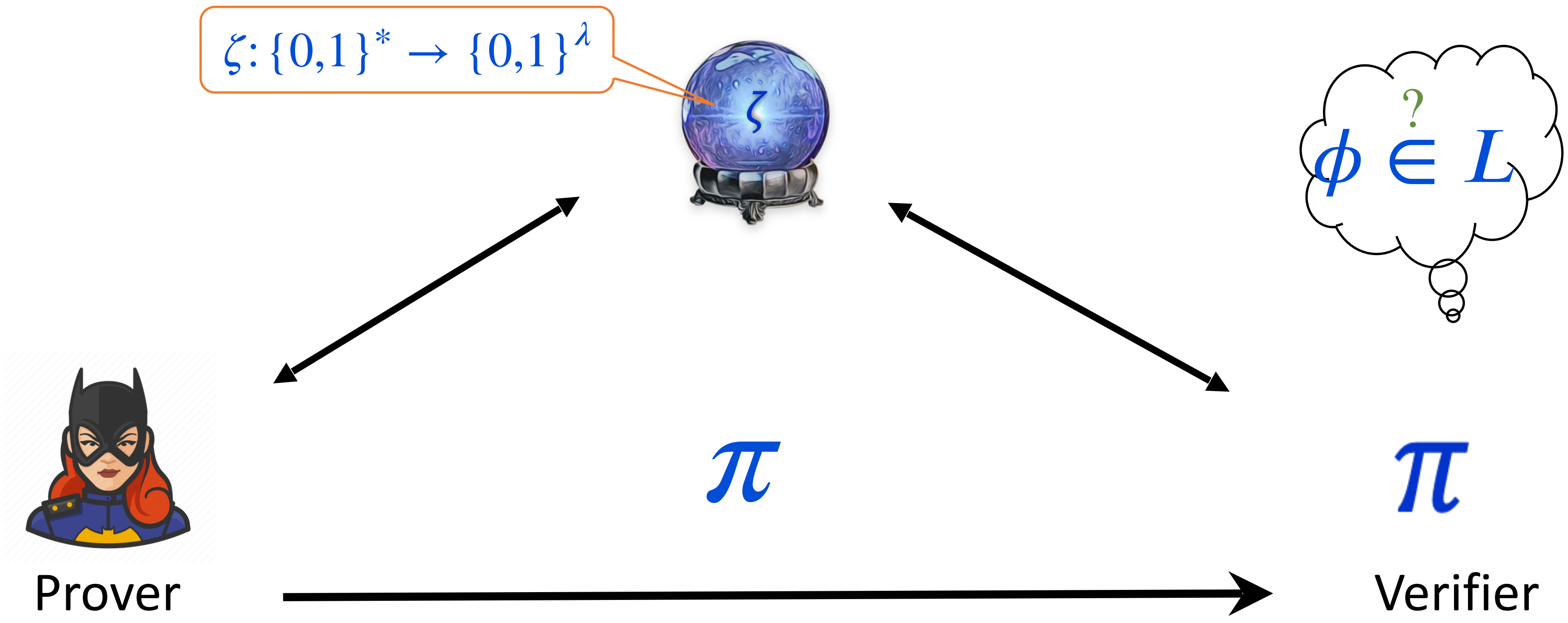

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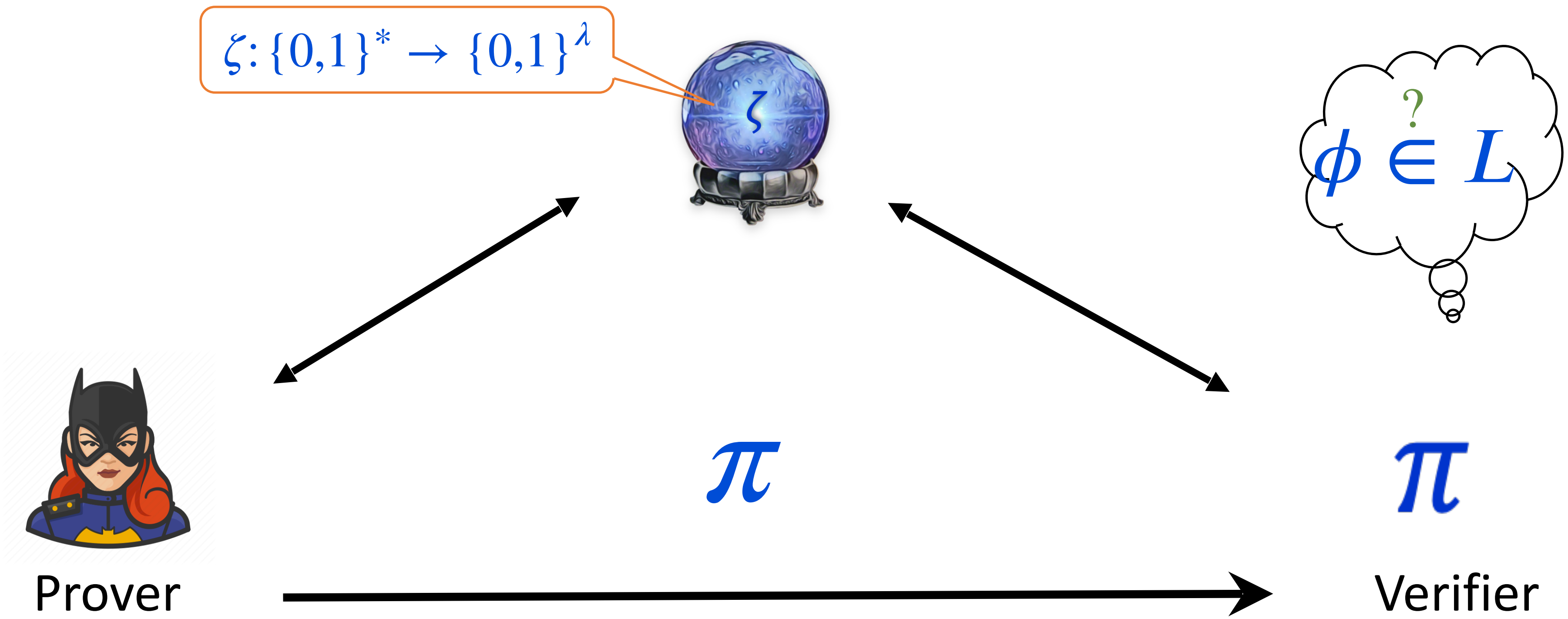

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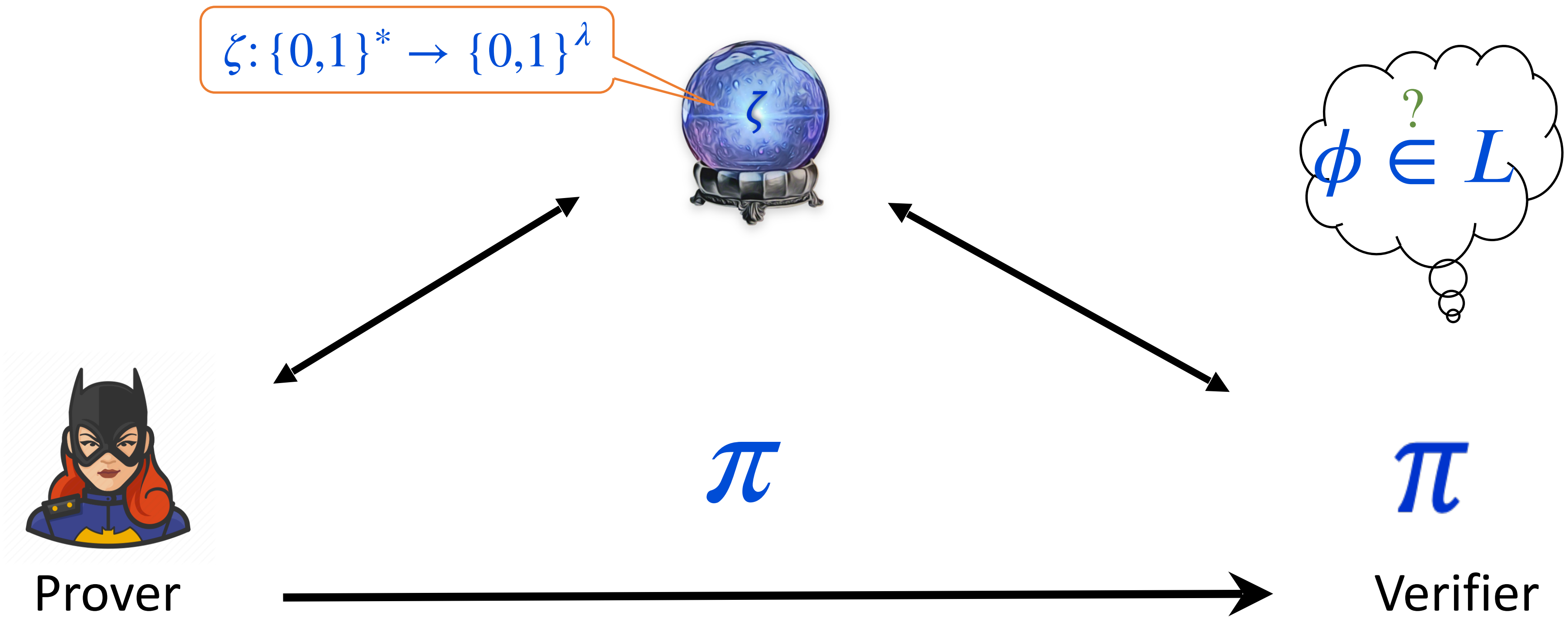
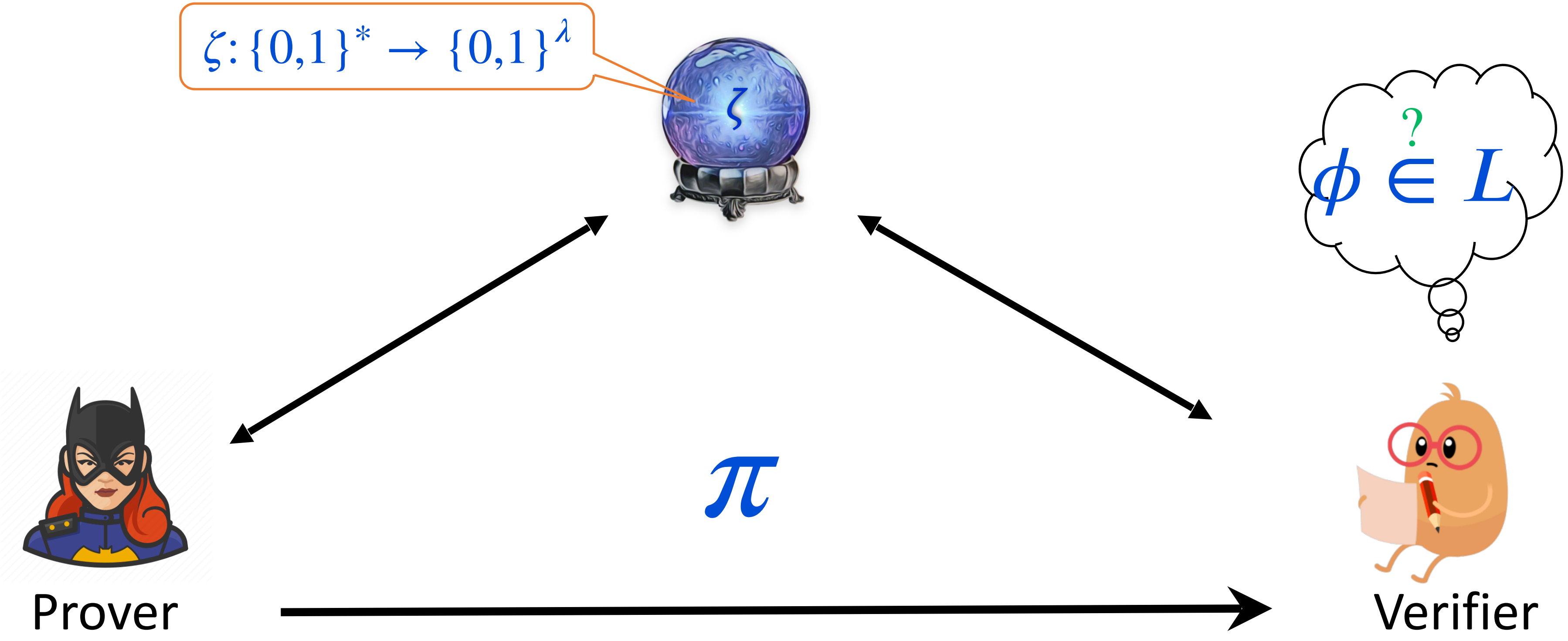


- Soundness against (computationally unbounded) query bounded provers
- $2^\lambda \gg$ instance size (n) and cheating prover running time (t)

Completeness



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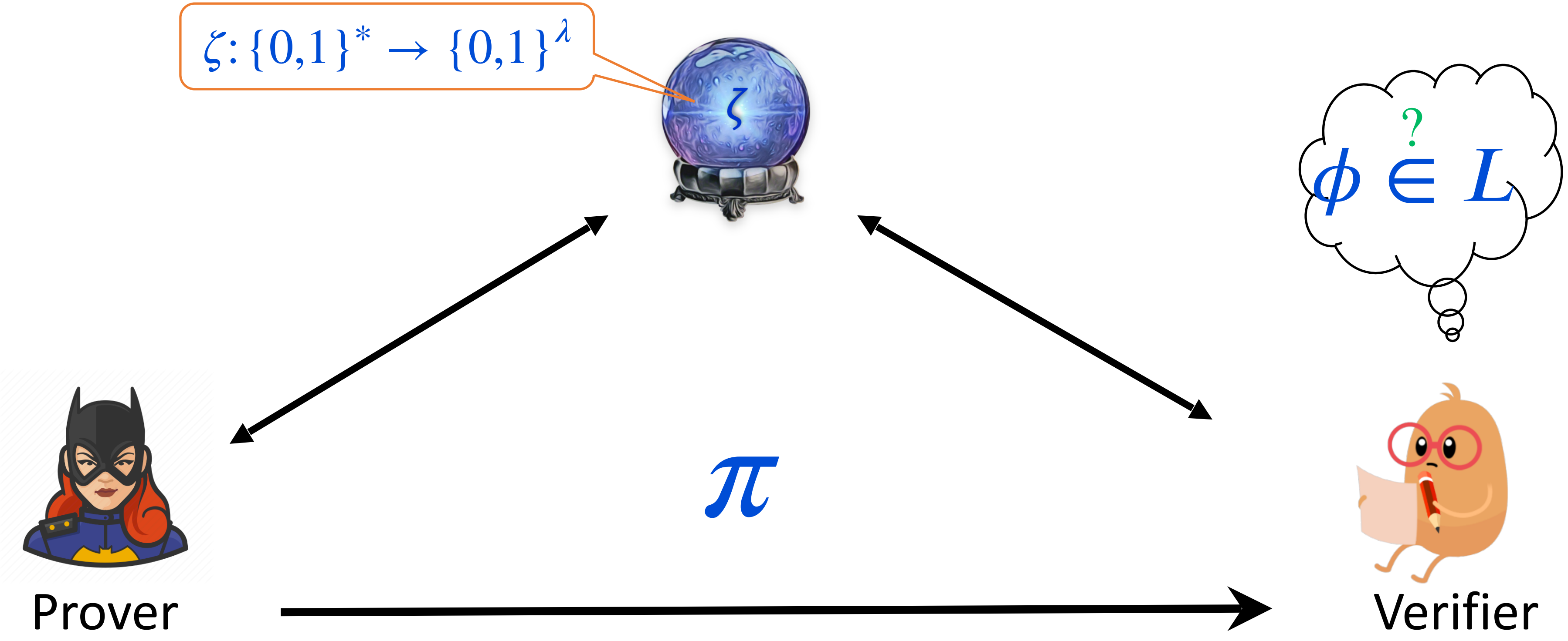
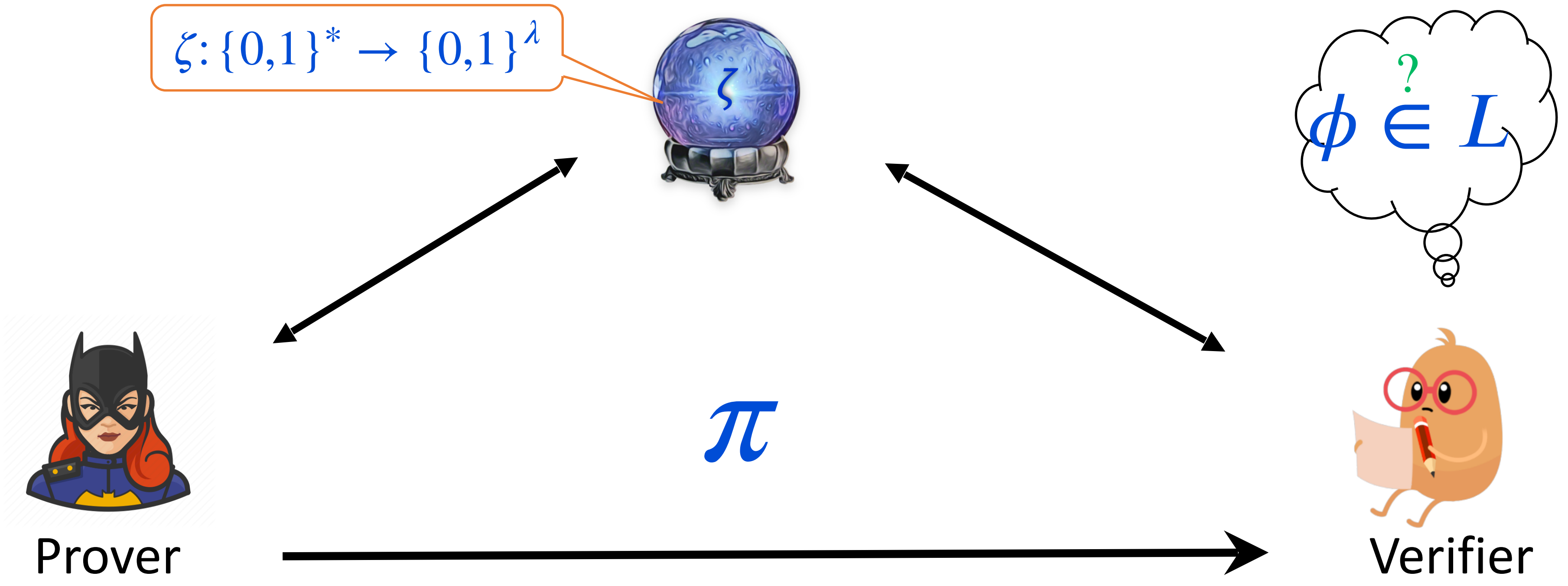


α-completeness: for every $\phi \in L$:

$$\Pr_{\zeta}\left[V^{\zeta}(\phi, \pi) = 1 : \pi \leftarrow P^{\zeta}\right] \geq \alpha$$

(t, ϵ)-soundness



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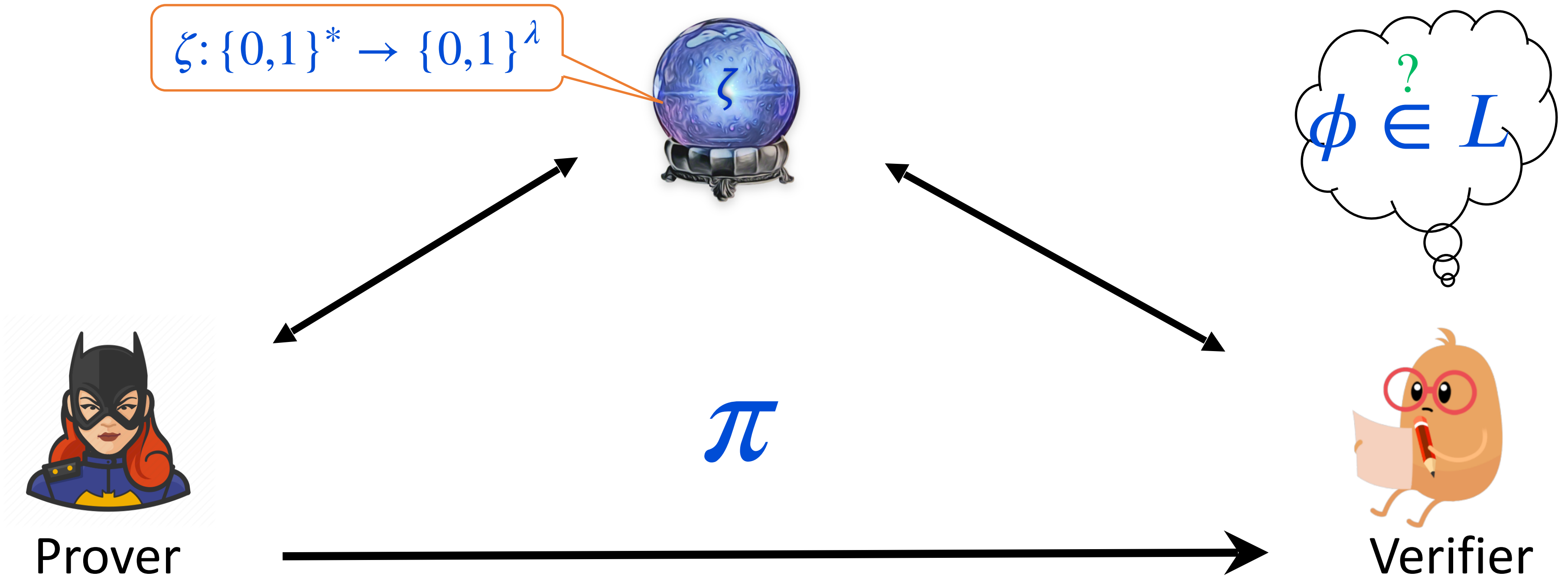

(t, ϵ)**-soundness:** for any $\phi \notin L$ and t-query (comp. unbounded) $\tilde{P}$:

$$\Pr_{\zeta}\left[V^{\zeta}(\phi, \pi) = 1 : \pi \leftarrow \tilde{P}^{\zeta}\right] \leq \epsilon$$

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- Widely used in practice

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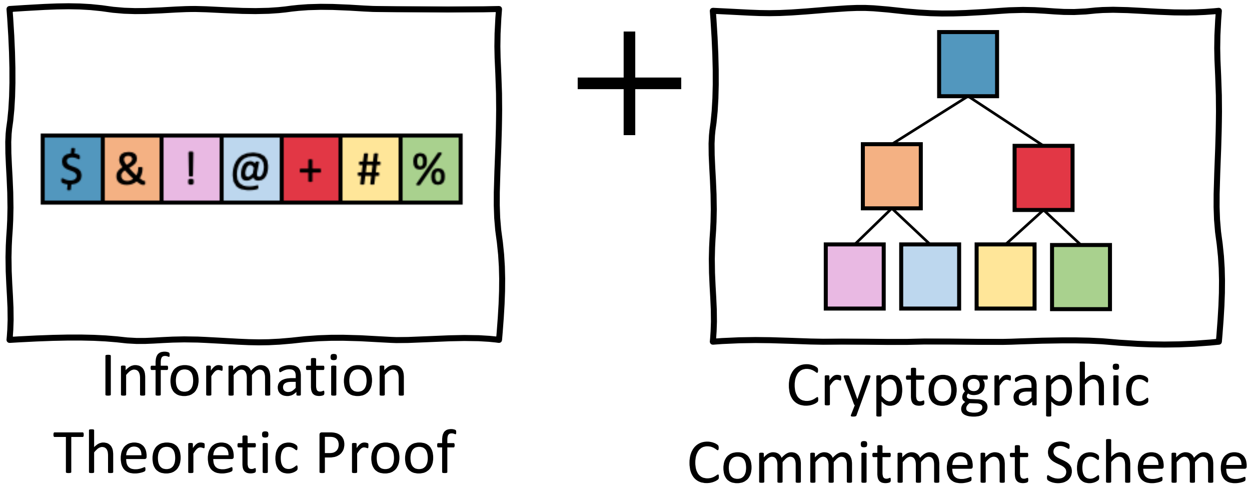

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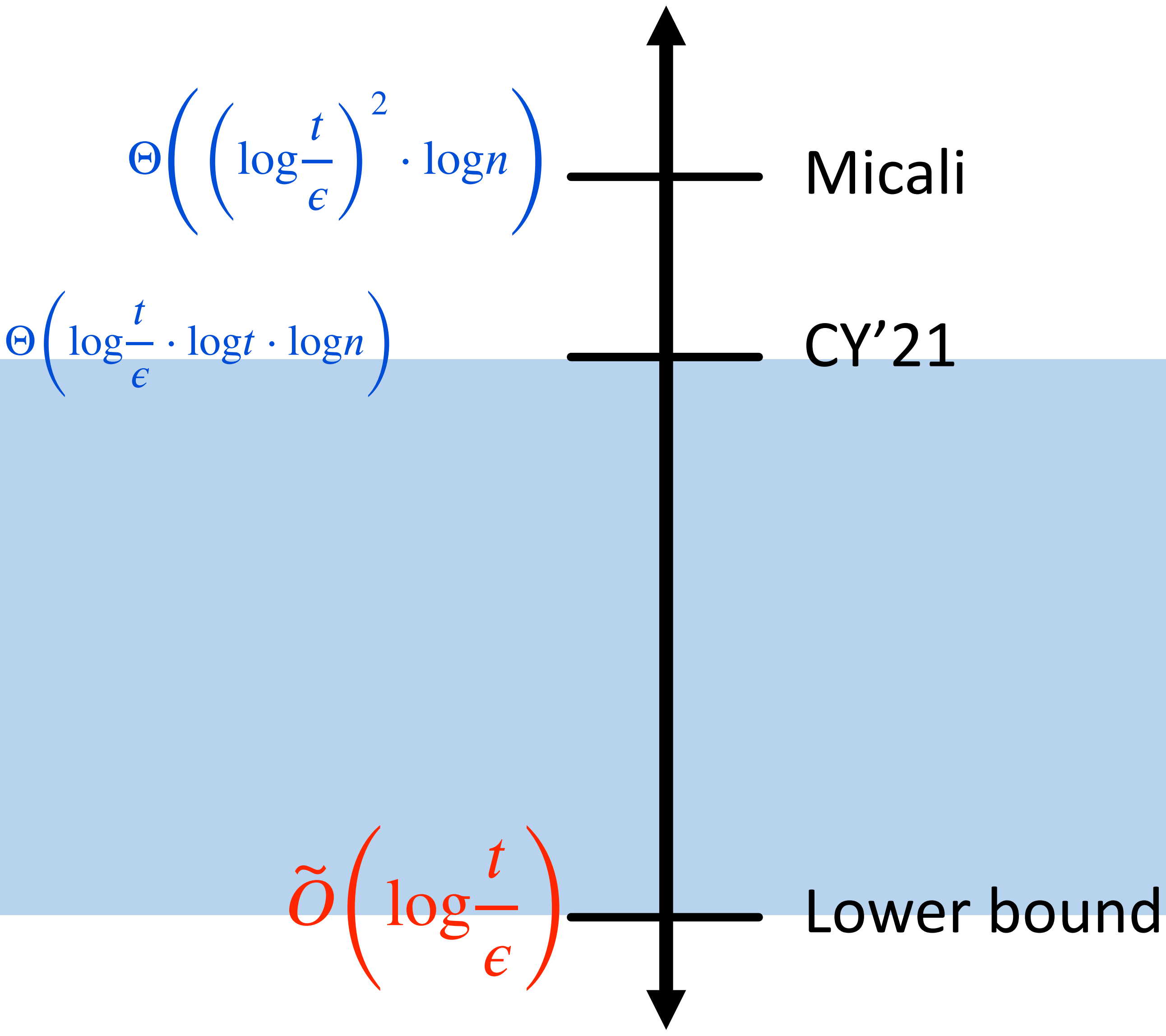

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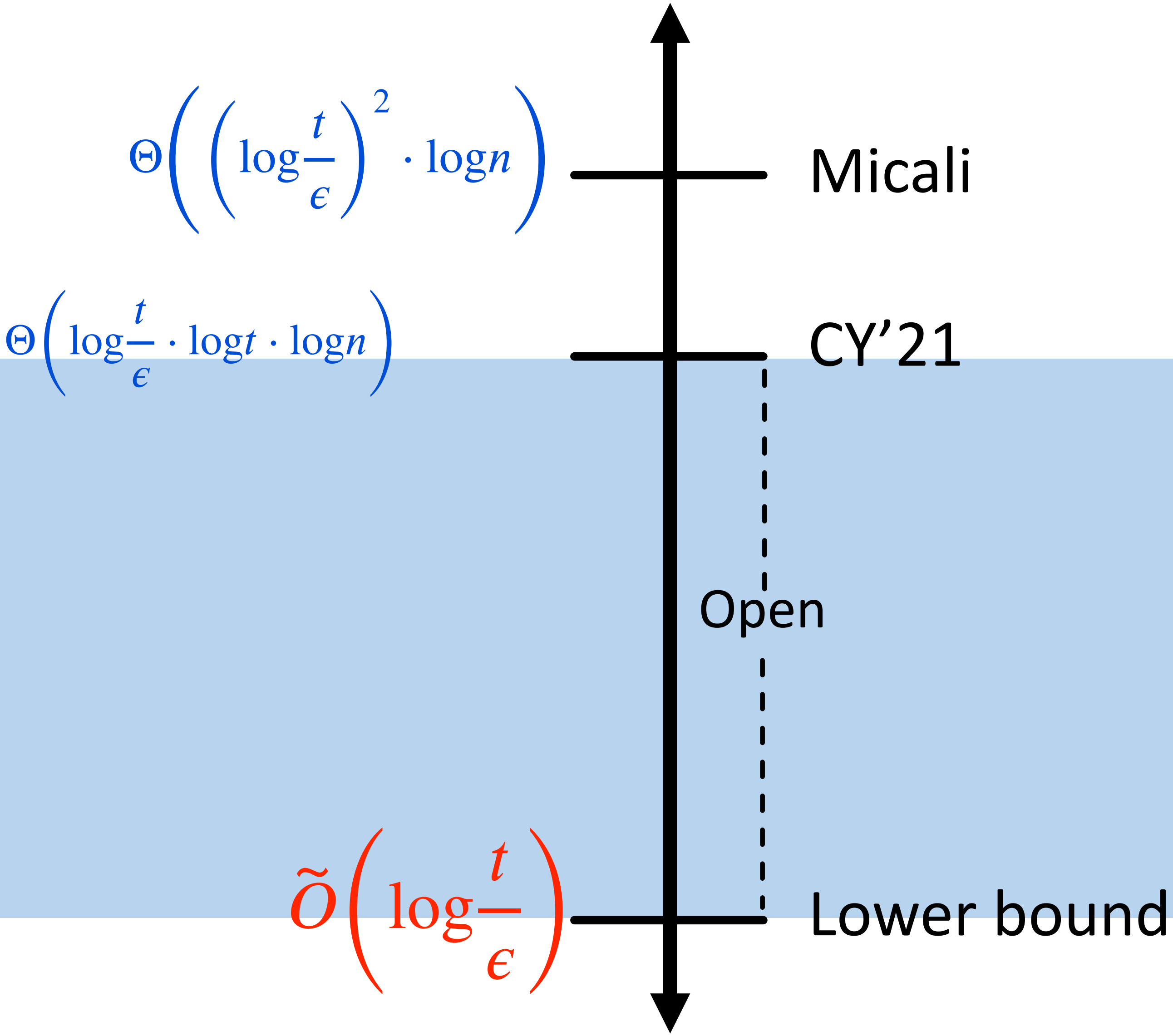

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Thm: Assuming rnd ETH, any "natural" ROM-SNARG (P, V) of (t, ϵ)-soundness has proof size $\Omega\left(\log\frac{t}{\epsilon} \cdot \log t / \log q_P\right)$

Tight up to $\log \mathrm{n} \cdot \log \mathrm{q}_P$ term ([CY'21] proof size is $\Theta\left(\log\frac{t}{\epsilon} \cdot \log t \cdot \log n\right)$)

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All known (non-contrived) constructions are natural

Proof size for natural constructions

- $\Theta\left(\left(\log\frac{t}{\epsilon}\right)^2 \cdot \log n\right)$ — Micali
- $\Theta\left(\log\frac{t}{\epsilon} \cdot \log t \cdot \boldsymbol{\log n}\right)$ — CY’21
- $\Omega\left(\log\frac{t}{\epsilon} \cdot \log t / \boldsymbol{\log q_P}\right)$ — This work
- $\Omega\left(\log\frac{t}{\epsilon}\right)$ — Lower bound

Lower bound on ROM SubVector Commitment

Subvector commitment (SVC) – **non-interactive** cmt with local opening.

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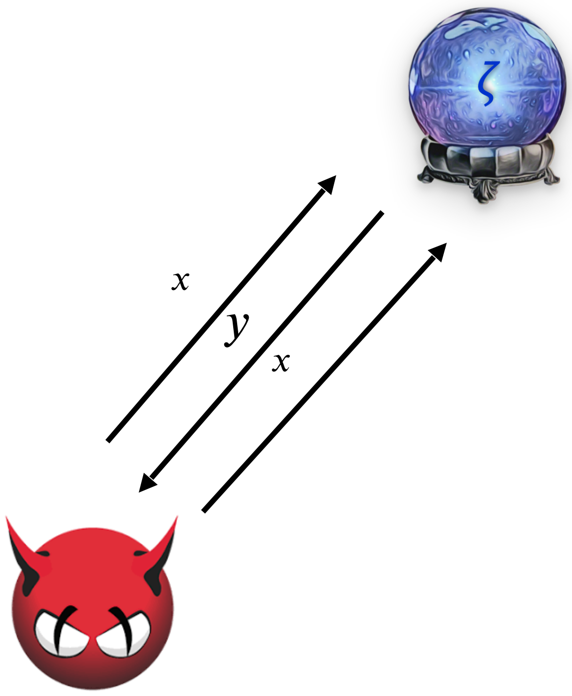

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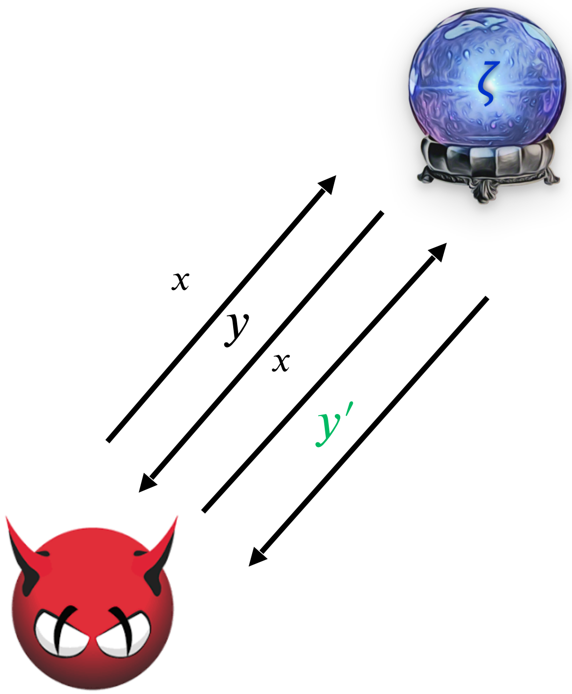

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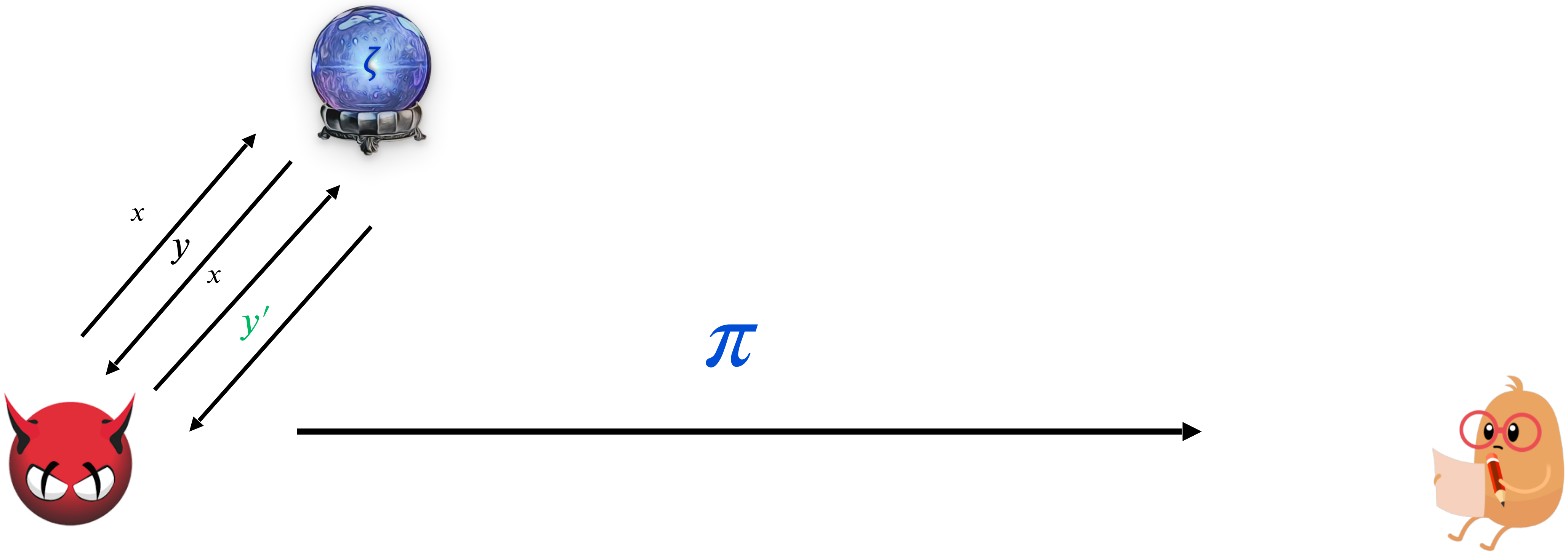

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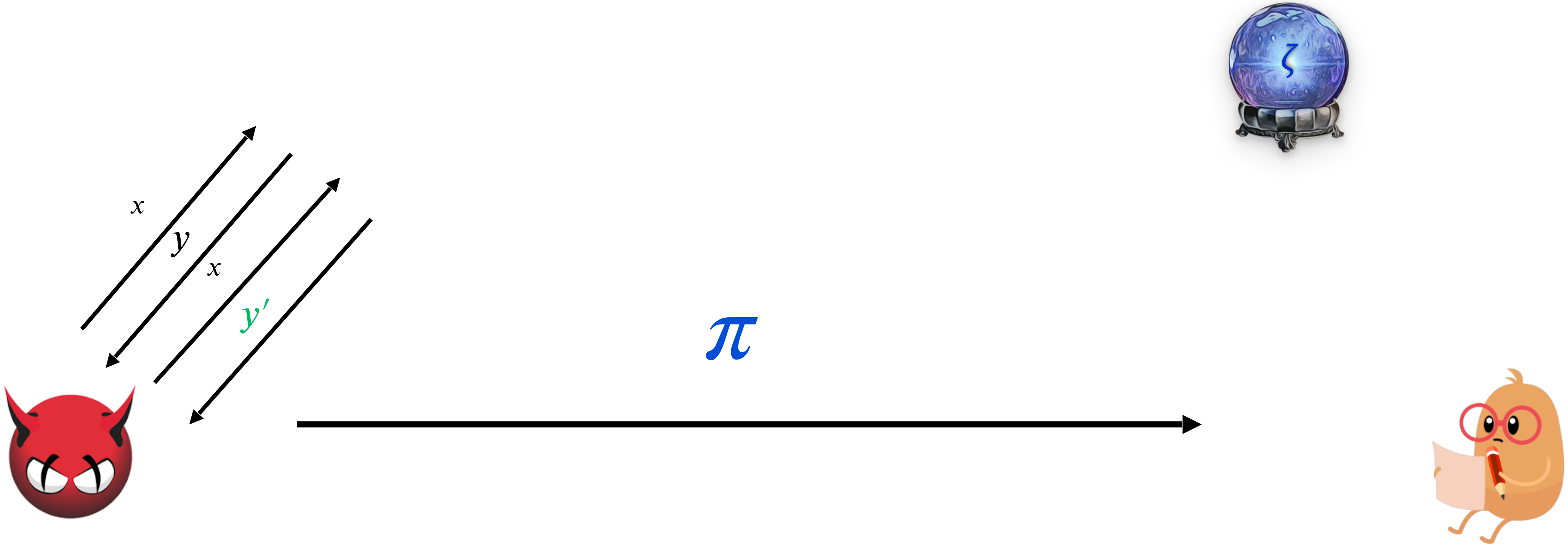

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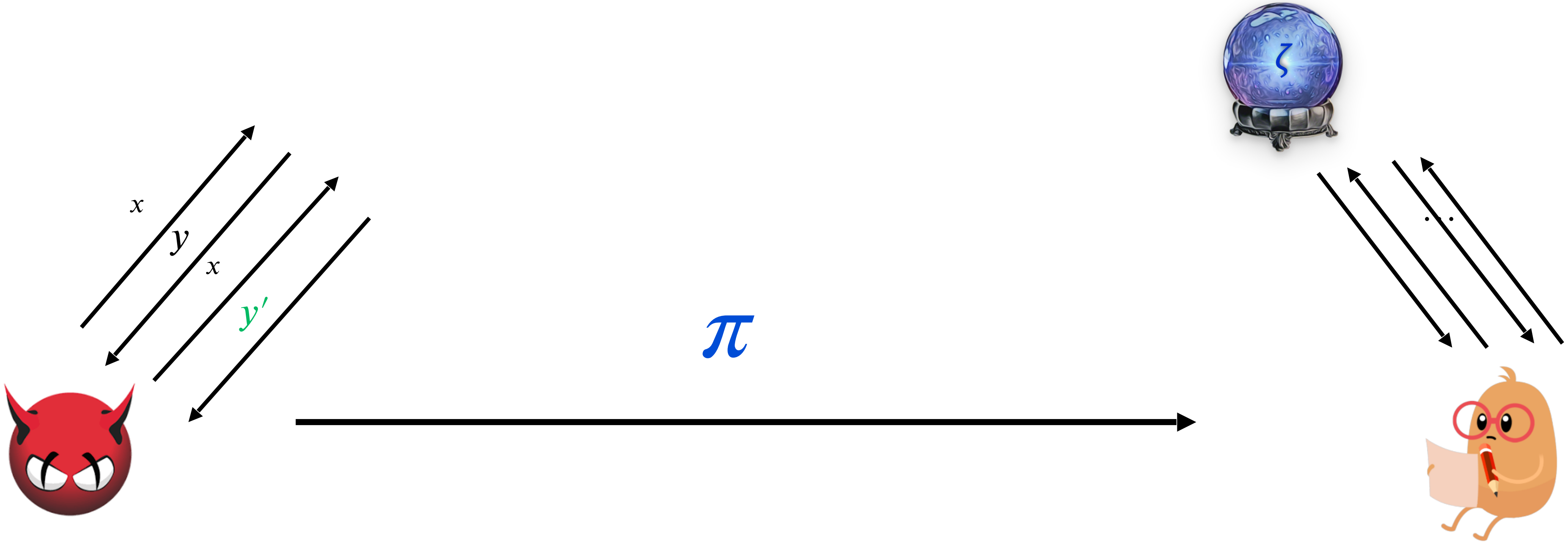

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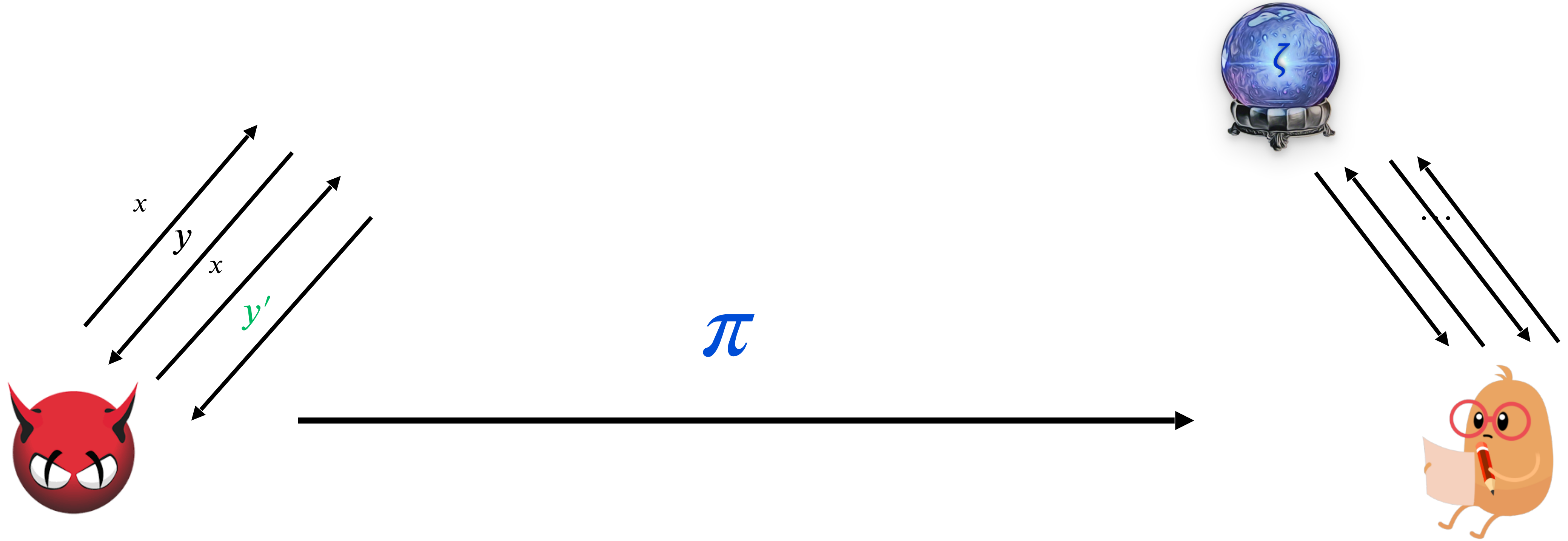

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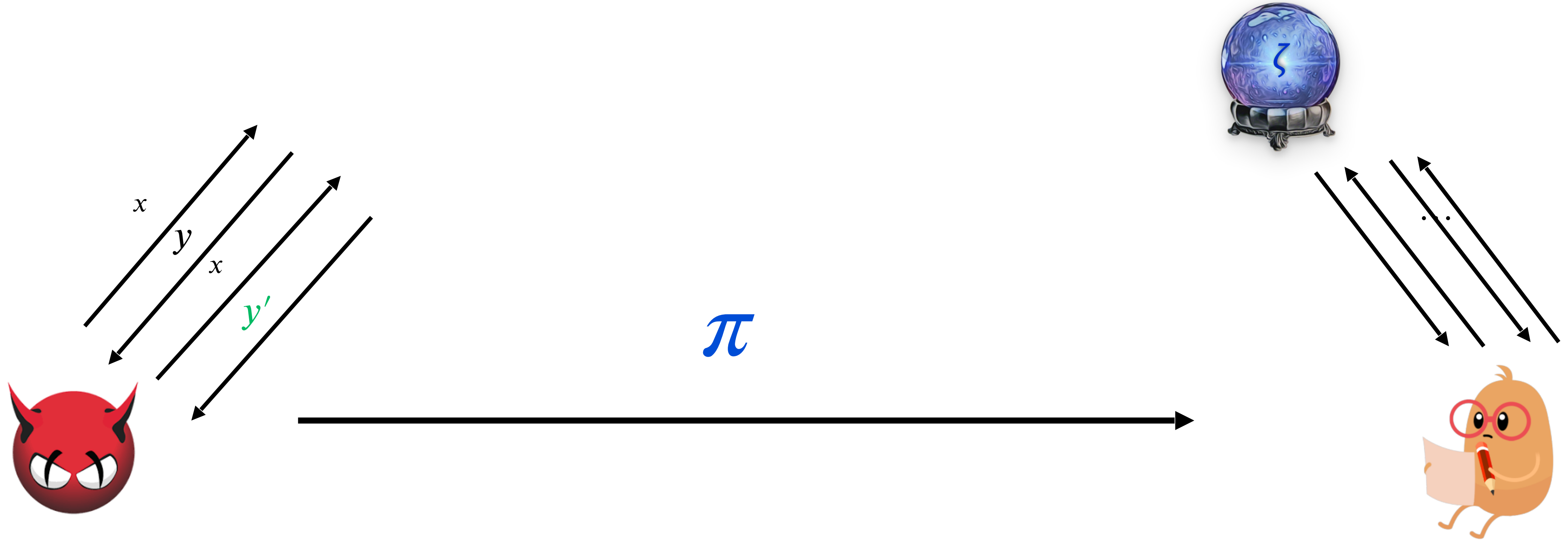

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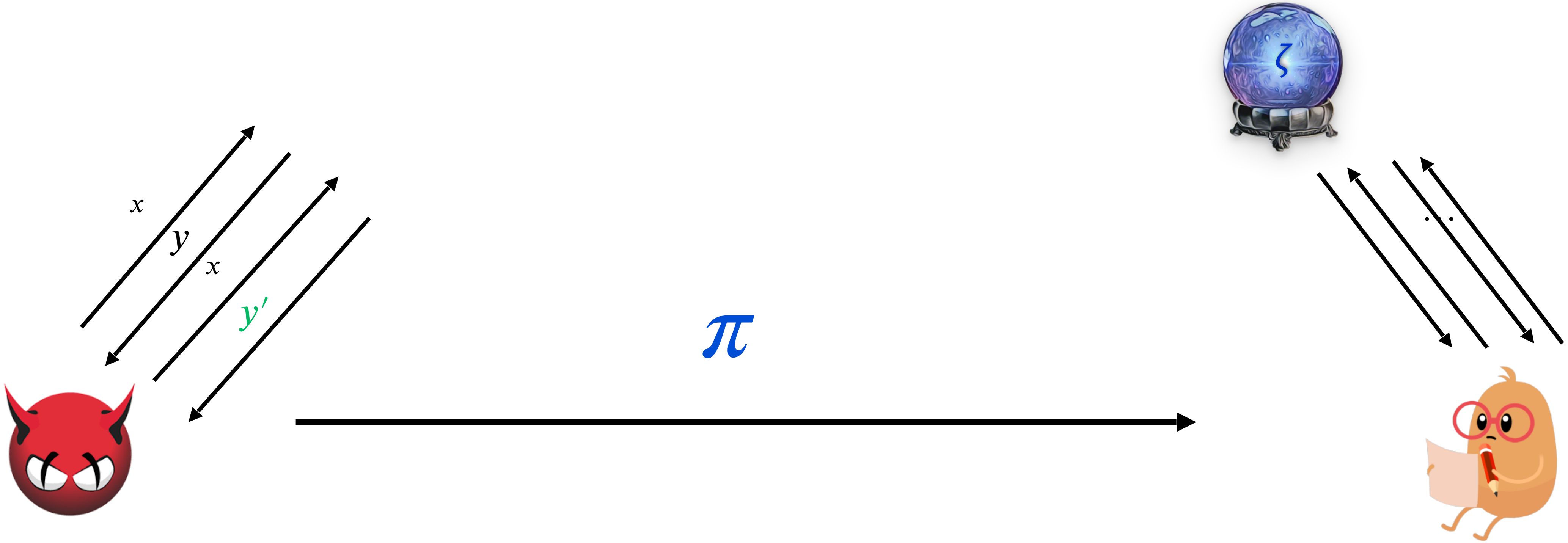

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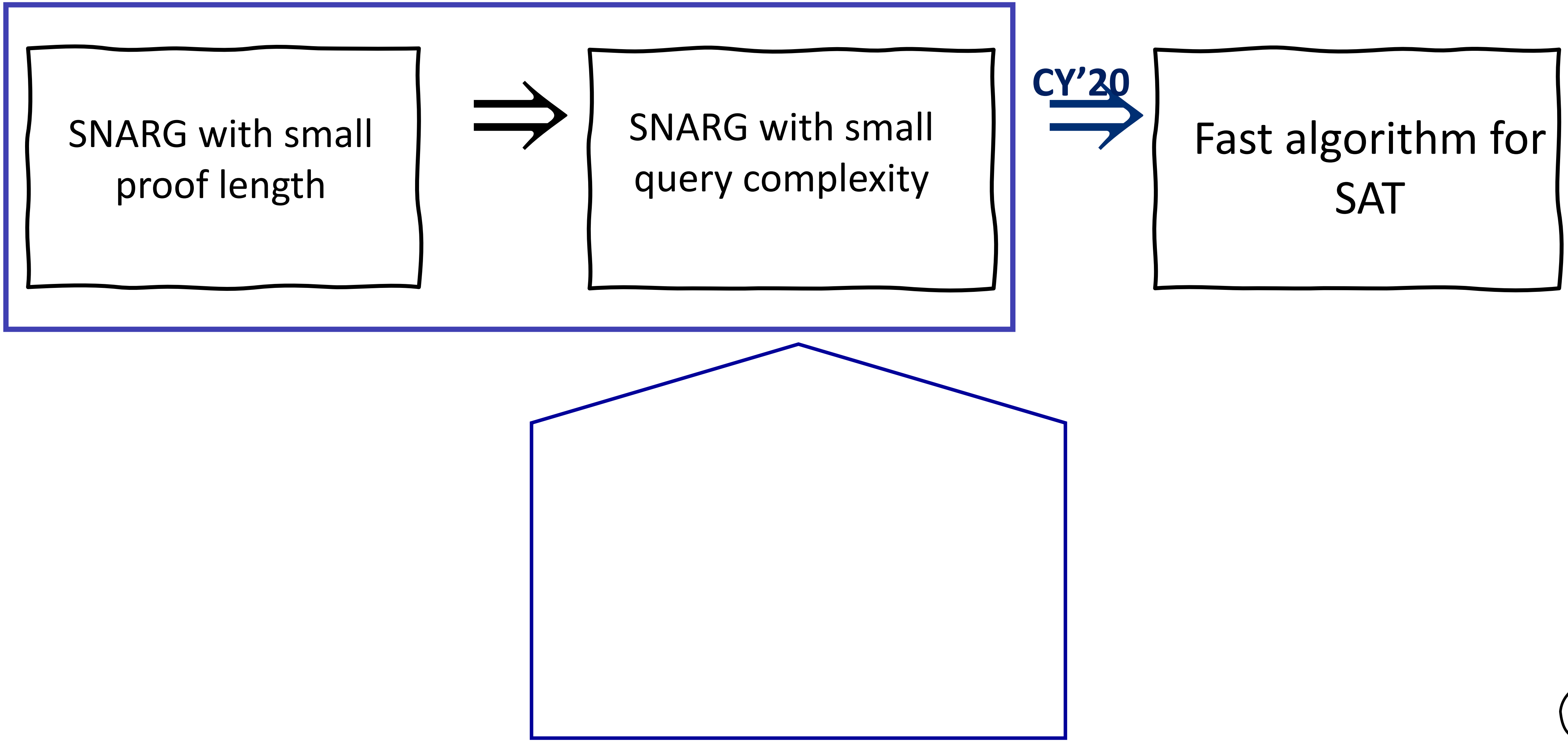
- Seems hard to get rid of w/o making the verifier adaptive

Short SNARGS to Fast Algorithms

SNARG with small proof length ⇒ SNARG with small query complexity

CY'20

Short SNARGS to Fast Algorithms



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⇒

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CY'20

⇒

Fast algorithm for SAT

- Proof size is unchanged

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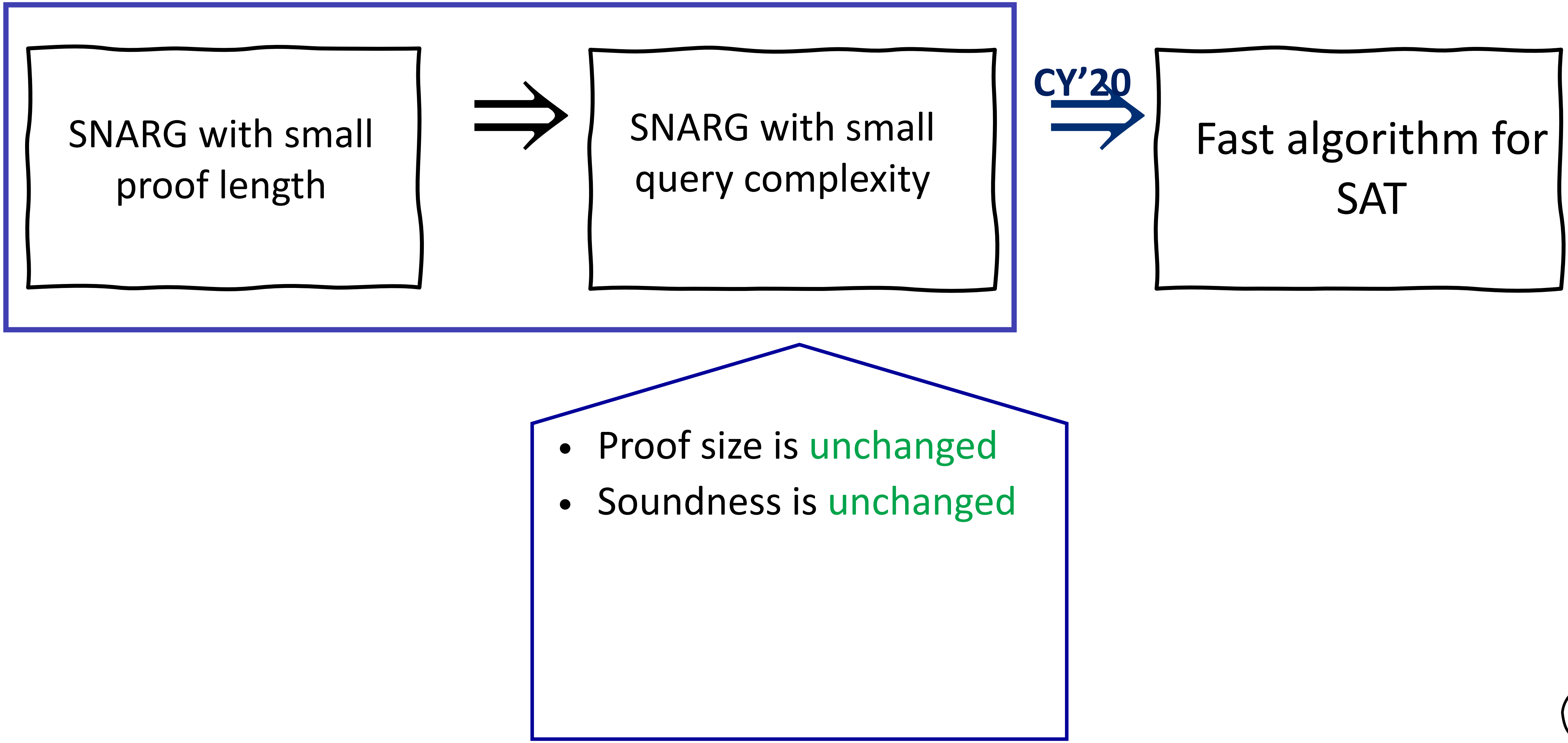

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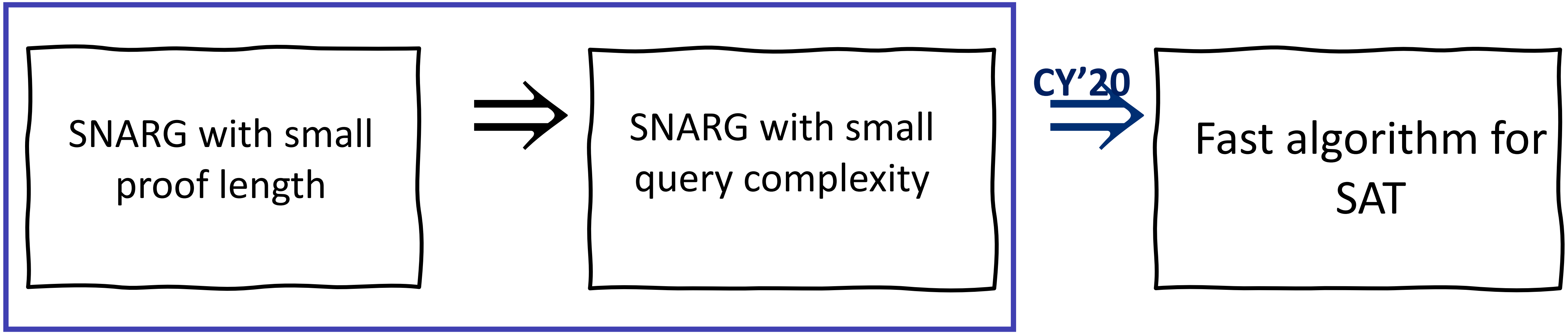

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Short SNARGs to Low-query SNARGs

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5. Accept if any combination of these answers makes $V(\phi, \pi)$ accept

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- We conclude by showing that the expected size of B is $O(\ell/\gamma)$

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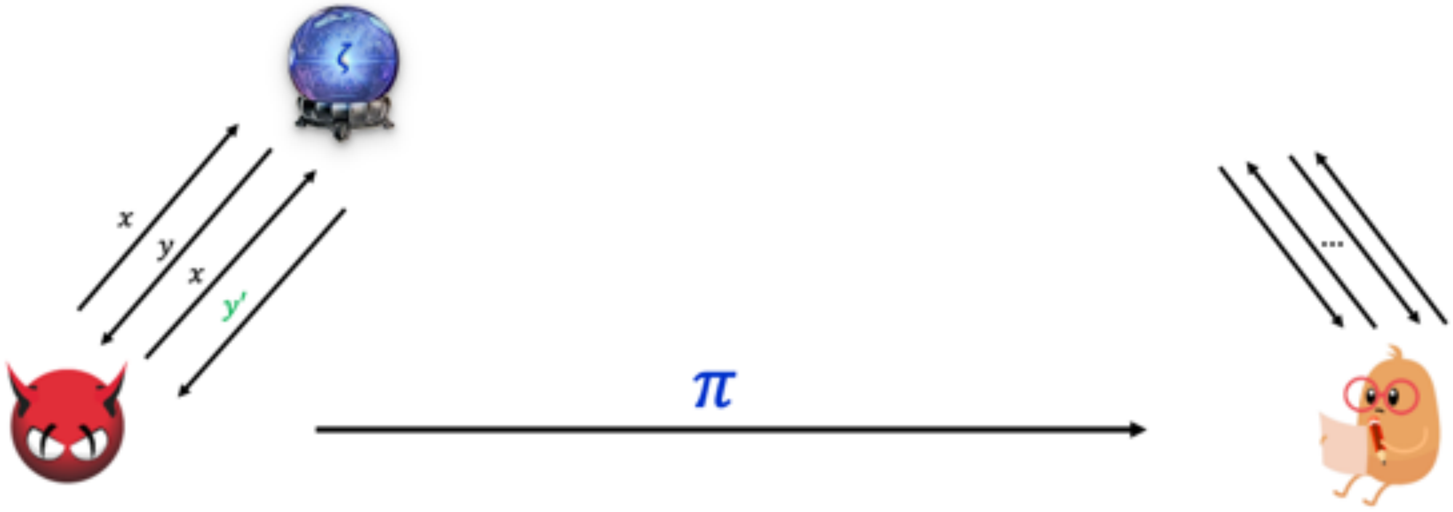

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Thank You!