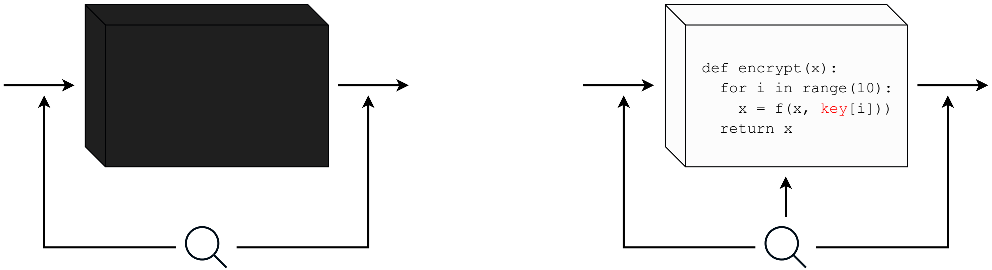
Implicit White-Box Implementations: White-Boxing ARX Ciphers

Adrián Ranea, Joachim Vandersmissen, and Bart Preneel

White-Box Cryptography



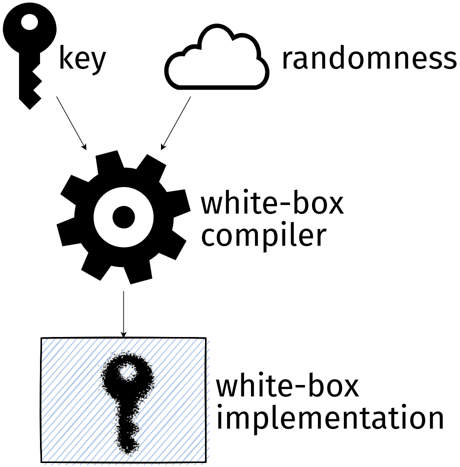
WBC: securing software crypto. implementations in the white-box model.

Applications: DRM, mobile payments, ...

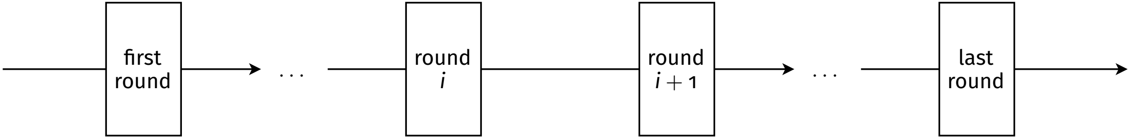
White-Box Implementations of Block Ciphers

Academic white-box implementations:

- Fixed hard-coded cipher key.
- Compiler/method public.
- Security goal: key-extraction resistance.



CEJO Implementations



Chow S, Eisen P, Johnson H & Van Oorschot PC. *White-Box Cryptography and an AES Implementation.* SAC 2002.

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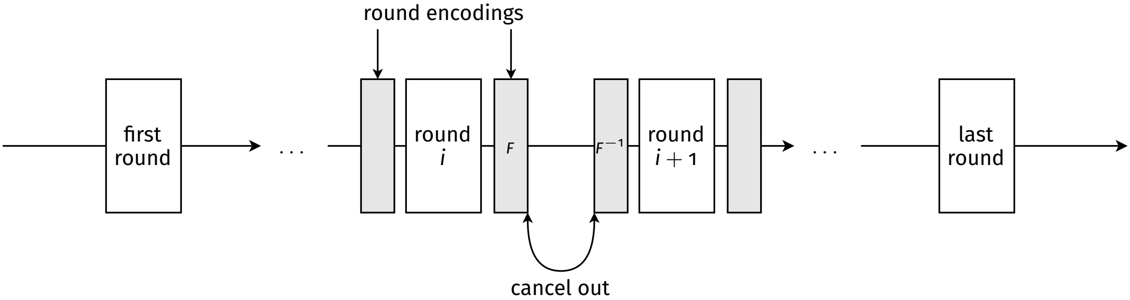

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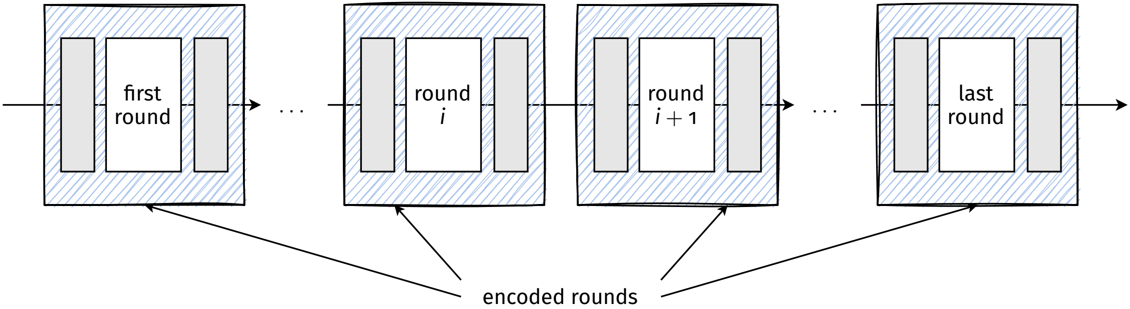

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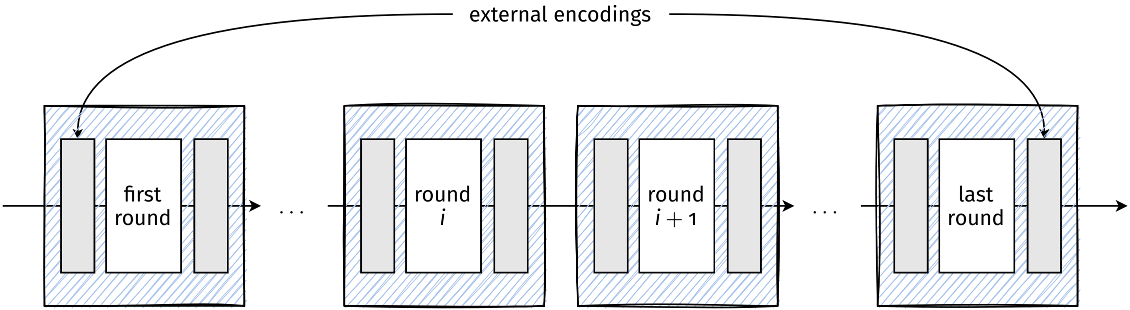

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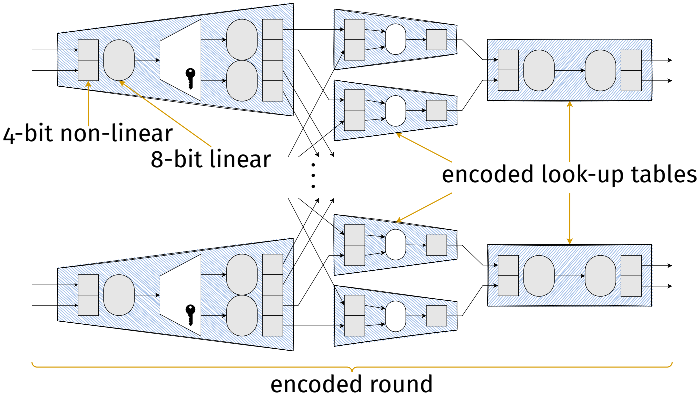
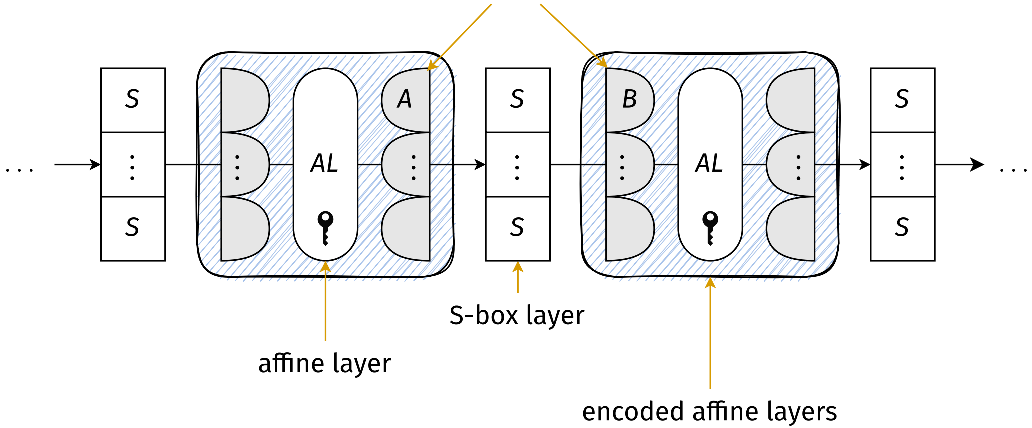
CEJO round



All CEJO implementations have been broken.

Self-Equivalence Implementations

(A, B) self-equivalence of $S \iff S = B \circ S \circ A$



Self-Equivalence (SE) Implementations

- No look-up tables,
  SE efficient with large encodings.
- CEJO can be reduced to SE,
  but the converse doesn't hold.

- **Problem:** difficult to find
  non-linear layers with
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Self-Equivalence (SE) Implementations

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- **Problem:** difficult to find
  non-linear layers with
  many and large self-equivalences.

Candidate non-linear layer: permuted modular addition $x, y \mapsto (x \boxplus y, y)$

Finding Self-equivalences of $x, y \mapsto (x \boxplus y, y)$

F is CCZ-equivalent to G if the graph of F, $\{(x, F(x))\}$, is equal to the graph of G up to an affine permutation.

- $F(x, y) = (x \boxplus y, y)$ is CCZ-equivalent to a quadratic function G.

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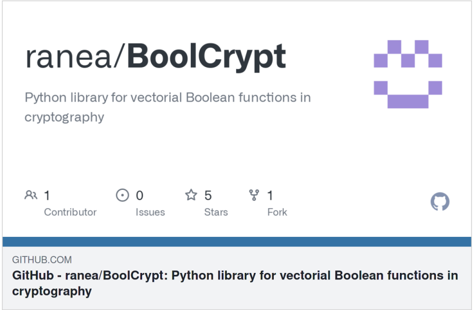
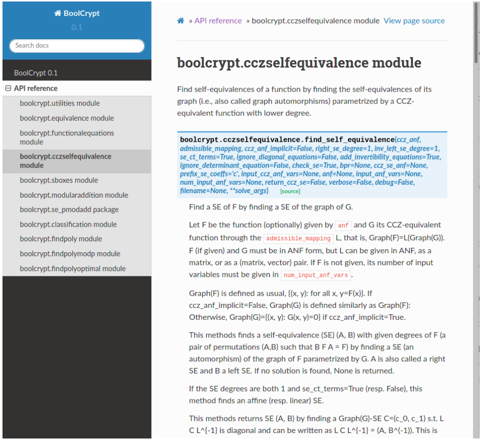
A graph automorphism of F is an affine permutation mapping the graph of F to itself.

CCZ-based method to find self-equivalences of F:

- Find graph automorphisms of low-degree G by solving a functional equation.
- Transform graph automorphisms of G into self-equivalences of F using CCZ-equivalence.

github.com/ranea/Boolcrypt





Self-Equivalences of the Permuted Modular Addition

SE found for wordsize $4 \leq n \leq 64$:

Type	$\#\{(A,B) : S = B \circ S \circ A\}$
Linear	$3 \times 2^{2n+2}$
Affine	$3 \times 2^{2n+8}$
Affine-quadratic	$3^2 \times 2^{3n+14} - 3 \times 2^{2n+8}$

Open problem: prove these SE subsets are the full SE groups for $n \geq 4$.

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$$
\left(\begin{array}{ccccc|ccccc}
\star & & & & & \star & & & & \\
\star & 1 & & & & \star & & & & \\
\vdots & & \ddots & & & \vdots & & & & \\
\star & & & 1 & & \star & & & & \\
\star & & & \star & 1 & \star & \star & \dots & \star & \star \\
\hline
\star & & & & & \star & & & & \\
\star & & & & & \star & 1 & & & \\
\vdots & & & & & \vdots & & \ddots & & \\
\star & & & & & \star & & & 1 & \\
\star & & & & & \star & & & \star & 1
\end{array}\right)
$$

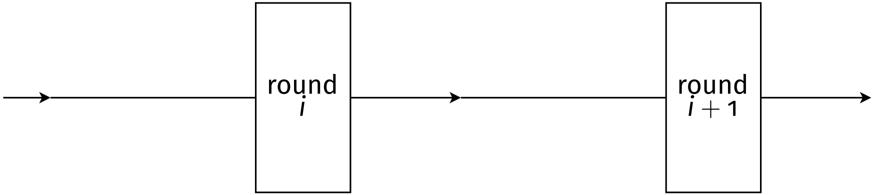
$$
\left(\star \; b \; \cdots \; b \; \star \; \star \,\middle|\, \star \; b \; \cdots \; b \; \star \; \star\right)
$$

Implicit framework

Implicit implementation:

- an encoded implementation where
- the round encodings are the composition of affine permutations and affine-nonlinear self-equivalences,
- and the encoded round functions are implemented by systems of low-degree equations.

Round Encodings in an Implicit Implementation



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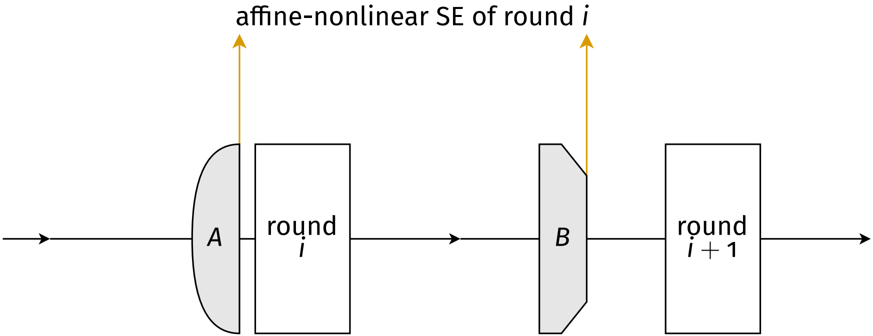

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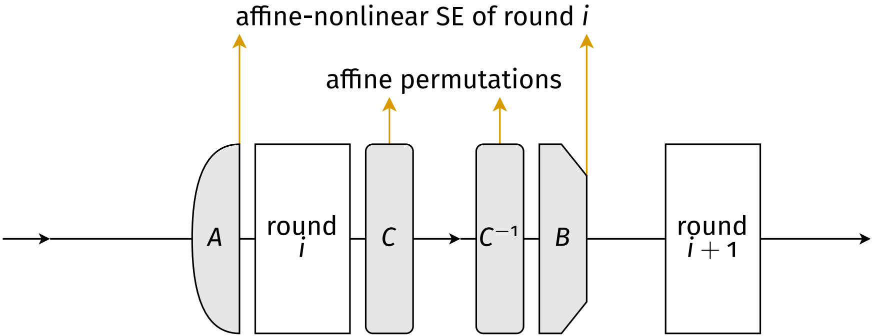

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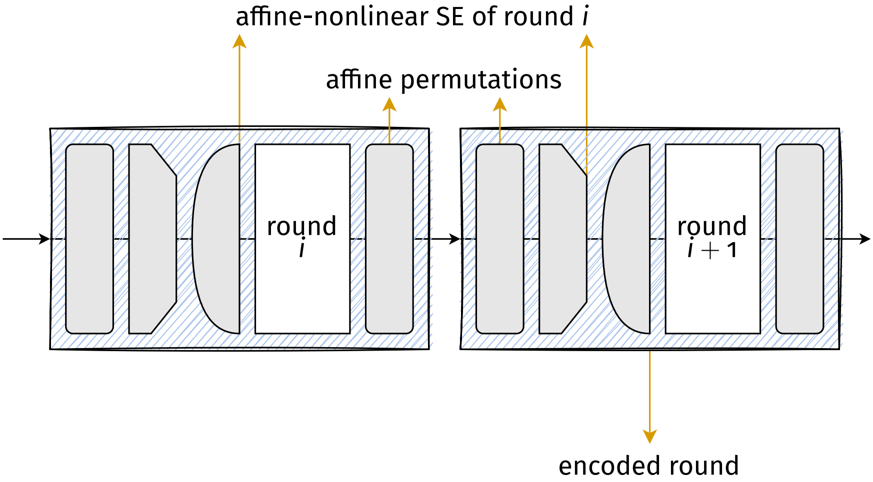

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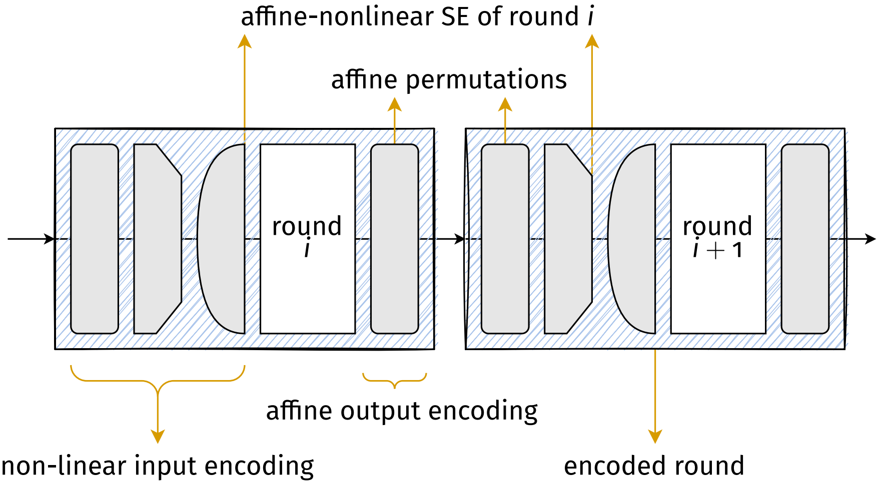


Implicit Round Functions

Implemented by low-degree quasilinear implicit functions:

- P is an implicit function of F if

$$P(x, y) = 0 \iff y = F(x)\,.$$

- Evaluate $F(x_0)$ by substituting $x_0 = x$ and solving $P(x_0, y) = 0$ for y.
- Fast solving if P is quasilinear:

$$\forall x, \text{ the function } y \mapsto P(x, y) \text{ is affine}\,.$$

Permuted modular addition has a quasilinear quadratic implicit function!

Size of an Implicit Round Function

Upper bound on the size of a $(2n, n)$-bit P for an n-bit cipher.

Cipher blocksize	Degree of P	Size of P
64	2	0.05 MB
64	3	1.42 MB
64	4	6.50 MB
128	2	0.40 MB
128	3	22.50 MB
128	4	193.19 MB

For the permuted modular addition, P is cubic or quartic if affine-quadratic self-equivalences are used.

Security of Implicit Implementations

Security goal: key-extraction resistance.

An implicit implementation is secure against known generic attacks if

- quadratic input encodings OR
- large non-linear layer

Implicit framework cannot secure SPN ciphers with affine encodings.

Security of Implicit Implementations

New generic attack (reduction to self-equivalence implementations) based on functional equations (affine equivalence problems):

known functions

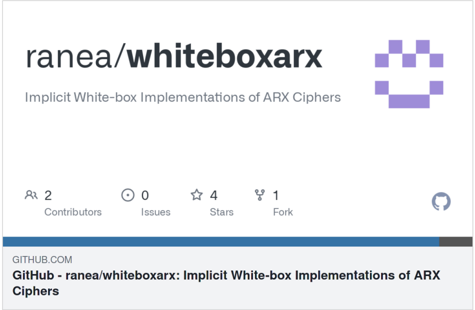
$$G = Y \circ F \circ X$$

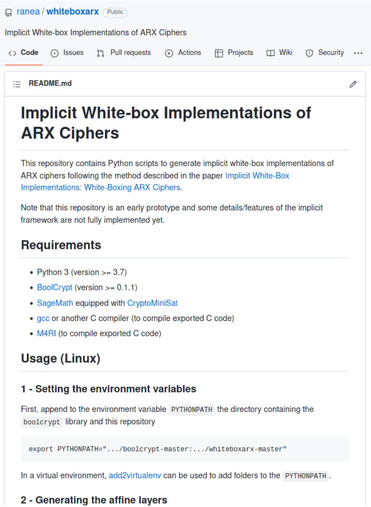
unknown affine permutations

Security of Implicit Implementations

function	$\overline{E} = O \circ (L \circ S) \circ \oplus_k \circ I$	$P = V \circ T \circ U \circ (\oplus_k \parallel L^{-1}) \circ (I \parallel O^{-1})$
equation	$\overline{E} = Y \circ (L \circ S) \circ X$	$P = Y \circ T \circ X$
degree	high	low
access	black-box	white-box
goal	find any solution	find any solution and guess U

github.com/ranea/whiteboxarx





Implicit White-box Implementations of ARX Ciphers

This repository contains Python scripts to generate implicit white-box implementations of ARX ciphers following the method described in the paper Implicit White-Box Implementations: White-Boxing ARX Ciphers.

Note that this repository is an early prototype and some details/features of the implicit framework are not fully implemented yet.

Requirements

- Python 3 (version >= 3.7)
- BoolCrypt (version >= 0.1.1)
- SageMath equipped with CryptoMiniSat
- gcc or another C compiler (to compile exported C code)
- M4RI (to compile exported C code)

Usage (Linux)

1 - Setting the environment variables

First, append to the environment variable `PYTHONPATH` the directory containing the `boolcrypt` library and this repository

```
export PYTHONPATH=".../boolcrypt-master:.../whiteboxarx-master"
```

In a virtual environment, add2virtualenv can be used to add folders to the `PYTHONPATH`.

2 - Generating the affine layers

Conclusion

- Implicit framework: new design that prevents generic attacks, first method applicable to ARX ciphers.
- New method to find self-equivalences based on the CCZ-equivalence, applied to the permuted modular addition.
- Two open-source tools: `BoolCrypt` and `whiteboxarx`.
- Future work: new attacks, other non-linear layers, …