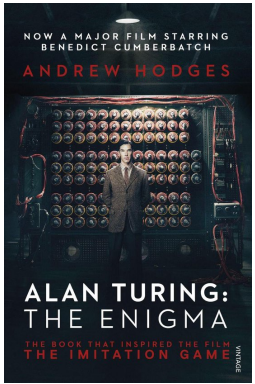
Provably Secure Reflection Ciphers

Tim Beyne and **Yu Long Chen**

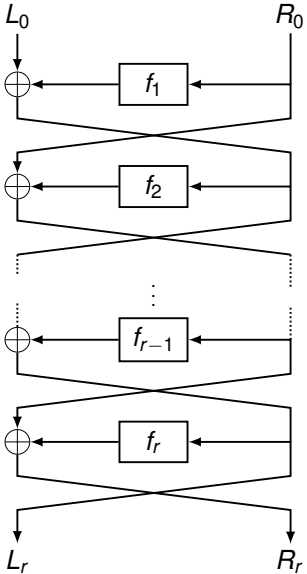
August 18, 2022

Self-Inverse Encryption Schemes



- Fascinating feature: self-inverse
- Enigma: encryption and decryption operations are identical
- Enabled by middle reflector (Umkehrwalze)
- Encryption device considerably more compact

Feistel Ciphers



DES block cipher (Horst Feistel (IBM) + NSA)

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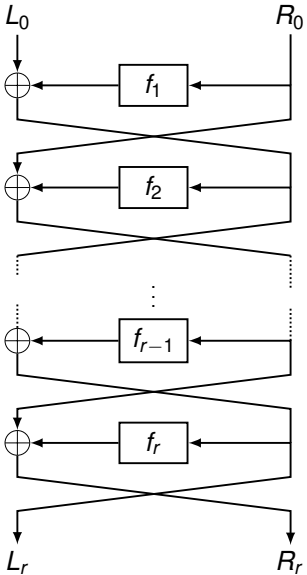

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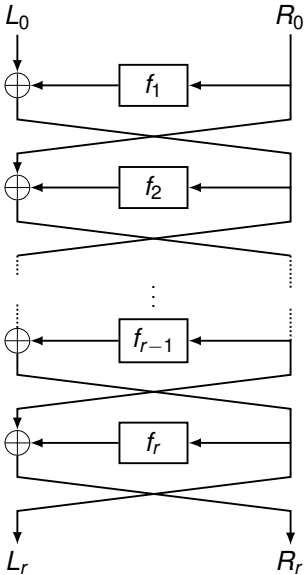

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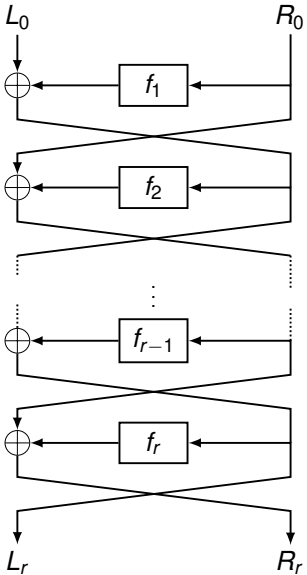

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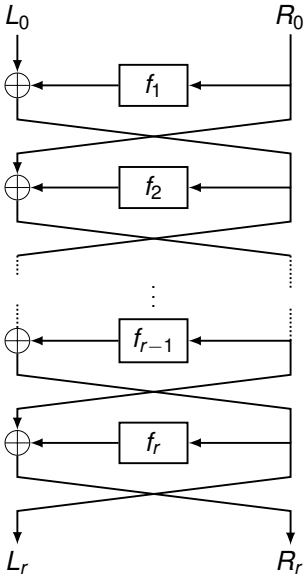

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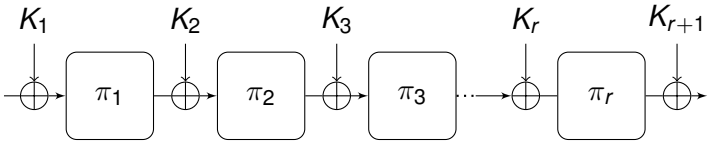
- Luby and Rackoff (1985): generic security

Key-Alternating Cipher



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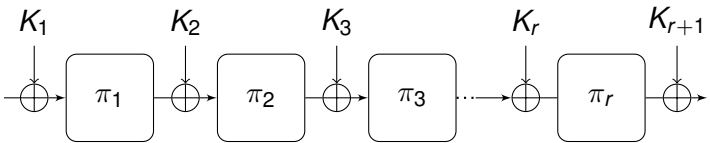

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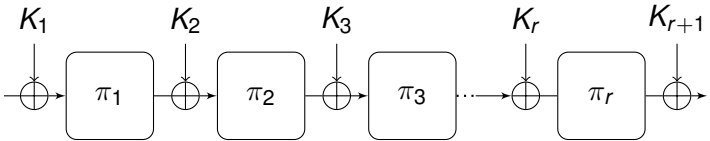

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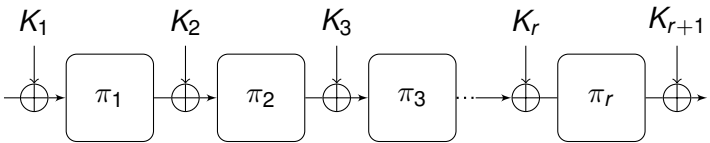

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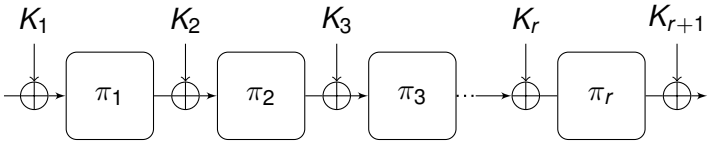

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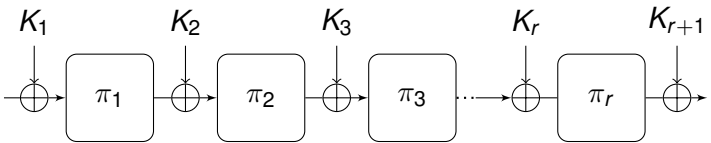

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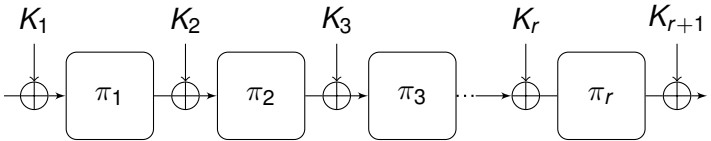

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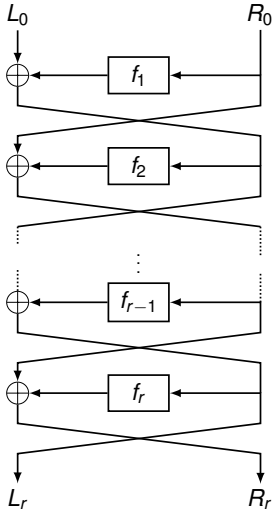

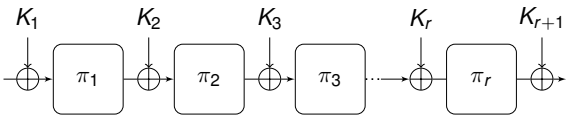

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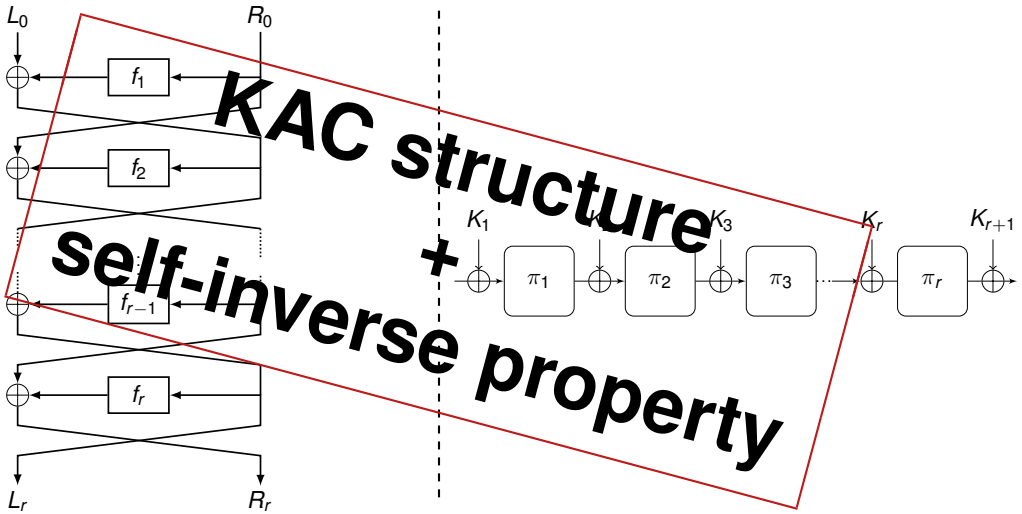
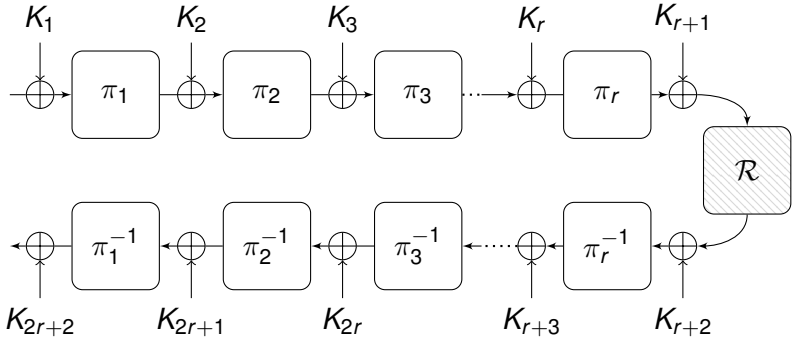


Reflection Ciphers



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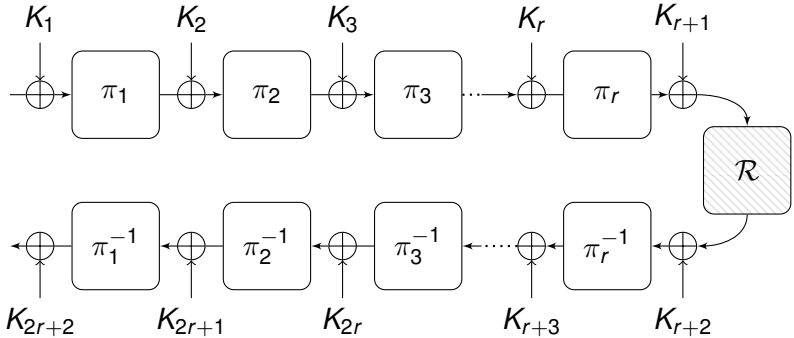

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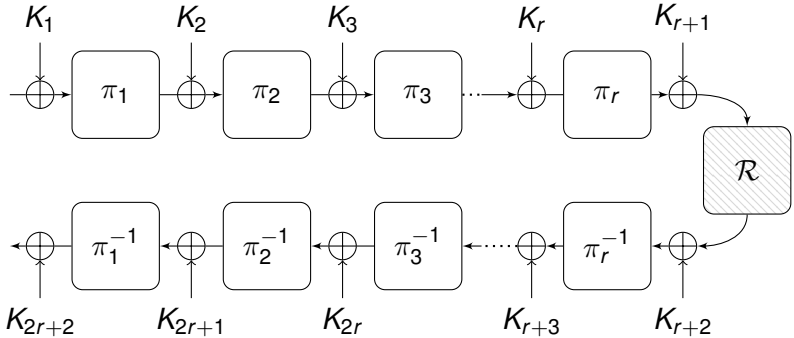

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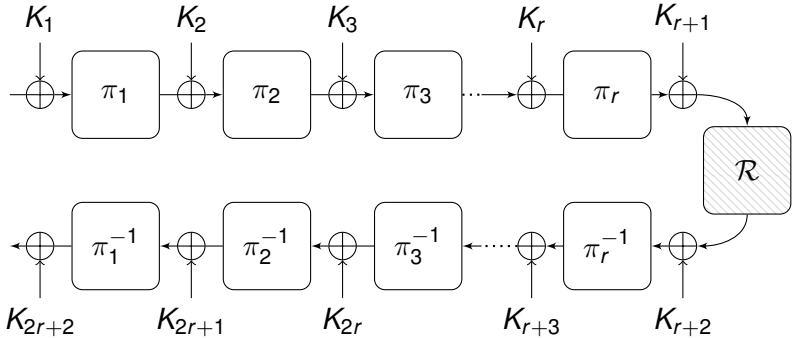

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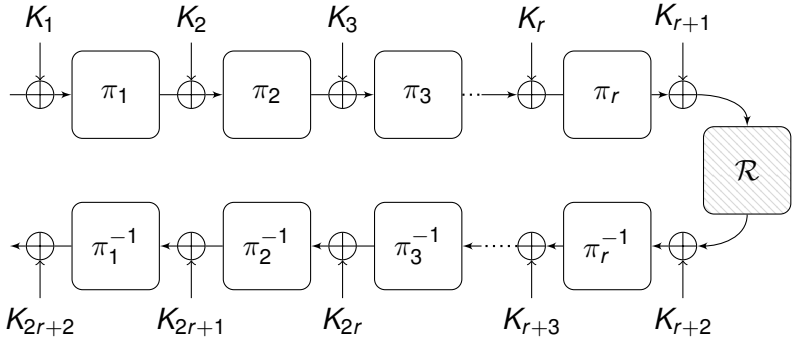

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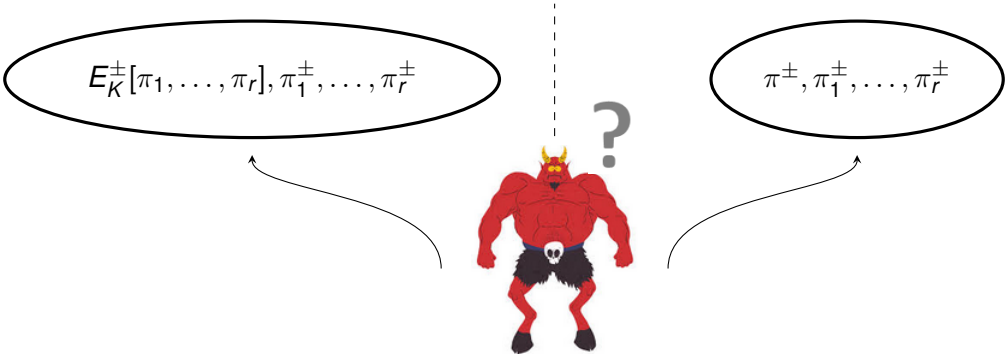
- Study of generic security is missing

Generic Security of Block Ciphers



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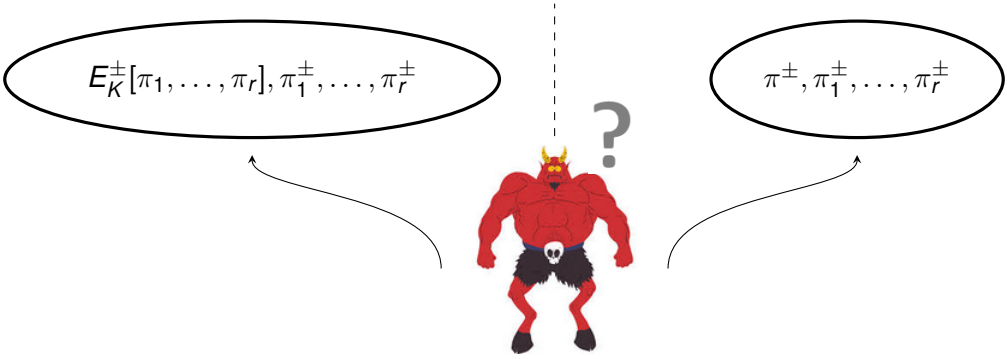

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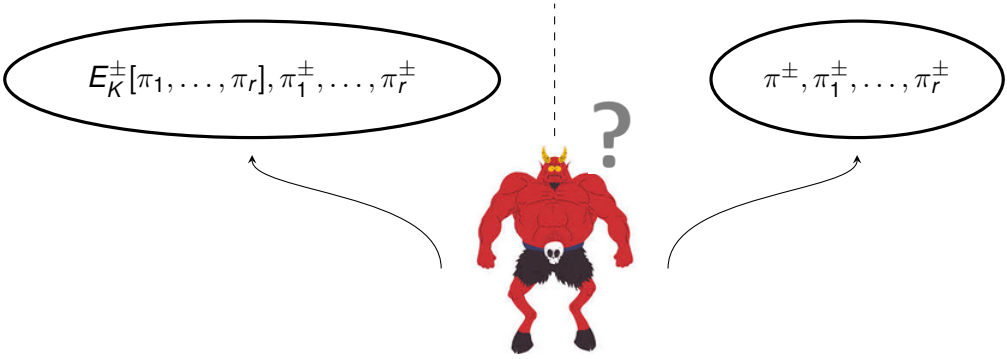

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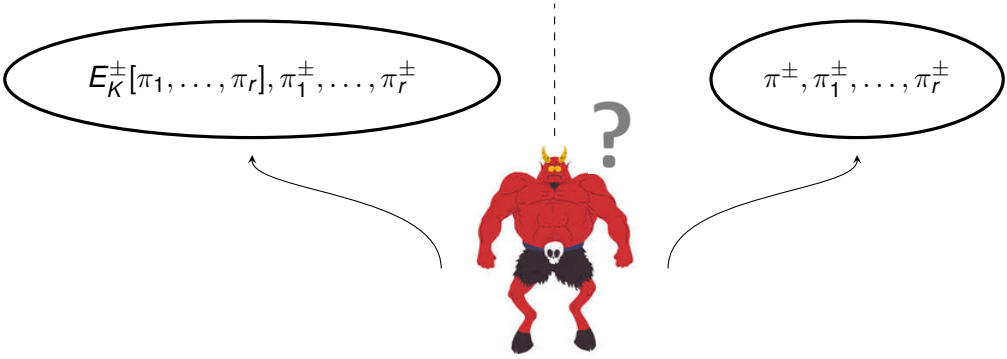

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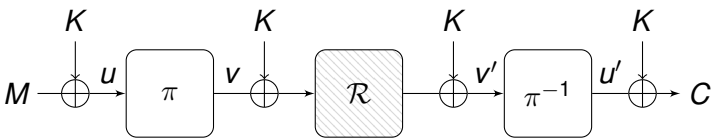
- E_K is secure $\iff$ $\mathbf{Adv}_E^{\text{sprp}}(\mathcal{A})$ is negligible

2-round Key-Alternating Reflection Cipher (Trivial Key Schedule 1)



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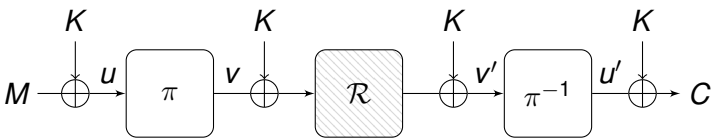

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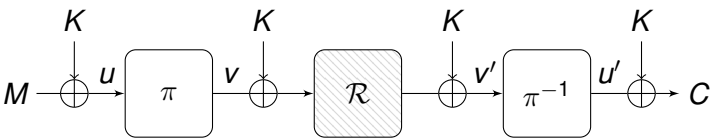

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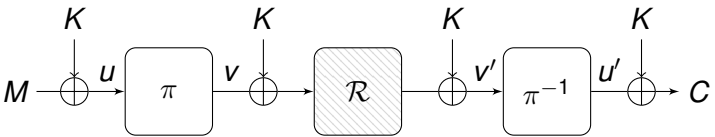

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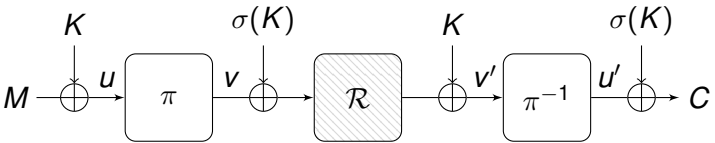

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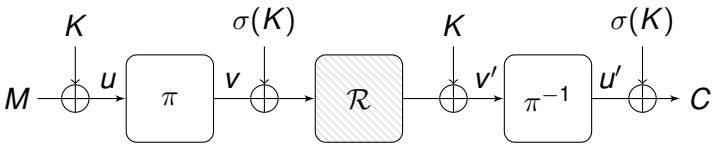

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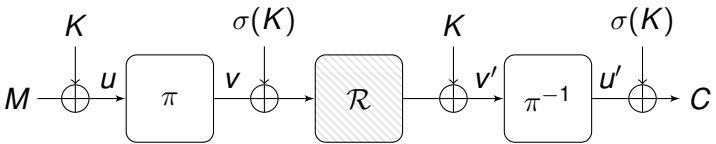

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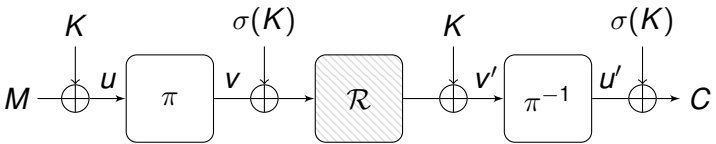

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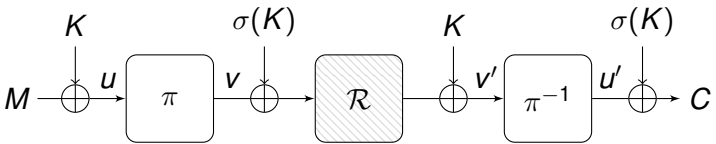

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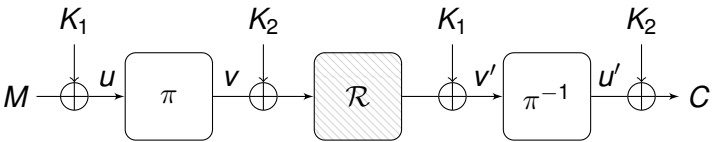
Security of 2-round Key-Alternating Reflection Cipher



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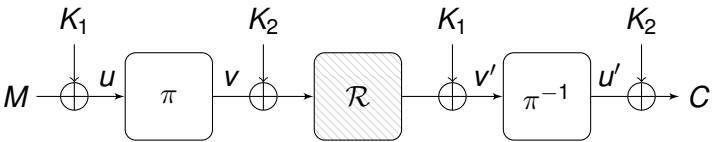

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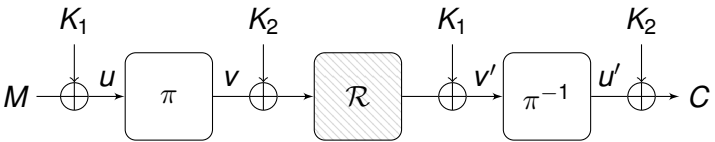

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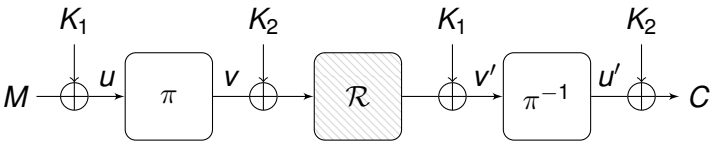

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- Linear reflector $\mathcal{R}$
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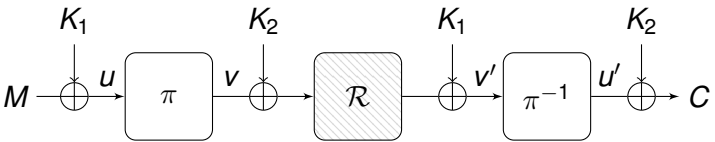
- Decryption = encryption uses swapping keys K_1 and K_2

Information-theoretic attack for $qp^2/2^{2n}$



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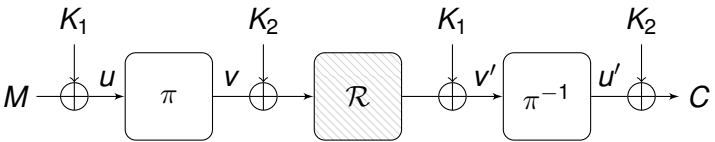

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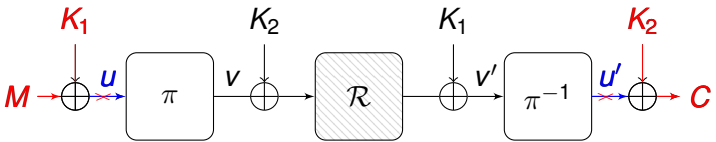

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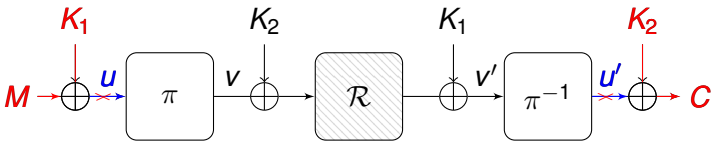

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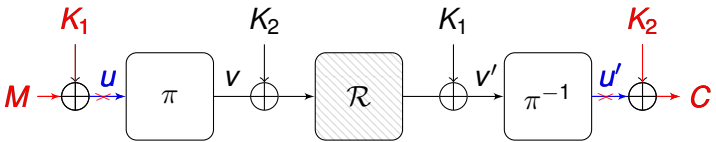
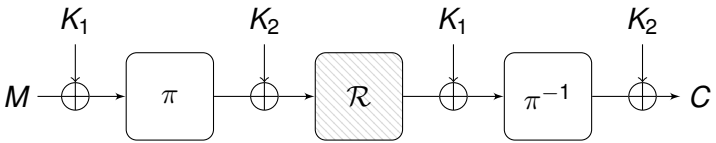
- For each pair, check if $\mathcal{R}(v) \oplus v' = K_1 \oplus \mathcal{R}(K_2)$

Information-theoretic attack for $qp^2/2^{2n}$



- $2q$ construction queries and p primitive queries such that $p^2q = 2^{2n}$
- For each possible pair K_1 and K_2
- Find a pair primitive queries such that $M \oplus K_1 = u$ and $C \oplus K_2 = u'$
- For each pair, check if $\mathcal{R}(v) \oplus v' = K_1 \oplus \mathcal{R}(K_2)$
- Attack uses $O(2^{2n})$ table lookups $\rightarrow$ impractical

Practical attack for $q^2/2^n$



Mirror slide attack of Dunkelman, Keller and Shamir (EC 2012)

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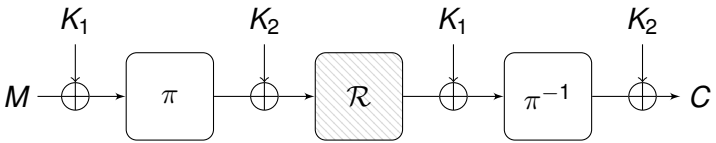

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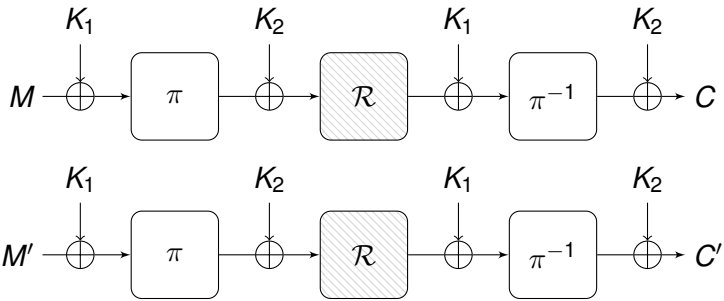

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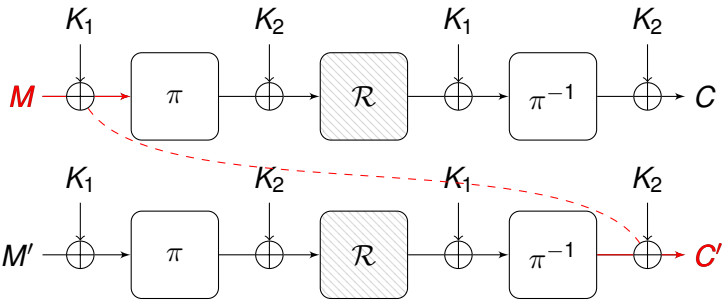

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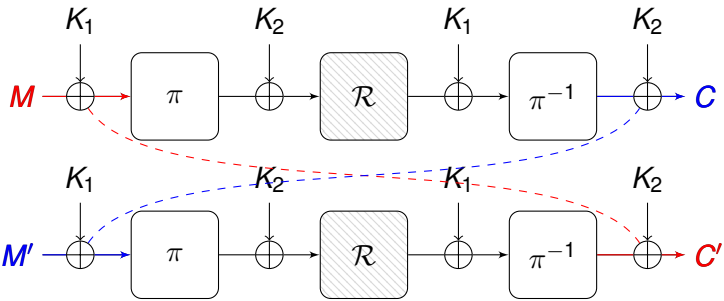

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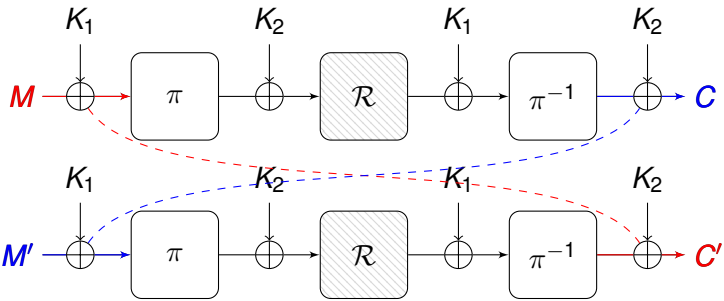

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- Find a pair of construction queries such that $M \oplus C' = K_1 \oplus K_2 = M' \oplus C$
- Recovers the value of $K_1 \oplus K_2$

Security Analysis

- Patarin's H-coefficient technique

$$\frac{\Pr(X_{\mathcal{O}} = \tau)}{\Pr(X_{\mathcal{P}} = \tau)} \geq 1 - \epsilon$$

$$\mathbf{Adv}(\mathcal{A}) \leq \epsilon + \Pr(X_{\mathcal{P}} \in \mathcal{T}_{\text{bad}})$$

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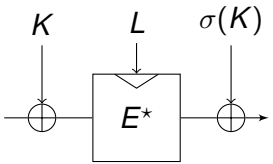
- Ideal permutation model $\rightarrow \mathcal{A}$ has access to π
- Single permutation case $\rightarrow$ domain separation needed
- Domain separation is covered by a bad event $\rightarrow$ leads to $q^2/2^n$ term

Key-Length Extension



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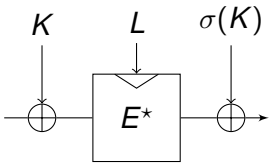
Key-Length Extension



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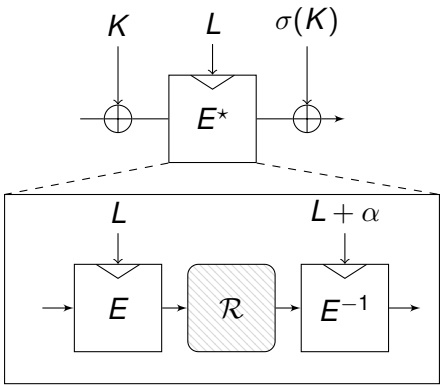

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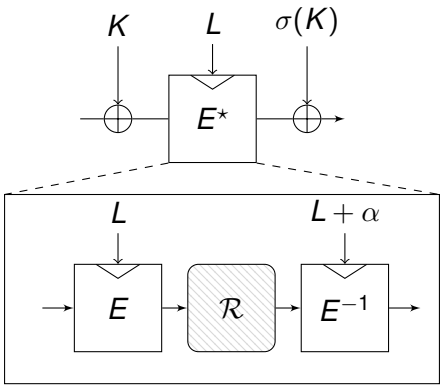

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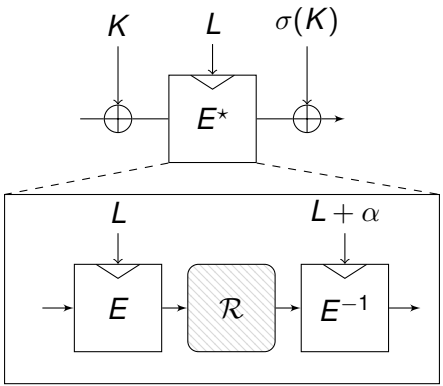

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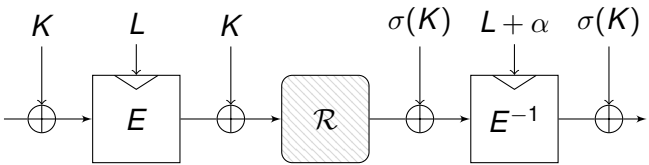
- PRINCE v2: alternating round keys + modify the reflector

Our New Key-Length Extender



$$\mathbf{Adv}^{\mathrm{sprp}} \leq p\sqrt{q}/2^{2n}$$

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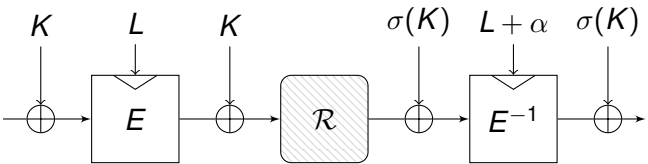

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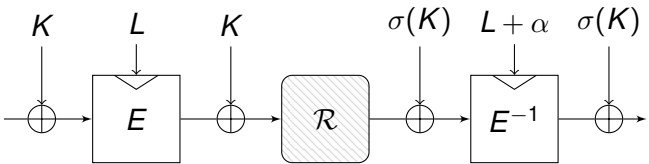

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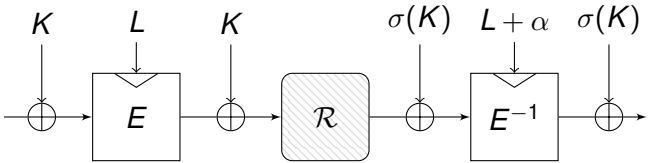

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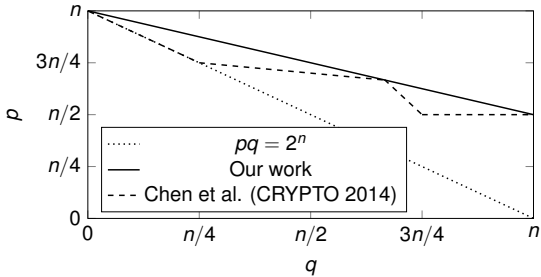

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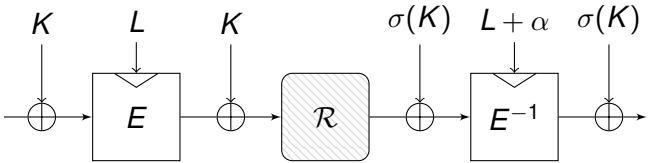

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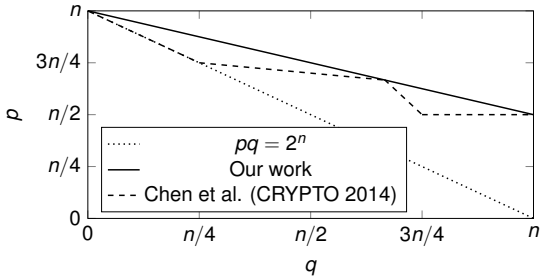
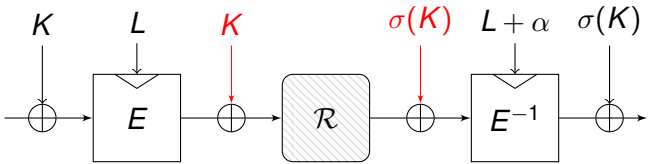


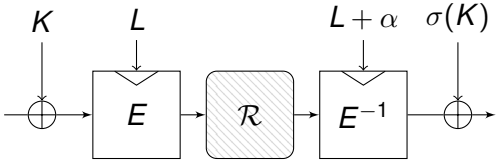
Our New Key-Length Extender: Comparison With FX

Our Construction



$\mathbf{Adv}^{\mathrm{sprp}} \leq p^2 q / 2^{256}$

FX



$\mathbf{Adv}^{\mathrm{sprp}} \leq pq / 2^{128}$

Our New Key-Length Extender: Comparison With PRINCE v2

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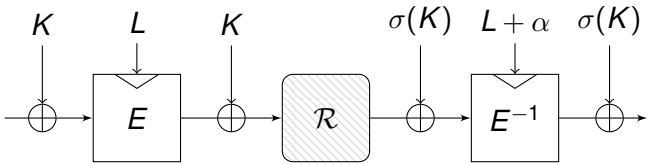
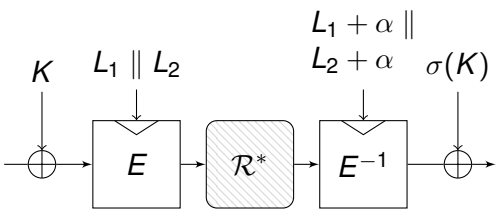


Minimalist change from PRINCE

PRINCE v2



Two alt keys + $\mathcal{R}^*$ does not preserve refl. property

Conclusion

New results

- First generic treatment of reflection ciphers
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I am looking for a new job position $\rightarrow$ yulongchen92@gmail.com