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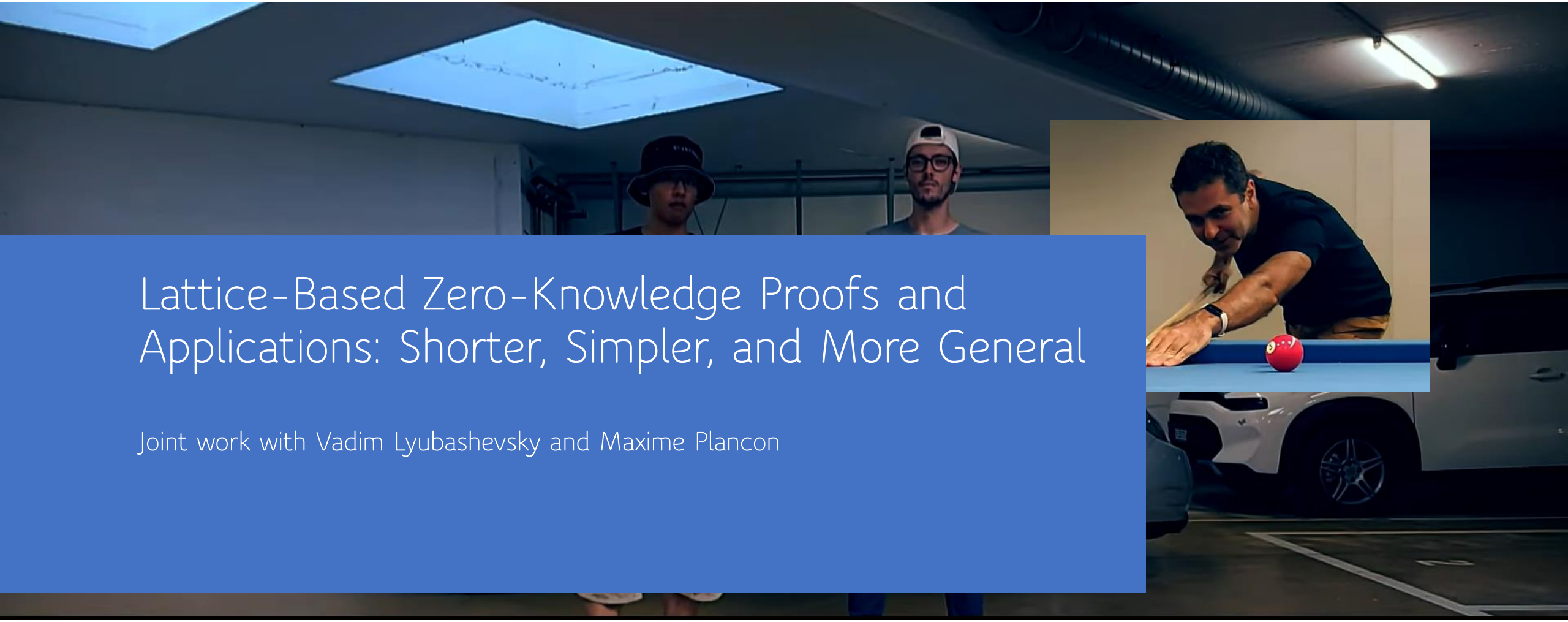

Lattice-Based Zero-Knowledge Proofs and Applications: Shorter, Simpler, and More General

Joint work with Vadim Lyubashevsky and Maxime Plancon

Zero-knowledge proofs

Zero-knowledge proofs

statement

"I got even funnier lines in the full version of the song!"

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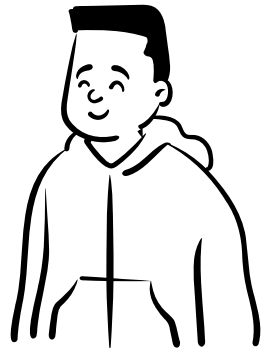
Prover

Verifier



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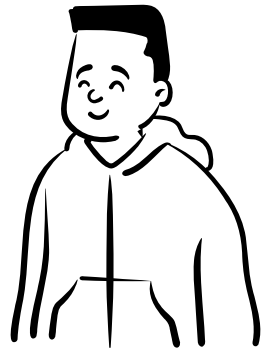

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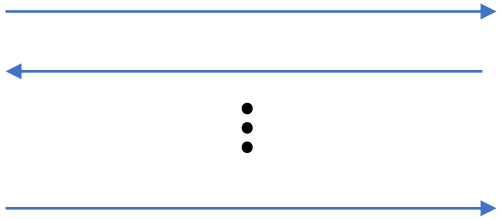

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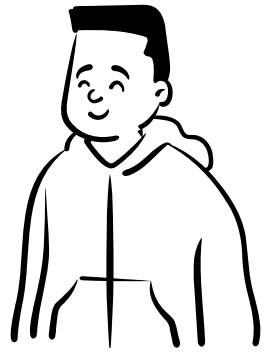

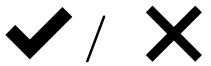

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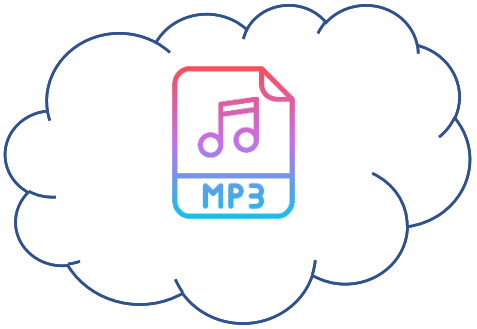

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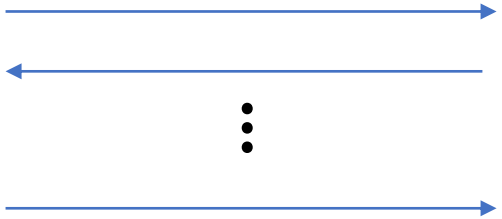

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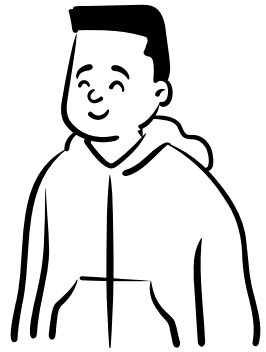

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if the witness is valid, the verifier accepts

Soundness:
if the witness is invalid, the verifier rejects

Zero-knowledge:
the verifier does not learn anything about the witness from the interaction

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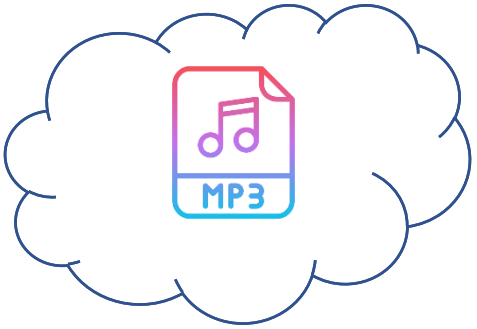

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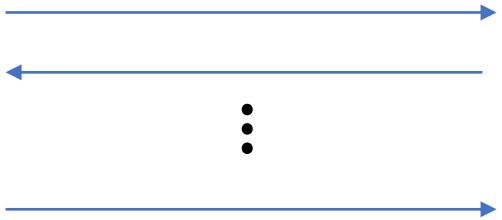

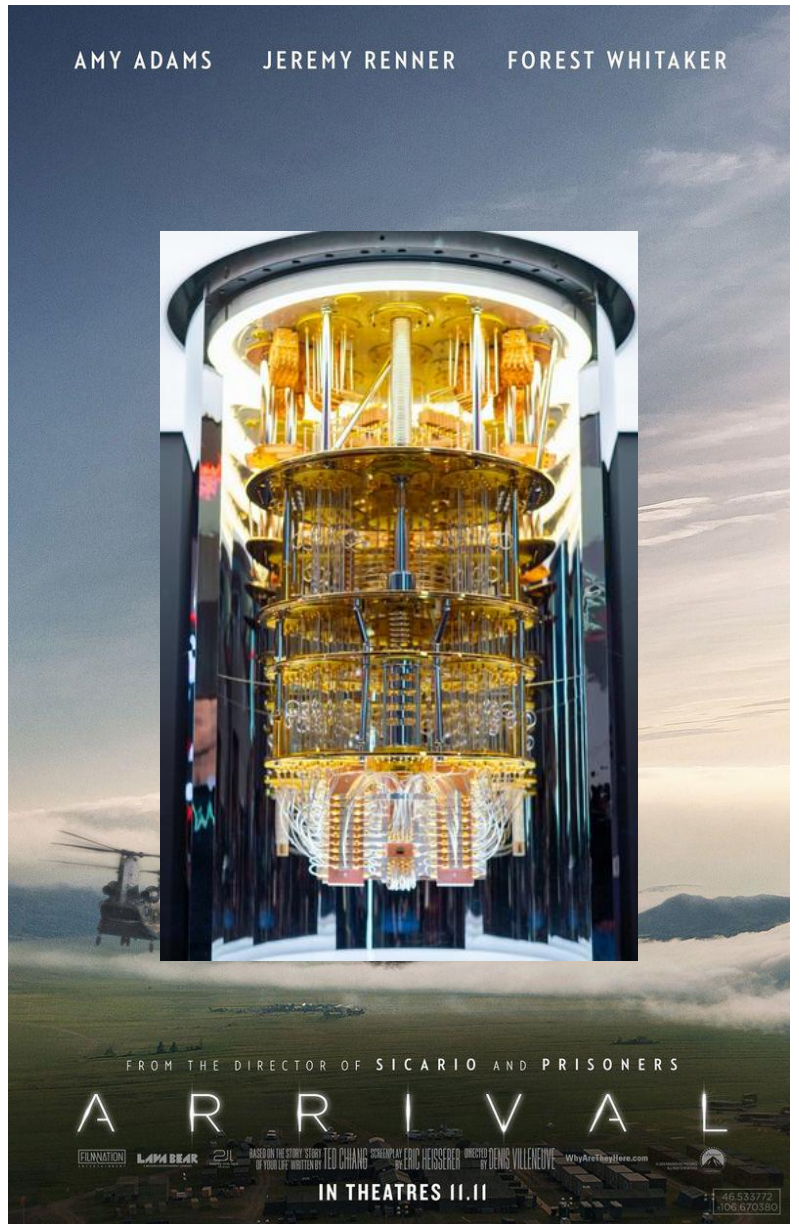

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Current state-of-the-art of Quantum-Safe ZK

Scheme	Structure	Asymptotic proof size (witness size = N)	Concrete proof size $N \approx 2^{20}$	Prover runtime
Ligero [AHIV17]	Hash functions	$O(\sqrt{N})$	9MB	38s
Aurora [BCR+19]	Hash functions	$O(\log^2 N)$	170KB	304s
Fractal [COS20]	Hash functions	$O(\log^2 N)$	215KB	184s
[BLNS20]	Lattices	$O(N^{1/d})$	-	-
Lattice Bulletproofs [BLNS20,AL21,ACK21]	Lattices	$O(\log^2 N)$	-	-

Sizes and runtimes taken from [ACMLT22,ISW21, NS22]

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[ACLMT22]	Lattices	$polylog(N)$	32MB	-

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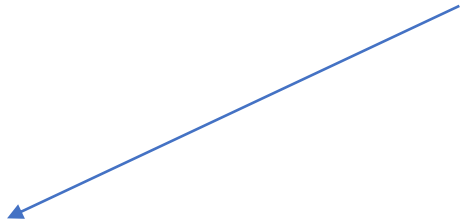
Up next!

Possible research directions (in the lattice world)



Construct succinct (e.g. logarithmic-sized) ZK proofs

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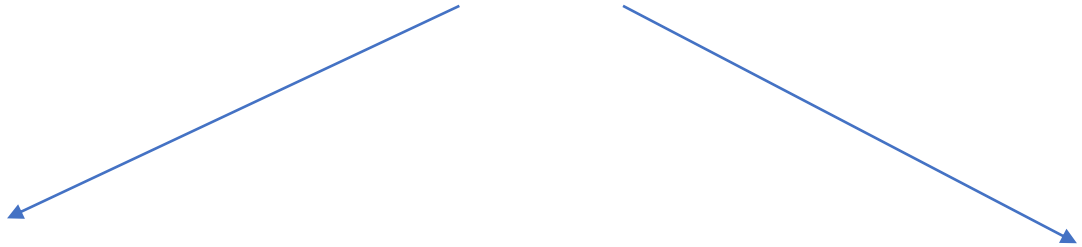

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In this talk

What are the interesting statements?

$$\boldsymbol{As = u}$$

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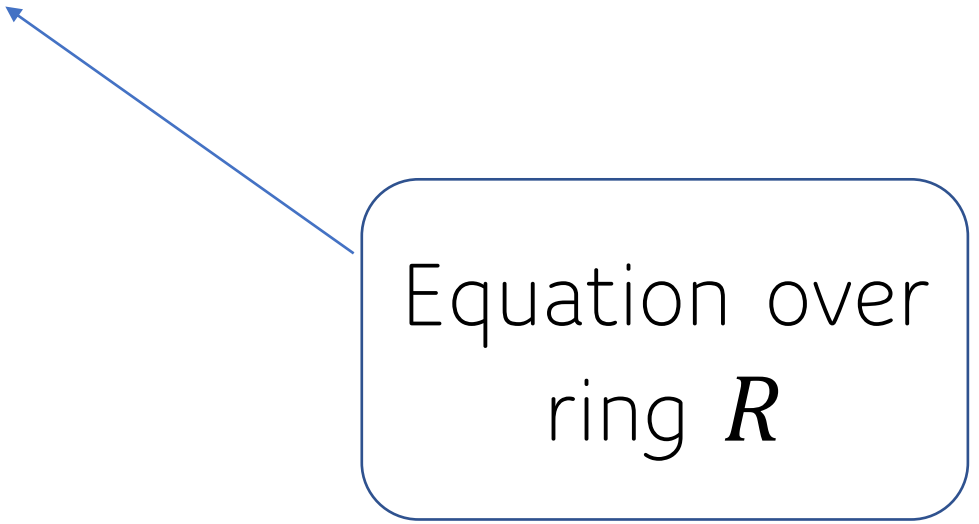

What are the interesting statements?

$$\boldsymbol{As = u}$$

Vector $\boldsymbol{s}$ has *small* coefficients e.g. {-1,0,1}

Equation over ring $\boldsymbol{R}$

Exact proofs

Benchmark: prove $\boldsymbol{As} = \boldsymbol{u}$ where $s_i \in \{-1,0,1\}$ and $A \in \mathbb{Z}_q^{1024\times 2048}$ for $q \approx 2^{32}$.

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NTT/CRT-packing proofs

Scheme	Proof size
[BLS19,YAZ+19]	384KB
[ALS20,ENS20]	47KB
[LNS21]	33KB

Sizes taken from [ENS20,LNS21]


it's time for some technical overview

Exact proofs via NTT packing

- Consider the standard polynomial ring $R_q = \mathbb{Z}_q[X]/(X^d + 1)$ where d is a power-of-two and $q = 1 \, (mod \, 2d)$.
- Then, we can write:

$$X^d + 1 = (X - r_1)(X - r_2) \ldots (X - r_d) \, mod \, q.$$

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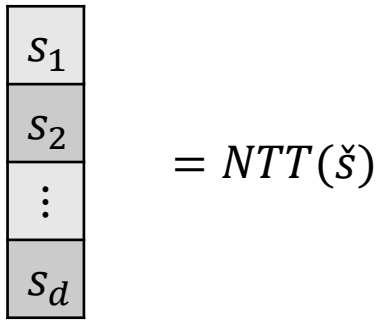
Given a polynomial $a = a_0 + a_1 X + \ \ldots + a_{d-1} X^{d-1} \in R_q$, define $\mathbf{NTT}(a)$ as the vector $\hat{a} = \left(a(r_1), \ldots, a(r_d)\right) \in \mathbb{Z}_q^d$.

By definition, $\mathbf{NTT}(ab) = \mathbf{NTT}(a) \circ \mathbf{NTT}(b)$.

Exact proofs via NTT packing

We want to prove $\boldsymbol{s} \in \{0,1\}^d$.



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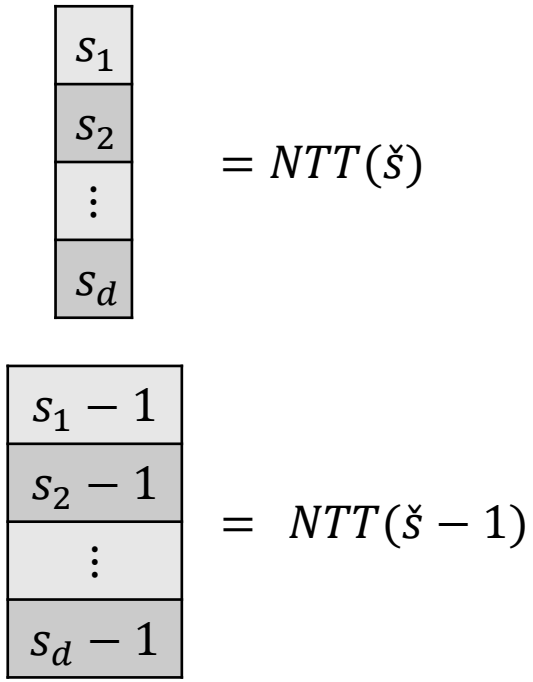

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$$\begin{bmatrix} 0 \\ 0 \\ \vdots \\ 0 \end{bmatrix} = \begin{bmatrix} s_1 - 1 \\ s_2 - 1 \\ \vdots \\ s_d - 1 \end{bmatrix} \circ \begin{bmatrix} s_1 \\ s_2 \\ \vdots \\ s_d \end{bmatrix} \quad = NTT(\check{s}) \circ NTT(\check{s} - 1) = NTT(\check{s}(\check{s} - 1))$$

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We want to prove $\check{s}(\check{s} - 1) = 0$.

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Note that $\check{s}$ has large coefficients.

Hence, we commit to it using the [BDLOP18] homomorphic commitment.

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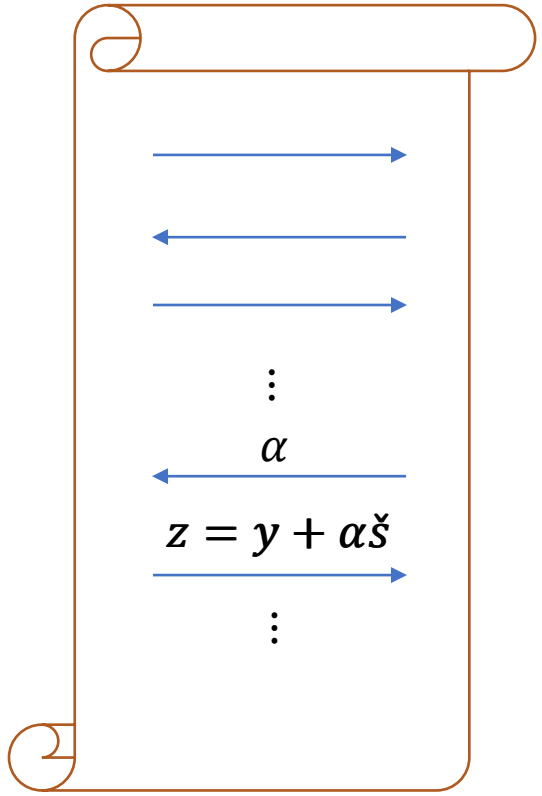

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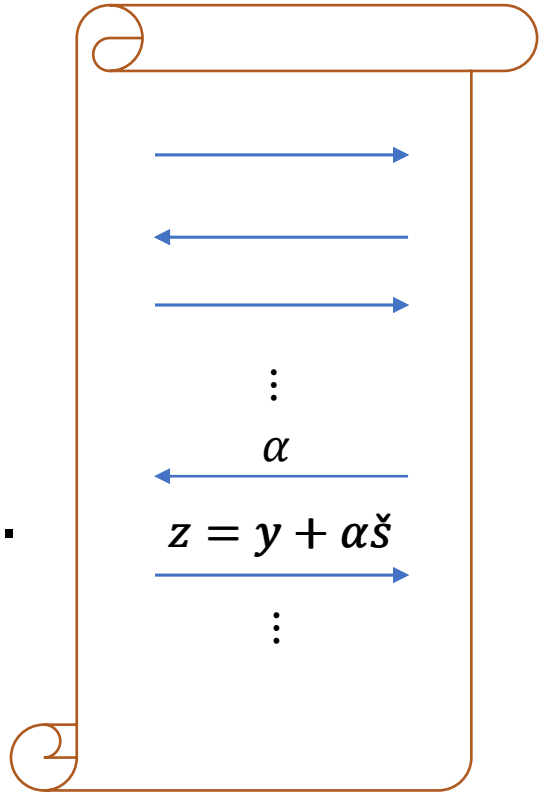
The verifier can thus compute:

$$z(z-\alpha) = y^2 + \alpha\cdot(2\check{s}y - y) + \alpha^2\cdot\check{s}(\check{s}-1).$$



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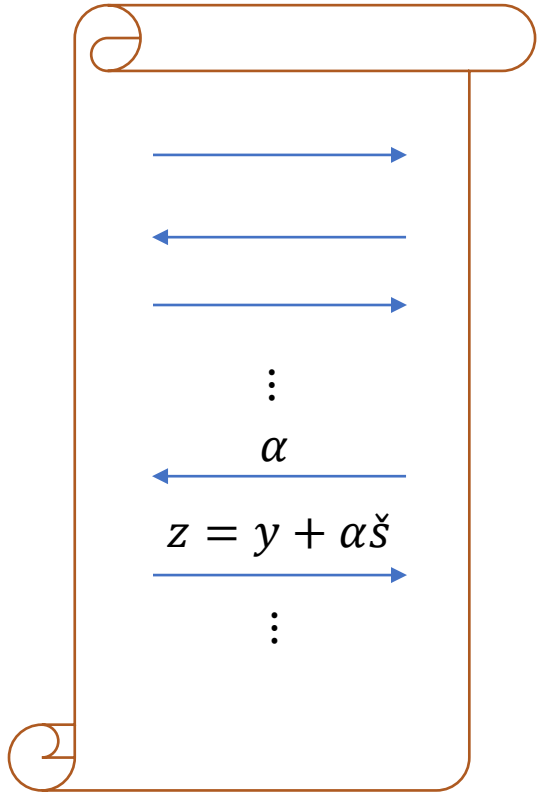

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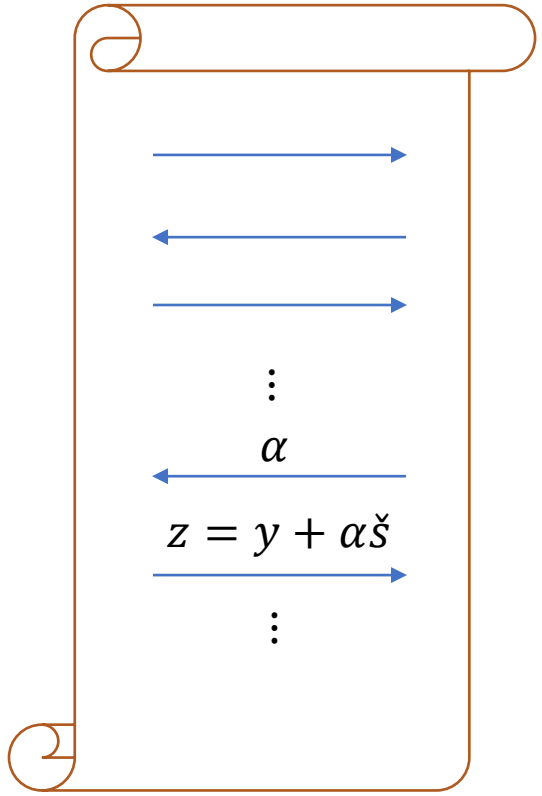
3. Send commitments

   $t_0 = Com(y^2)$ and $t_1 = Com(2\check{s}y - y)$.



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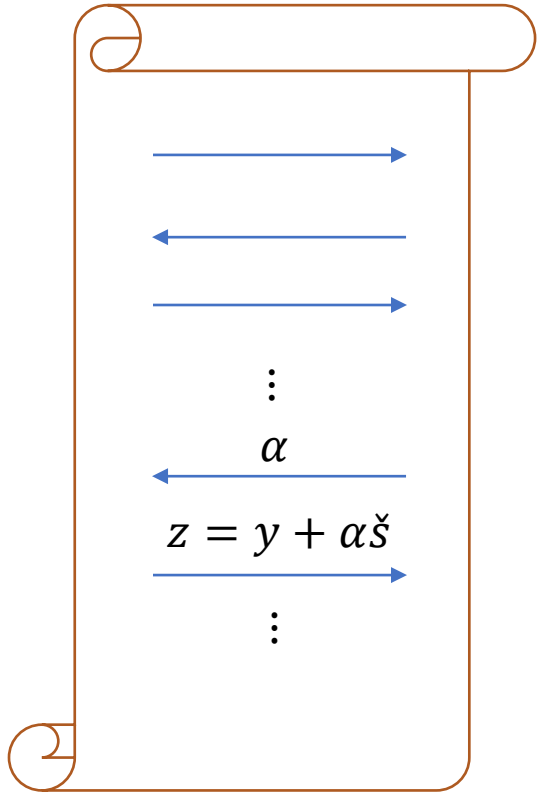
4. Given a challenge α, output $z = y + \alpha\check{s}$.



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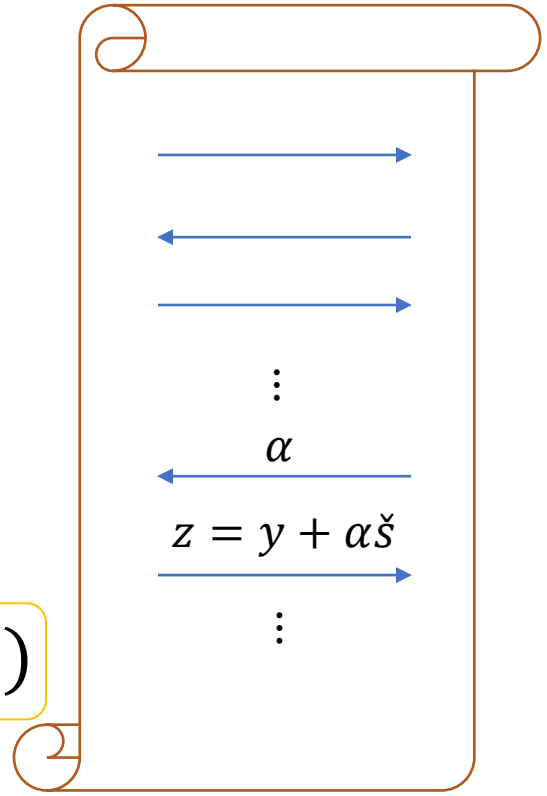
5. Prove: (i) $z - (t_y + \alpha t_s)$ and (ii) $z(z-\alpha) - (t_0 + \alpha t_1)$

are commitments to zero.



Exact proofs via NTT packing

Proving linear relations, i.e. $\boldsymbol{As = u}$:

$$z = y + \alpha\check{s}$$

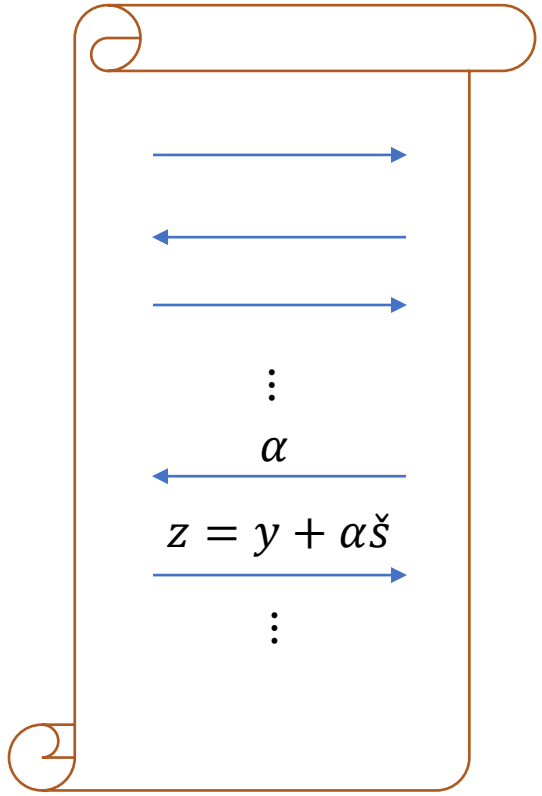
$$\downarrow \quad \alpha \in \mathbb{Z}_q$$

$$\hat{z} = \hat{y} + \alpha\boldsymbol{s}$$

$$\downarrow$$

$$A\hat{z} = A\hat{y} + \alpha\boldsymbol{u}$$

Here, $\hat{x} = NTT(x)$.



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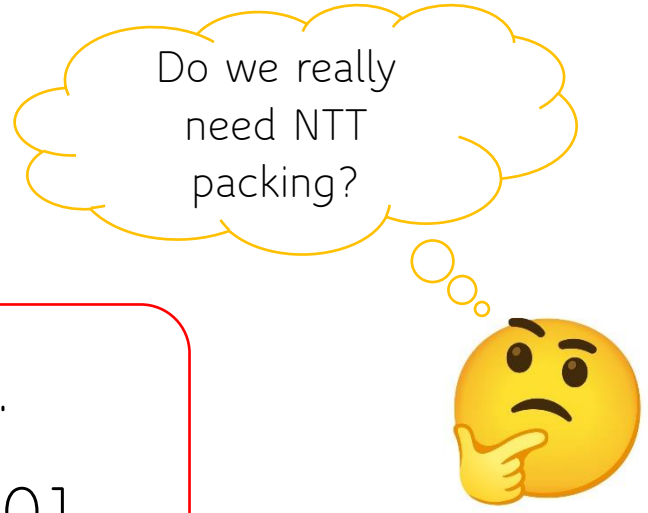

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Lattice-based zero-knowledge proofs aye!

- Does not rely on NTT packing
- (Almost) one-shot
- Compressing commitment

Roadmap

ABDLOP commitment

- It combines the Ajtai [Ajt96] and BDLOP [BDLOP18] commitments into one.
- It puts the long commitment to $\boldsymbol{s}$ into the "Ajtai" part of the commitment scheme.
- The BDLOP part of the commitment scheme is then used for low-dimensional auxiliary elements that will need to be committed to later in the protocol.

Roadmap

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"are you still with me?"

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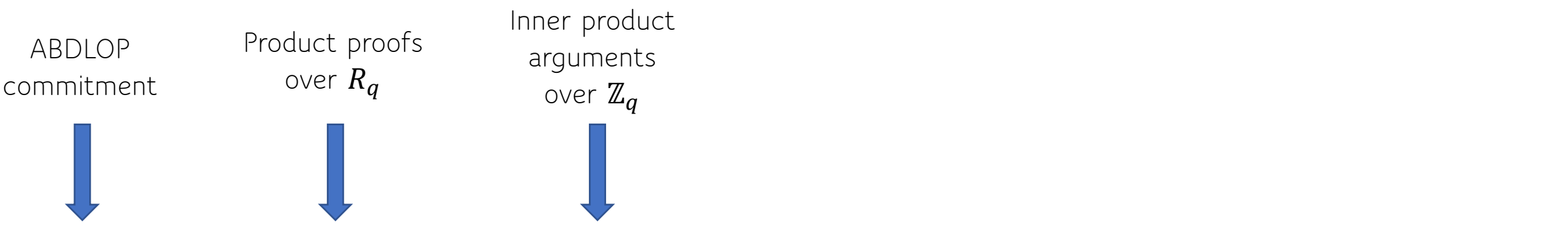
ABDLOP commitment

Product proofs over R_q

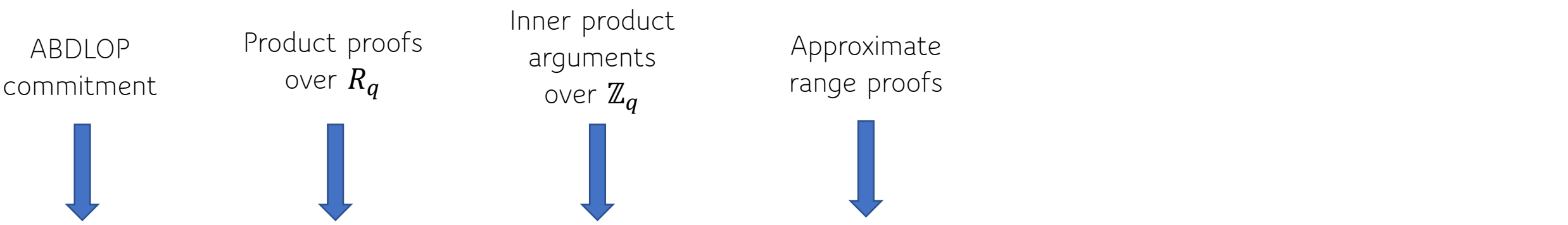
- Simple adaption of the [ALS20] protocol.
- It can be used to prove product relations, e.g. $\boldsymbol{s^T s = 0}$.
- Extended to also prove quadratic equations involving R_q-automorphisms, e.g. $\boldsymbol{s^T \sigma(s) = 0}$.

Roadmap



- We prove inner products over $\mathbb{Z}_q$, e.g. $\langle \boldsymbol{s}, \boldsymbol{v} \rangle = 0$ or $\langle \boldsymbol{s}, \boldsymbol{s} \rangle = B$.
- Fact: There is an automorphism σ such that $\langle \boldsymbol{x}, \boldsymbol{y} \rangle \in \mathbb{Z}_q$ is the constant coefficient of the polynomial $\boldsymbol{x}^T \sigma(\boldsymbol{y})$.
- Proving the const. coeff. of a polynomial is zero: [ENS20] + Product proof (with automorphisms) over R_q.

Roadmap



- We know how to prove equations modulo q.
- But how to prove relations **over integers**?
- $||s||^2 = \langle s, s \rangle = B \ (mod\ q)$ and s *approximately* small coefficients $\Longrightarrow$ $||s||^2 = B$ over integers!

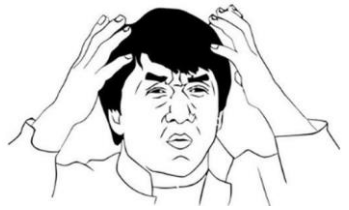
Roadmap

ABDLOP commitment

Product proofs over R_q

Inner product arguments over $\mathbb{Z}_q$

Approximate range proofs



Applications

- Proving knowledge of a Module-LWE sample

Scheme	Proof size
Stern proofs (e.g. [Ste93,LNSW13])	3MB
[Beu20]	233KB
[BLS19,YAZ+19]	384KB
Ligero [AHIV17]	157KB
Aurora [BCR+19,BCOS20]	72KB
[ALS20,ENS20]	47KB
[LNS21]	33KB
This work	14KB

"Proof size so small, made my mom impressed!"

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More applications in the paper…

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Thank you!

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