Triangulating Rebound Attack on AES-like Hashing

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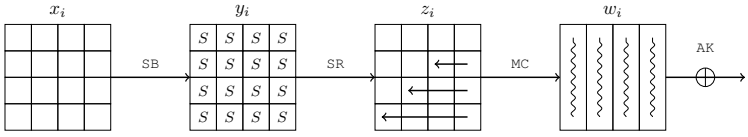
- Improve Rebound Attack with enlarging the Inbound phase to SuperInbound (multiple Inbound phases)
- Solve the system of equations of SuperInbound by an automatic way with Triangulation Algorithm
- Apply new method to obtain new semi-freestart collision attacks on AES-like Hashings
- Find the first 6-round semi-freestart collision of AES-128-MMO

AES-like Hashing



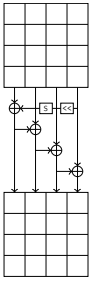


Figure 1: The AES round-function and key-schedule

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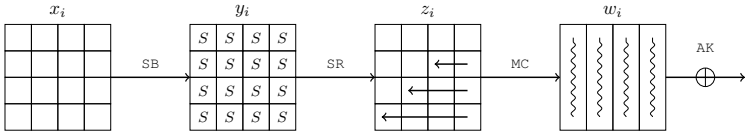

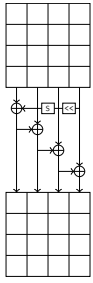

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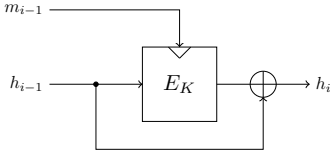
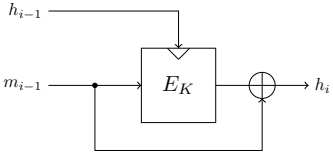
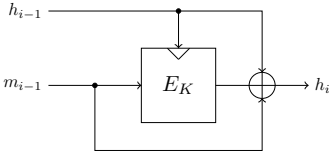


(DM) Davies-Meyer



(MMO) Matyas-Meyer-Oseas



(MP) Miyaguchi-Preneel

AES System of Equations

$$R_i : \begin{cases} y_i \oplus S(w_{i-1} \oplus k_i) = 0 \\ w_i \oplus \begin{pmatrix} 02 & 03 & 01 & 01 \\ 01 & 02 & 03 & 01 \\ 01 & 01 & 02 & 03 \\ 03 & 01 & 01 & 02 \end{pmatrix} \times \begin{pmatrix} y_i[0] & y_i[4] & y_i[8] & y_i[12] \\ y_i[5] & y_i[9] & y_i[13] & y_i[1] \\ y_i[10] & y_i[14] & y_i[2] & y_i[6] \\ y_i[15] & y_i[3] & y_i[7] & y_i[11] \end{pmatrix} = 0 \end{cases} \tag{1}$$

$$KS_i : \begin{cases} k_i[j] \oplus k_i[j-4] \oplus k_{i-1}[j] = 0, & j = 4, \ldots b-1 \\ k_i[0] \oplus k_{i-1}[0] \oplus S(k_{i-1}[b-3]) \oplus \mathbf{RCON}_i = 0 \\ k_i[1] \oplus k_{i-1}[1] \oplus S(k_{i-1}[b-2]) = 0 \\ k_i[2] \oplus k_{i-1}[2] \oplus S(k_{i-1}[b-1]) = 0 \\ k_i[3] \oplus k_{i-1}[3] \oplus S(k_{i-1}[b-4]) = 0 \end{cases} \tag{2}$$

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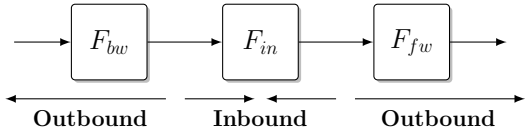
- Quantum complexity without quantum memory: Rho algorithm with $2^{n/2}$

Rebound Attack



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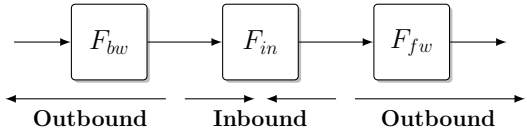

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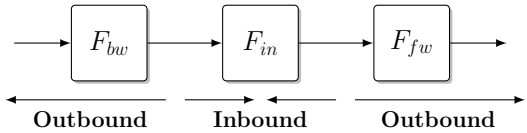

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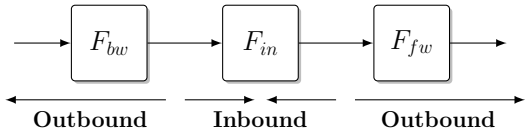

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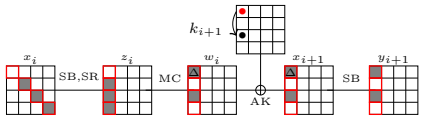
  - **Complexity**: p is the probability of outbound trail, then the complexity is roughly $1/p$ in classic setting with $p > 2^{-n/2}$, $1/\sqrt{p}$ in quantum setting $p > 2^{-2n/3}$ with quantum memory, or $p > 2^{-n}$ without quantum memory

Connecting multiple inbound parts



- Active bytes of x_i, x_{i+1} are known, random $k_{i+1}[0] \rightarrow k_{i+1}[2]$ is computed.

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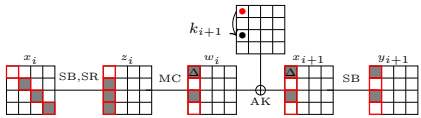

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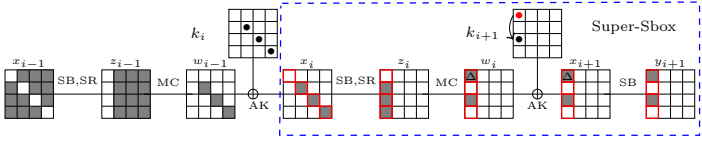

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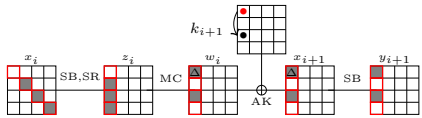

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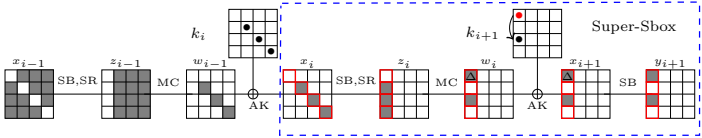

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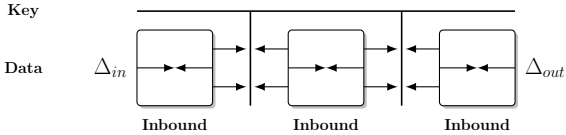
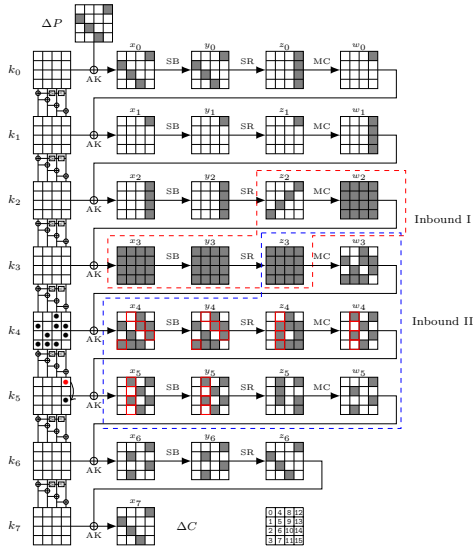


Figure 2: Super-Inbound: bridging multiple local inbound parts

New 7-round AES-128-MMO trail

- Choose the valid differences for z_2, w_3, w_4, w_5
- Assess differential distribution table (DDT) to obtain values of 31 active bytes of x_3, x_4, x_5
- Use k_4, k_5 to connect these known bytes
- Outbound probability: 2^{-56} for MC $w_1 \rightarrow z_1$ and $\Delta_P = \Delta_C$
- Complexity: 2^{56}



Triangulation Algorithm

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- Automatically detect a way to solve the nonlinear system.
- Models an AES-like block cipher as a system of key schedule and round function equations, and the state bytes and key bytes as variables.

Triangulation Algorithm

$$\begin{cases} F(x_1 \oplus x_2) \oplus 2 \times a & = 0 \\ x_1 \oplus 3 \times x_3 \oplus G(x_4) & = 0 \\ x_2 \oplus x_4 \oplus a & = 0 \end{cases}$$

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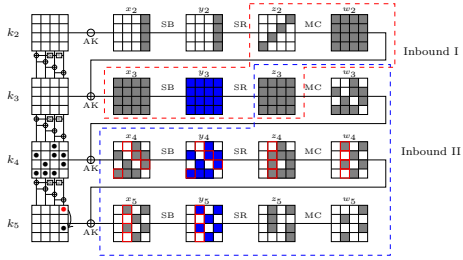
4. Return to Step 3 if there still exist non-processed equations.
5. Return all non-processed variables as "free variables".

Triangulation Rebound Attack



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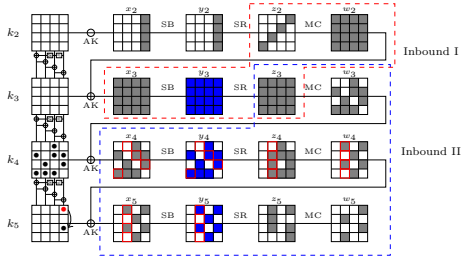

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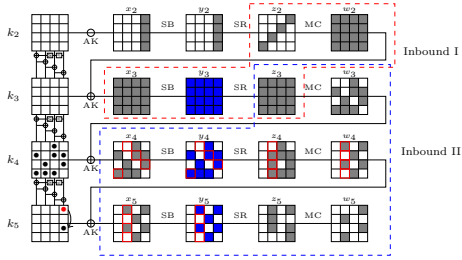

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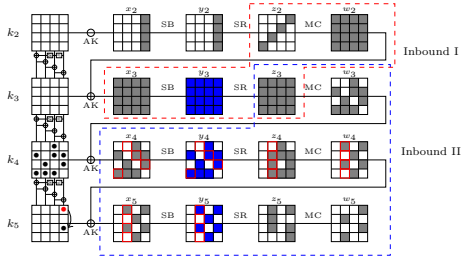

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4. For AES, a basis of $n + k$ variables, if m bytes are fixed, the algorithm should output $n + k - m$ bytes

Comparison with previous work

Previous work:

- Inbound pairs are found manually [GP10, LMR$^+$09, Nay11]
- Only quantum collision attack on 7-round AES-hash [HS]
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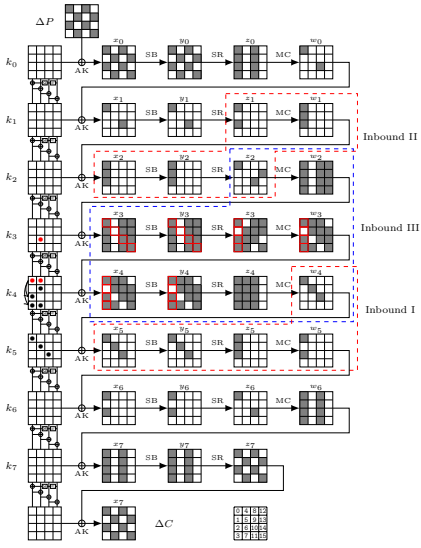
Our work:
- Inbound part is opened to **3 to 4** rounds for AES
- The first semi-free-start collision attack on 7-round AES-MMO in classic setting
- Automatic way to find the inbound pairs with Triangulation algorithm

New 8-round AES-128-MMO trail

- 30 active bytes in x_2, x_3, x_4, x_5 are known due to DDT.
- Use k_3, k_4, k_5 to connect these known bytes
- Triangulation algorithm output 3 free bytes: $k_3[6], k_4[0,4]$ and 1 filter
- Outbound probability: 2^{-64} for $\Delta_P = \Delta_C$ and 1 filter 2^{-8}
- Complexity: 2^{34} in quantum setting



First 6-round semi-freestart collision on AES-128-MMO

- By cutting the first and last round of 8-round trail

Plaintext			
P	13622301 f4ad7096 c7cea69e e26646e5	K_{-1}	6aa0c09f 92854210 9411daed 8db5f736
P'	eb622301 f4ad7096 c7cedd9e e26646e5		
Ciphertext			
C	ede24da9 8f76183b b23536aa 34dbd668		
C'	15e24da9 8f76183b b2354daa 34dbd668		
Hash output after feedback			
H	fe806ea8 7bdb68ad 75fb9034 d6bd90cd		
H'	fe806ea8 7bdb68ad 75fb9034 d6bd90cd		

Table 1: Pair found by semi-free-start collision attack on 6-round AES-128

Attack on SKINNY-hash-128-384

- MC operation is non-MDS:

$$MC\begin{pmatrix} a \\ b \\ c \\ d \end{pmatrix} = \begin{pmatrix} a \oplus c \oplus d \\ a \\ b \oplus c \\ a \oplus c \end{pmatrix}$$

- The values of k_i are the XOR of subkeys and constants of AK operator.
- Linear key-schedule

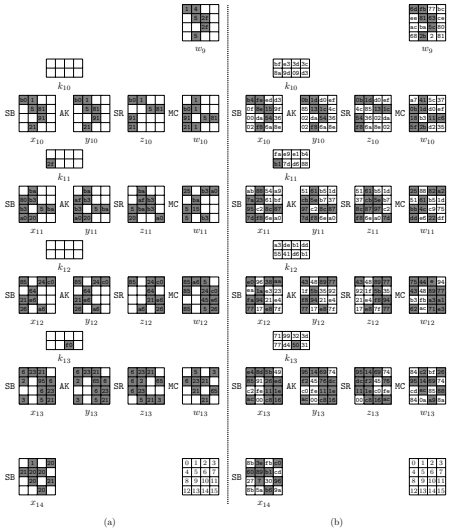


Figure 3: (a) Differences, (b) Values.

Results

Target	Attack	Rounds	Time	C-Mem	qRAM	Setting	Ref.
AES Hash	Collision	6/10	2^{56}	-	0	Quantum	[GP10, LMR+09]
		8/10	$2^{55.53}$	0	0	Quantum	Ours
Compression function AES-128-MMO/MP	Semi-free	6/10	2^{24}	-	-	Classic	Ours
	Semi-free	7/10	2^{56}	2^{16}	-	Classic	Ours
	Semi-free	8/10	2^{34}	-	-	Quantum	Ours
Compression function Saturnin-hash	Semi-free	7/16	2^{86}	-	-	Quantum	Ours
	Free-start	8/16	$2^{122.5}$	-	-	Quantum	[DZS+21]
	Free-start	8/16	$2^{89.65}$	-	-	Quantum	Ours
	Free-start	10/16	$2^{127.2}$	-	-	Quantum	Ours
Compression function Grostl	Semi-free	6/14	2^{180}	2^{64}	-	Classic	[Sch11]
		7/14	2^{214}	-	-	Quantum	
Compression v0	Semi-free	8/14	2^{244}	-	-	Quantum	Ours
Compression function SKINNY-hash	Free-start	16/56	$2^{59.8}$	-		Quantum	[DZS+21]
		19/56	$2^{51.2}$	-		Classic	Ours
		21/56	$2^{46.2}$	-		Quantum	Ours

Table 2: A summary of the results.

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