# Gossiping for Communication-Efficient Broadcast

G. Tsimos, Julian Loss, Charalampos Papamanthou









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• A designated sender s w. input value  $u_s$ • s wants to broadcast  $u_s$  to all n parties s might be dishonest Honest parties want to agree on the same value.

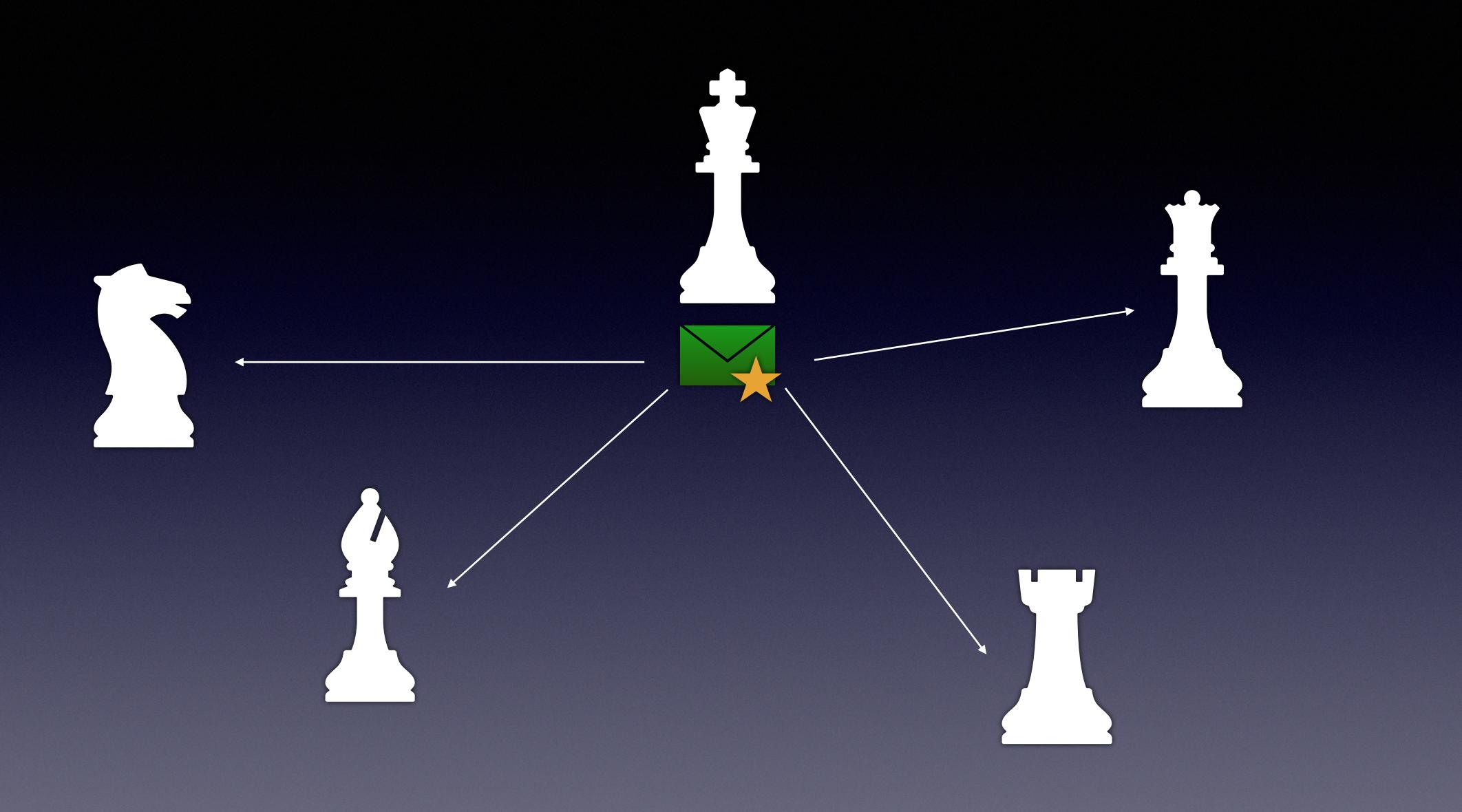
- Authenticated Broadcast:
  - Broadcast **BUT** with Use of a Public Key Infrastructure (PKI) •
    - Bulletin Board \ Trusted PKI
  - Each party can sign with a signature each message they send
    - $P_i$  holds  $(pk_i, sk_i)$  and posts  $pk_i$  publicly











All honest parties output the same message If S is honest, all honest parties output S's message



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  - static/adaptive Adversary

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### Metrics:

### Communication Complexity (CC)

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Amount of bits shared by honest parties

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  - Dolev-Strong protocol [DS'83] with  $O(n^3)$  Communication

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 $\mathcal{O}(n^3)$  CC using **bulletin board PKI**, against  $t < (1 - \epsilon)n$ **adaptive** corruptions.

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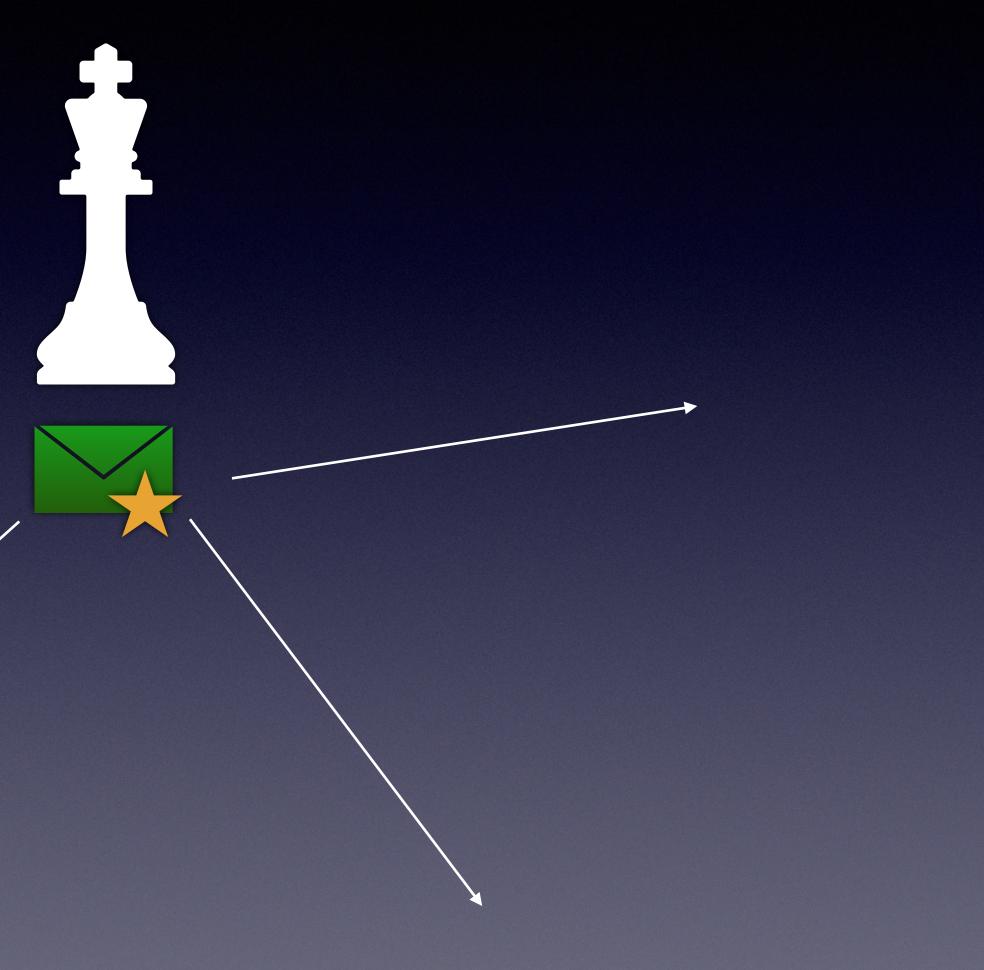
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 $\mathcal{O}(n^3)$  CC using **bulletin board PKI**, against  $t < (1 - \epsilon)n$ adaptive corruptions.

 $\mathcal{O}(n^2)$  CC using **trusted PKI**, against  $t < (1 - \epsilon)n$  adaptive corruptions.

### Dolev-Strong

### S sends $\sim$ with S's signature $\star$ to all parties



### For each $r \leq t + 1$ : p checks if it received some new "valid"





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  - With  $\mathcal{O}(n^3\kappa)$  Communication

# If so, it adds its signature 🗡 and sends



# Our Observation

- What do parties want to achieve with sending?
  - needed for the property.

• Perhaps, sending to everyone takes more communication than what

"Do

### "Do I send the message to party j?"



"Do If ⊢

#### "Do I send the message to party j?"

Flip a coin with prob. m/nIf Heads, then I send, else I don't

# Our Idea for BC

- Gossiping: •

  - (Ofc, this doesn't work single-shot.) Takes  $\sim O(\log n)$  rounds.

• Each honest party picks randomly  $\sim O(\log n)$  other parties to send to.

 $\bullet \quad \bullet \quad \bullet$ 





r=a<t+1

#### 



r=a+1

#### 

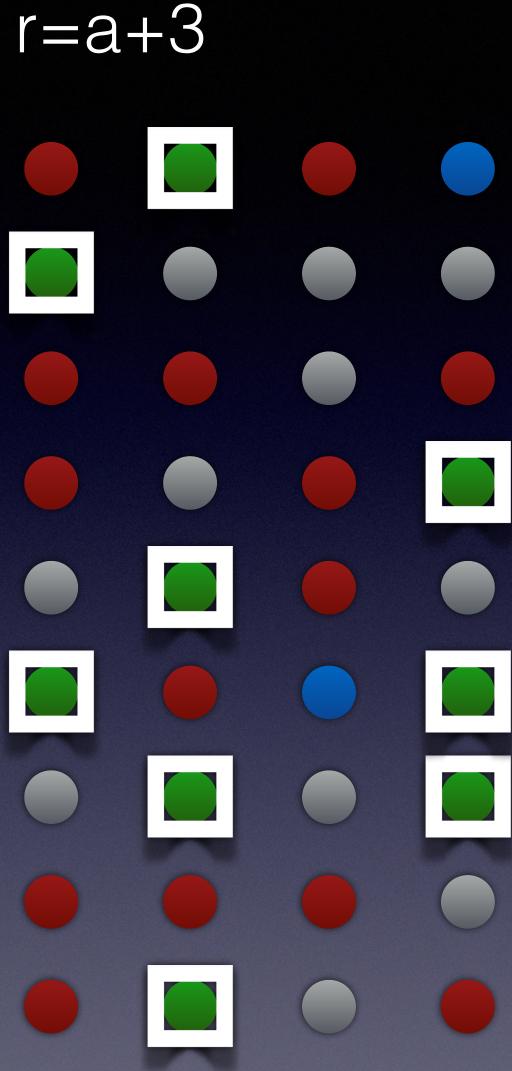
## 

S

r=a+2

#### 











r=a+5



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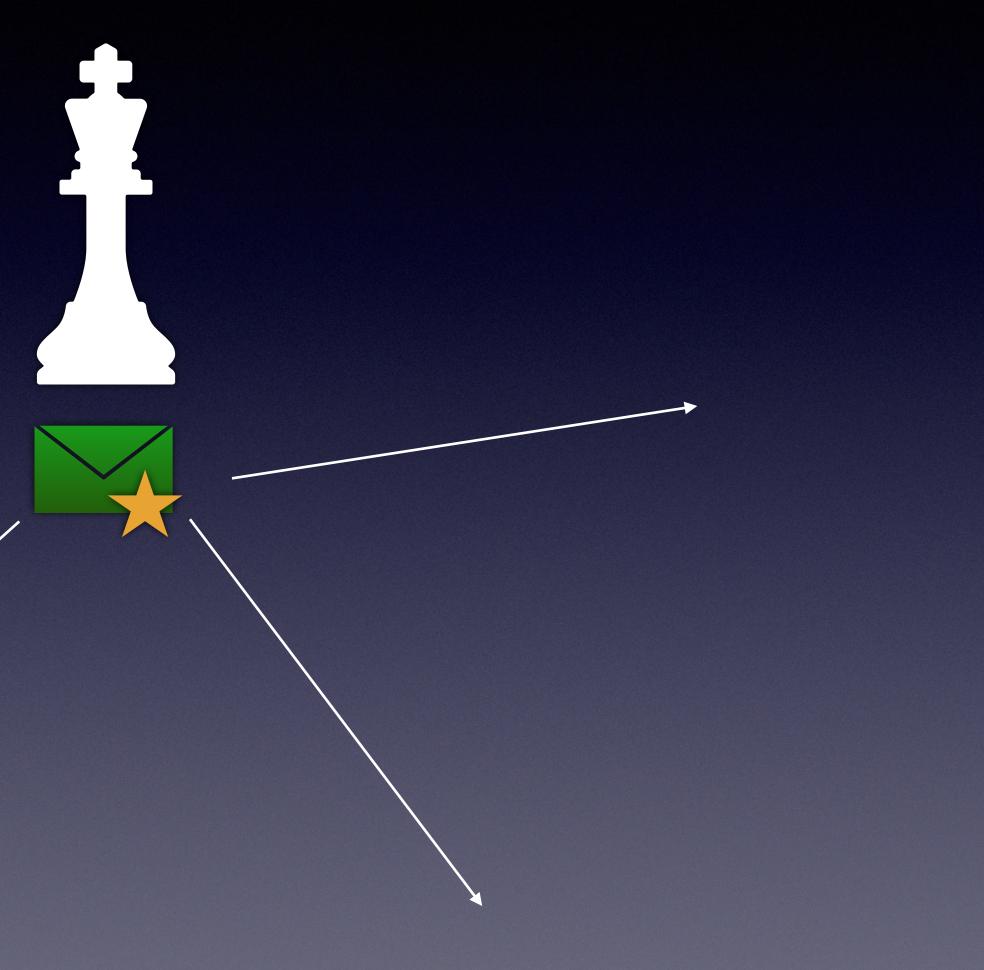
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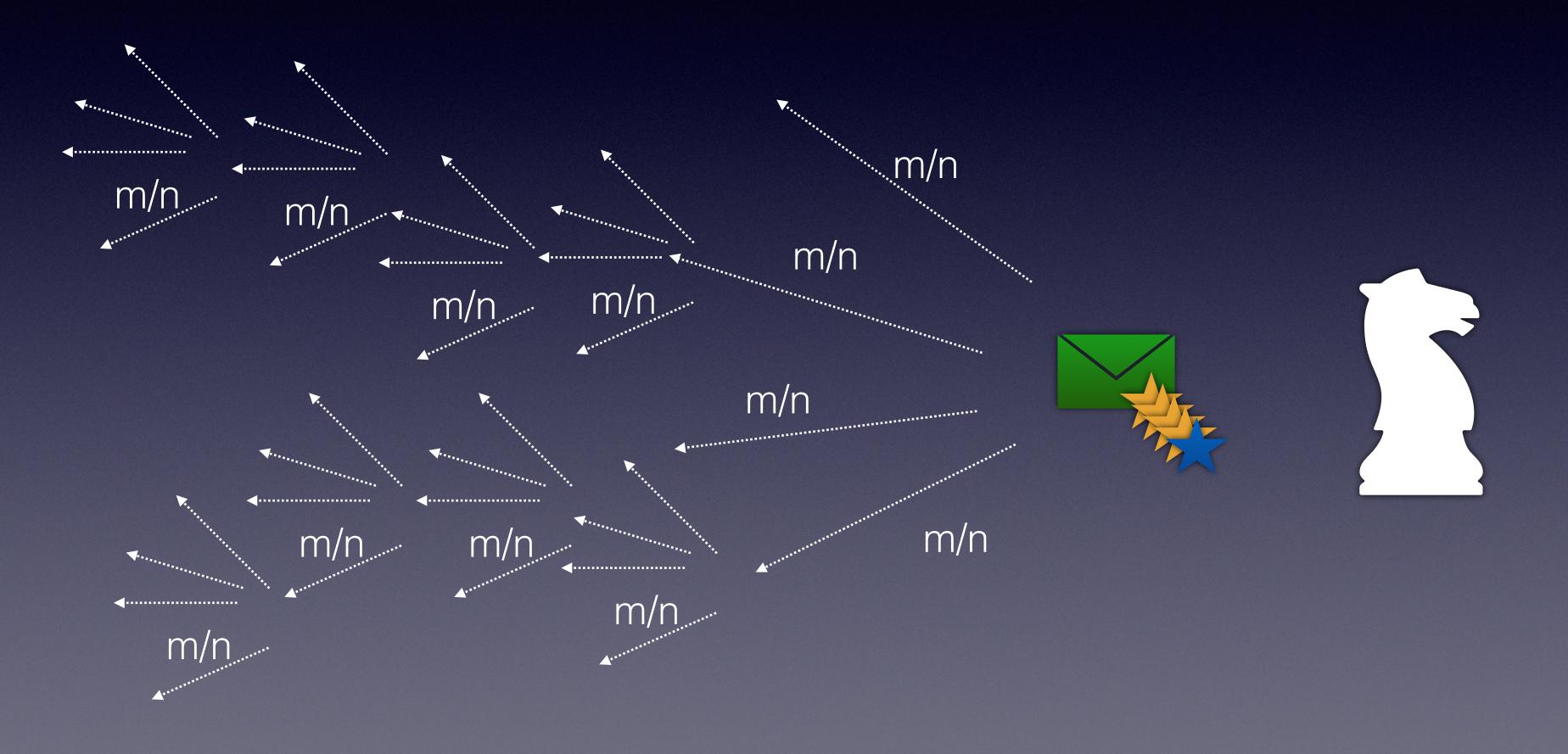
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## • Bulletin-Board PKI (NO trusted setup)

## • State-of-the-art Communication Complexity for t > n/2

Protocol	Model	CC
Dolev-Strong	Bulletin	$O(n^3\kappa)$
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Momose and Ren	Bulletin	$\tilde{O}(n^2\kappa)$

# Comparison

RC	Adversary	Corruptions	Тур
O(n)	Adaptive	< n	BC
O(n)	Static	$< (1 - \epsilon)n$	BC
<b>O</b> (1)	Adaptive	< n/2	BC
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# Limitations so far

- Static vs Adaptive adversary:
- An adaptive adversary can break the security of the process.
  - Any ideas how?

## But... Broadcast?

- Back to our motivation:
  - broadcast values.
    - (E.g. MPC, VSS applications)

### • Many times in practical uses of Broadcast, we require all parties to

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• Consistency: For each slot  $s_i$ , all honest parties output the same bit

### 5. Alexander





### 4. Napoleon







### 2. Washington



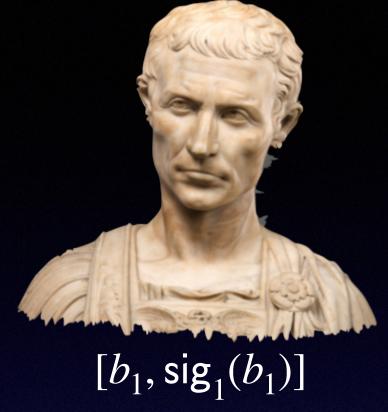
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96

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 $[b_1, \mathsf{sig}_1(b_1)]$ 

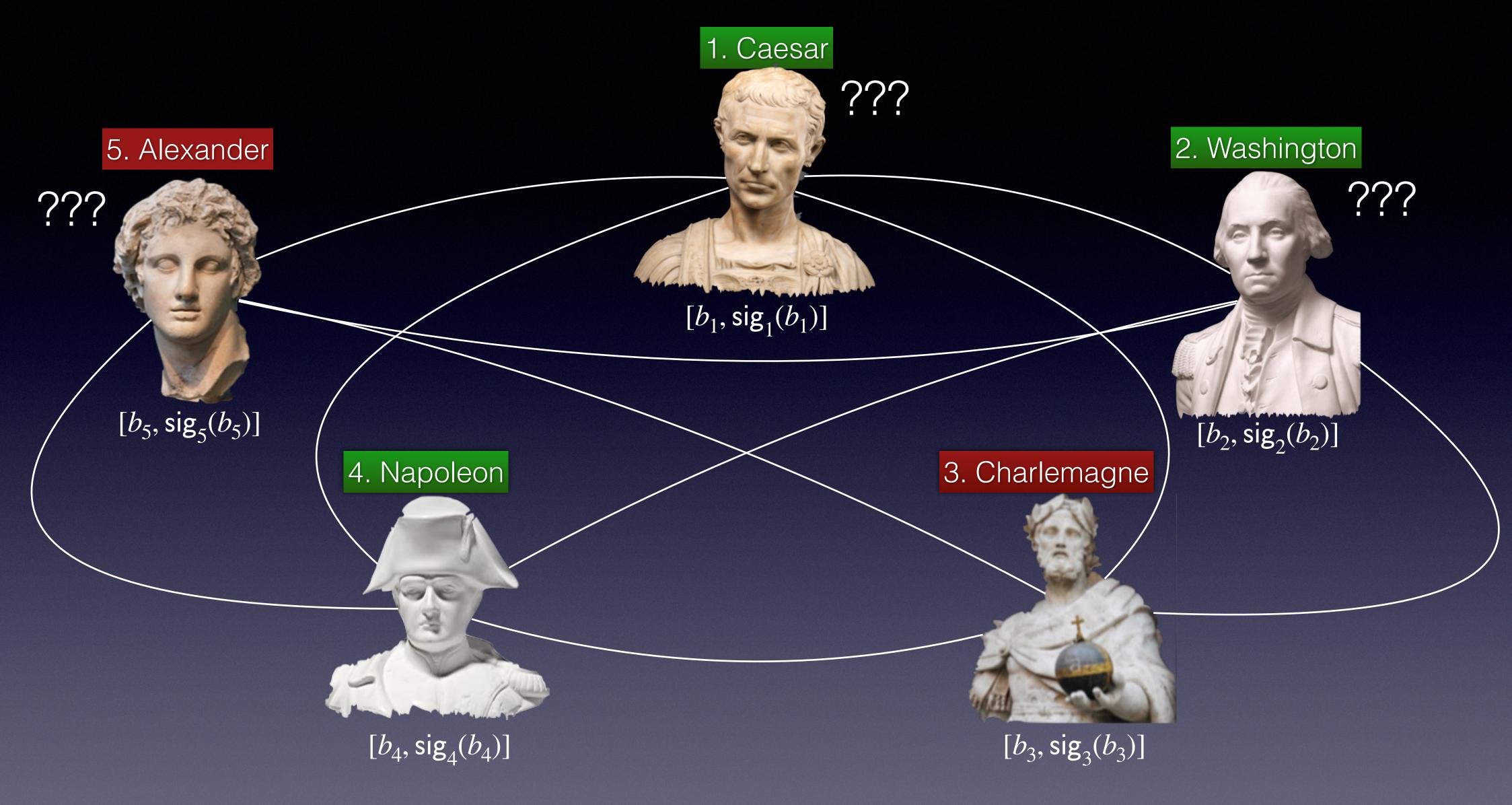
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- Can we do **better**?

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# Authenticated Broadcast with $O(n^2)$ CC using bulletin

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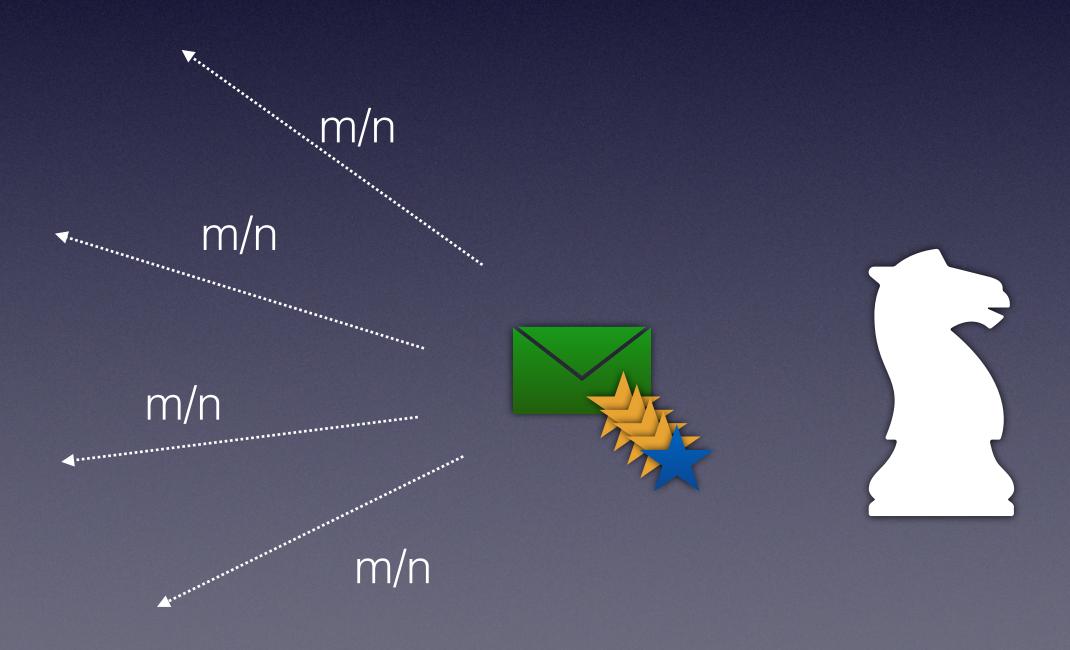
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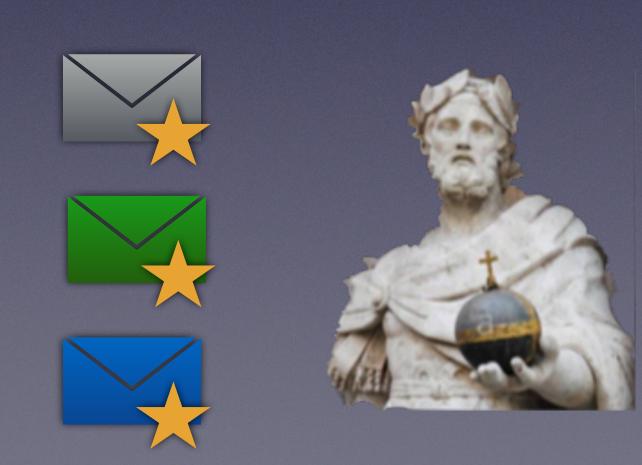


## Before: Party gossips a specific message to a few other parties randomly



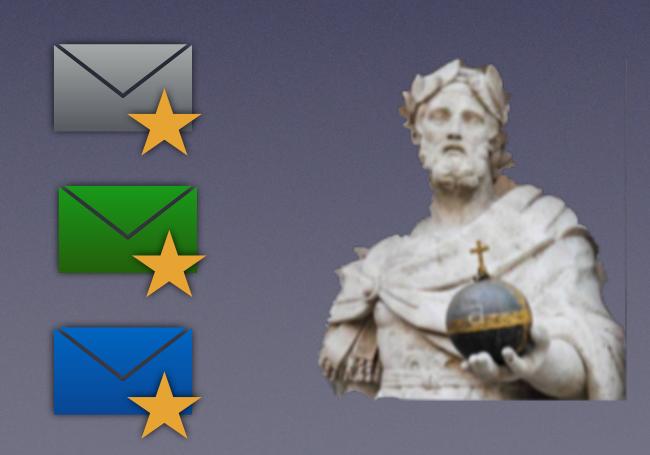


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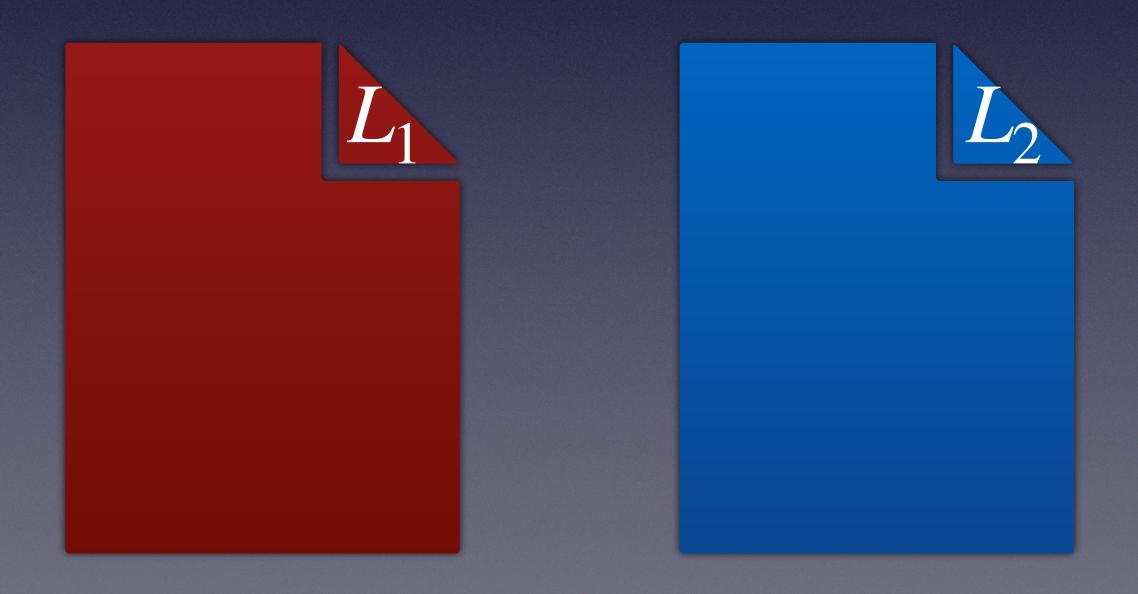
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- The adversary doesn't gain any advantage by observing the execution of the protocol





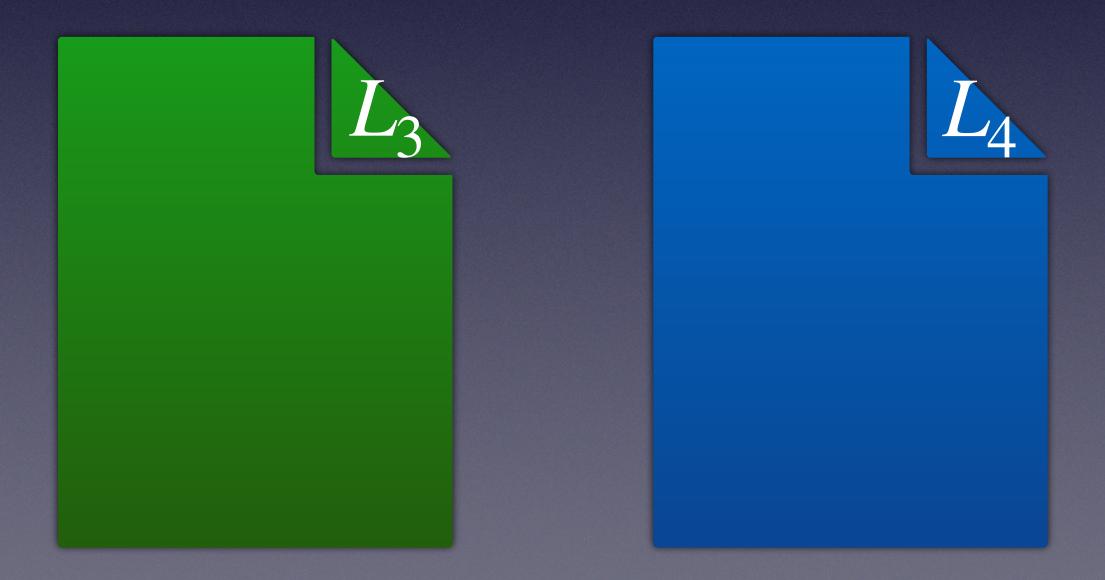




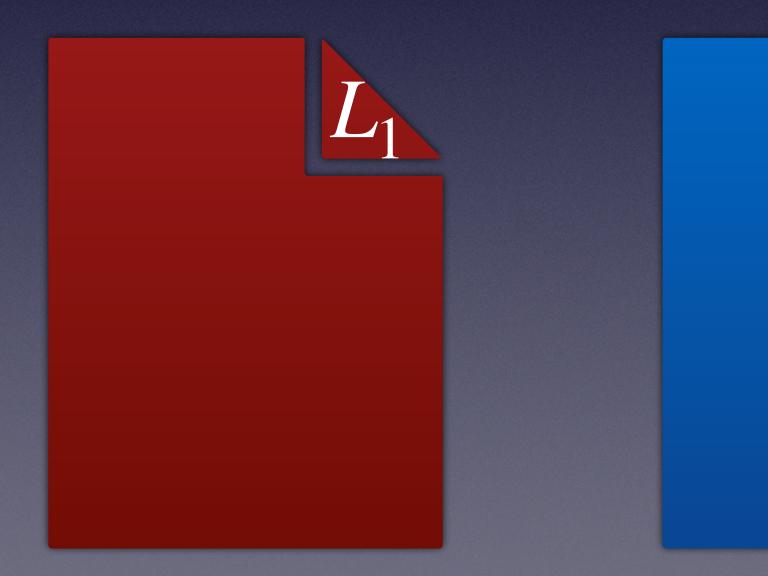








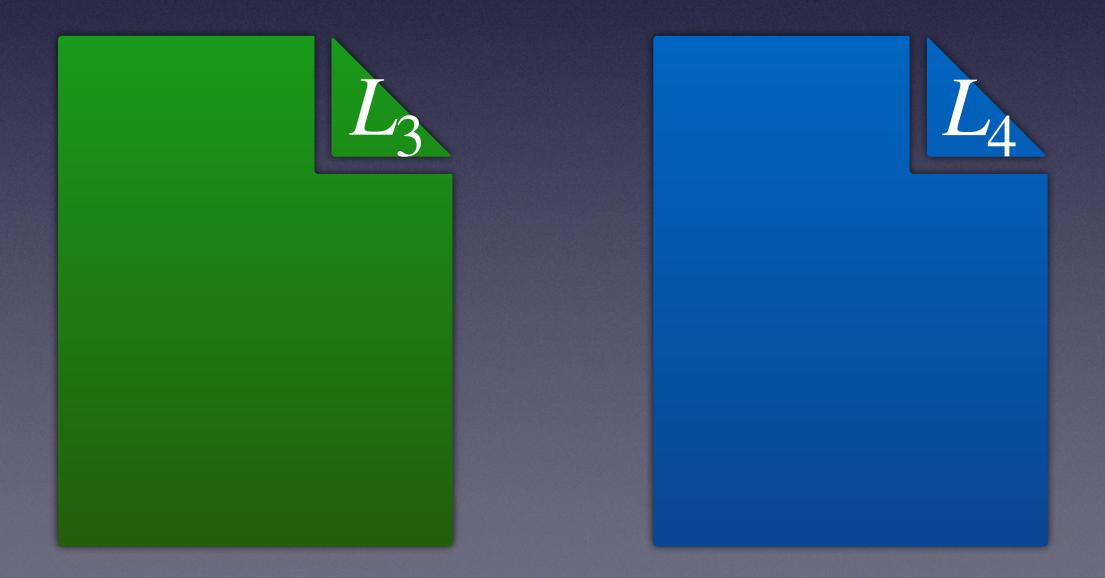




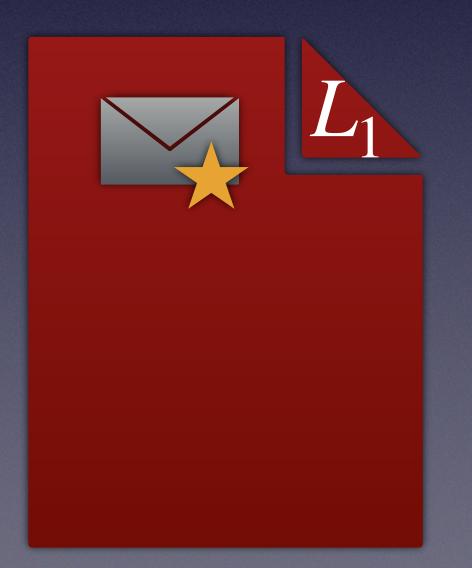


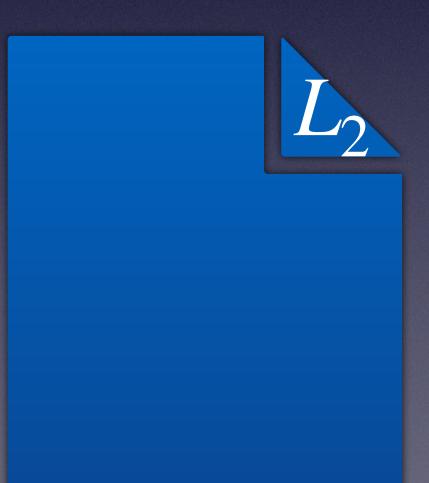






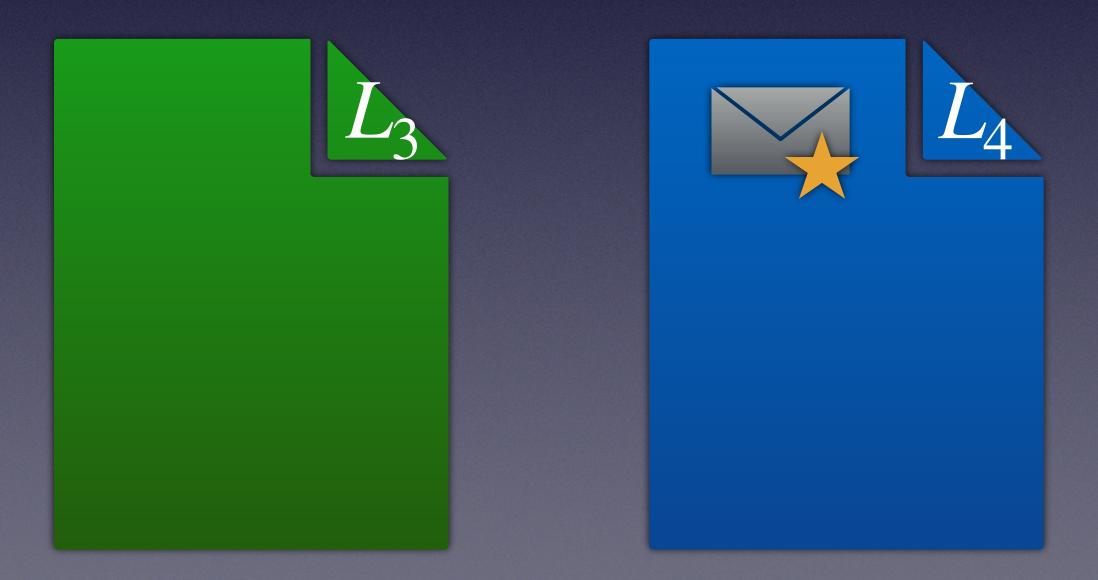




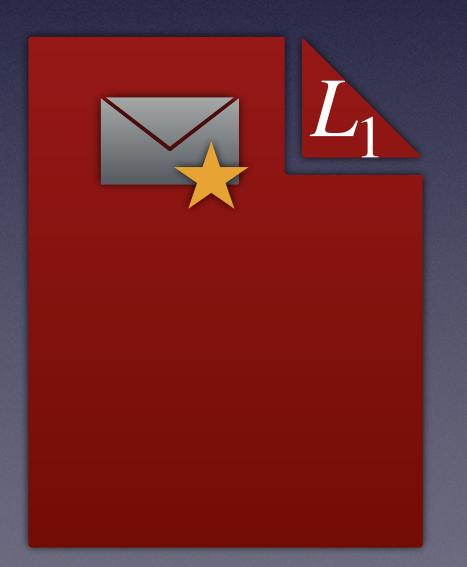








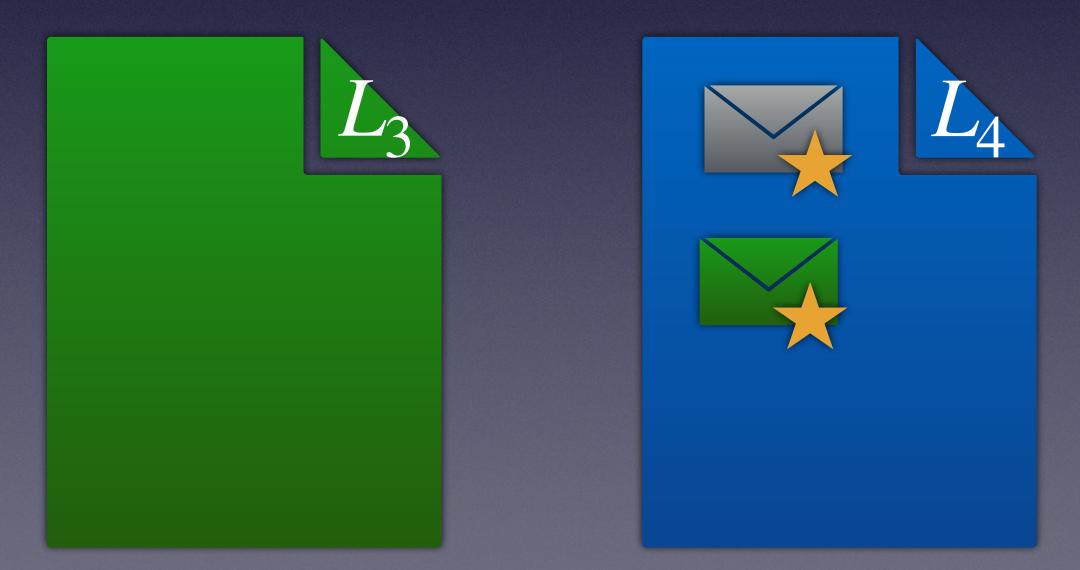








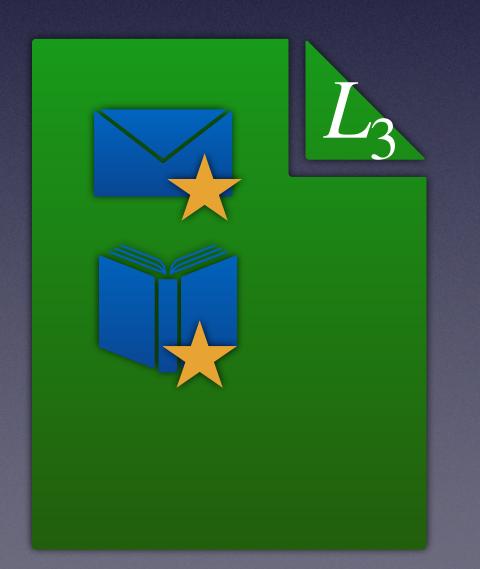


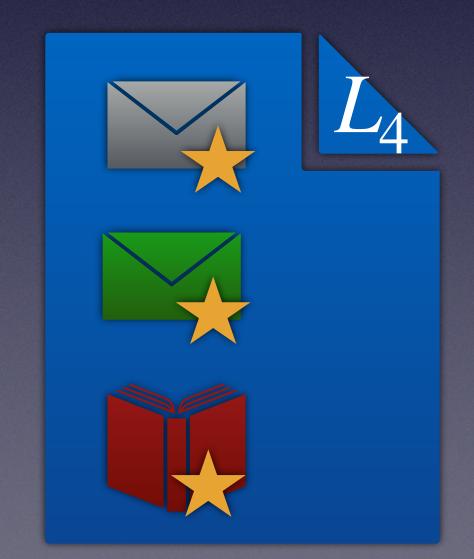










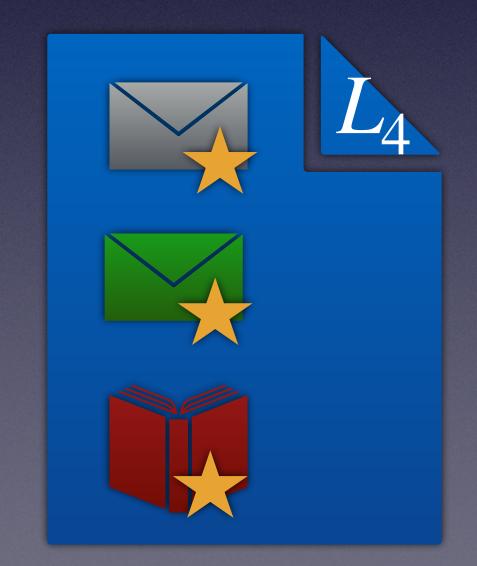


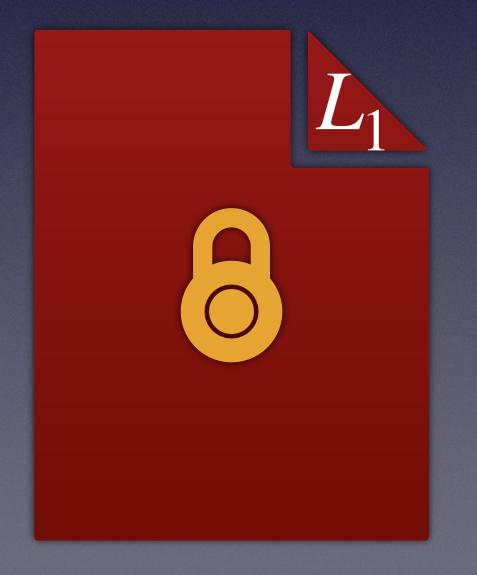














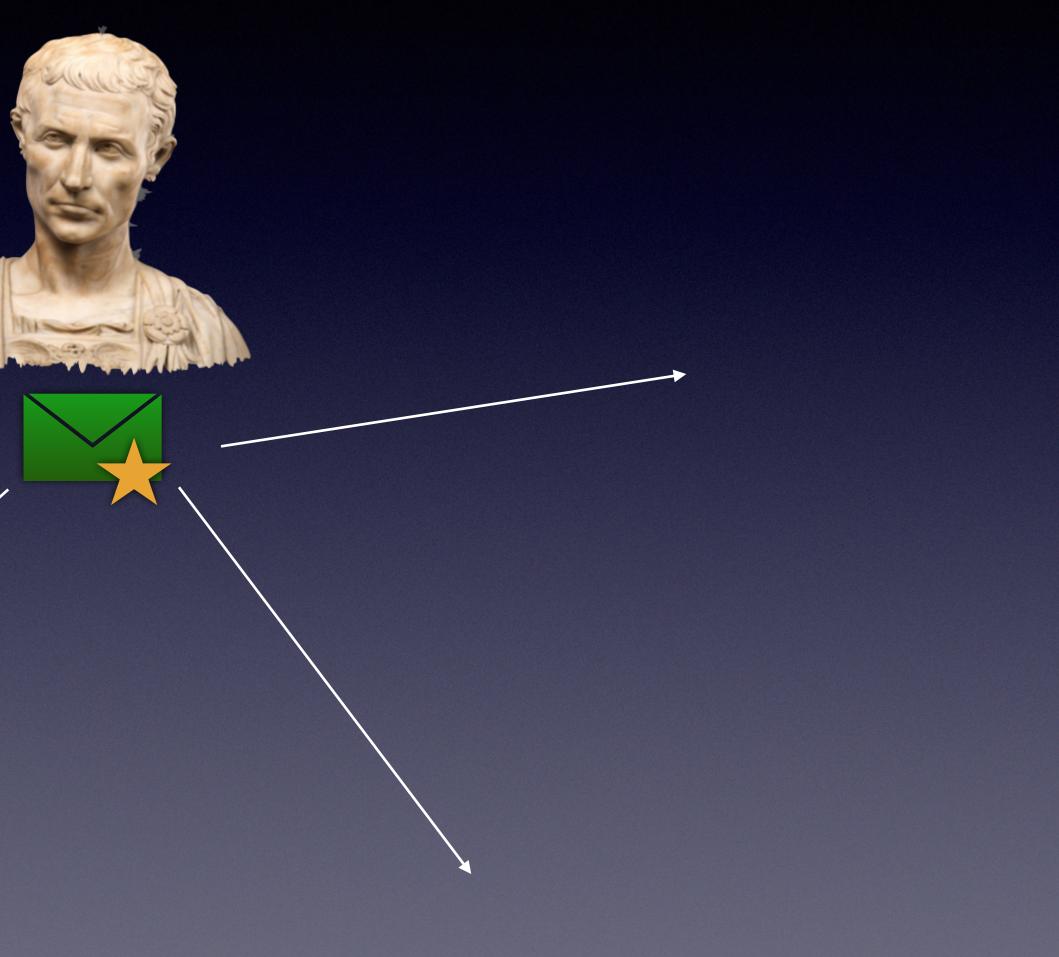






#### Our Bulletin PBC Protocol

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#### Stage 1:

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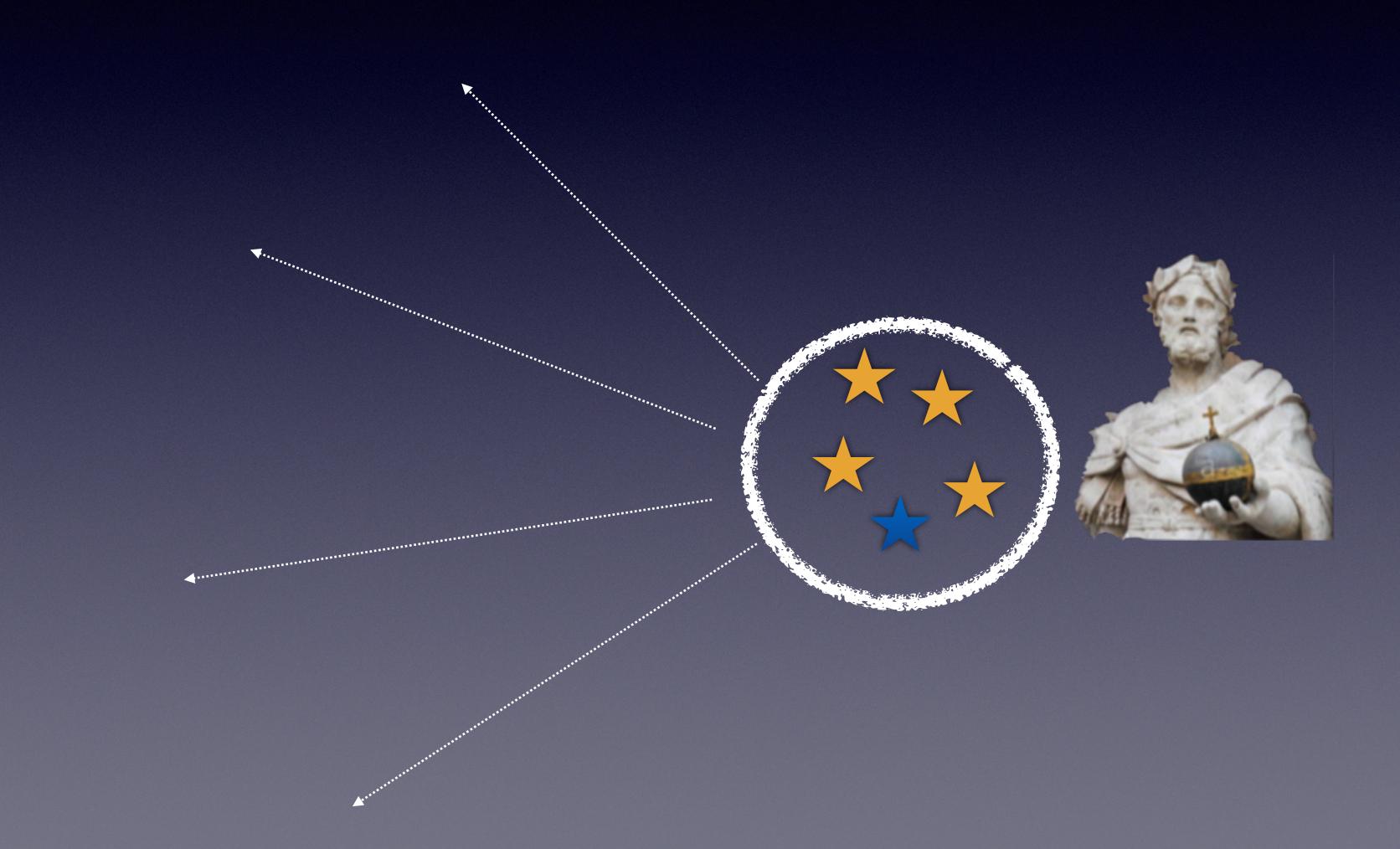
Valid bit at round r: At least *r* distinct signatures, where one is from s.

no non

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#### Stage 2:

#### For each $r \leq t + 1$ : *p* calls **Converge** on $M_p$ : received signatures



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- Message space  $\mathcal{M}$ : signatures on [b,s],  $|\mathcal{M}| = O(n^2)$
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- Total CC:  $\tilde{O}(n^3)$  (Amortized  $\tilde{O}(n^2)$  per broadcast)

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In each round, message propagation follows Converge instead of Send-

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Momose and Ren	Bulletin	$\tilde{O}(n^2\kappa)$	O(n)	Adaptive	< n/2	BC
BulletinPBC	Bulletin	$\tilde{O}(n^2\kappa^2)^*$	$O(n \log n)$	Adaptive	$< (1 - \epsilon)n$	PBC
TrustedPBC	Trusted	$\tilde{O}(n\kappa^4)$ *	$O(\kappa \log n)$	Adaptive	$< (1 - \epsilon)n$	PBC

## Comparison

\* refers to amortized Complexity per sender



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TrustedPBC	Trusted	$\tilde{O}(n\kappa^4)$ *	$O(\kappa \log n)$	Adaptive	$< (1 - \epsilon)n$	PBC

### Comparison

\* refers to amortized Complexity per sender



Protocol	Model	CC	RC	Adversary	Corruptions	Туре
Dolev-Strong	Bulletin	$O(n^3\kappa)$	O(n)	Adaptive	< n	BC
BulletinBC	Bulletin	$\tilde{O}(n^2\kappa^2)$	O(n)	Static	$< (1 - \epsilon)n$	BC
Abraham et al.	Trusted	$\tilde{O}(n\kappa)$	<i>O</i> (1)	Adaptive	< n/2	BC
Chan et al.	Trusted	$O(n^2\kappa^2)$	$O(\kappa)$	Adaptive	$< (1 - \epsilon)n$	BC
Momose and Ren	Bulletin	$\tilde{O}(n^2\kappa)$	O(n)	Adaptive	< n/2	BC
BulletinPBC	Bulletin	$\tilde{O}(n^2\kappa^2)^*$	$O(n \log n)$	Adaptive	$< (1 - \epsilon)n$	PBC
TrustedPBC	Trusted	$\tilde{O}(n\kappa^4)^*$	$O(\kappa \log n)$	Adaptive	$< (1 - \epsilon)n$	PBC

## Comparison

\* refers to amortized Complexity per sender



#### Contributions

#### Contributions Introduced gossiping, Converge & 3 SOA protocols: $\tilde{\mathcal{O}}(n^2\kappa^4)$ Parallel Broadcast using trusted PKI, against $t < (1 - \epsilon)n$ adaptive corruptions

 $\tilde{O}(n^2\kappa^4)$  Parallel Broadcast using trusted PKI, against  $t < (1 - \epsilon)n$  adaptive corruptions

 $\tilde{O}(n^3\kappa^2)$  Parallel Broadcast using bulletin board PKI, against  $t < (1 - \epsilon)n$  adaptive corruptions



 $\tilde{O}(n^2\kappa^4)$  Parallel Broadcast using trusted PKI, against  $t < (1 - \epsilon)n$  adaptive corruptions

 $\tilde{O}(n^3\kappa^2)$  Parallel Broadcast using bulletin board PKI, against  $t < (1 - \epsilon)n$  adaptive corruptions

 $\tilde{O}(n^2\kappa^2)$  single sender Broadcast using bulletin board PKI, against  $t < (1 - \epsilon)n$  static corruptions



 $\tilde{O}(n^2\kappa^4)$  Parallel Broadcast using trusted PKI, against  $t < (1 - \epsilon)n$  adaptive corruptions

 $\tilde{O}(n^3\kappa^2)$  Parallel Broadcast using bulletin board PKI, against  $t < (1 - \epsilon)n$  adaptive corruptions

 $\tilde{O}(n^2\kappa^2)$  single sender Broadcast using bulletin board PKI, against  $t < (1 - \epsilon)n$  static corruptions

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