Short Leakage Resilient and Non-malleable Secret Sharing Schemes

Sruthi Sekar

Berkeley
UNIVERSITY OF CALIFORNIA

Nishanth Chandran

Microsoft Research

Bhavana Kanukurthi

IISc

Sai Lakshmi Bhavana Obbattu

Microsoft Research

Secret Sharing Schemes

Shamir and Blakely (1979)

Secret Sharing Schemes

Shamir and Blakely (1979)

Bob

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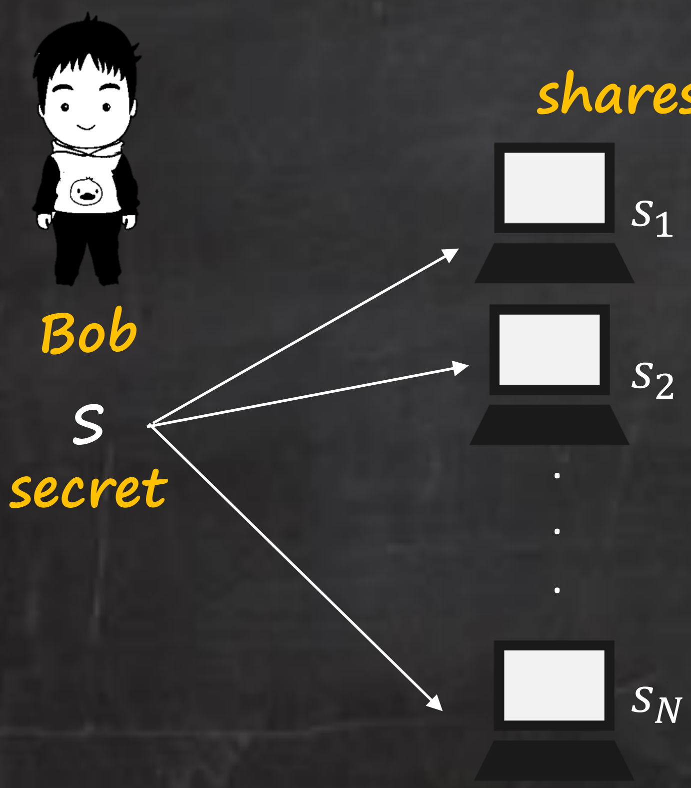
Secret Sharing Schemes

Shamir and Blakely (1979)



Secret Sharing Schemes

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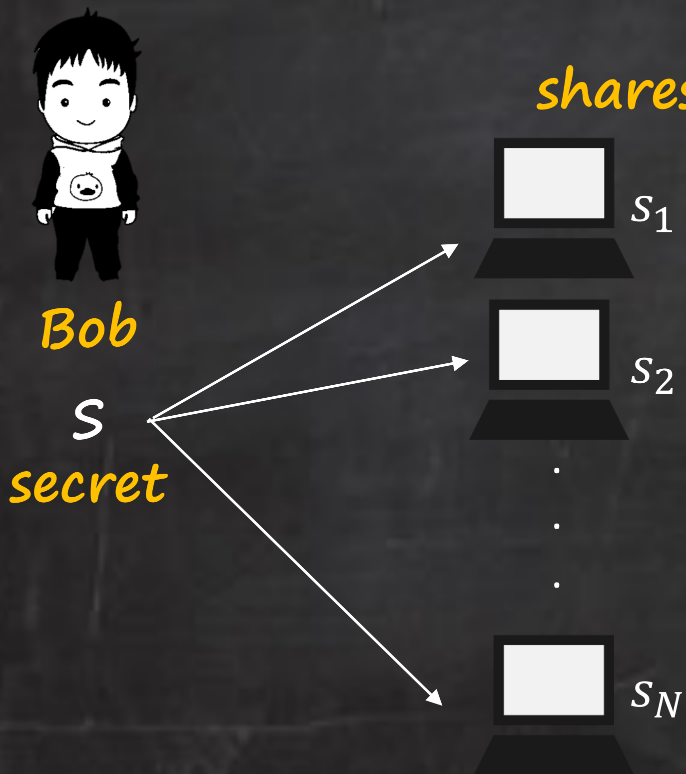

<u>Correctness</u>:
$\geq t$ shares give s.

Secret Sharing Schemes

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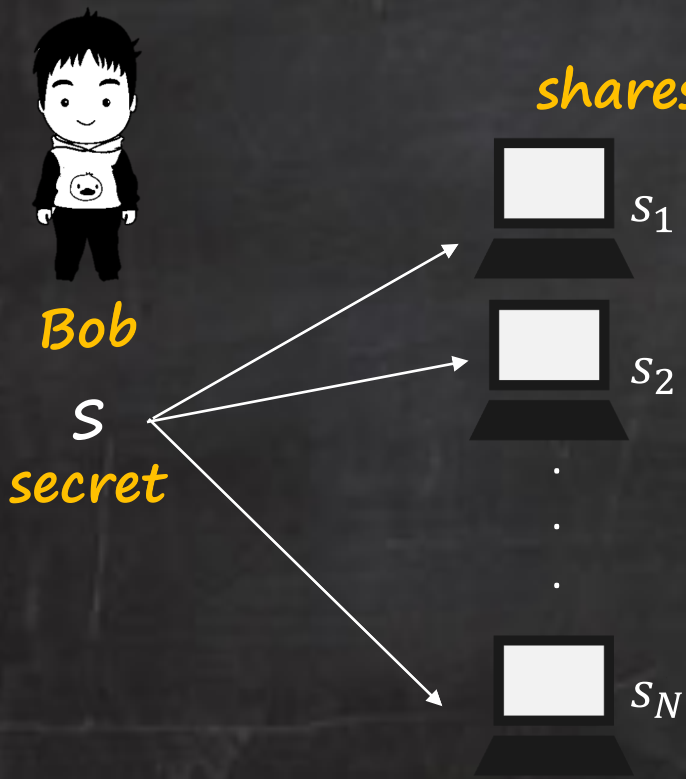

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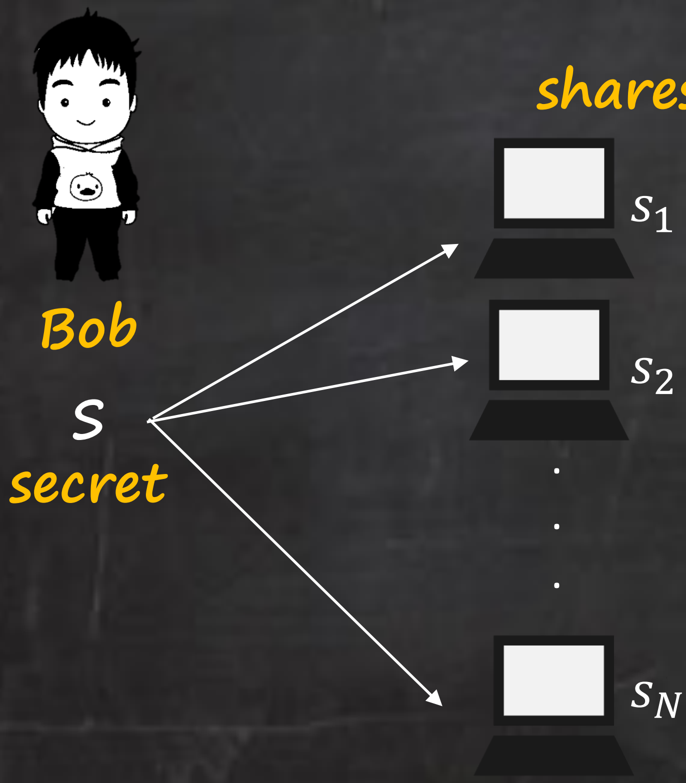

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$$(< t \text{ shares of } s) \approx (< t \text{ shares of } s')$$

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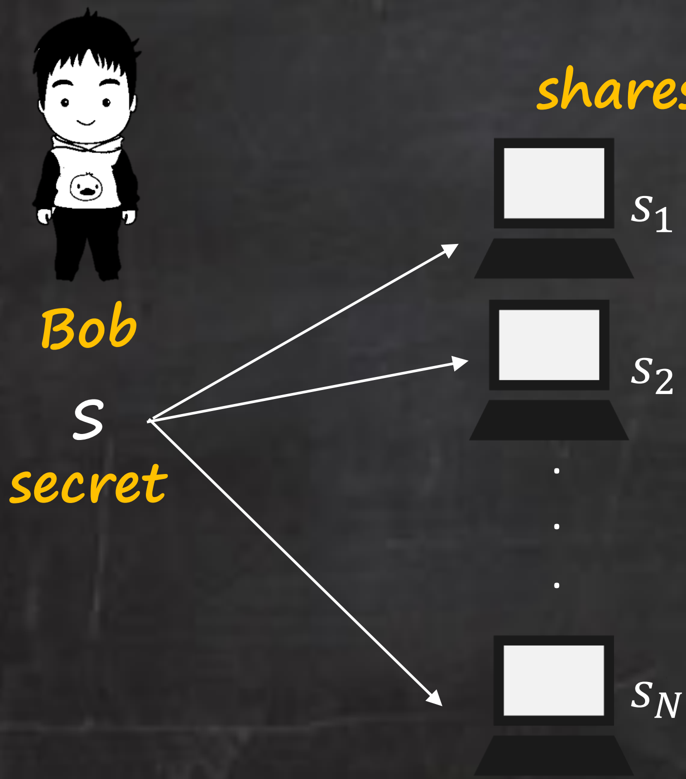

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Statistical distance

Secret Sharing Schemes

Shamir and Blakely (1979)

B

secr

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Secret Sharing Schemes

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LEAKAGE ATTACKS [Kocher(1996)]

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What if in addition to $t-1$ shares, adversary gets (arbitrary) bounded # of bits from other shares too?

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<u>LEAKAGE ATTACKS [Kocher(1996)]</u>

What if in addition to $t-1$ shares, adversary gets (arbitrary) bounded # of bits from other shares too?

[Guruswami, Wooters (2016)]: Shamir SS breaks, given 1-bit leakage on remaining shares.

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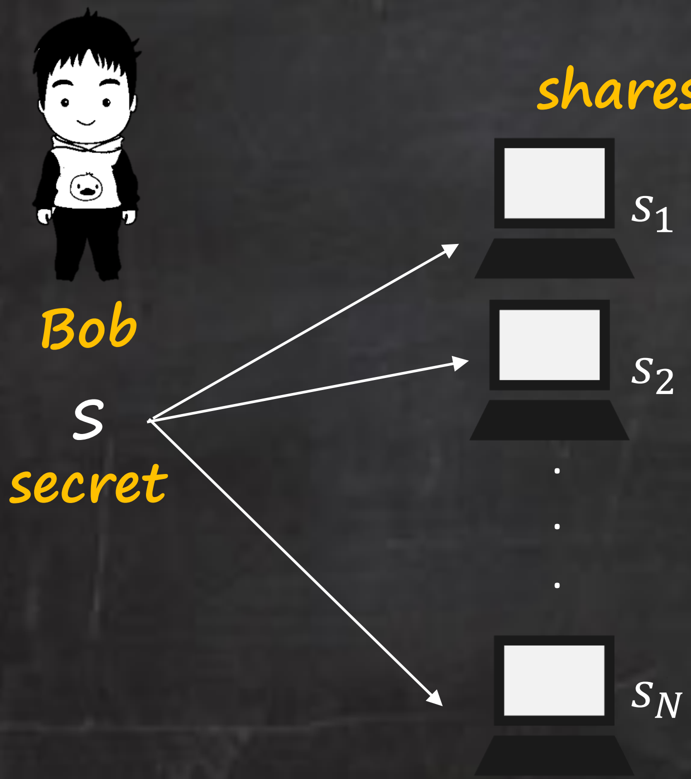
Leakage Resilient Secret Sharing

Dziembowski and Pietrzak (2007)



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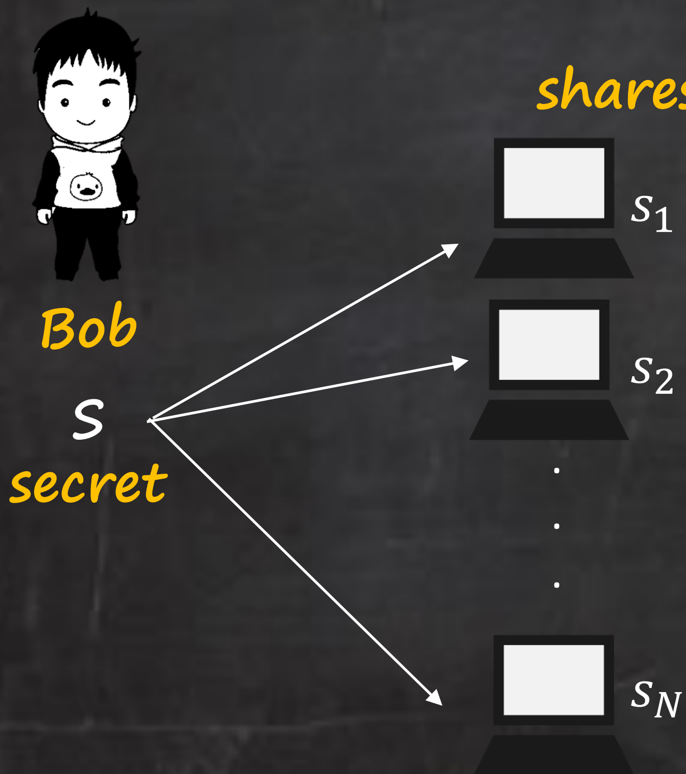

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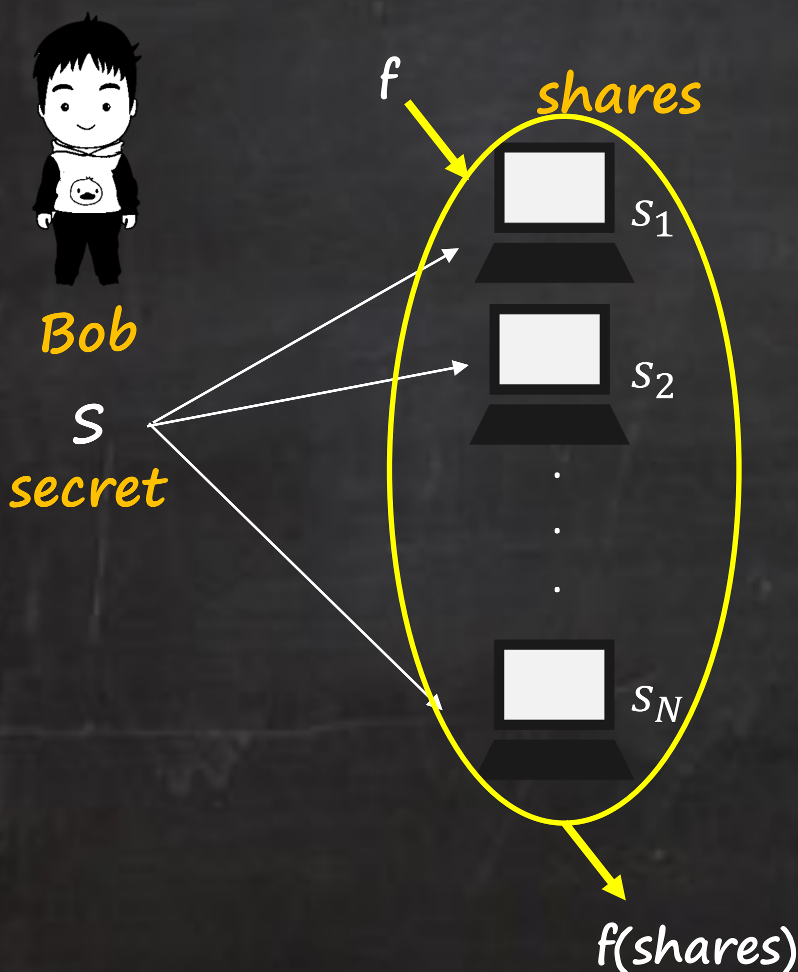

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Leakage Resilience:

$\mathcal{F}$ be some function family.
For function f $\in \mathcal{F}$, f(shares) gives no information about s.

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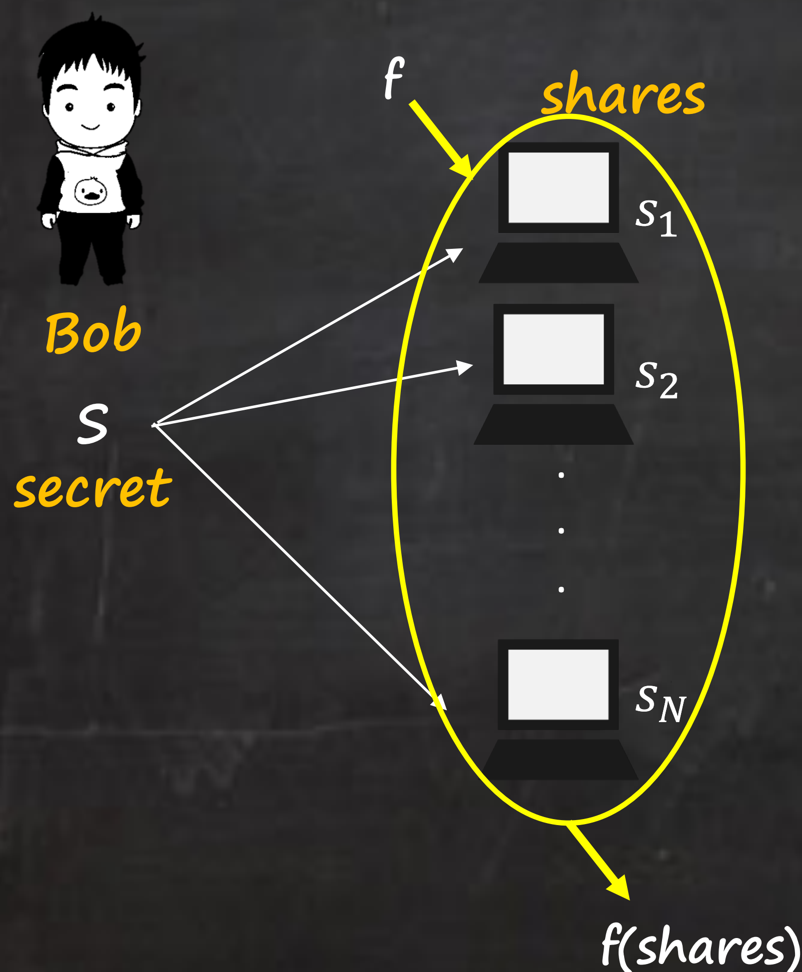

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- Share Size: size of the largest share amongst the N shares. Best share size one can hope for an LRSS: $message\ length + \mu$, where μ is the #leakage-bits per share.

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Adversary

$\mathsf{f} = (f_1, f_2, \ldots, f_N)$

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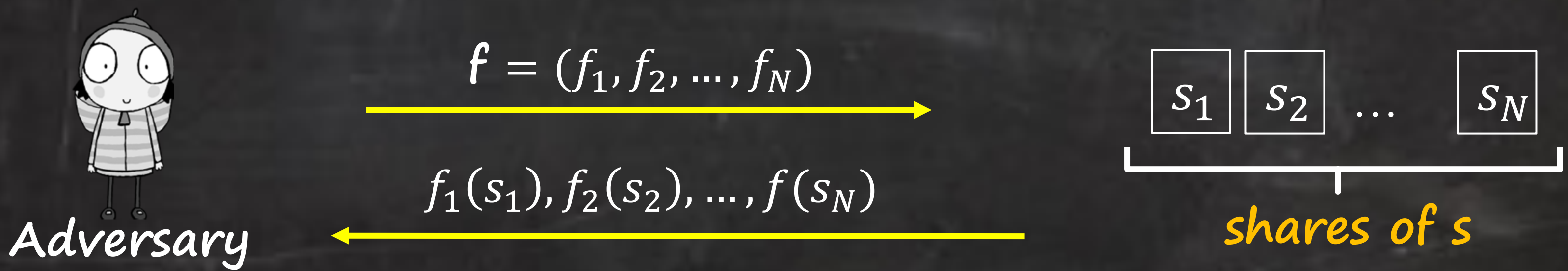

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$\mathsf{f} = (f_1, f_2, \dots, f_N)$ →

← $f_1(s_1), f_2(s_2), \dots, f(s_N)$

$t - 1$ of these are full shares,
rest arbitrary functions outputting μ bits each.

s_1	s_2	…	s_N

shares of s

Leakage Resilient Secret Sharing

Prior Works

Leakage Resilient Secret Sharing

Prior Works

- Long line of research: [DDV10, LL12, GK18, BDIR18,GK18, BS19,SV19, ADN+19,KMS19,FV19,BFV19, LCG+19,CGG+20, BFO+20, CKOS21,MPSW21,MNP+21]

Leakage Resilient Secret Sharing

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Most of these works focus on stronger leakage models (adaptive, joint)

However, the share size of these schemes is $\omega(message\ length)$!

Leakage Resilient Secret Sharing

Prior Works: Local Leakage Model

1. Leakage resilience of Shamir SS [BDIR18]

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This is the best one can hope from Shamir SS—[NS20]

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2. Generic Compiler [ADN+19, SV19]

- Best known for arbitrary N and t [SV19]:
  Share size is $(3 . message\ length + \mu)$, with μ-bits of leakage per share ($\mu \leq (1 - o(1))\ . message\ length$).

Leakage Resilient Secret Sharing

Prior Works: Local Leakage Model

1. Leakage resilience of Shamir SS [BDIR...]
For a large characteristic f... umber of
full share co...

- If $t \geq$ (N... e.

- If ...

Can we get an LRSS scheme
with optimal share size and leakage?

2. Ge...

-Best known ...
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$(\mu \leq (1 - o(1))\,.\,message\ length)$.

Leakage Resilient Secret Sharing

Our Results

Leakage Resilient Secret Sharing

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- We build the first information-theoretic LRSS scheme for the **threshold access structures** against the **local leakage model** (allowing μ bits of leakage per share), with a share size of $message\ length + \mu$!

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OUR CONSTRUCTION

Building Blocks

Linear Extractors

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RANDOMNESS EXTRACTORS

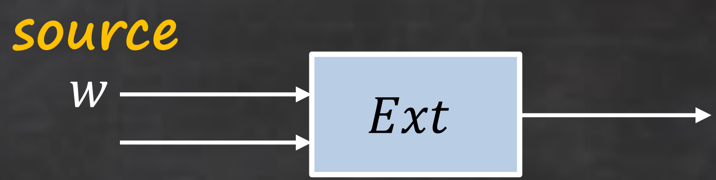
[Nisan and Zuckerman, 1996]

Building Blocks

Linear Extractors

$w \leftarrow W$

$H_\infty(W|Z) \geq k$



RANDOMNESS EXTRACTORS

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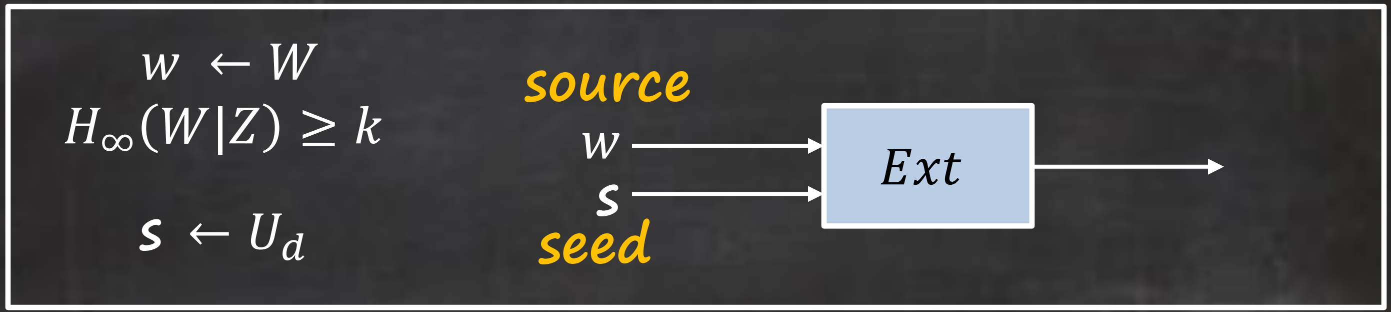
Building Blocks

Linear Extractors



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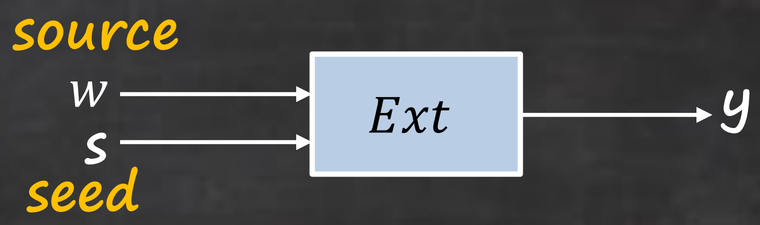

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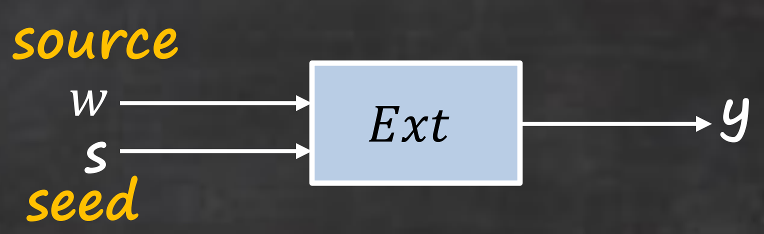

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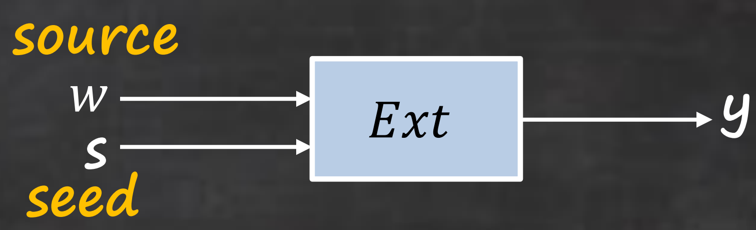

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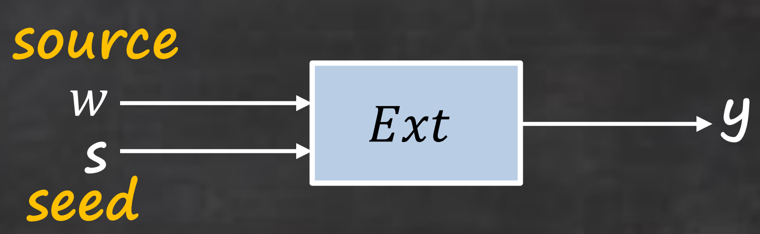

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We use invertibility of such linear extractors!

Building Blocks

Linear Extractors: Invertibility

$w \leftarrow U_\eta$

$H_\infty(U_\eta | Z) \geq k$

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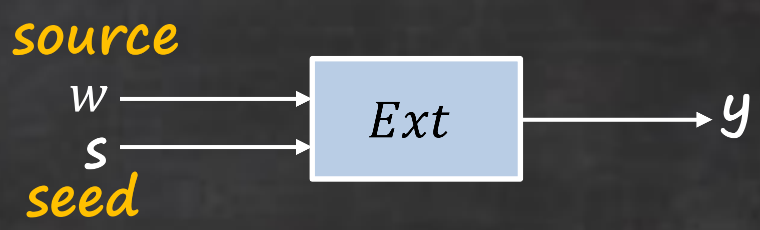

LINEAR RANDOMNESS EXTRACTORS

For the above Ext, there exists efficient $InvExt$ such that:

Building Blocks

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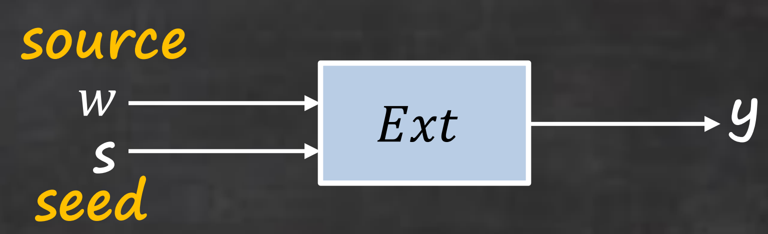

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1. $InvExt(Ext(U_\eta; U_d), U_d), U_d, Ext(U_\eta, U_d) \equiv U_\eta, U_d, Ext(U_\eta, U_d)$

Can invert and get a "correct" source string w, given a seed s and an extractor output y.

Building Blocks

Linear Extractors: Invertibility

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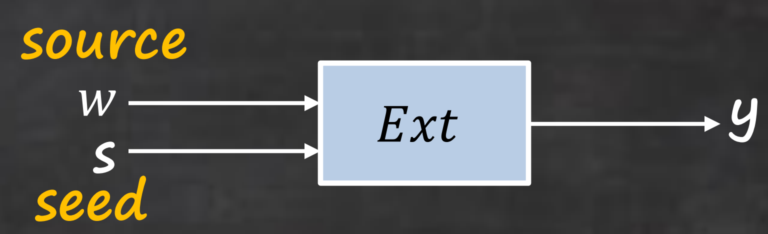

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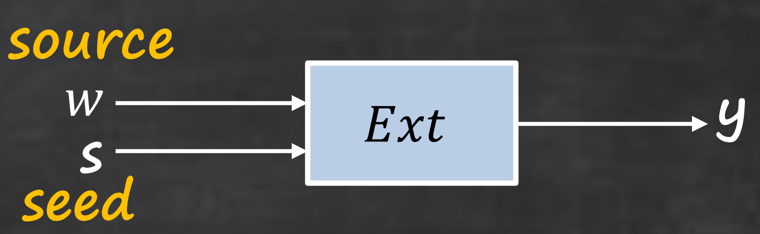

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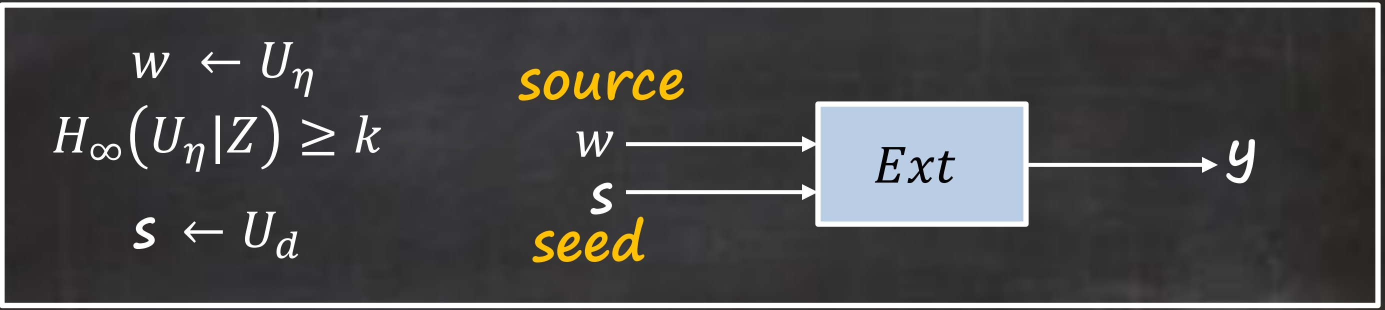

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   - **Else** $InvExt(y,s) = \perp$ **w.p. 1.**

Our Construction

Optimal Threshold LRSS

m

secret

Our Construction

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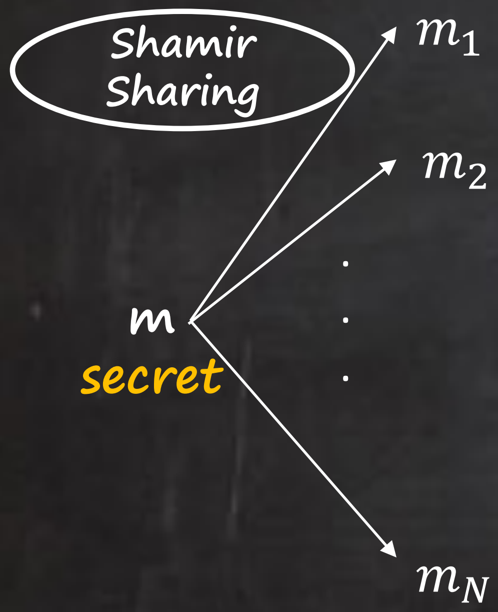

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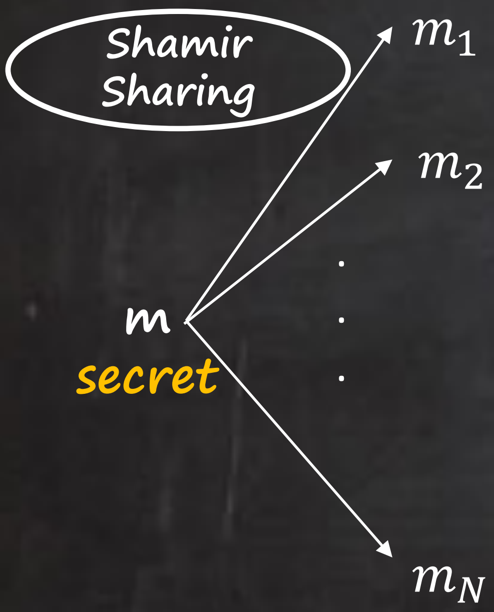

Sample $s \leftarrow U_d$

Our Construction

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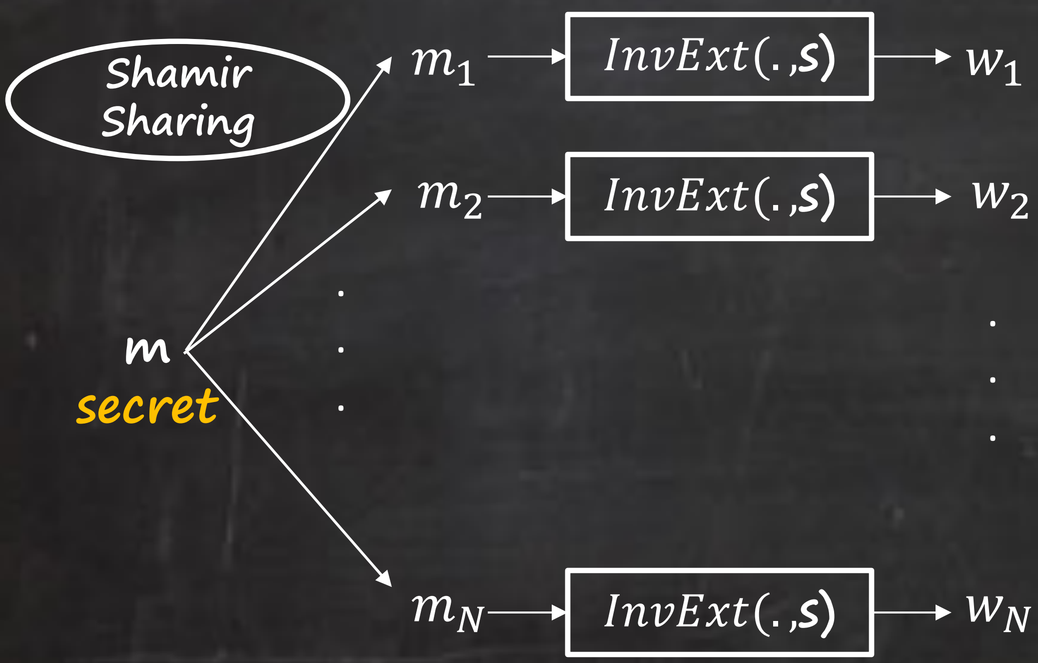

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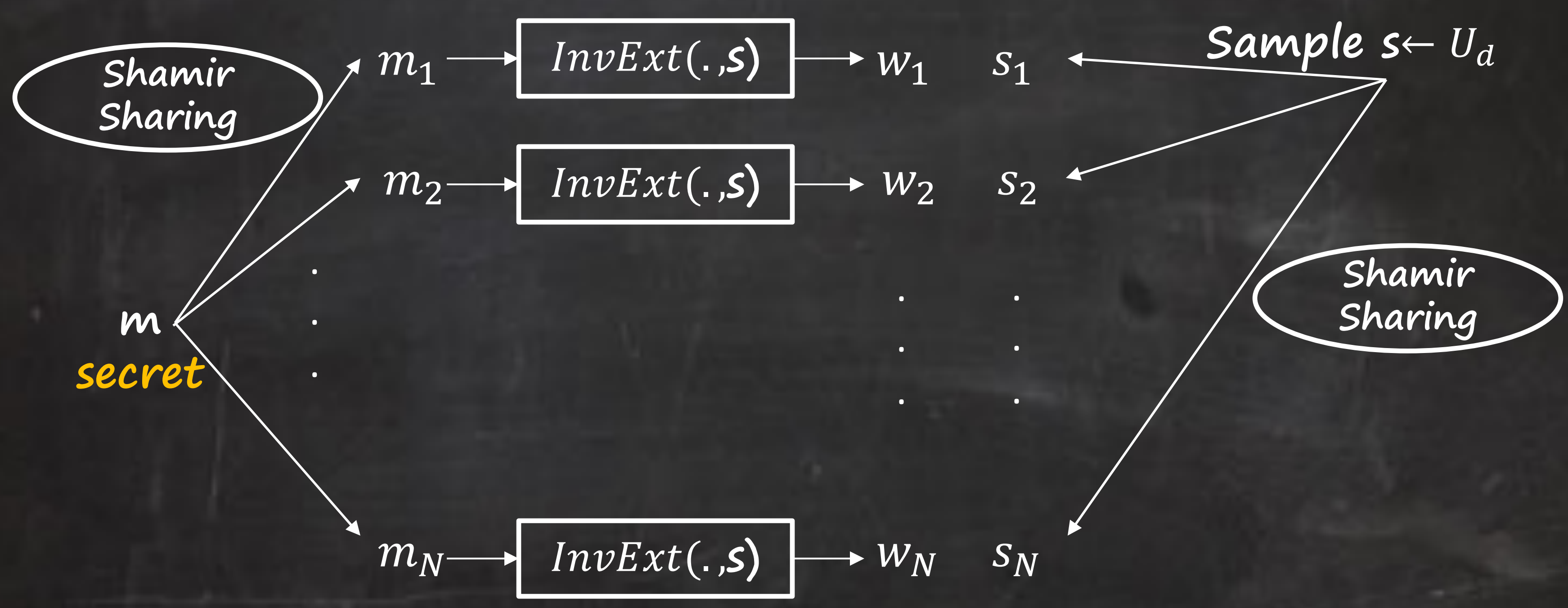

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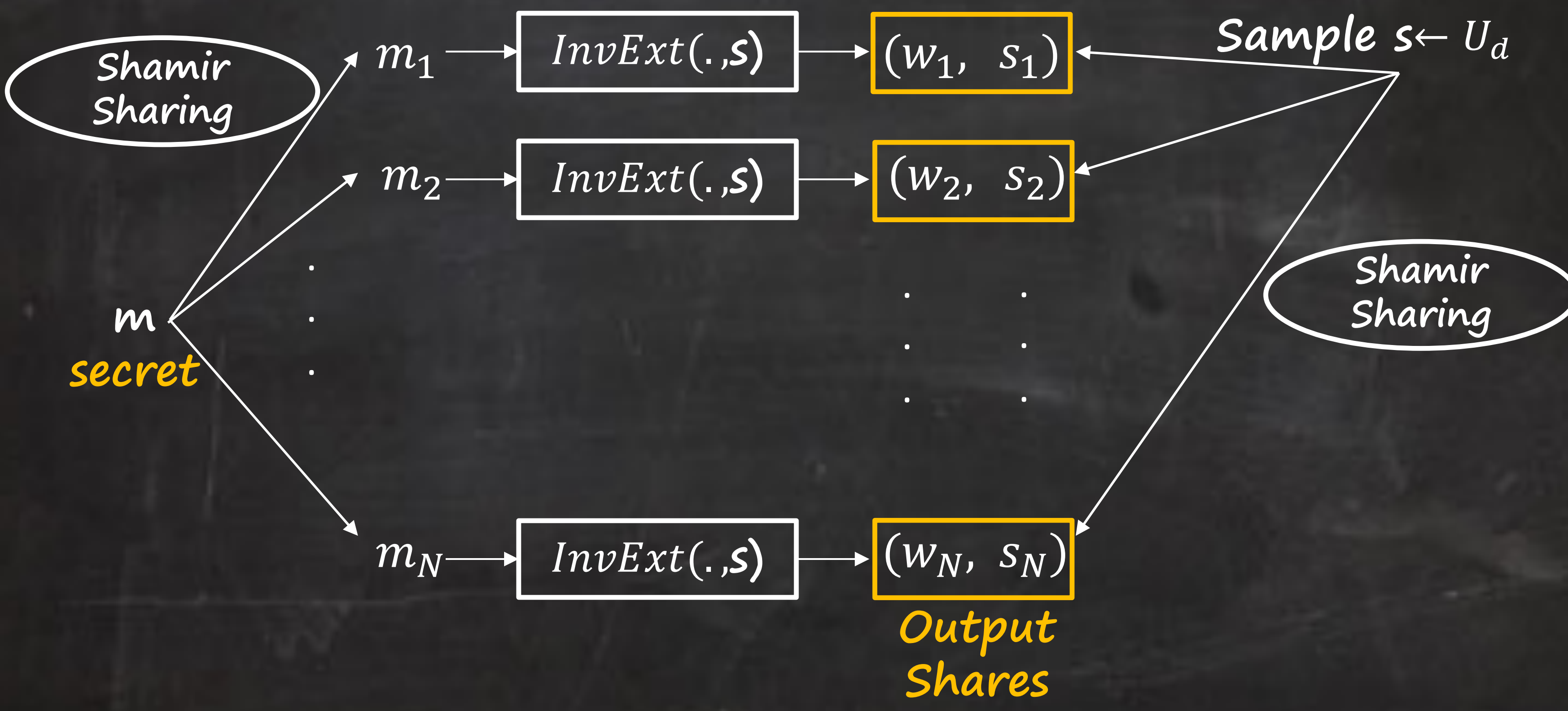

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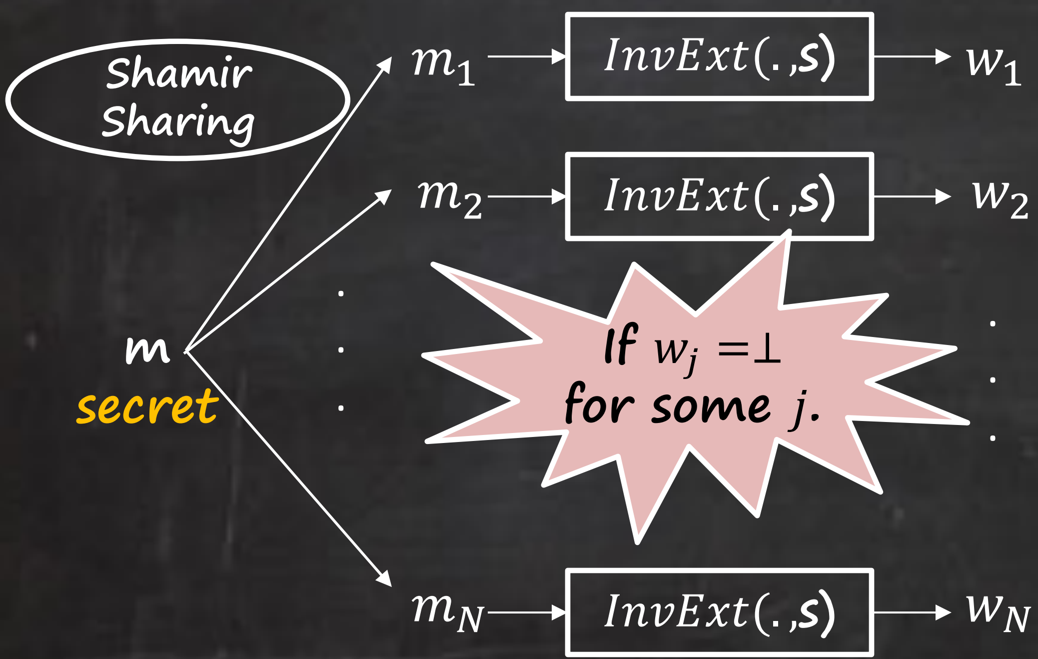

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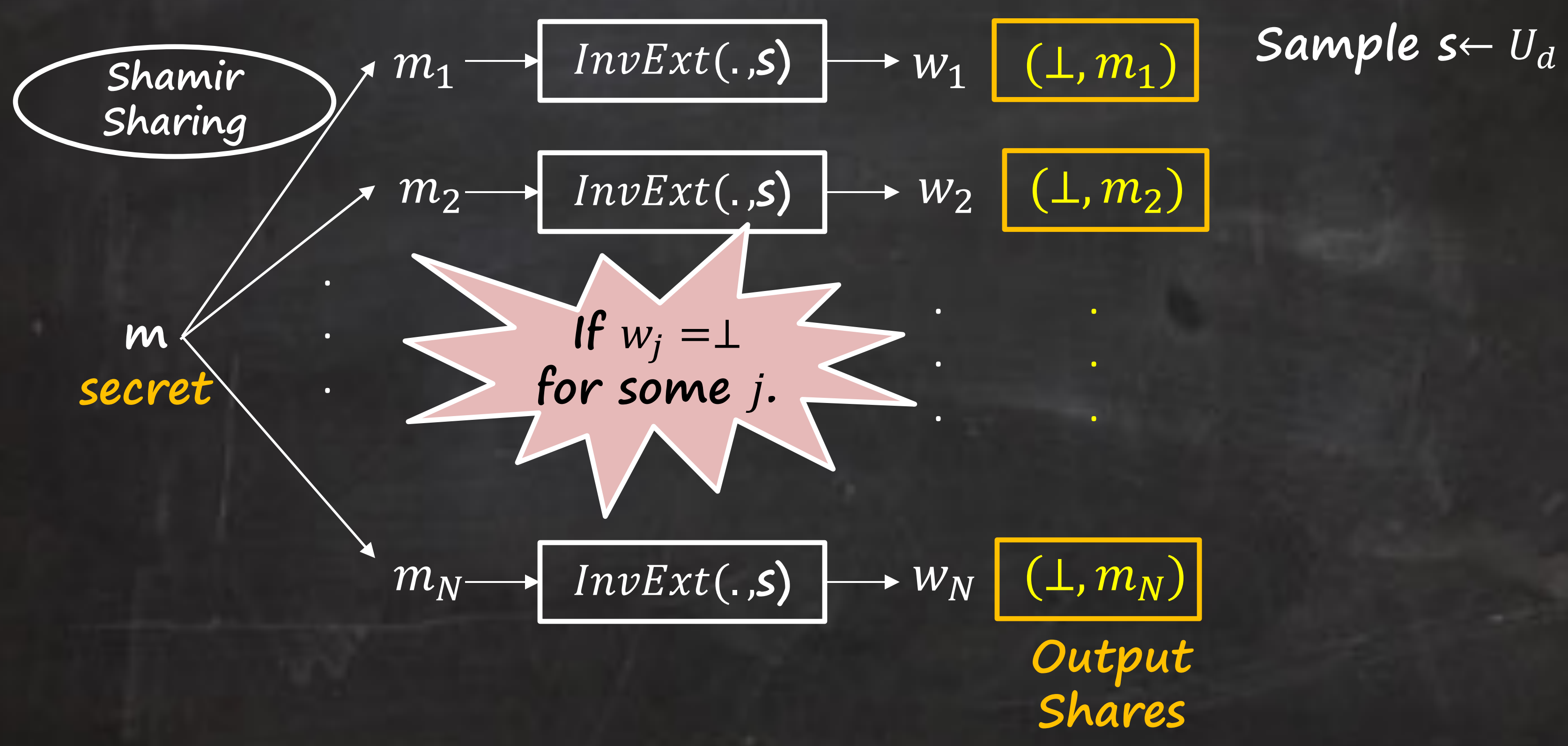

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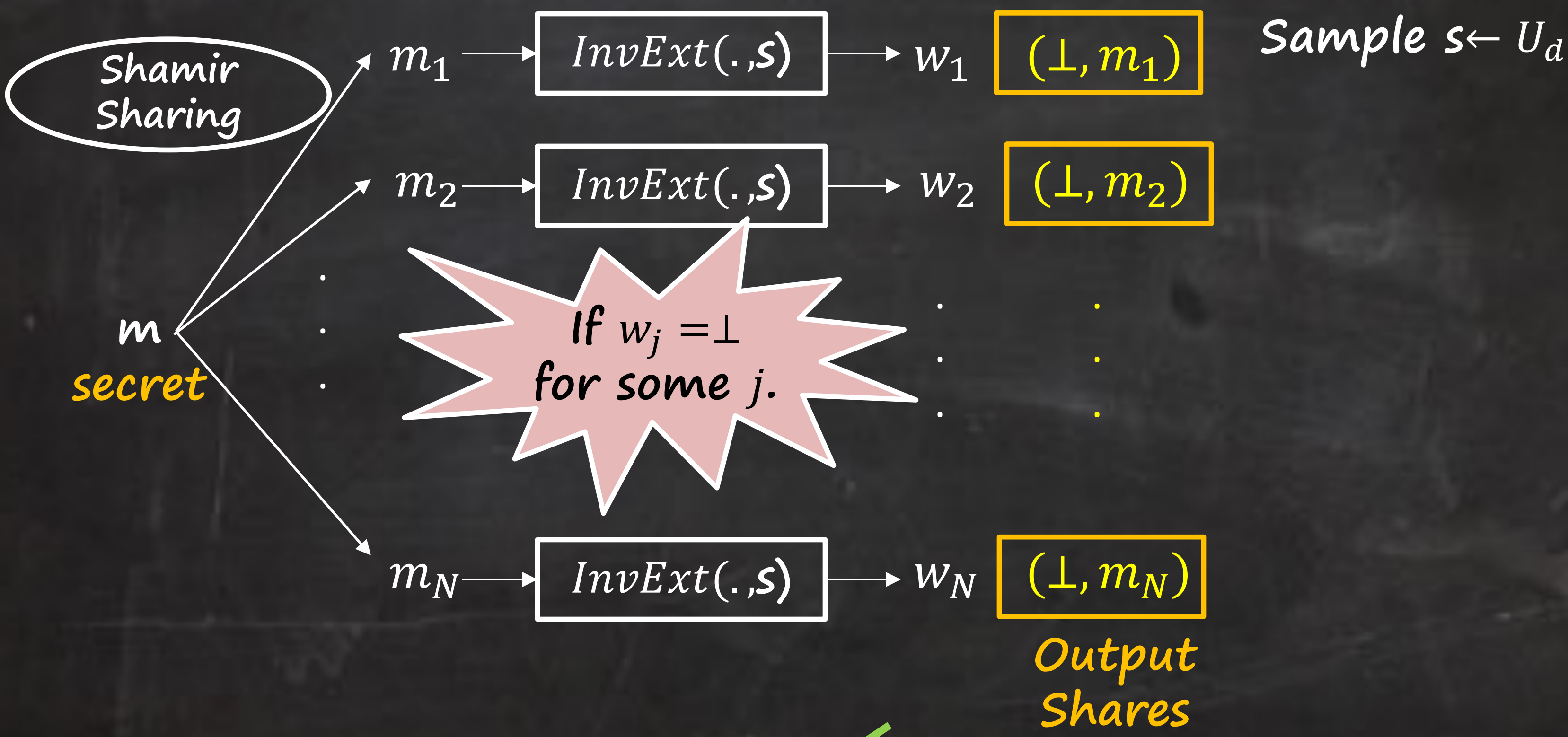


CORRECTNESS ✓

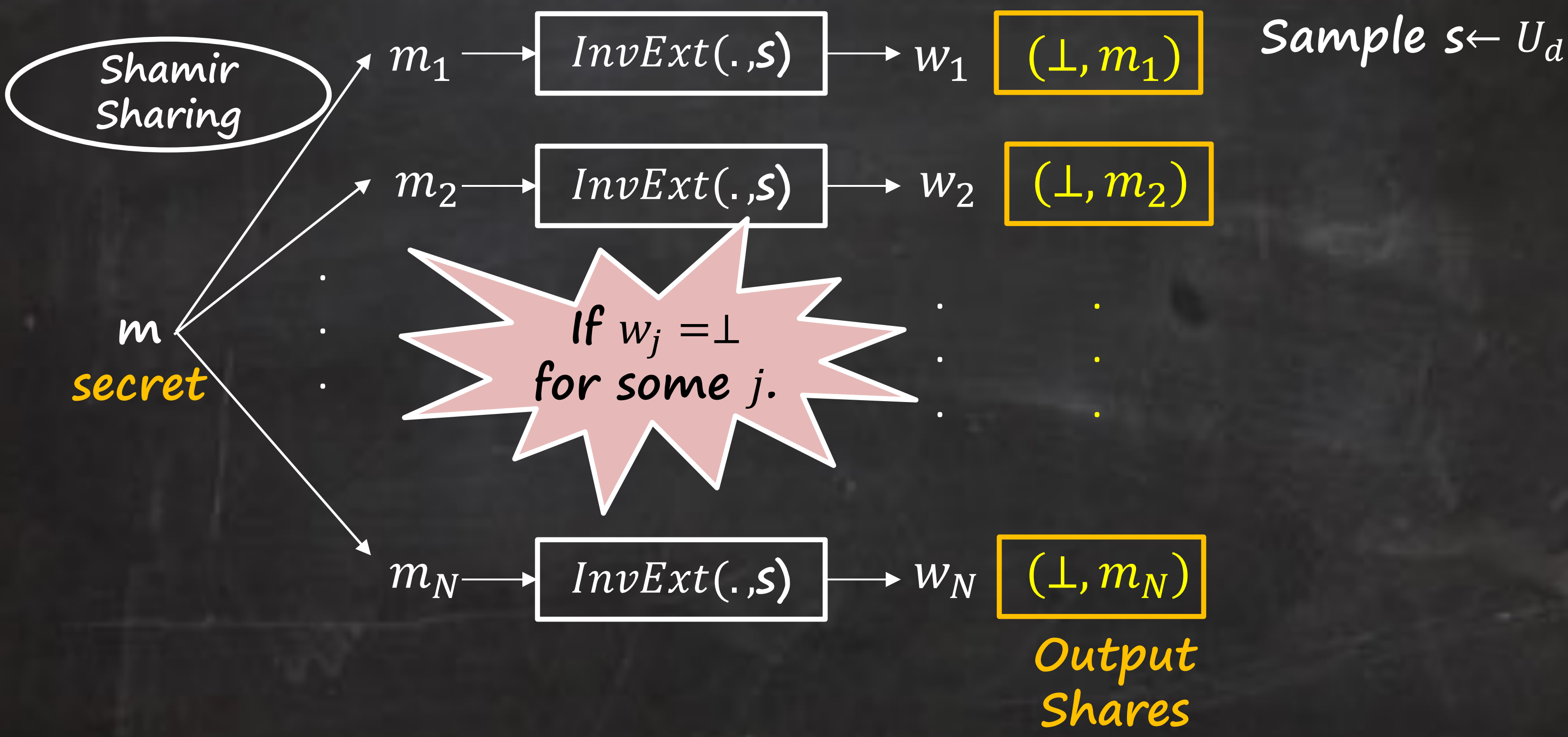
Our Construction

Optimal Threshold LRSS: Leakage Resilience

Our Construction

Optimal Threshold LRSS: Leakage Resilience

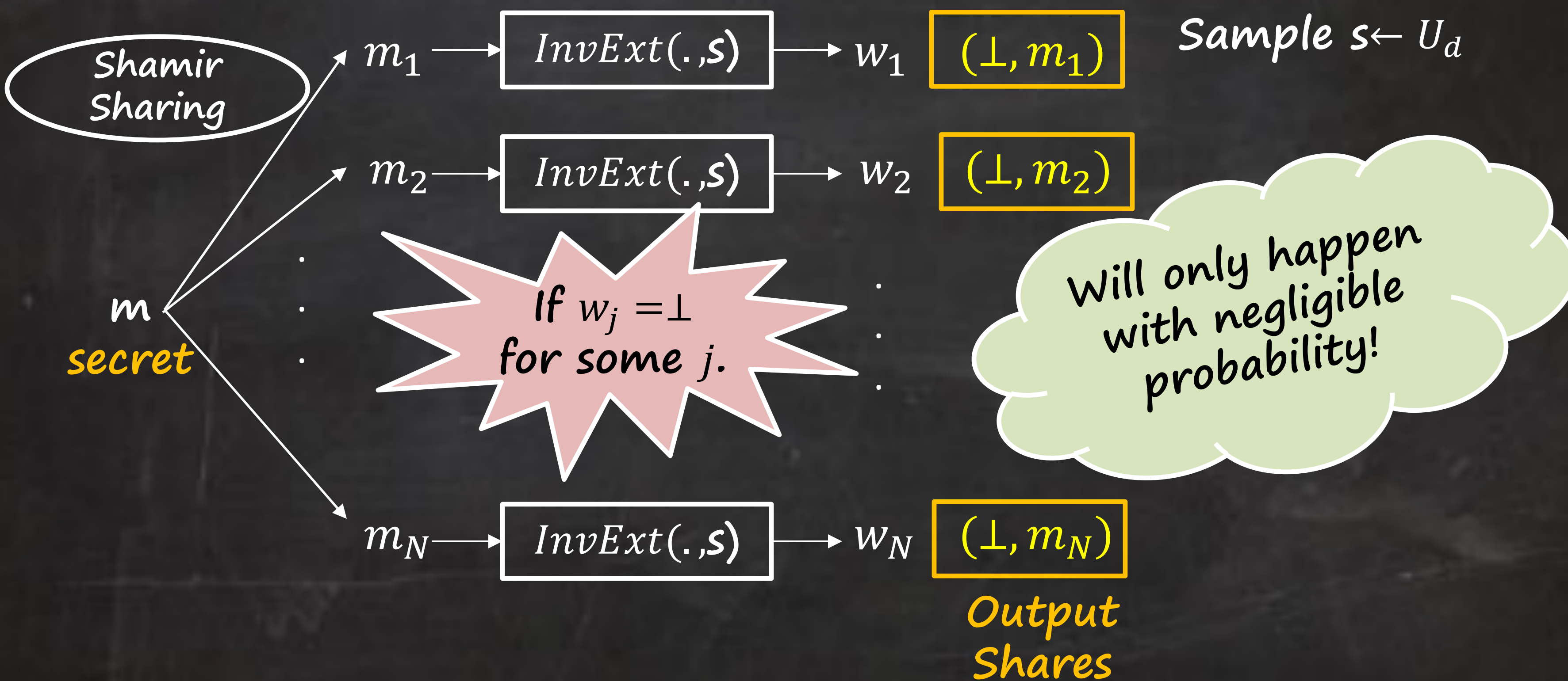
CASE I



Our Construction

Optimal Threshold LRSS: Leakage Resilience

CASE I



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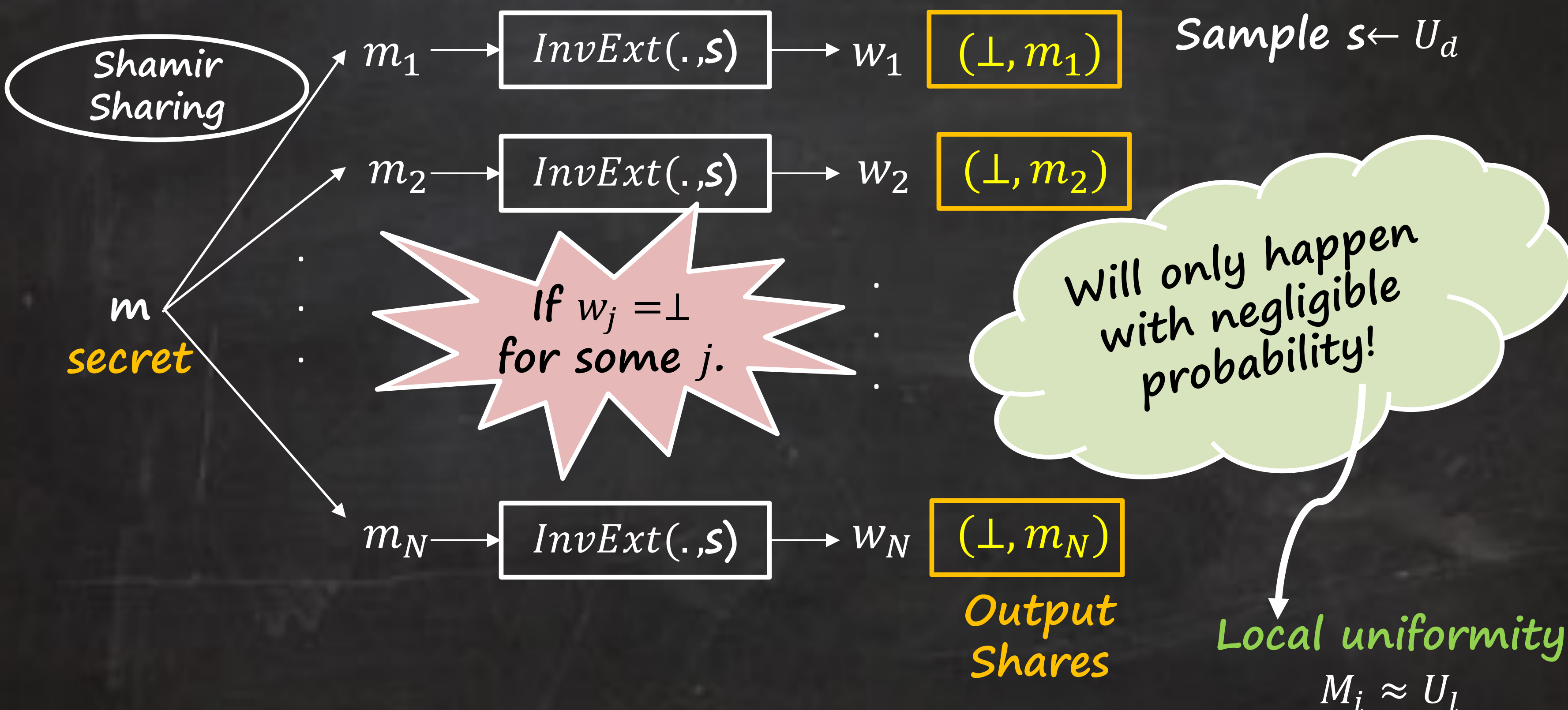

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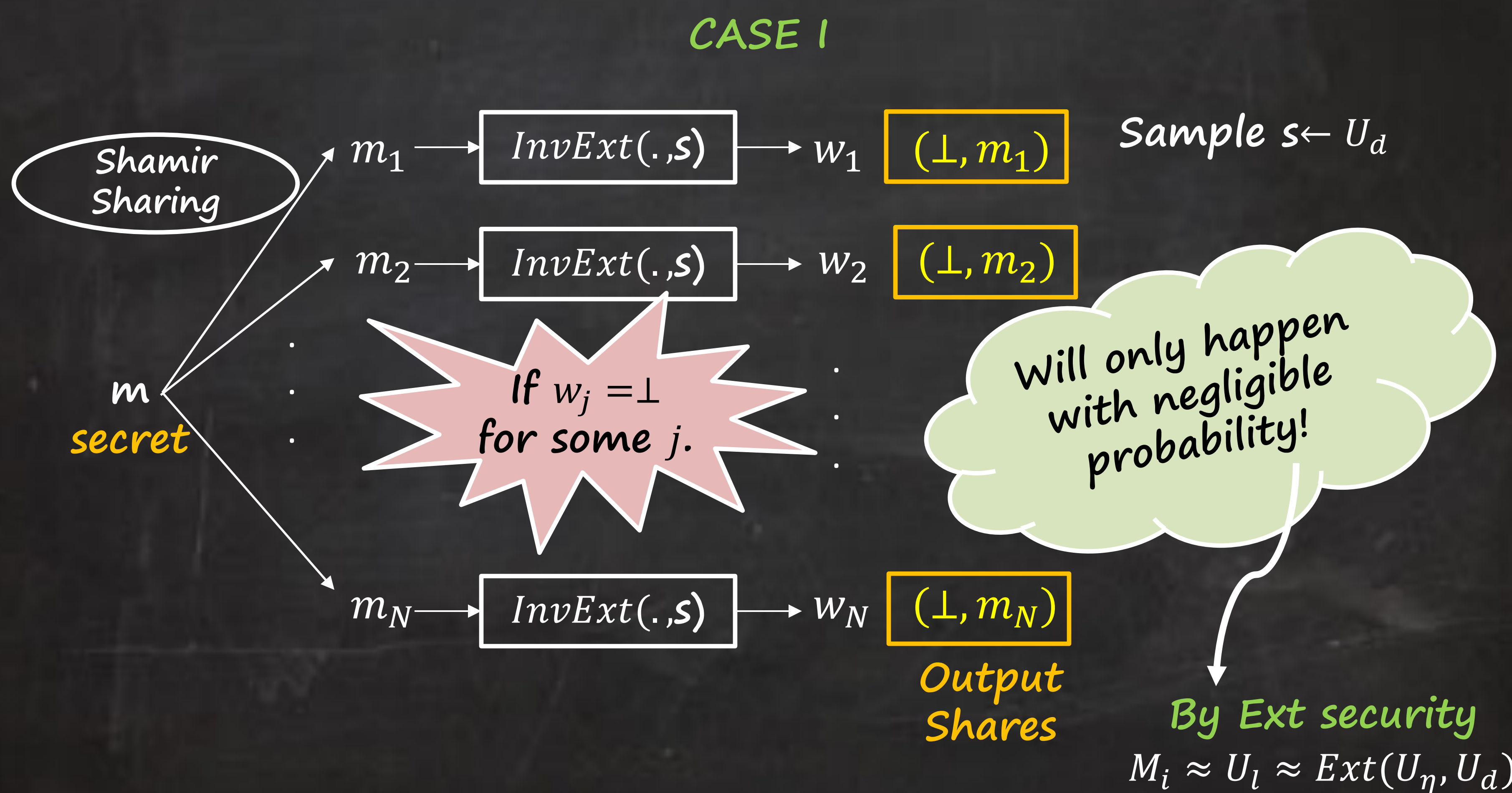

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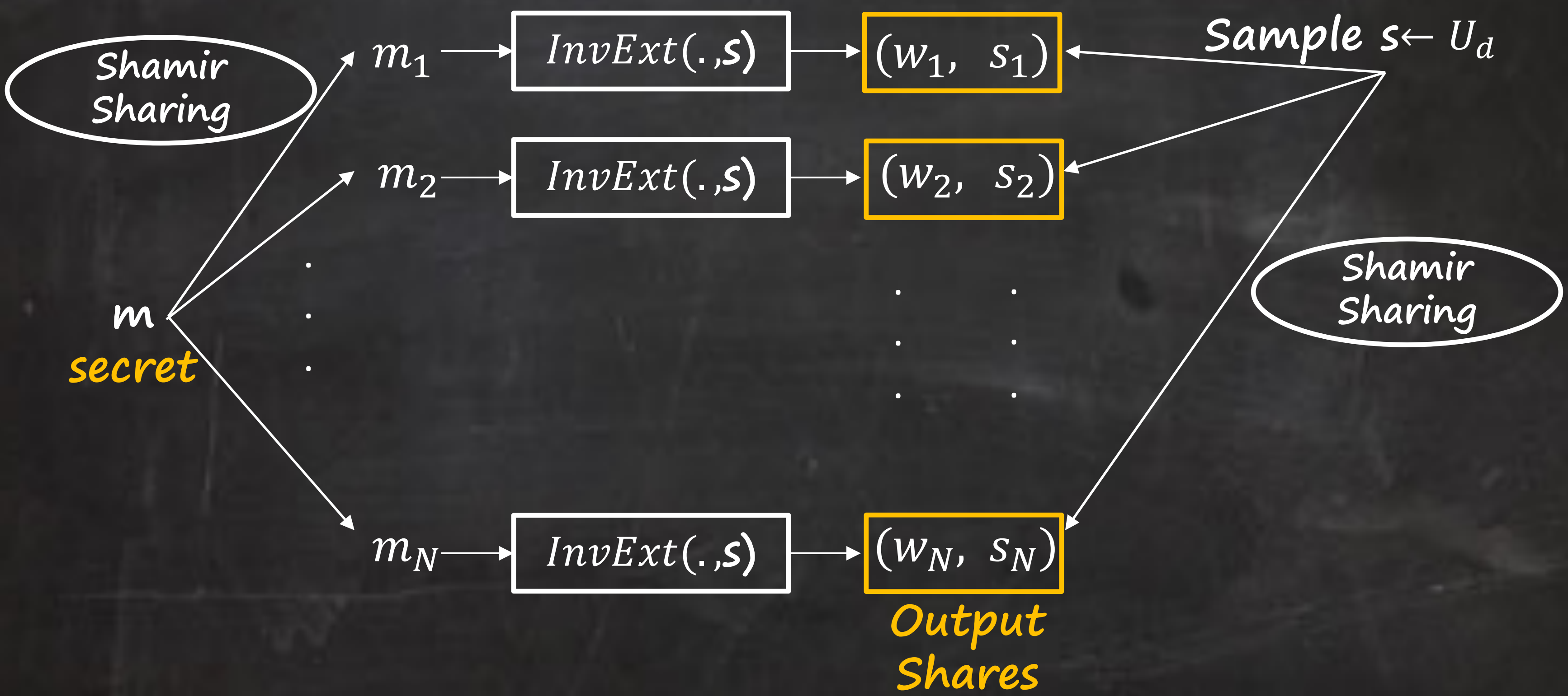
CASE II



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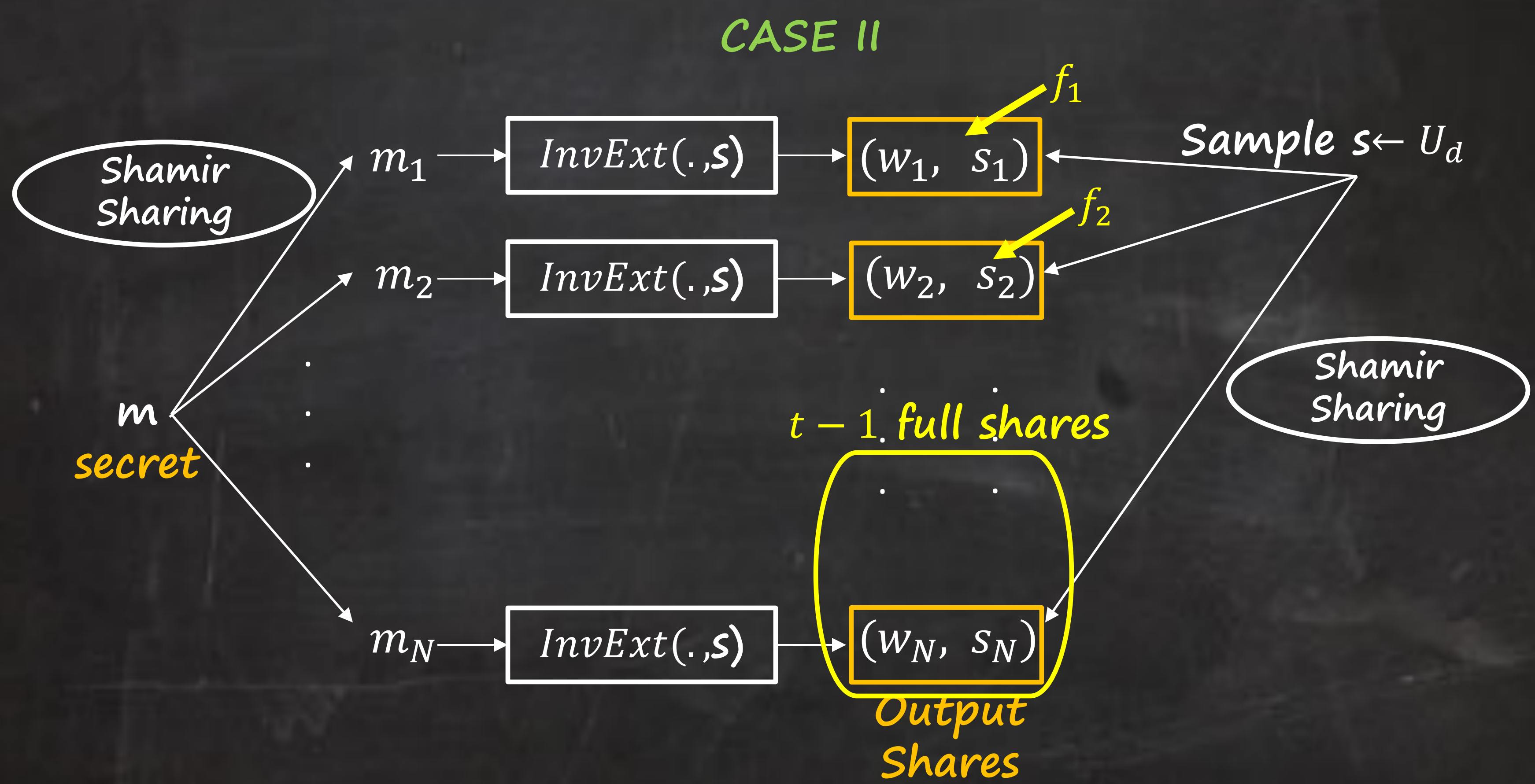

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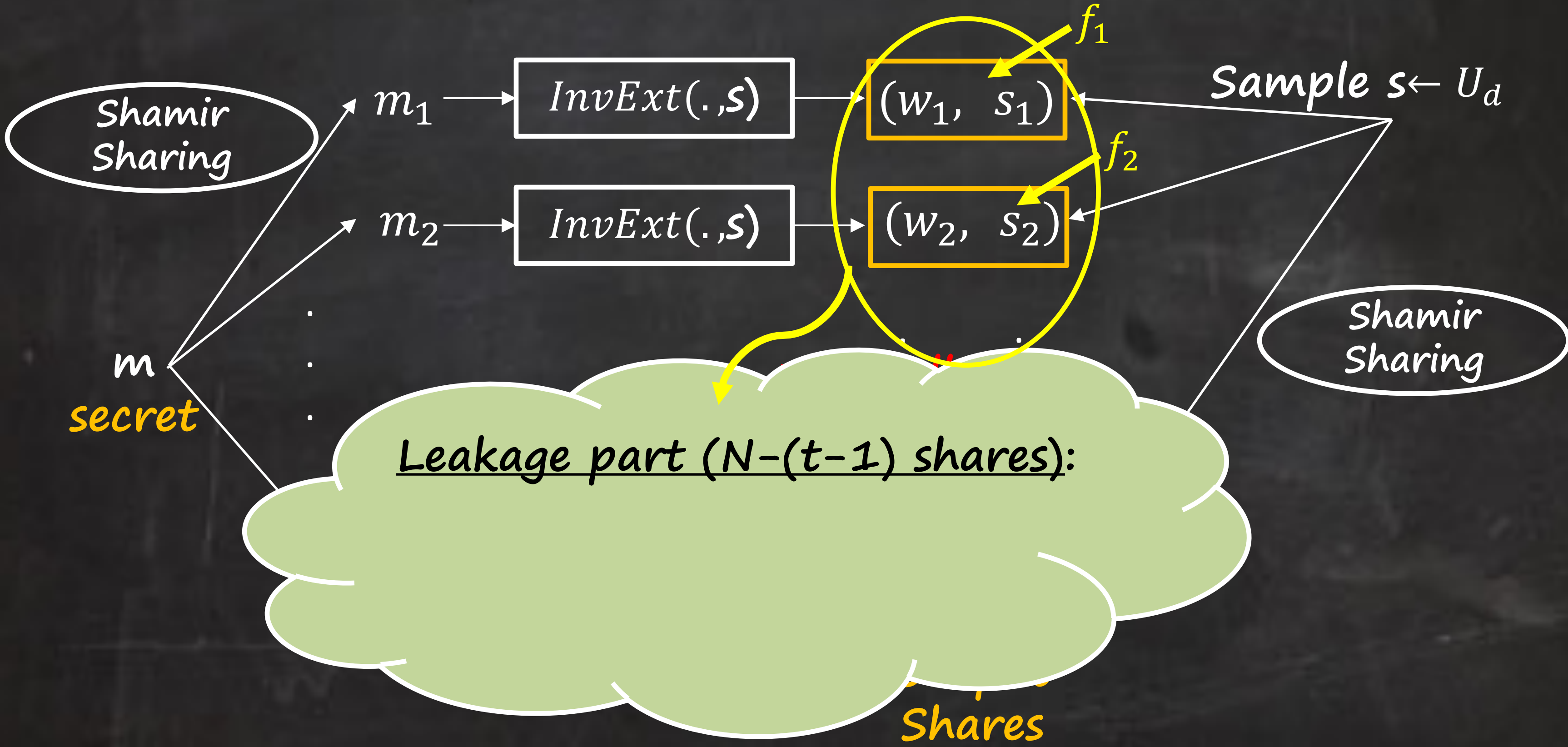

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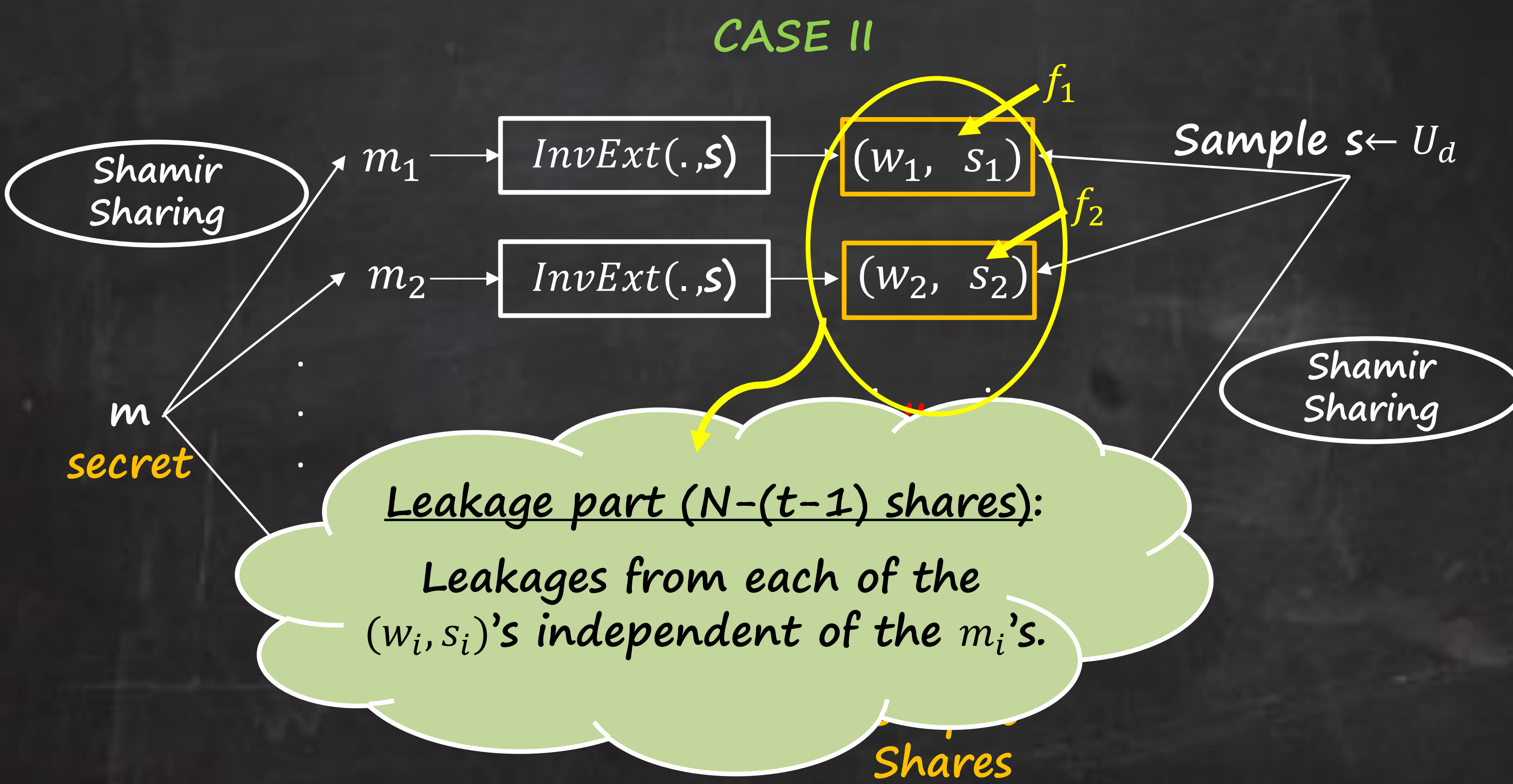

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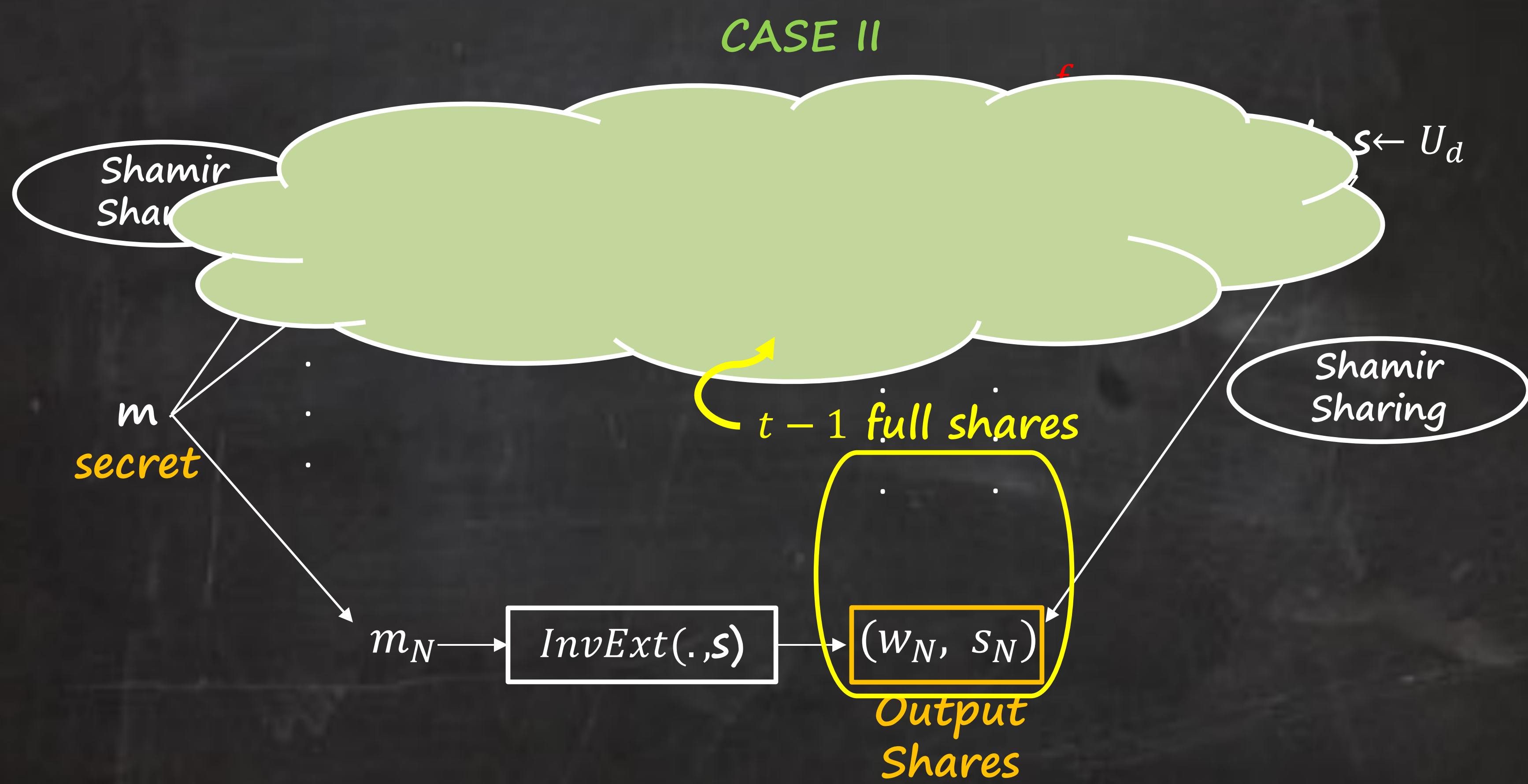

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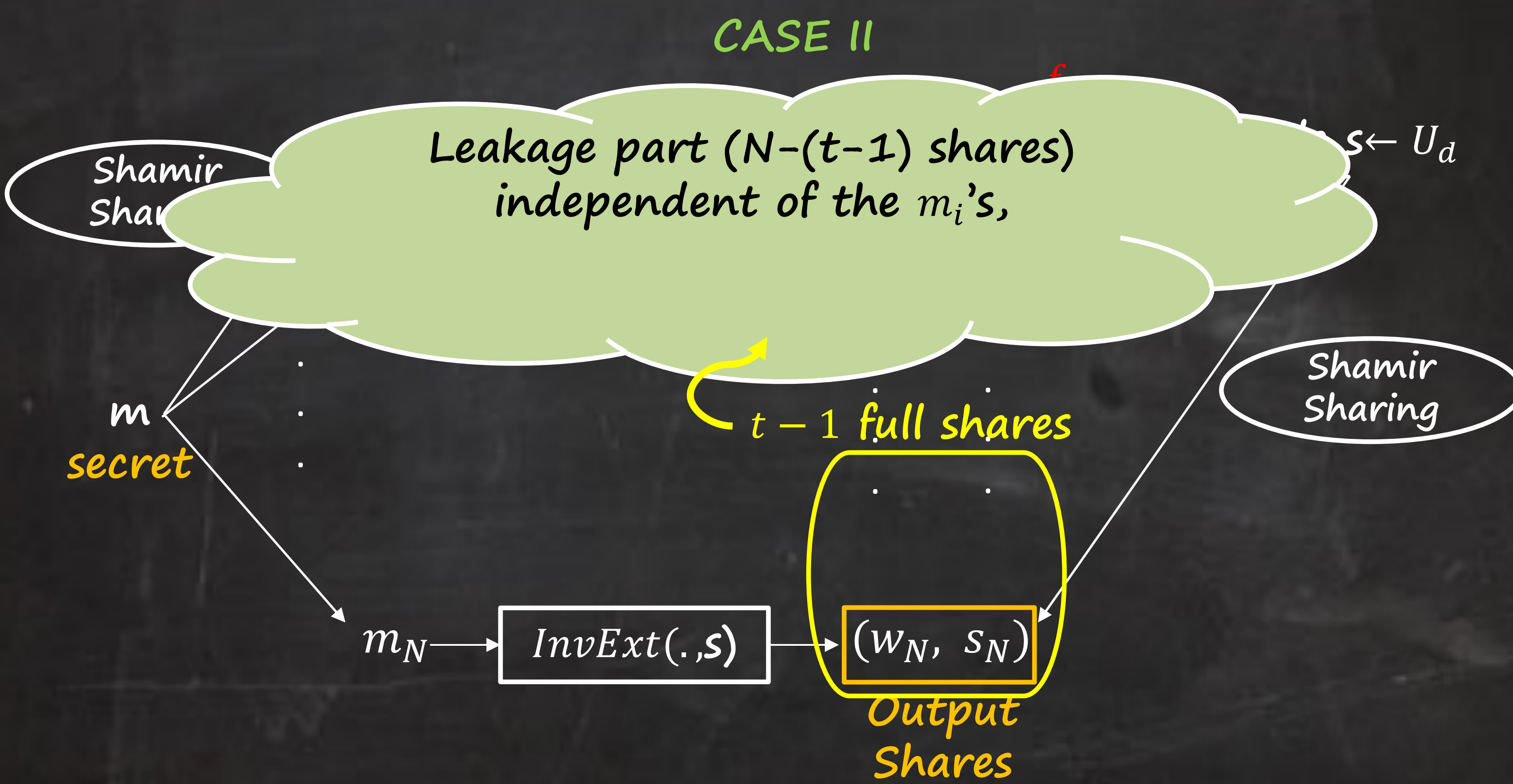

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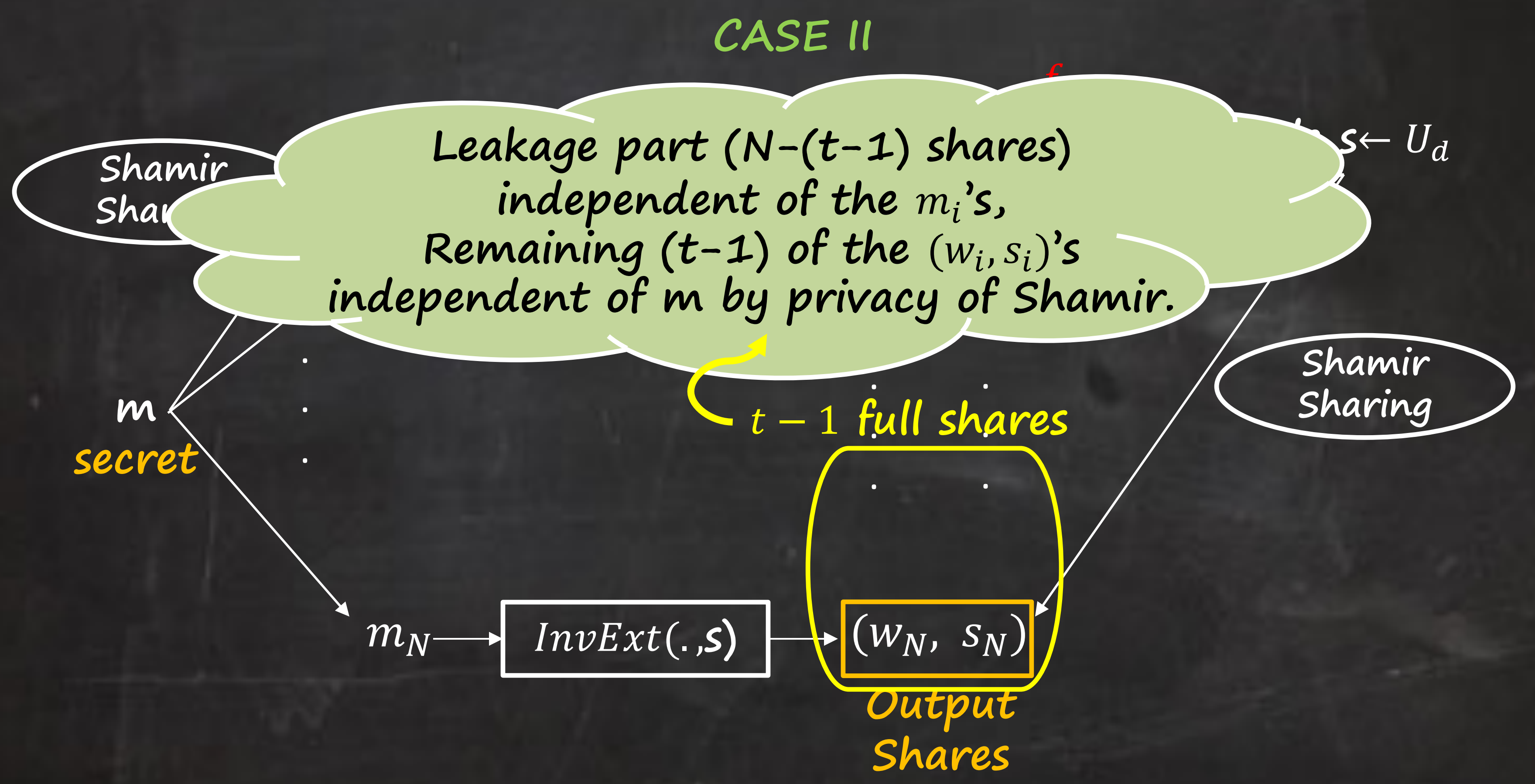

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THANK YOU

eprint/2022/216