CISPA
HELMHOLTZ CENTER FOR INFORMATION SECURITY

(Nondeterministic) Hardness vs. Non-Malleability

Marshall Ball (NYU), Dana Dachman-Soled (UMD), **Julian Loss (CISPA)**

Error Correcting Codes

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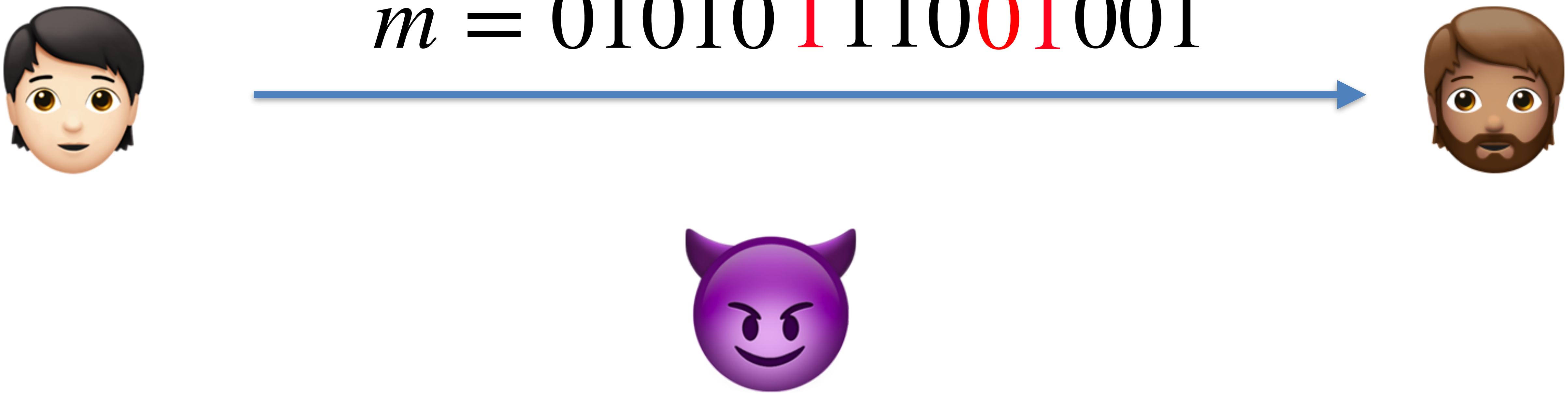

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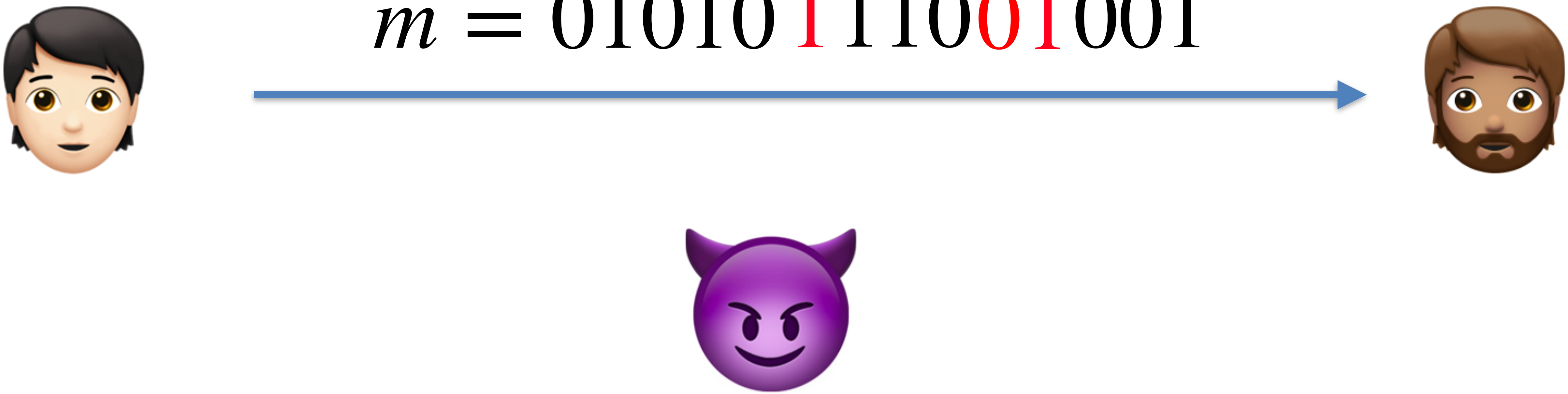

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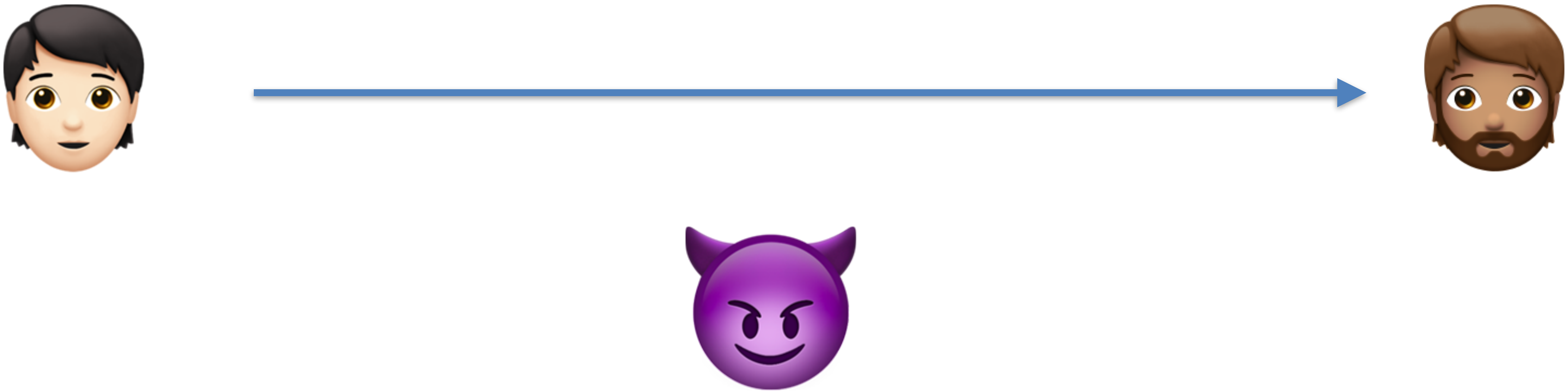

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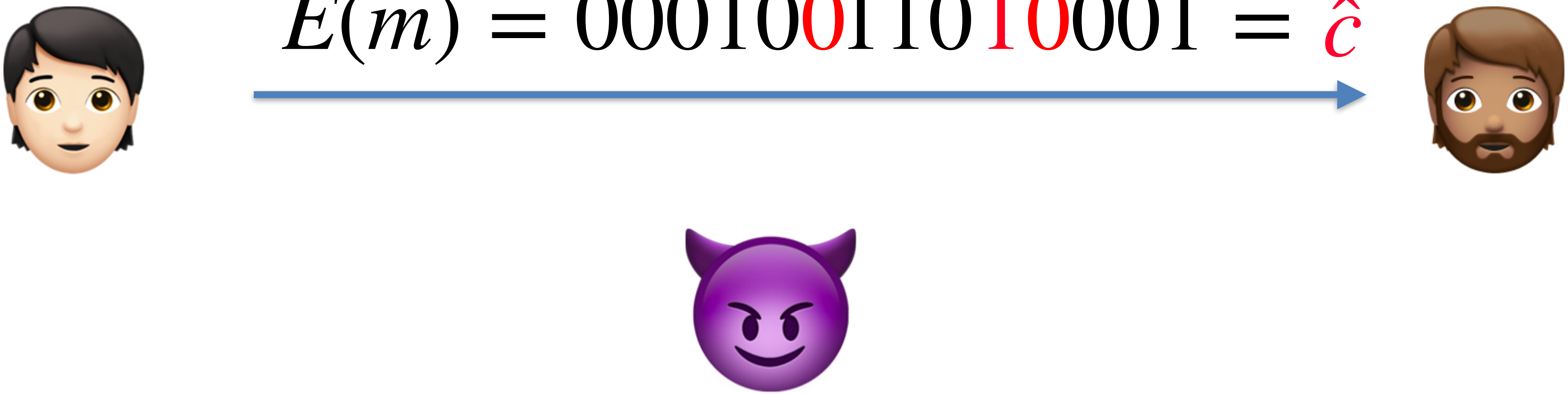

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- **What if $\hat{c}$ doesn't decode to m?**

$$E(m) = 0001001101010001 = \hat{c}$$

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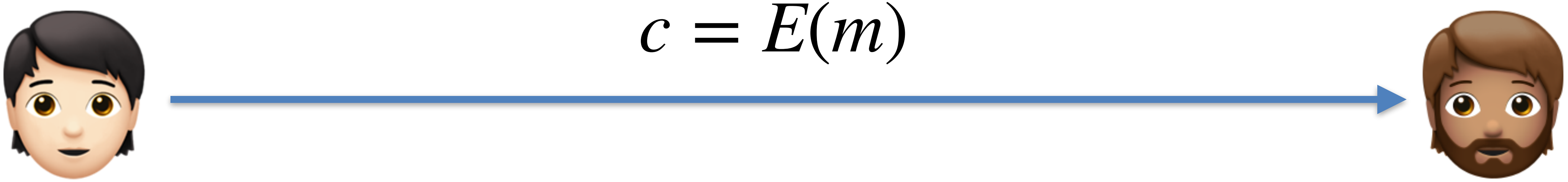
Tampering Attack

$$c = E(m)$$



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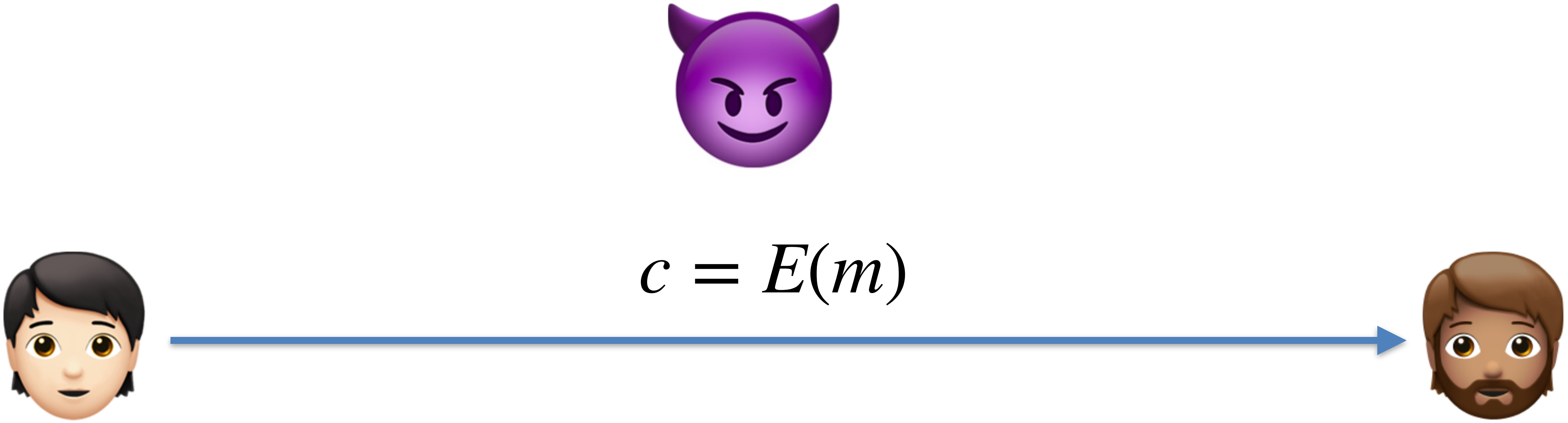

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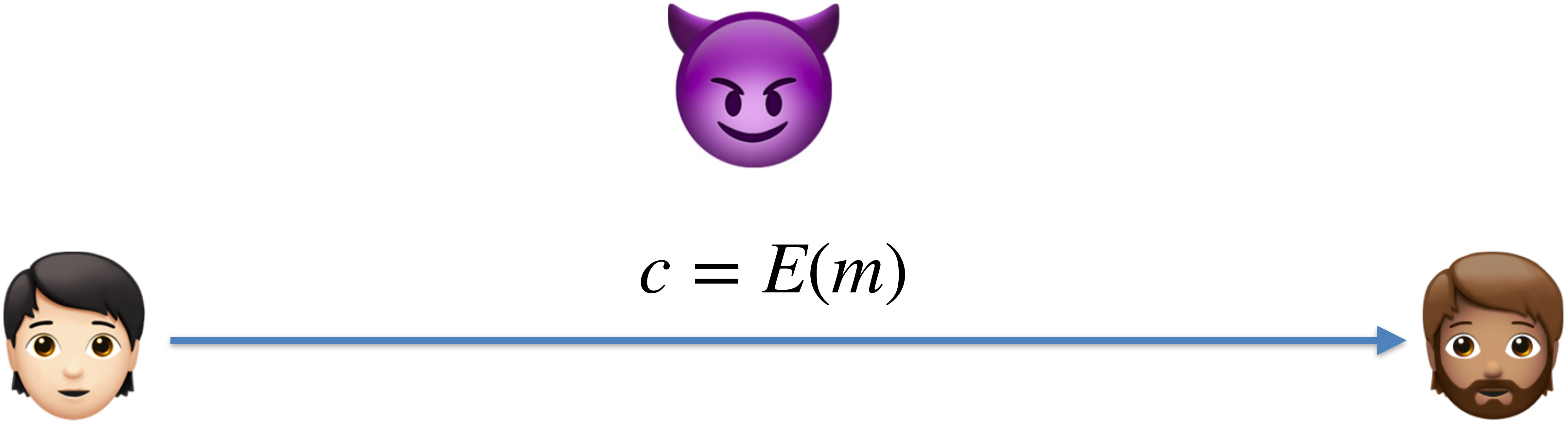

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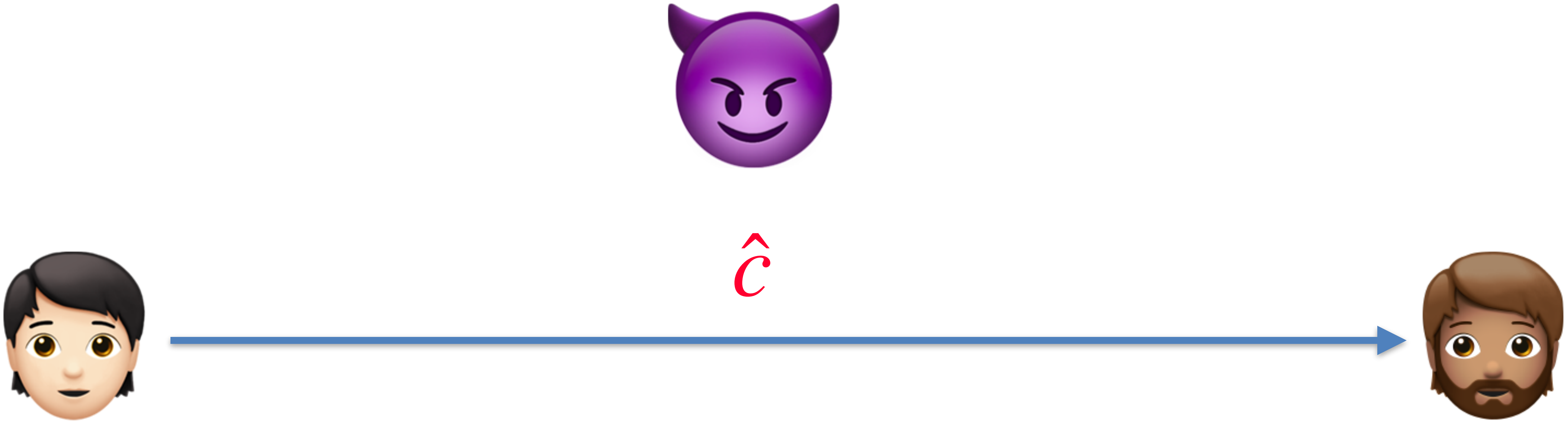

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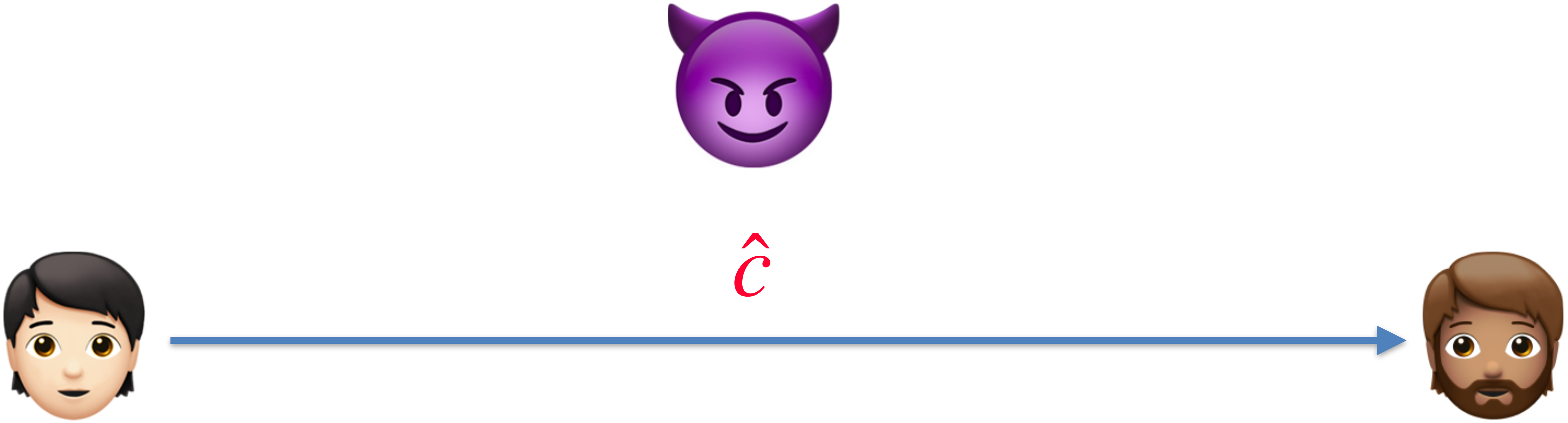

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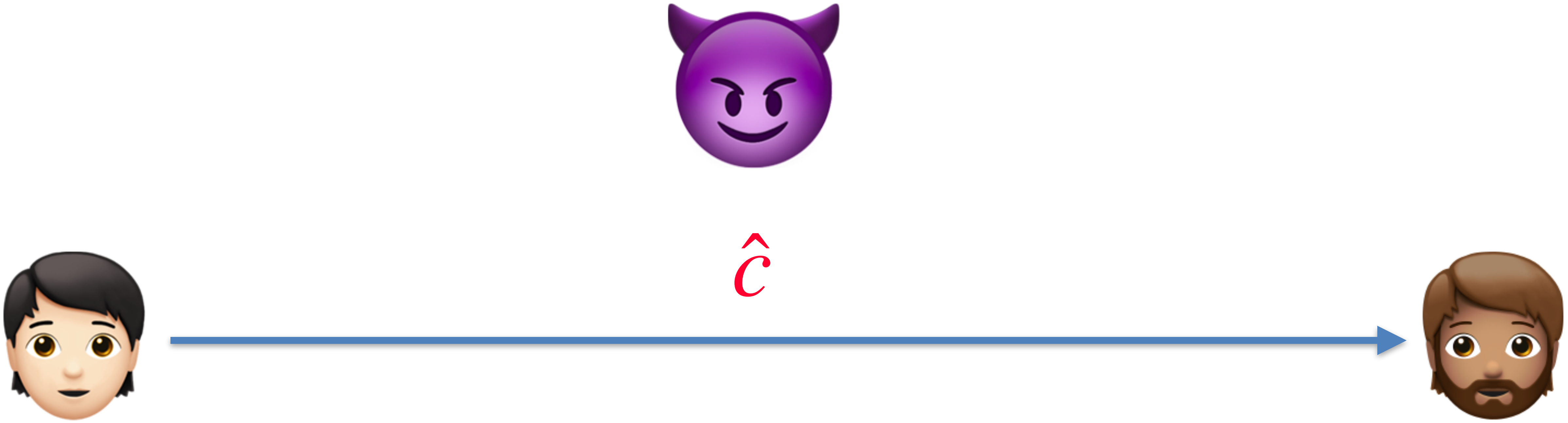

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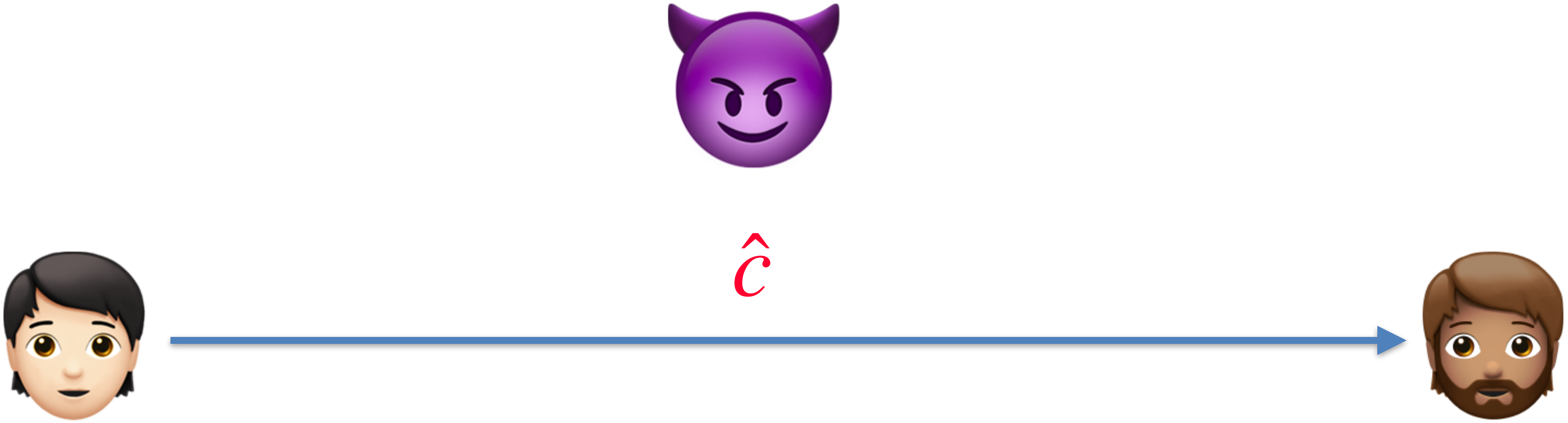

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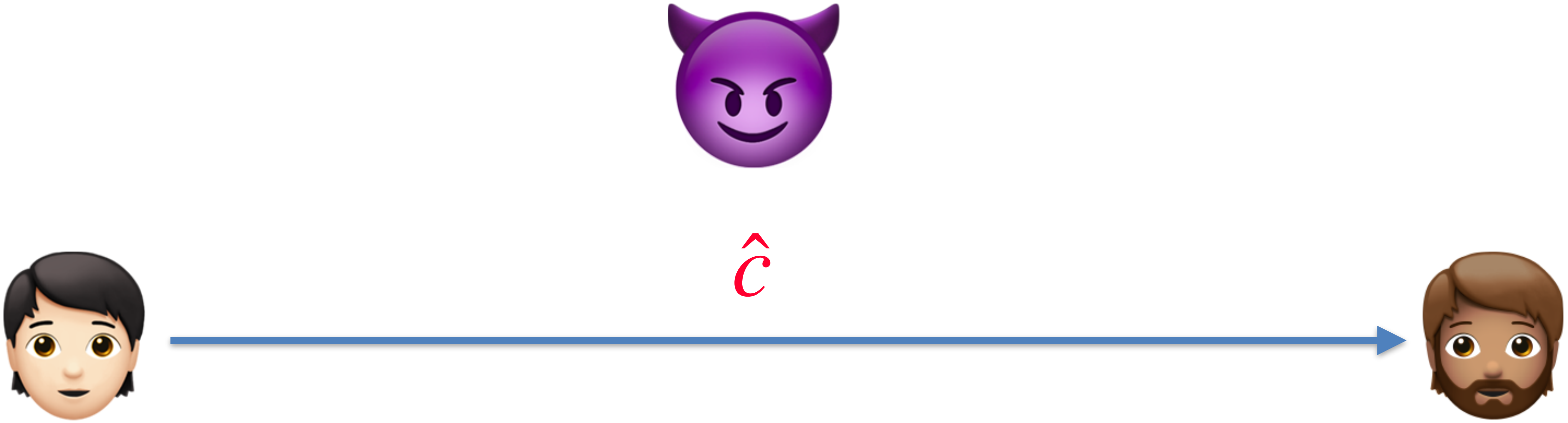

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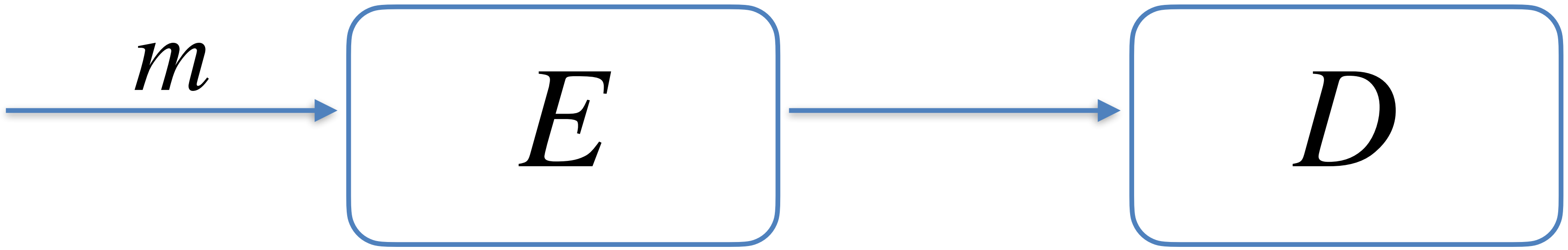

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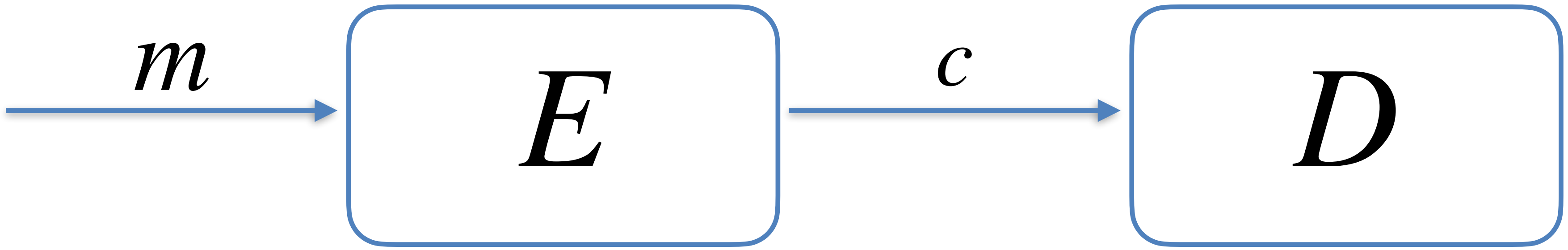

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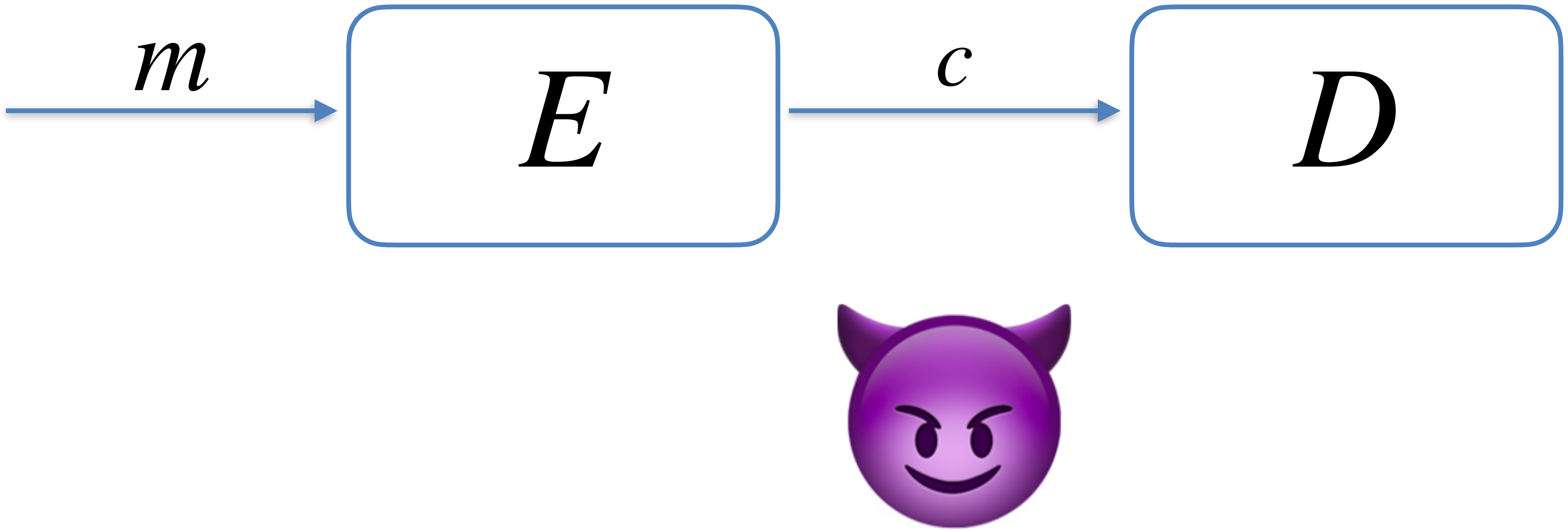

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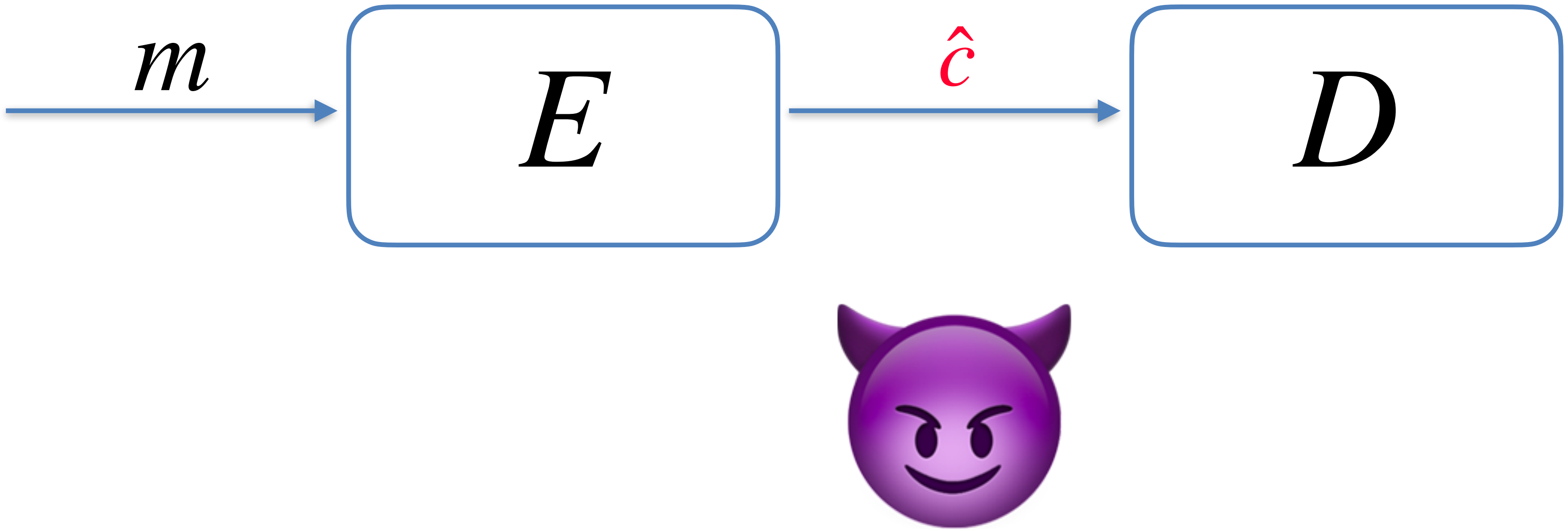

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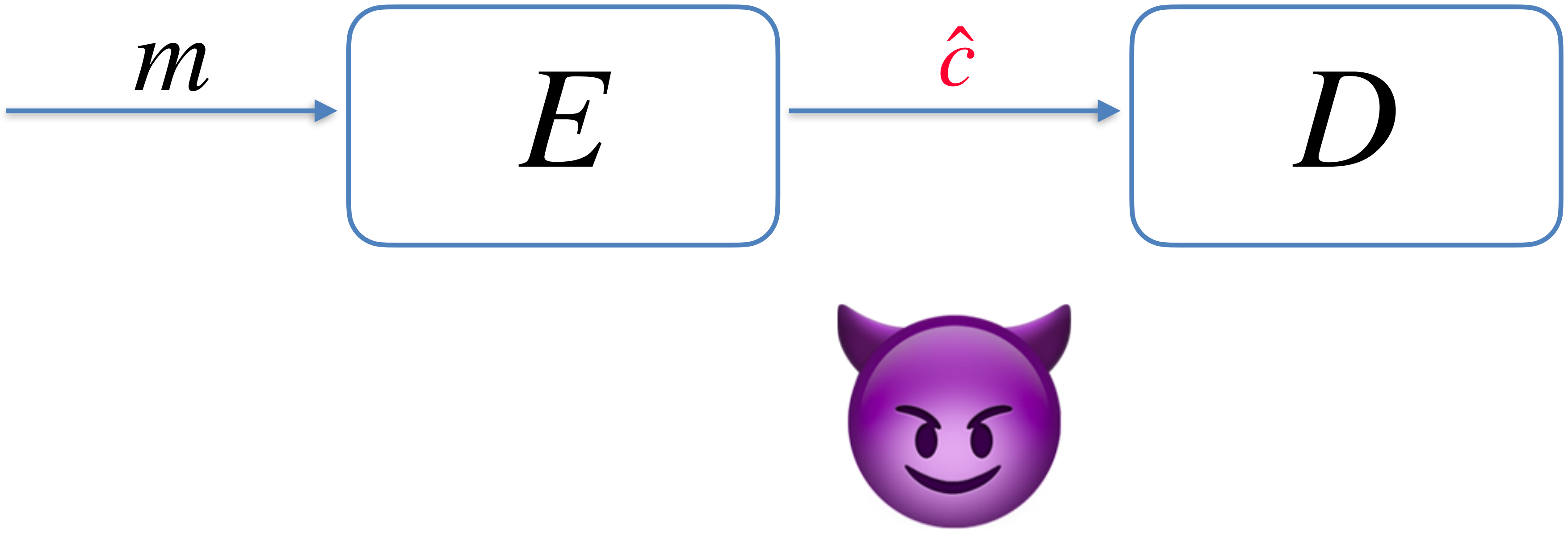

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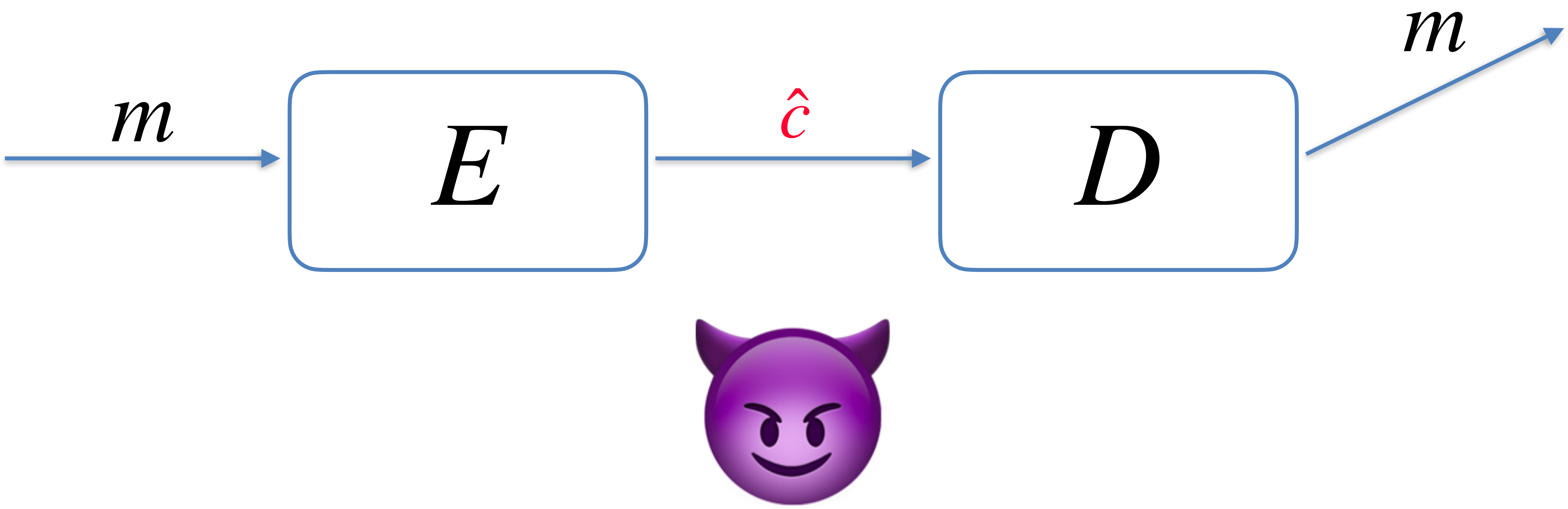

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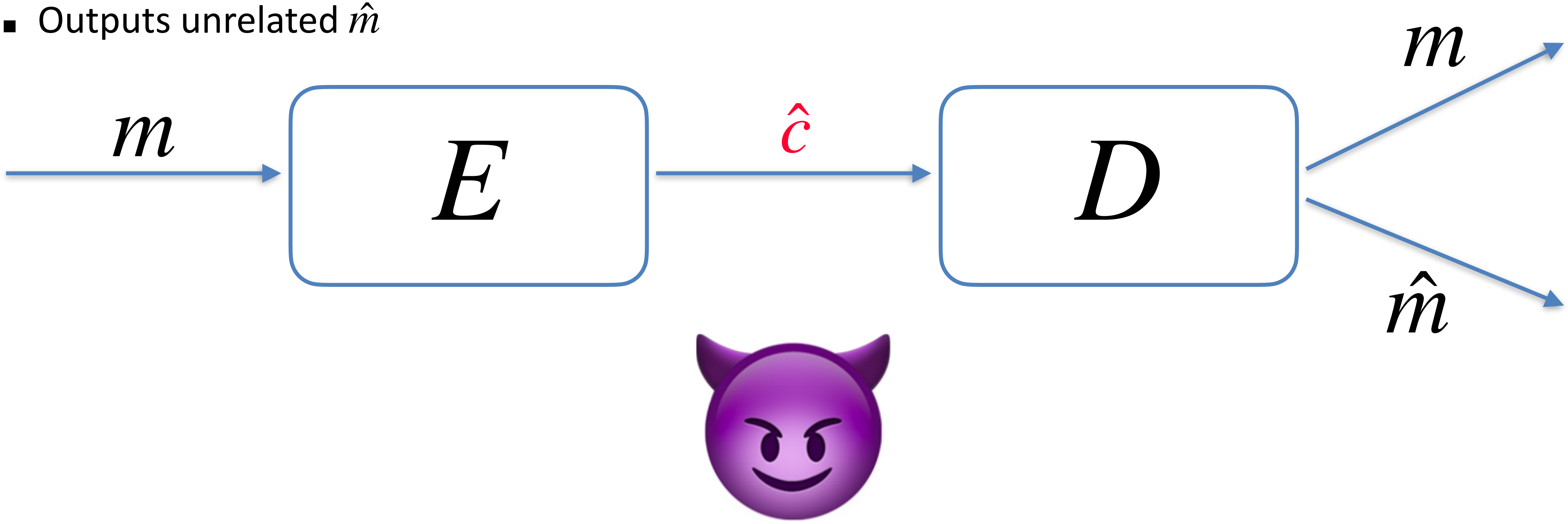

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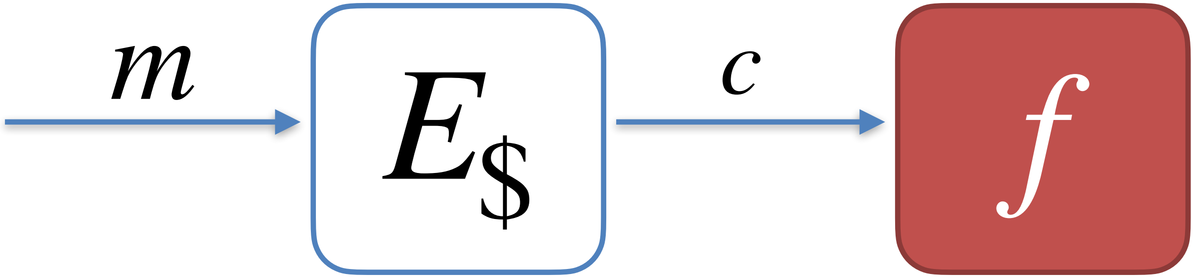

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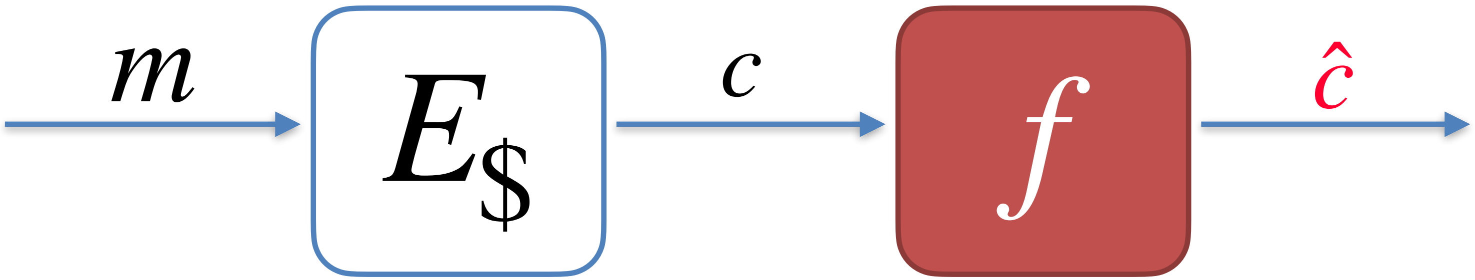

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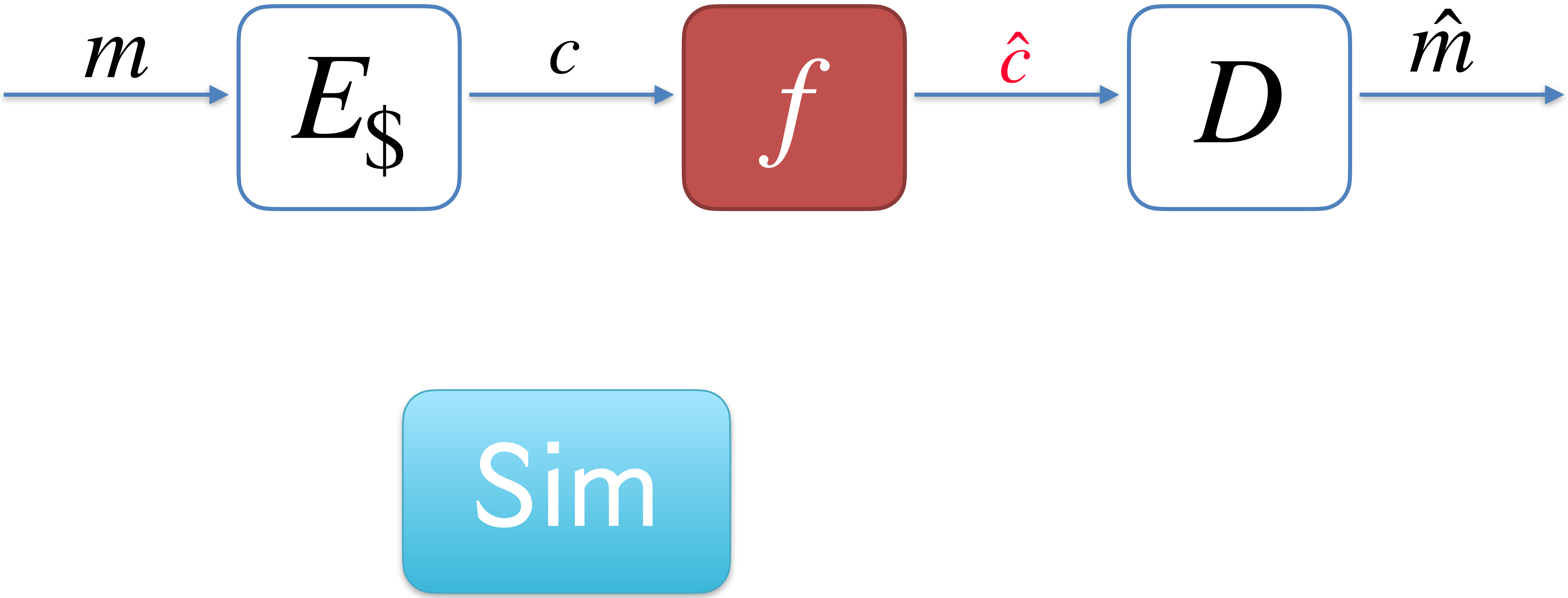

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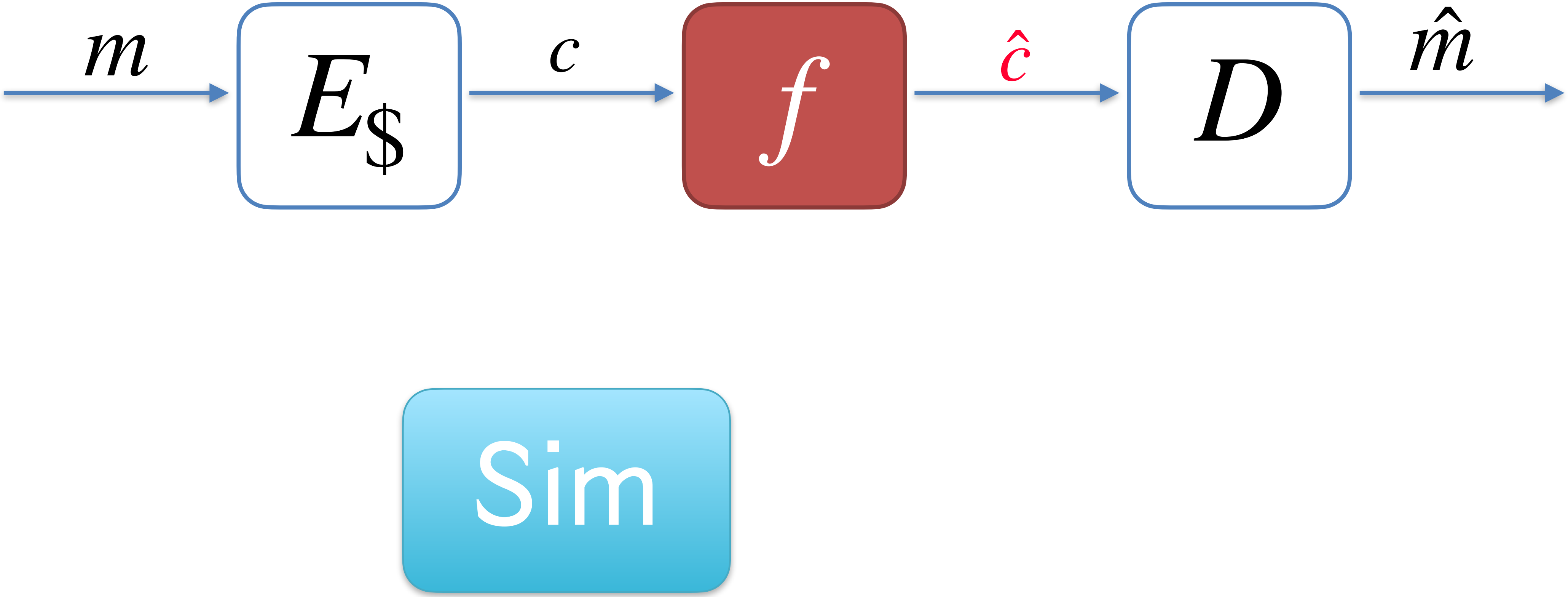

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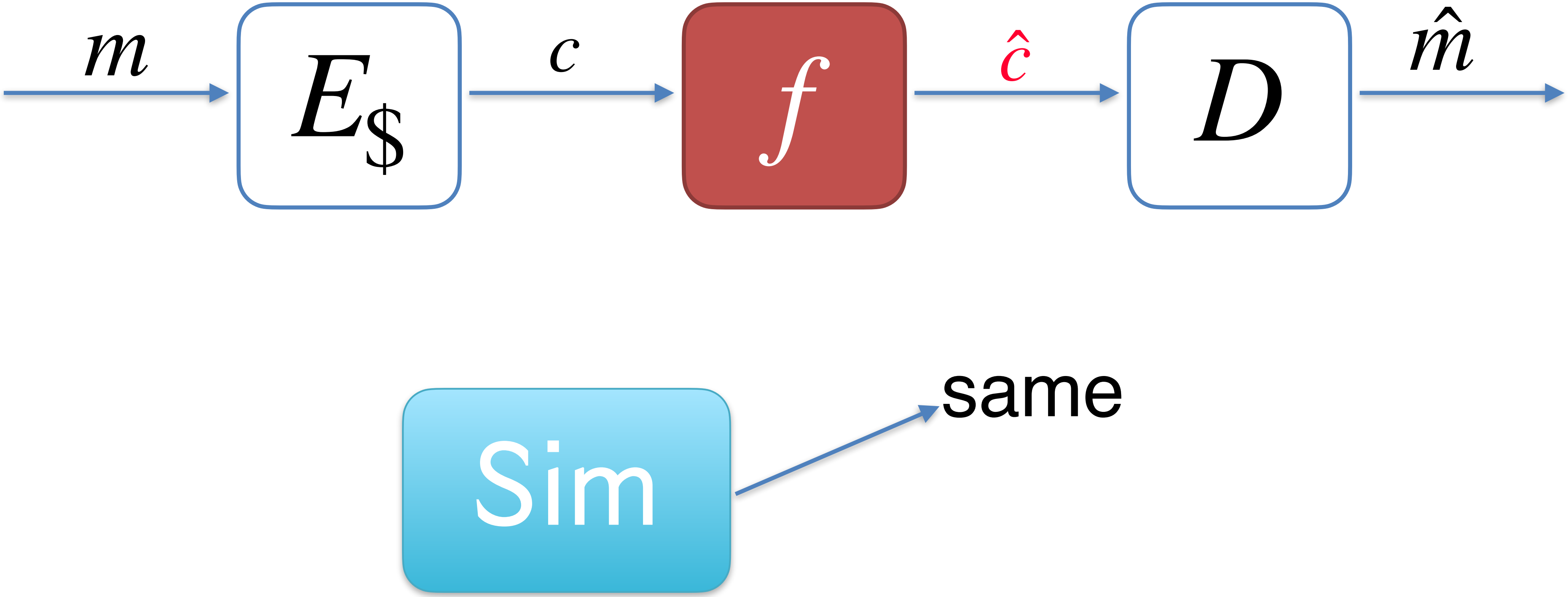

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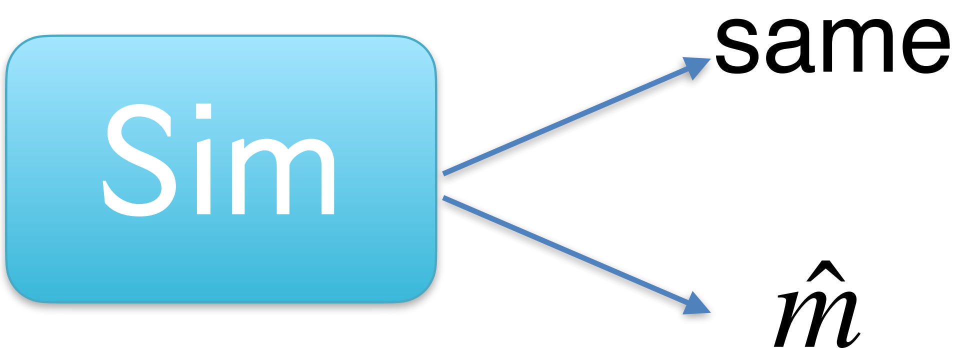

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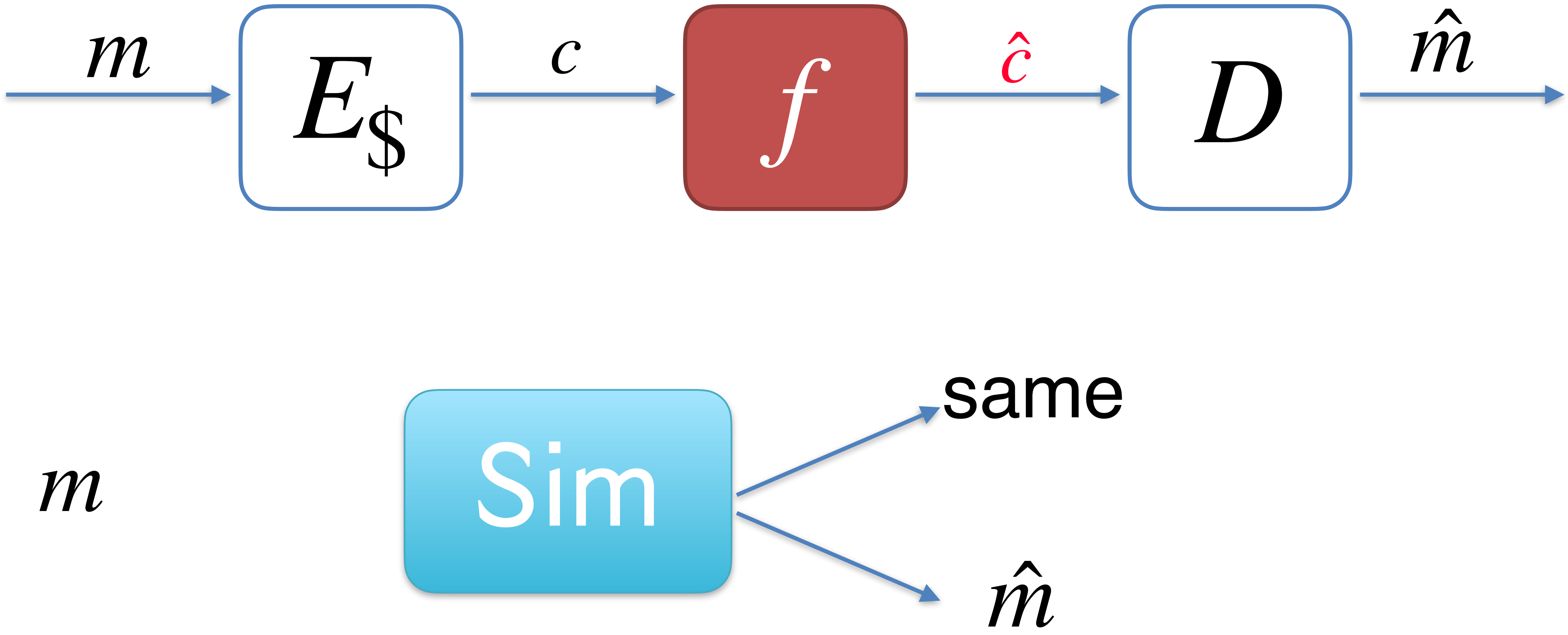

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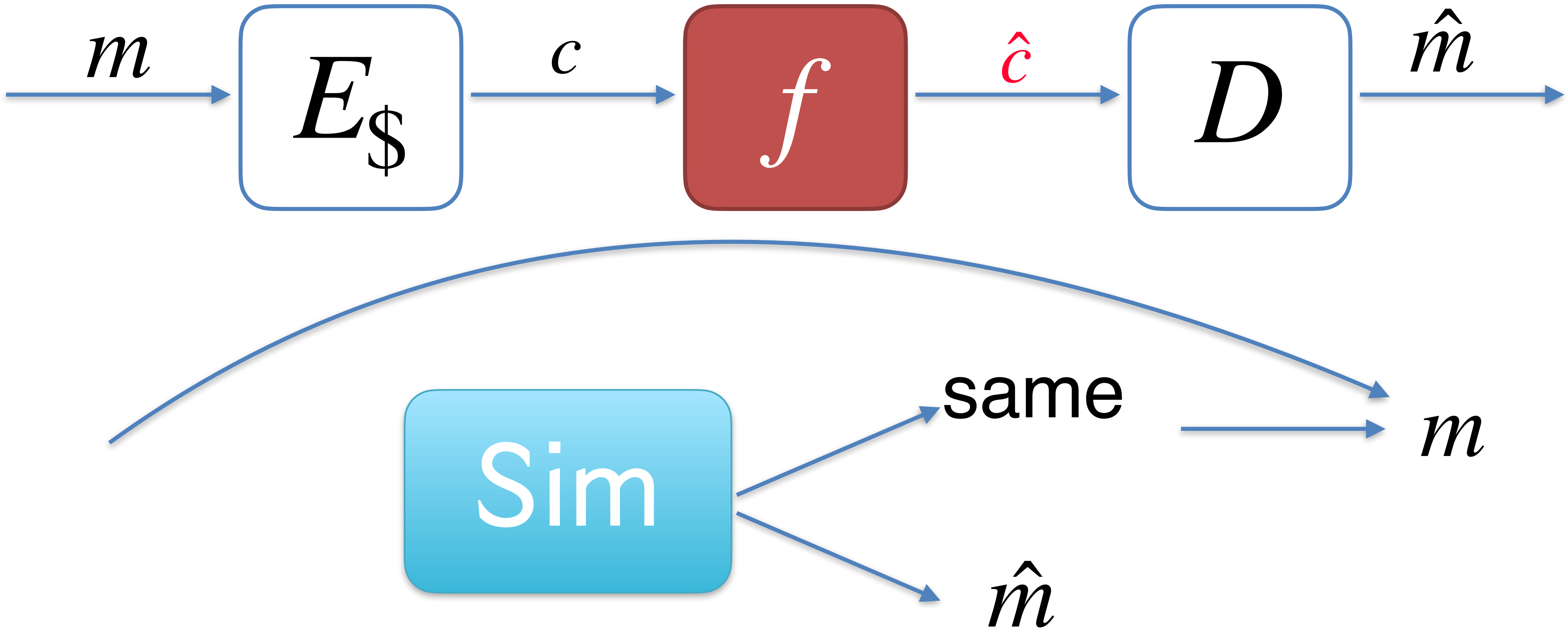

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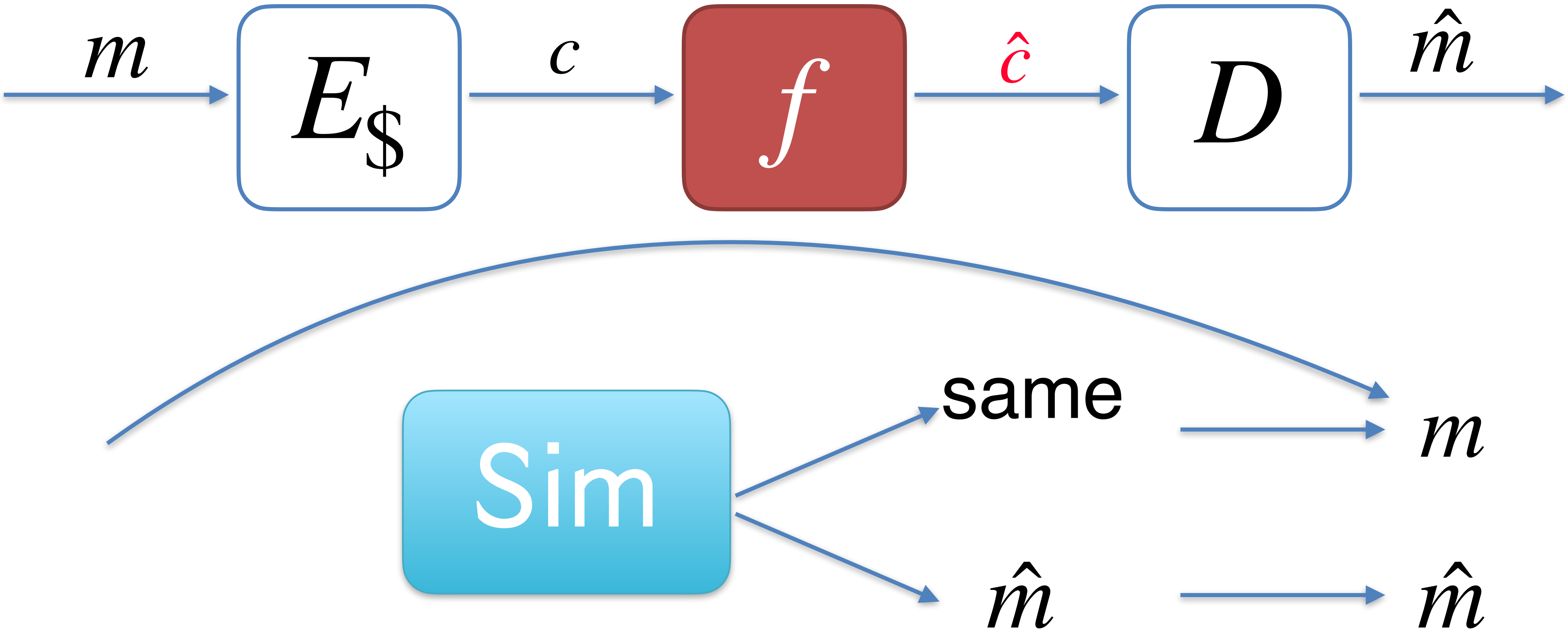

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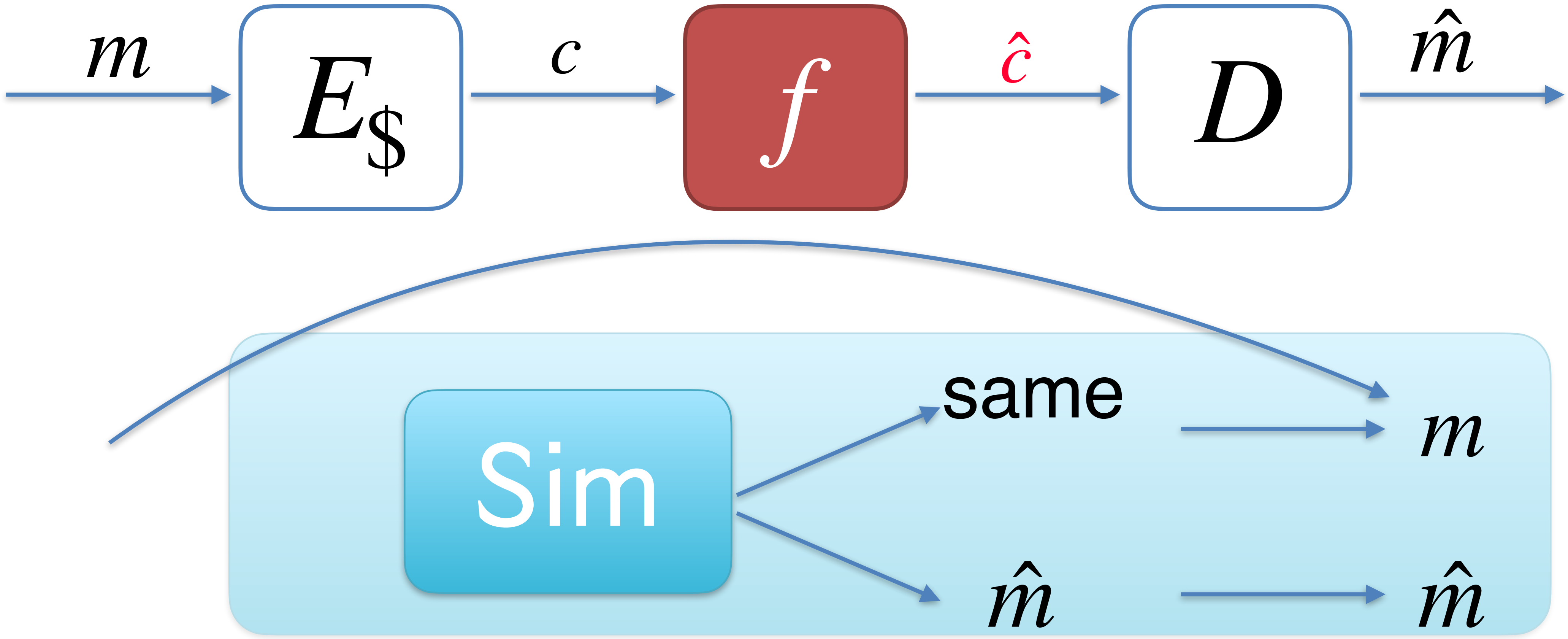

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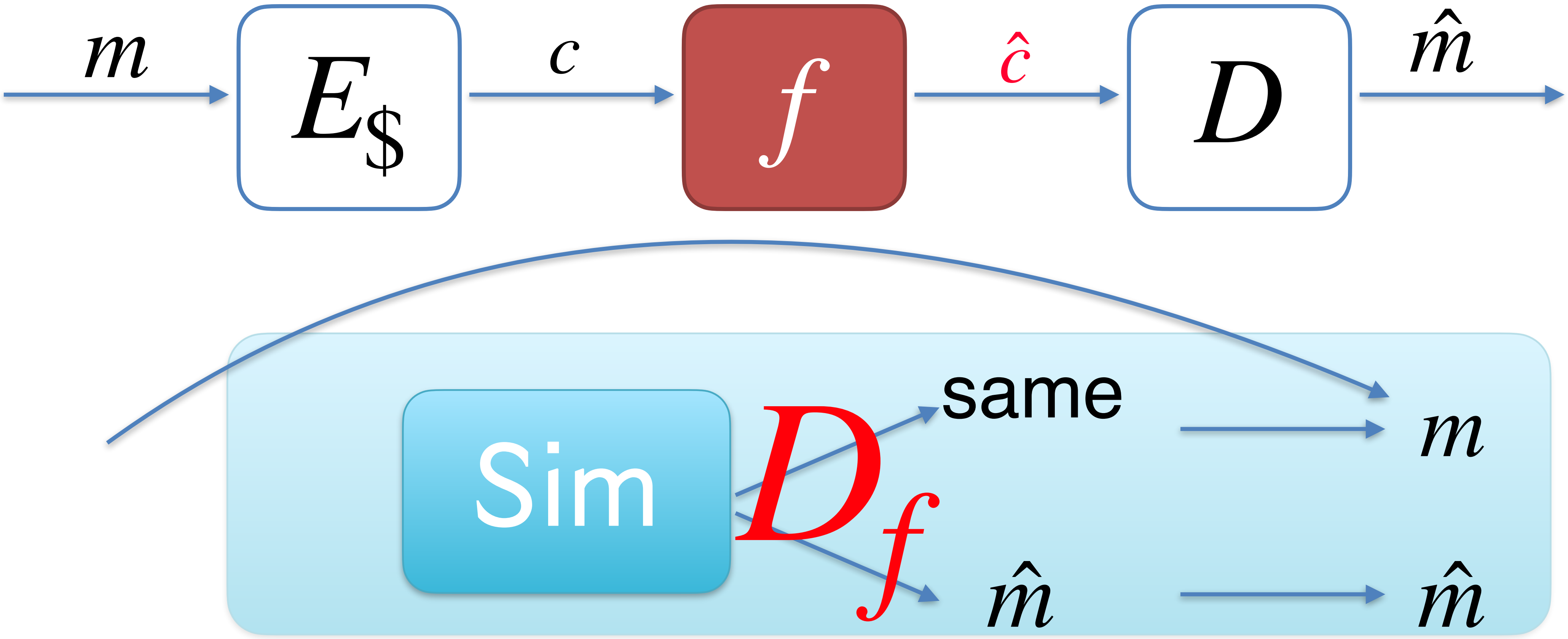

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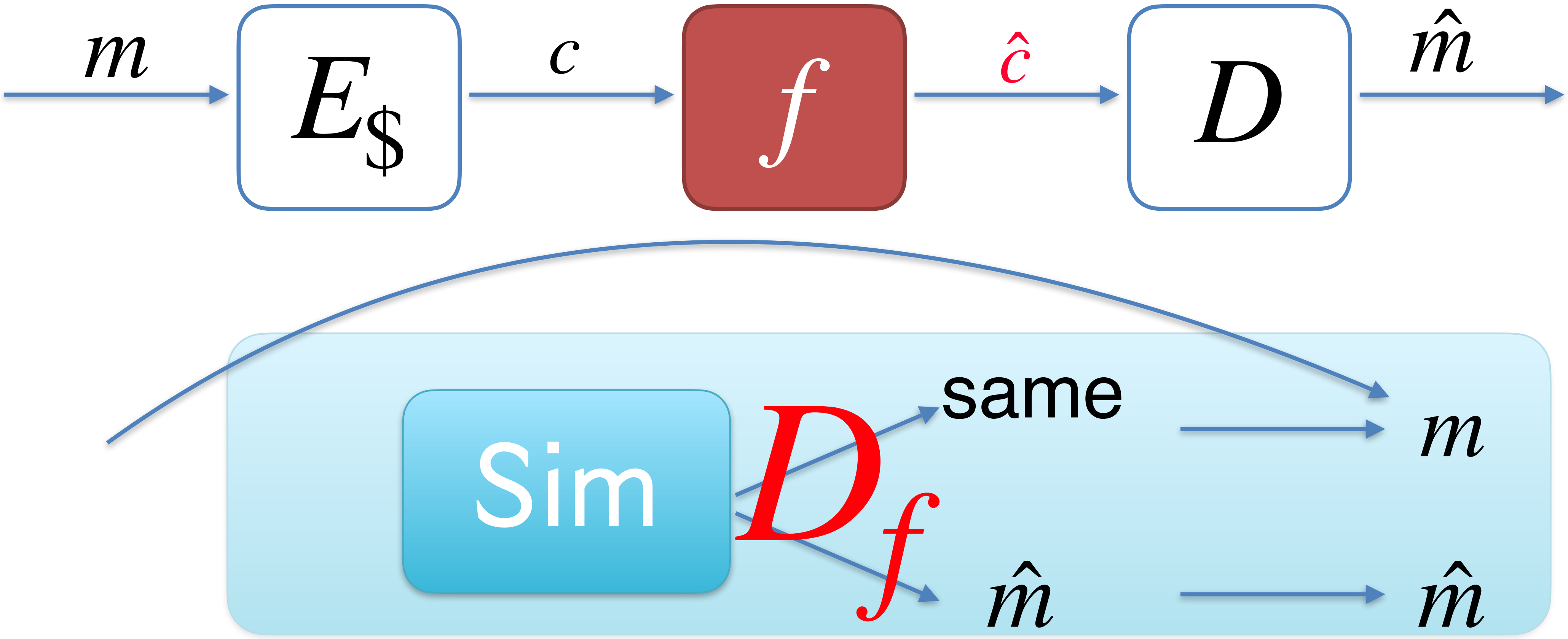

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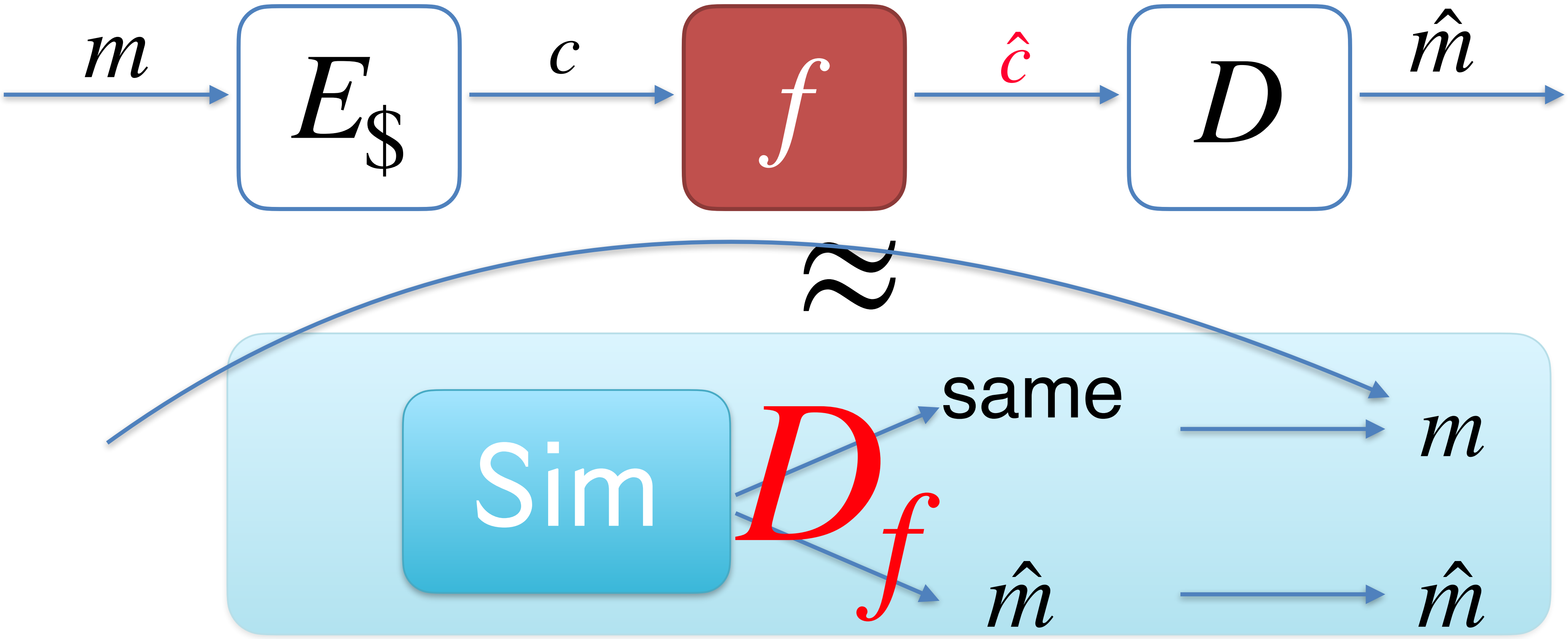

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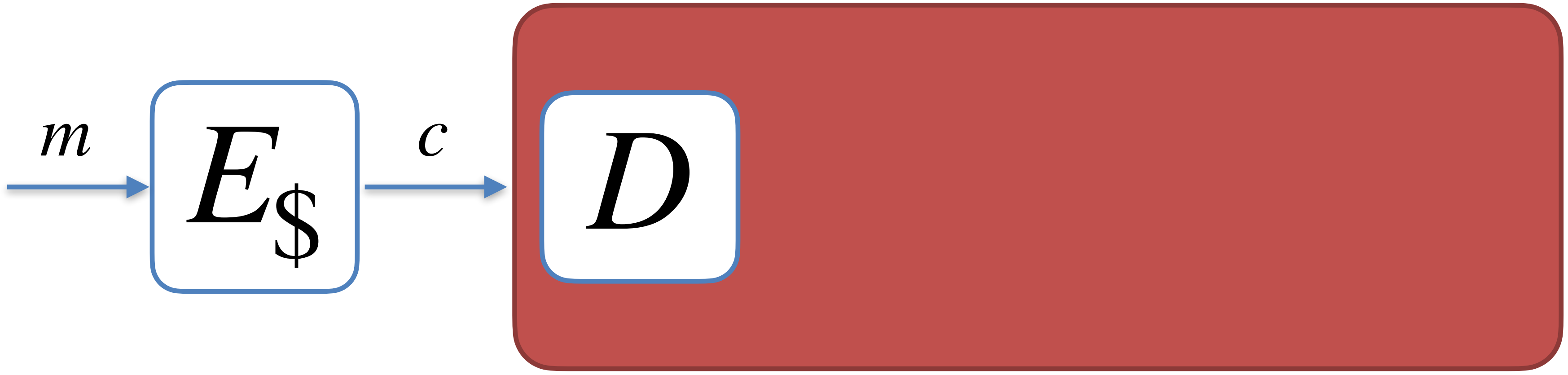

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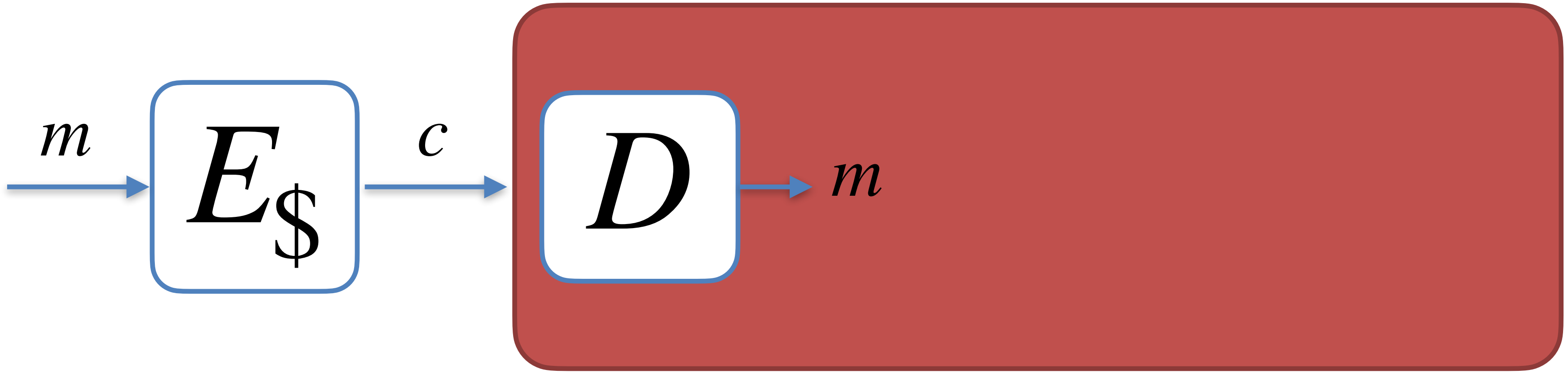

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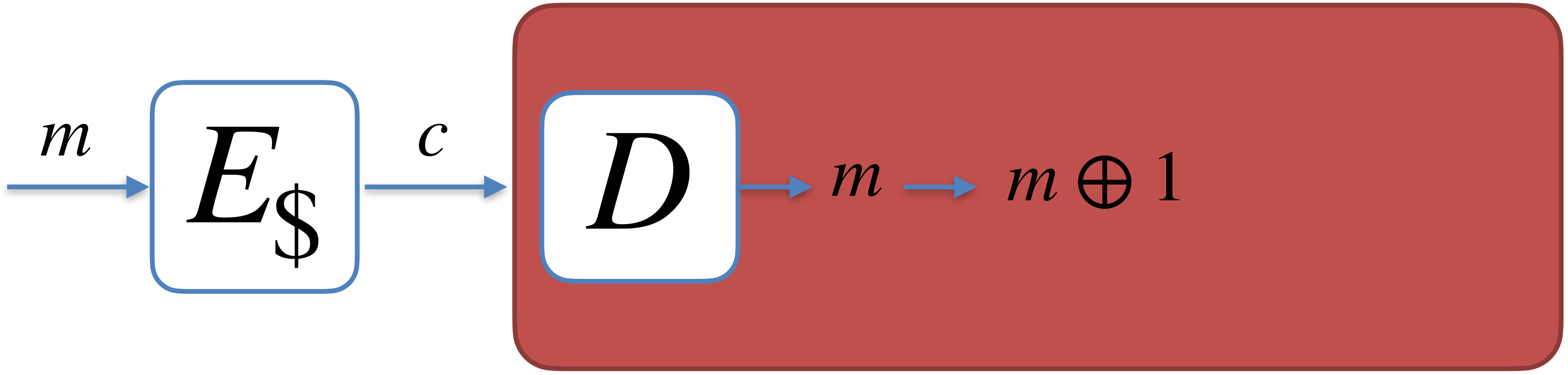

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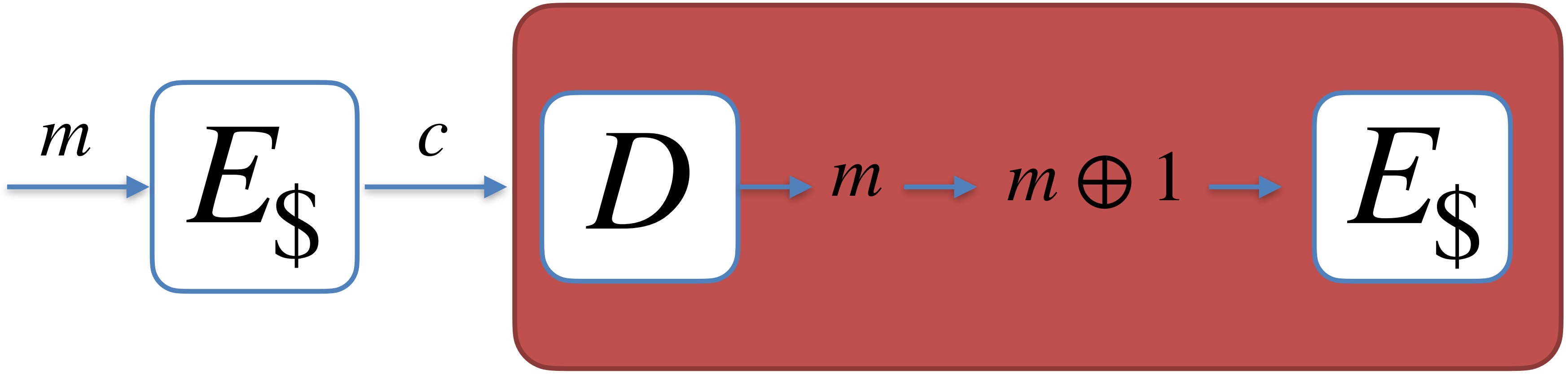

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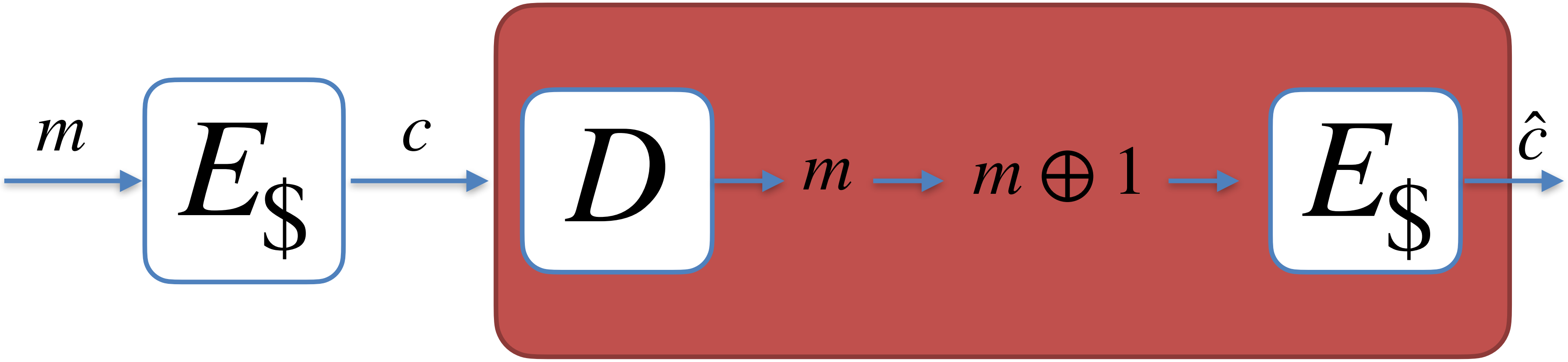

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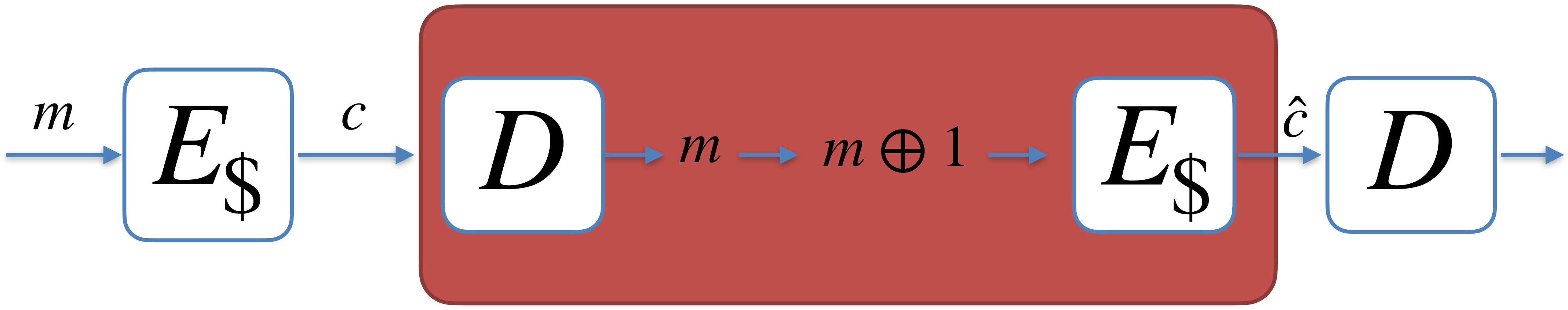

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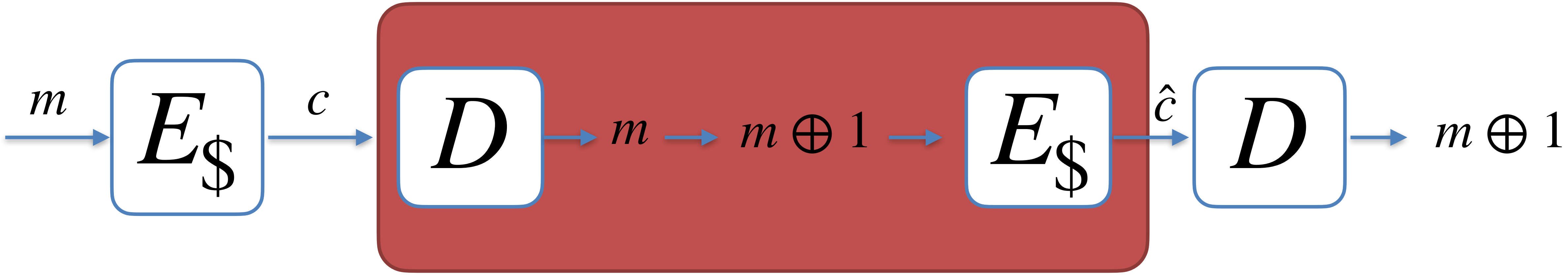

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- **Solution:** assume such lower bounds!

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- Properties:
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  - Orthogonal to crypto (to the best of our knowledge)
- **Theorem:** Suppose that Conjecture 1 is true. Then, for all constants c, there exists an (explicit) n^{-c}- NMC for n^c-sized circuits.

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- **Solution:** Non-deterministic reduction + strong statistical tool

Bounded Communication Tampering

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$$E(m)$$

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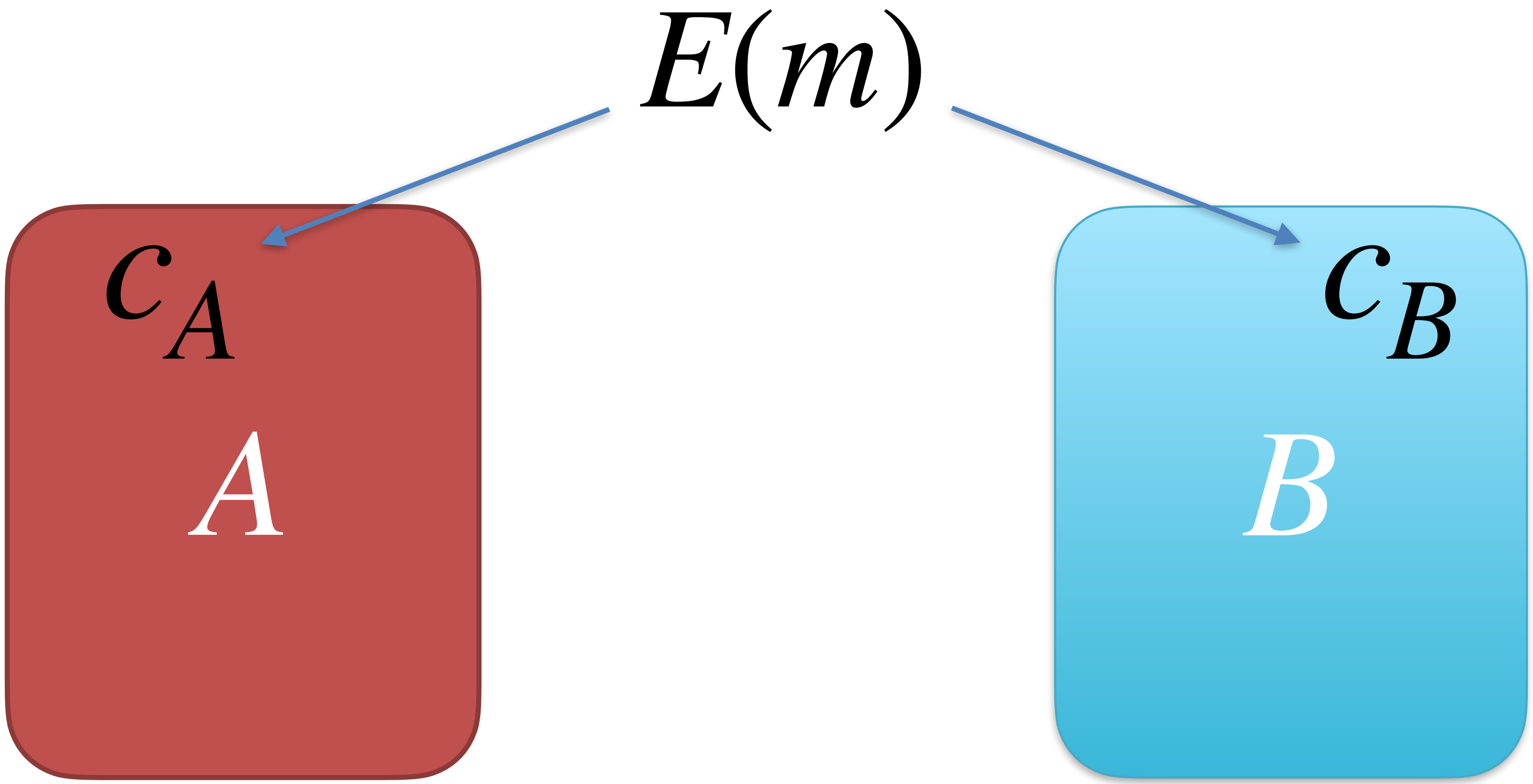

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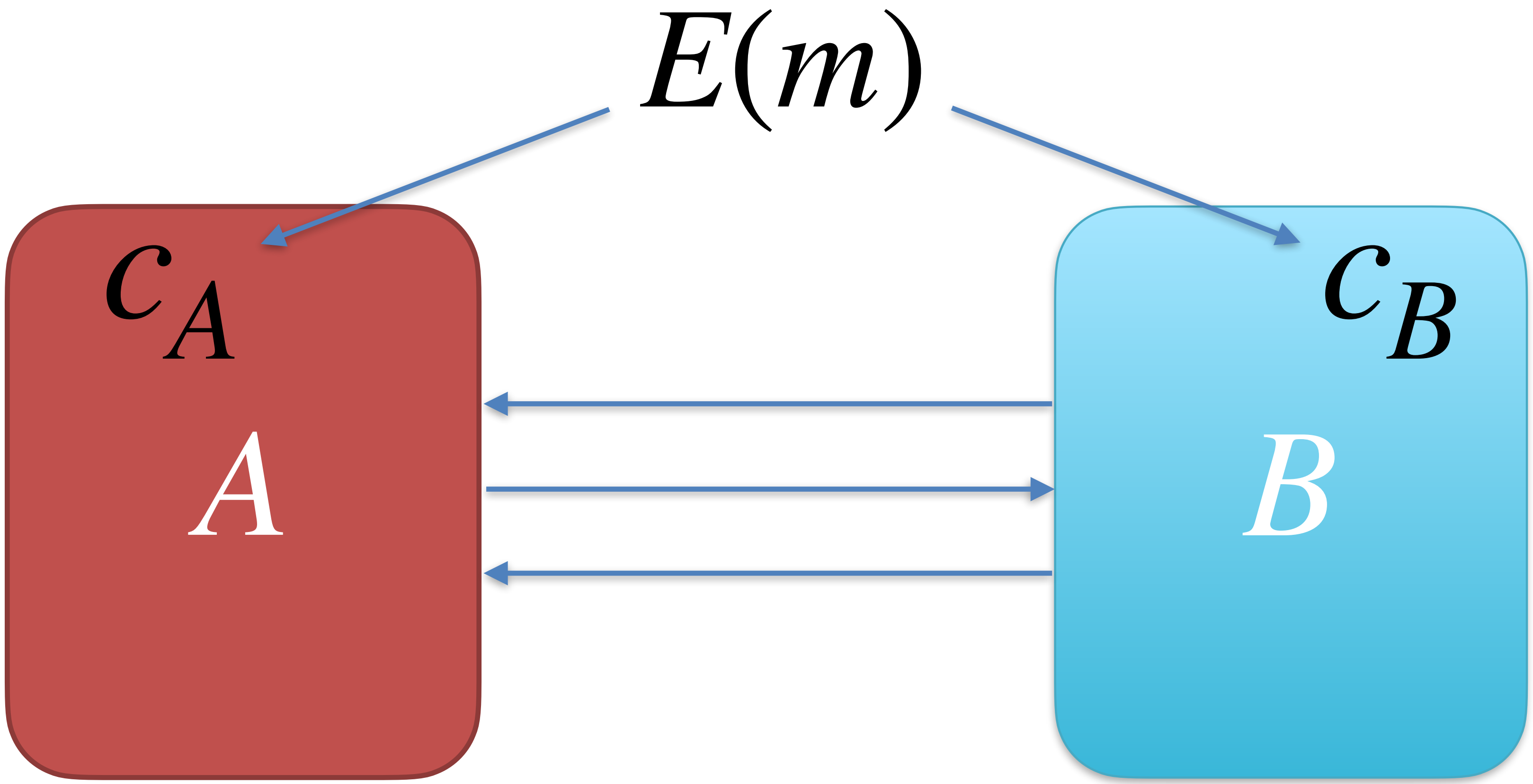

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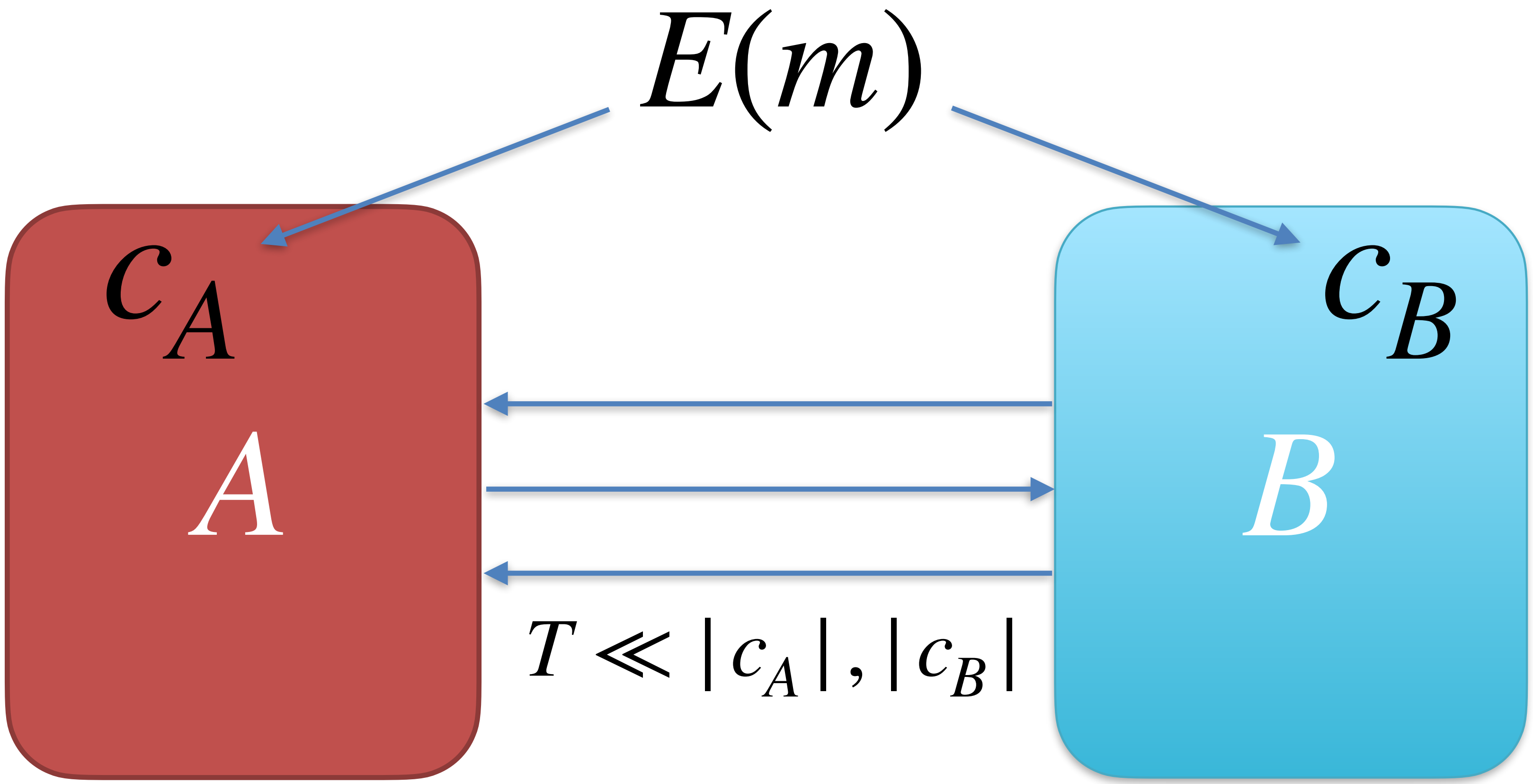

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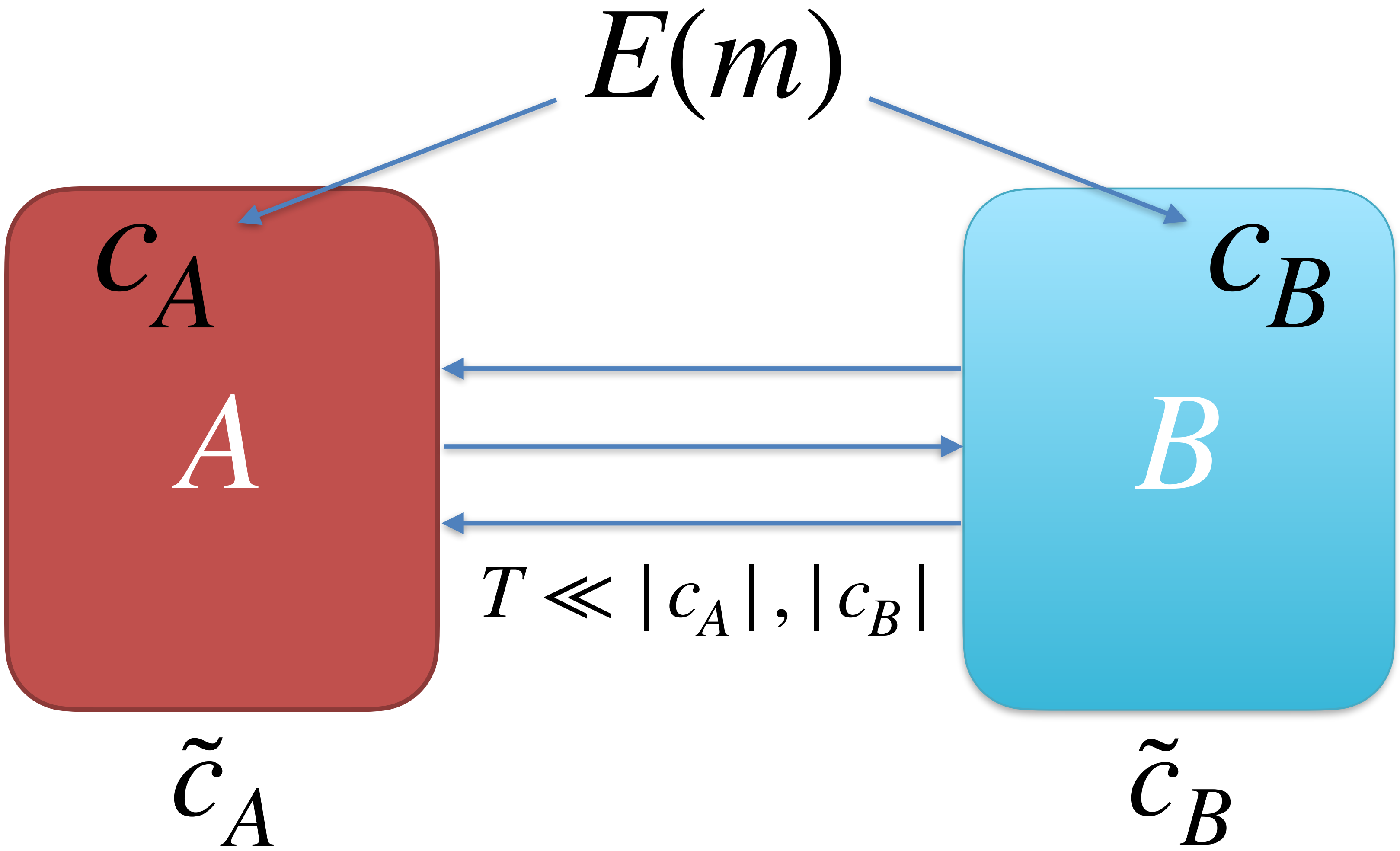
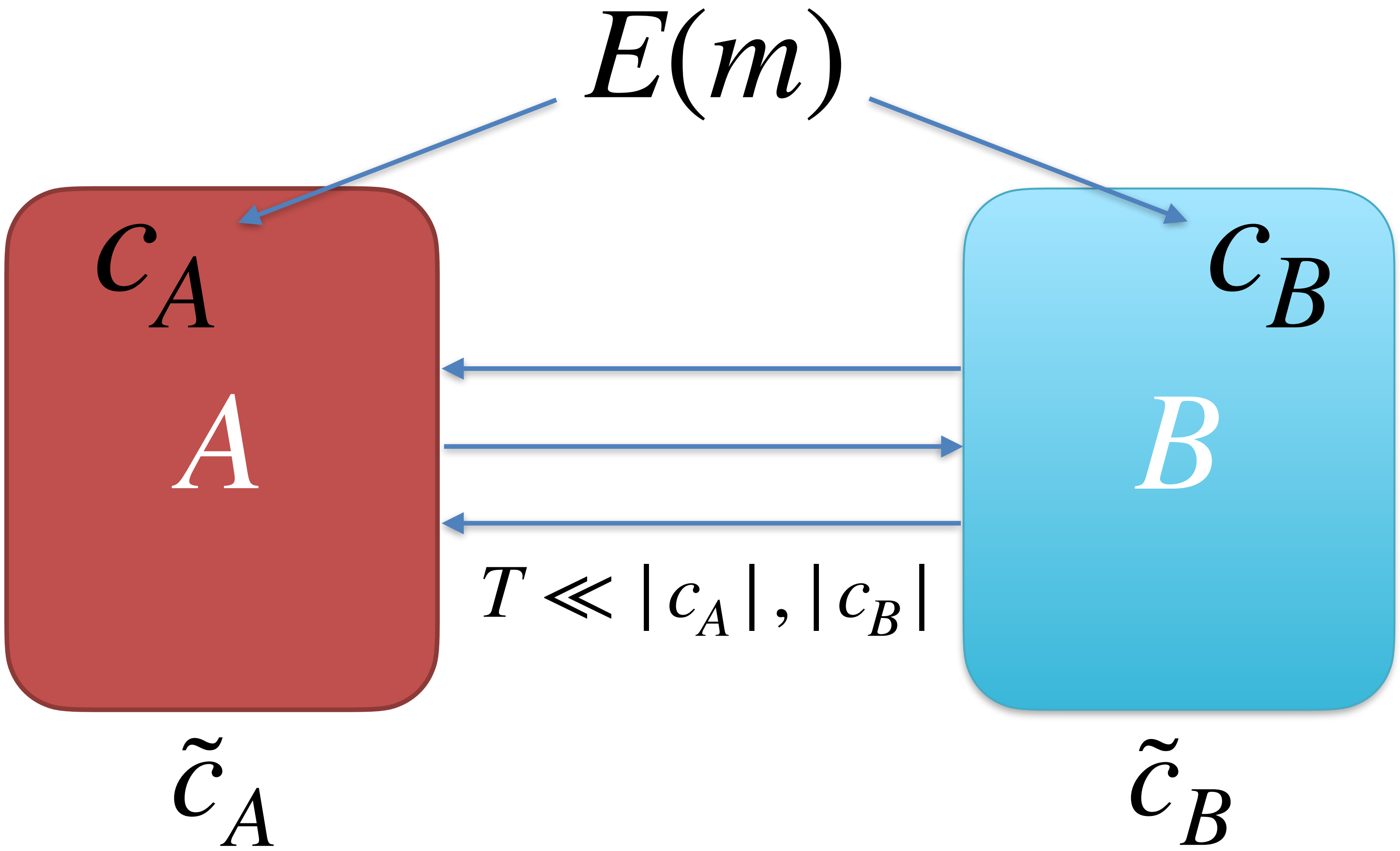


- Main ingredient: split state tampering with bounded communication
- Known NMCs for this tampering class in the standard model



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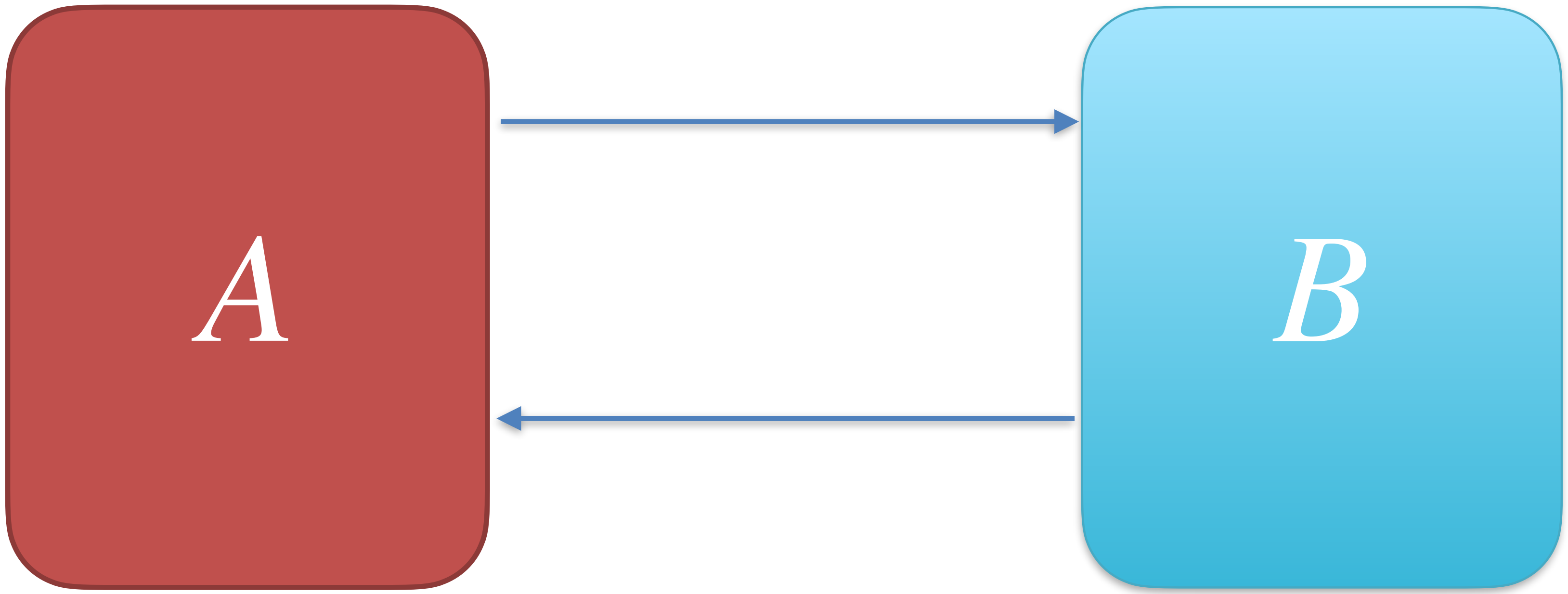
- $D(s', c'_B) = \tilde{D}(\mathsf{PRG}(s'), c'_B)$

Proof Idea



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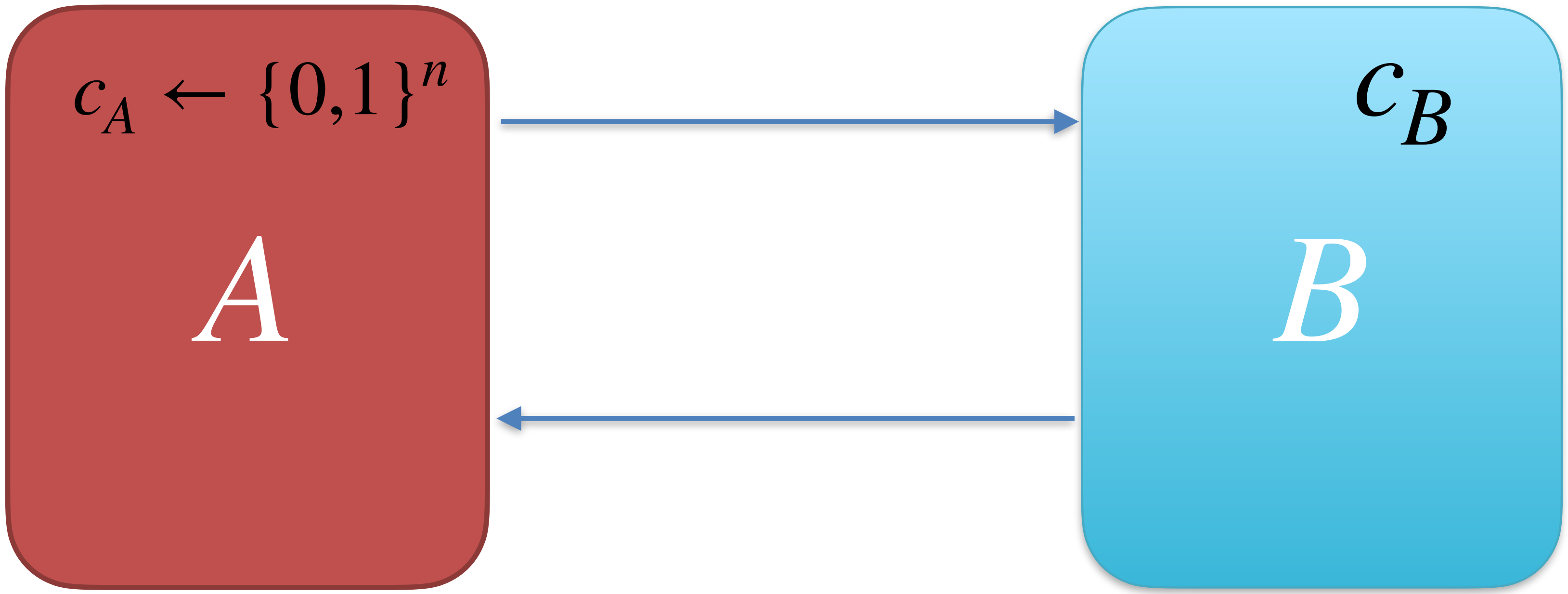

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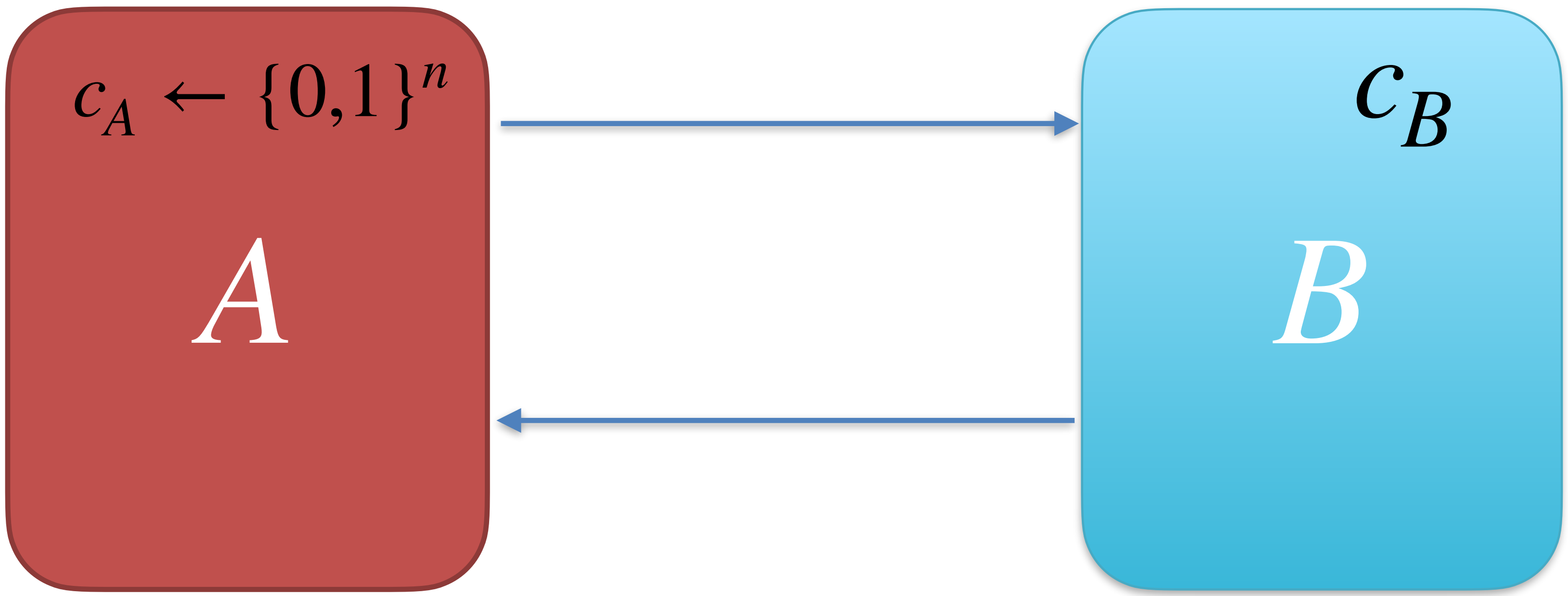

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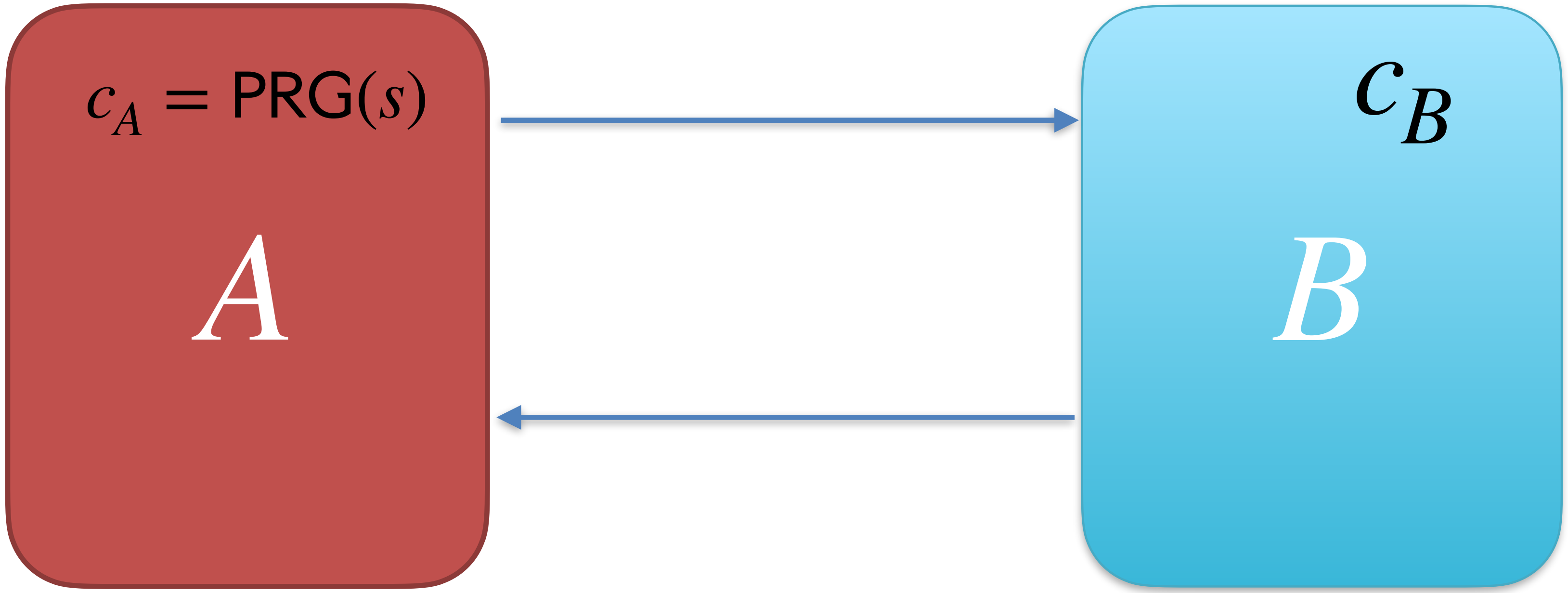

Proof Idea

- Code is secure if c_A is random
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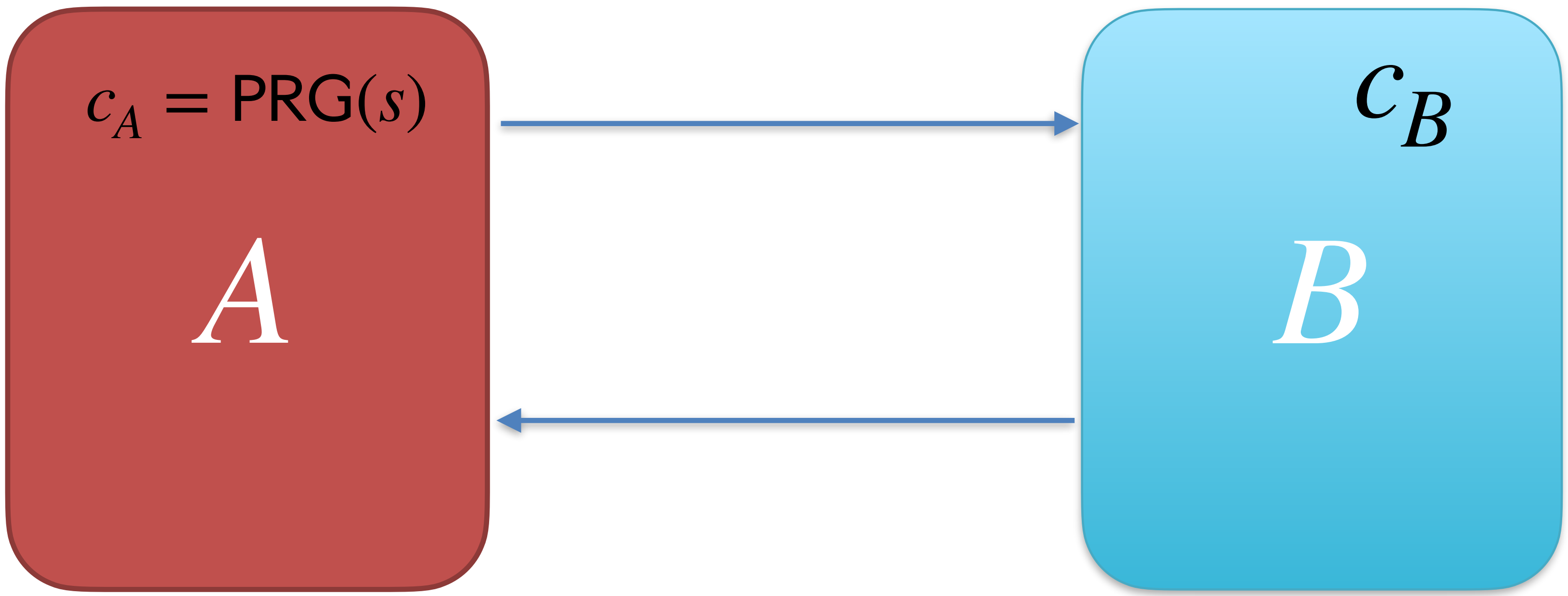
- Then there exists efficient tampering f



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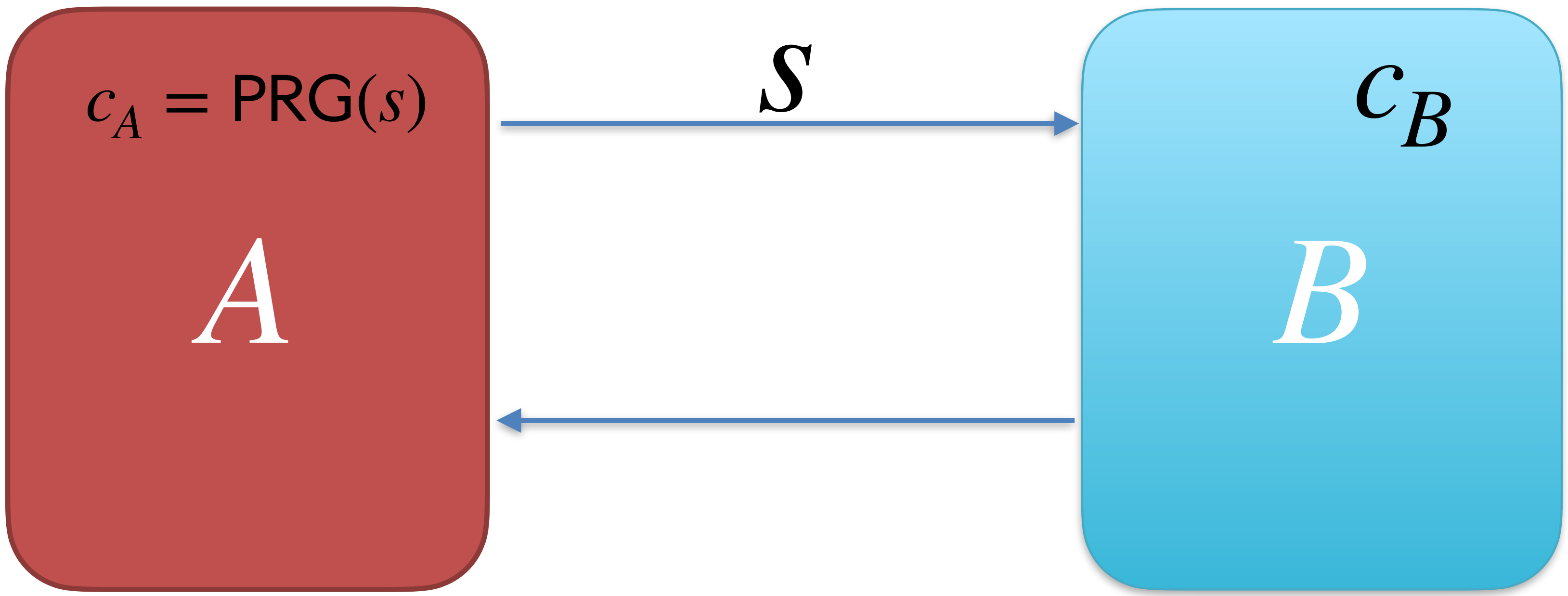

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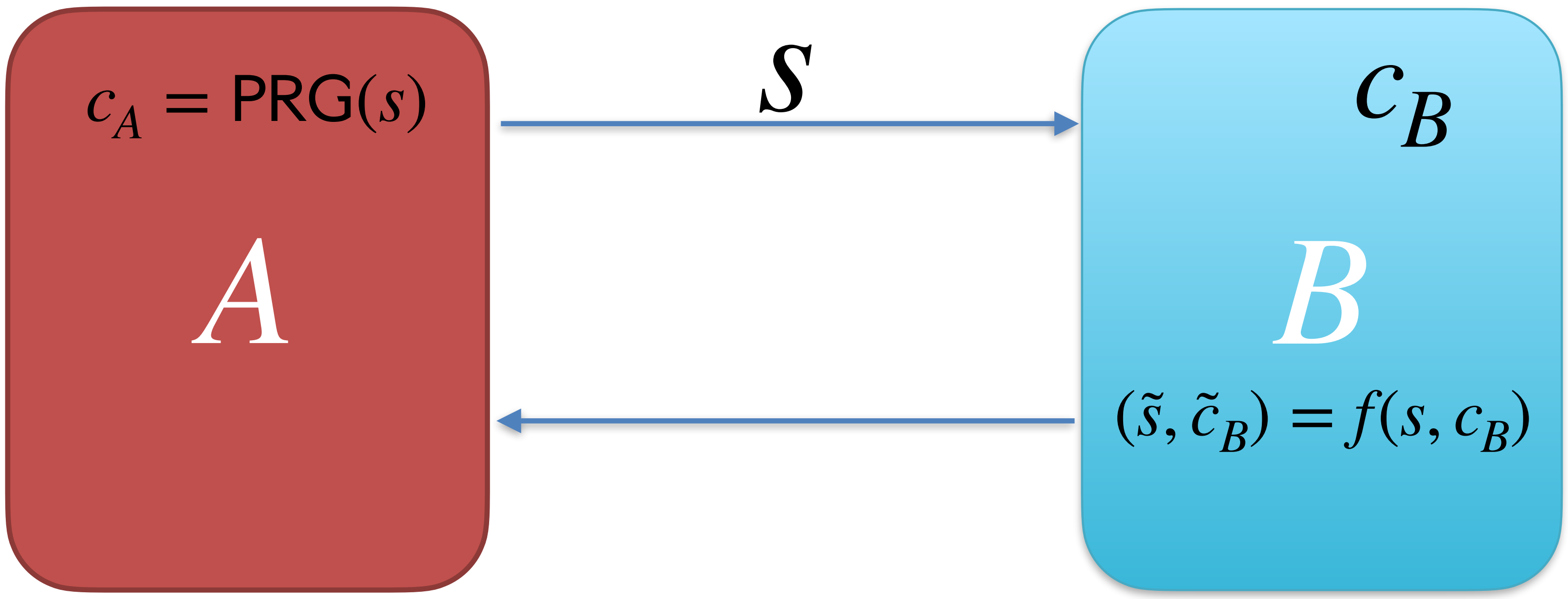

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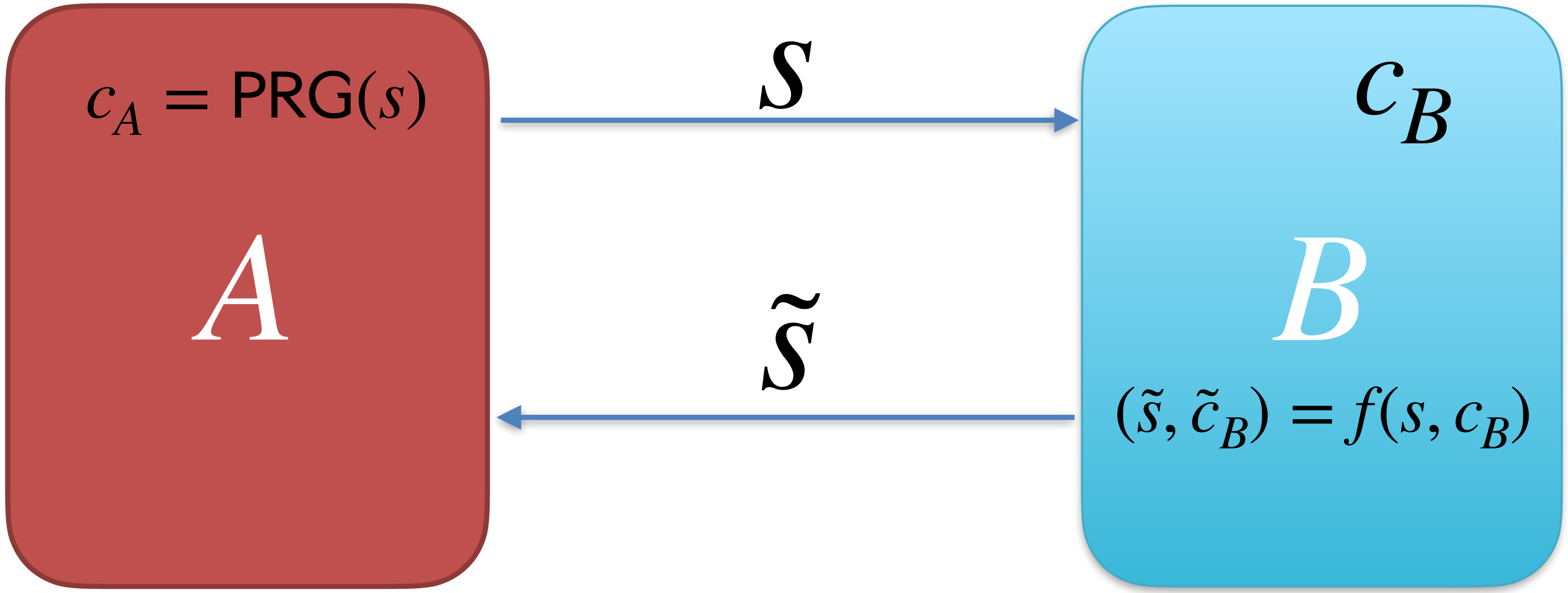

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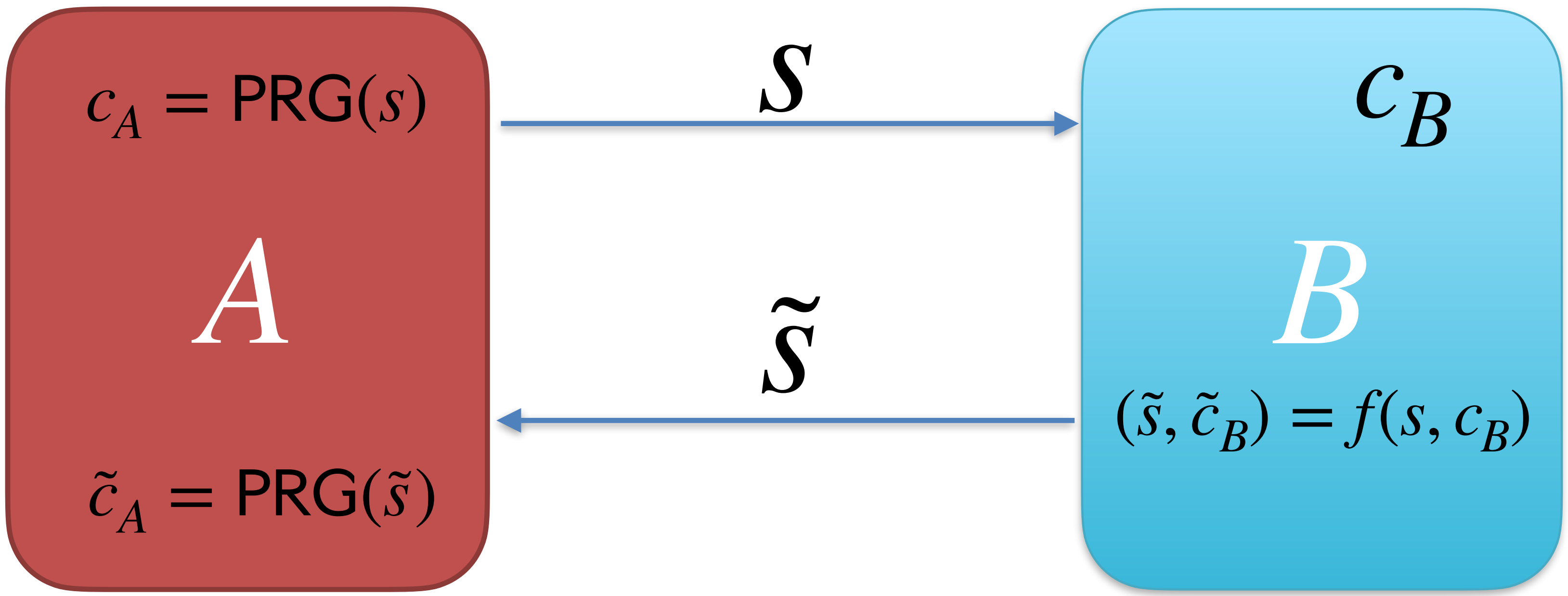

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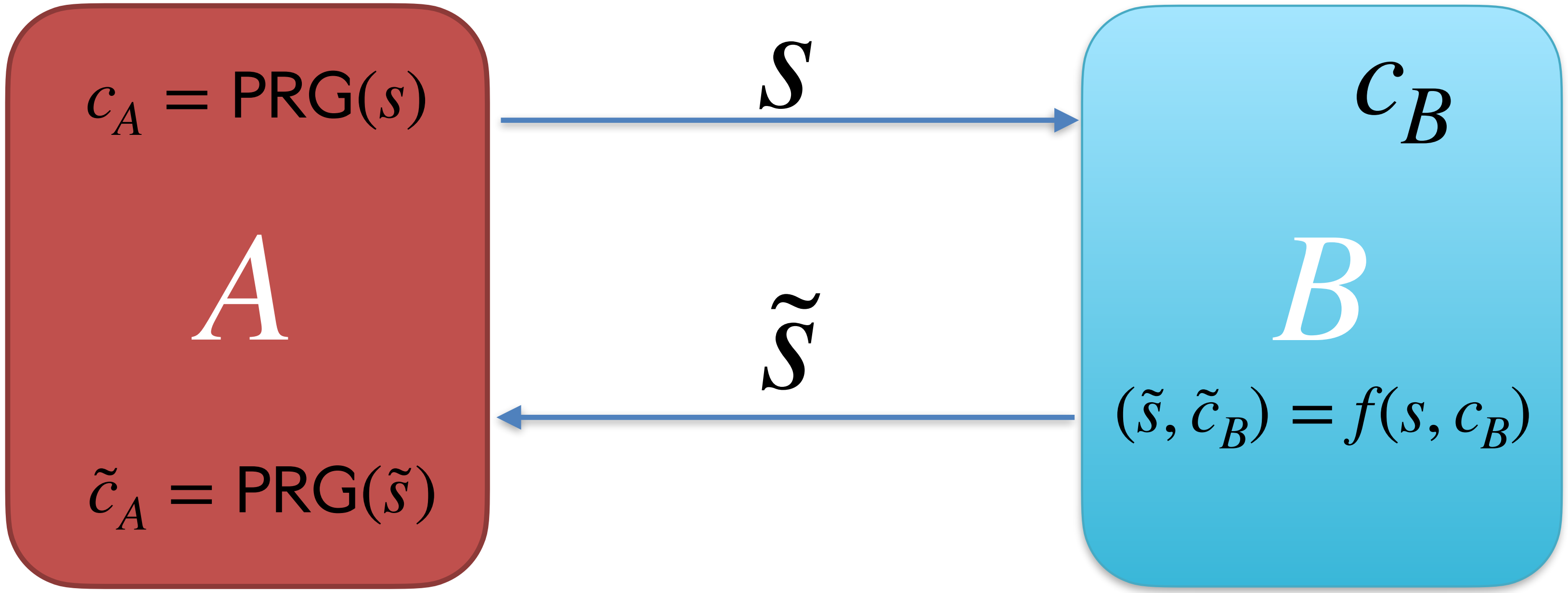
- Leads to a distinguisher on PRG



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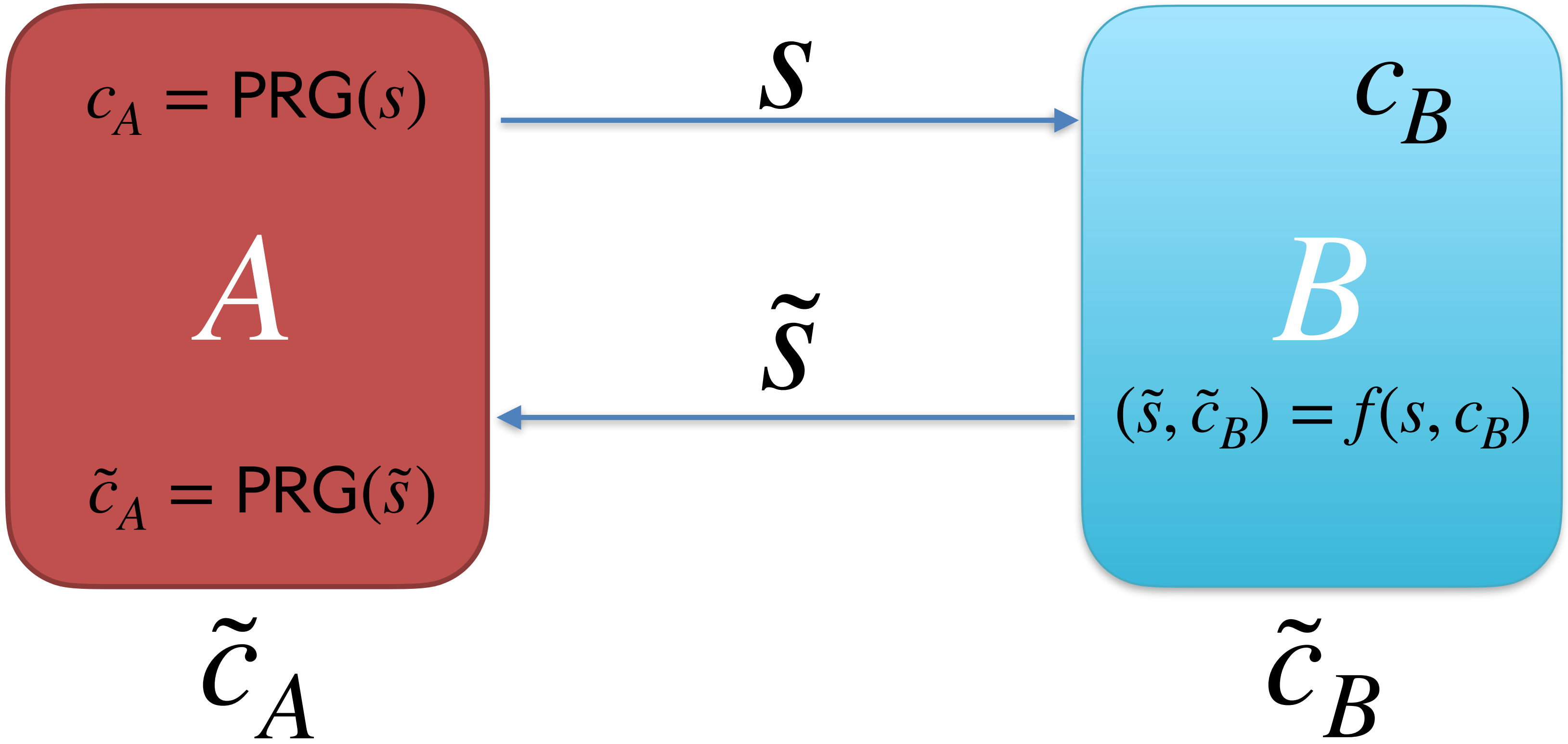
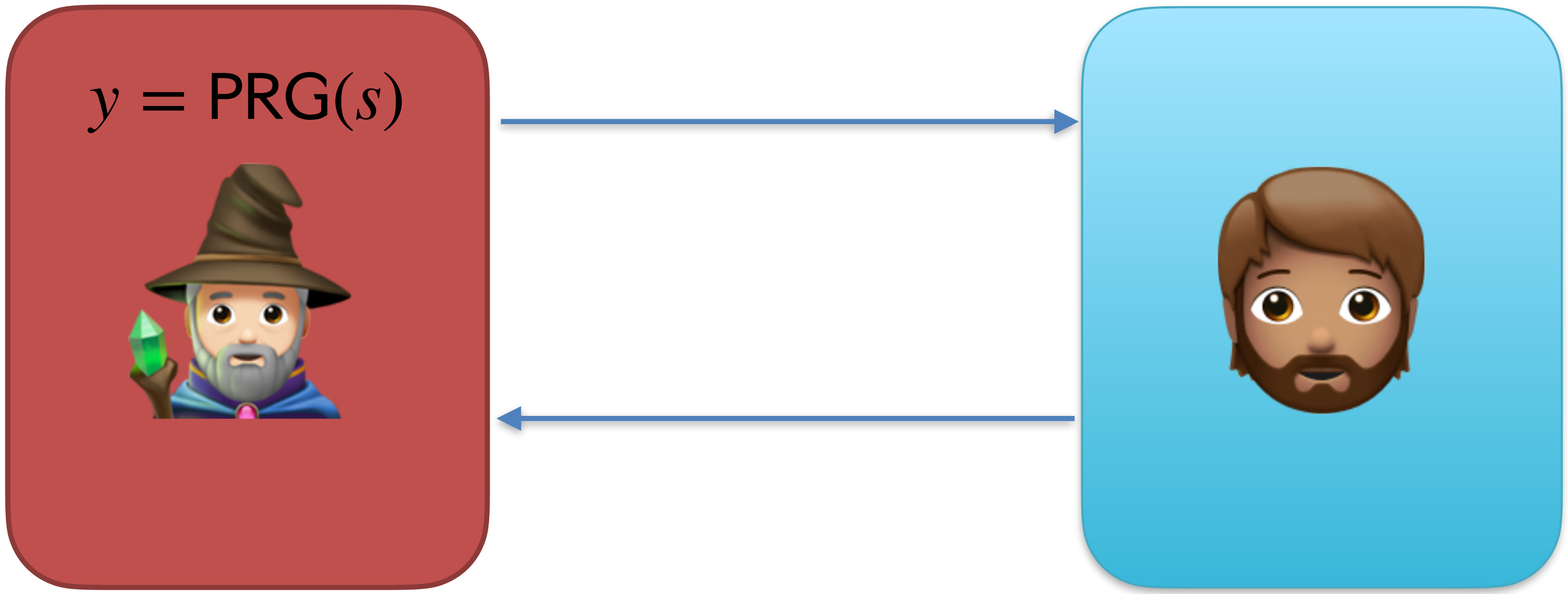
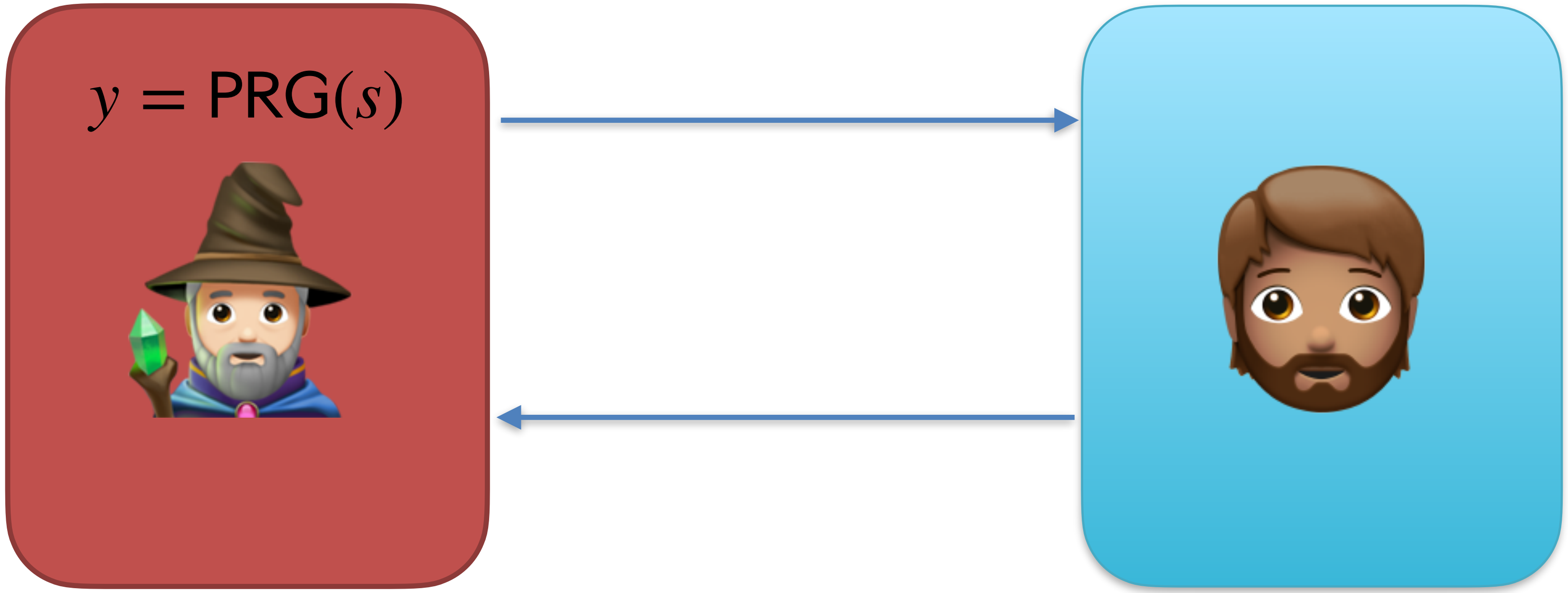


Merlin-Arthur Protocol



Merlin-Arthur Protocol

- Protocol accepts $\left(s, \mathrm{PRG}(s)\right)$ and rejects (s, U)



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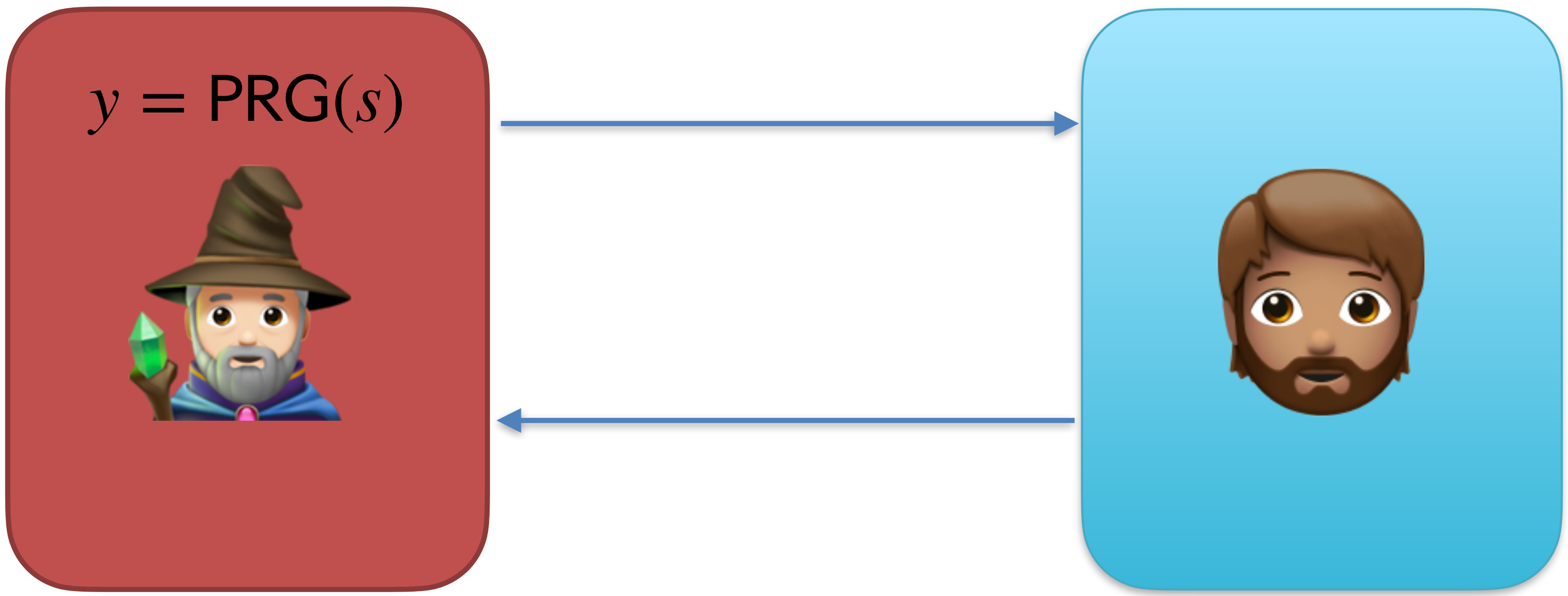

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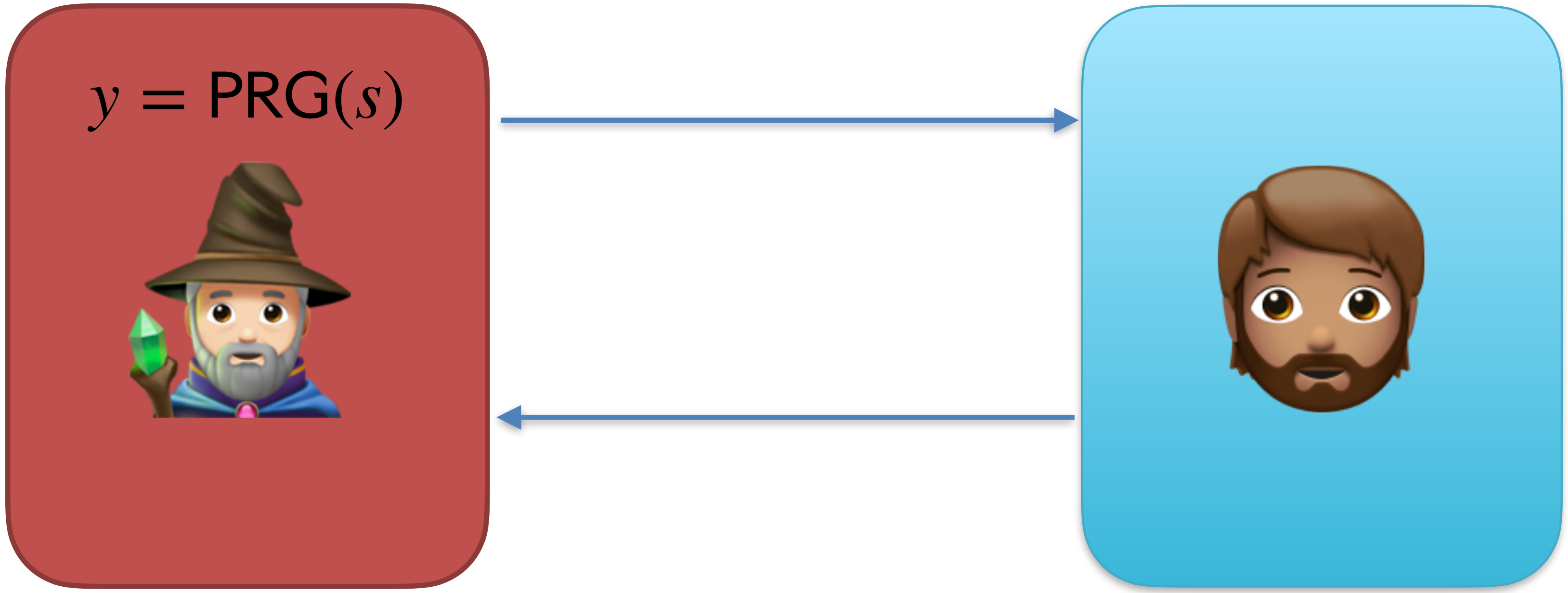

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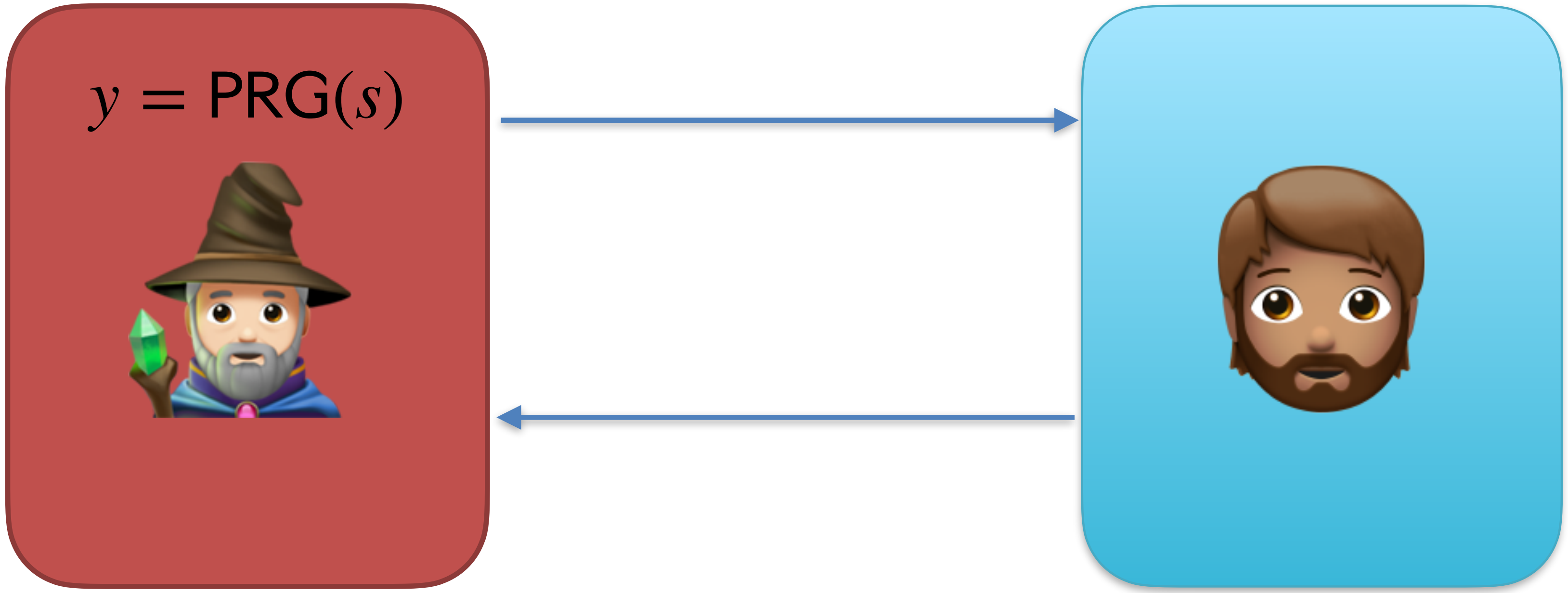
- Turn into a non-deterministic distinguisher for PRG via known techniques



Thanks!