Securing Approximate Homomorphic Encryption using Differential Privacy

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FHE (informal)

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$$\mathsf{Enc}_{\mathsf{pk}}(m) \times \mathsf{Enc}_{\mathsf{pk}}(m') \mapsto \mathsf{Enc}_{\mathsf{pk}}(m \times m').$$

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- Our focus — approximately correct FHE [CKKS17]

q-IND-CPA^D Security [LM21]

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  - q-IND-CPA^D: Restrict IND-CPA^D to q decryption queries

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  - Important to have formal definition

Approximate FHE: Error Definition

Definition: Ciphertext Error

Let Π be an FHE scheme with message space $\mathcal{M}$ that is a subset of a normed space $\widetilde{\mathcal{M}}$. For a ciphertext c, message m, and secret key sk, define the plaintext error to be

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- Dynamically Correct: $\mathsf{PtxtError}(c, m, \mathsf{sk}) \leq f(c, \mathsf{sk})$ is computable during decryption

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  - Give novel weakening of concrete security that enables smaller parameters

Norm KL Differential Privacy

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A family of randomized algorithms (indexed by $t \geq 0$) is said to be ρ-KL differentially private if, for all x, x' such that $\|x - x'\| \leq t$

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- Generalization of (Renyi) DP from the Hamming norm to general norms
- Vital that $M_t(x)$ takes as input t

Gaussian Noise Flooding

Discrete Gaussian Mechanism

Let $\rho \geq 0$. The mechanism that, on input $x \in \mathbb{Z}^n$, returns a Gaussian sample $x + \mathcal{N}_{\mathbb{Z}^n}(0, (t^2/2\rho)I_n)$ is ρ-KLDP.

- It (and uniform noise flooding) are exceedingly common in the literature

c-Bit Security

Definition: c-Bits of Security [MW18]

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- For decision games $\mathrm{adv}_G^A = O(\Delta(\cdot,\cdot)^2)$
  - Resolves several paradoxes (see [MW18])

Achieving q-IND-CPA^D Security: Main Theorem

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If CKKS is $(c + \log_2 24)$-bit IND-CPA-secure, then modifying CKKS to decrypt via

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Nearly Matching Attack

for $i \in \{0, \ldots, 44\}$ **do**
  $C_i \leftarrow \mathsf{Enc}_{\mathsf{pk}}(m_i^{(0)} = 0, m_i^{(1)} = B)$;
end for
for $i \in \{45, \ldots, 59\}$ **do**
  $c_i \leftarrow \mathsf{Enc}_{\mathsf{pk}}(m_i^{(0)} = 0, m_i^{(1)} = -B)$;
end for
$c_{60} \leftarrow \mathsf{Eval}_{\mathsf{pk}}(g, \{0, \ldots, 59\})$ for $g(x_0, \ldots, x_{59}) = \sum_{i=0}^{29}(x_i \cdot x_{30+i})$
$m' \leftarrow \mathsf{Dec}_{\mathsf{sk}}(60)$
$V_0 = 30\sigma^4 + O(wn) + \sigma_s^2$
$V_1 = 30\sigma^4 + 60B^2\sigma^2 + O(wn) + \sigma_s^2$
if $|\varphi(m')_0| < \sqrt{\frac{\log(V_1/V_0)V_0V_1}{V_1 - V_0}}$ **then**
  return 0
else
  return 1
end if

- Analysis implies that $c/4 - \tilde{O}(1)$ bits of additional noise is required

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(c, s)-Bit Security

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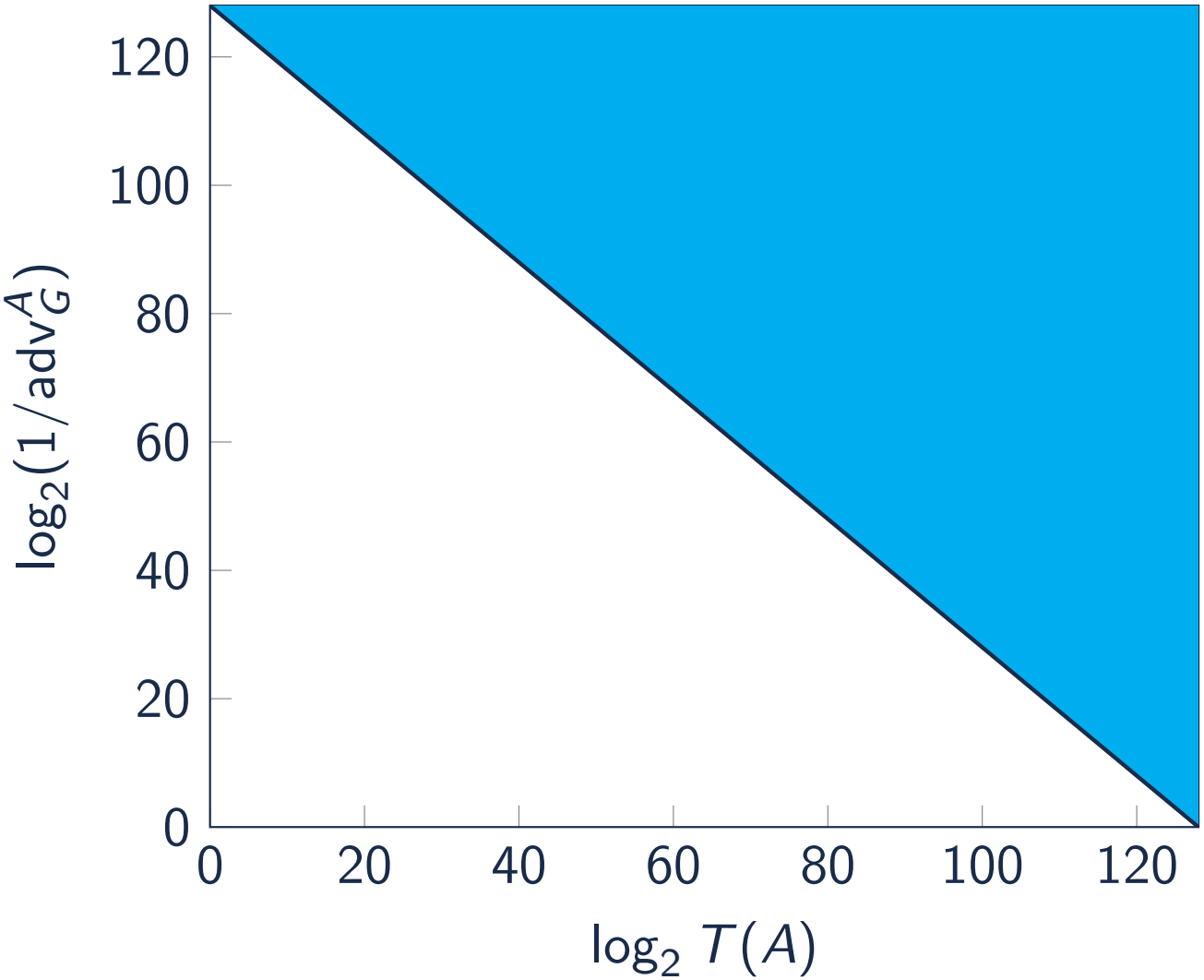
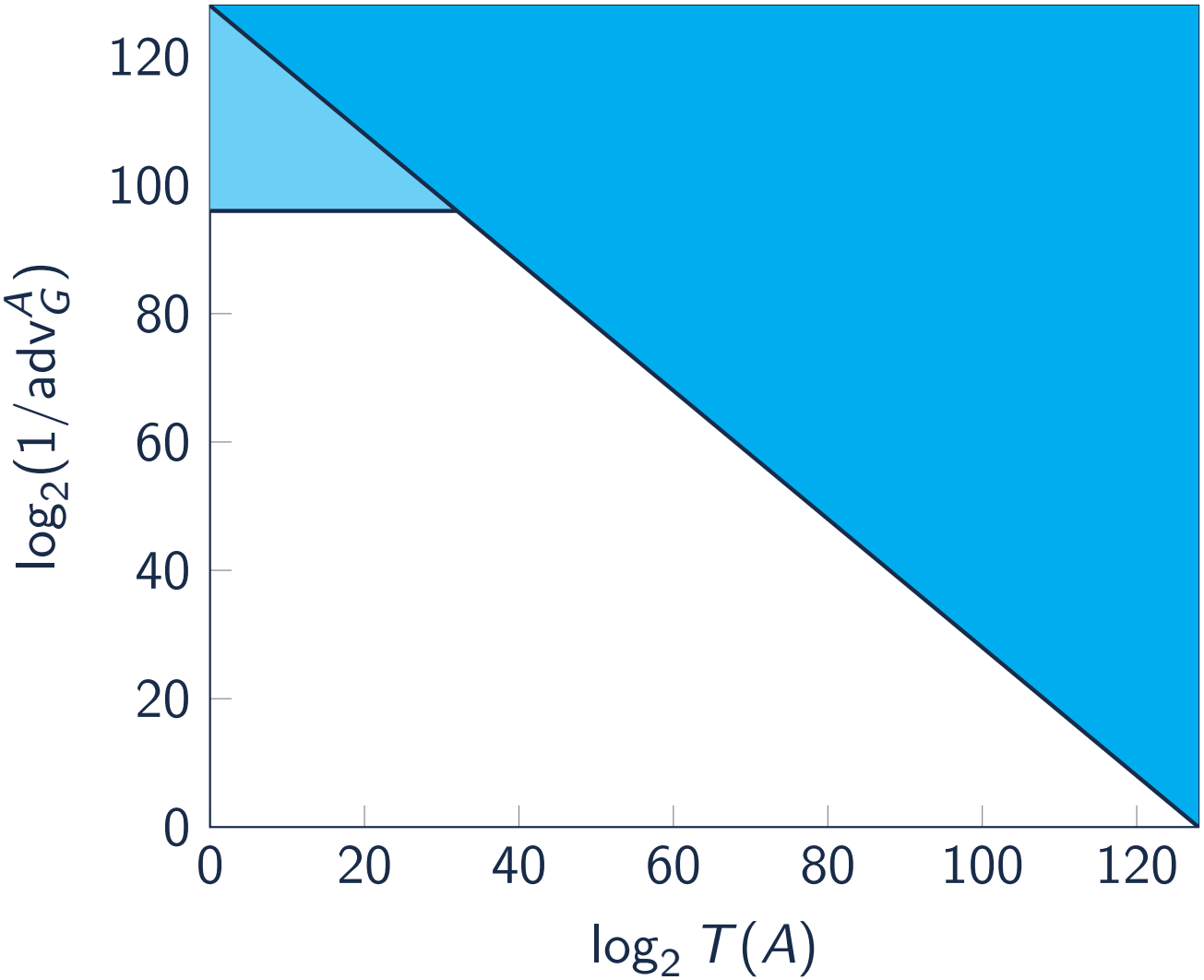
  - often can compute adv_G^A explicitly
- Can plausibly set $s \ll c$

128-Bit Security Graphically



(128, 96)-Bit Security Graphically



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- Saves $(c - s)/2$ bits of noise compared to the c-bit security

Parameters for $(128, s)$-bits of q-IND-CPA^D security

$s \backslash q$	1	2^5	2^{10}	2^{15}
128	73.79	76.29	78.79	81.29
112	65.79	68.29	70.79	73.29
96	57.79	60.29	62.79	65.29
80	49.79	52.29	54.79	57.29
64	41.79	44.29	46.79	49.29
48	33.79	36.29	38.79	41.29
32	25.79	28.29	30.79	33.29

Table 1: Table of additional Gaussian noise (in bits) to add to achieve $(128, s)$-bit security with lattice dimension $n \leq 2^{15}$

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    - Can maybe support constrained (8 or 16 bit) computations

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- Rules out generic transformation from IND-CPA-secure (dynamically correct) FHE to any form of security that implies KR^D

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