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Tight Preimage Resistance of the Sponge Construction

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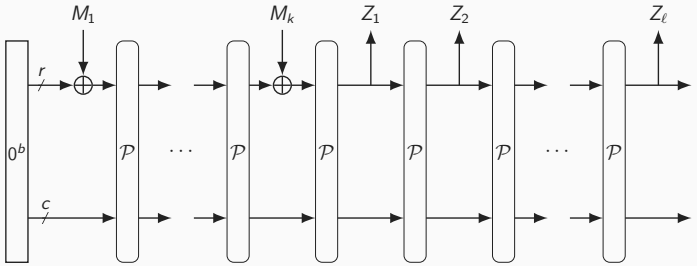
ESCADA

A generalized sponge construction [Bertoni et al., 2007]



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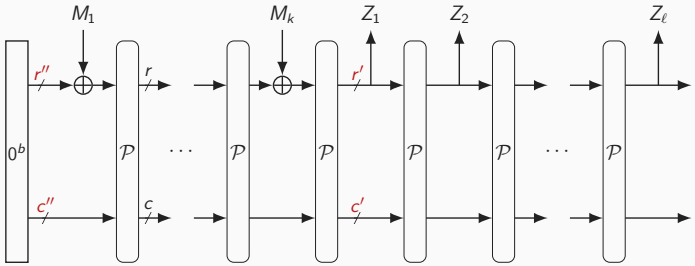
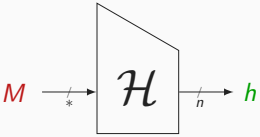


- $M_1 \| \cdots \| M_k$ is the message padded into r-bit blocks
- Variable-length digest, if n bits required, the digest is the first n bits of $Z_1 \| \cdots \| Z_\ell$
- The first message block can be larger, can squeeze at a larger rate
  [Guo et al., 2011, Naito and Ohta, 2014]

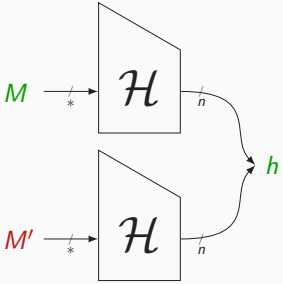
Classical security requirements

Preimage



Given h, find M

Second Preimage



Given M, find $M' \neq M$

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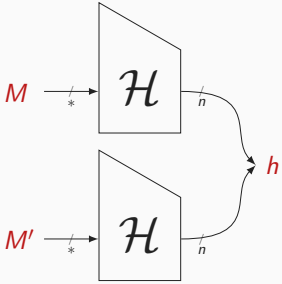

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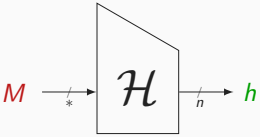

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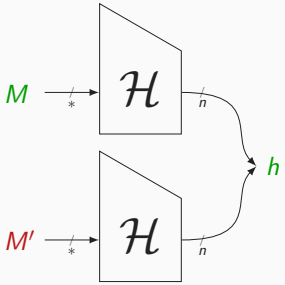

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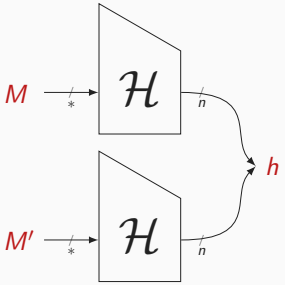
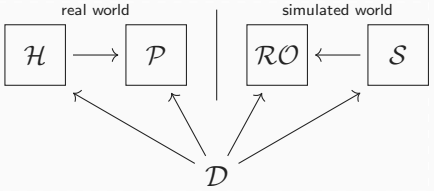


Find $M \neq M'$

- Not always sufficient, e.g., $MAC(k, M) = \mathcal{H}(k \| M)$ with $\mathcal{H}$ = plain MD
- Hash function should behave like a random oracle

Indifferentiability [Maurer et al., 2004, Coron et al., 2005]



- $(\mathcal{H}^{\mathcal{P}}, \mathcal{P})$ for a random primitive $\mathcal{P}$ should behave like a random oracle $\mathcal{RO}$ paired with a simulator $\mathcal{S}$ that maintains construction-primitive consistency
- $\mathcal{H}$ is indifferentiable from $\mathcal{RO}$ for some simulator $\mathcal{S}$ whenever any $\mathcal{D}$ can distinguish the two worlds only with a negligible probability

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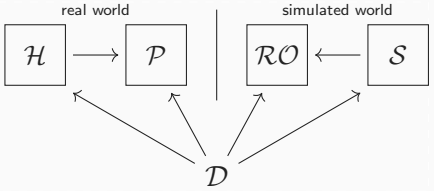

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- Indifferentiability $\Longrightarrow$ Pre/SecPre/Col security [Andreeva et al., 2010]

Indifferentiability of the sponge construction

- The (generalized) sponge construction was proven $\mathcal{O}\left(\frac{q}{2^{c/2}}\right)$ indifferentiable (provided r' and r'' are not too large) [Bertoni et al., 2008, Naito and Ohta, 2014]

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(q is number of primitive queries)

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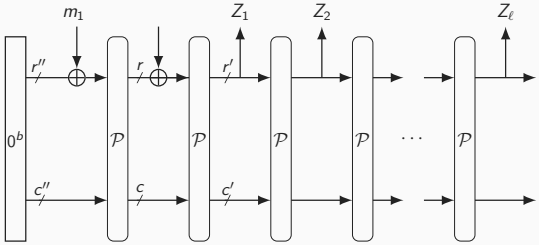
This work: Tight security bound for Pre

Collision attack with $q \approx 2^{c/2}$ queries [Bertoni et al., 2011]



- Query $\mathcal{P}(m_1 \| 0^c)$ for $2^{c/2}$ different m_1's, store them in a list L

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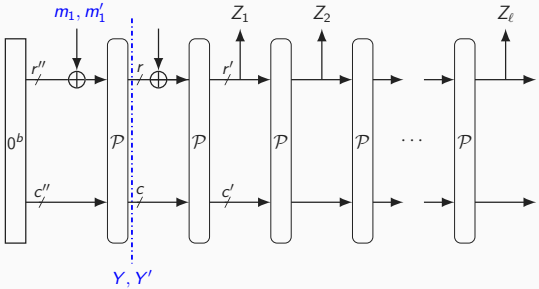

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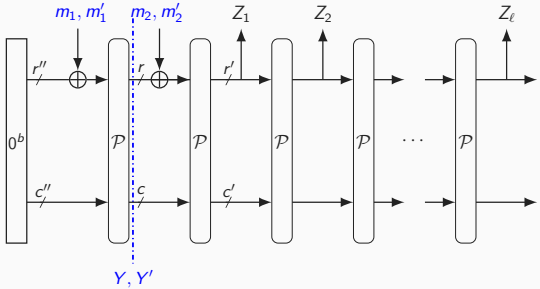

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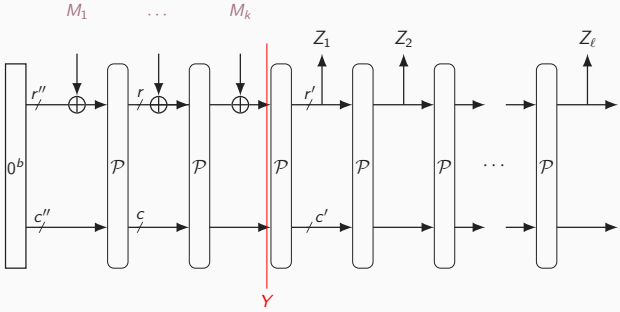
$\Longrightarrow$ Take $m_2 = \text{outer}_r(Y), m_2' = \text{outer}_r(Y')$

$\Longrightarrow$ This gives $\mathcal{H}(\text{unpad}(m_1 \| m_2)) = \mathcal{H}(\text{unpad}(m_1' \| m_2'))$

Second preimage attack with $q \approx 2^{c/2}$ queries [Bertoni et al., 2011]



- Let M be the first preimage, $M_1 \| \cdots \| M_k := \mathrm{pad}(M)$
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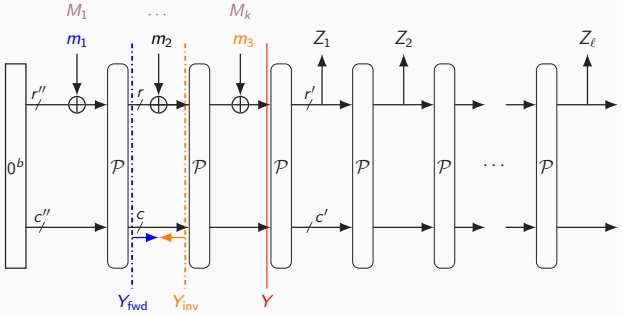
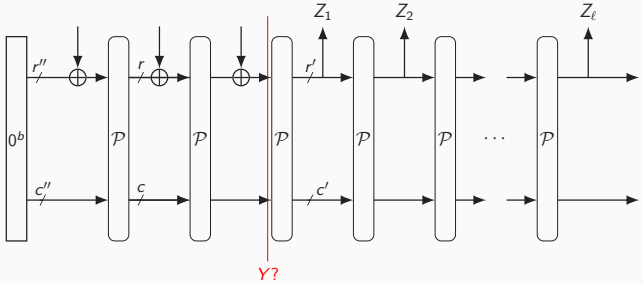


- Let M be the first preimage, $M_1 \| \cdots \| M_k := \text{pad}(M)$
- Compute the state before the first squeeze, call it Y
- Reach Y with an inner forward/backward collision, compensate the outer part with $m_2 = \text{outer}_r(Y_{\text{inv}}) \oplus \text{outer}_r(Y_{\text{fwd}})$

$\Longrightarrow$ This gives $\mathcal{H}(\text{unpad}(m_1 \| m_2 \| m_3)) = \mathcal{H}(M)$

First preimage attack $q \approx 2^{n-r'} + 2^{c/2}$ queries [Bertoni et al., 2011]



- Let $Z = Z_1 \| \cdots \| Z_\ell$ be the image
- No intermediate state Y: we need to find it before applying the same attack
- More precisely, we need Y s.t., $\forall i = 1, \ldots, \ell$, $\text{outer}_{r'}(\mathcal{P}^i(Y)) = Z_i$

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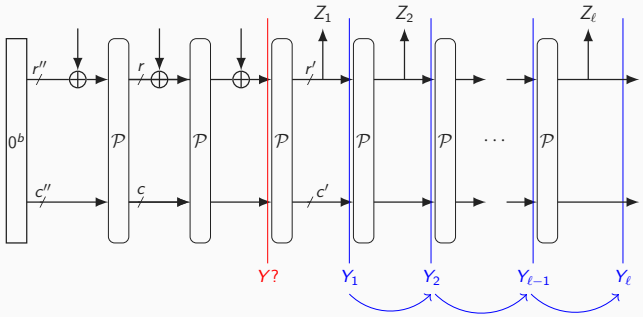

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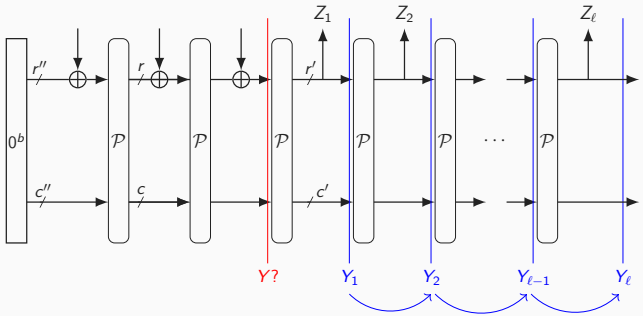

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$\Longrightarrow$ Total attack cost $\approx 2^{n-r'} + 2^{c/2}$ queries

Improved first preimage resistance

- The best known first preimage attack succeeds after $\approx \min\{2^{n-r'} + 2^{c/2}, 2^n\}$ queries, while the indifferentiability bound guarantees preimage security up to $\approx \min\left\{2^{c/2}, 2^n\right\}$ queries
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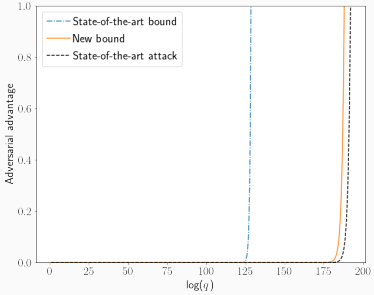
- Our contribution: we prove preimage resistance with bound

$$\mathcal{O}\left(\frac{q}{2^n} + \min\left\{\frac{q}{2^{n-r'}}, \frac{q}{2^{c/2}}\right\}\right)$$

$\Longrightarrow$ Optimality of the attack

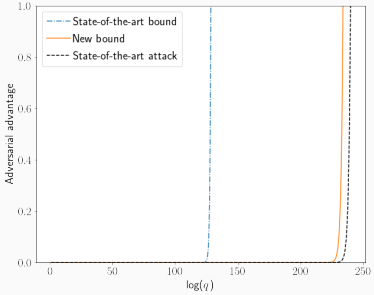
Improved first preimage resistance

Adversarial advantage upper bound according to number of queries



Ascon

$(b, c, r, r', n) = (320, 256, 64, 64, 256)$

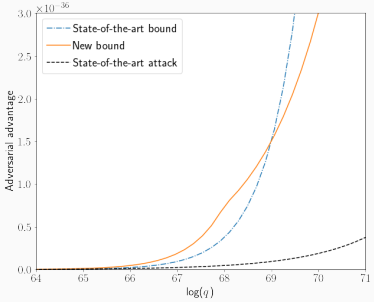


Spongent largest mode

$(b, c, r, r', n) = (272, 256, 16, 16, 256)$

Improved first preimage resistance

Closeup



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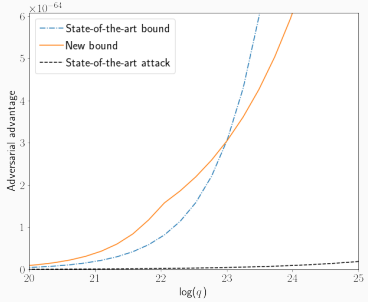

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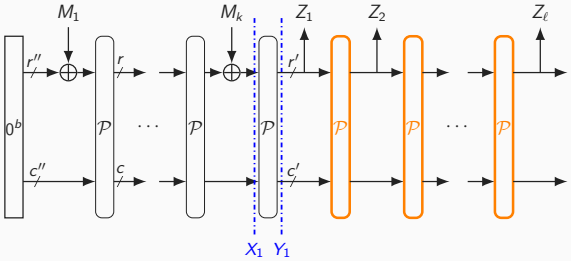

- To find a preimage, the adversary must find a **cascade** of $\ell - 1$ permutation evaluations giving $Z_2, \ldots, Z_\ell$
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- Depending on the direction of the query $X_1 \to Y_1$, there are two scenarios:

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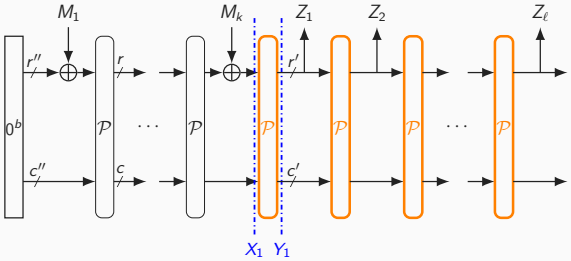

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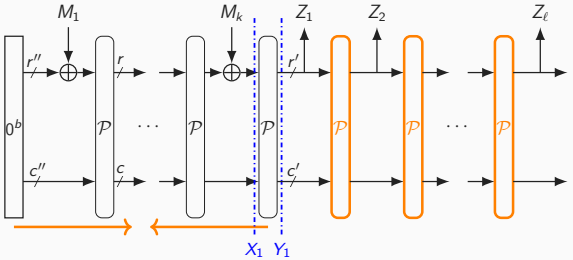

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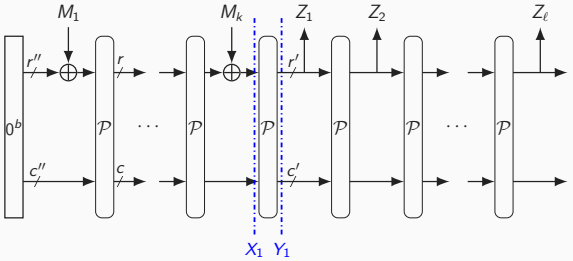
  - Inverse direction: an inner collision must have been found as well

Proof idea – probability computation



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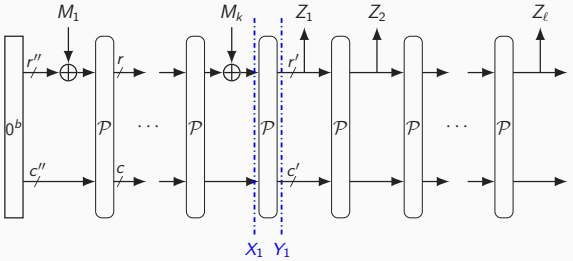

- Forward direction: adversary must guess a "good" $X_1 \implies \mathcal{O}\left(\frac{q}{2^n}\right)$
- Inverse direction:
  - Cascade: more involved, since the queries can appear in any order, any direction within the cascade $\implies \mathcal{O}\left(\frac{q}{2^{n-r'}}\right)$
  - Inner collision: upper bounded by $\frac{q(q+1)}{2^c} + \frac{q}{2^{c''}}$

Impact on the generic security of a few schemes

Scheme	Parameters						Security bound		Note
	b	c	r	r'	n	ℓ	Old	New	
Spongent	272	256	16	16	256	16	2^{128}	2^{240}	ISO/IEC standard
PHOTON	288	256	32	32	256	8	2^{128}	2^{224}	ISO/IEC standard
T-QUARK	256	224	32	32	224	7	2^{112}	2^{192}	
ACE-Hash	320	256	64	64	256	4	2^{128}	2^{192}	NIST LWC round 2
KNOT Hash	256	224	32	128	256	2	2^{112}	2^{128}	NIST LWC round 2
SKINNY-tk2-Hash	256	224	32	128	256	2	2^{112}	2^{128}	NIST LWC round 2
Subterranean 2.0	257	248	9	32	256	8	2^{124}	2^{224}	NIST LWC round 2
Ascon-Hash	320	256	64	64	256	4	2^{128}	2^{192}	NIST LWC finalist
PHOTON-Beetle-Hash	256	224	32	128	256	2	2^{112}	2^{128}	NIST LWC finalist

Conclusion

- We derived a tight security bound for the first preimage of the sponge construction
- This bound has direct implications on the proven security of lightweight cryptographic sponges, such as Ascon-Hash, Spongent, PHOTON, ACE, Subterranean 2.0, and QUARK

Thank you for your attention!

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