

Password-Authenticated Key Exchange from Group Actions

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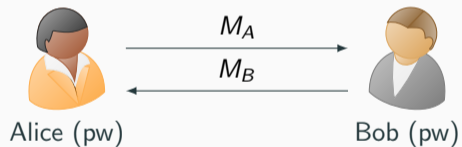
Alice (pw)



Bob (pw)

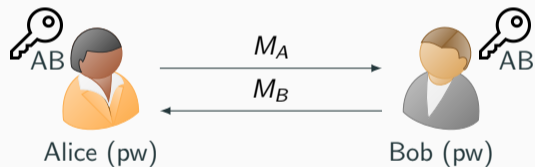
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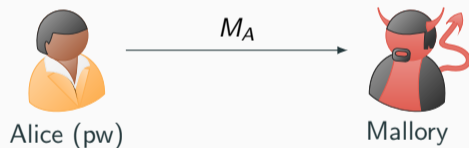
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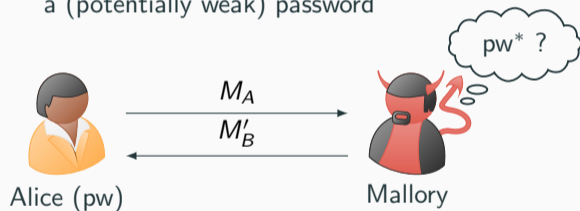
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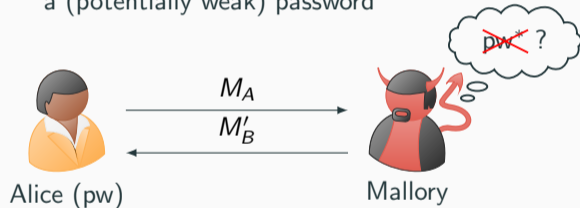
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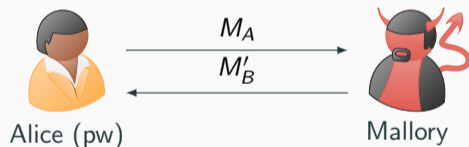
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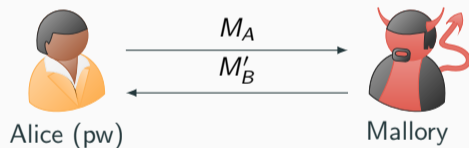
Group Actions

- Abstraction is close to the classical DH-setting

Motivation

Password Authenticated Key Exchange

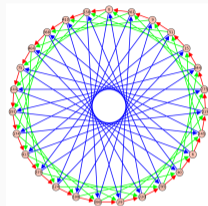
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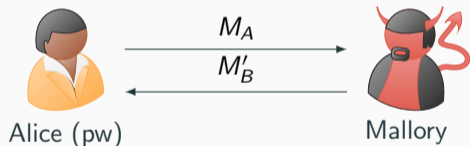
Group Actions

- Abstraction is close to the classical DH-setting
- CSIDH as candidate for post-quantum security
- Public-Key Encryption, Signatures, Oblivious Transfer, ...



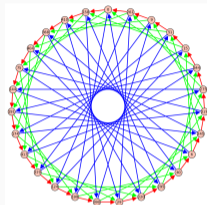
Password Authenticated Key Exchange from Group Actions ?

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Difficulties (e.g., [AJK⁺20])

- Limited structure of the group action
 - Special properties of CSIDH
- ⇒ Known DH-based constructions cannot be directly translated to the group action setting

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Generic Constructions

- Quite inefficient construction using OT
- Unclear how to use the HPS of [ADMP20]

Cryptographic Group Actions

Cryptographic Group Actions [ADMP20]

Group Action

Let (\mathcal{G}, \cdot) be a group with identity element $id \in \mathcal{G}$, and \mathcal{X} a set. A map $\star : \mathcal{G} \times \mathcal{X} \rightarrow \mathcal{X}$ is a group action if it satisfies the following properties:

1. Identity: $id \star x = x$ for all $x \in \mathcal{X}$.
2. Compatibility: $(g \cdot h) \star x = g \star (h \star x)$ for all $g, h \in \mathcal{G}$ and $x \in \mathcal{X}$.

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Technical Assumptions

- \mathcal{G} and \mathcal{X} are finite.
- \mathcal{G} is commutative.
- $\star : \mathcal{G} \times \mathcal{X} \rightarrow \mathcal{X}$ is regular.
- A distinguished element $\tilde{x} \in \mathcal{X}$ (“origin”).

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⚠ In general, we cannot combine two elements of the set \mathcal{X} !

The CSIDH Group Action

CSIDH [CLM⁺18] can be seen as a *restricted effective* group action:

\mathcal{G} = corresponds to isogenies between elliptic curves

\mathcal{X} = supersingular elliptic curves over \mathbb{F}_p

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Computational Assumptions

- DLOG: Given $g \star \tilde{x} \in \mathcal{X}$, it is hard to find $g \in \mathcal{G}$.
- CDH: Given $(g \star \tilde{x}, h \star \tilde{x}) \in \mathcal{X}^2$, it is hard to find $z = gh \star \tilde{x} \in \mathcal{X}$.
- DDH: Given $(g \star \tilde{x}, h \star \tilde{x}, gh \star \tilde{x}) \in \mathcal{X}^3$ or $(g \star \tilde{x}, h \star \tilde{x}, u \star \tilde{x}) \in \mathcal{X}^3$, decide which is the case.

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- Strong/Gap CDH: same as CDH but with access to a decision oracle DDH, where

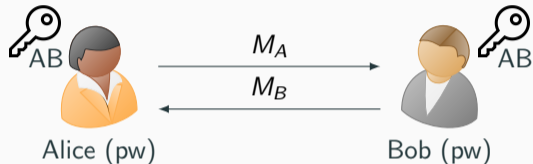
$$\text{DDH}(x, y, z) = \begin{cases} 1 & \text{CDH}(x, y) = z \\ 0 & \text{otherwise} \end{cases}$$

Password-Authenticated Key Exchange

Password-Authenticated Key Exchange (PAKE)

Focus

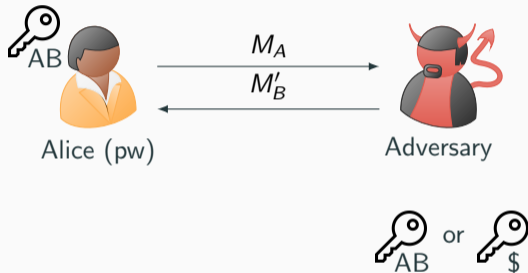
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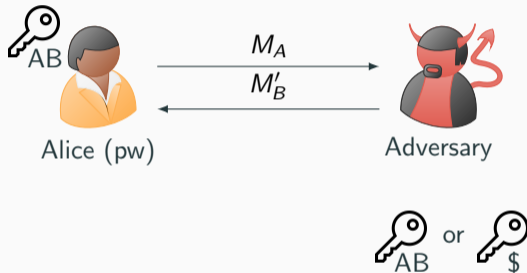
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- BPR security model (game-based) with extension to multiple test queries



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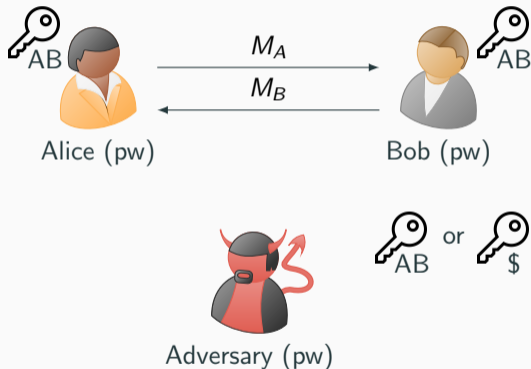
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- perfect forward secrecy



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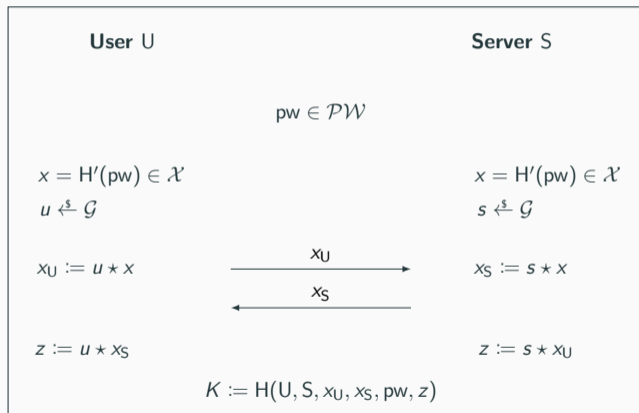
Focus

- balanced PAKE
- BPR security model (game-based) with extension to multiple test queries
- perfect weak forward secrecy



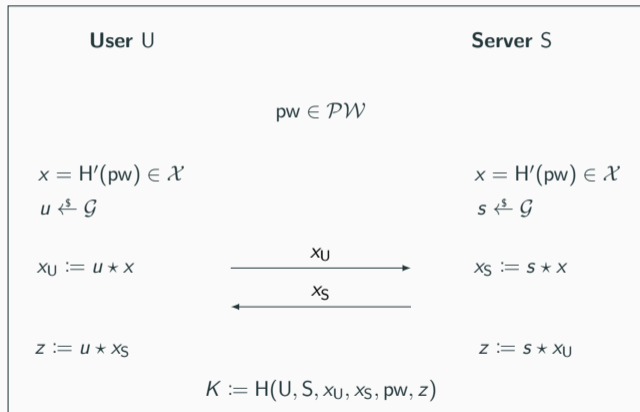
Our First Protocol

Simple Password Exponential Key Exchange [Jab96]



Starting Point: SPEKE

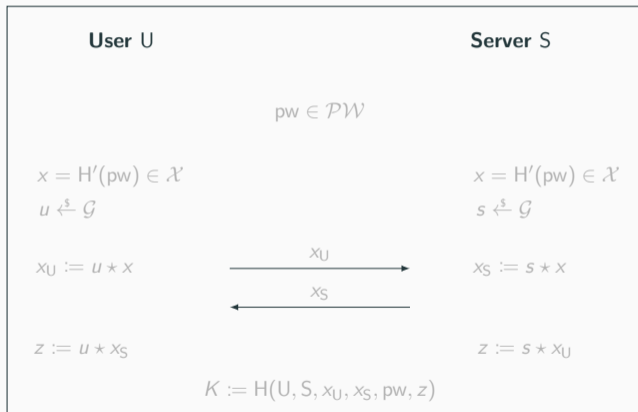
Simple Password Exponential Key Exchange [Jab96]



Problem: A hash function $H' : \mathcal{PW} \rightarrow \mathcal{X}$ is still an open problem for CSIDH [BBD⁺22, MMP22].

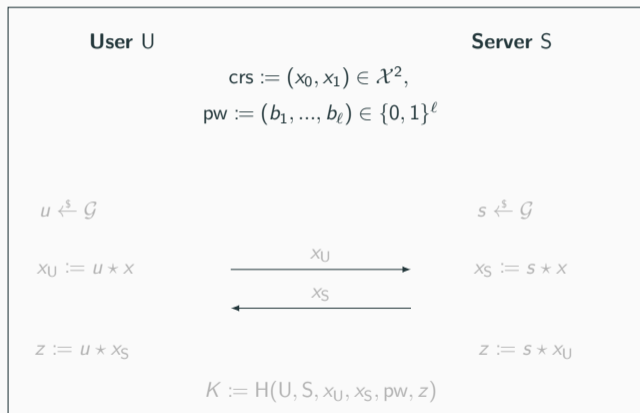
Our Group Action PAKE

Idea: Replace the hash function by a bit-by-bit approach.



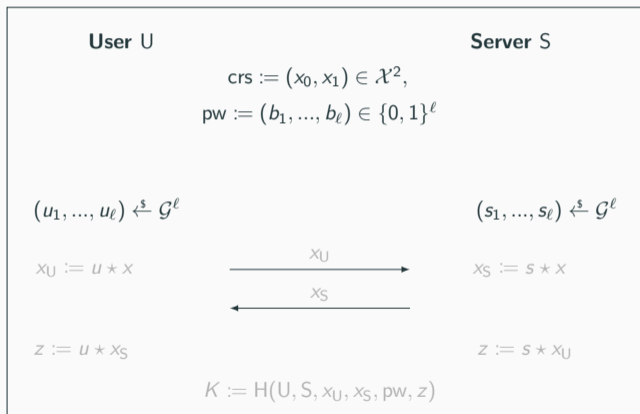
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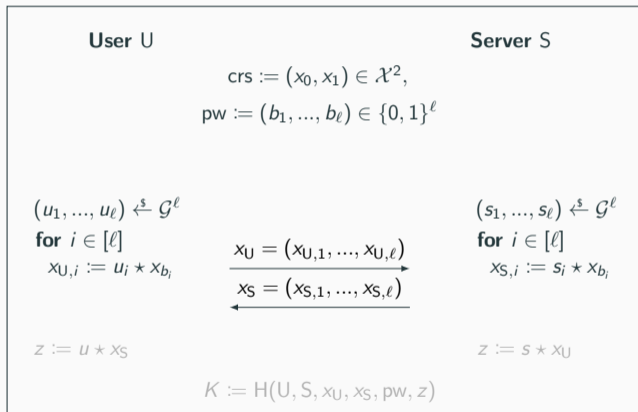
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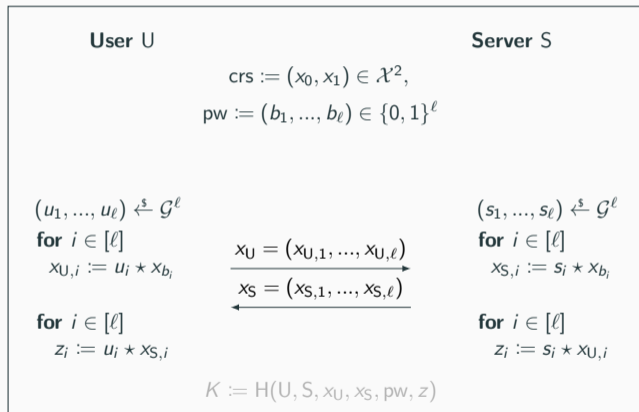
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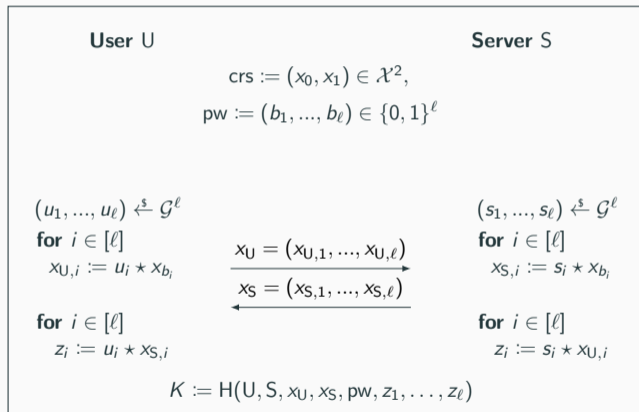
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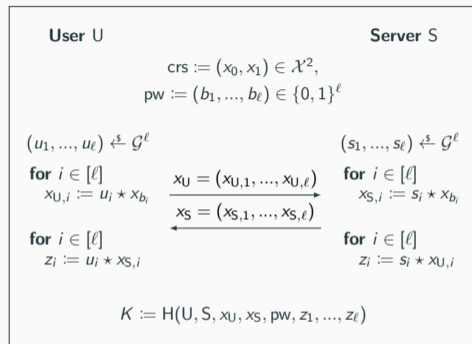
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(In)Security of our Protocol

Security against Passive Adversaries

- secure under Strong CDH + ROM



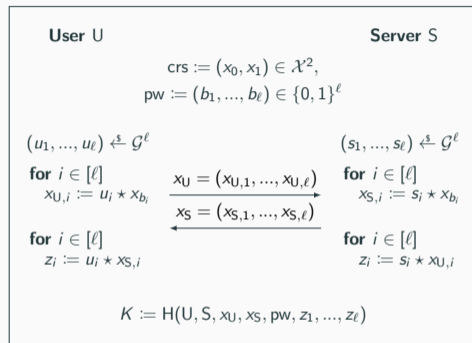
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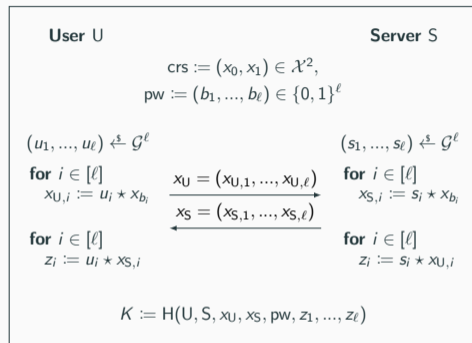
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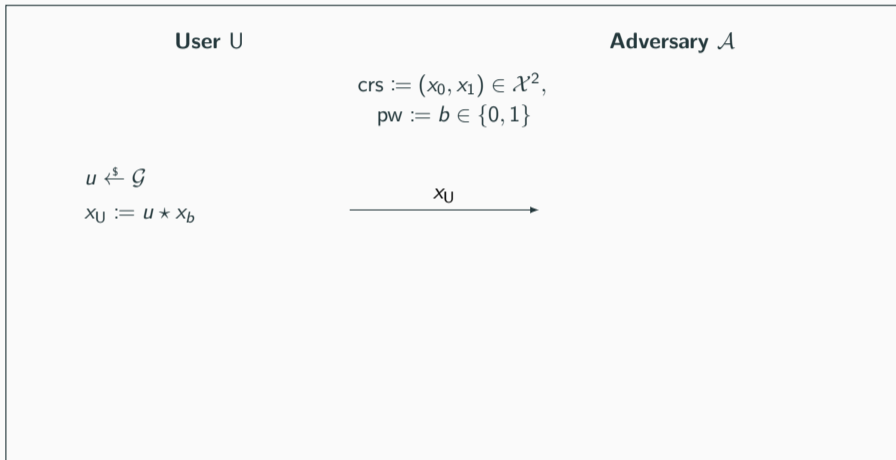
Additional Structure of the CSIDH Group Action

- For any $x \in \mathcal{X}$, we can efficiently compute its *twist* denoted by x^t .
- Let $x = g \star \tilde{x}$, then $x^t = g^{-1} \star \tilde{x}$. In particular $\tilde{x}^t = \tilde{x}$.



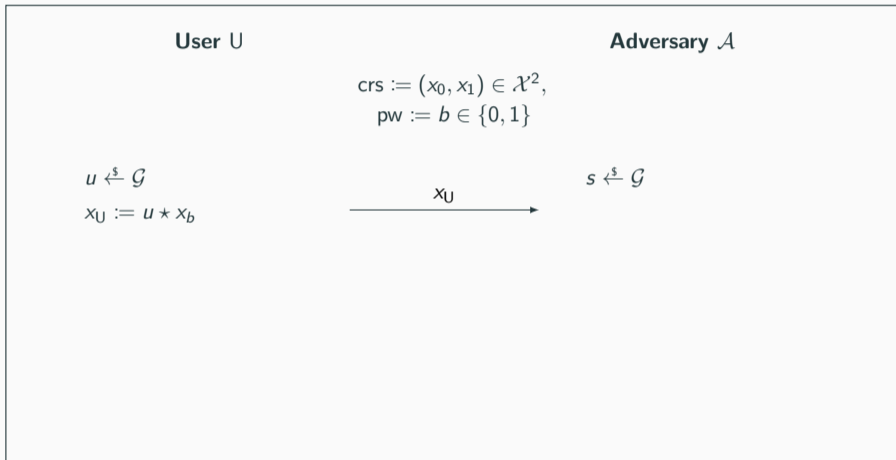
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Twists yield an Offline Dictionary Attack!



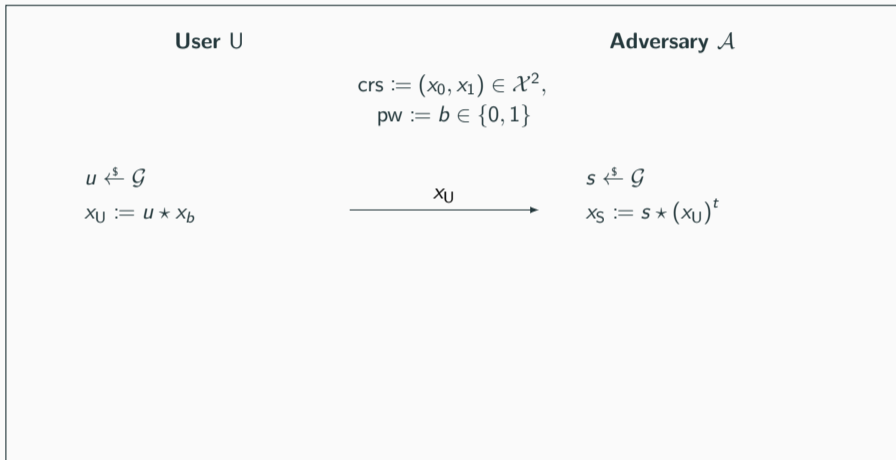
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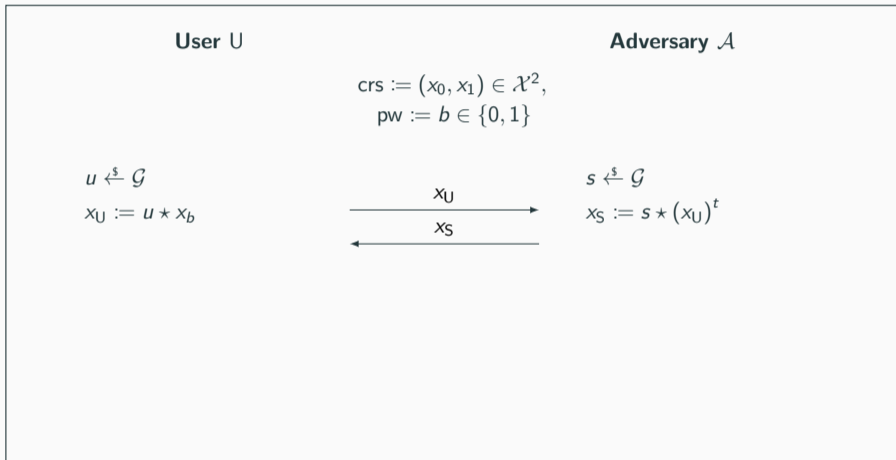
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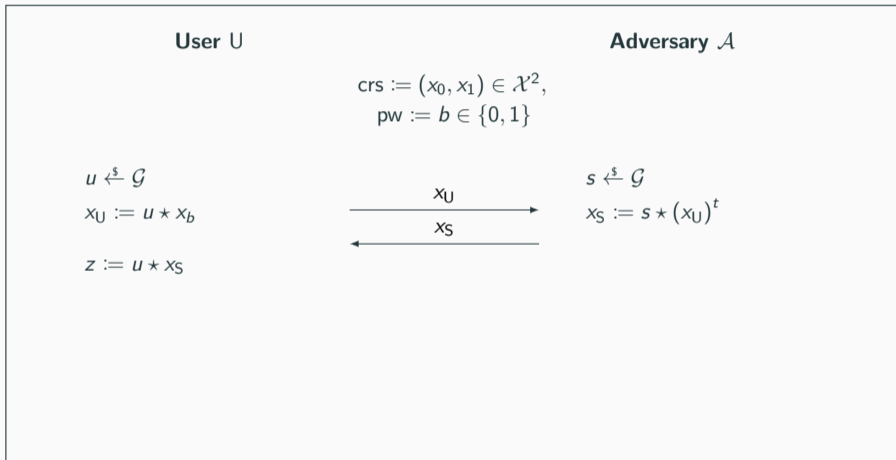
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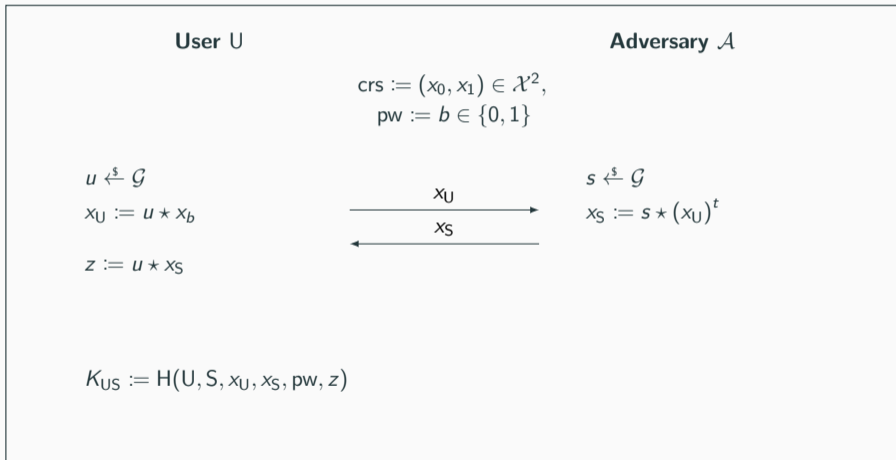
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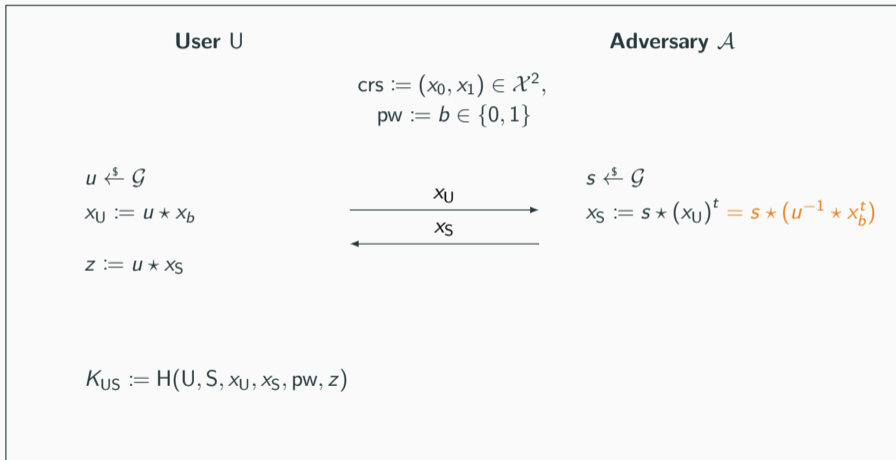
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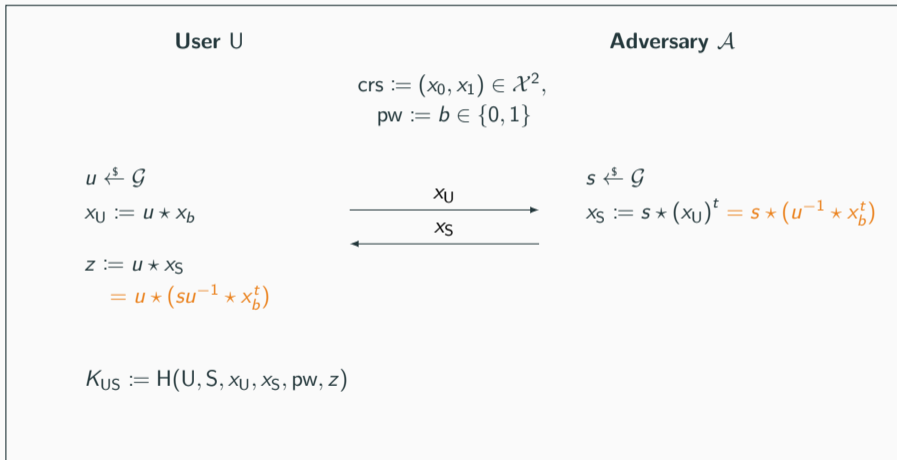
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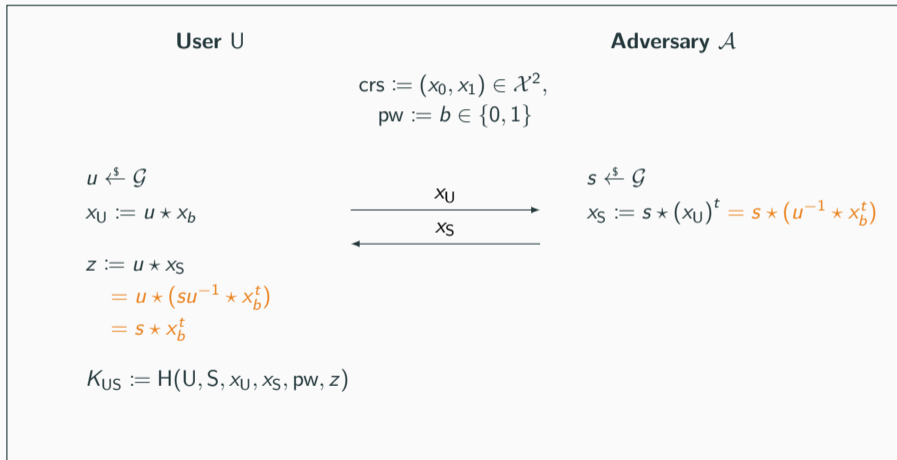
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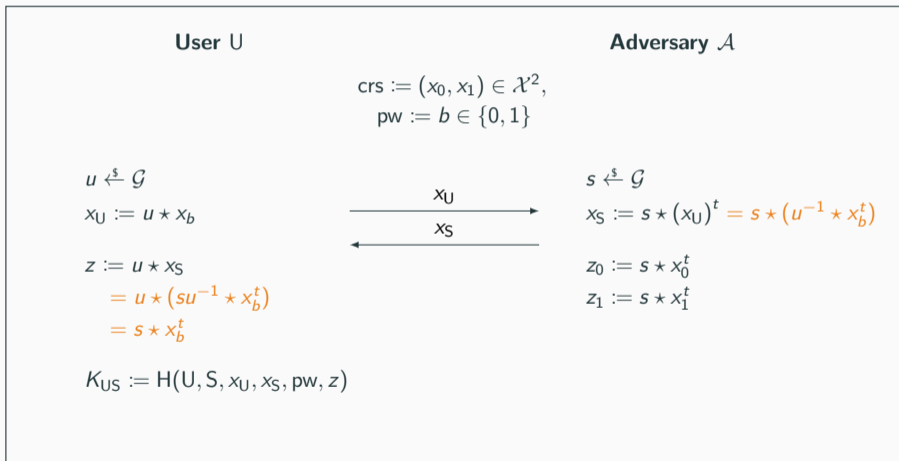
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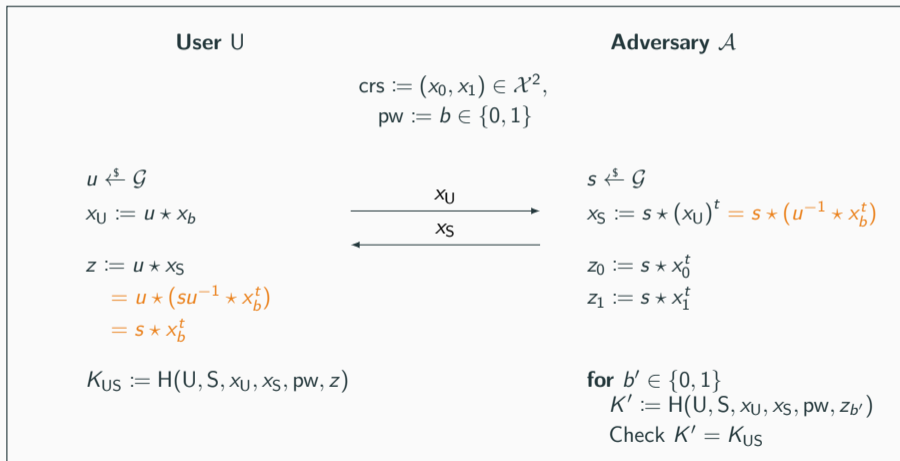
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Two New PAKE Protocols

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1. Use a Commitment (Com-GA-PAKE)

- The server commits on its message using a hash function (random oracle).
- An adversary cannot choose x_S depending on the user's message.
- Active security is based on GapCDH.

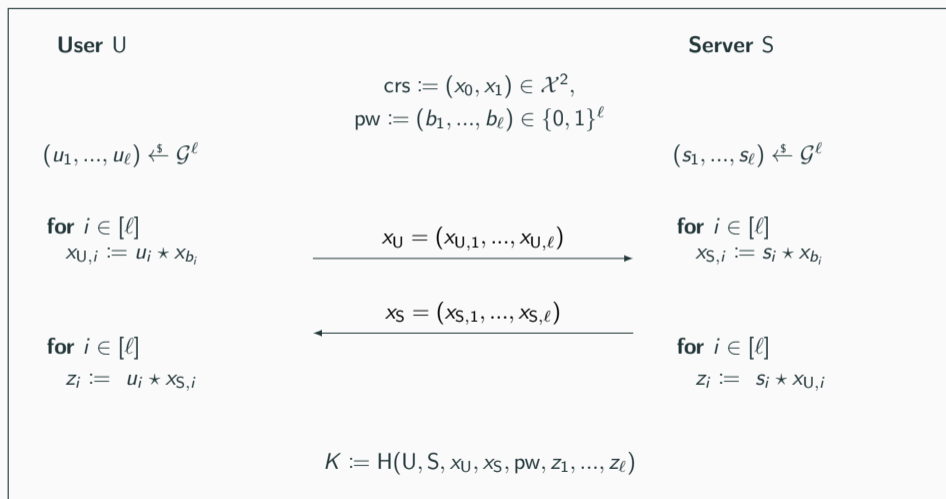
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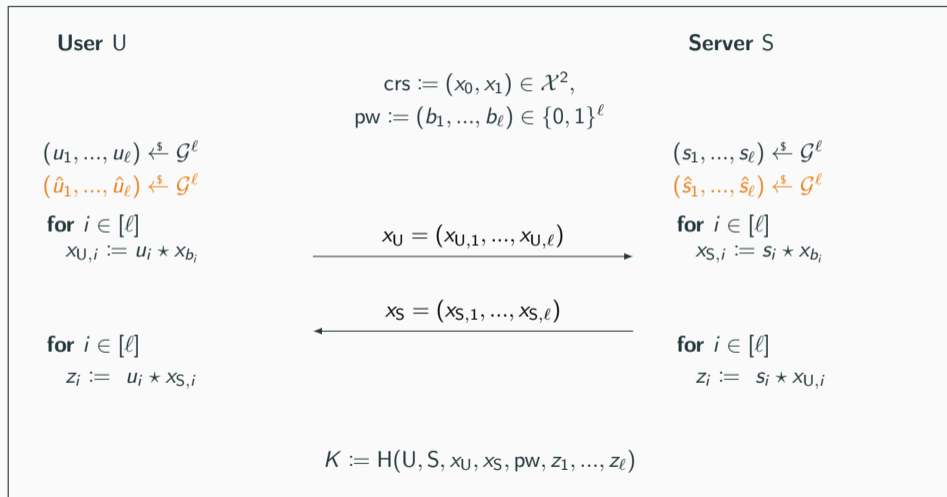
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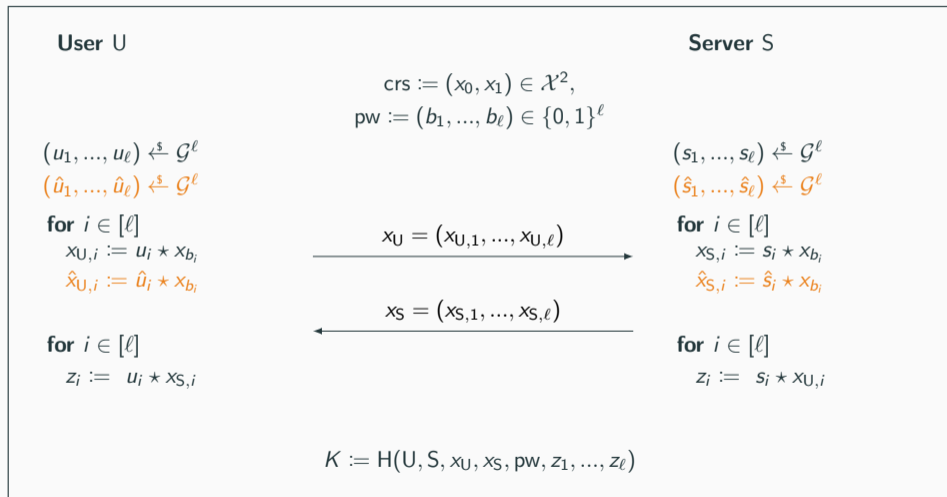
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2. Use “Cross-Terms” (X-GA-PAKE)

- Double the communication and combine elements in three ways.
- \mathcal{A} can compute at most two of the three combinations.







User U

$$(u_1, \dots, u_\ell) \xleftarrow{s} \mathcal{G}^\ell$$

$$(\hat{u}_1, \dots, \hat{u}_\ell) \xleftarrow{s} \mathcal{G}^\ell$$

for $i \in [\ell]$

$$x_{U,i} := u_i \star x_{b_i}$$

$$\hat{x}_{U,i} := \hat{u}_i \star x_{b_i}$$

for $i \in [\ell]$

$$z_i := u_i \star x_{S,i}$$

Server S

$$(s_1, \dots, s_\ell) \xleftarrow{s} \mathcal{G}^\ell$$

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for $i \in [\ell]$

$$z_i := s_i \star x_{U,i}$$

$$\text{crs} := (x_0, x_1) \in \mathcal{X}^2,$$

$$\text{pw} := (b_1, \dots, b_\ell) \in \{0, 1\}^\ell$$

$$x_U = (x_{U,1}, \dots, x_{U,\ell}, \hat{x}_{U,1}, \dots, \hat{x}_{U,\ell})$$

$$x_S = (x_{S,1}, \dots, x_{S,\ell}, \hat{x}_{S,1}, \dots, \hat{x}_{S,\ell})$$

$$K := H(U, S, x_U, x_S, \text{pw}, z_1, \dots, z_\ell)$$

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for $i \in [\ell]$

$$z_i := (u_i \star x_{S,i}, \hat{u}_i \star x_{S,i}, \\ u_i \star \hat{x}_{S,i})$$

Server S

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$$x_U = (x_{U,1}, \dots, x_{U,\ell}, \hat{x}_{U,1}, \dots, \hat{x}_{U,\ell})$$

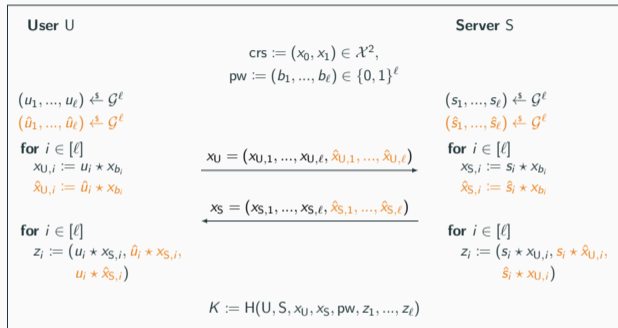
$$x_S = (x_{S,1}, \dots, x_{S,\ell}, \hat{x}_{S,1}, \dots, \hat{x}_{S,\ell})$$

$$K := H(U, S, x_U, x_S, \text{pw}, z_1, \dots, z_\ell)$$

Security of X-GA-PAKE

Security against Passive Adversaries

- secure under Strong CDH + ROM



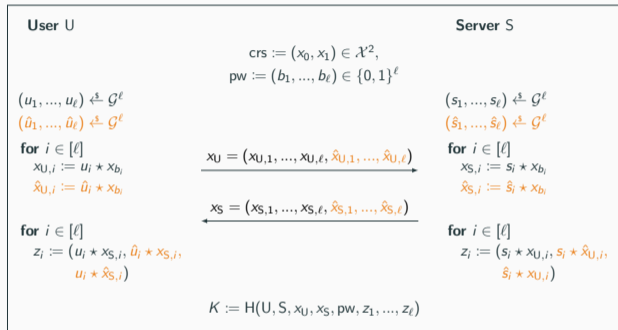
Security of X-GA-PAKE

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Security against Active Adversaries

- secure under Strong Square-Inverse DH + ROM



Security of X-GA-PAKE

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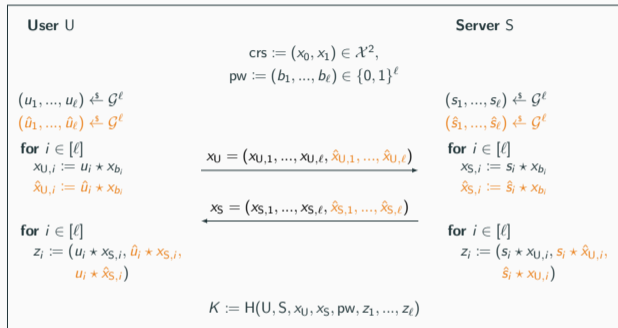
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Security against Active Adversaries

- secure under Strong Square-Inverse DH + ROM
- given $(g \star \tilde{x})$ compute (y, z_0, z_1) such that

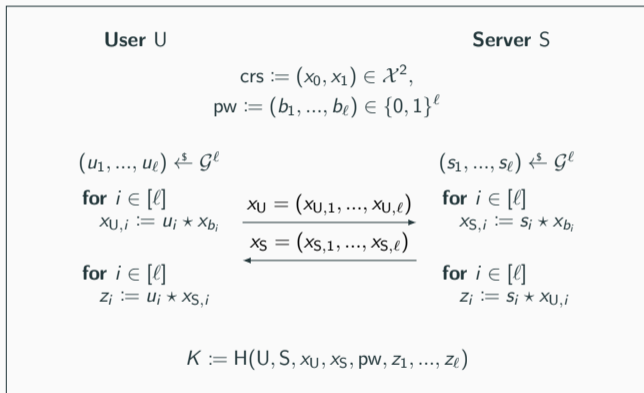
$$z_0 = g^2 \star y$$

$$z_1 = g^{-1} \star y$$

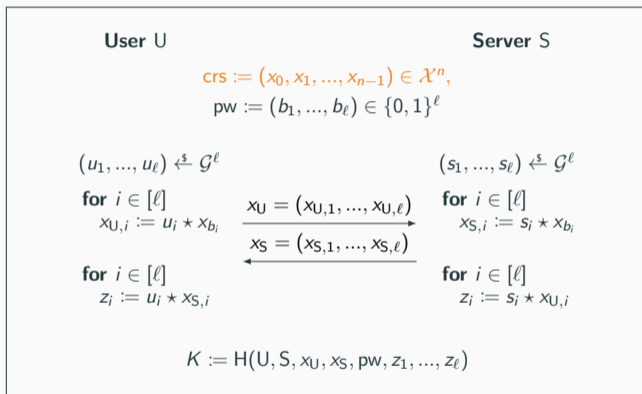


Optimizations

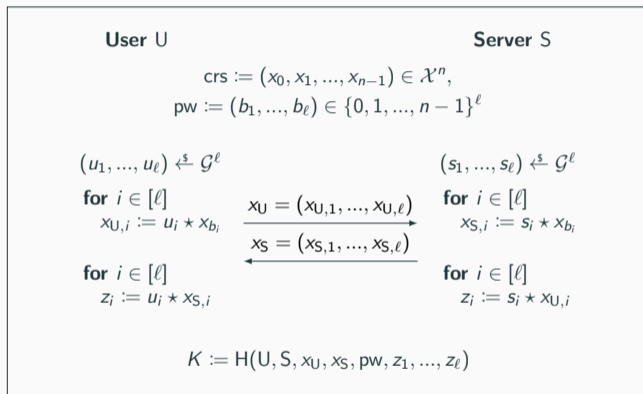
Decrease the Communication and Computation Cost



Decrease the Communication and Computation Cost



Use Twists in the Setup



Comparison of Com-GA-PAKE and X-GA-PAKE

	Using OT [CDVW12, LGd21]	Com-GA-PAKE	X-GA-PAKE
Set Elements	384	16	32
Evaluations	1408	32	80
Rounds	4	3	1
Security Assumption	CDH	Gap CDH	Strong Square-Inverse
Tight	no	no	yes

Here we assume $\mathcal{PW} \subset \{0, 1\}^{128}$ for all schemes, i.e.,

- $\ell = 128$ for the OT-based construction
- $\ell = 16$, $n = 8$ for our optimized variants

Comparison of Com-GA-PAKE and X-GA-PAKE

	Using OT [CDVW12, LGd21]	Com-GA-PAKE	X-GA-PAKE
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Results

- Group actions with twists as abstraction for CSIDH
- The first direct constructions and provably secure PAKE protocols from CSIDH
- Better efficiency than generic constructions, e.g. based on OT

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Thank you!

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