Faster Sounder Proofs

Justin Holmgren
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Ron Rothblum
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Paper

Slides

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Proof Systems

Proof Systems: the Theory Perspective

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Super-efficient verification of complex statements:

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Proofs spurred the development of lots of deep theory!

Practical Motivation:
Blockchains & Cryptocurrencies

Efficient ZK proofs have amazing applications.

"Perhaps the most powerful cryptographic technology to come out of the last decade is general-purpose succinct zero knowledge proofs"

- Vitalik Buterin, Ethereum co-founder (2021)

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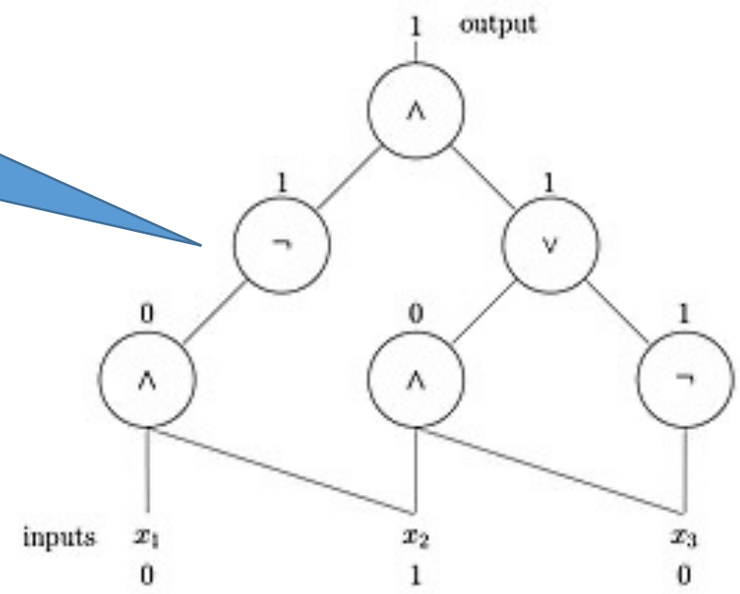
Looking at very small factors ⇒ the computational model matters!

Today: Boolean Circuits

Constant fan-in, arbitrary gates



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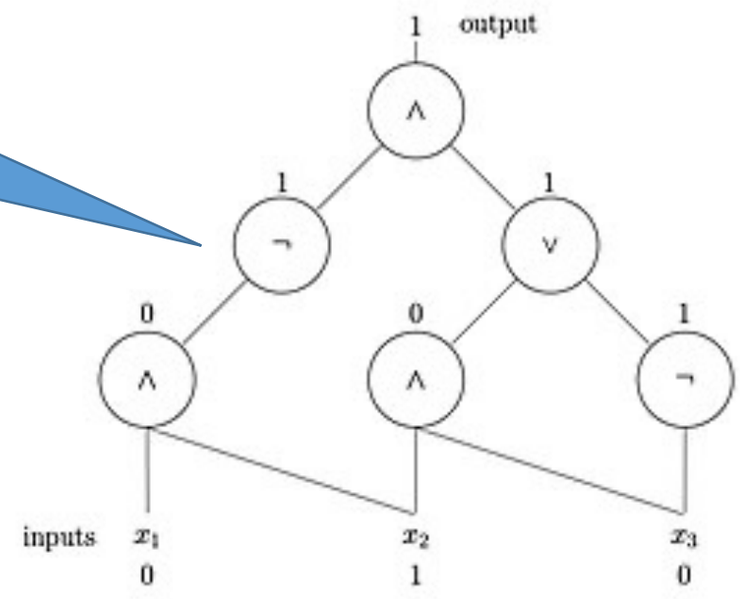

Why Boolean circuits? [IKOS08]

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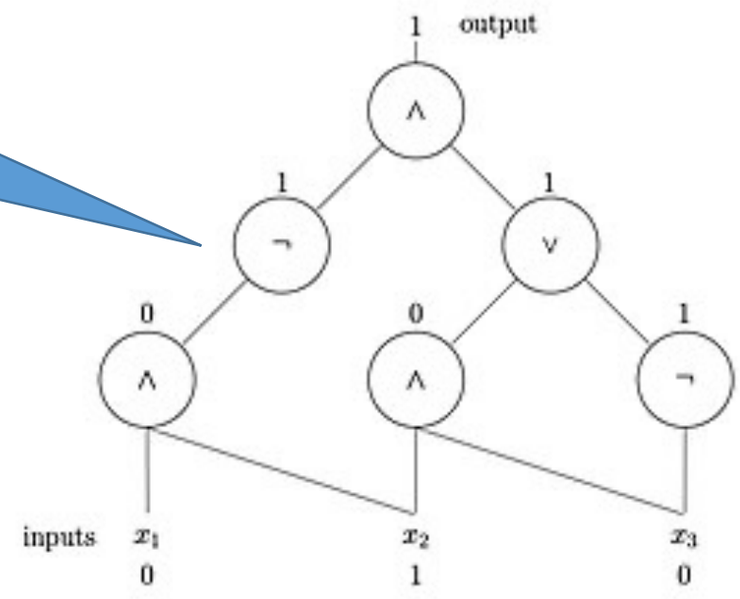

Why Boolean circuits? [IKOS08]

- Natural, fundamental, and physically motivated model of computational complexity
- Includes interesting non-arithmetic computation (e.g, SHA).

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Parameterized by:

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Main Result

TL;DR: succinct proofs for "nice circuit" satisfiability with $S \cdot \text{polylog}(\lambda)$ size prover, soundness error $2^{-\lambda}$, and sublinear verification

hiding additive $\text{poly}(\lambda)$

exact complexity depends on niceness

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- **More generally,** we define a "tensor CSP", and say that C is nice if it has a small tensor CSP. Ask me for details, if you dare :D

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"nice" Boolean (repetitive structure)	Boolean size $S \cdot \text{polylog}(\lambda)$ [this work]
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Pointwise multiplication

Bonus Time:

Technical Tour

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- $M : \mathbb{F}^\lambda \to \mathbb{F}^{O(\lambda)}$ is a **multiplication code** encodable in size $\lambda \cdot \text{polylog}(\lambda)$ (e.g. Reed-Solomon + FFT)

Final Boss Slide

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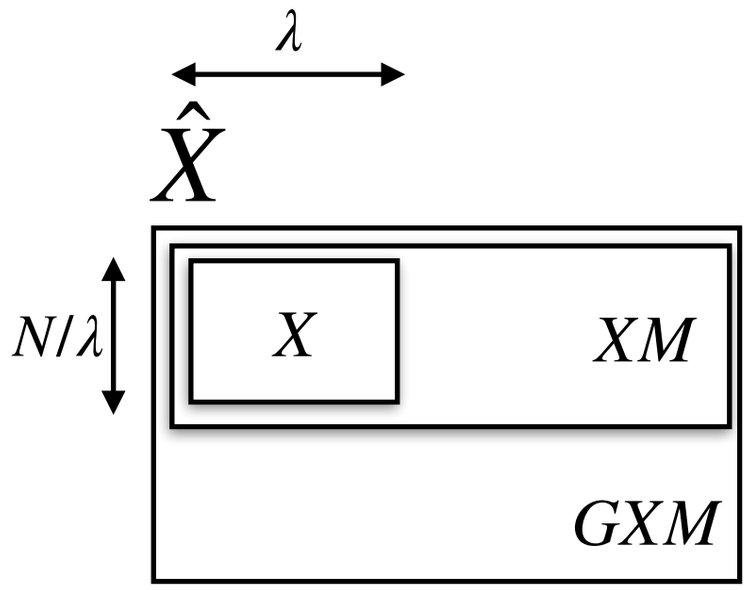

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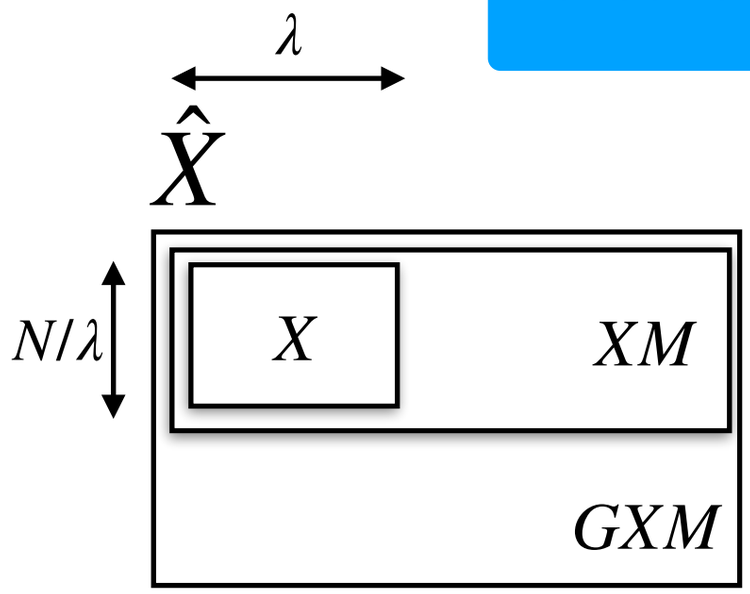

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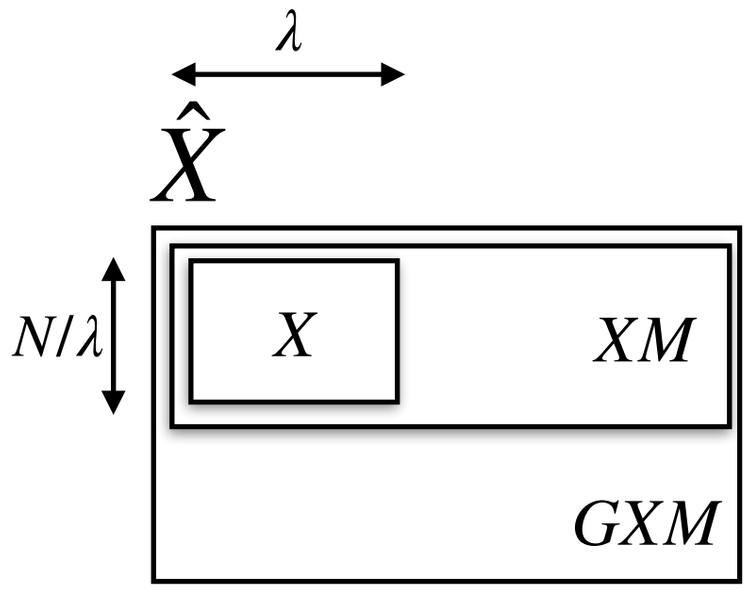

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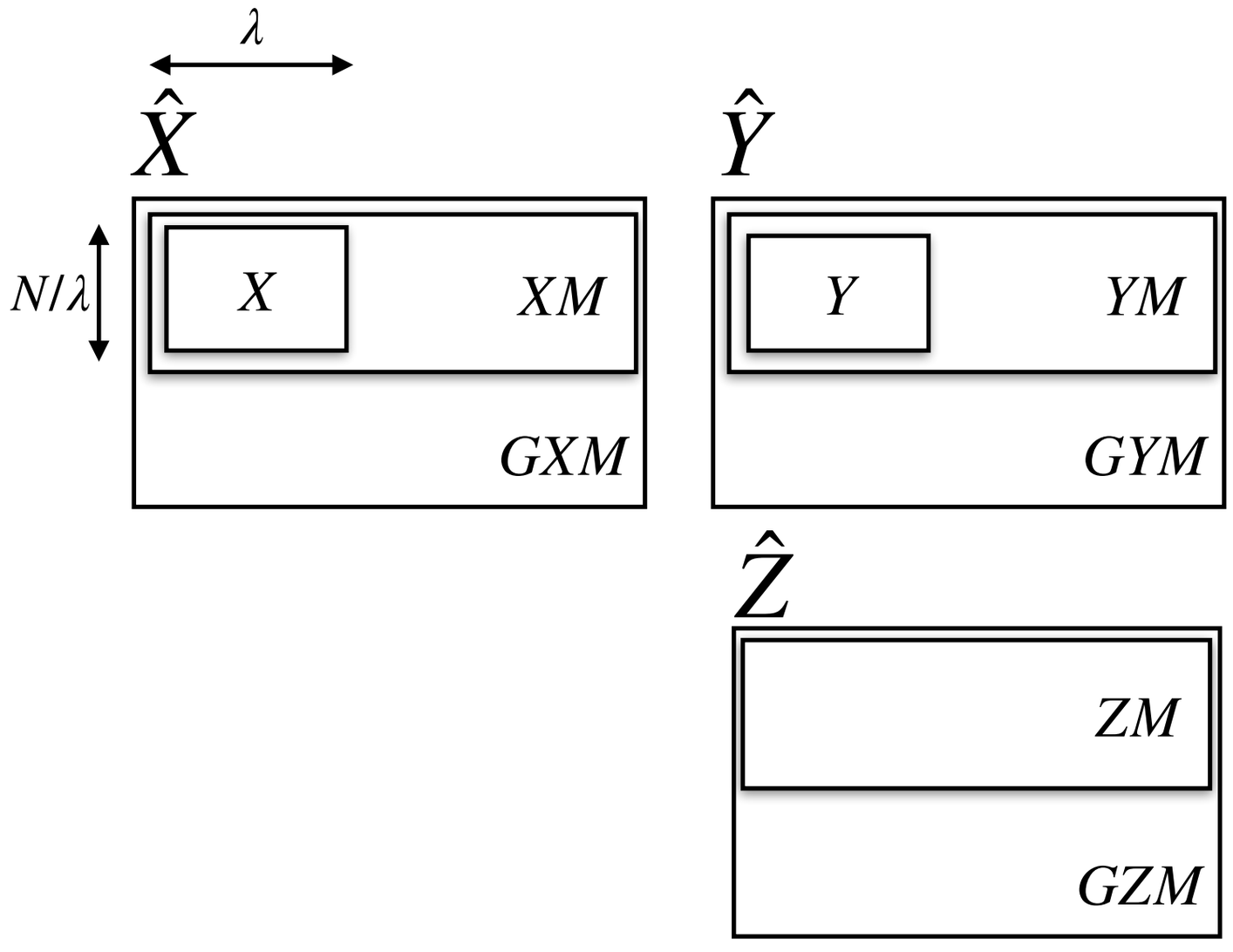

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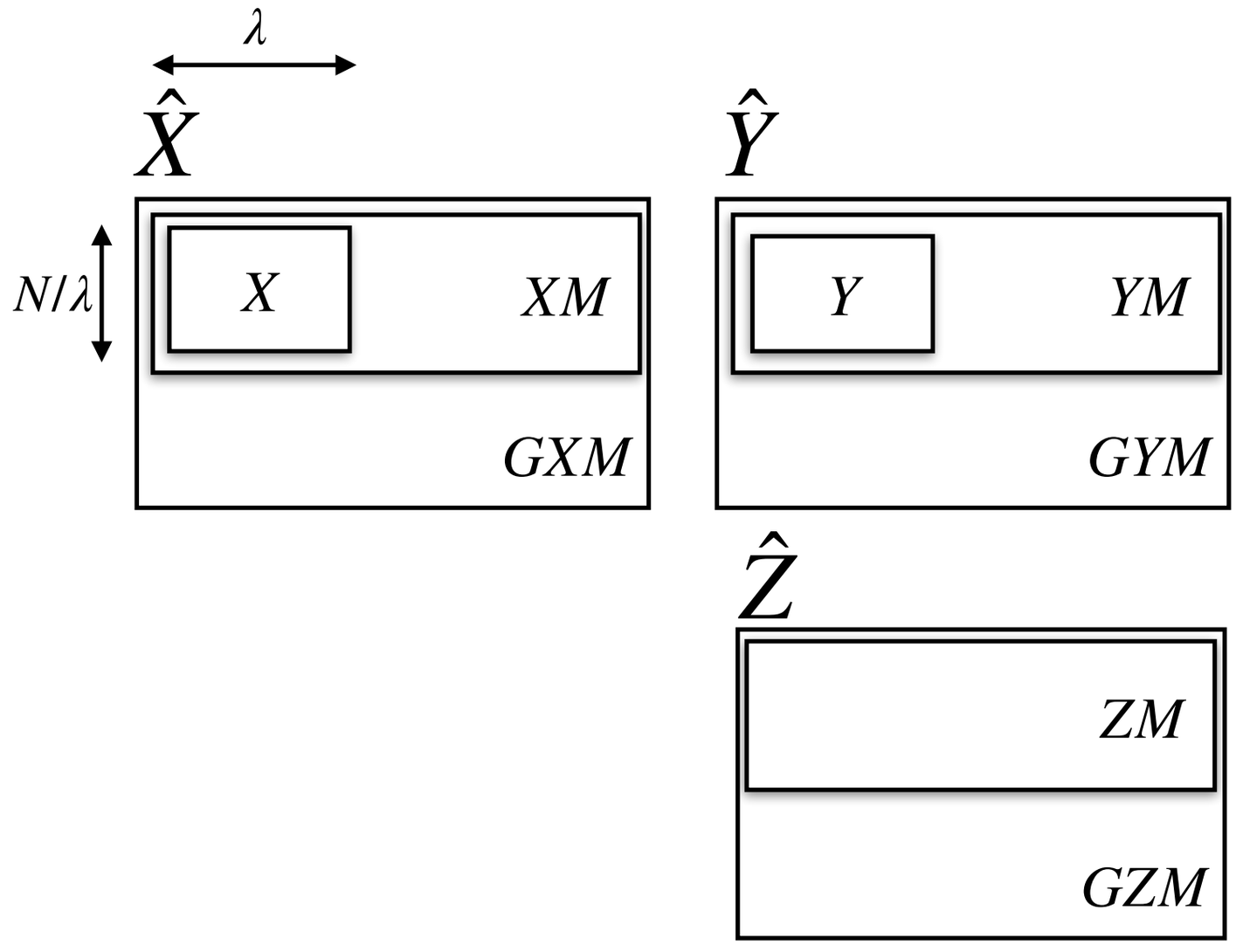

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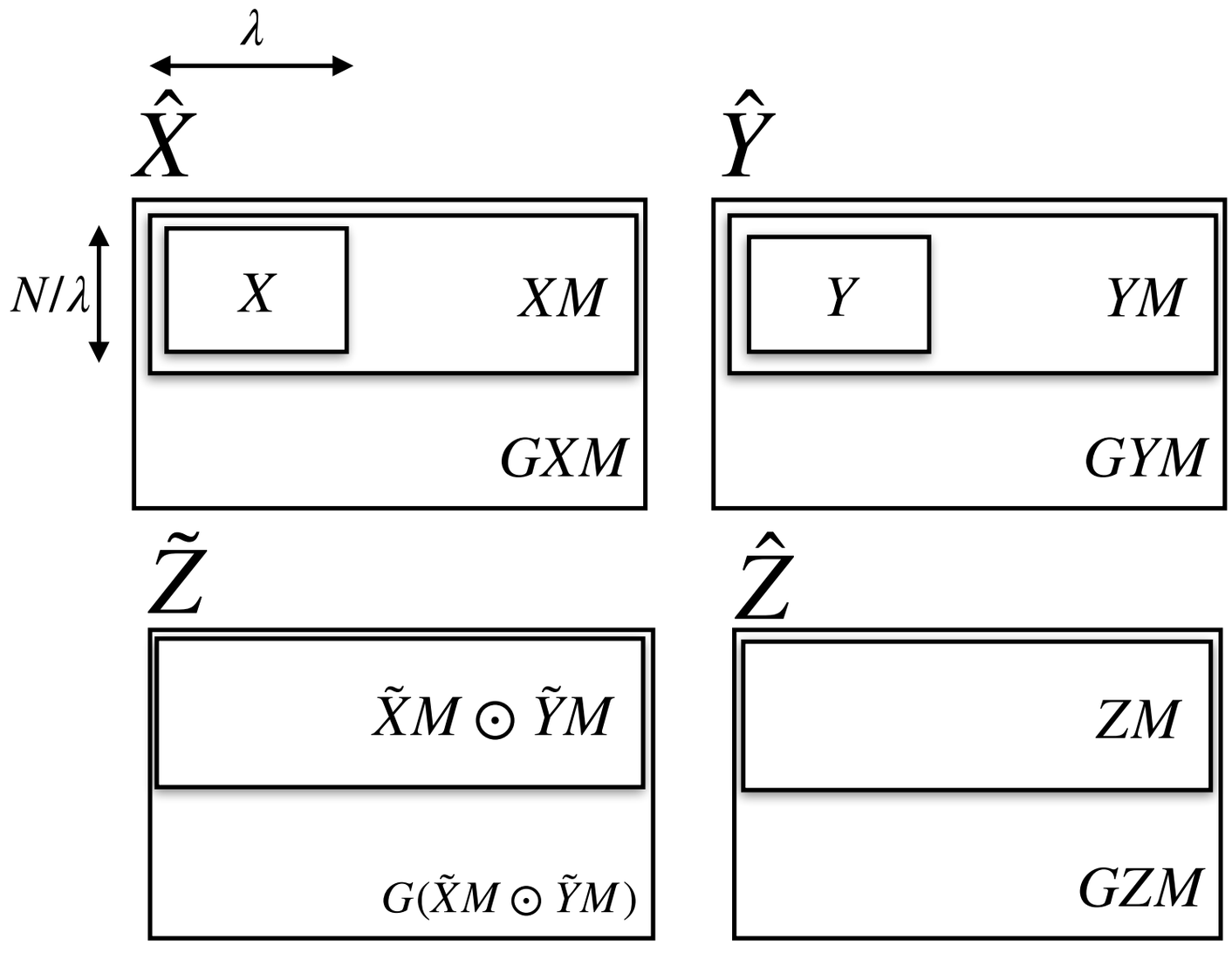

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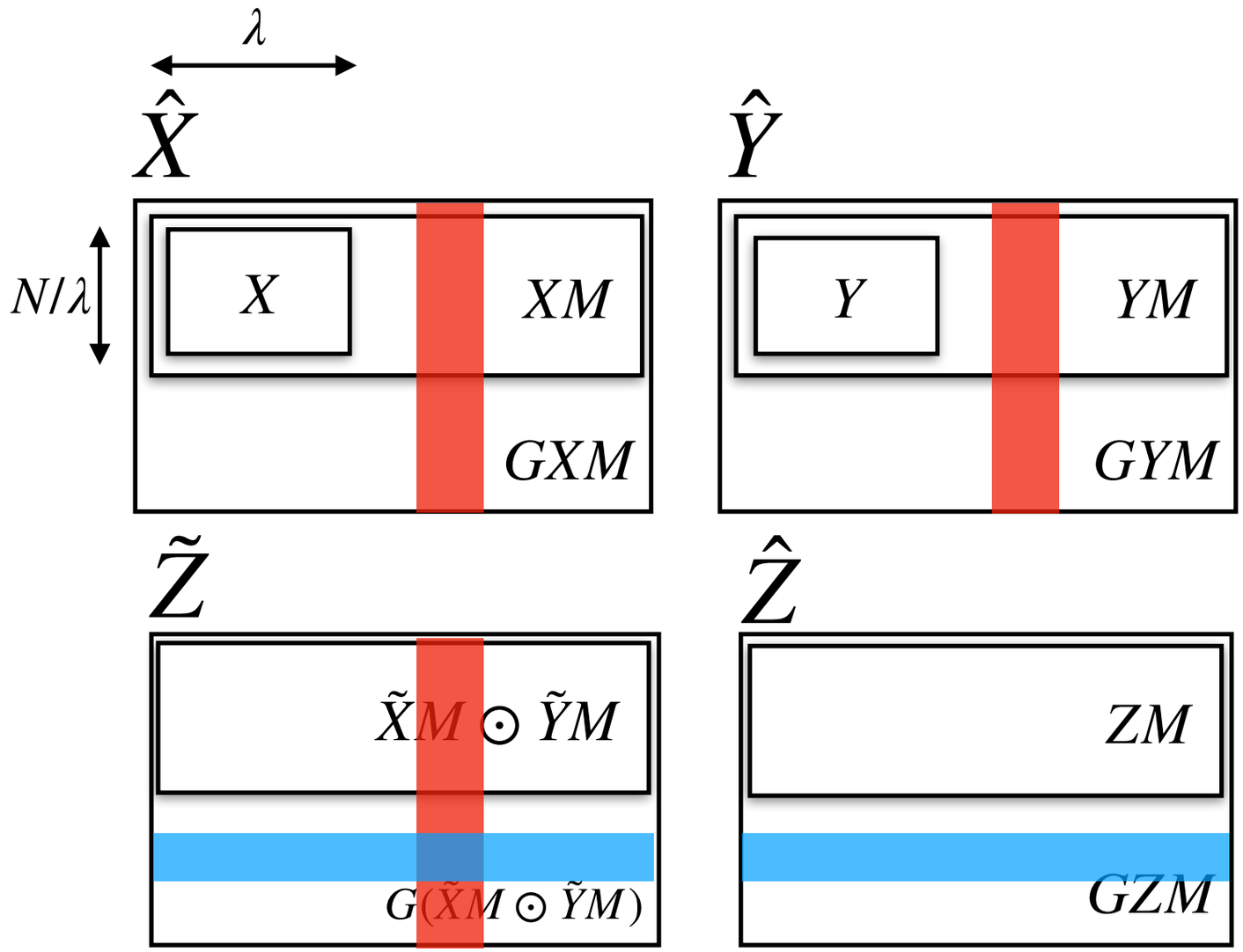

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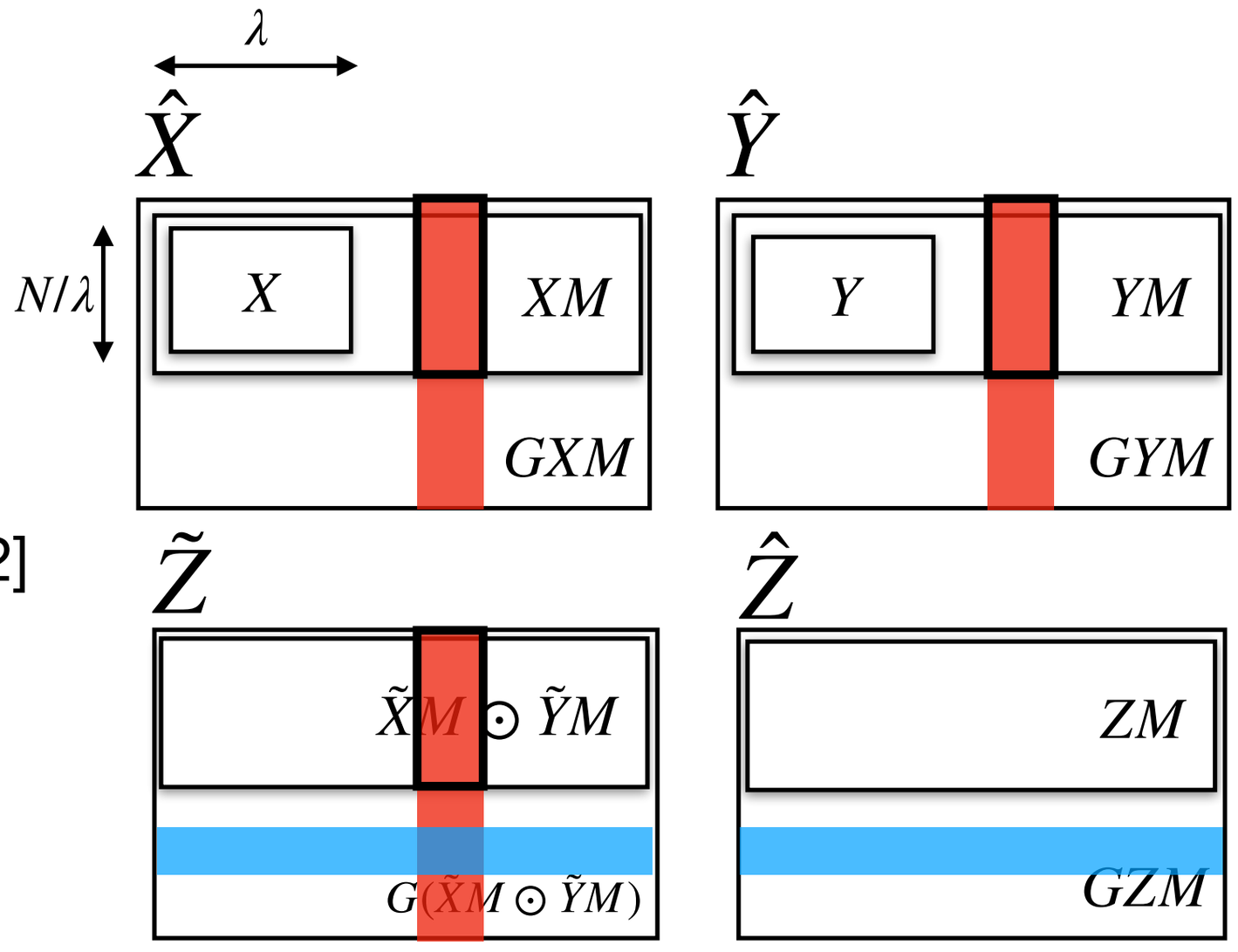

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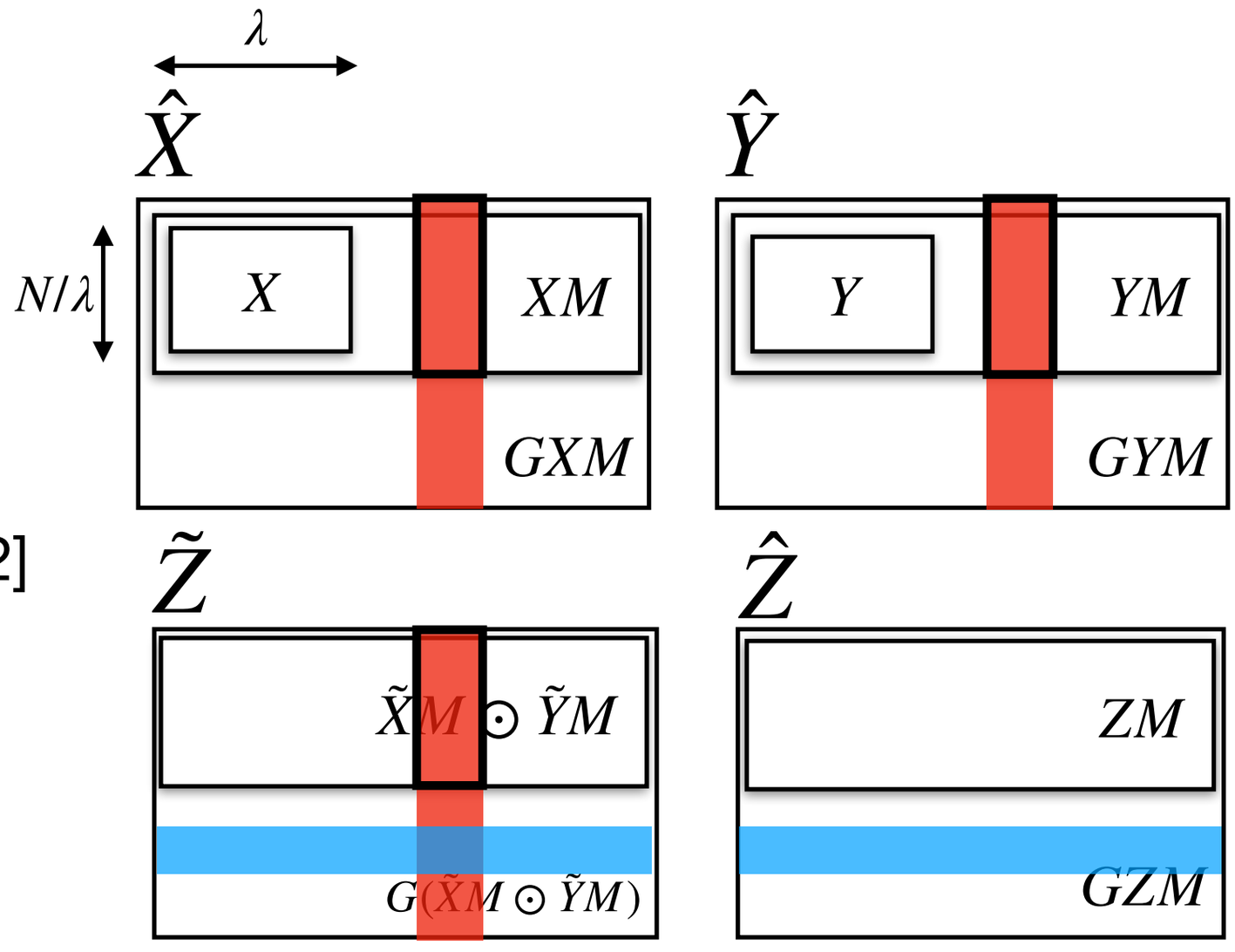

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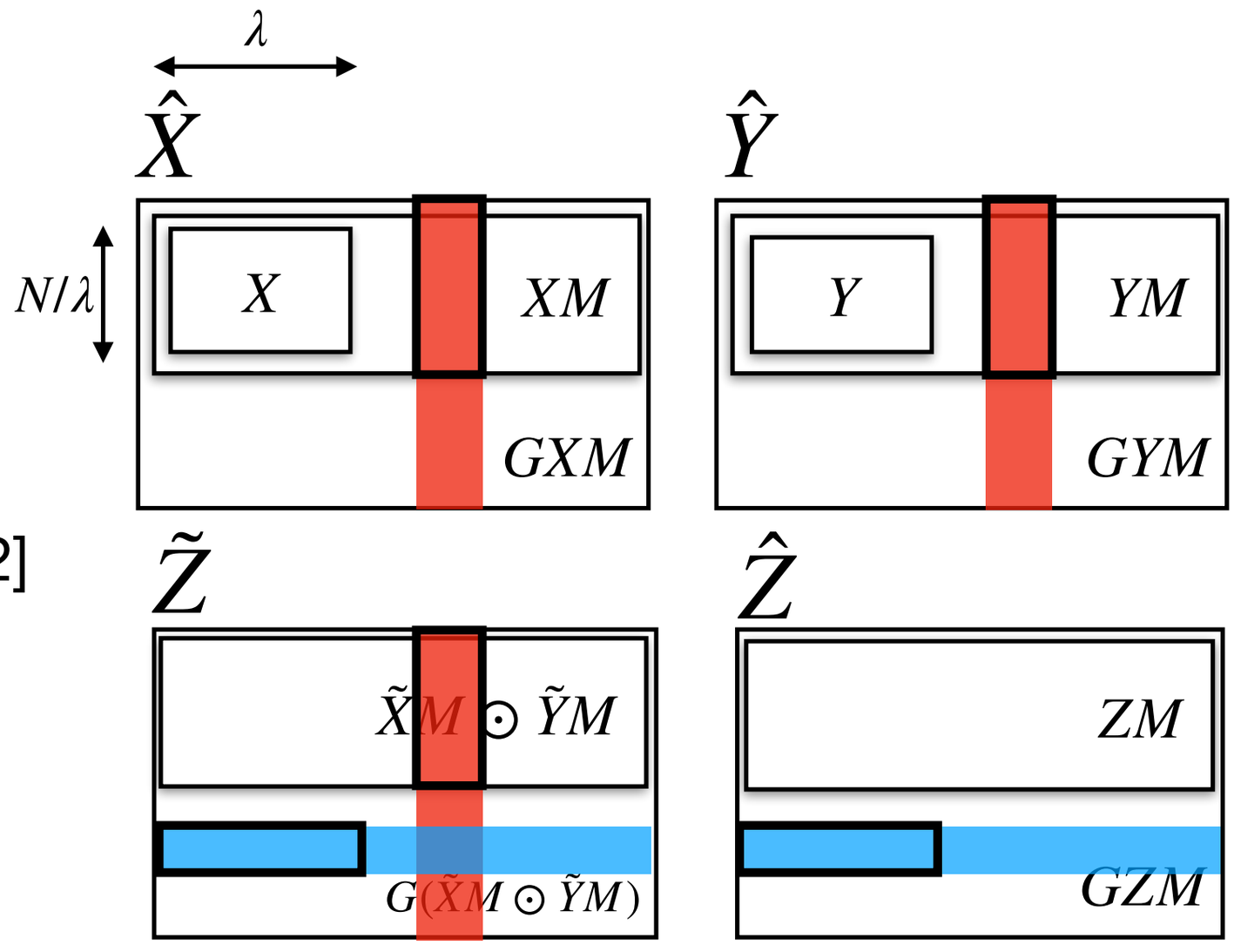
b) repeats λ times



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Epilogue

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