

Better than Advertised Security for Non-Interactive Threshold Signatures

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Stefano Tessaro, **Chenzhi Zhu**

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Threshold Signatures [DY90]

Single Key



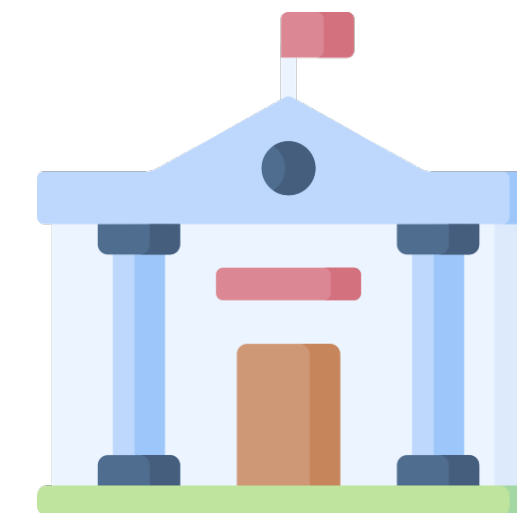
Distributed Key

Single point of failure

Tolerates some fraction of
corrupt signers

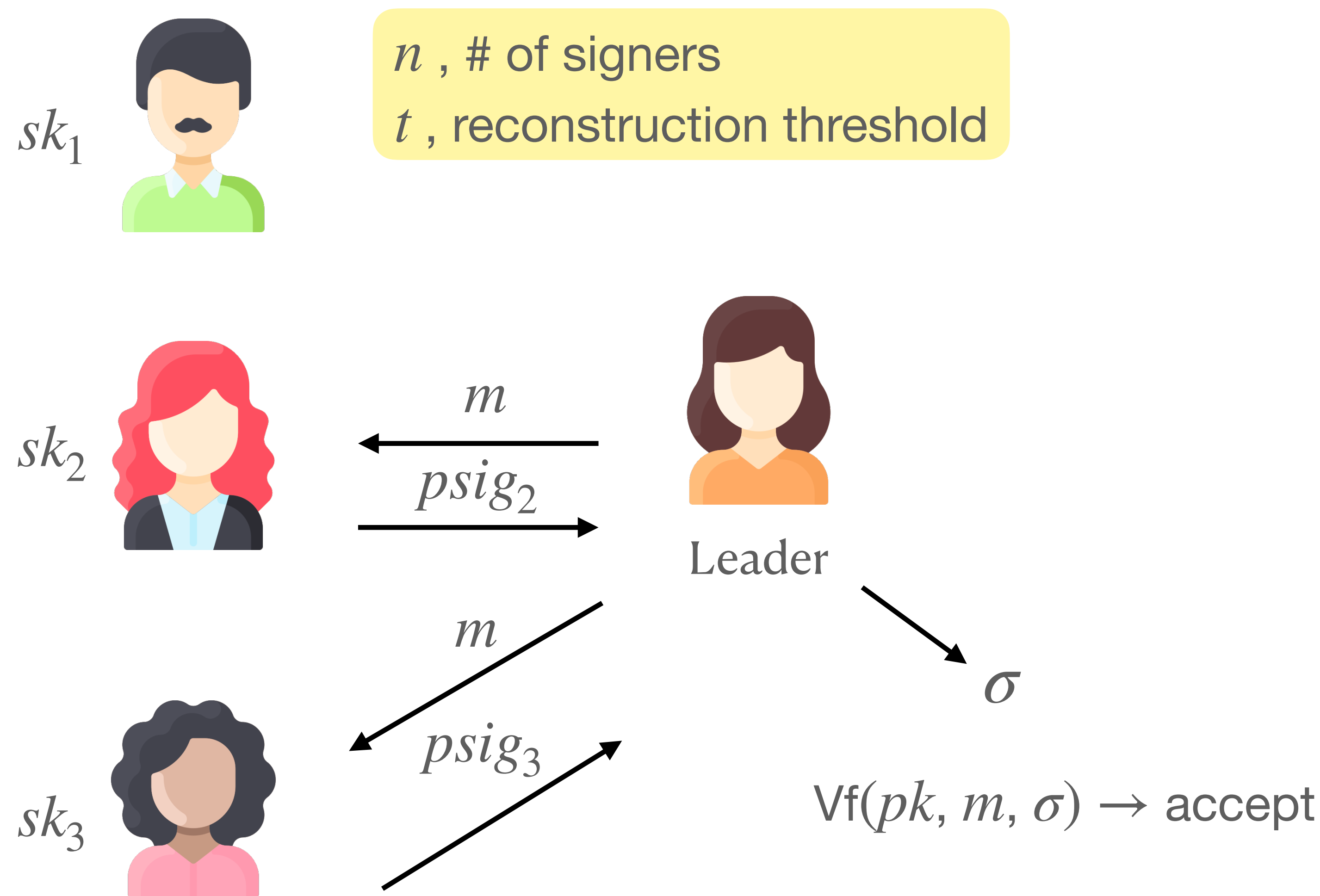


Cryptocurrency Wallets

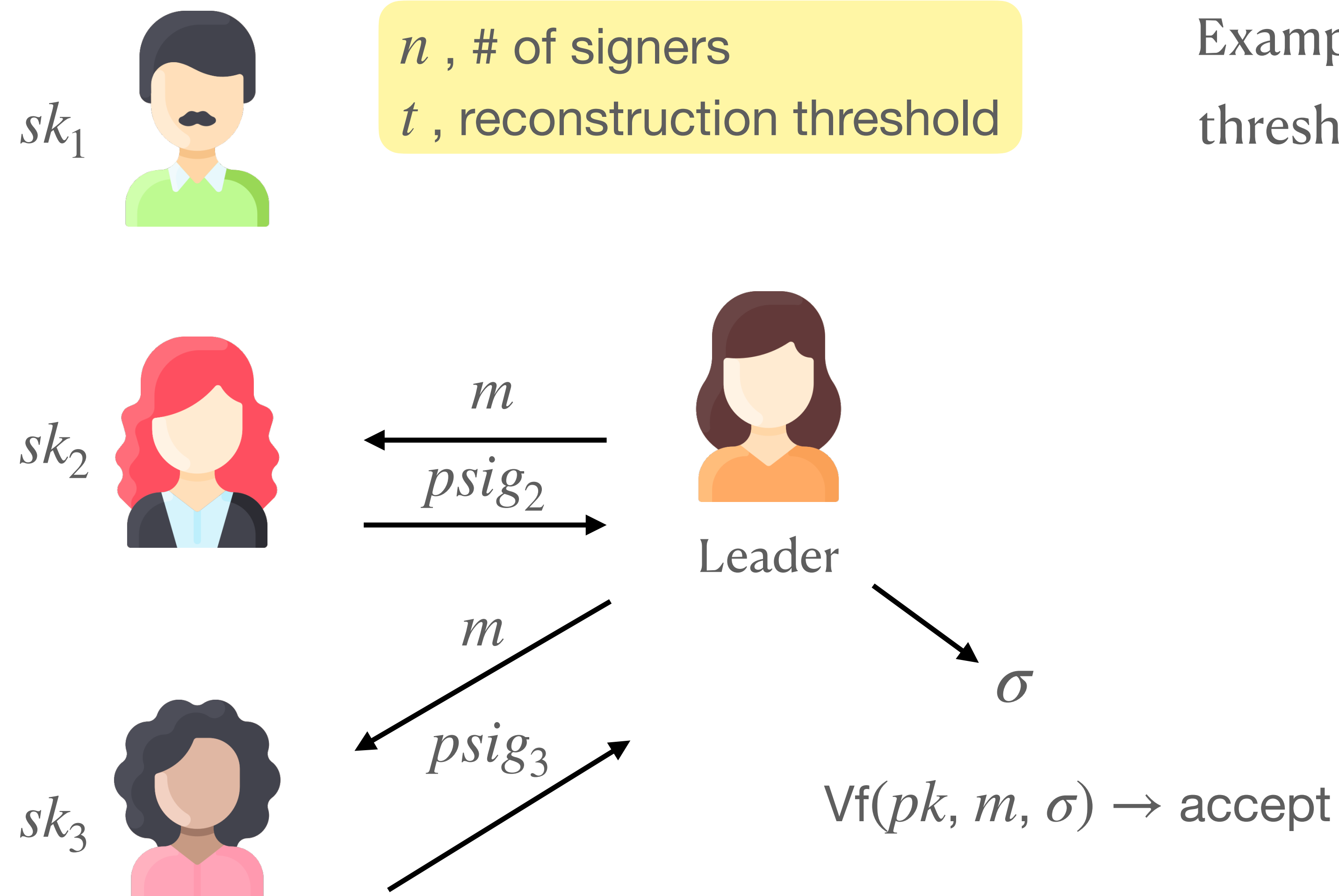


NIST Standardization

Our Focus: Non-Interactive Threshold Signatures



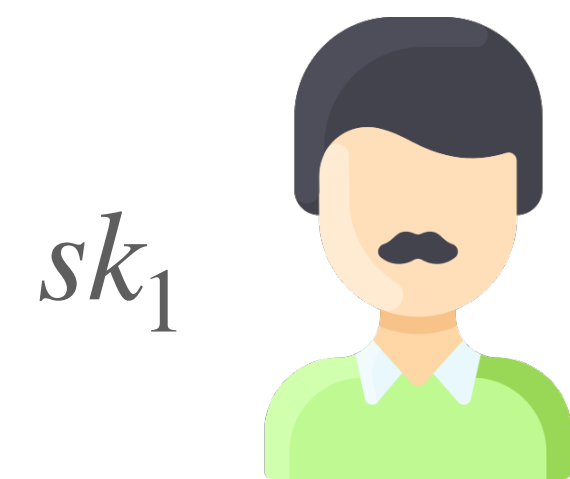
Our Focus: Non-Interactive Threshold Signatures



Examples: threshold BLS [BoI03]

threshold RSA [DSD+94, GJKR00, Sho00]

Our Focus: Non-Interactive Threshold Signatures



n , # of signers
 t , reconstruction threshold

Examples: threshold BLS [BoI03]
threshold RSA [DSD+94, GJKR00, Sho00]



m
 $psig_2$



Leader

σ



m
 $psig_3$

$Vf(pk, m, \sigma) \rightarrow \text{accept}$

What about DL-based?
(Pairing-free)

DL-based Threshold Signatures

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No fully non-interactive scheme yet

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Schnorr Signatures

Single message-dependent
round: FROST [KG20]

A **single-round** message-
independent pre-processing

“Partially non-interactive”

DL-based Threshold Signatures

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A **multi-round** message-
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Not covered

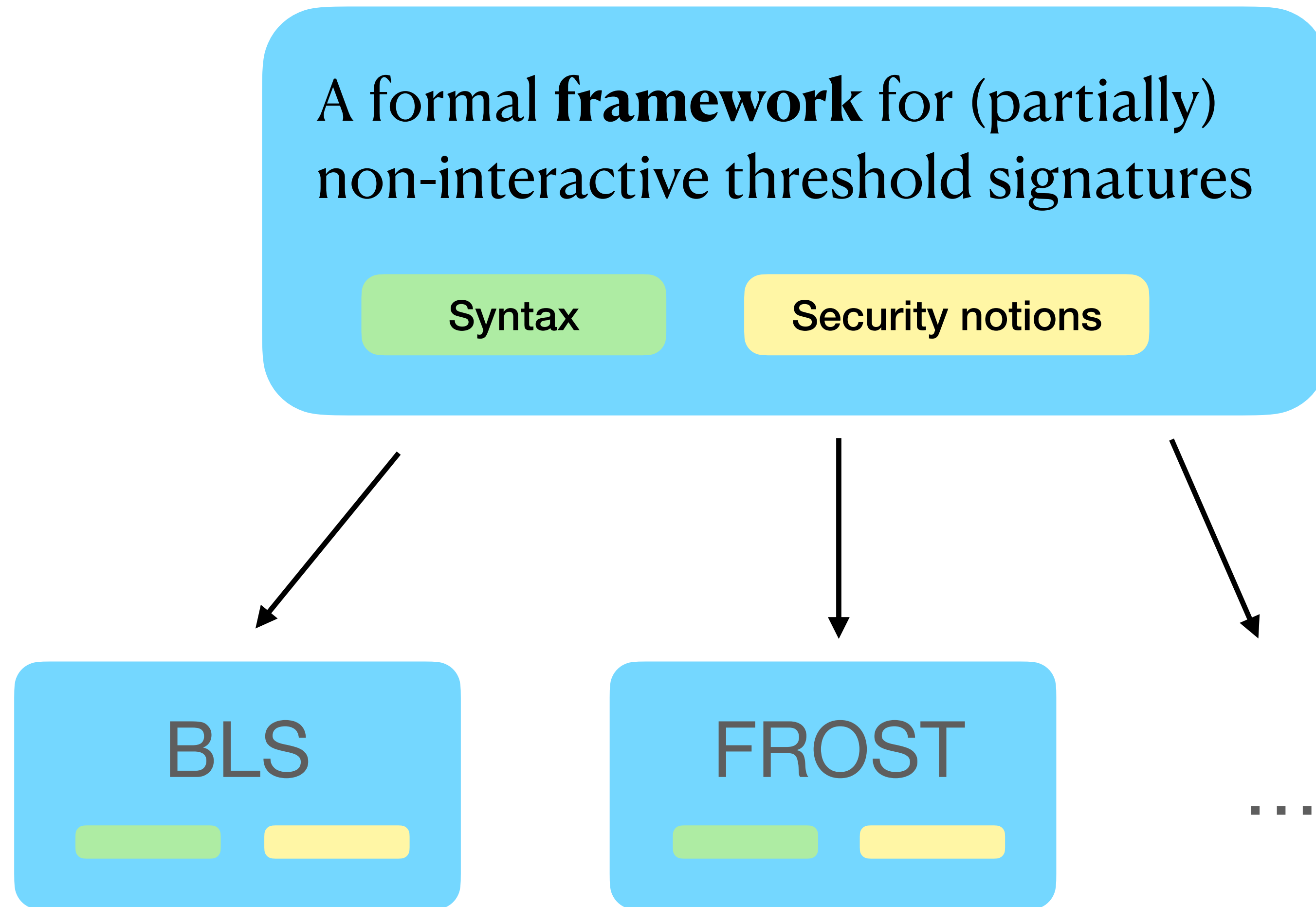
This Paper

A formal **framework** for (partially)
non-interactive threshold signatures

Syntax

Security notions

This Paper

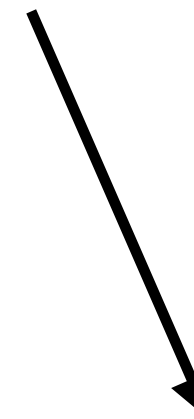
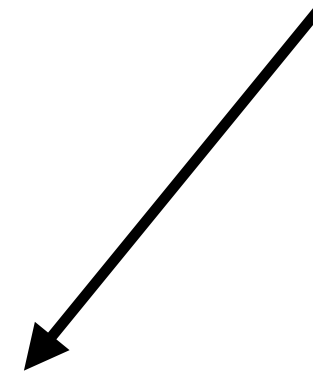


This Paper

A formal **framework** for (partially)
non-interactive threshold signatures

Syntax

Security notions



BLS



FROST



...

Why is this needed?

What's Missing & Our Contributions

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- No formalization for partially non-interactive schemes

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- Analysis of FROST based on our security hierarchy

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- Fine-grained security hierarchy with stronger notions

Analysis of BLS for stronger security
Concurrent work [Gro21]

- Original proof for FROST [KG20] relied on heuristic assumptions

- Analysis of FROST based on our security hierarchy

Our Contributions

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- Introduce FROST2, an optimized version of FROST1
- Prove the security of FROST1/2 under OMDL + ROM in trusted DKG setting
- Prove the security of FROST2 together with PedPoP DKG under OMDL + ROM + AGM
- Show separation of security of FROST1 vs 2

FROST

FROST

- Flexible Round-Optimized Schnorr Threshold Signature [KG20]

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- Consists of:
 - PedPoP distributed key generation (DKG) protocol
 - Two-round, concurrently secure signing protocol
 - First round message-independent pre-processing
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 - Outputs standard, single-party Schnorr signature
- Prior attempts at two-round Schnorr threshold signatures -> ROS attacks [BLLOR21], https://github.com/mmaller/multi_and_threshold_signature_reductions

FROST



FROST [KG20]

FROST



FROST [KG20]



**Cryptocurrency
Wallets**

FROST



FROST [KG20]



**NIST
Standardization**



**Cryptocurrency
Wallets**

FROST



FROST [KG20]



**NIST
Standardization**



**Cryptocurrency
Wallets**



IETF Draft

FROST

FROST [KG20]

**NIST
Standardization**

**5+
Implementations**

**Cryptocurrency
Wallets**

IETF Draft

PedPoP Distributed Key Generation

- PedPoP [KG20] = Pedersen DKG + Proofs of Possession
- Proofs of possession are Schnorr signatures
 - Requires a Knowledge of Exponent assumption
 - For simplicity, instead of adding rounds to DKG
 - Allows any number of corrupt signers (not honest majority)

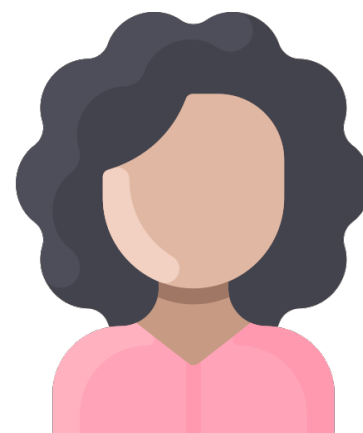
FROST1

sk_2



Leader

sk_3



FROST1

sk_2



$$R_2 = g^{r_2}, S_2 = g^{s_2}$$



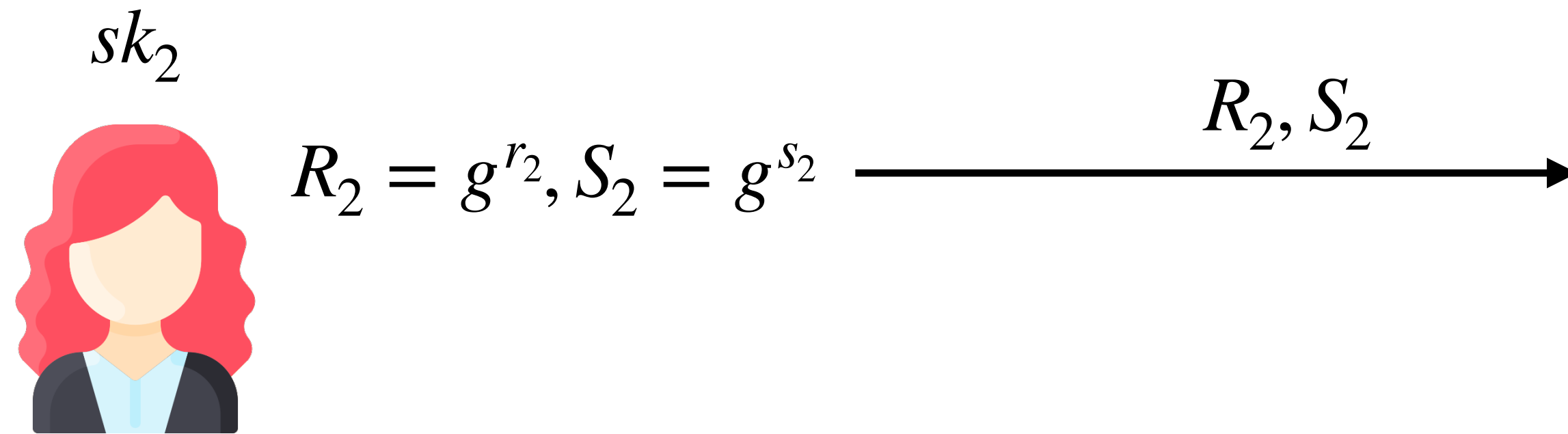
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sk_3

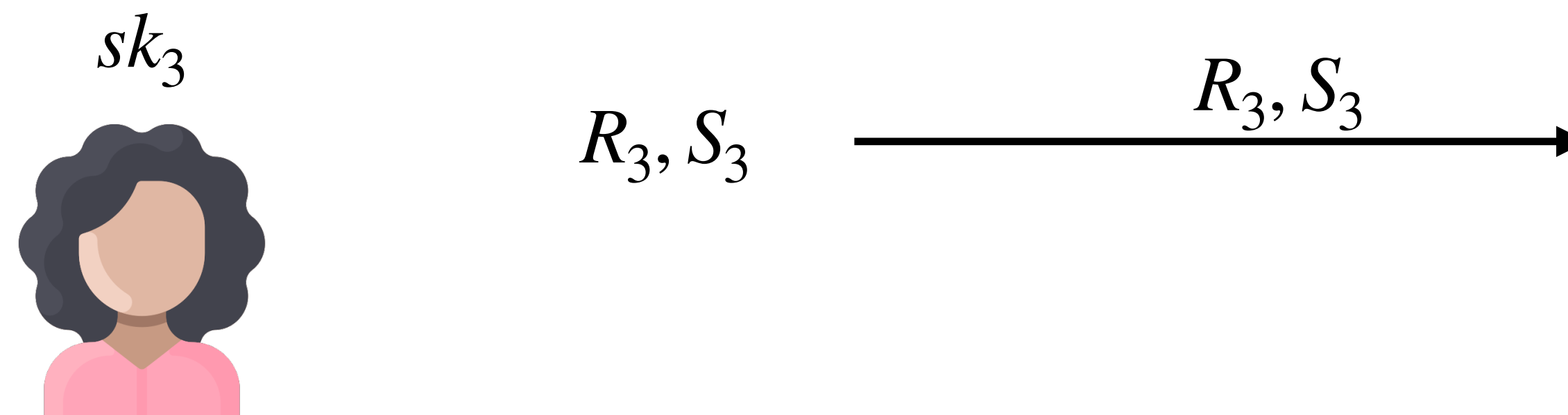


R_3, S_3

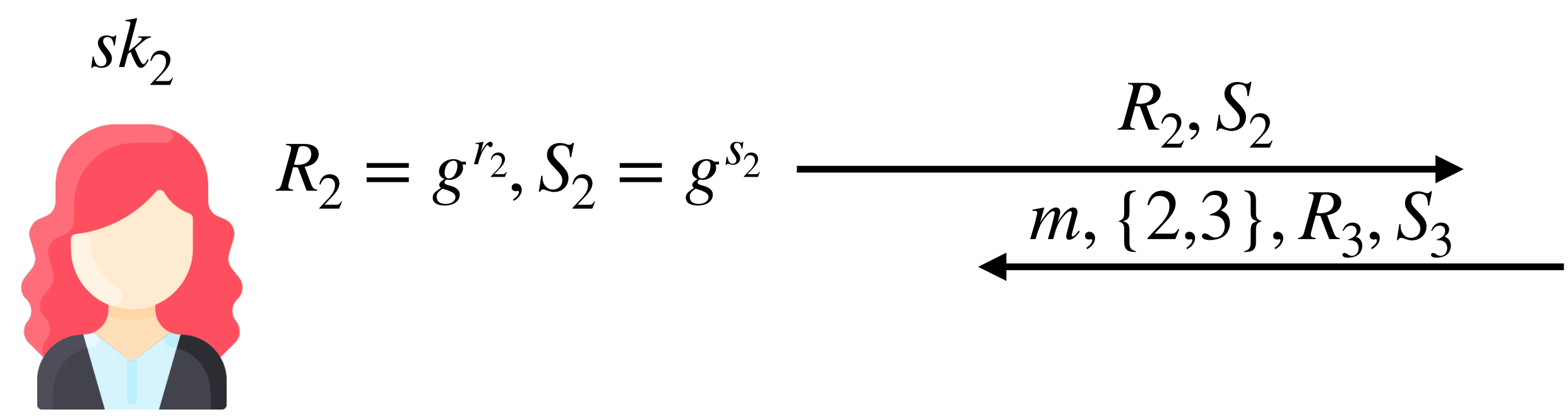
FROST1



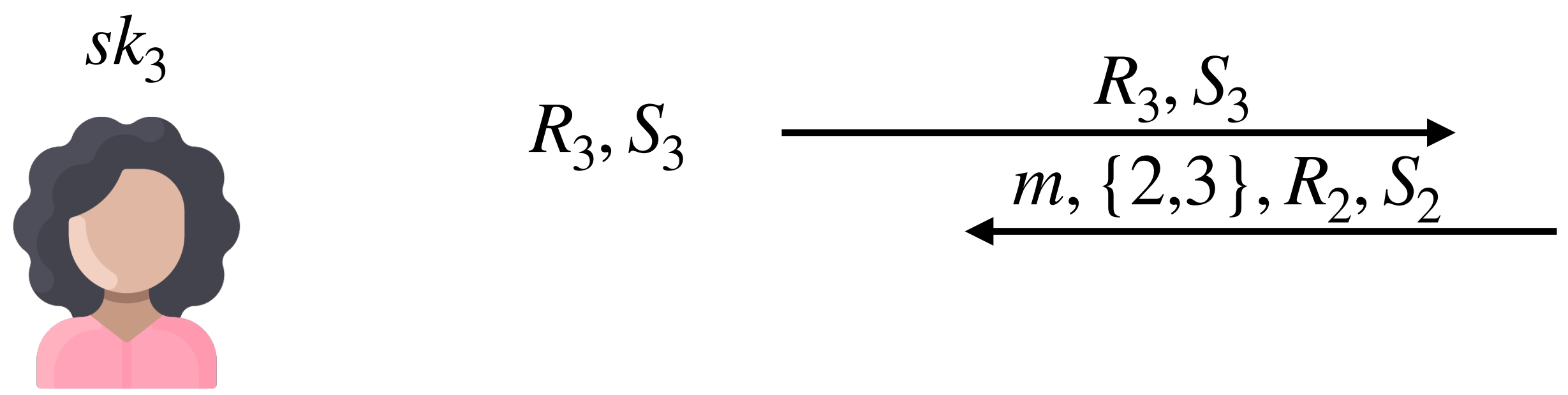
Leader



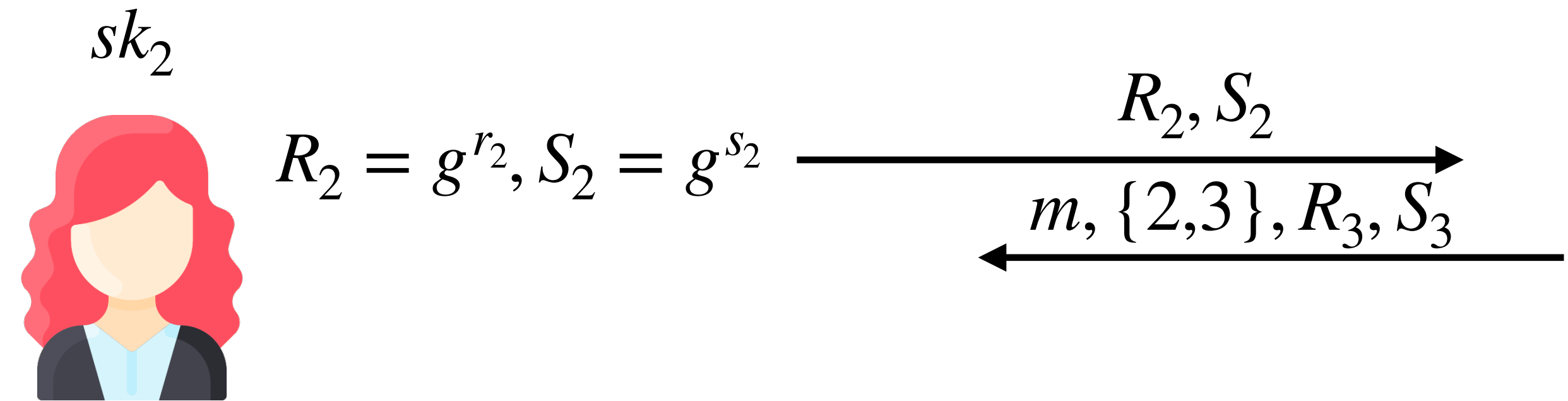
FROST1



Leader



FROST1

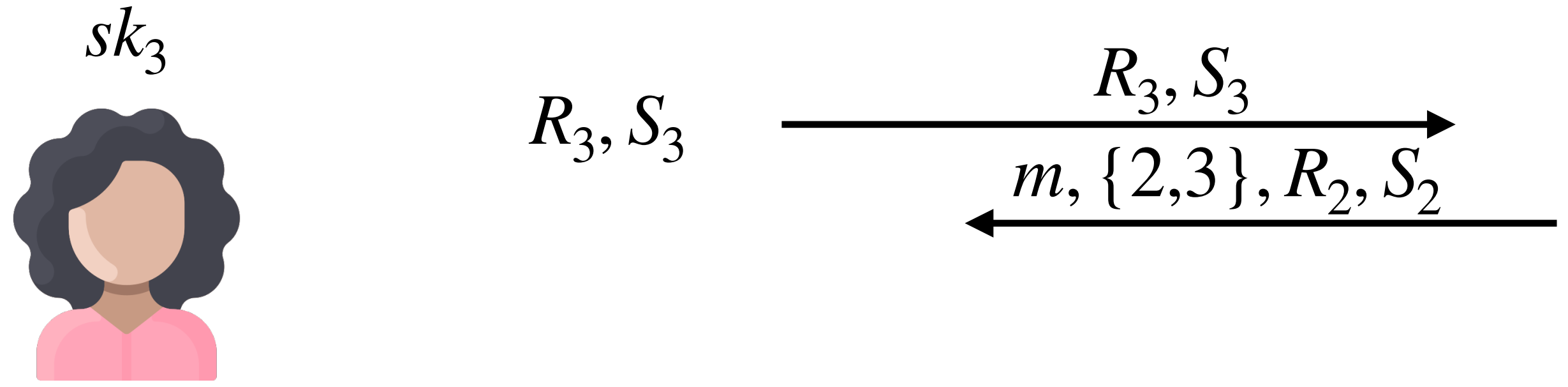


pk output by DKG

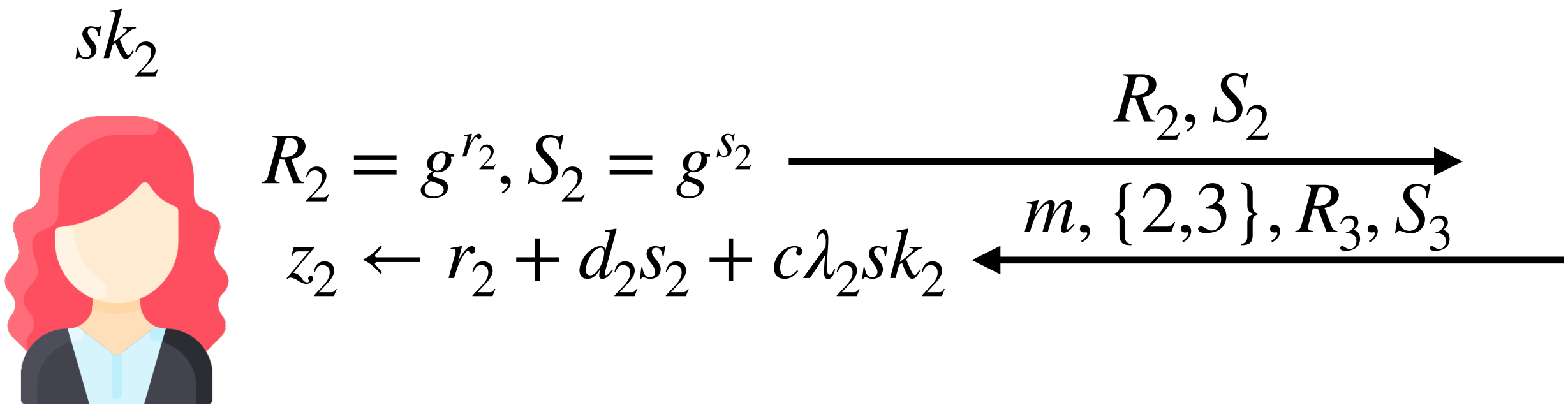
$$d_j = \tilde{H}(j, pk, m, \{R_j, S_j\}_2^3)$$
$$R = \Pi_2^3 R_j S_j^{d_j}$$
$$c \leftarrow H(pk, m, R)$$



Leader



FROST1



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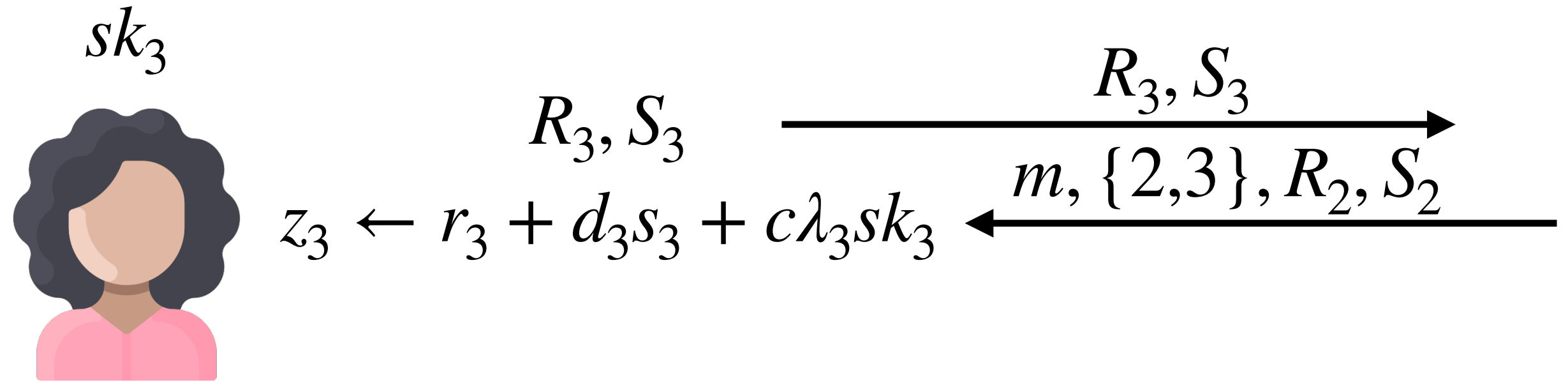
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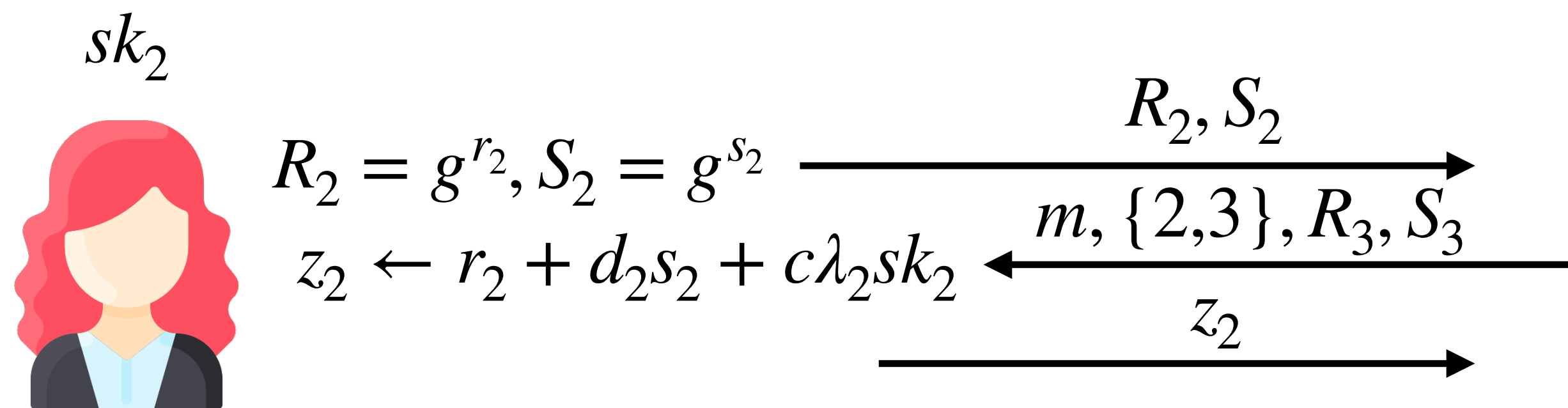
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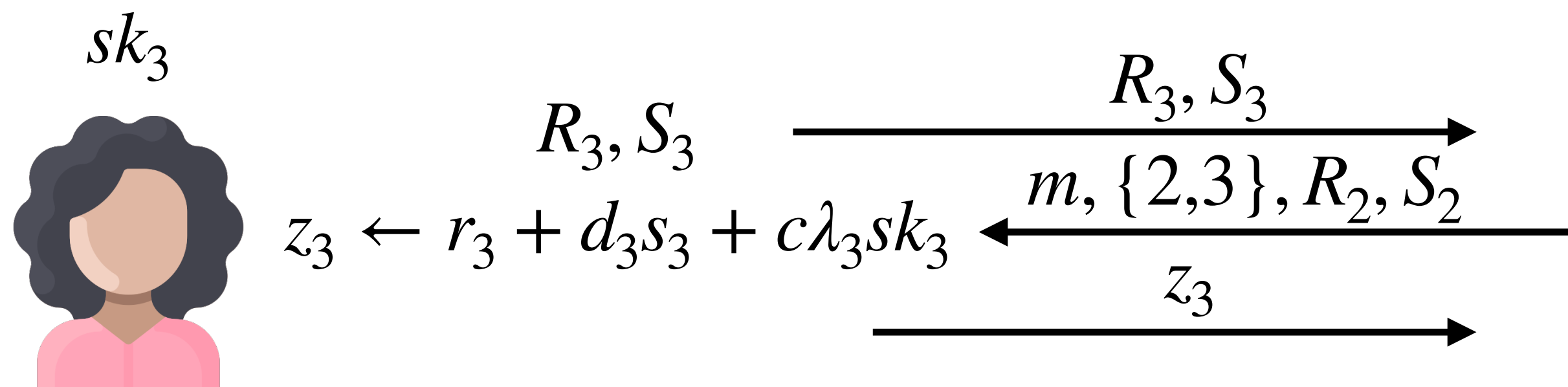
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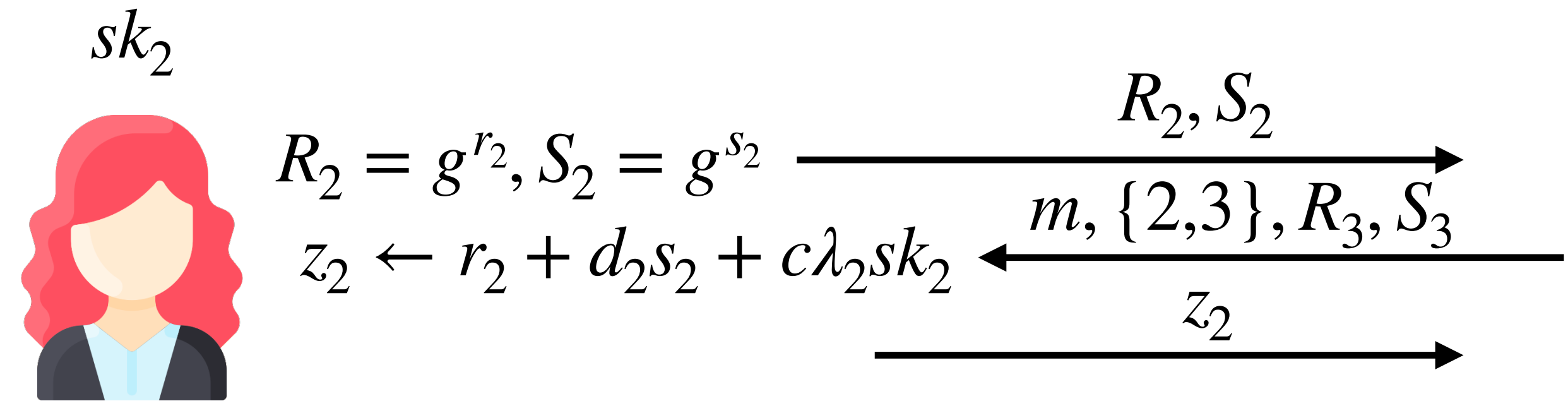
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FROST1

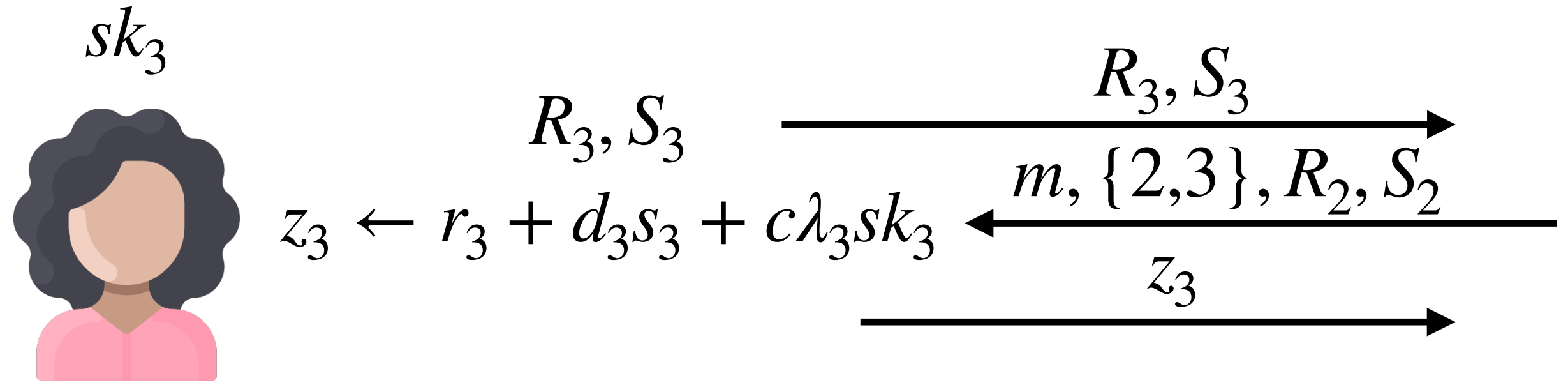


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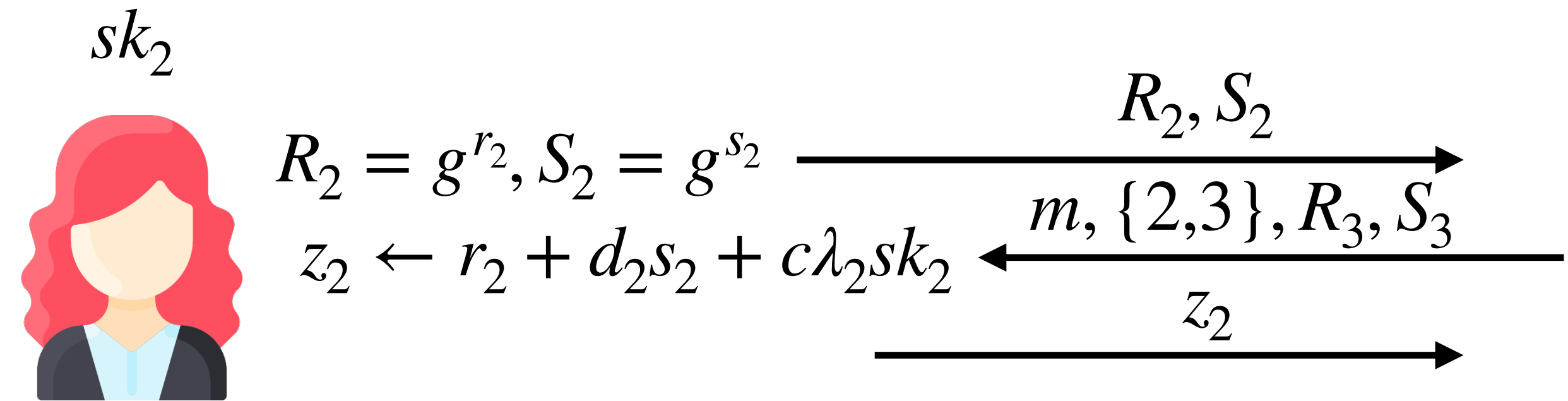
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Leader

$z \leftarrow z_2 + z_3$
 $\sigma = (R, z)$



FROST1



pk output by DKG

$$d_j = \tilde{H}(j, pk, m, \{R_j, S_j\}_2^3)$$

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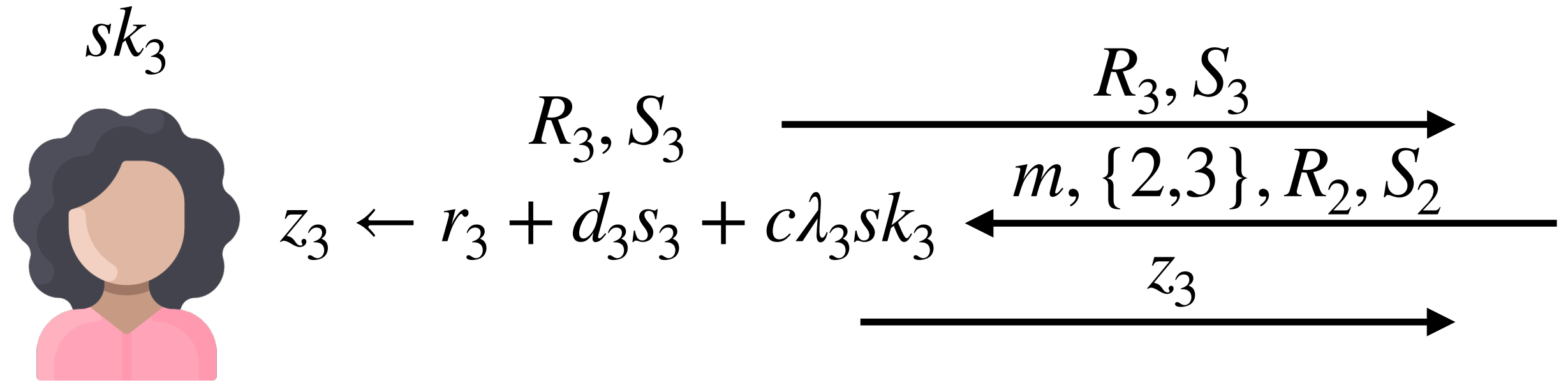
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Leader

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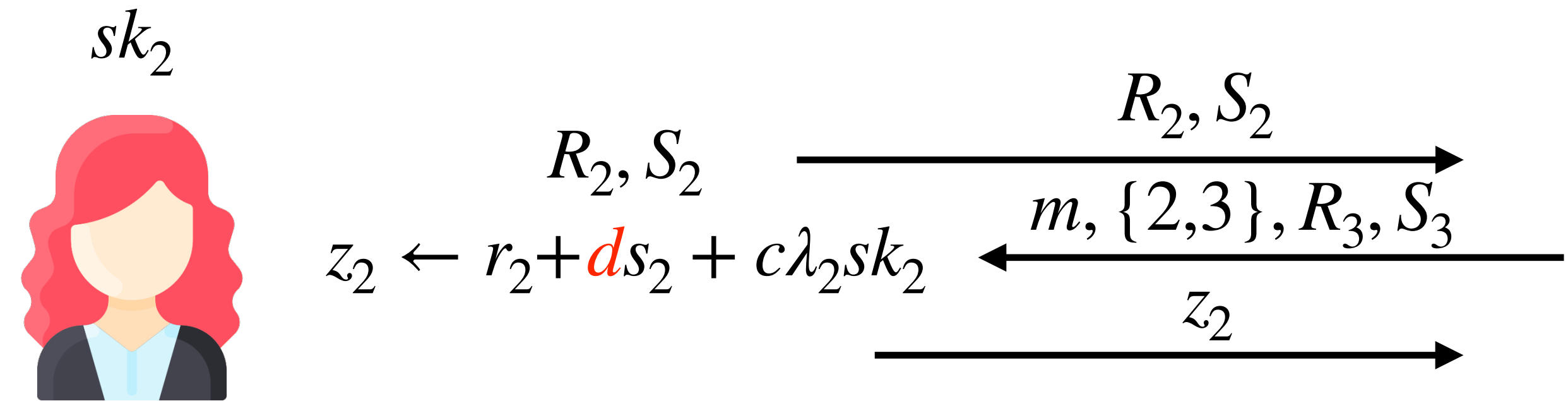


Verify:

$$c \leftarrow H(pk, m, R)$$

$$R \cdot pk^c \stackrel{?}{=} g^z$$

FROST2

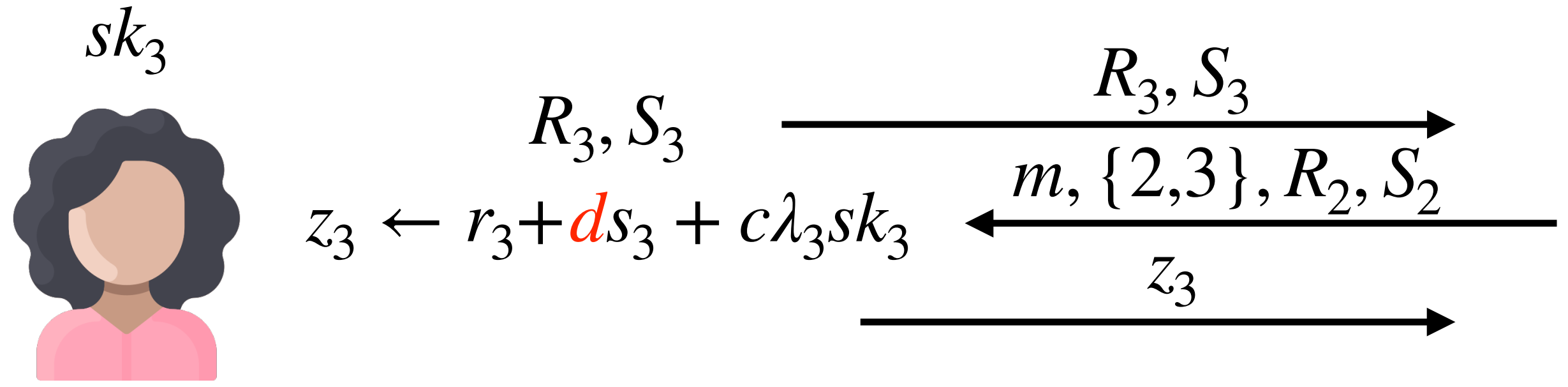


pk output by DKG
 $d = \tilde{H}(pk, m, \{R_j, S_j\}_2^3)$
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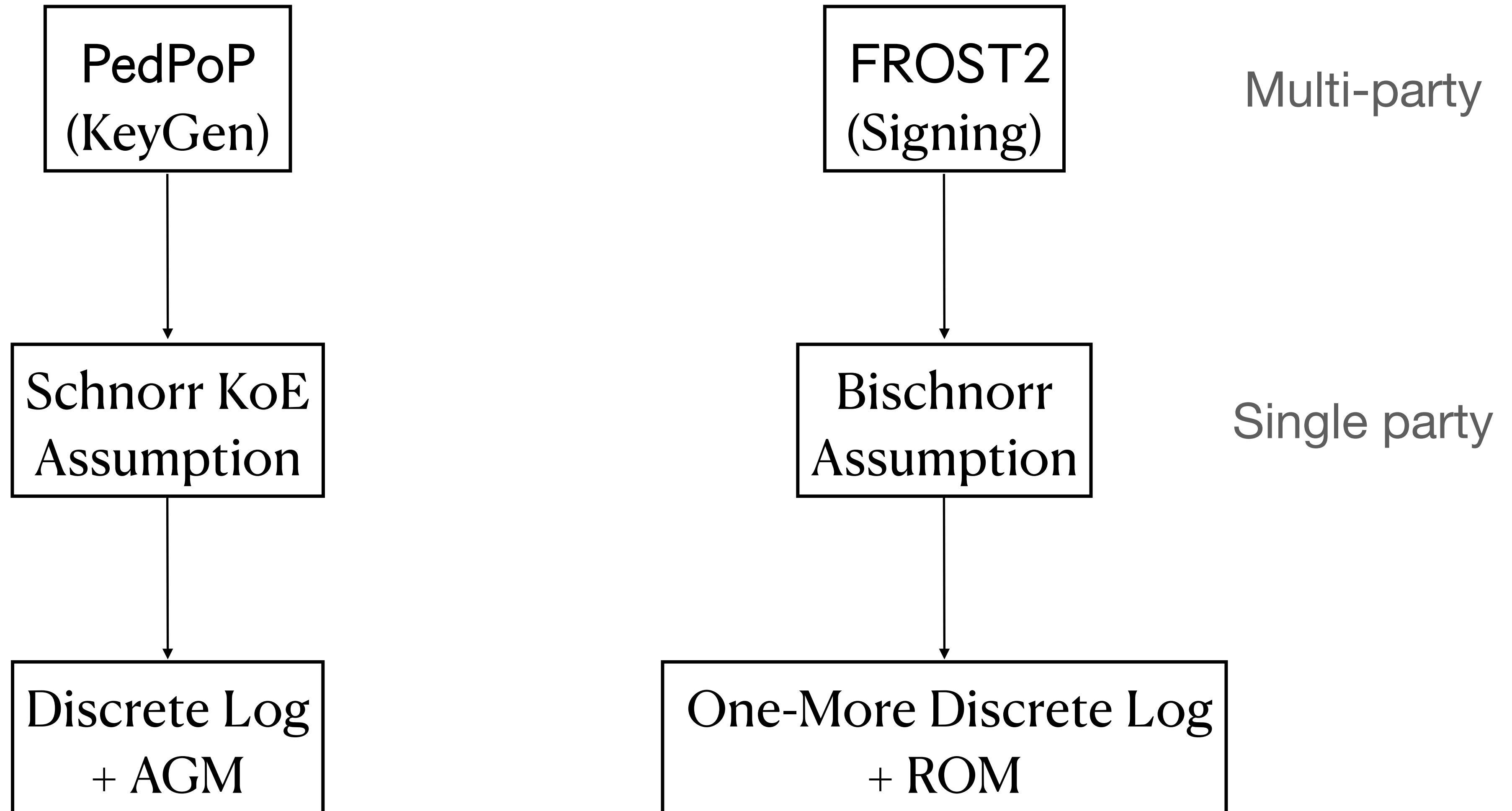


Verify:
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Proving the Security of FROST2 + PedPoP

- Security reductions for multi-party signatures have two moving parts:
 1. Simulating honest users interacting with the adversary
 2. Extracting a solution to some hard problem from the adversary's responses
- Idea: Separate the two parts for a more modular reduction

Proving the Security of FROST2 + PedPoP



Framework & Security Hierarchy

Motivations

- No formalization for partially non-interactive schemes
- Existing security notions are weaker than schemes can achieve

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- Most works: m is **considered signed** as long as **one** honest party signs it [GJKR96, KY02, Bol03, Wee11, KG20, BGG+18]
- Only very few consider stronger guarantees [Sho00, LJY14, Gro21]

(Partially) Non-interactive Threshold Sigs

$$n = 3, t = 2$$



Signer 1



Signer 2



Signer 3



Leader

(Partially) Non-interactive Threshold Sigs

$$n = 3, t = 2$$

sk_1



Signer 1

sk_2



Signer 2

sk_3



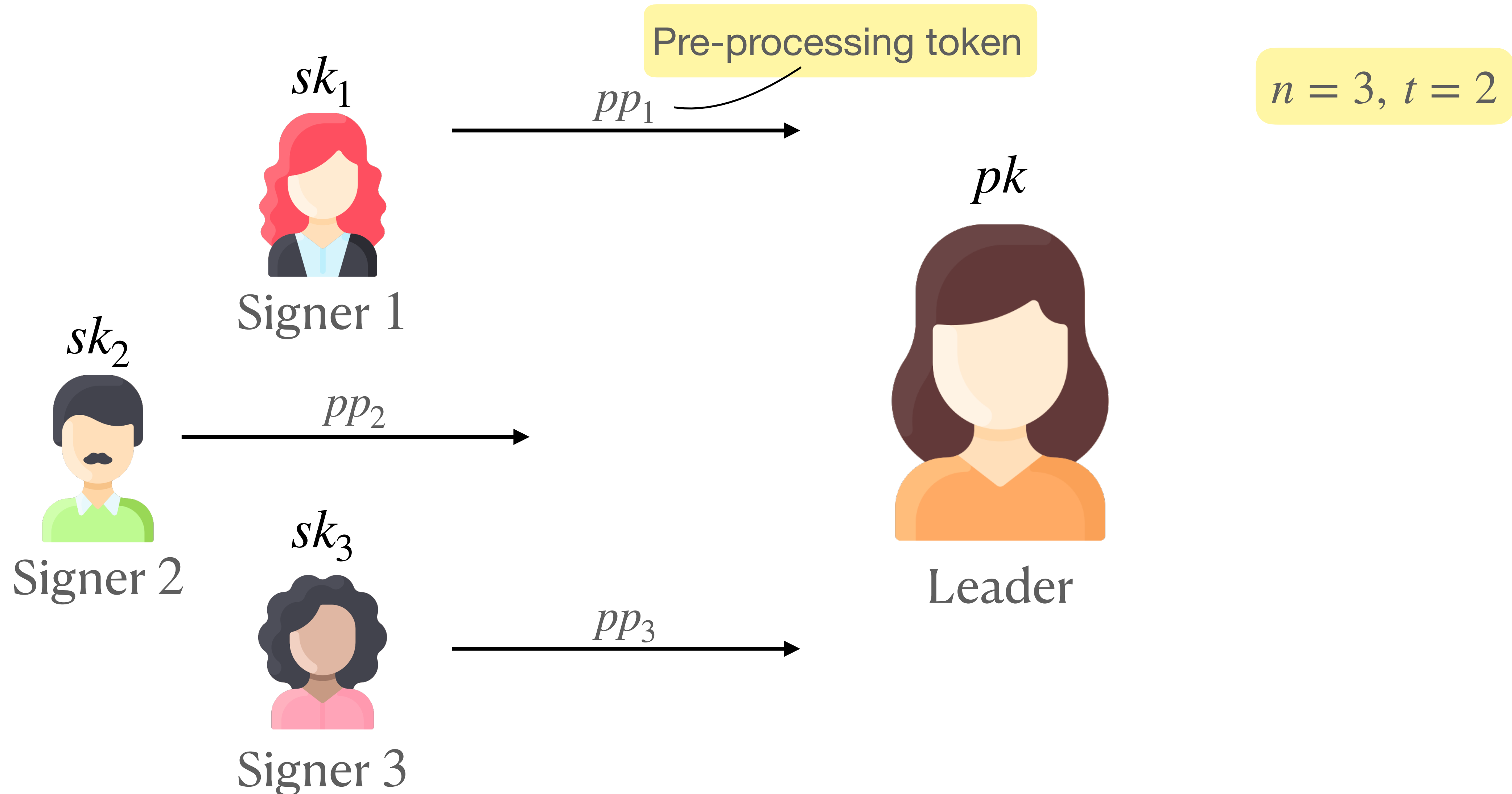
Signer 3

pk

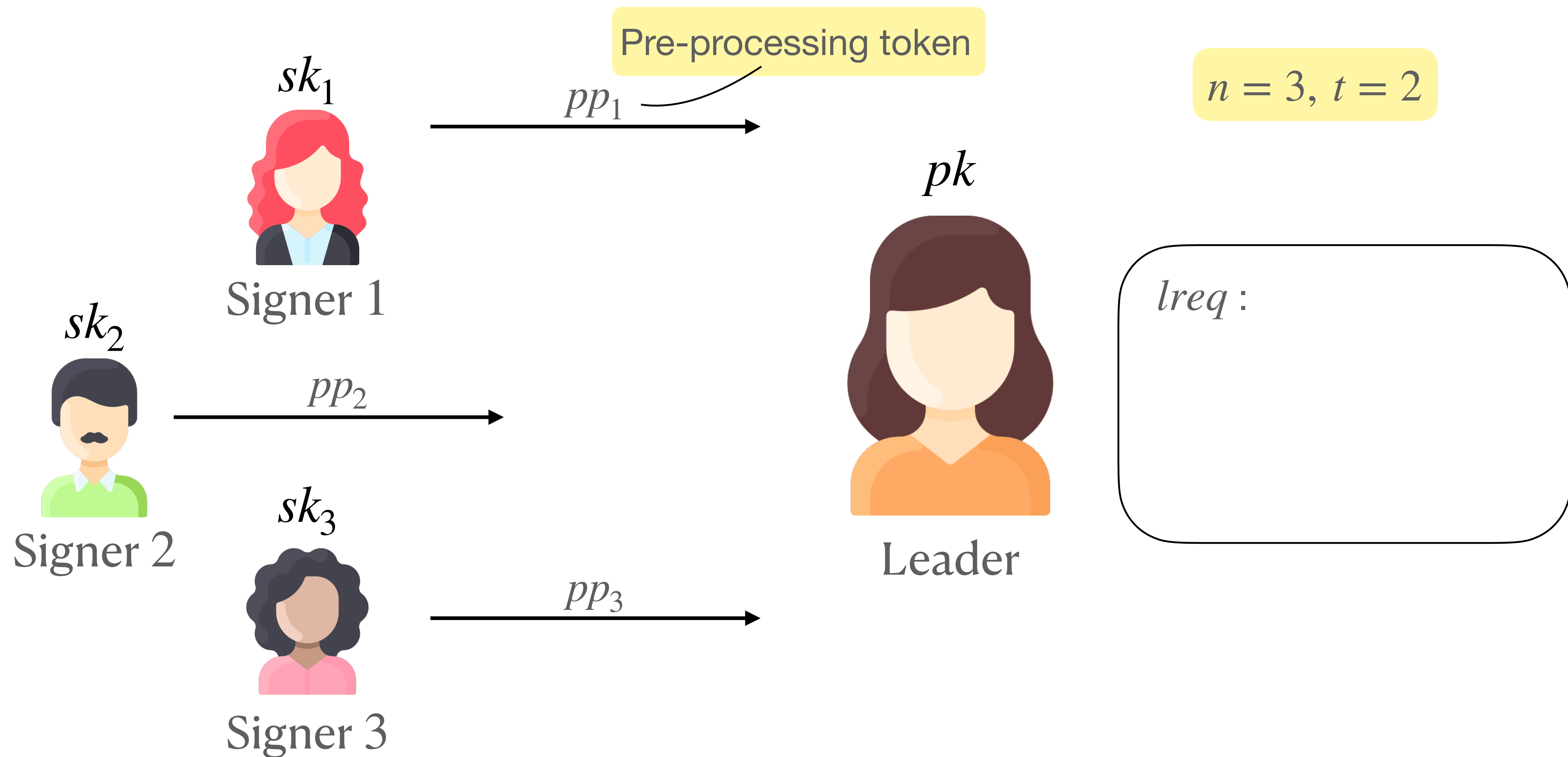


Leader

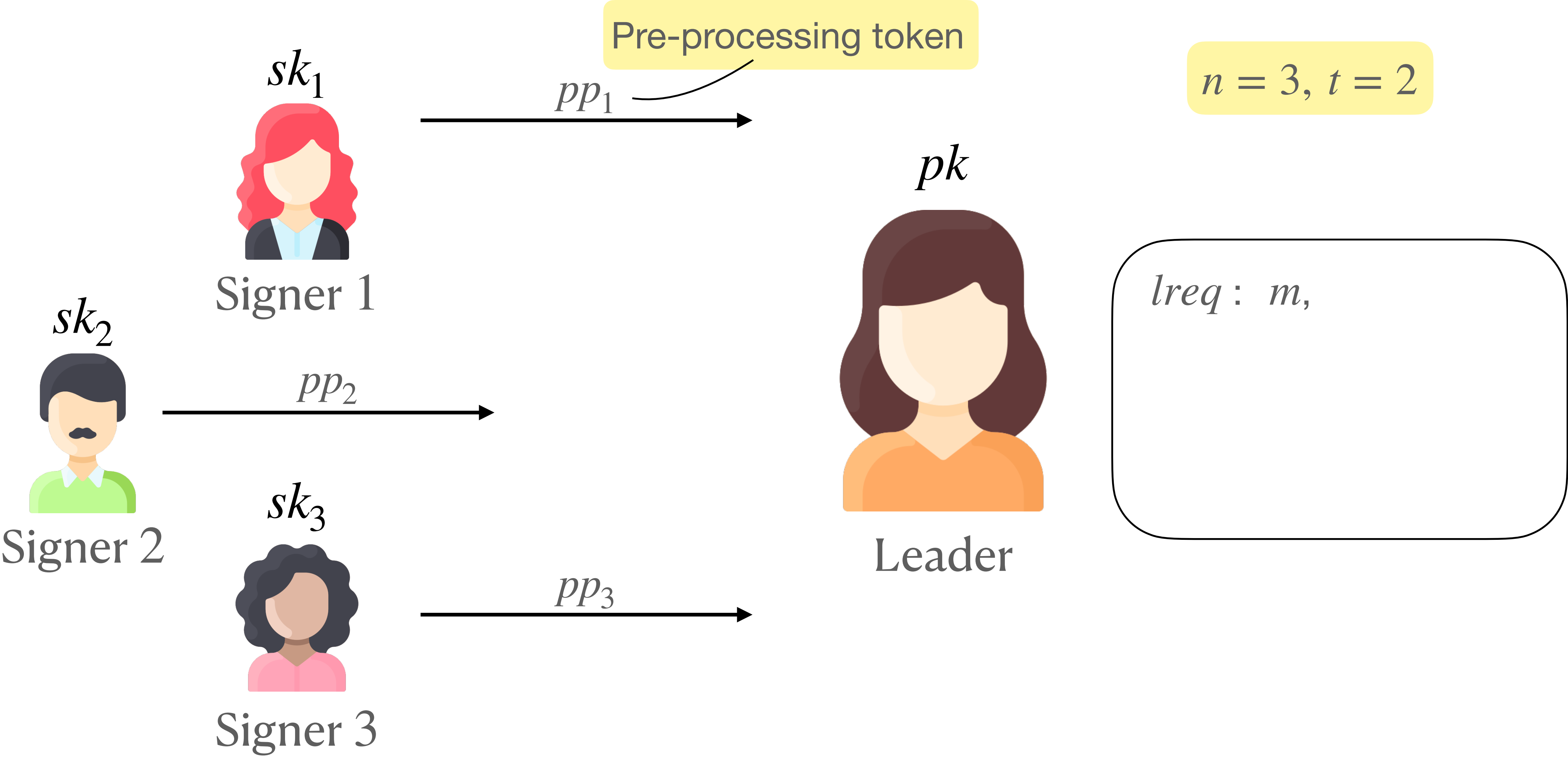
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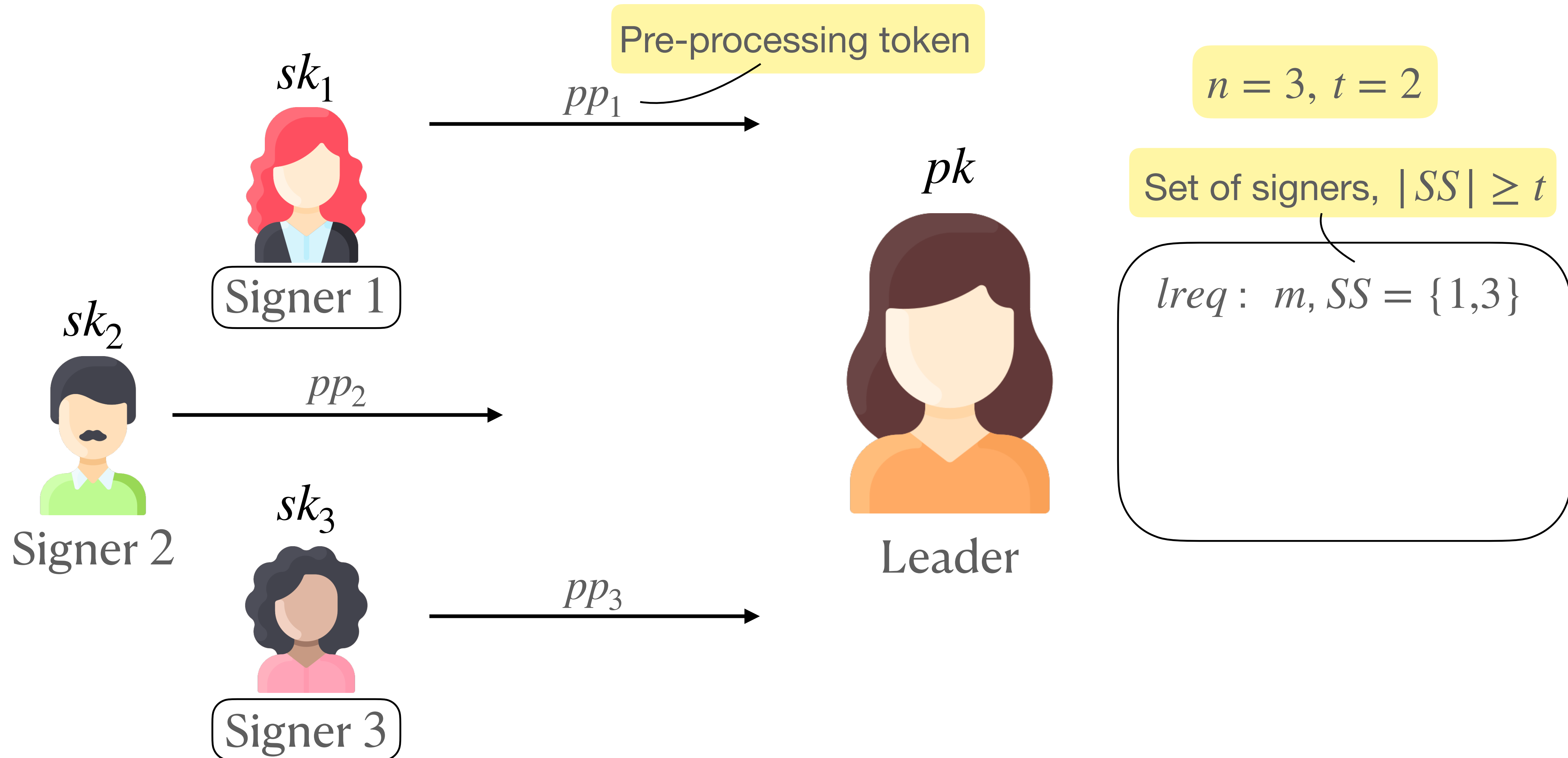
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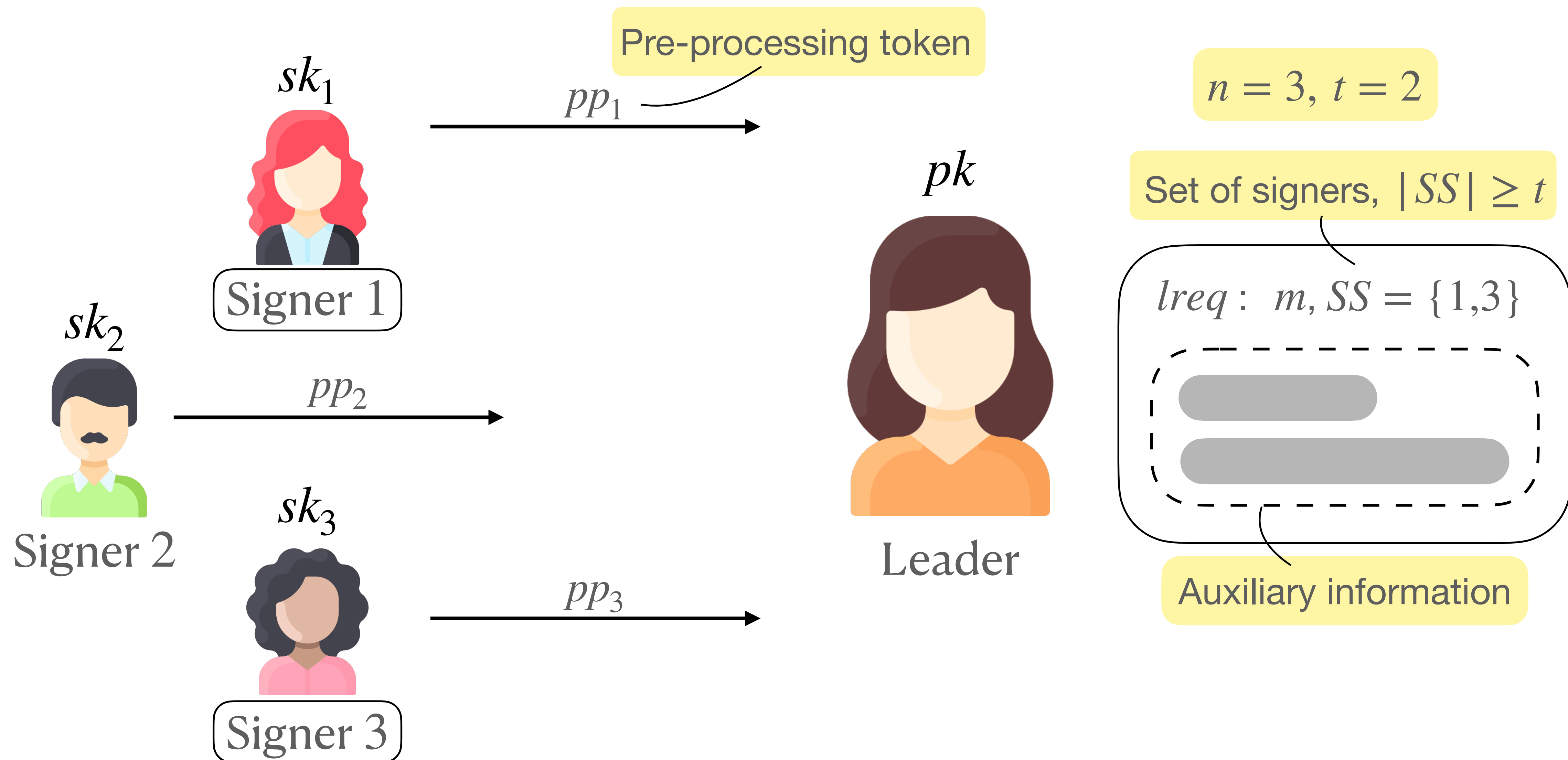
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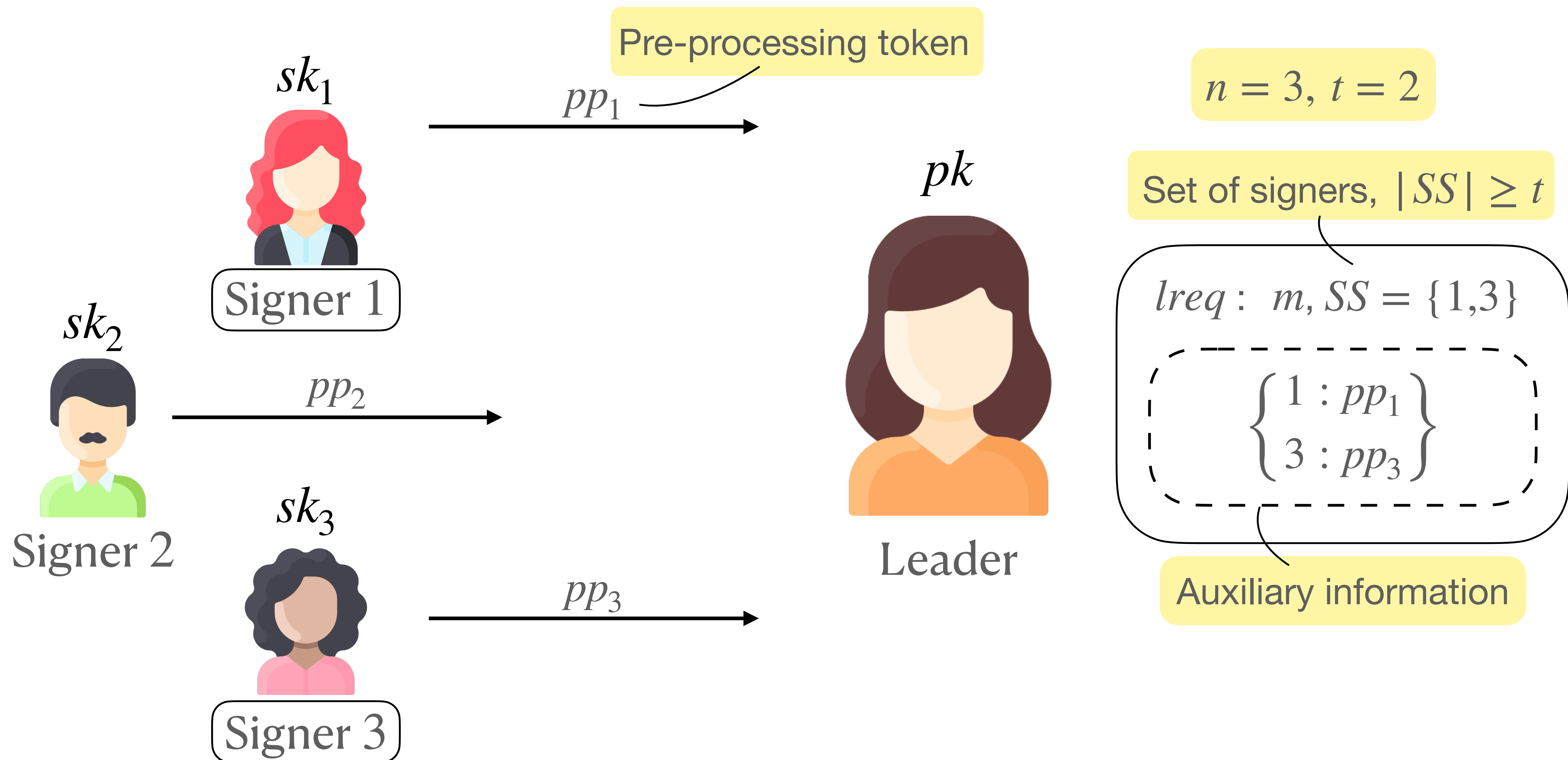
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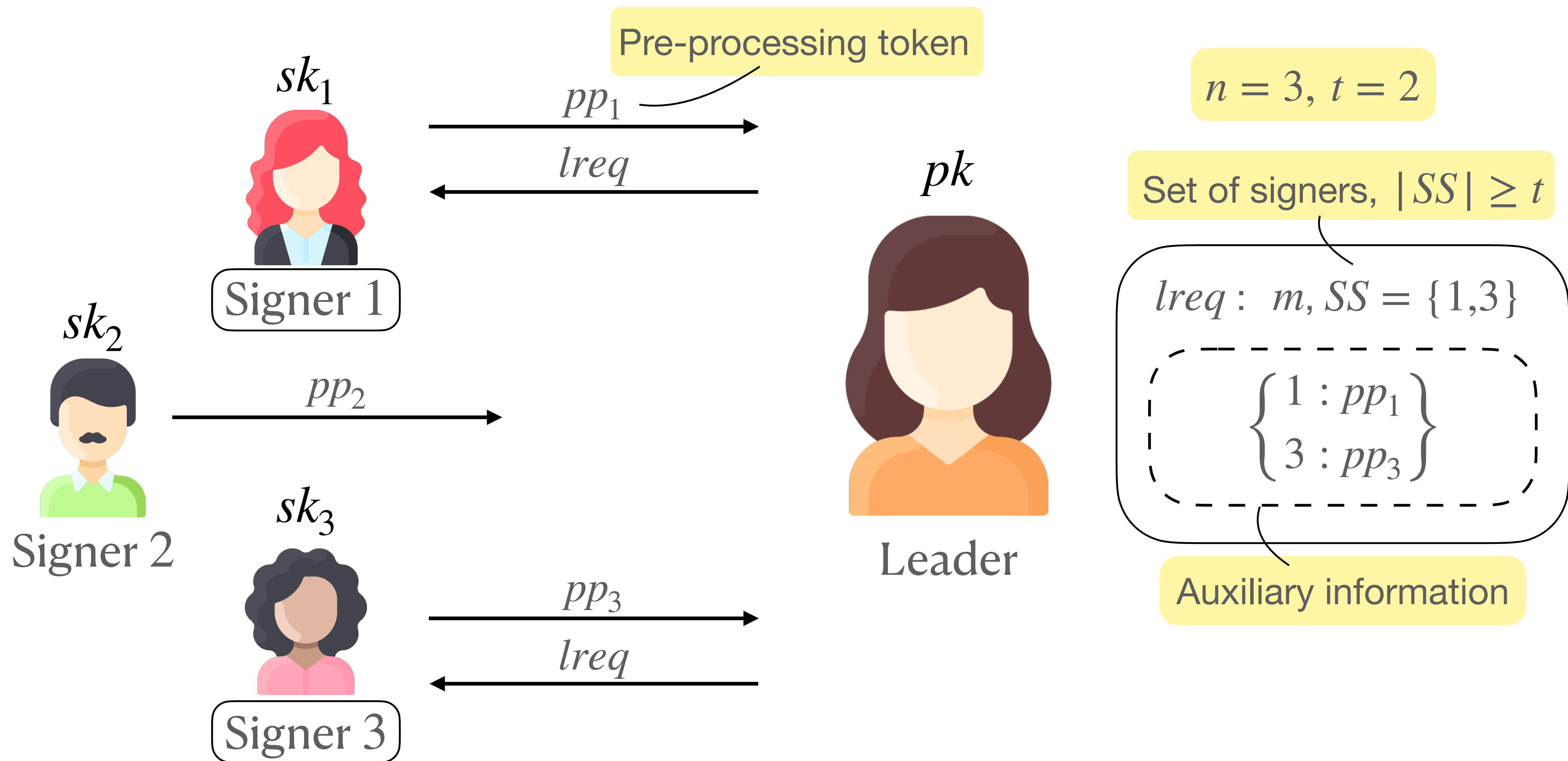
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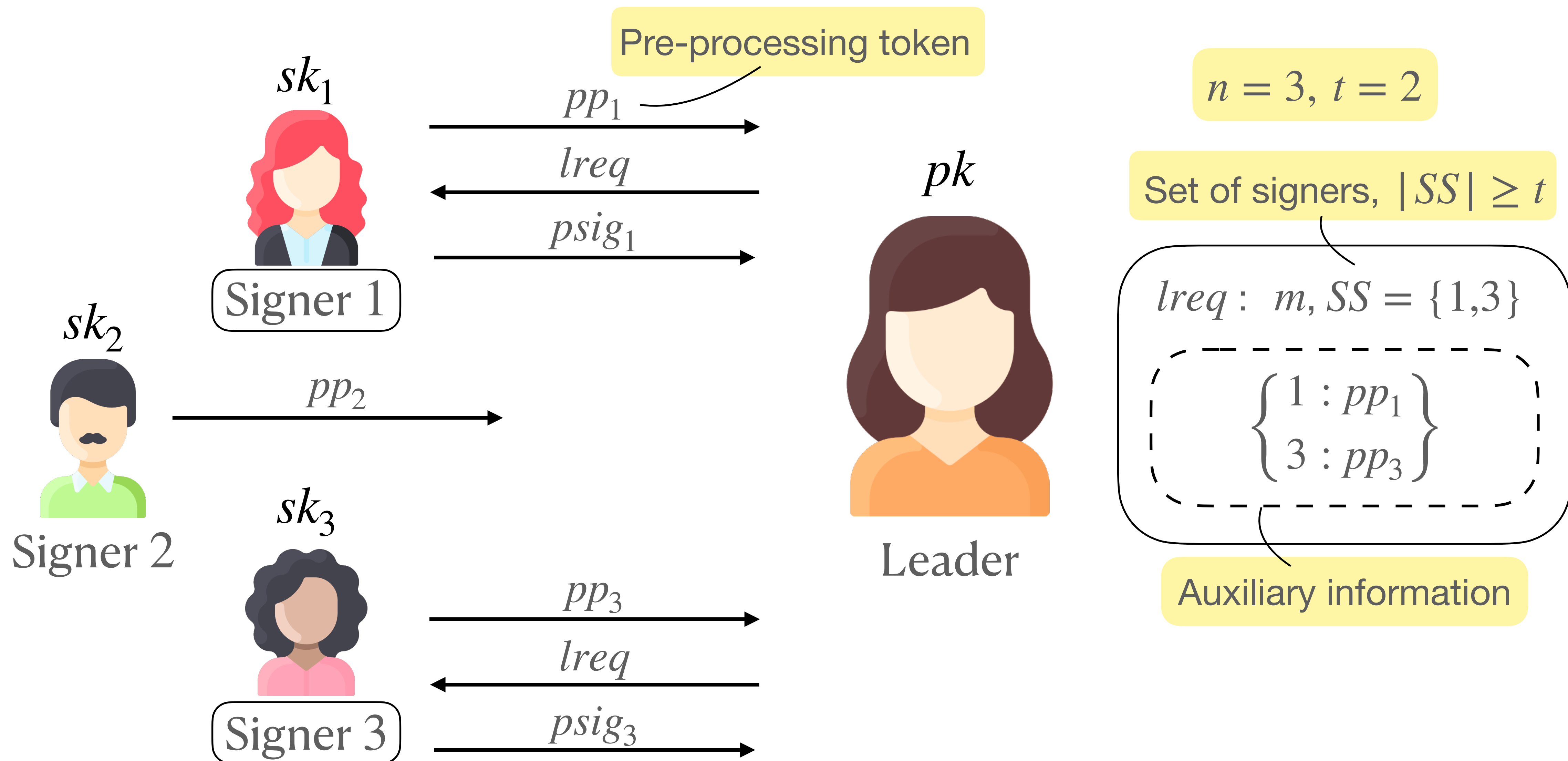
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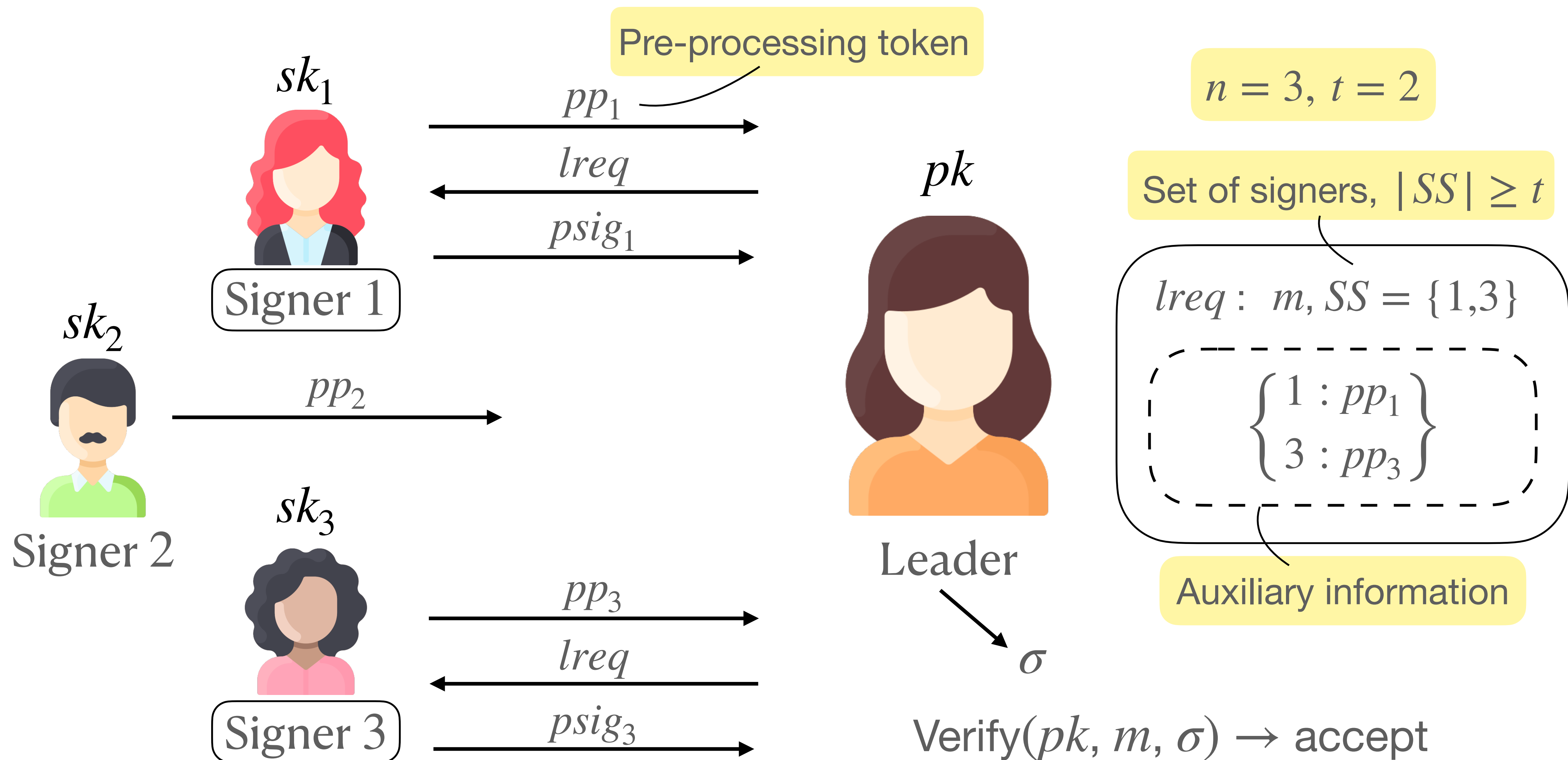
(Partially) Non-interactive Threshold Sigs



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(Partially) Non-interactive Threshold Sigs



Unforgeability

sk_1



Signer 1

sk_2



Signer 2

sk_3



Signer 3

pk



Leader

Unforgeability

sk_1



Signer 1

sk_2



Signer 2

sk_3



Signer 3

pk

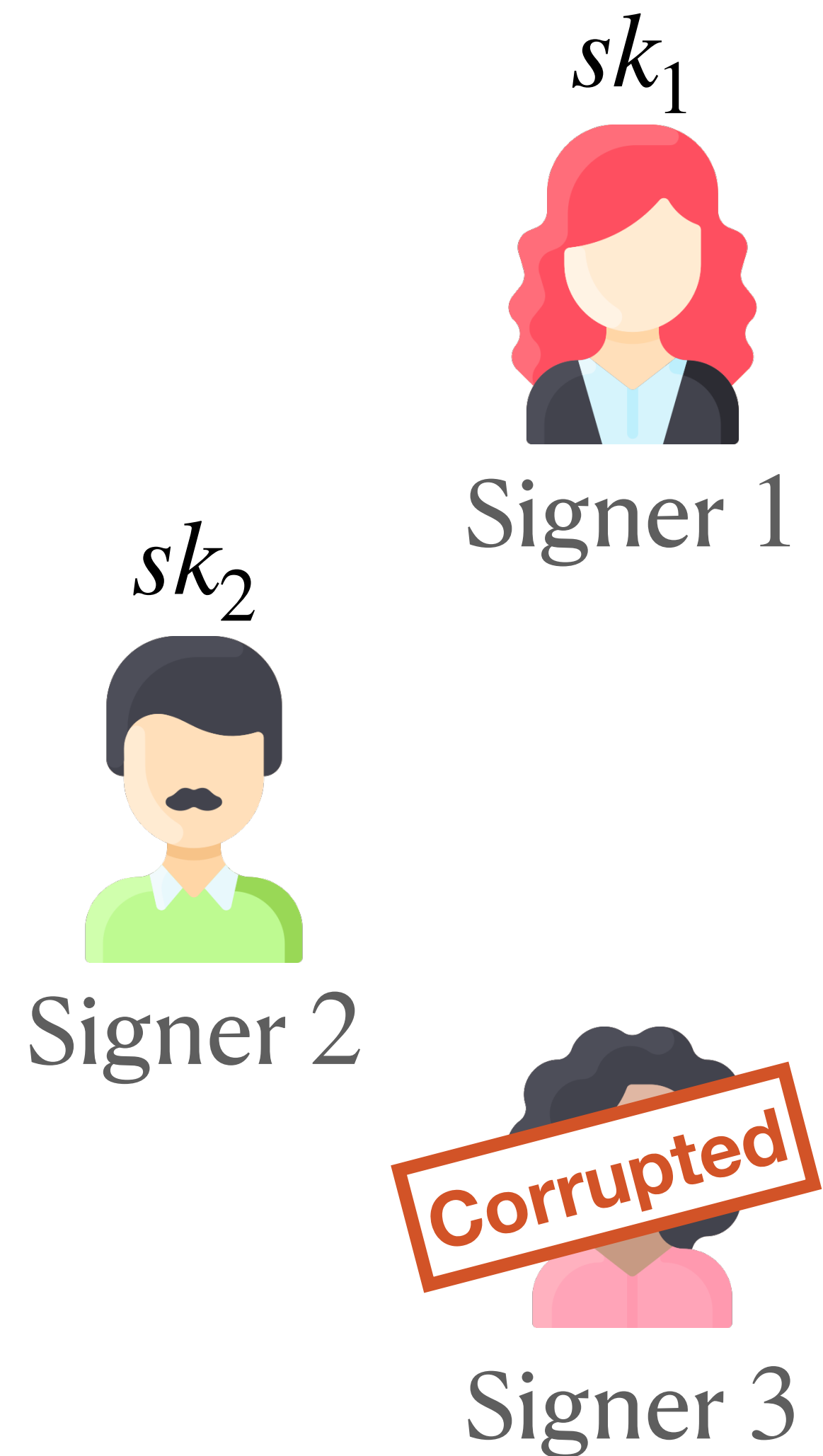


Adversary

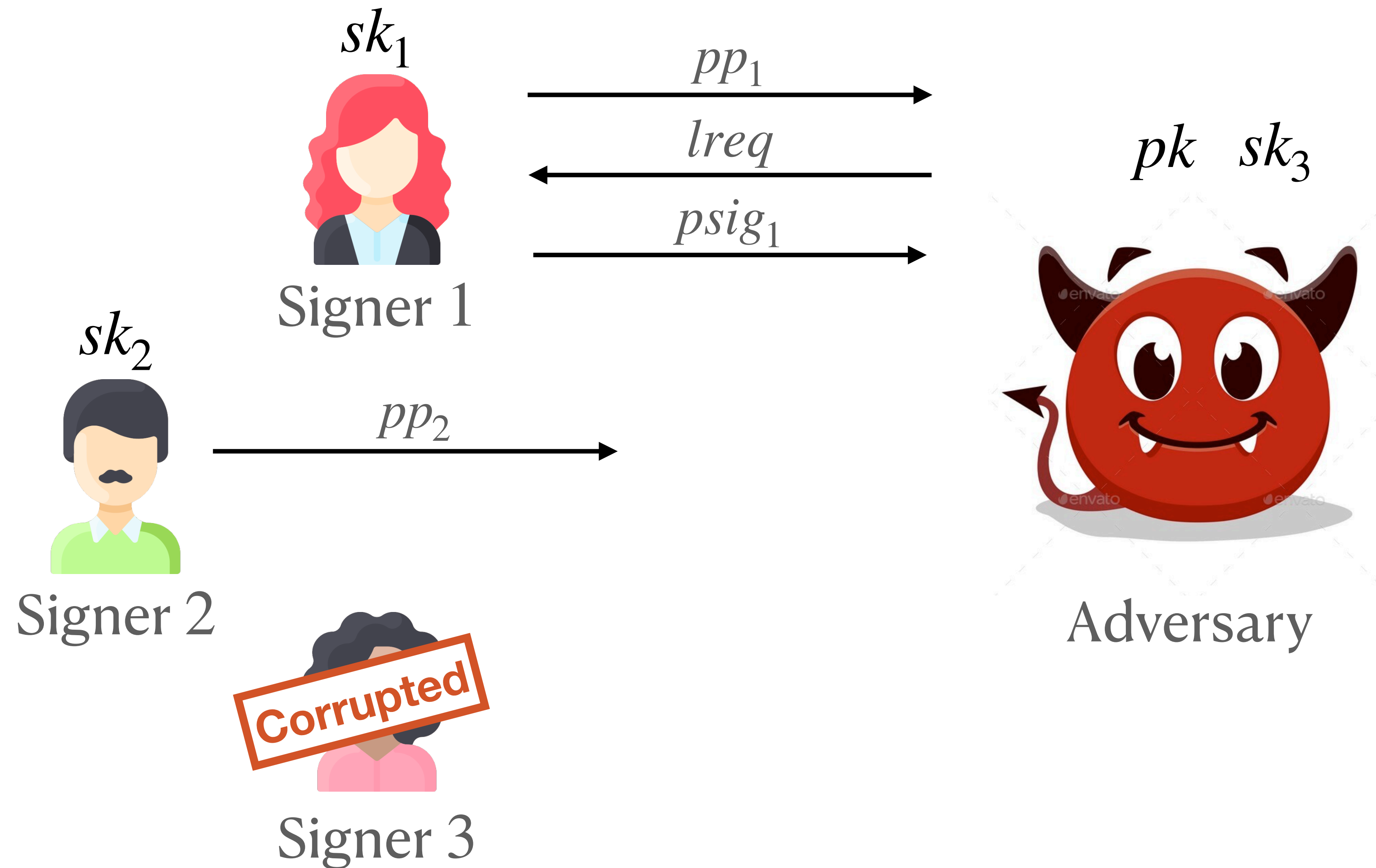
Unforgeability



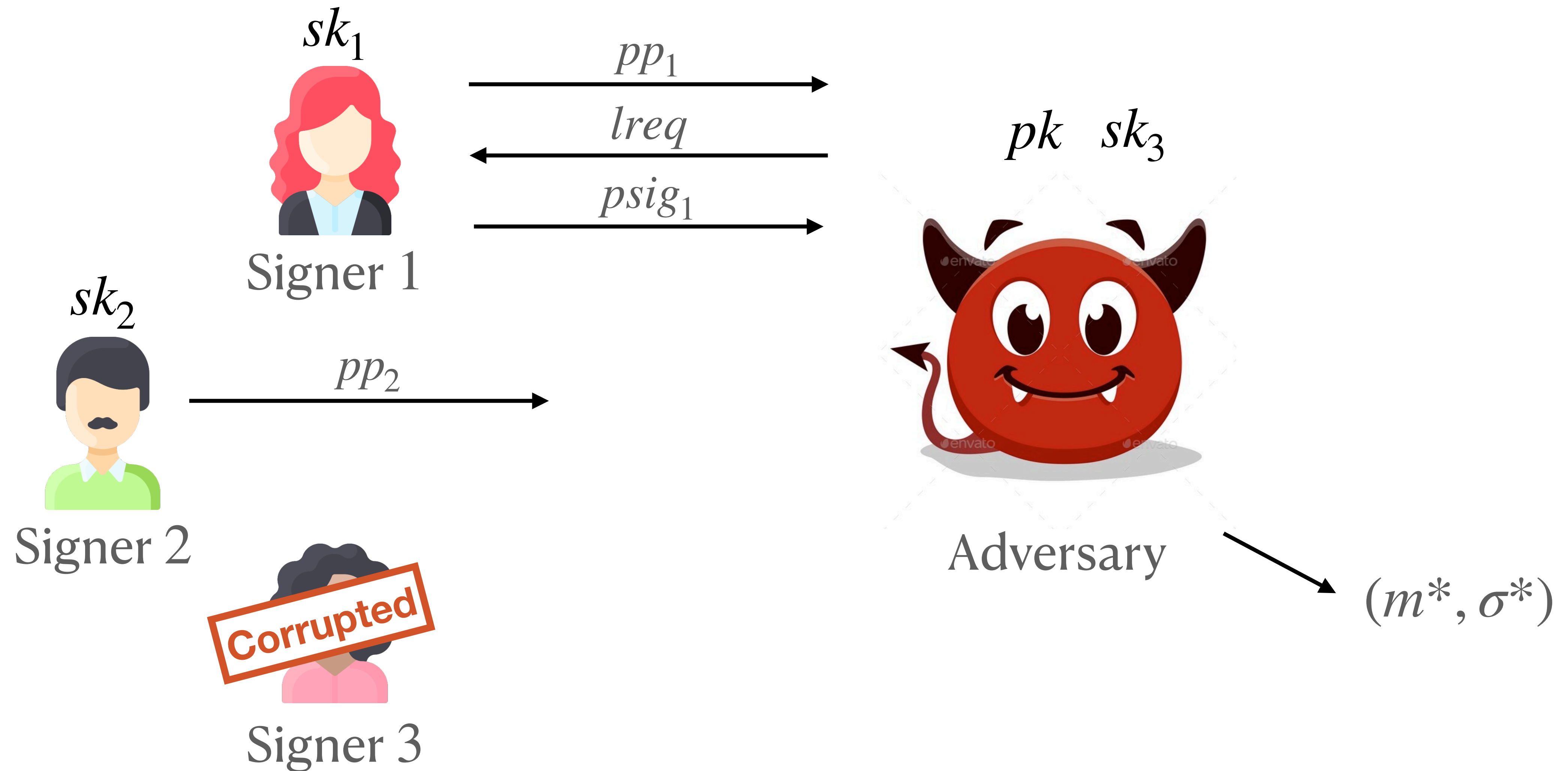
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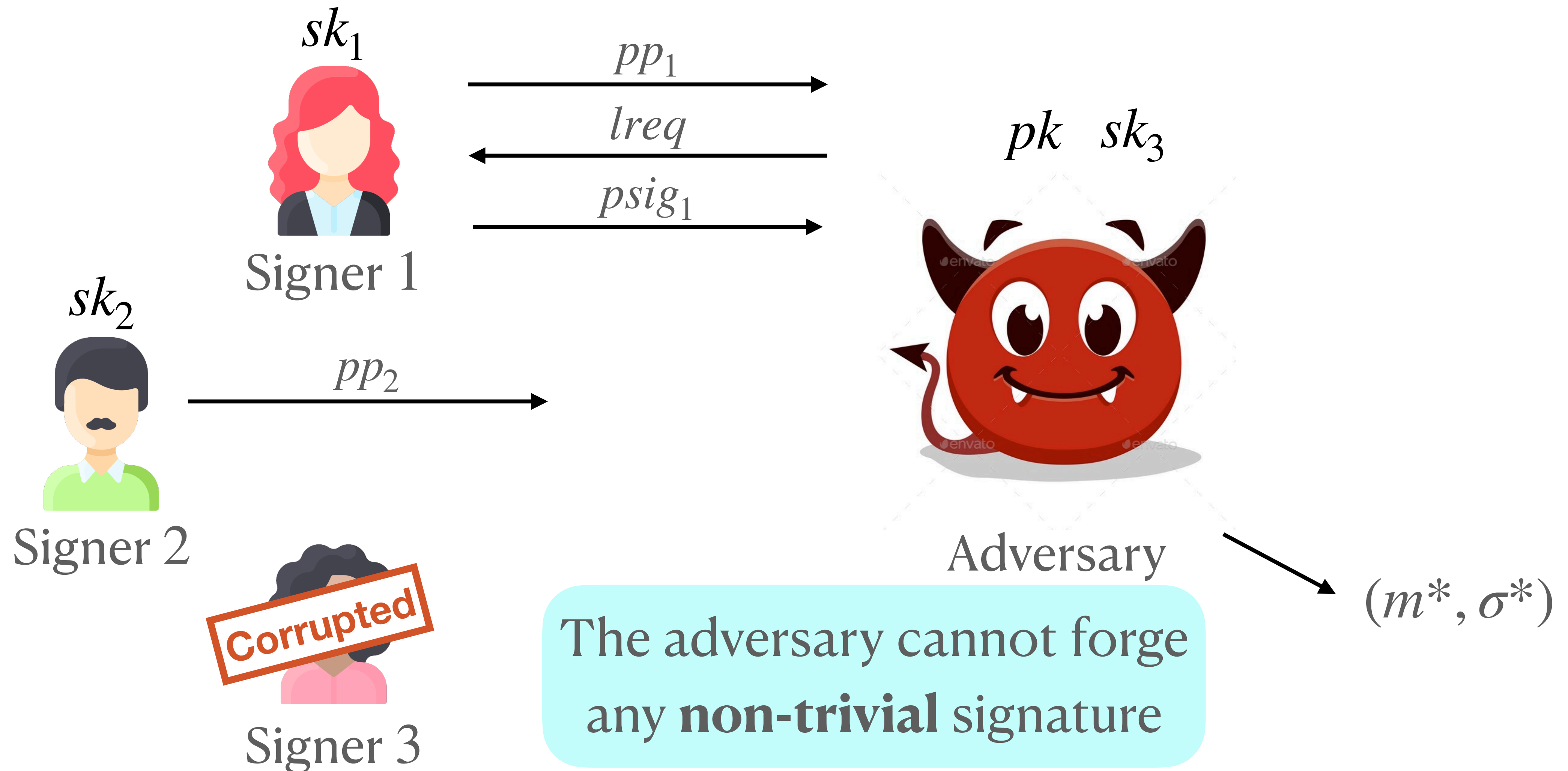
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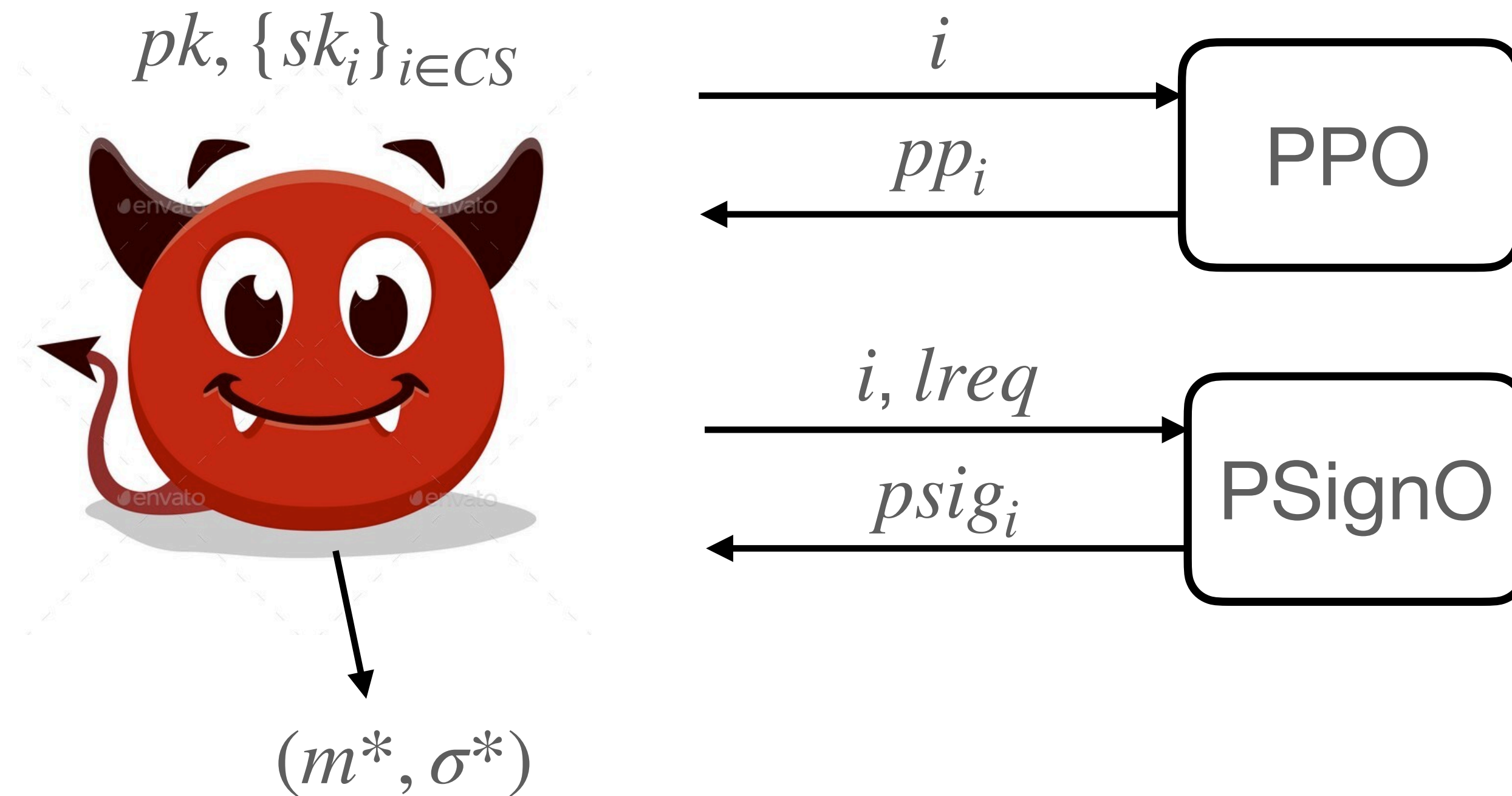
Unforgeability



(General) Unforgeability Game (TS-UF)

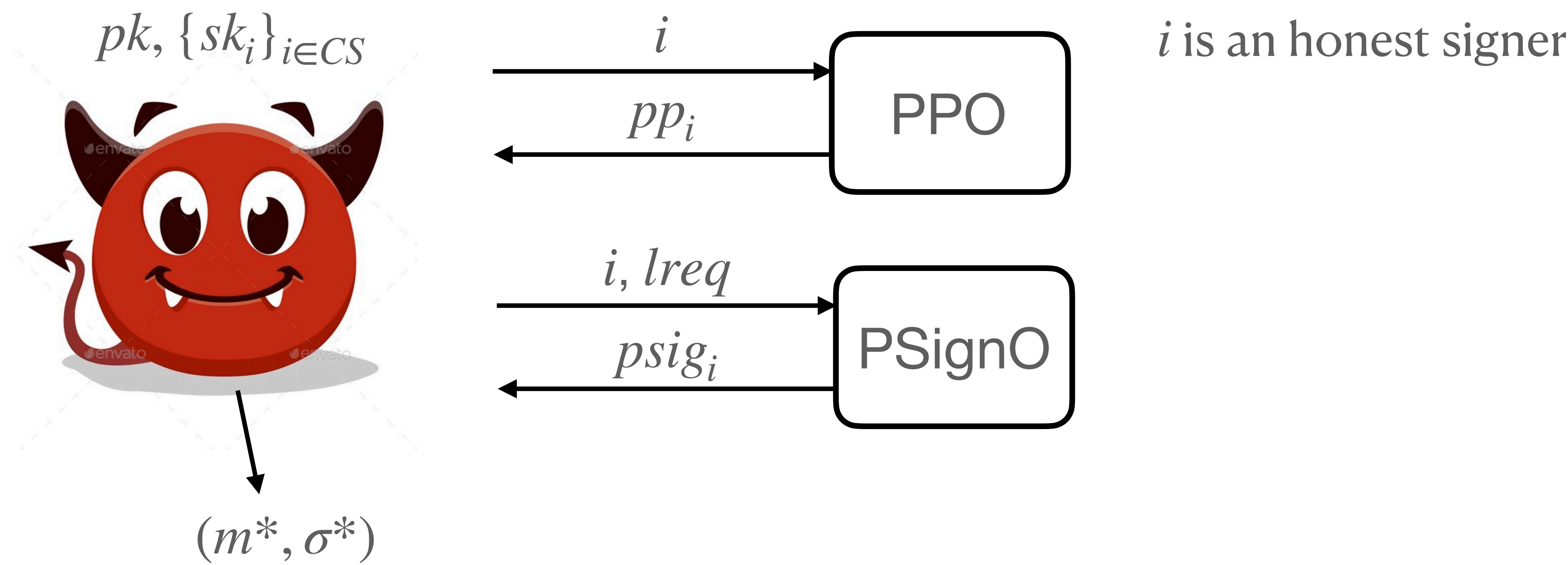
(General) Unforgeability Game (TS-UF)

CS , Set of corrupted signers



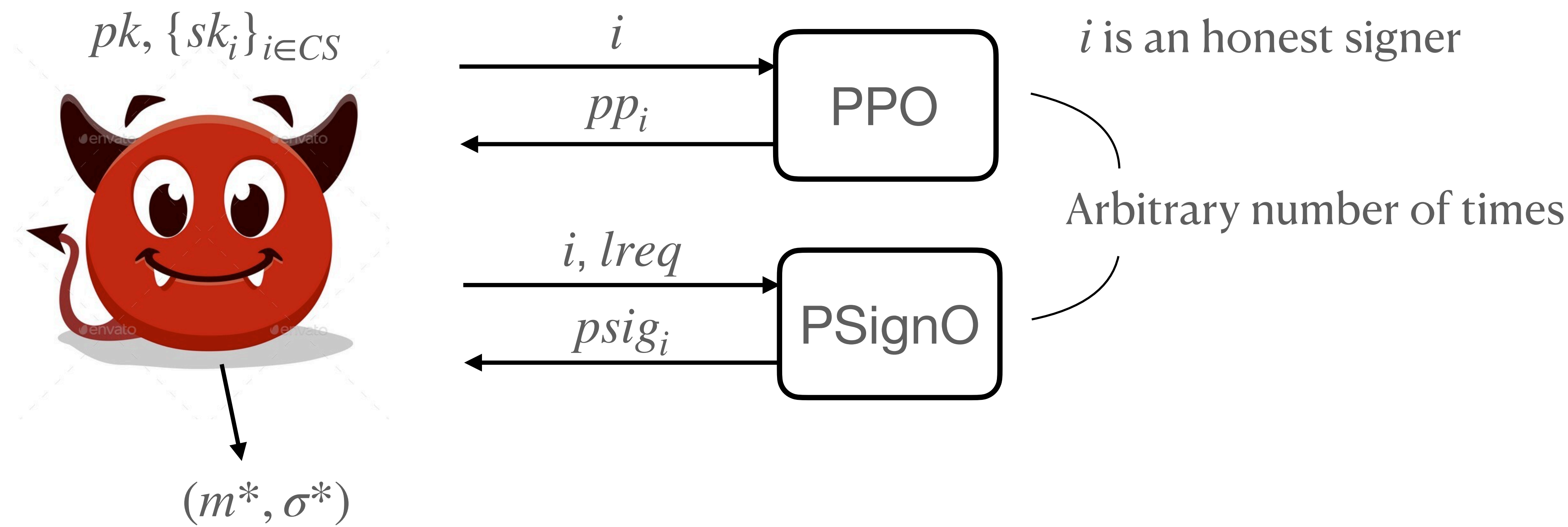
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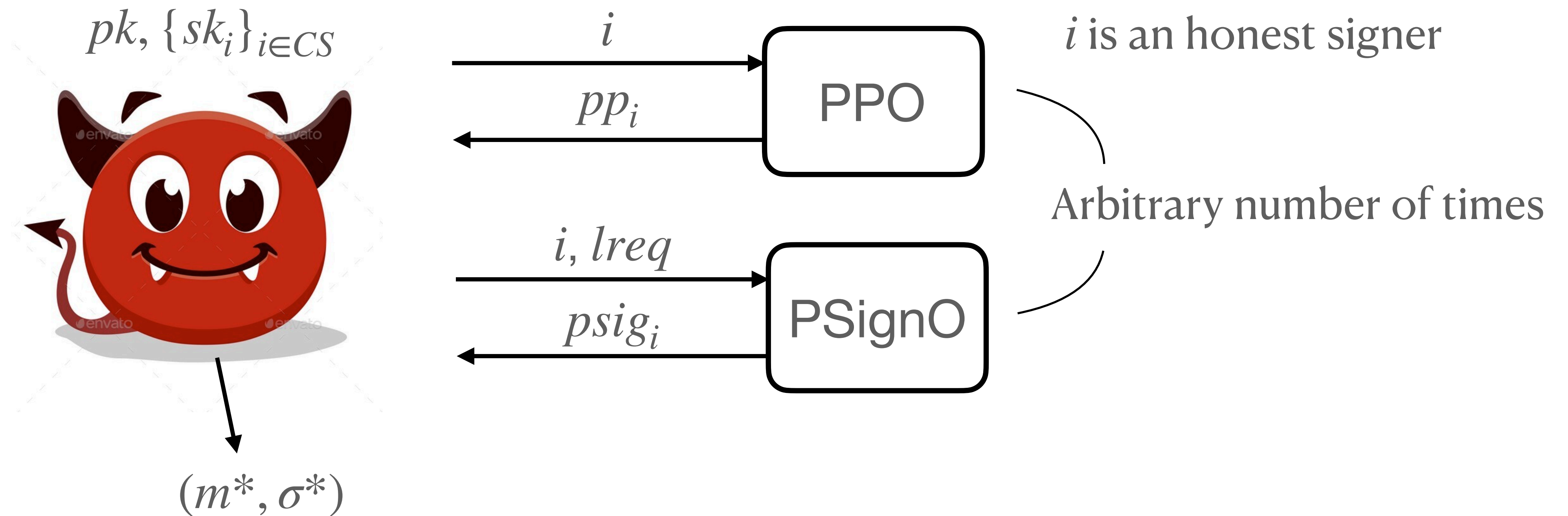
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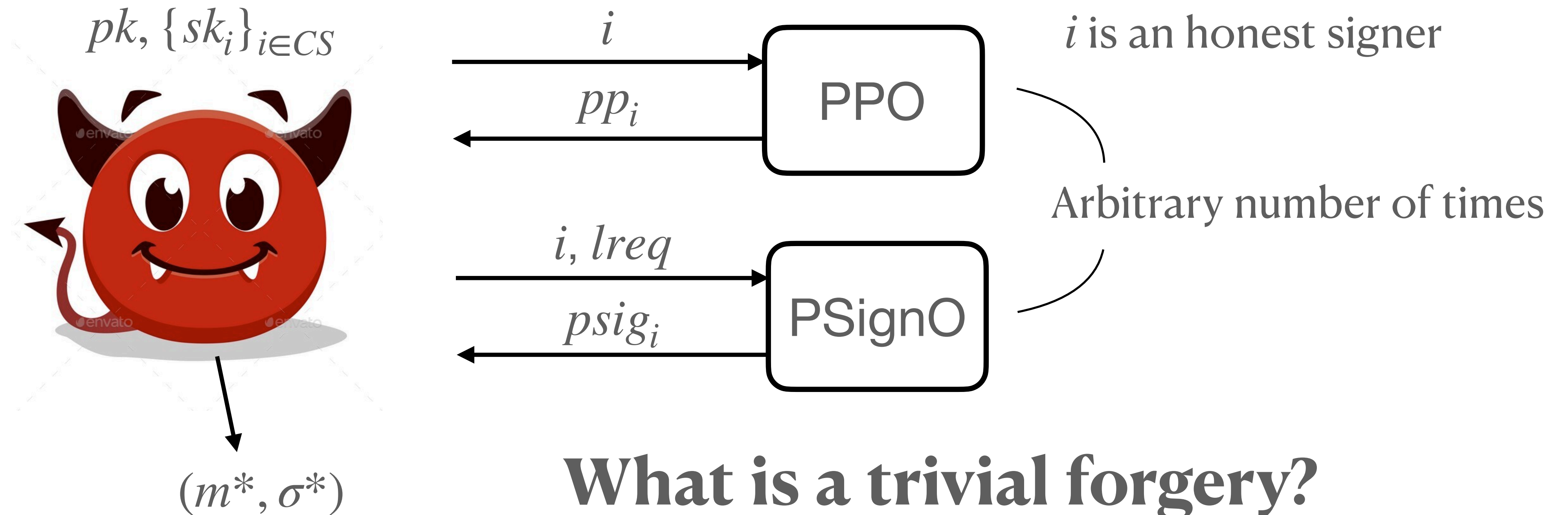
CS , Set of corrupted signers



Adversary wins iff (m^*, σ^*) is not a **trivial forgery**

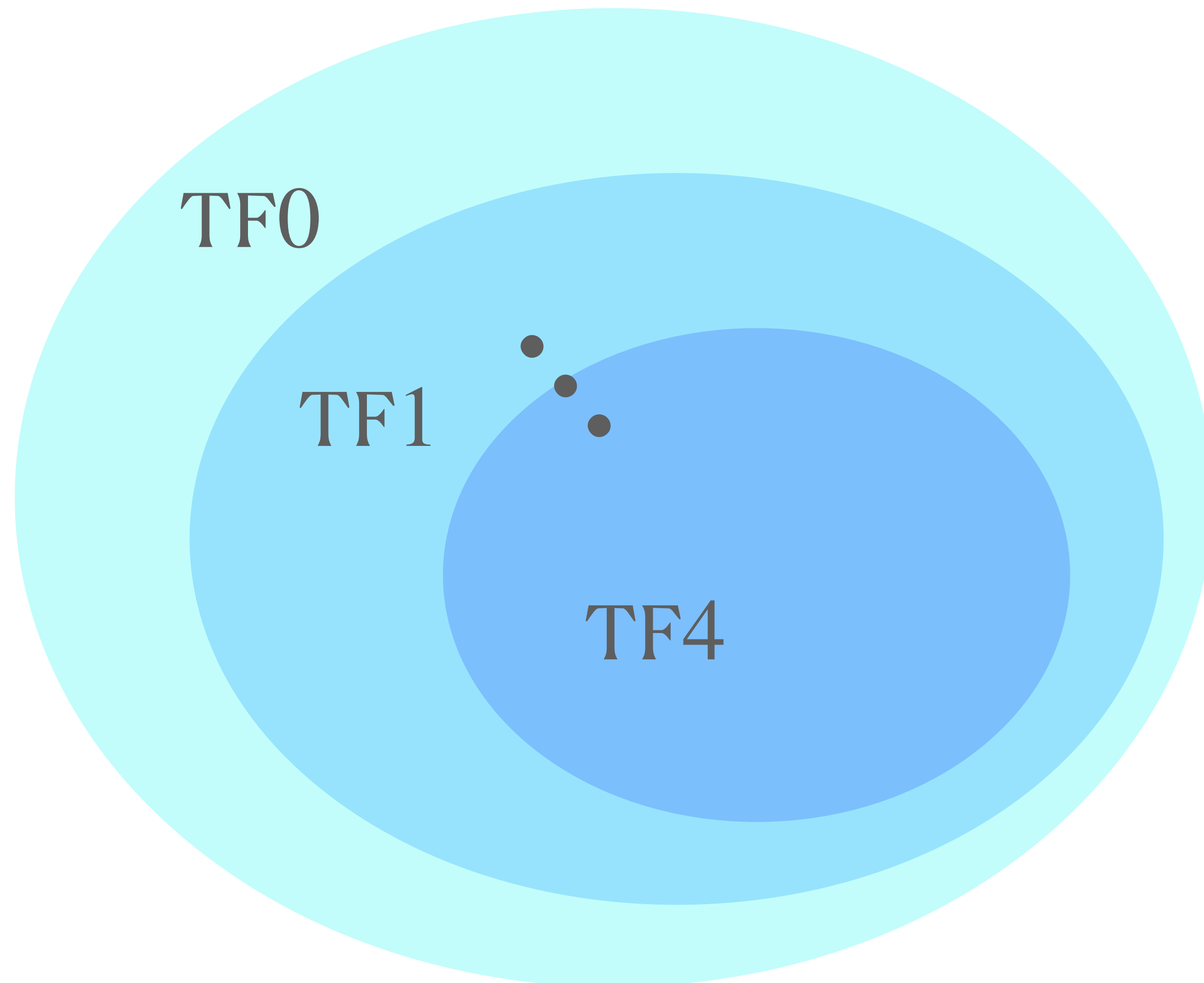
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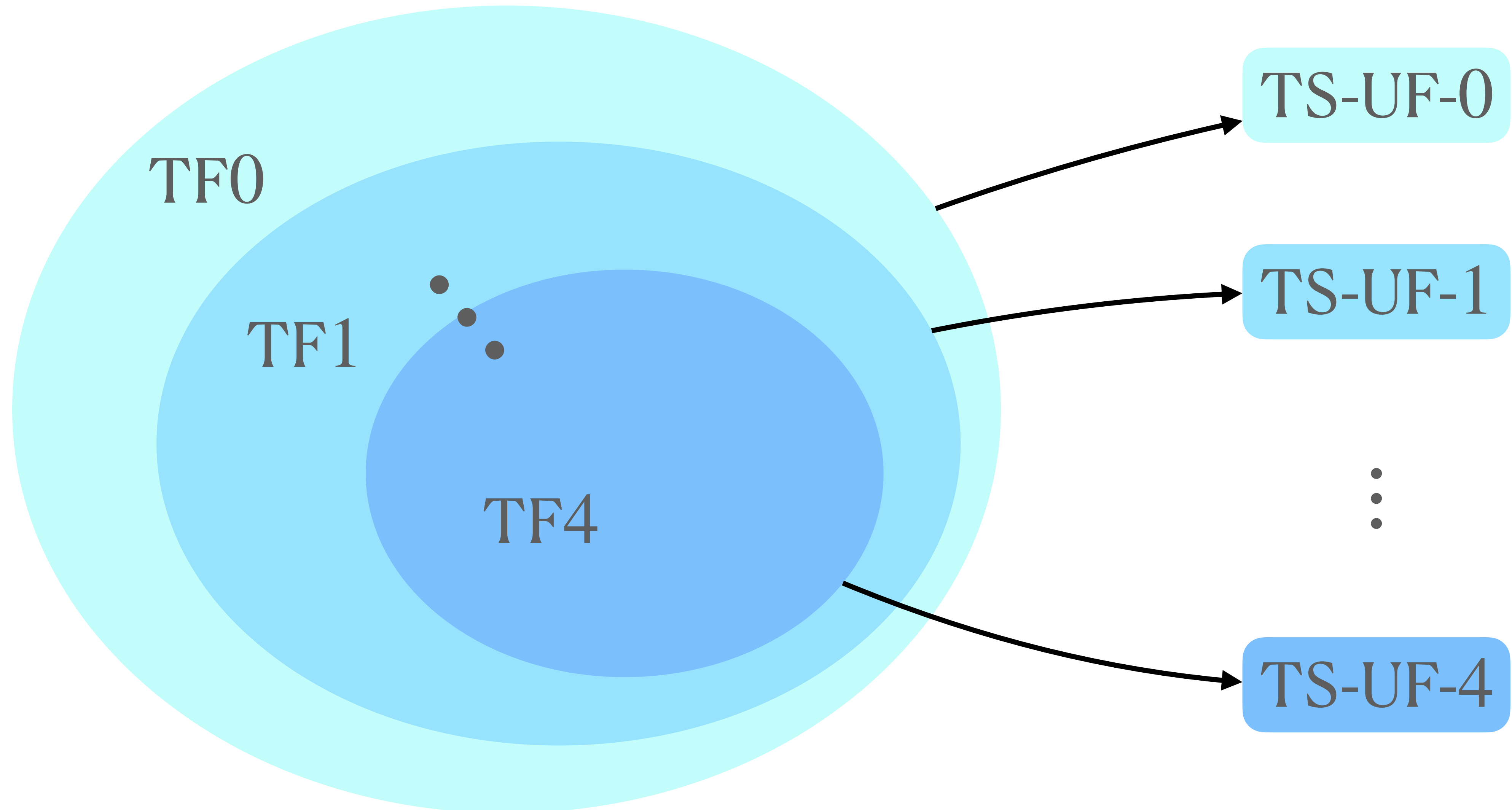
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Set of trivial forgeries



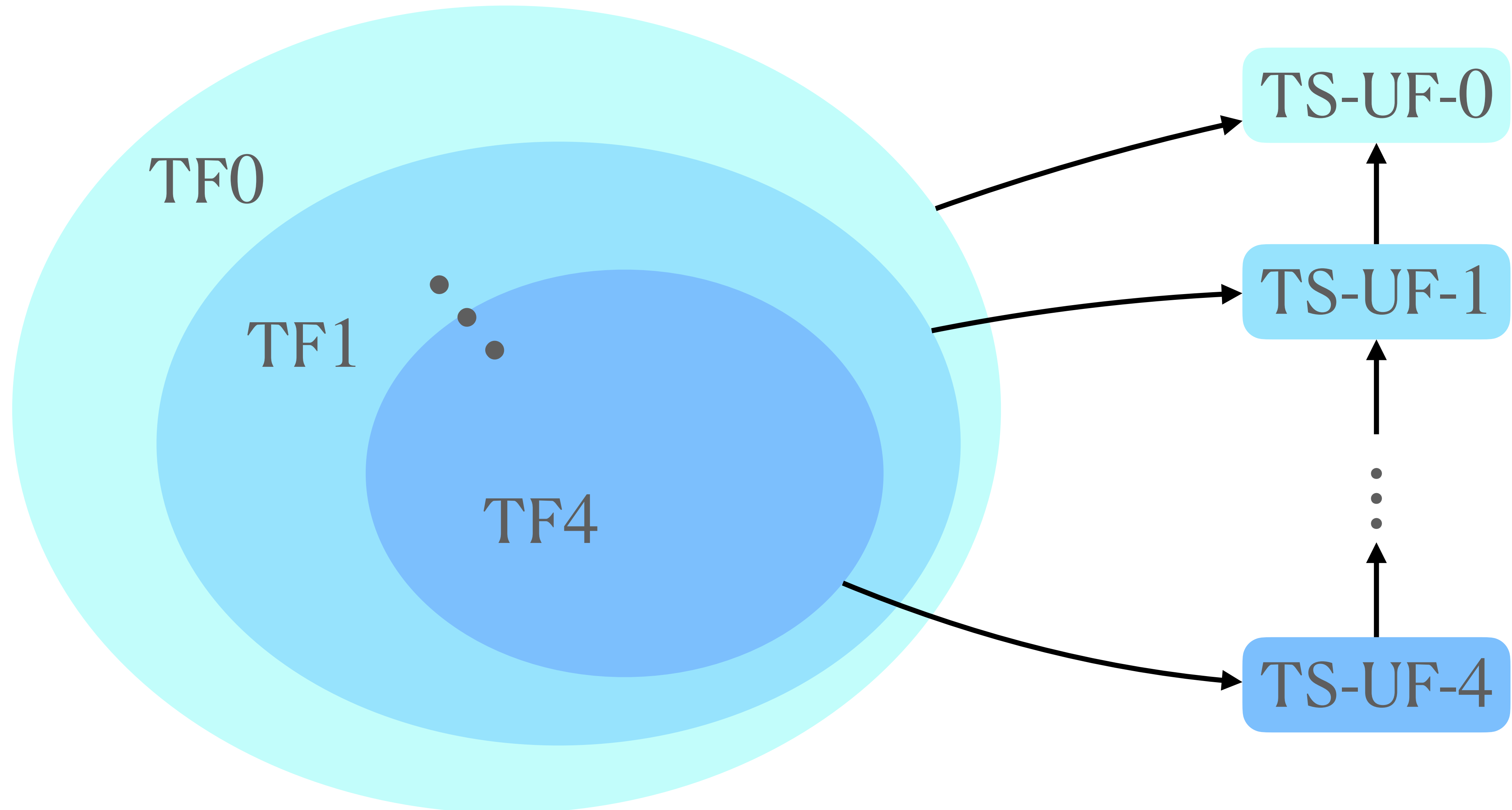
Set of trivial forgeries

Security levels



Set of trivial forgeries

Security hierarchy



The Simplest

L : set of $(i, lreq)$ queries to PSignO

TS-UF-0 (m^*, σ^*) is a trivial forgery :

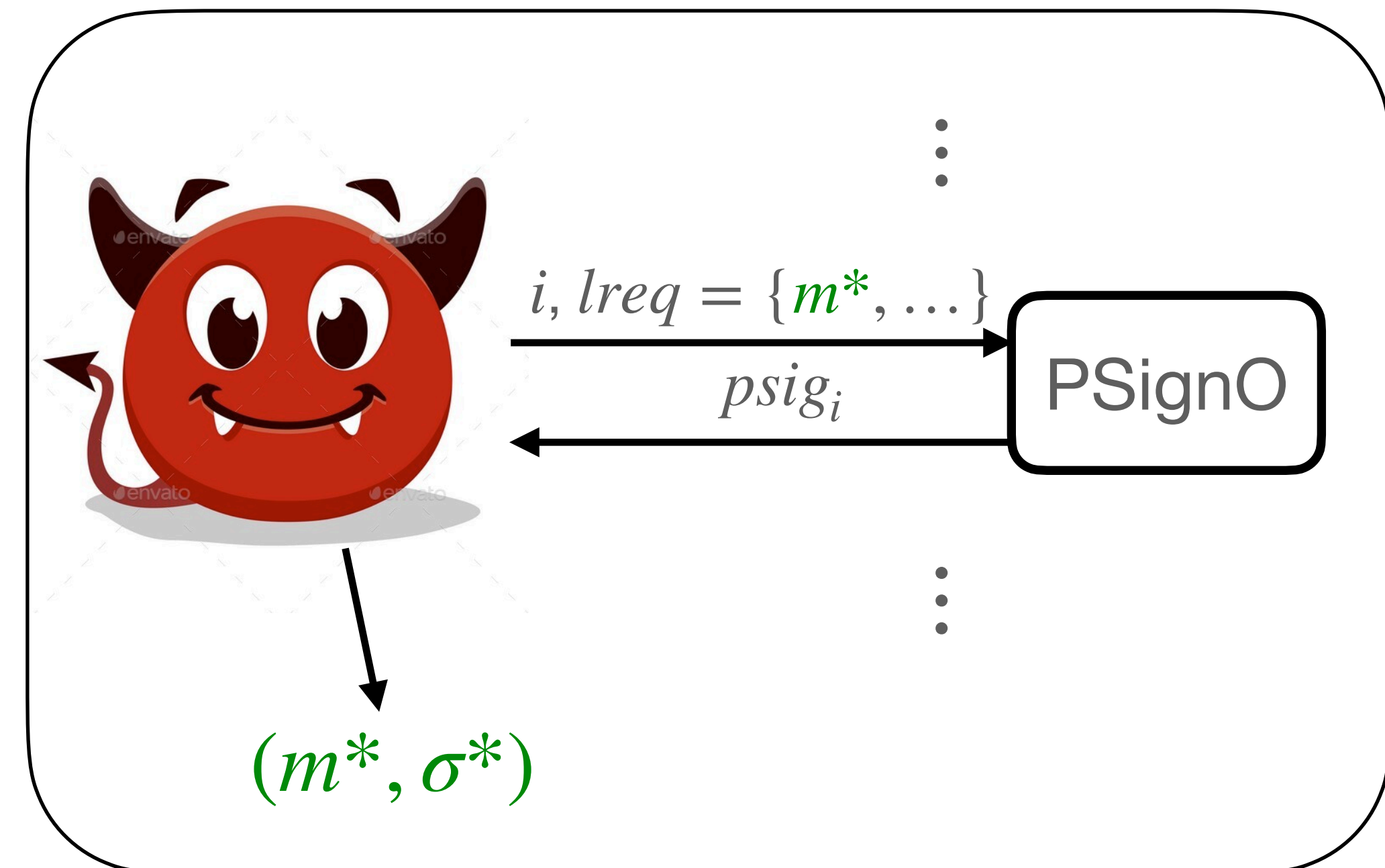
$\exists (i, lreq) \in L : lreq.msg = m^*$

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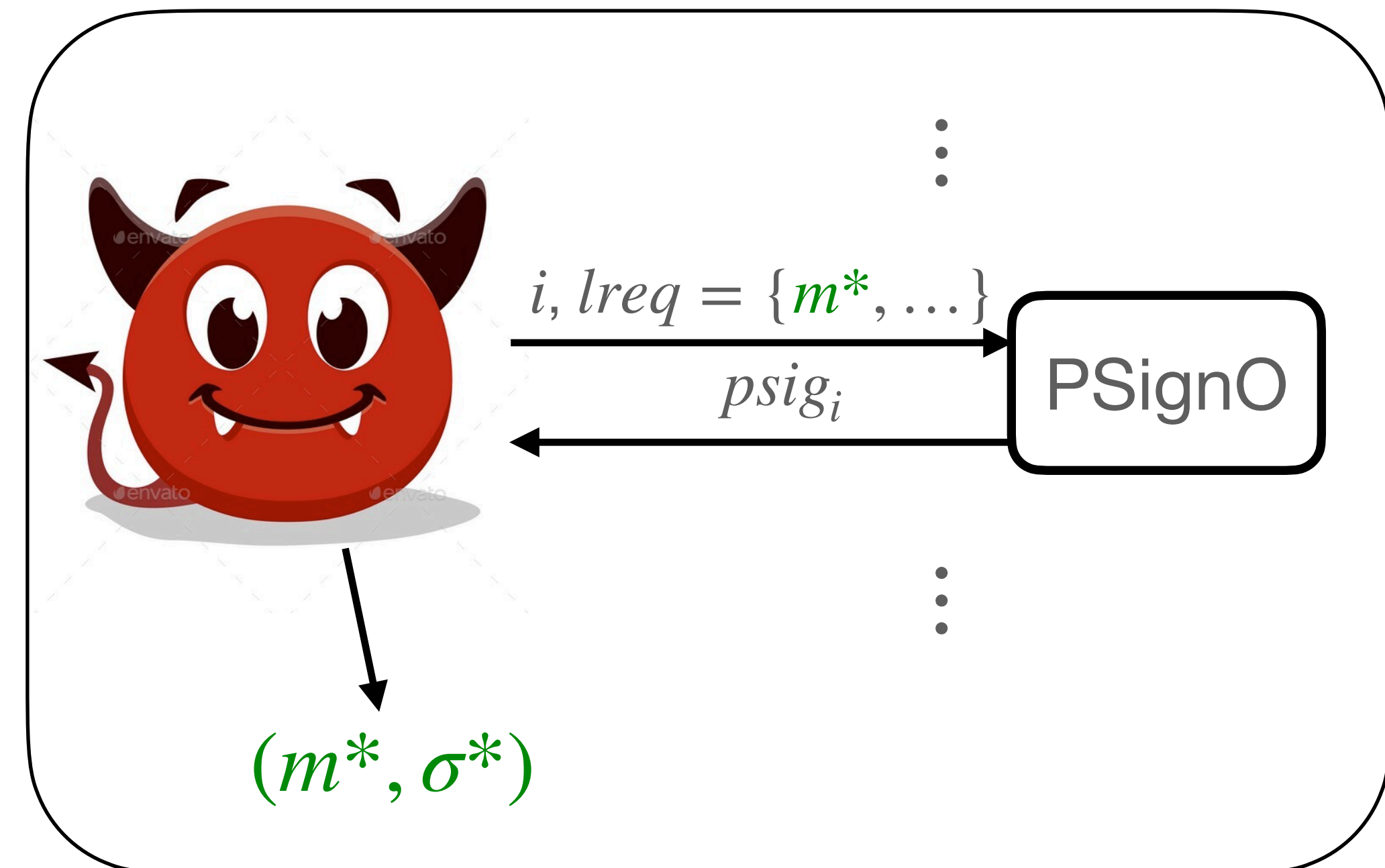
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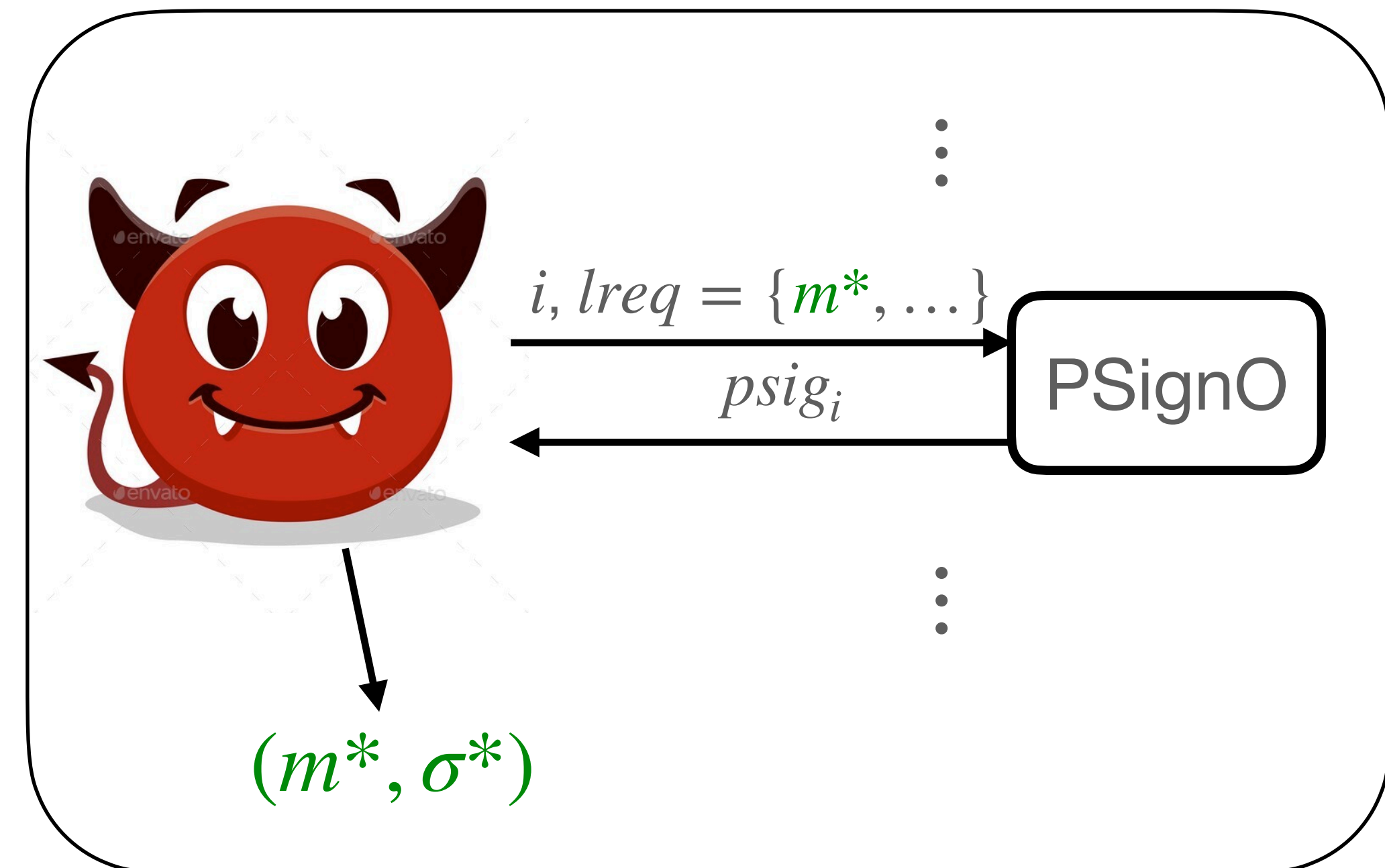
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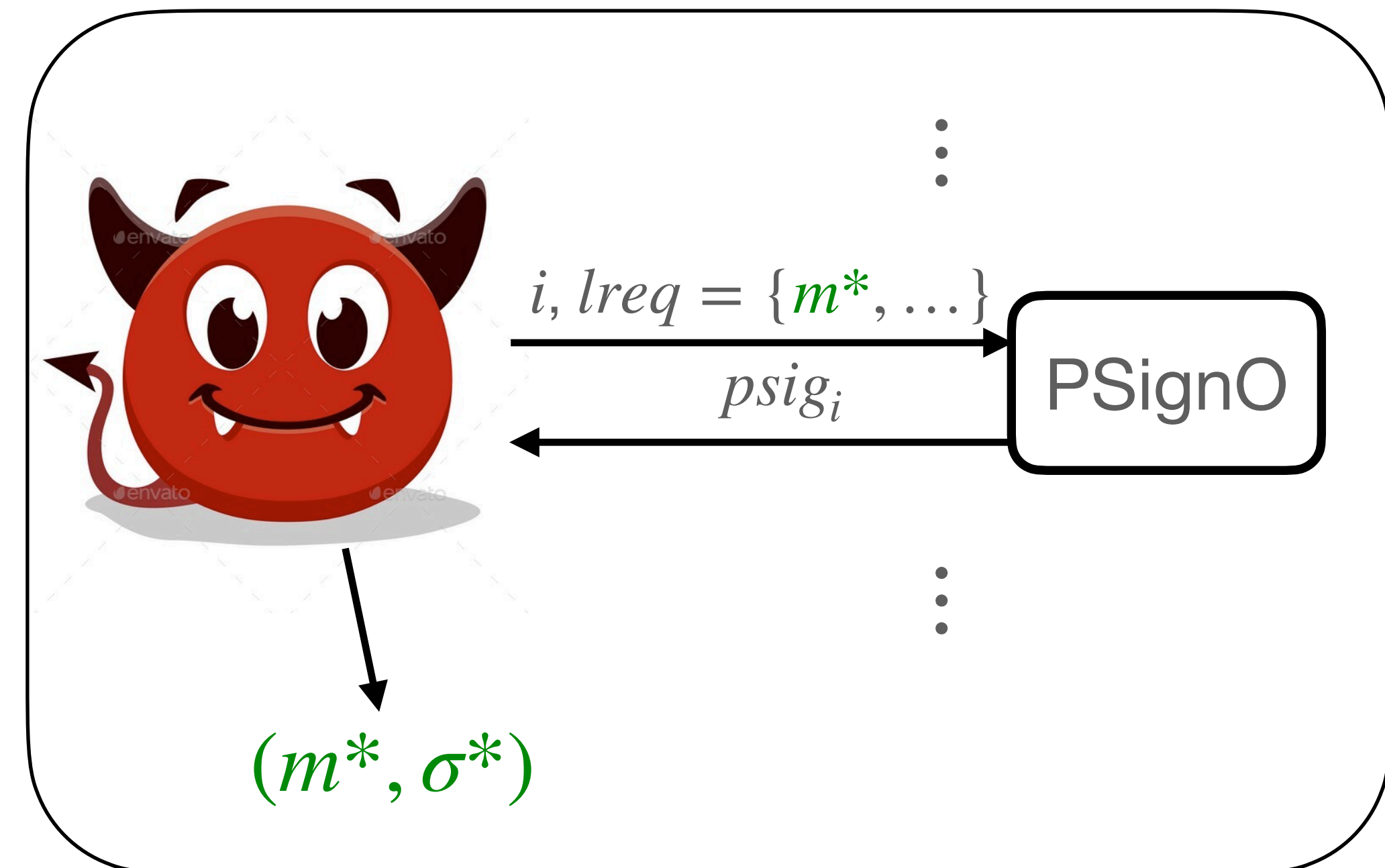
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When $|CS| < t - 1$,
there are **stronger**
security considerations



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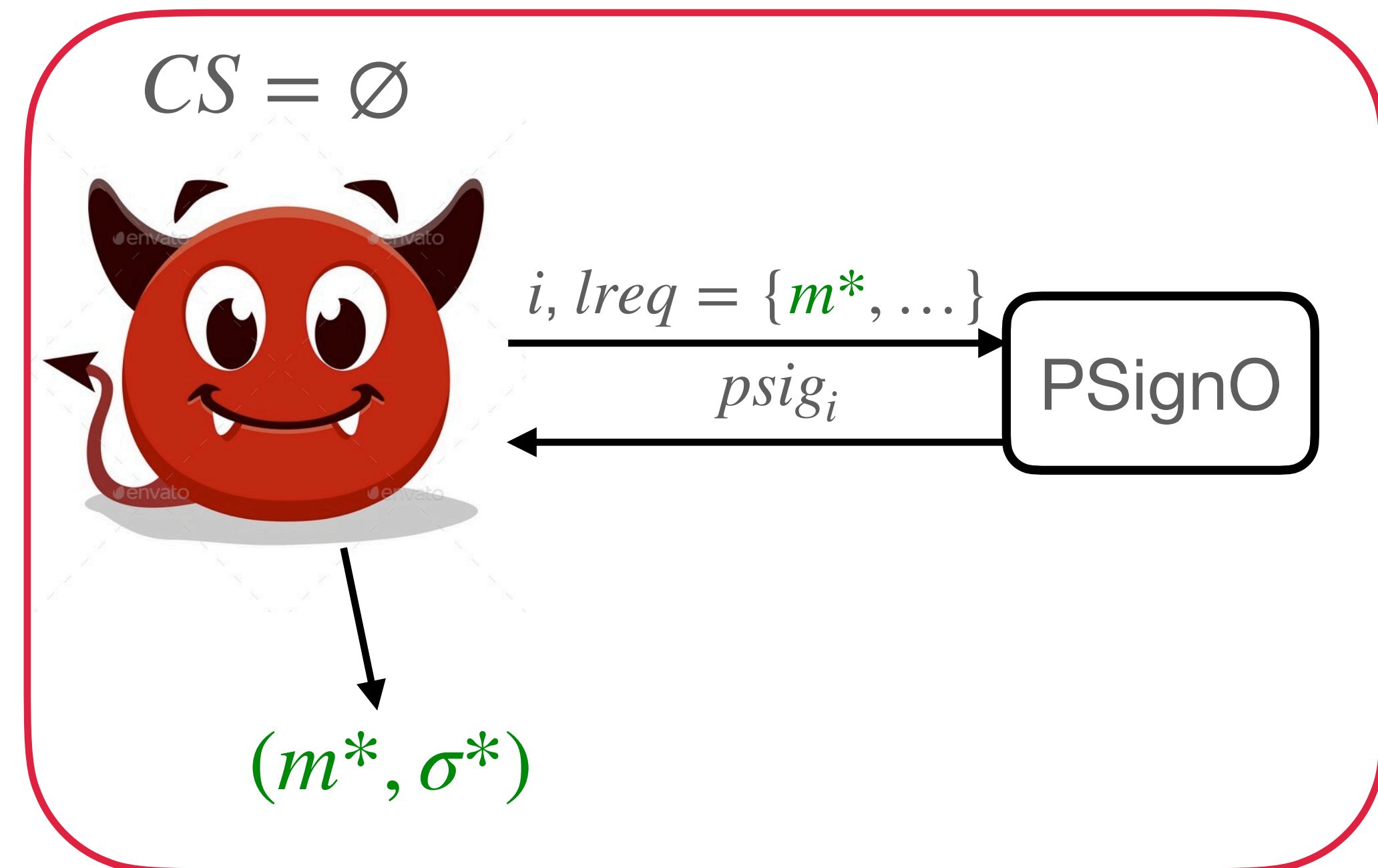
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At least one honest
signer signed m^*

Most of previous works consider this
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When $|CS| < t - 1$,
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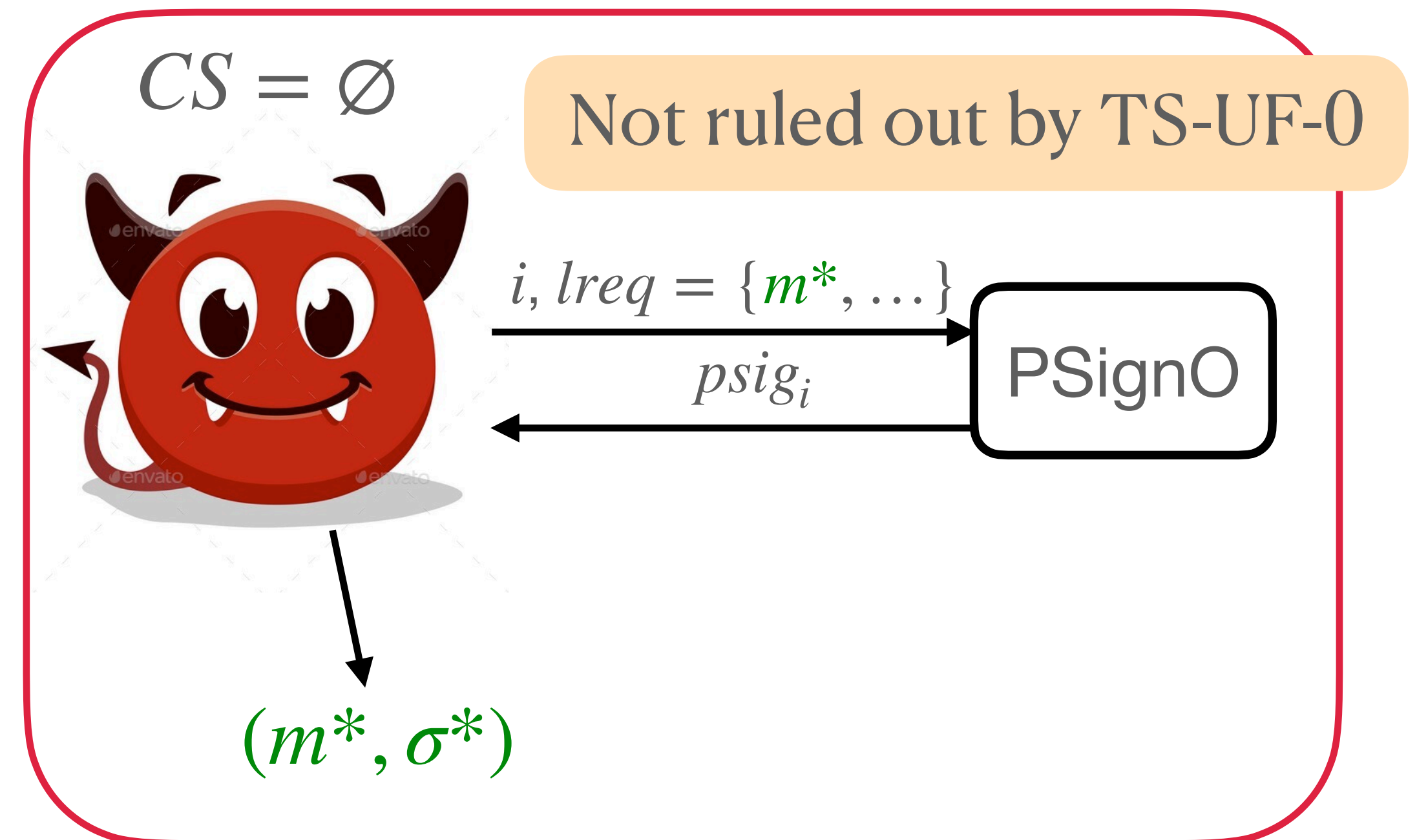
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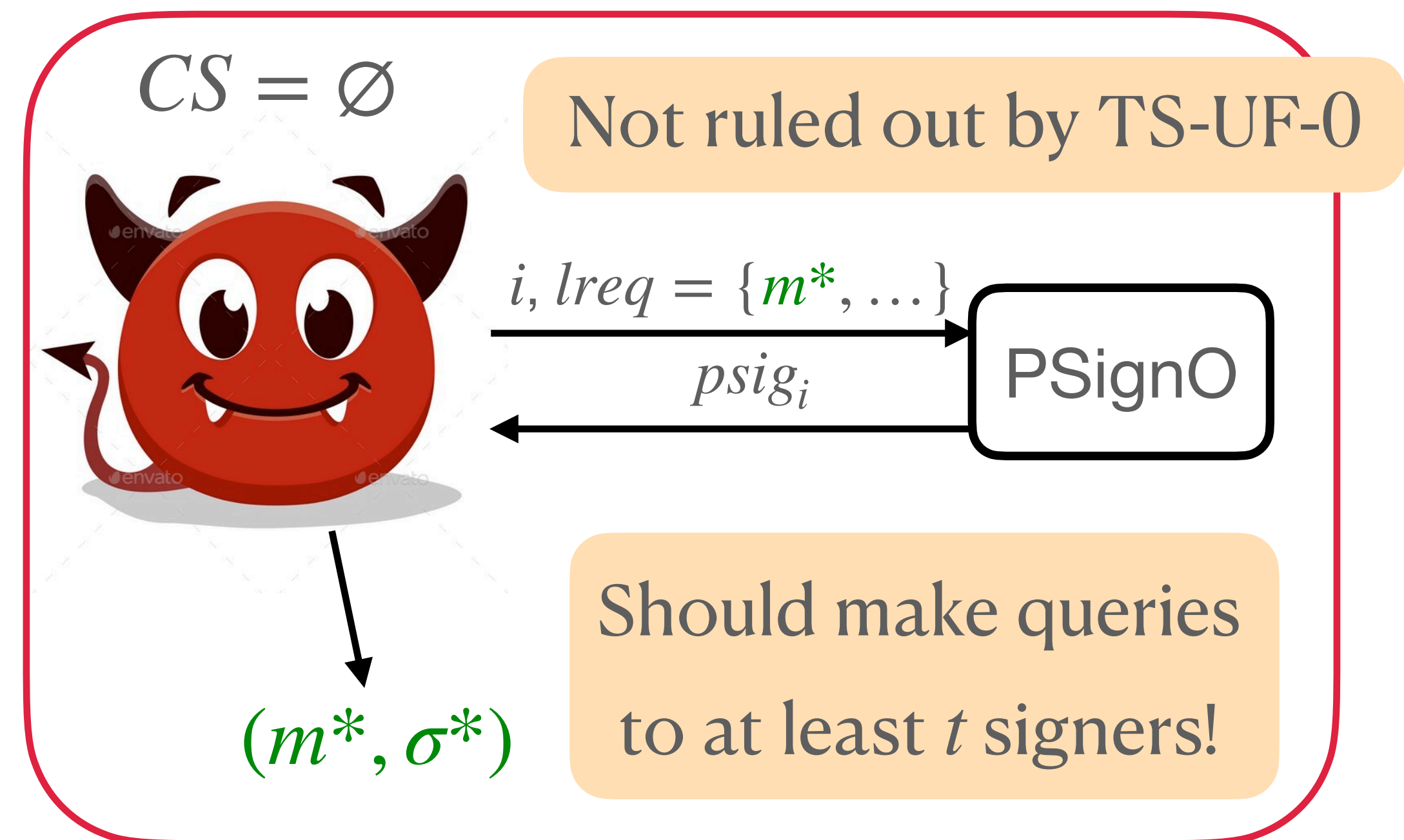
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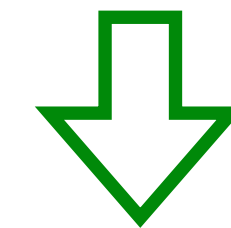
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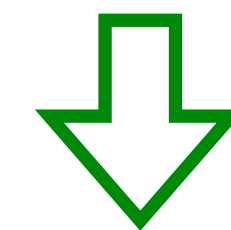
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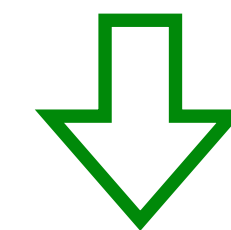
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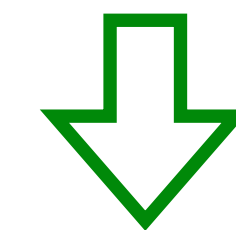
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Even stronger?

Partial sigs for m^* but from
different $lreq$ can be combined

Go Beyond

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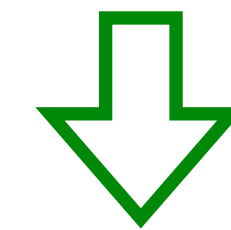
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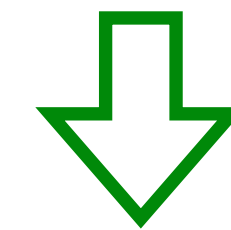
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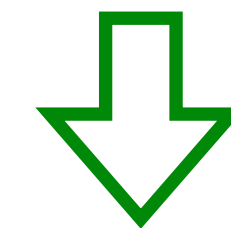
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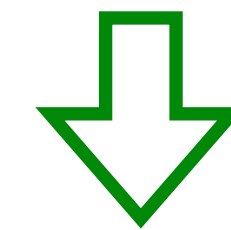
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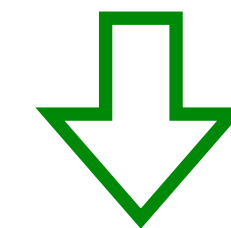
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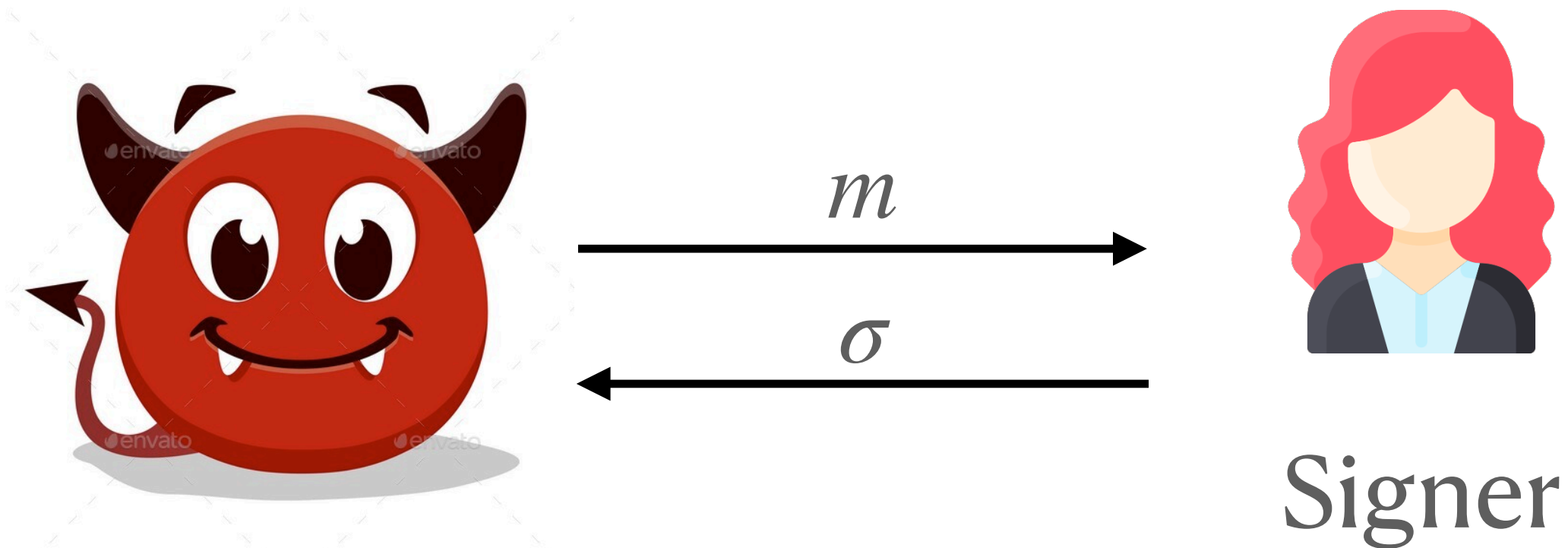
Our Highest

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Strong Unforgeability

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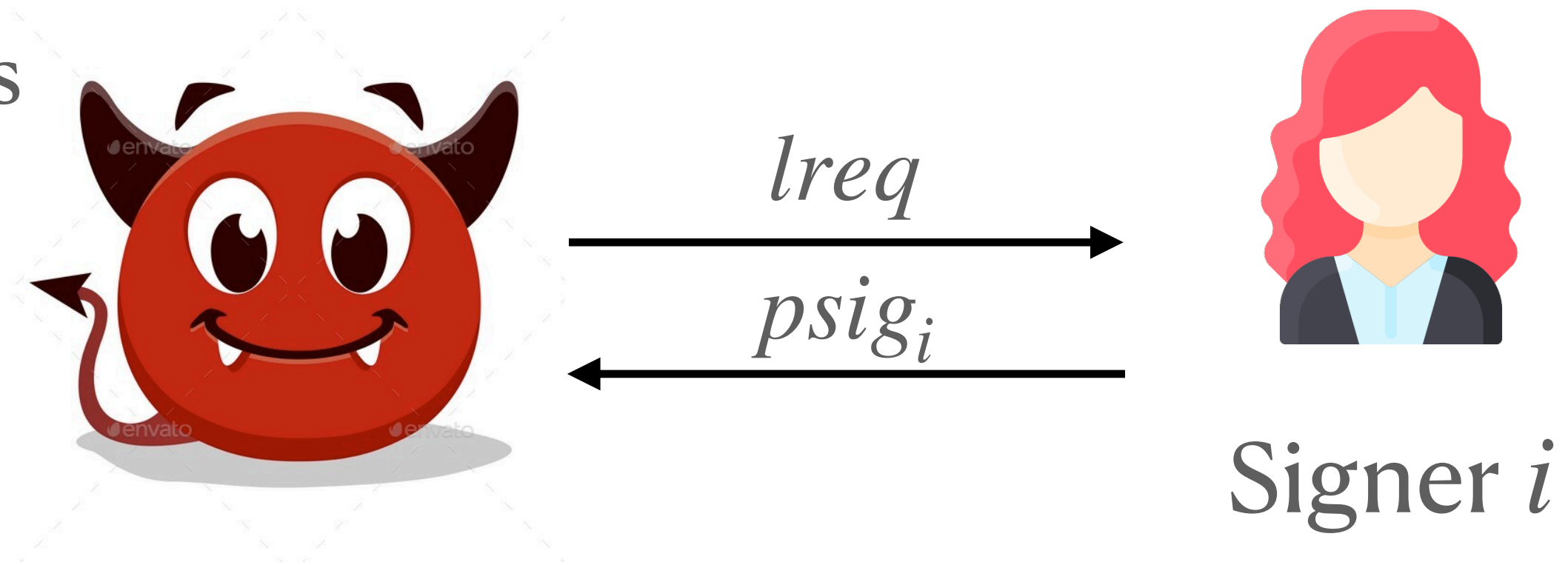
For signature schemes



The adversary cannot forge $\sigma^* (\neq \sigma)$ for m

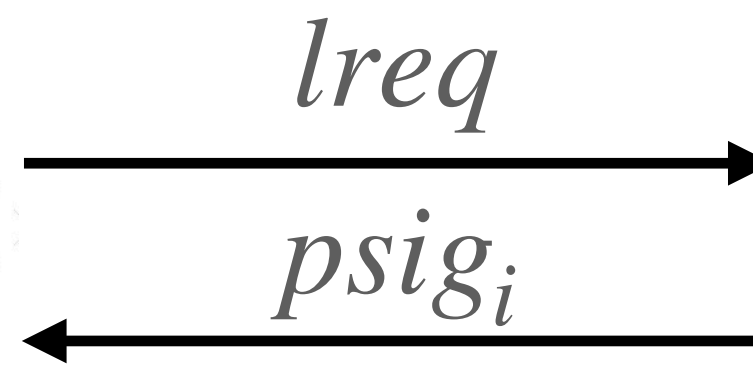
Strong Unforgeability

For threshold signatures



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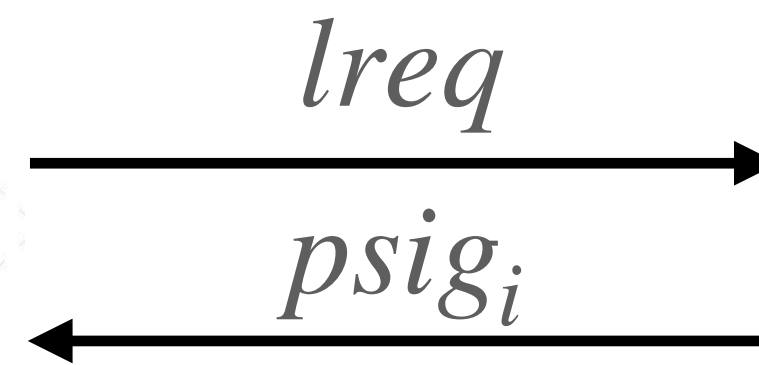


Signer i

What is the issued signature?

Strong Unforgeability

For threshold signatures



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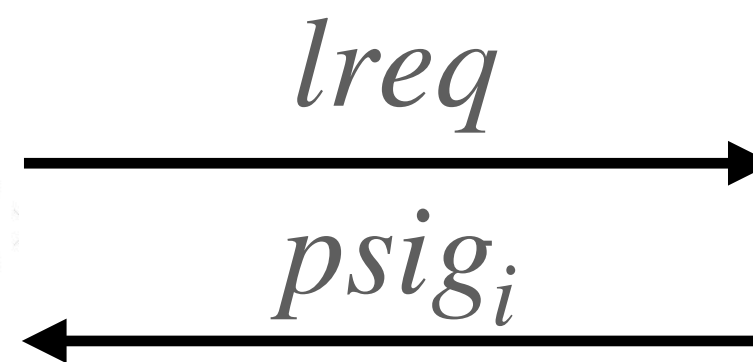
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For schemes with **deterministic signing**

$$(pk, lreq) \longrightarrow (m, \sigma)$$

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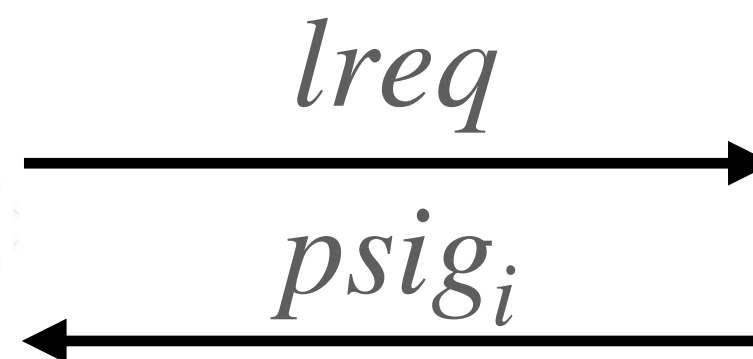
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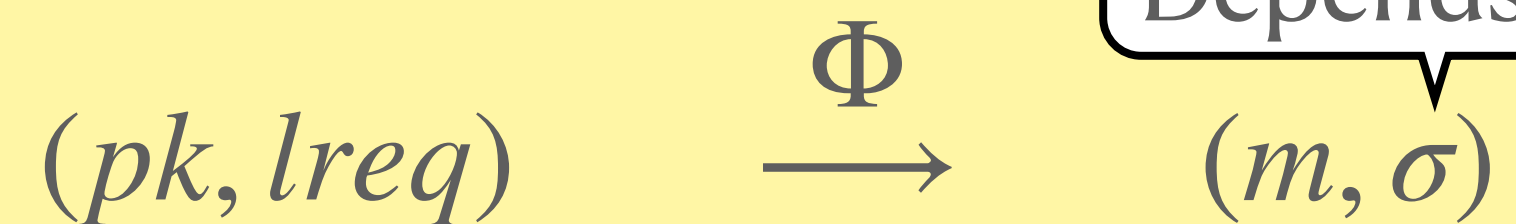
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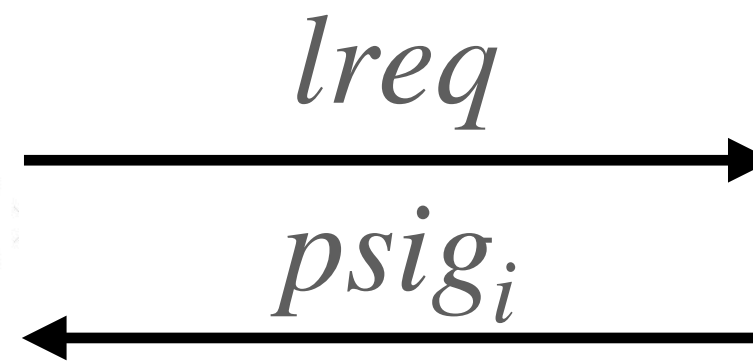


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(m, σ)

FROST_{1/2} have this property

$m = lreq.msg$

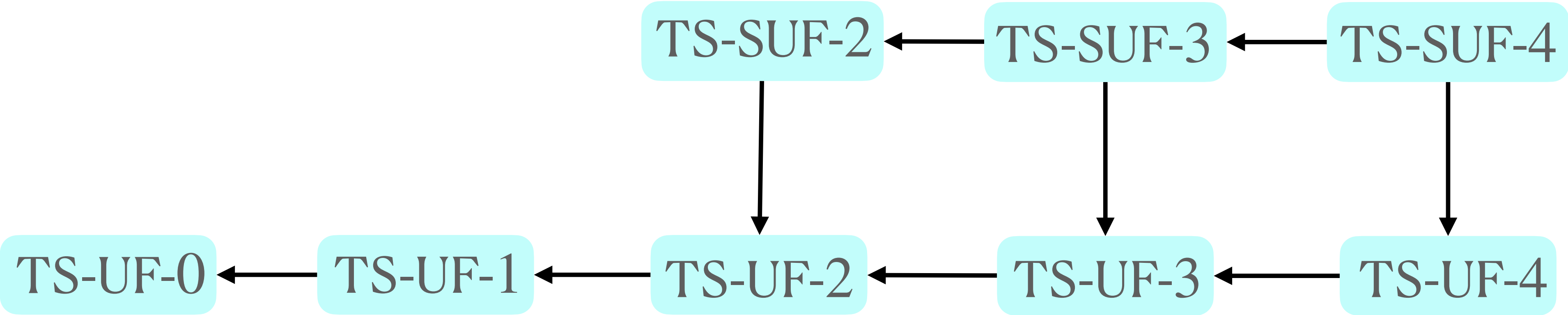
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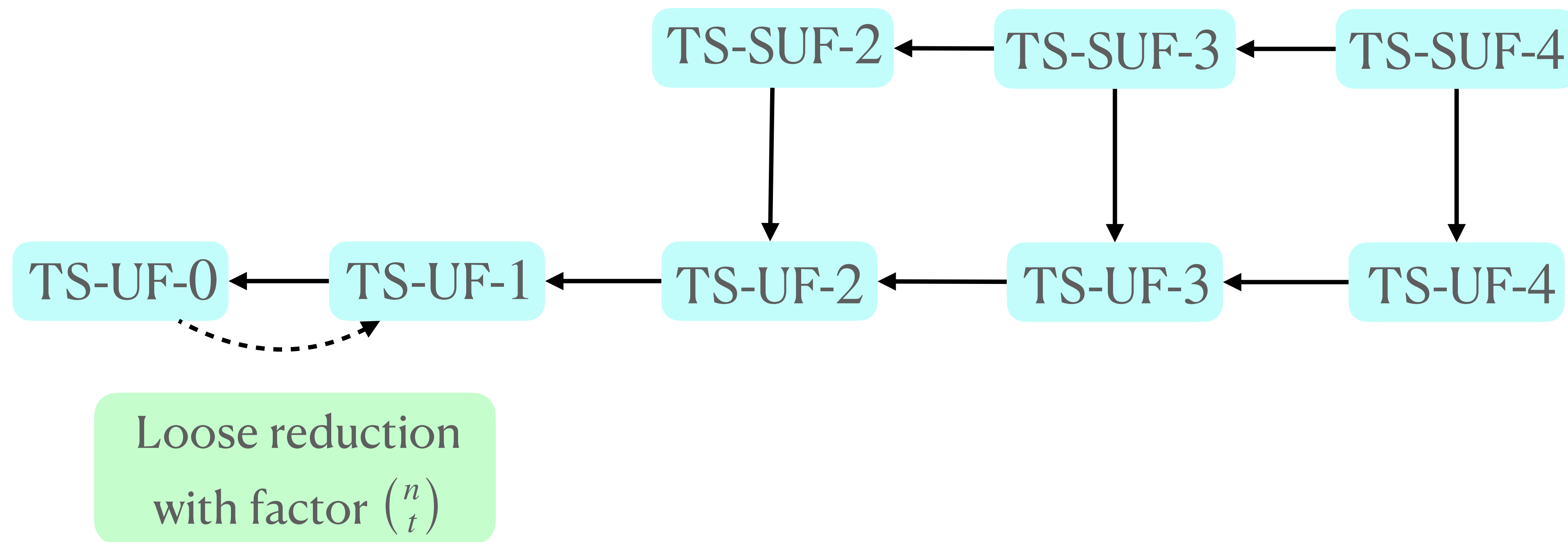
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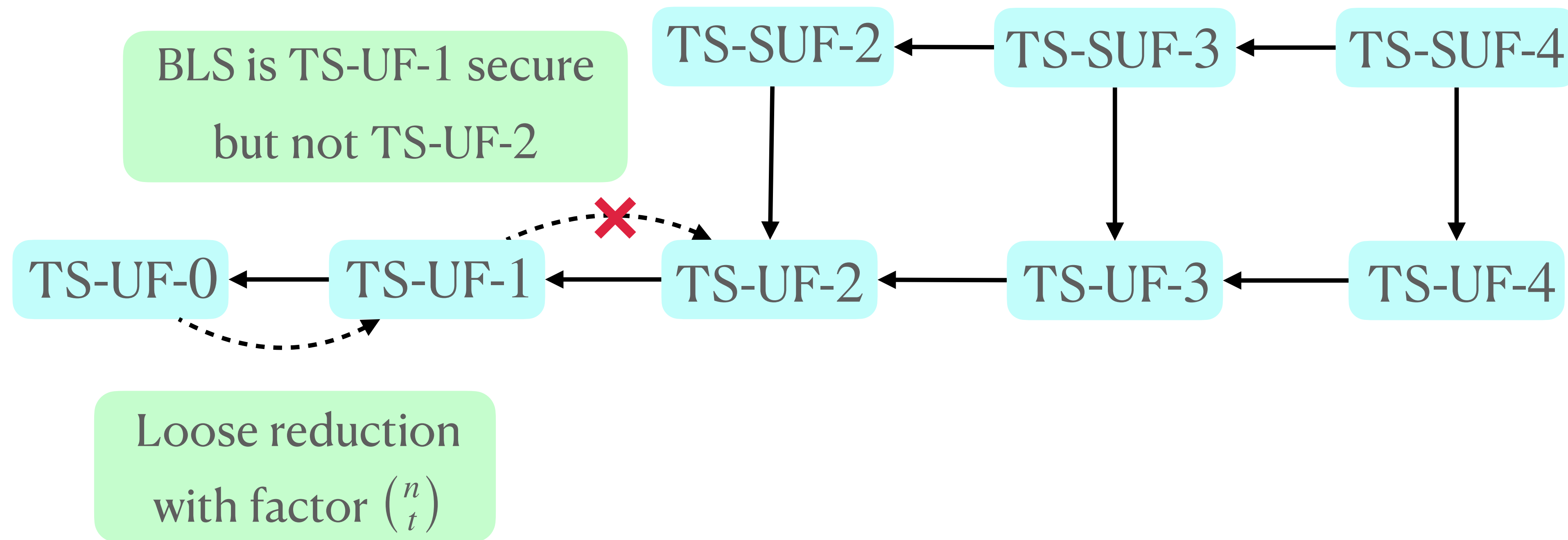
Full Picture



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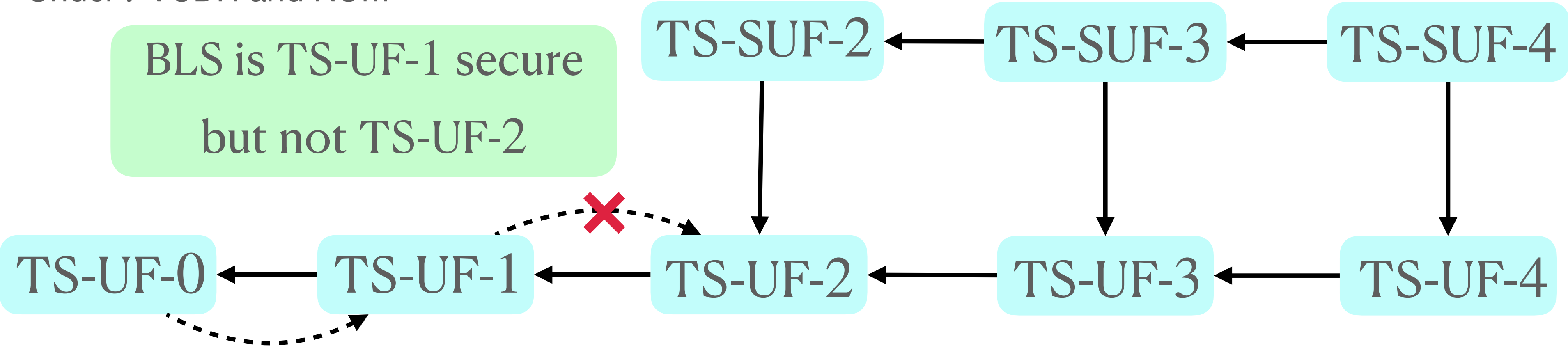


Full Picture

Hard in GGM
Loosely implied by CDH

Under t -VCDH and ROM

BLS is TS-UF-1 secure
but not TS-UF-2



Loose reduction
with factor $\binom{n}{t}$

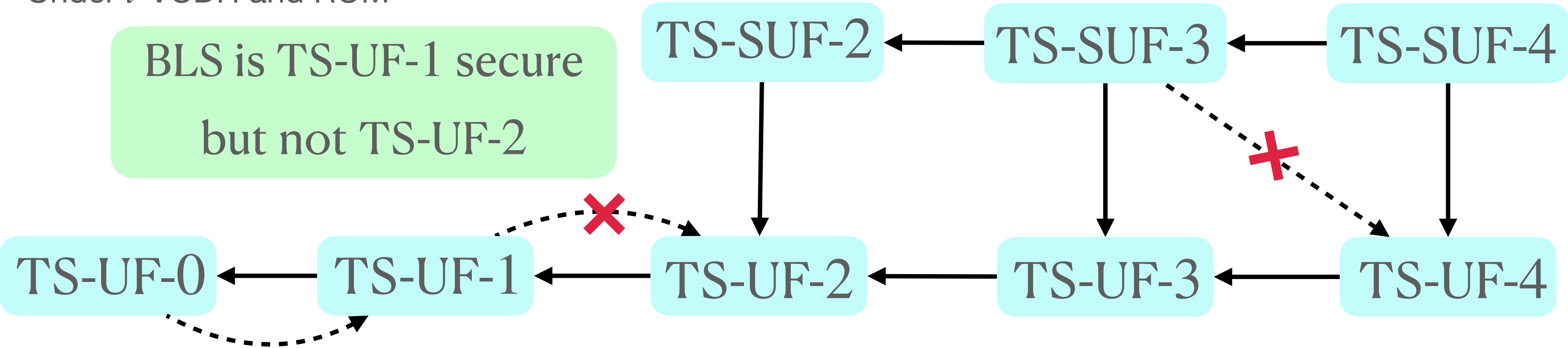
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FROST₁ is TS-SUF-3 secure
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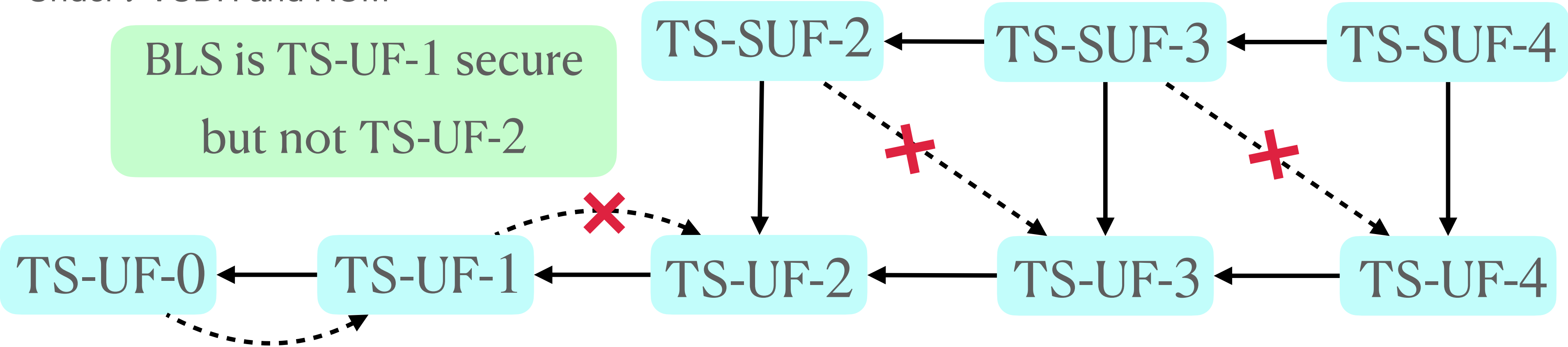
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FROST₂ is TS-SUF-2 secure
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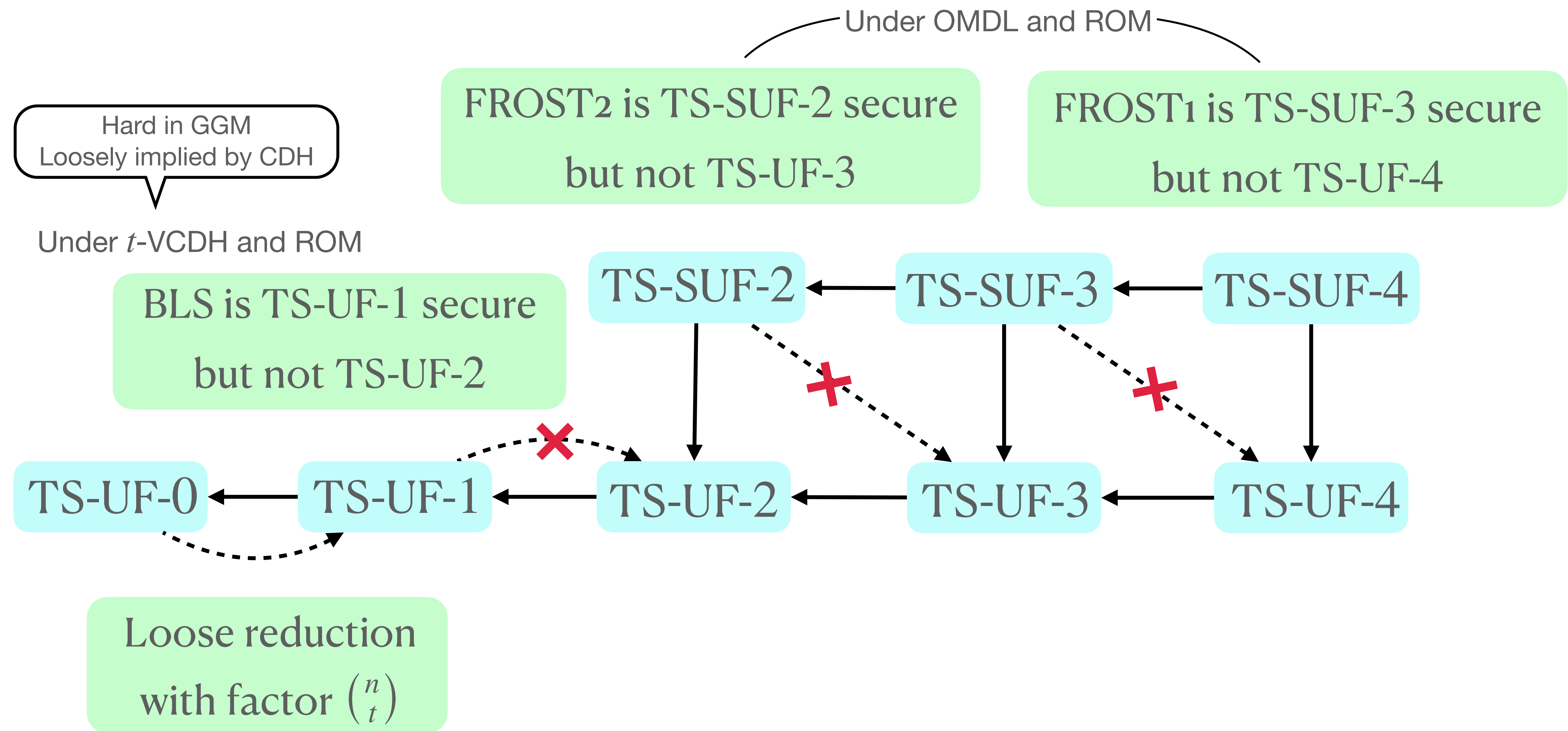
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Full Picture



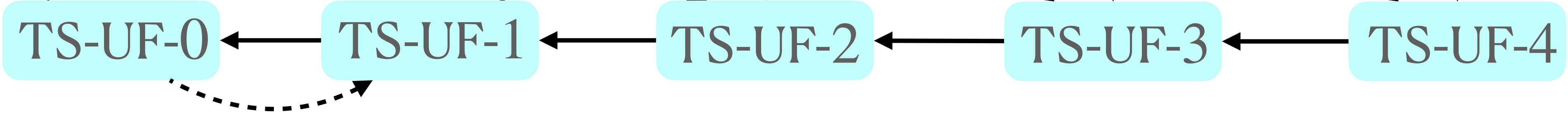
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Previous
works

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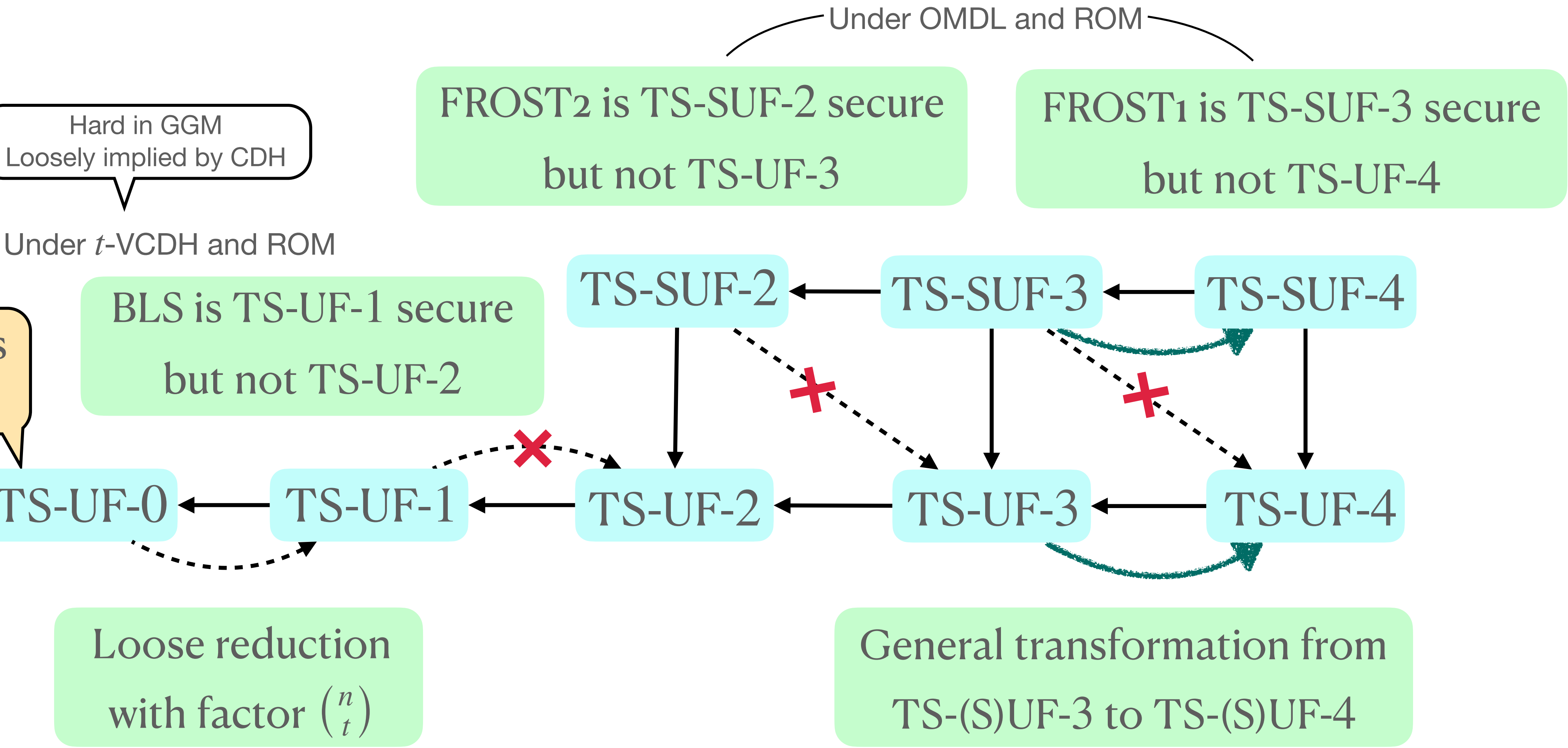


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Full Picture



Separation of FROST1/2

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An adversary for FROST₂



Separation of FROST1/2

An adversary for FROST₂

$n = 4, t = 3$



Separation of FROST1/2

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1 2 3 4



Separation of FROST1/2

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$n = 4, t = 3$

$CS = \{3, 4\}$

pk, sk_3, sk_4



Separation of FROST1/2

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1,2



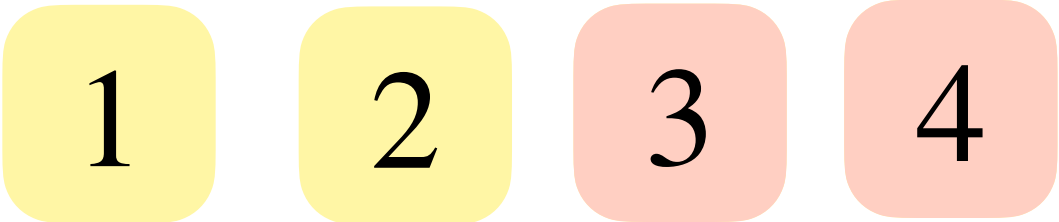
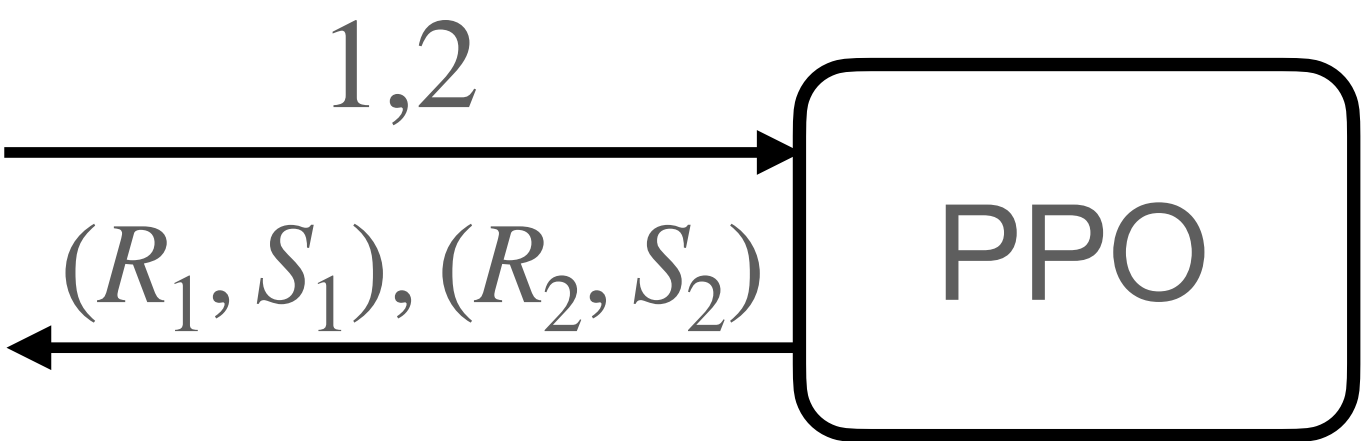
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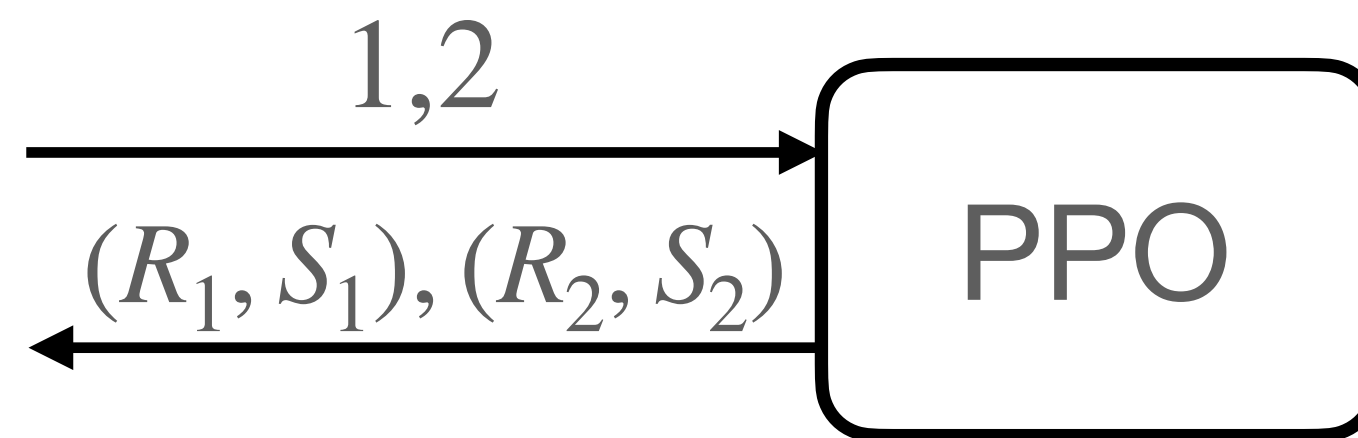
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3

4

$lreq : m^*, SS = \{1, 2, 3\}$

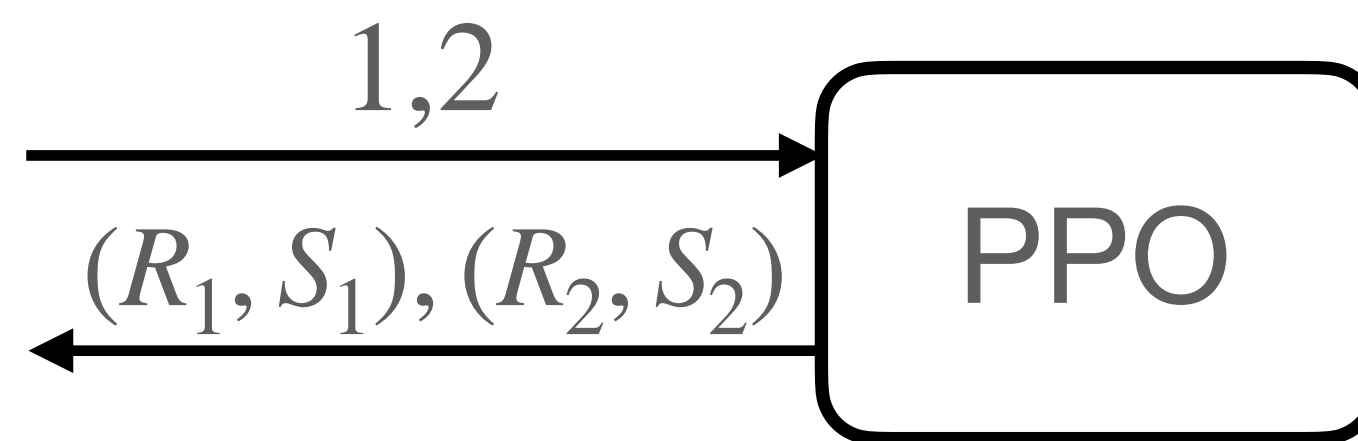
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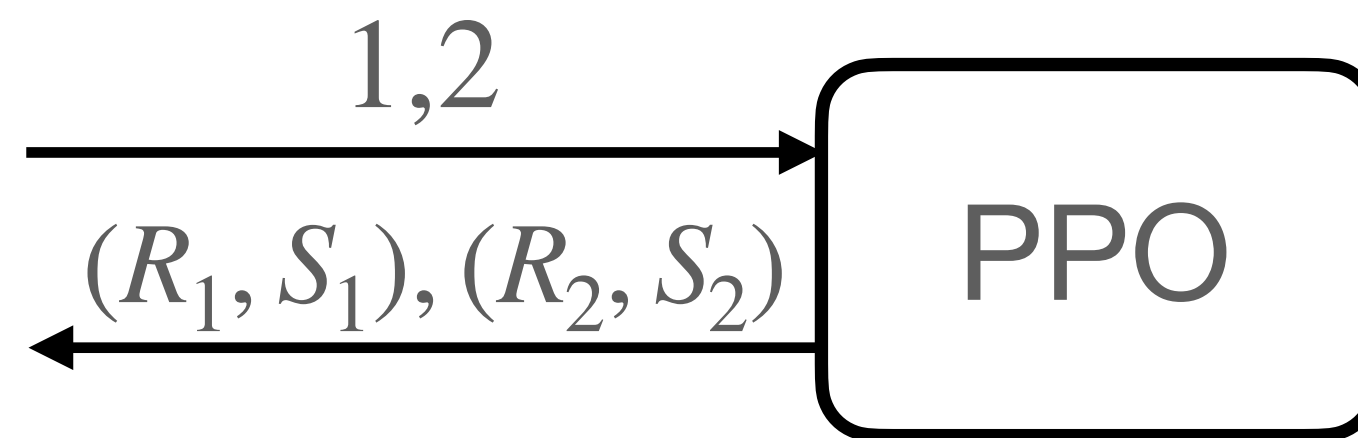
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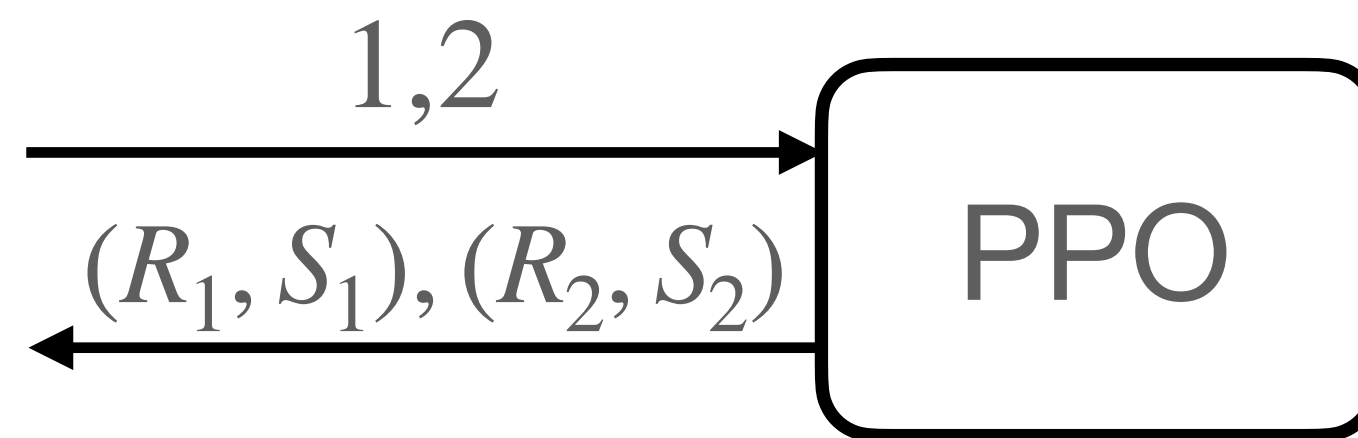
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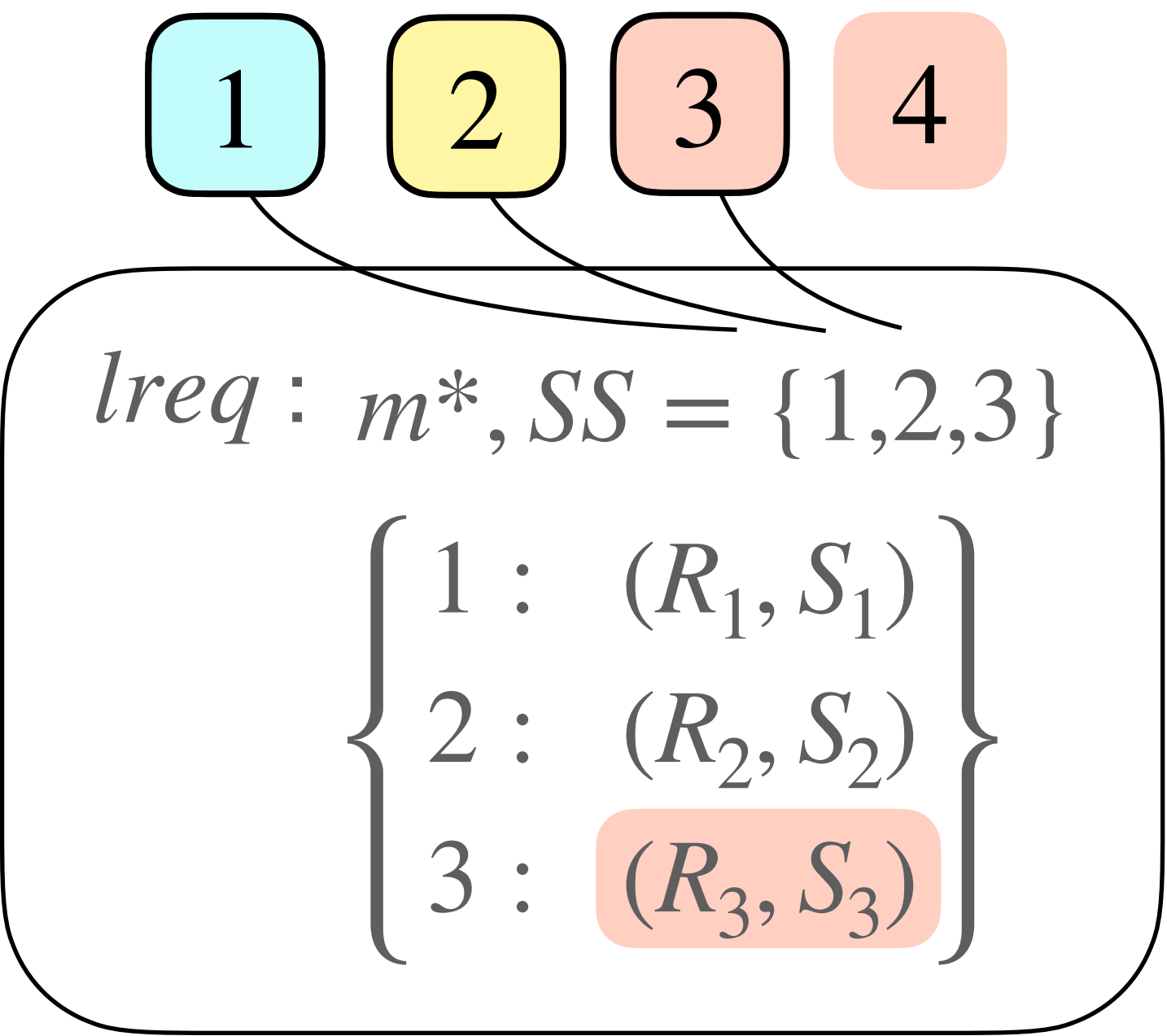
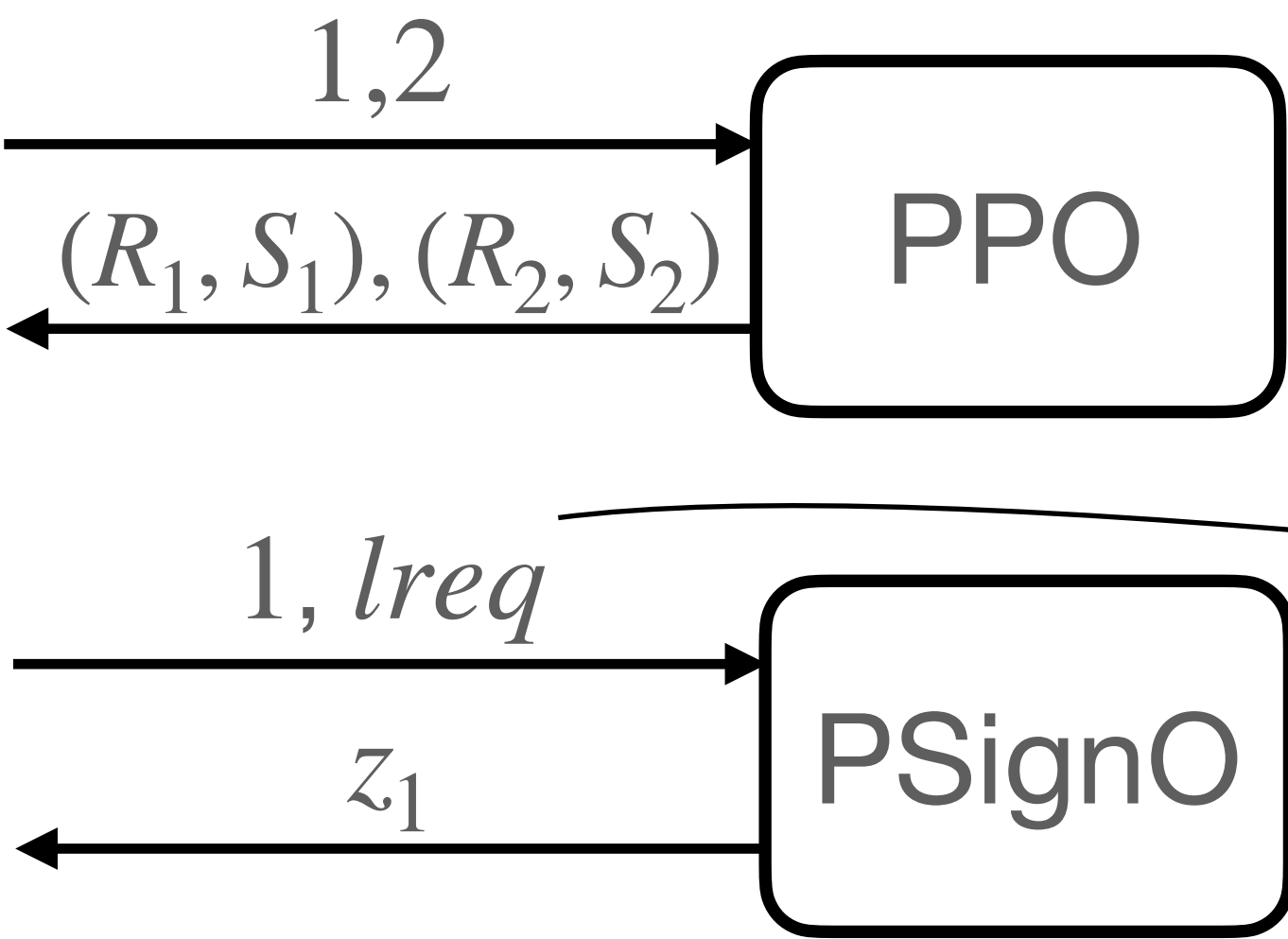
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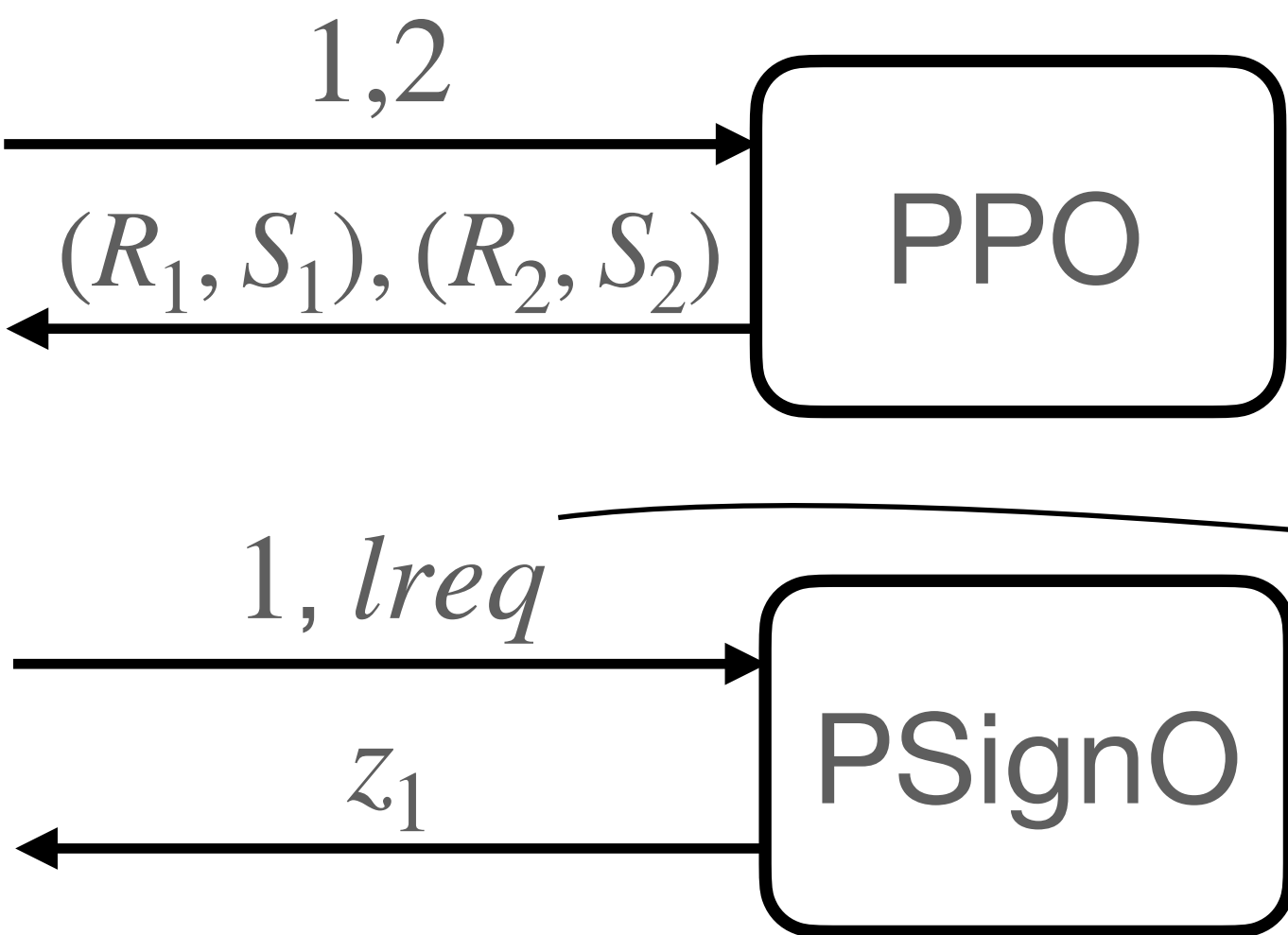
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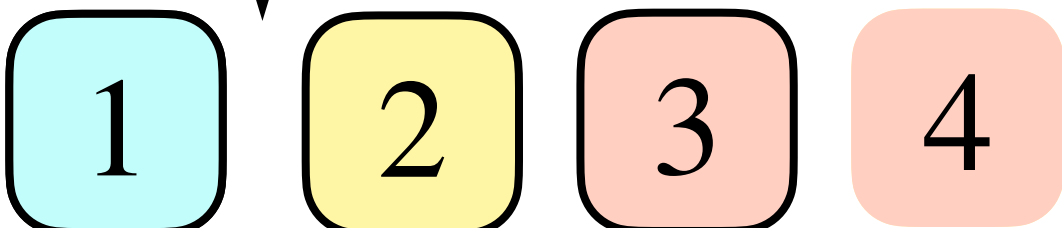
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$(m^*, (R^*, z^*))$



For FROST1, the adversary has to query **both signer 1 and 2**



$lreq : m^*, SS = \{1, 2, 3\}$

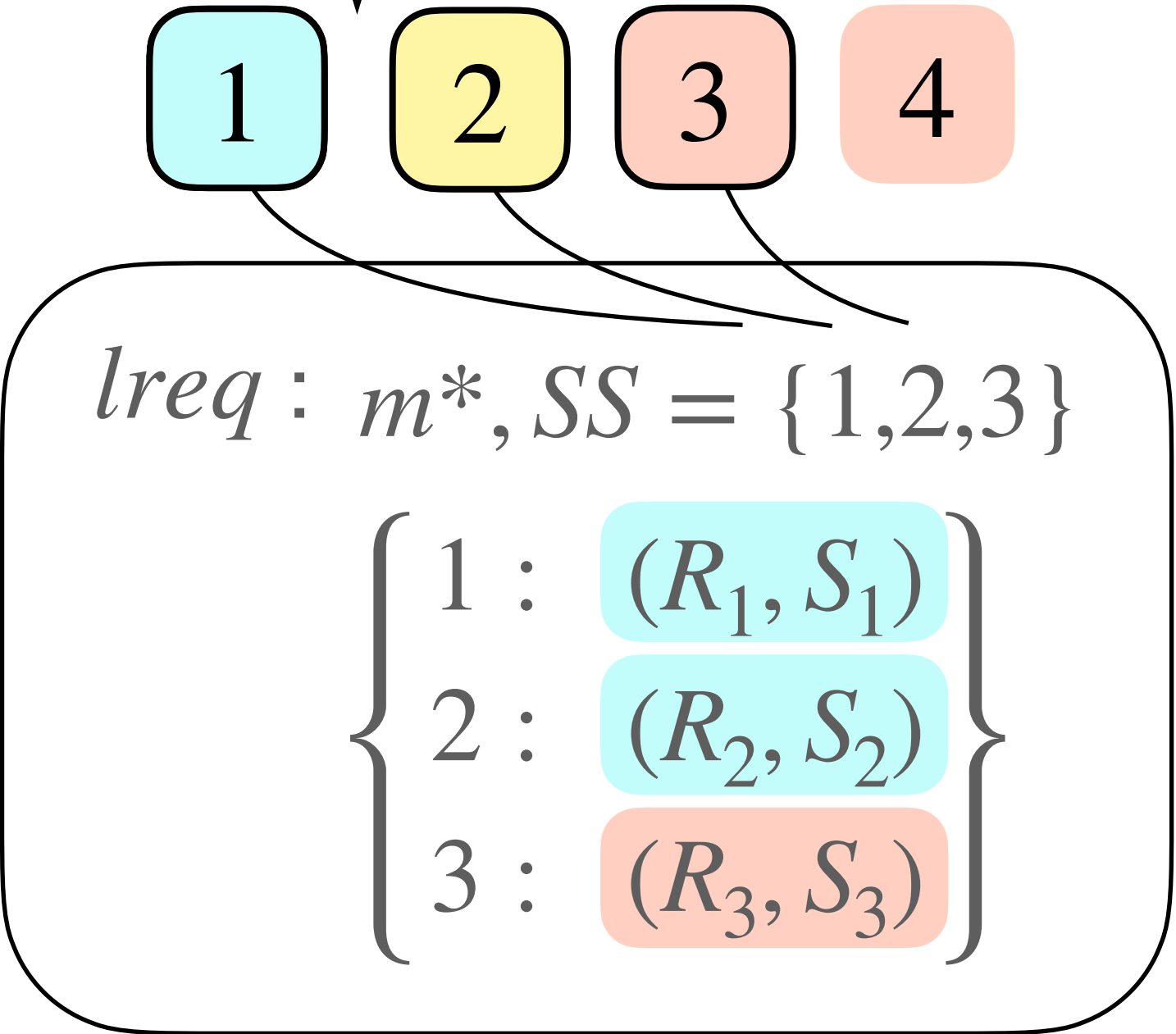
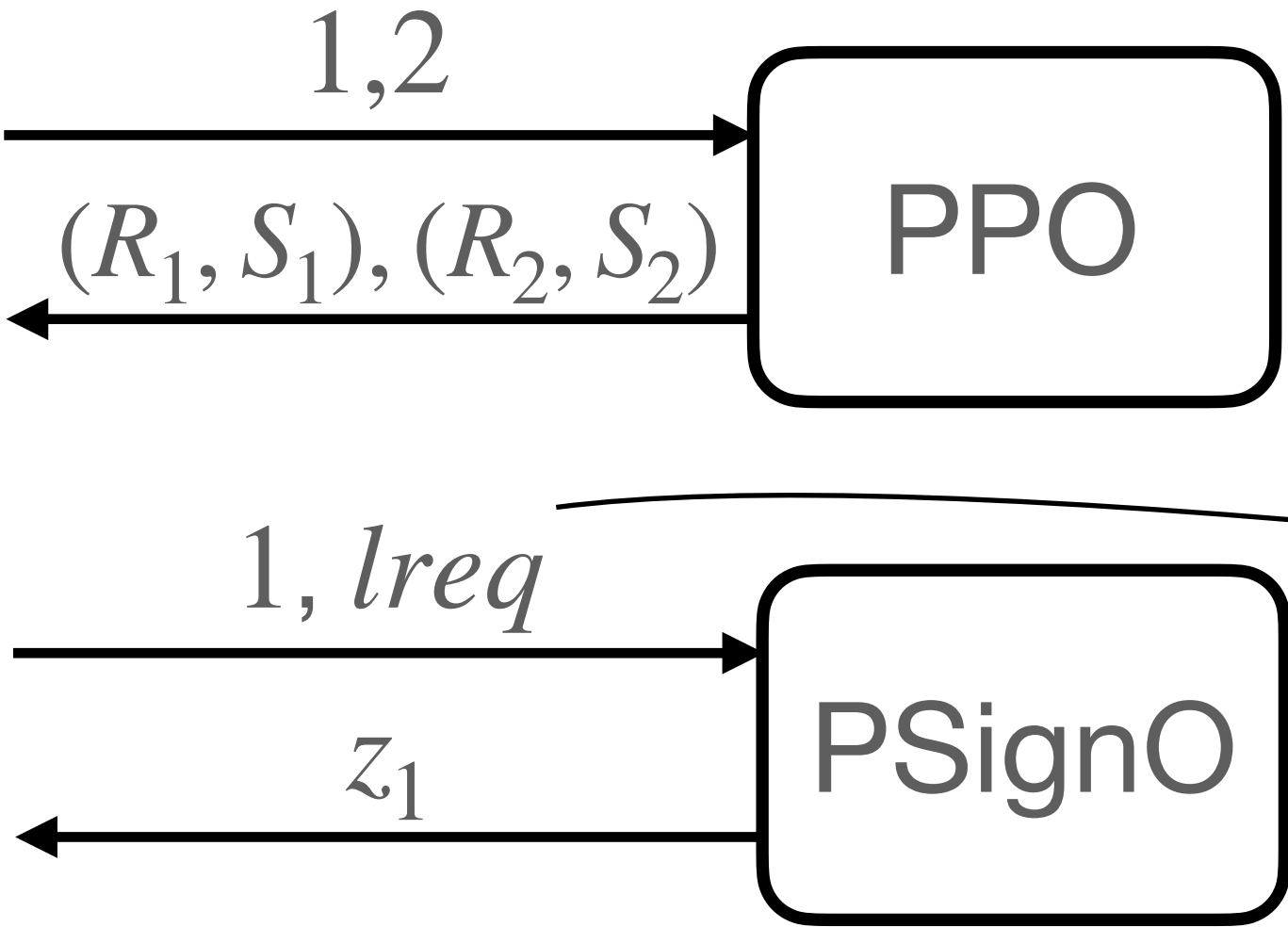
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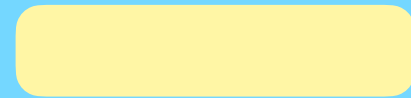
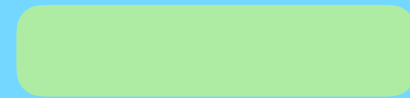
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Conclusion & Future work

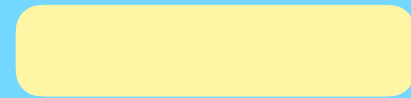
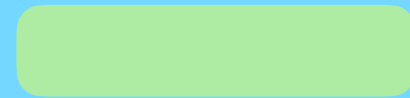
Conclusion & Future work

Framework &
Security Hierarchy



Conclusion & Future work

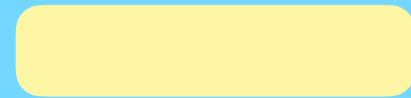
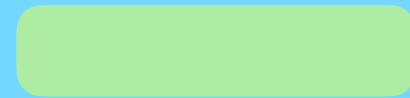
Framework &
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FROST₂

Conclusion & Future work

Framework &
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FROST₂

Better-than-advertised
security of BLS, FROST

Conclusion & Future work

Framework &
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Security with
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TS-UF- $X \leftrightarrow$ Applications

⋮

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Adaptive security?

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Framework for general DKG protocols?

Thank you!