

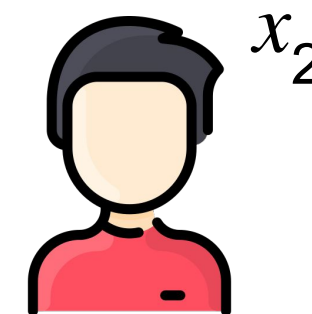
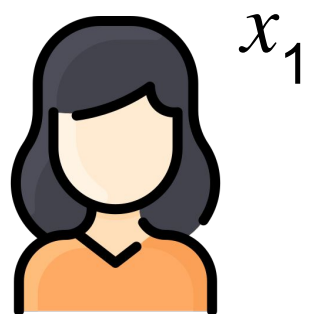
Round-Optimal and Communication-Efficient Multiparty Computation

Michele Ciampi, Rafail Ostrovsky,
Hendrik Waldner, Vassilis Zikas

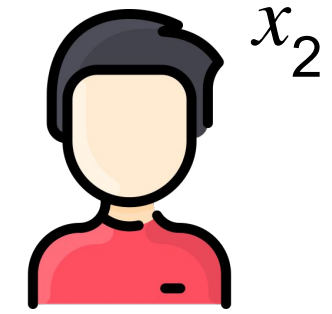
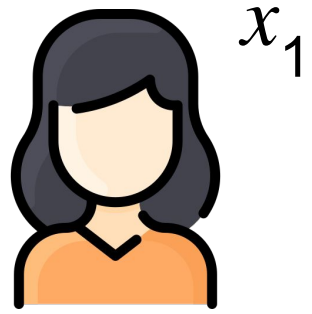


Multiparty Computation (MPC)

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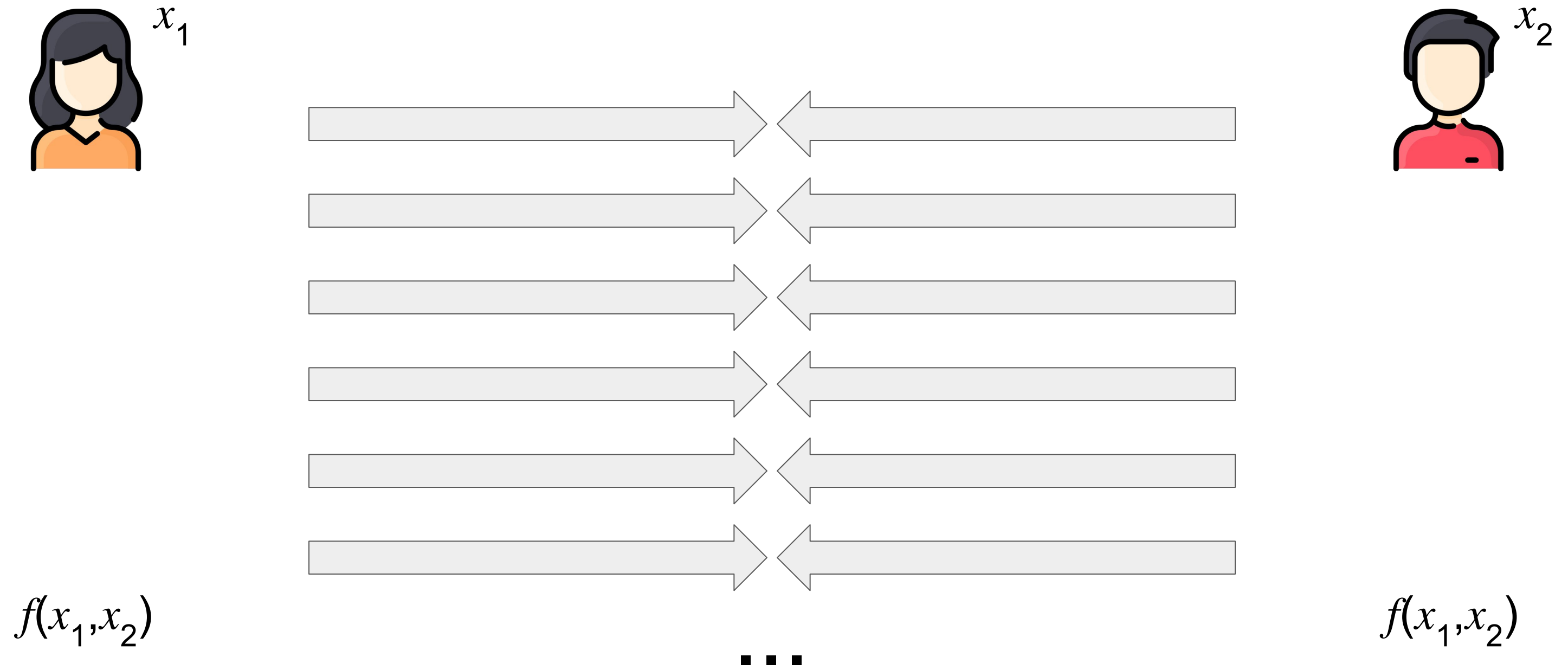
Multiparty Computation (MPC)



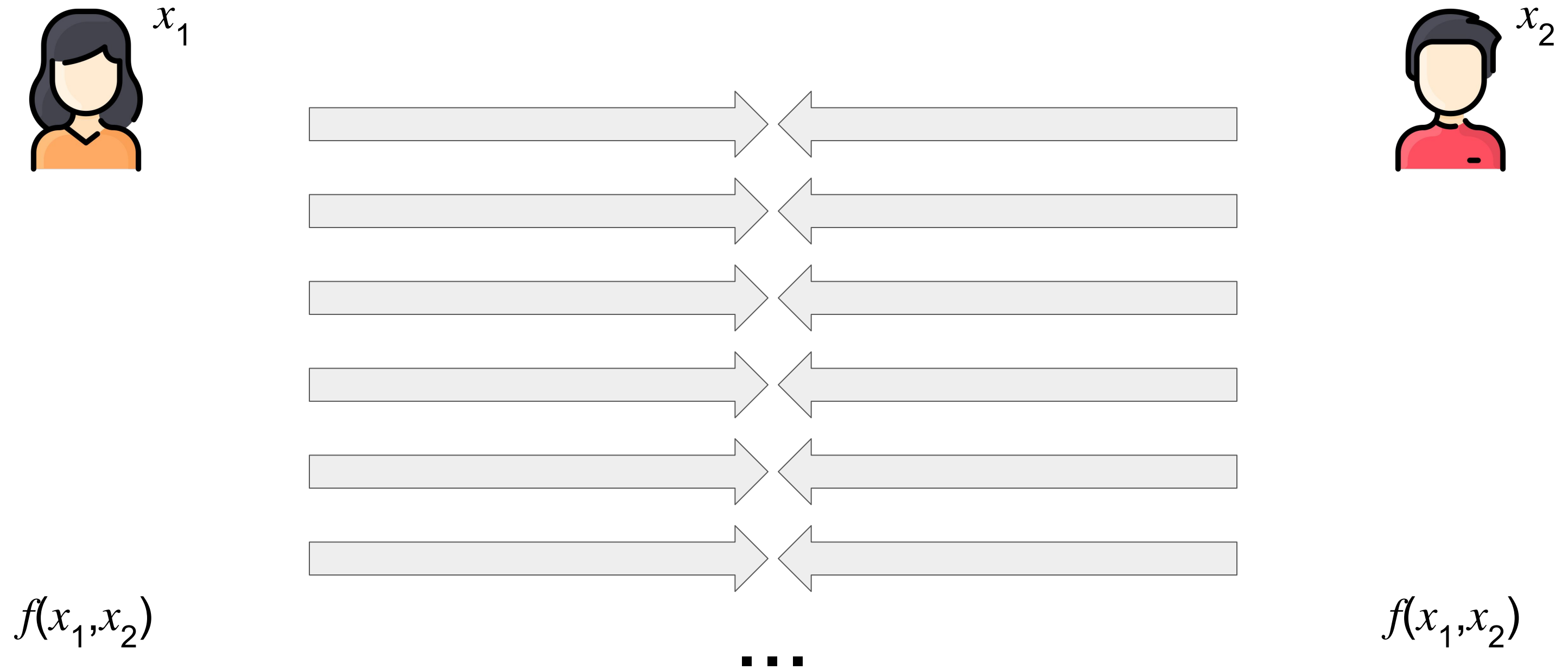
$$f(x_1, x_2)$$

$$f(x_1, x_2)$$

Multiparty Computation (MPC)

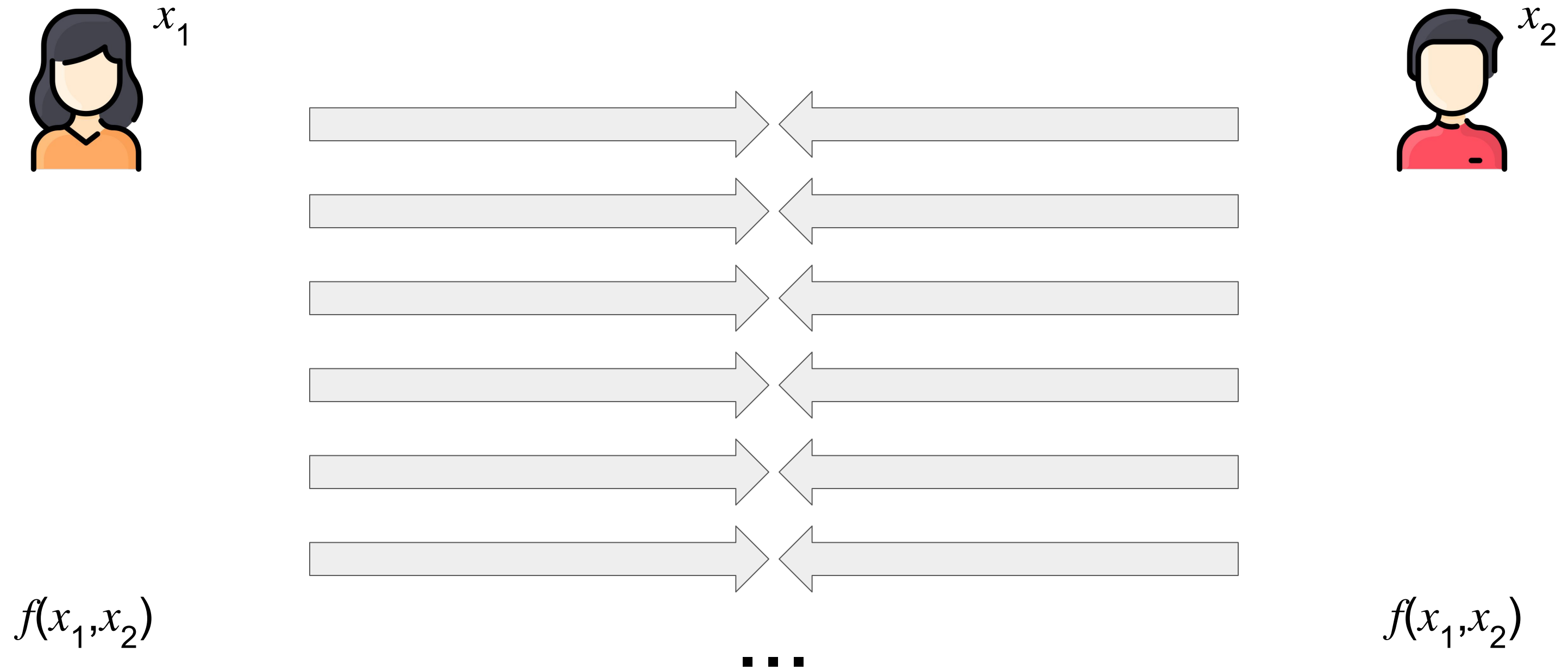


Multiparty Computation (MPC)



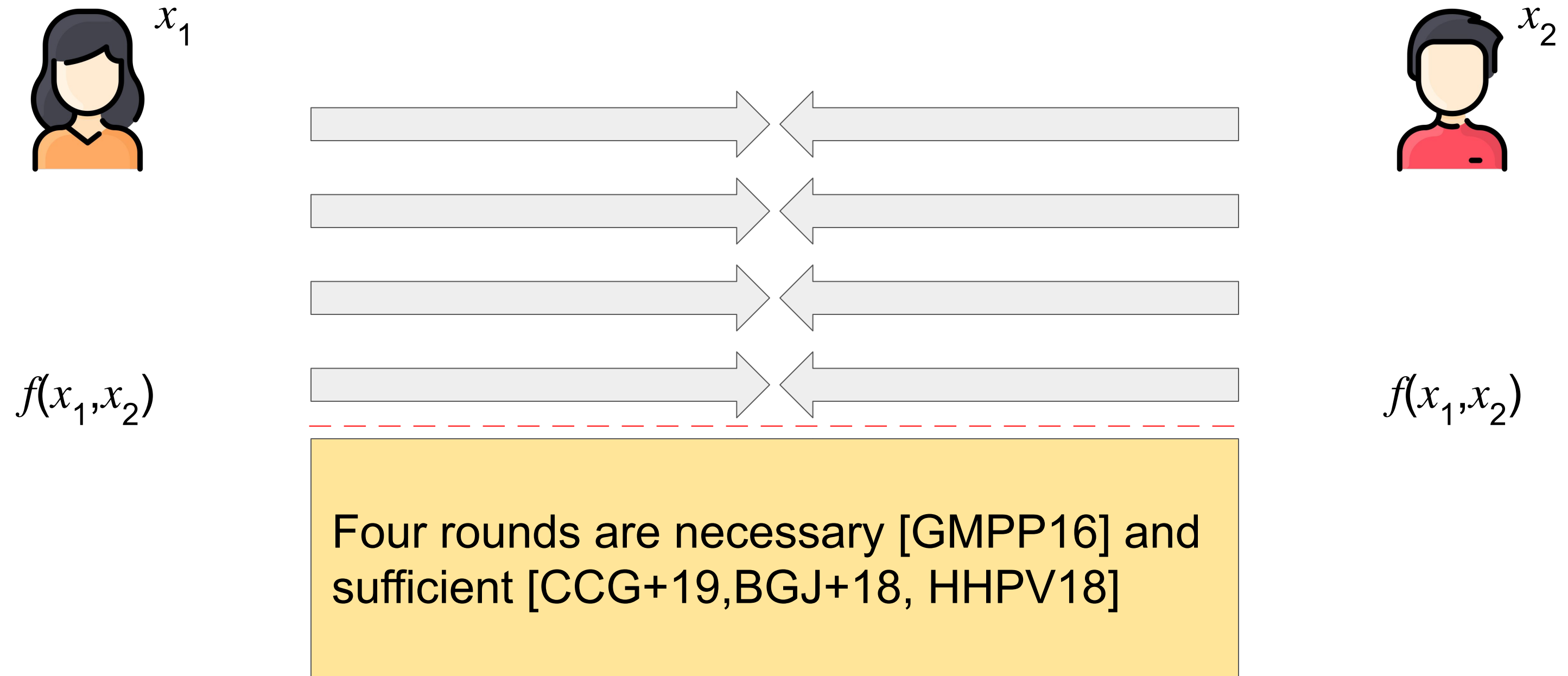
1. Number of Messages

Multiparty Computation (MPC)



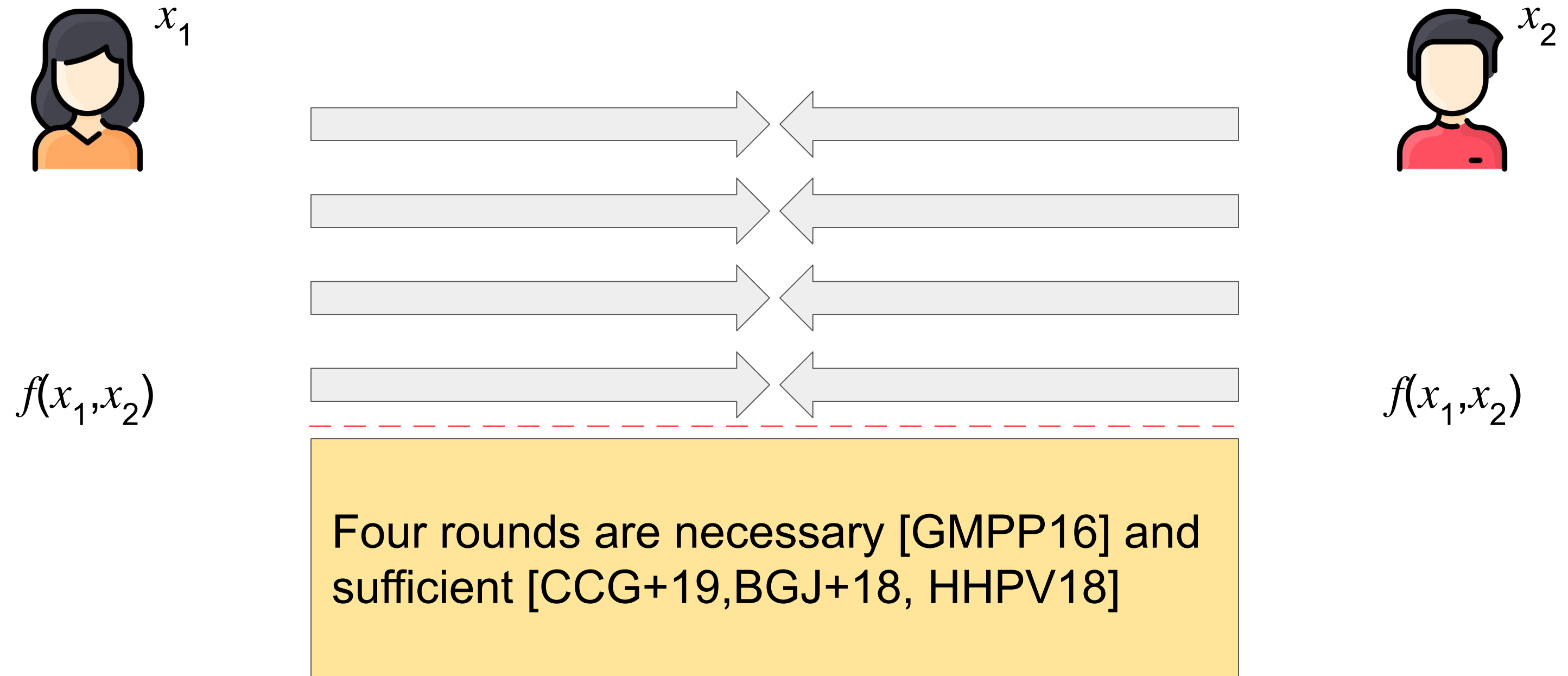
1. Number of Messages
2. Size of Messages

Multiparty Computation (MPC)



1. Number of Messages
2. Size of Messages

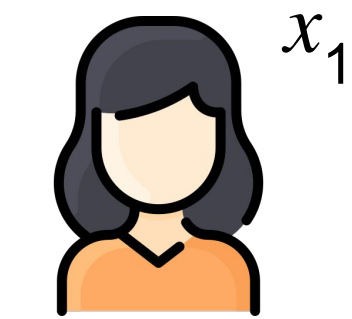
Multiparty Computation (MPC)



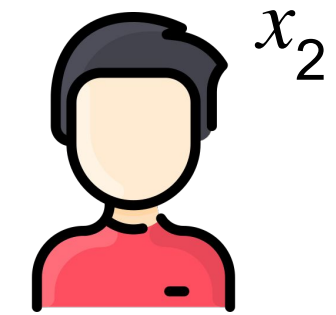
1. Number of Messages ✓

2. Size of Messages

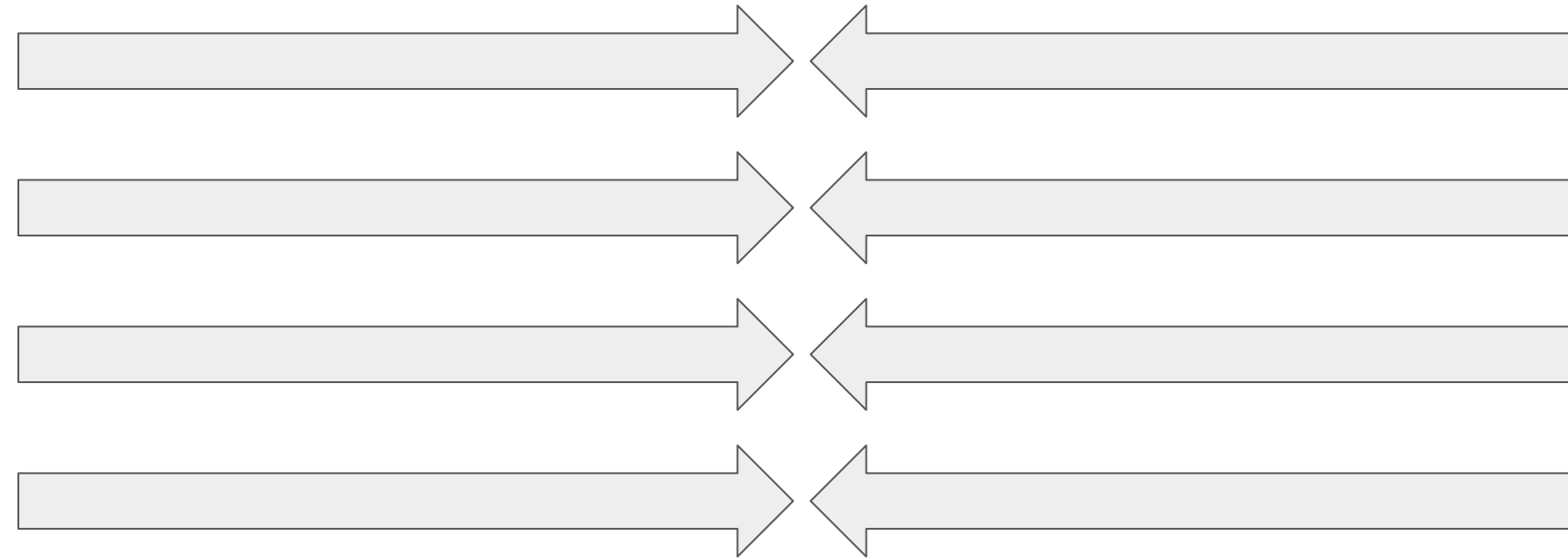
Multiparty Computation (MPC)



x_1



x_2



$f(x_1, x_2)$

$f(x_1, x_2)$

Four rounds are necessary [GMPP16] and sufficient [CCG+19, BGJ+18, HHPV18]

1. Number of Messages ✓

2. Size of Messages ←

Prior Work

Prior Work

	Semi-Honest		
	Work	Assumptions	Comm. Compl.
Round-Optimal Protocols	[BL18] [GS18]	OT	\mathcal{A}

Prior Work

	Semi-Honest			Malicious		
	Work	Assumptions	Comm. Compl.	Work	Assumptions	Comm. Compl.
Round-Optimal Protocols	[BL18] [GS18]	OT	\mathcal{A}	[BGJ+18]	DDH/Q-N Res.	\mathcal{A}

Prior Work

	Semi-Honest			Malicious		
	Work	Assumptions	Comm. Compl.	Work	Assumptions	Comm. Compl.
Round-Optimal Protocols	[BL18] [GS18]	OT	\mathcal{A}	[BGJ+18] [CCG+20]	DDH/Q-N Res. OT	\mathcal{A}

Prior Work

	Semi-Honest			Malicious		
	Work	Assumptions	Comm. Compl.	Work	Assumptions	Comm. Compl.
Round-Optimal Protocols	[BL18] [GS18]	OT	\mathcal{A}	[BGJ+18] [CCG+20]	DDH/Q-N Res. OT	\mathcal{A}
	[ABJ+19] [QWW18]	LWE	depth(f)			
With Improved Comm. Compl.						

Prior Work

	Semi-Honest			Malicious		
	Work	Assumptions	Comm. Compl.	Work	Assumptions	Comm. Compl.
Round-Optimal Protocols	[BL18] [GS18]	OT	\mathcal{A}	[BGJ+18] [CCG+20]	DDH/Q-N Res. OT	\mathcal{A}
	[ABJ+19] [QWW18]	LWE	depth(f)			
With Improved Comm. Compl.	[AJJM20]	R-LWE, DSPR & OT	L_{in} & L_{out}			

Prior Work

	Semi-Honest			Malicious		
	Work	Assumptions	Comm. Compl.	Work	Assumptions	Comm. Compl.
Round-Optimal Protocols	[BL18] [GS18]	OT	\mathcal{A}	[BGJ+18] [CCG+20]	DDH/Q-N Res. OT	\mathcal{A}
	[ABJ+19] [QWW18]	LWE	depth(f)	This Work		
With Improved Comm. Compl.	[AJJM20]	R-LWE, DSPR & OT	L_{in} & L_{out}			

Prior Work

	Semi-Honest			Malicious		
	Work	Assumptions	Comm. Compl.	Work	Assumptions	Comm. Compl.
Round-Optimal Protocols	[BL18] [GS18]	OT	\mathcal{A}	[BGJ+18] [CCG+20]	DDH/Q-N Res. OT	\mathcal{A}
	[ABJ+19] [QWW18]	LWE	$\text{depth}(f)$	This Work	LWE	$\text{depth}(f)$
With Improved Comm. Compl.	[AJJM20]	R-LWE, DSPR & OT	L_{in} & L_{out}			

Prior Work

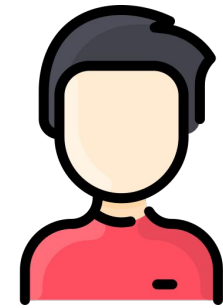
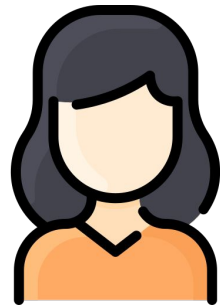
	Semi-Honest			Malicious		
	Work	Assumptions	Comm. Compl.	Work	Assumptions	Comm. Compl.
Round-Optimal Protocols	[BL18] [GS18]	OT	\mathcal{A}	[BGJ+18] [CCG+20]	DDH/Q-N Res. OT	\mathcal{A}
	[ABJ+19] [QWW18]	LWE	depth(f)	This Work	LWE	depth(f)
With Improved Comm. Compl.	[AJJM20]	R-LWE, DSPR & OT	L_{in} & L_{out}		R-LWE, DSPR & OT	L_{in} & L_{out}

Prior Work

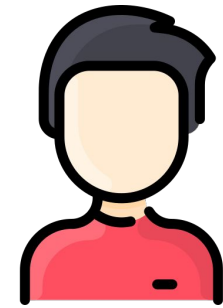
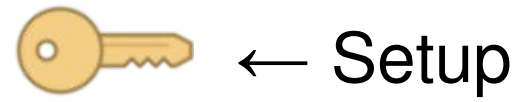
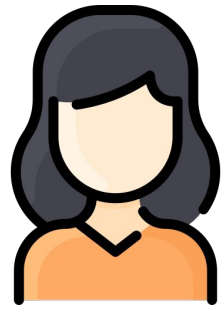
	Semi-Honest			Malicious		
	Work	Assumptions	Comm. Compl.	Work	Assumptions	Comm. Compl.
Round-Optimal Protocols	[BL18] [GS18]	OT	\mathcal{A}	[BGJ+18] [CCG+20]	DDH/Q-N Res. OT	\mathcal{A}
	[ABJ+19] [QWW18]	LWE	depth(f)	This Work	LWE	depth(f)
With Improved Comm. Compl.	[AJJM20]	R-LWE, DSPR & OT	L_{in} & L_{out}		R-LWE, DSPR & OT	L_{in} & L_{out}

⇒ Start from the work of Ananth et al. and Functional Encryption Combiners

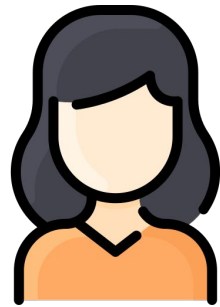
Functional Encryption [BSW11]




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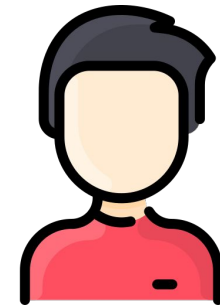


Functional Encryption [BSW11]

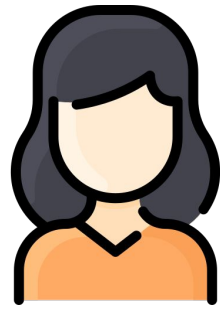



 \leftarrow Setup

 \leftarrow Keygen(, f)

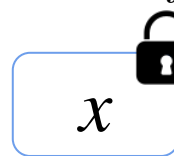



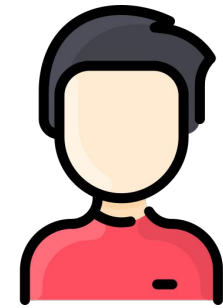
Functional Encryption [BSW11]



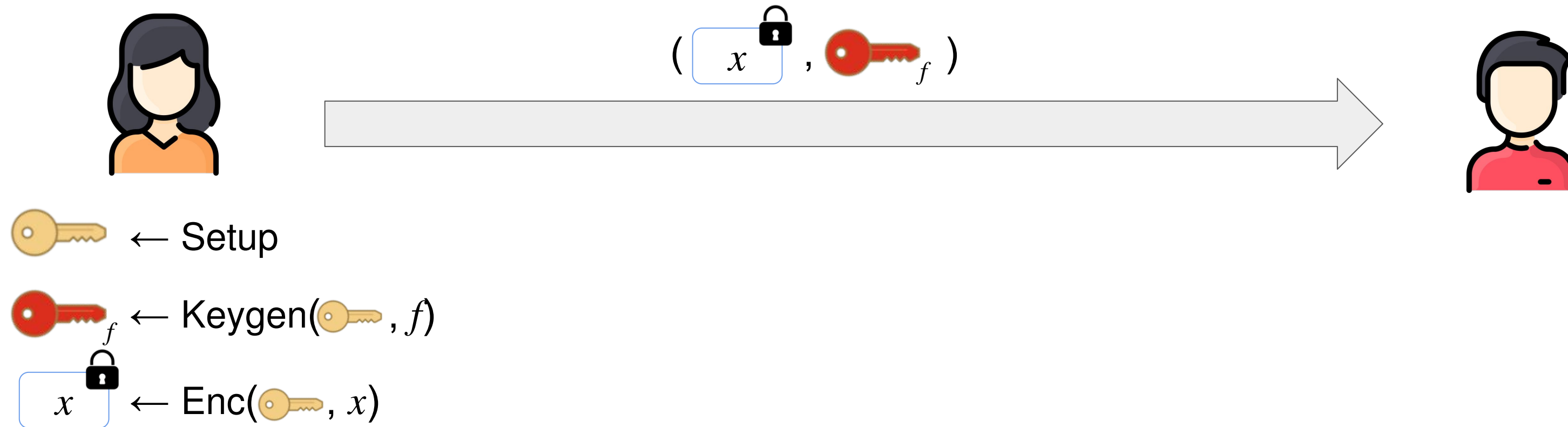
 \leftarrow Setup

 \leftarrow Keygen(, f)

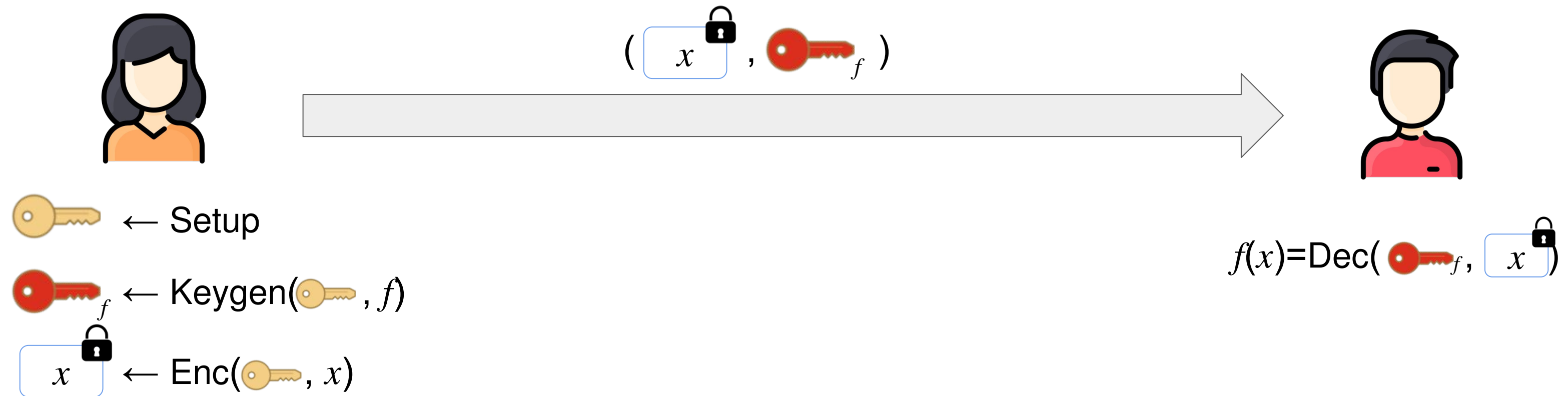
 \leftarrow Enc(, x)



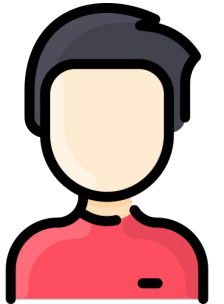
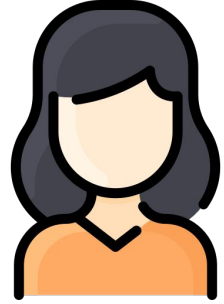
Functional Encryption [BSW11]



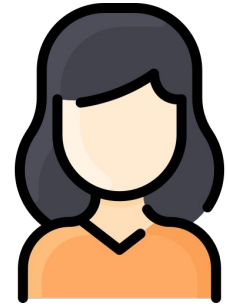
Functional Encryption [BSW11]




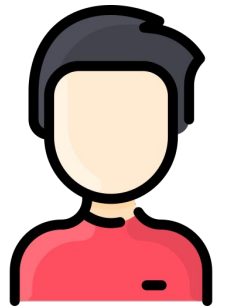
(Decomposable) Functional Encryption Combiner [ABJ+19]




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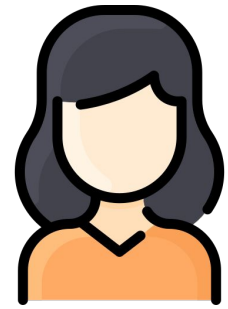



₁ ← Setup



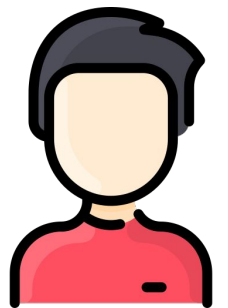
₂ ← Setup


(Decomposable) Functional Encryption Combiner [ABJ+19]



₁ ← Setup

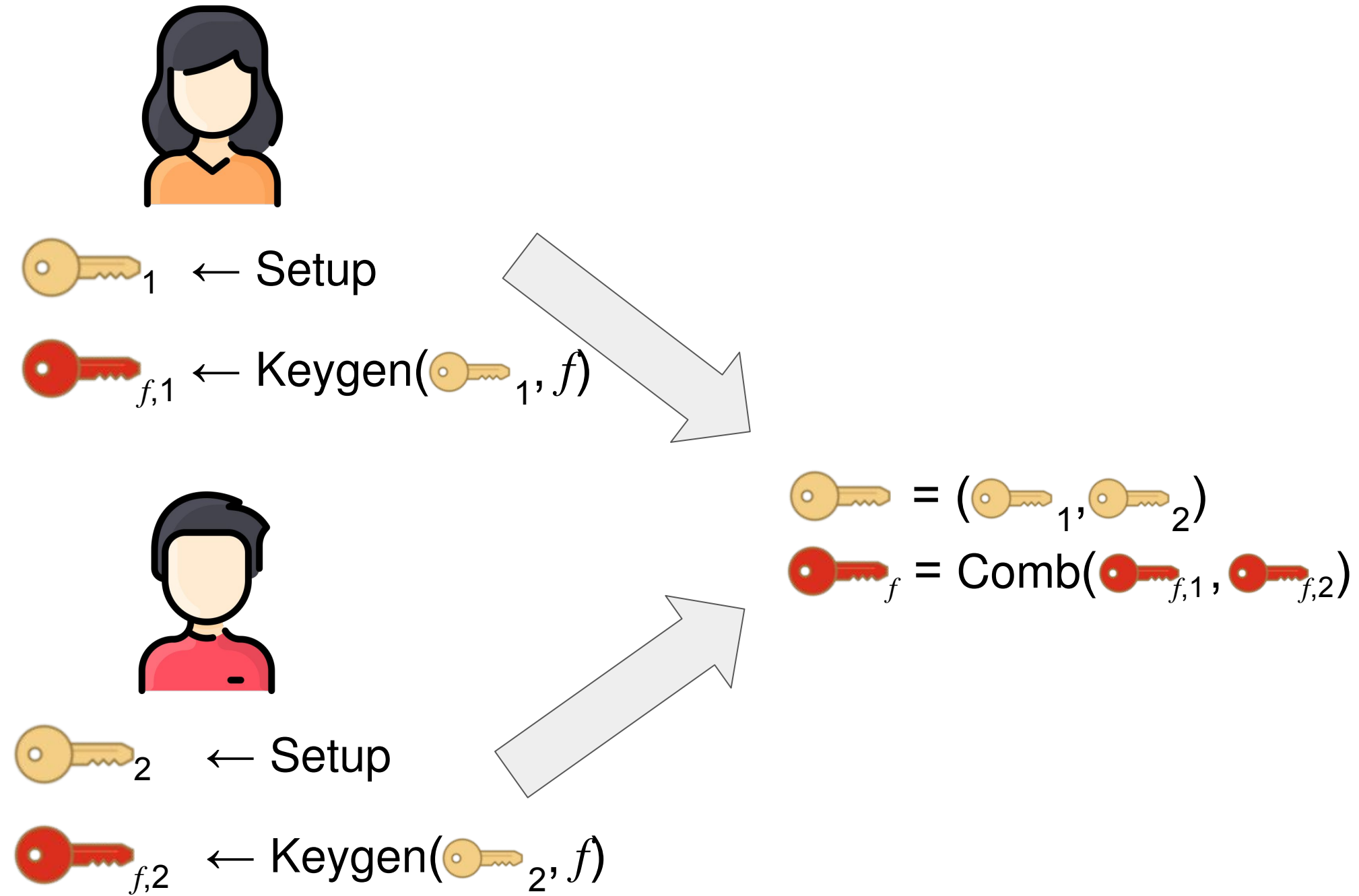
_{*f*,1} ← Keygen(₁, *f*)



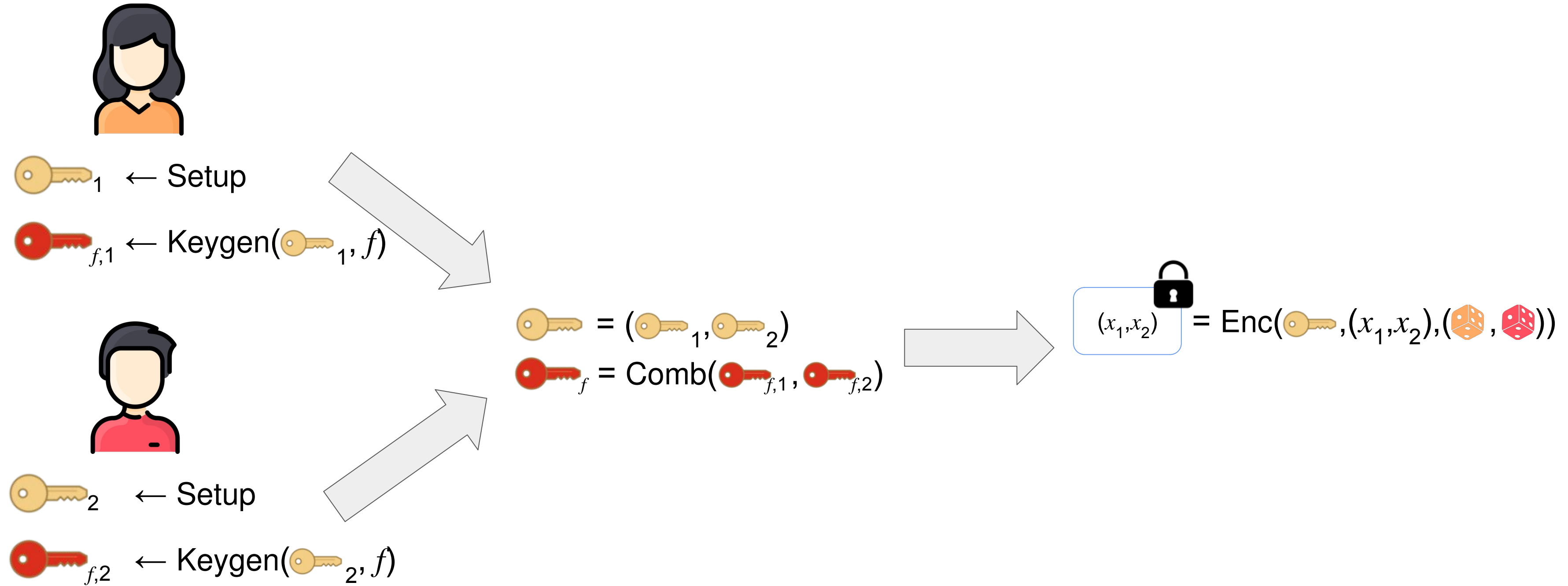
₂ ← Setup

_{*f*,2} ← Keygen(₂, *f*)

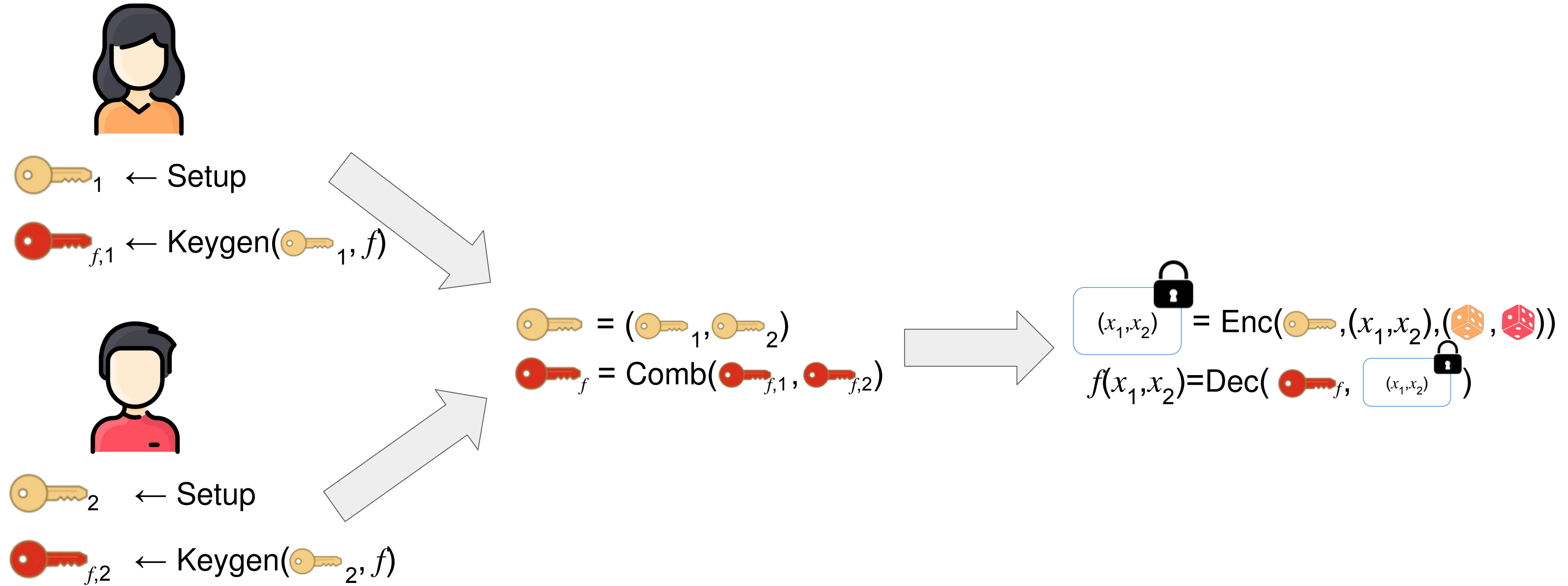
(Decomposable) Functional Encryption Combiner [ABJ+19]



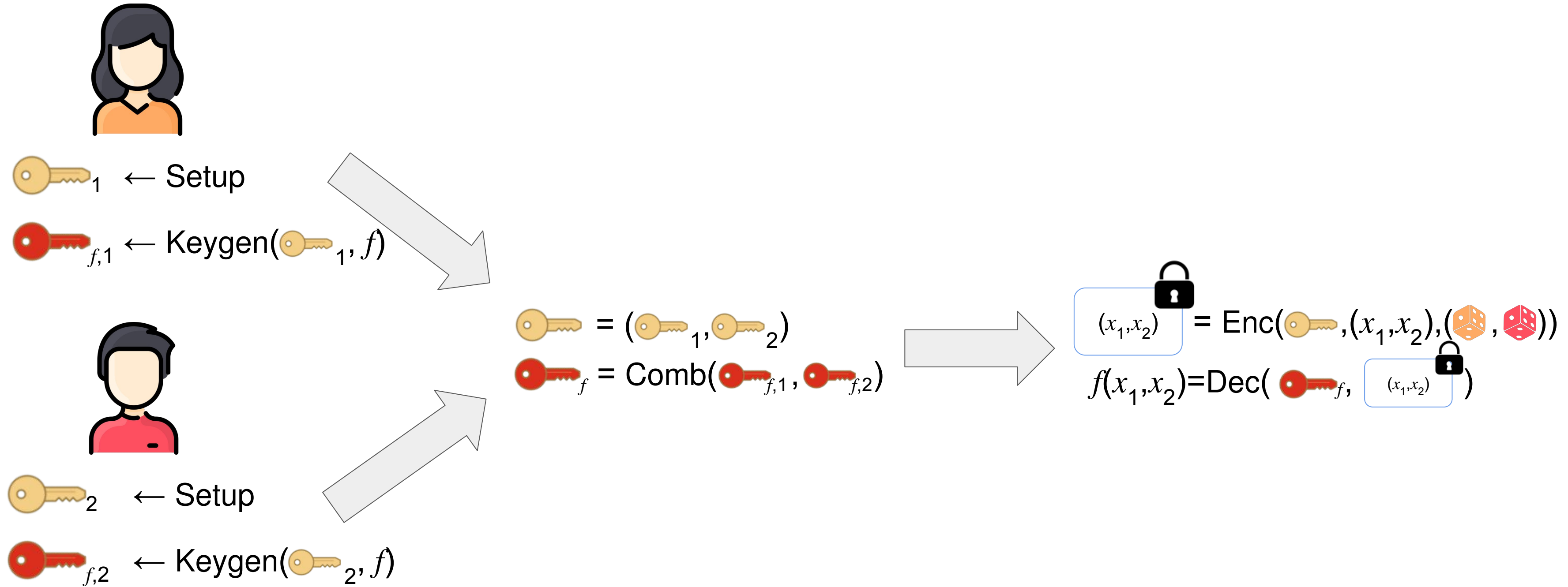
(Decomposable) Functional Encryption Combiner [ABJ+19]



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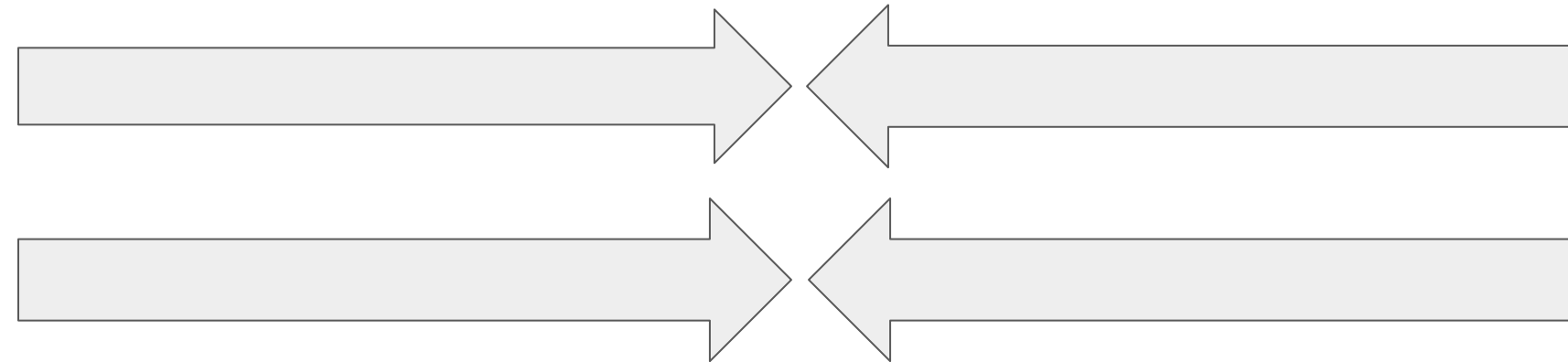
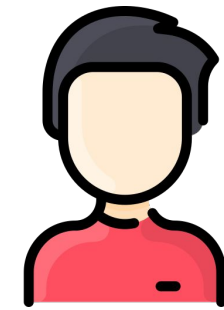
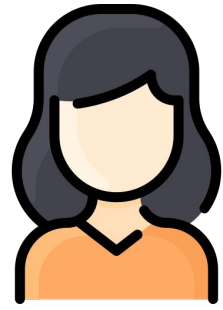
(Decomposable) Functional Encryption Combiner [ABJ+19]



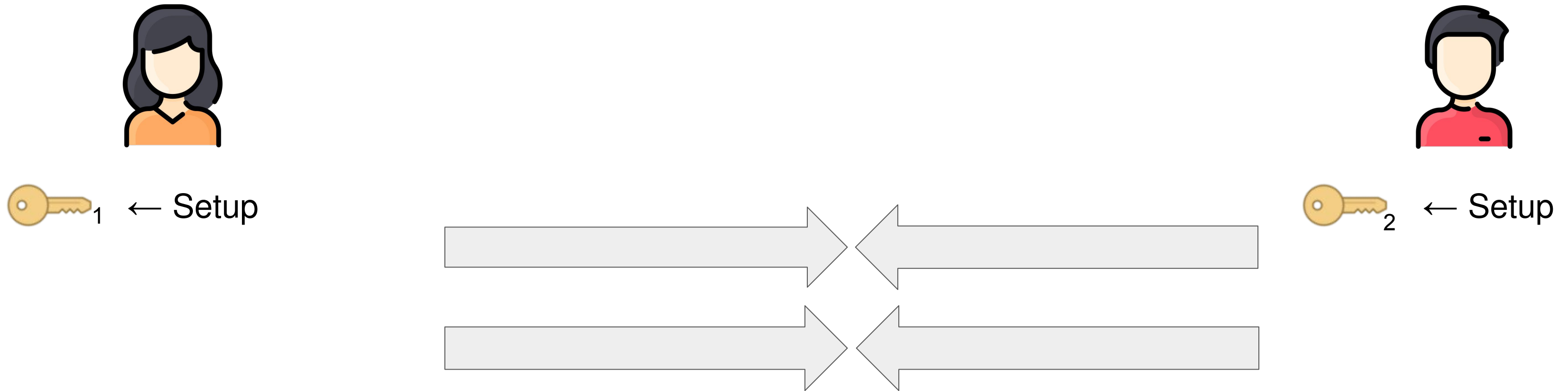
Succinctness: $|\text{key}_{f,i}| \leq \text{depth}(f)$

Protocol of Ananth et al. [ABJ+19]

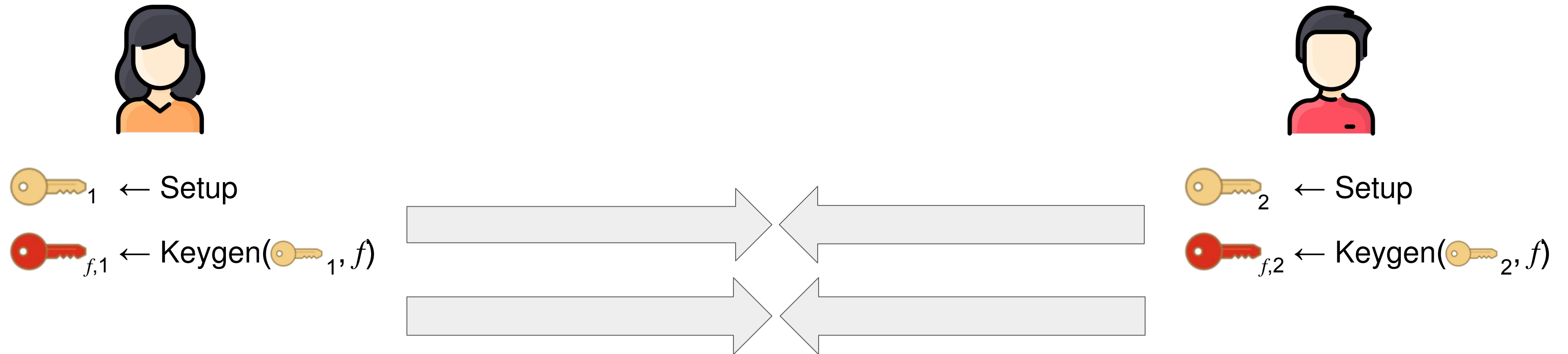
Protocol of Ananth et al. [ABJ+19]



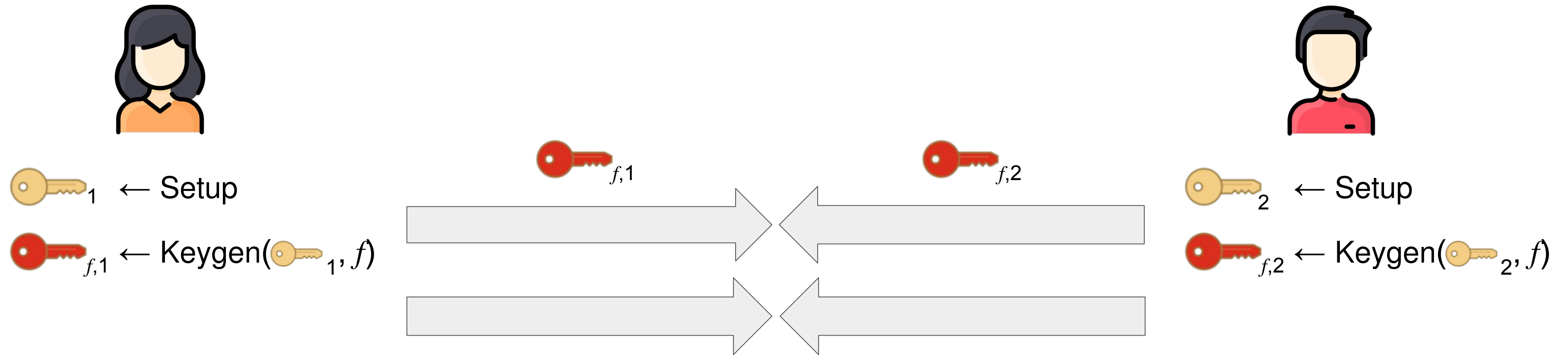
Protocol of Ananth et al. [ABJ+19]



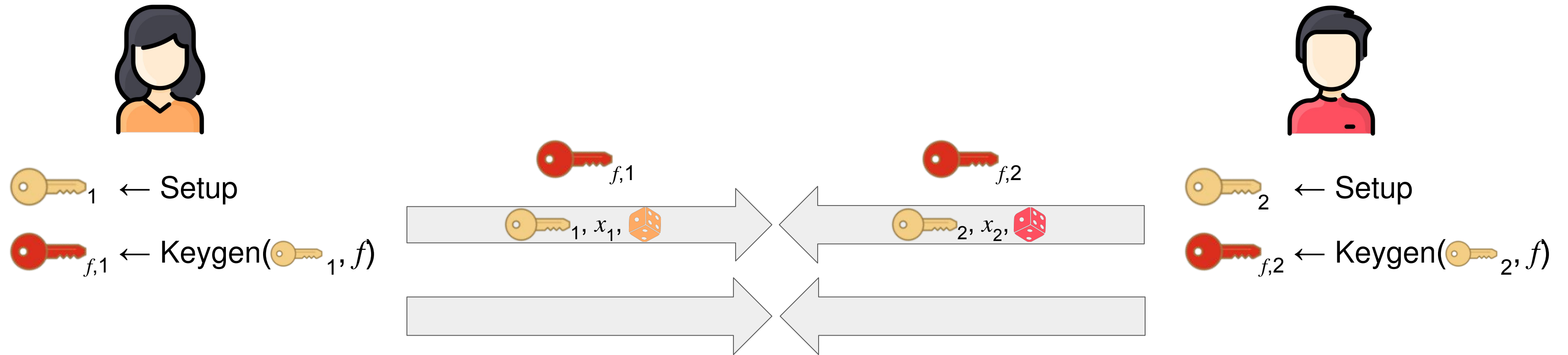
Protocol of Ananth et al. [ABJ+19]



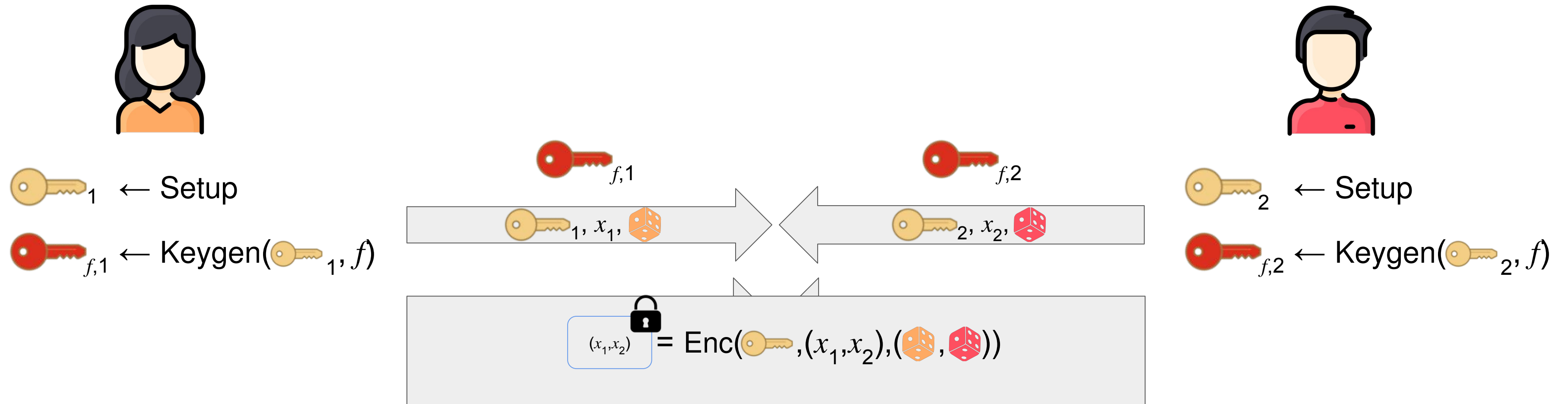
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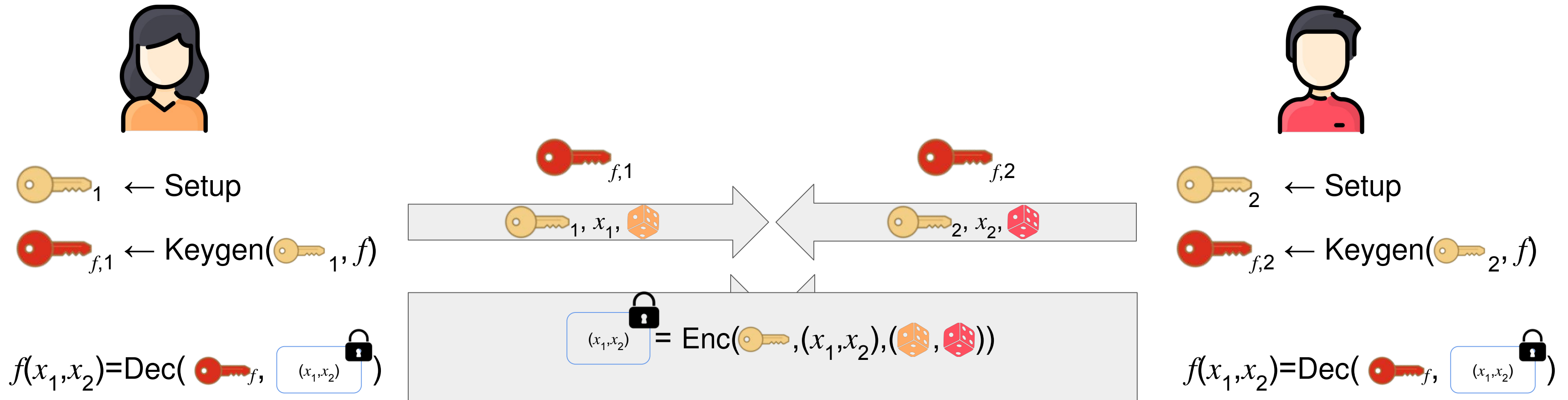
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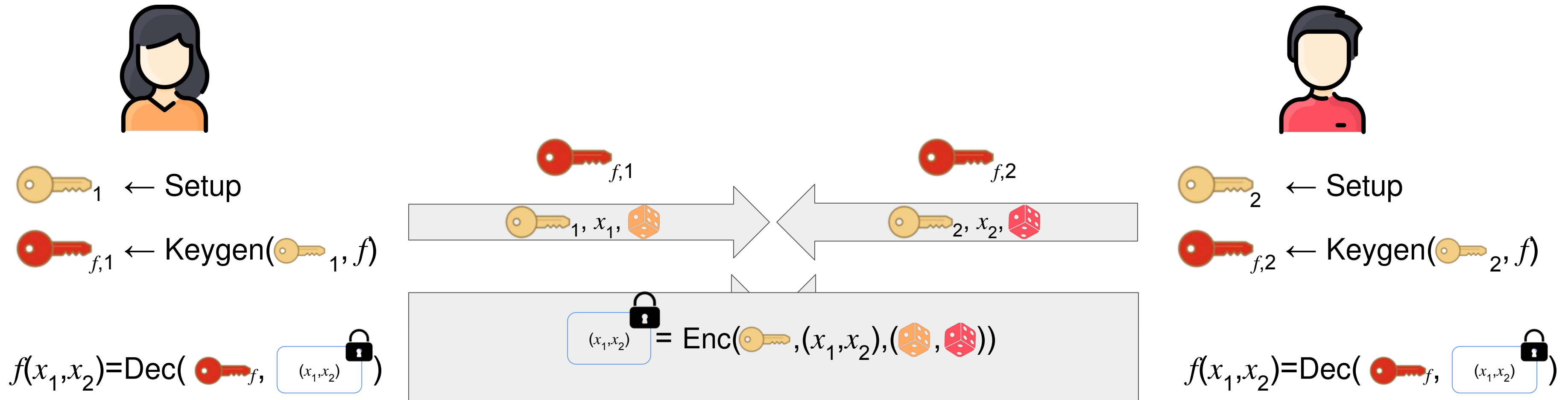
Protocol of Ananth et al. [ABJ+19]



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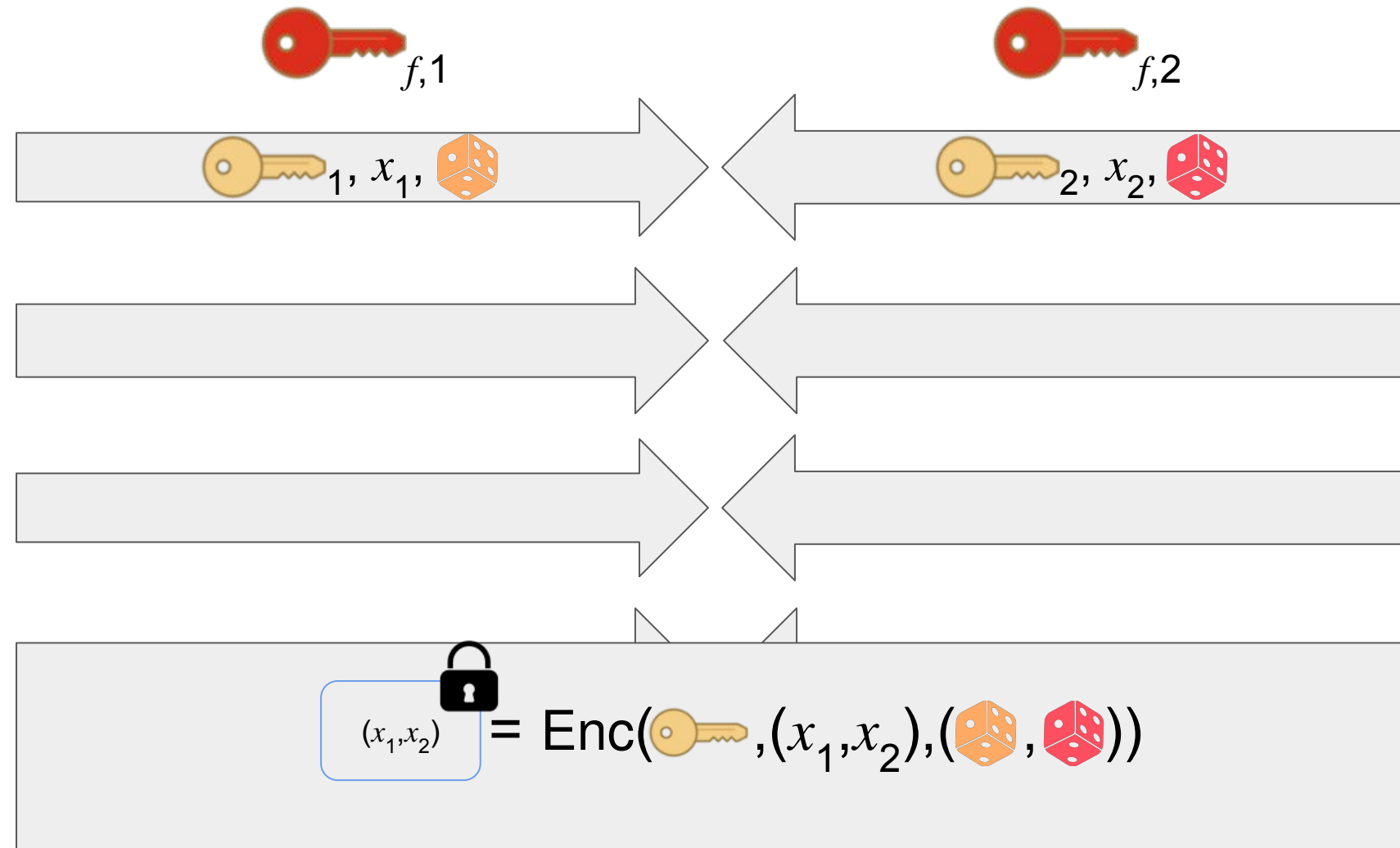
Protocol of Ananth et al. [ABJ+19]





⇒ Replace semi-honest protocol with maliciously secure protocol

First Approach

 $_1 \leftarrow \text{Setup}$
 $_{f,1} \leftarrow \text{Keygen}(\text{key}_1, f)$

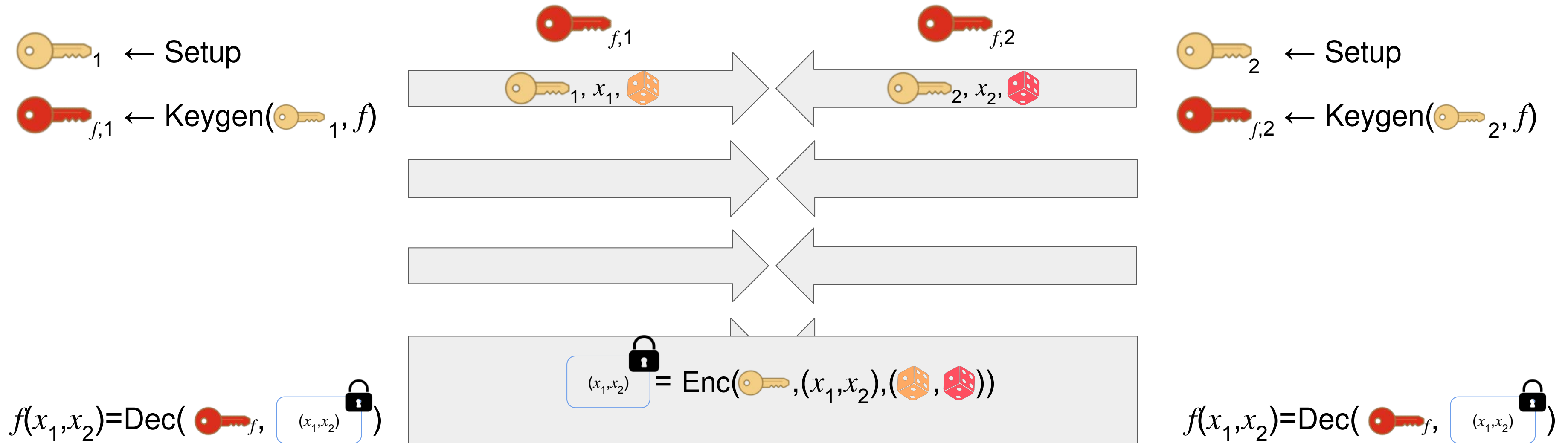


$f(x_1, x_2) = \text{Dec}(\text{key}_f, (x_1, x_2))$

 $_2 \leftarrow \text{Setup}$
 $_{f,2} \leftarrow \text{Keygen}(\text{key}_2, f)$

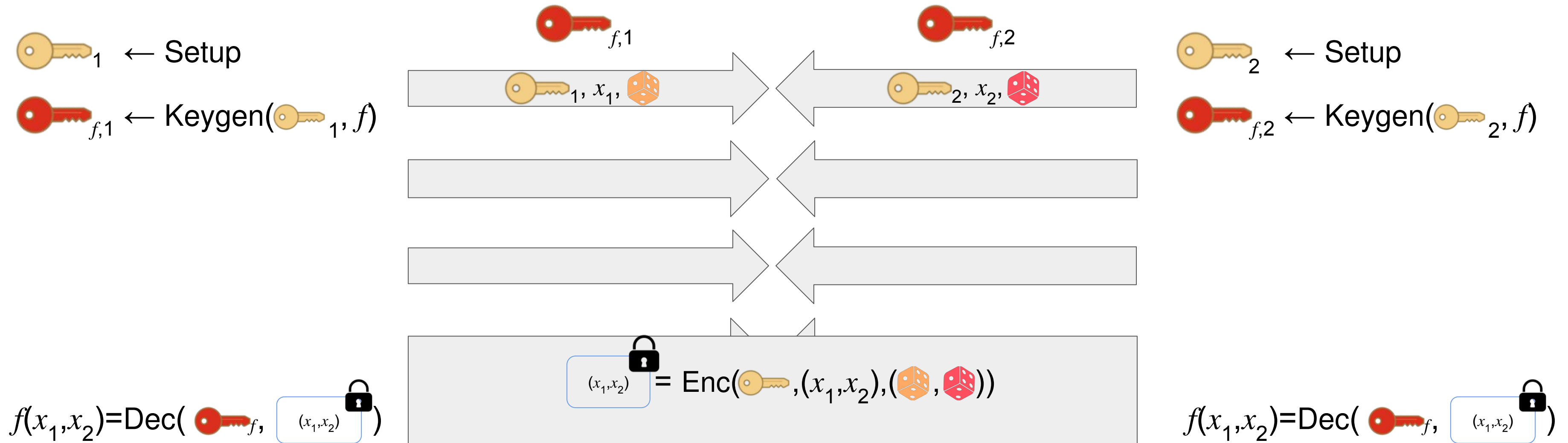
$f(x_1, x_2) = \text{Dec}(\text{key}_f, (x_1, x_2))$

First Approach



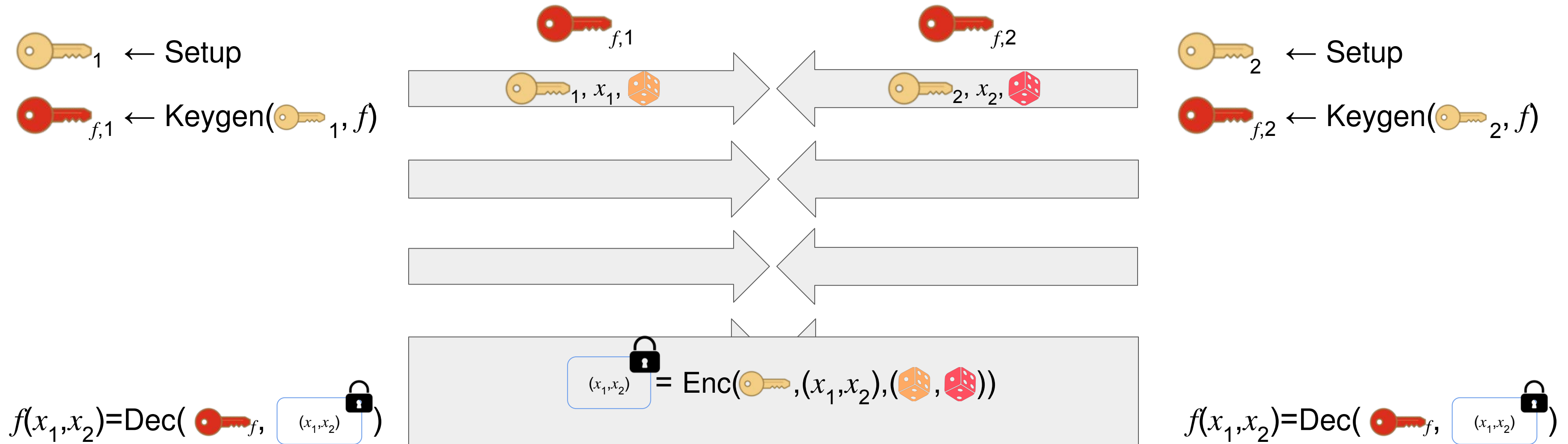
1. $k_{f,i}$ can be generated maliciously

First Approach



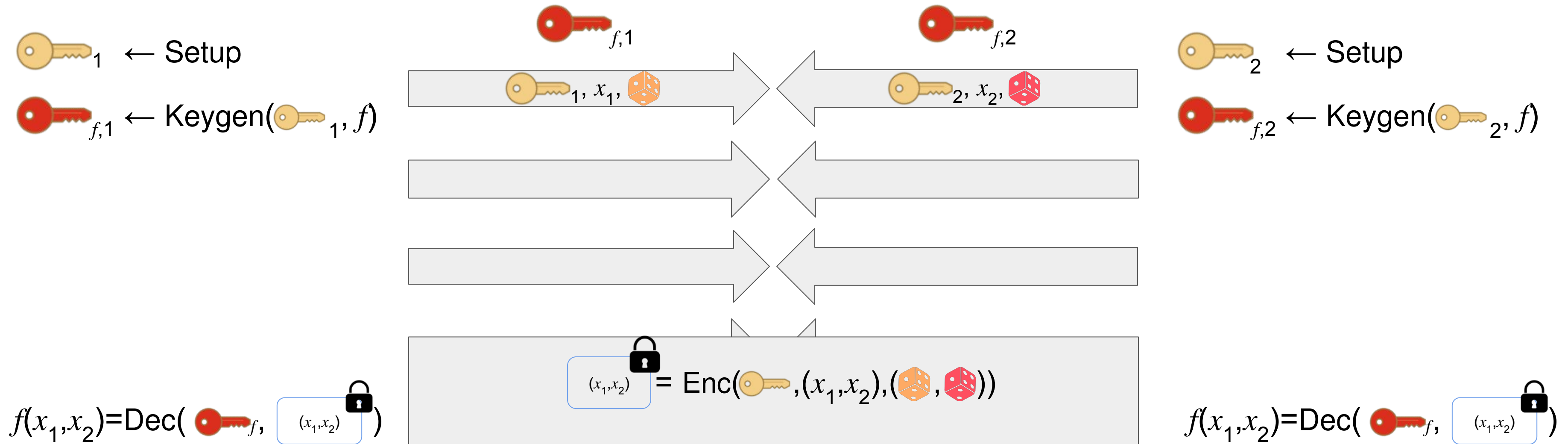
1. $\text{key}_{f,i}$ can be generated maliciously
2. $(\text{dice}_1, \text{dice}_2)$ used for encryption can be "bad"

First Approach



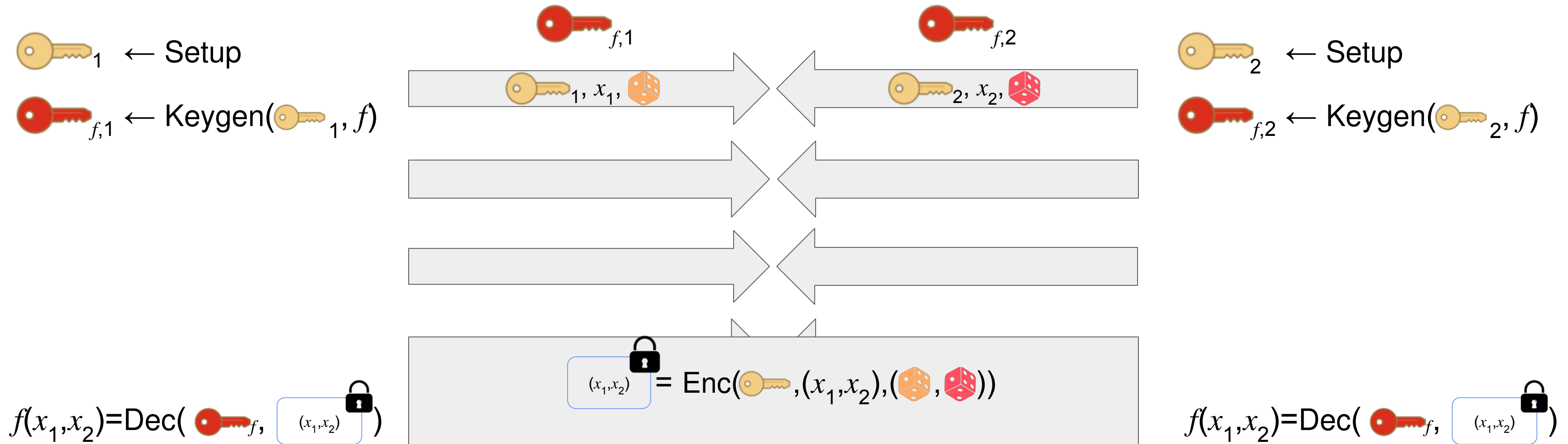
1. $k_{f,i}$ can be generated maliciously
2. $(\text{die}_1, \text{die}_2)$ used for encryption can be "bad"
3. k_i can be generated arbitrarily

First Approach



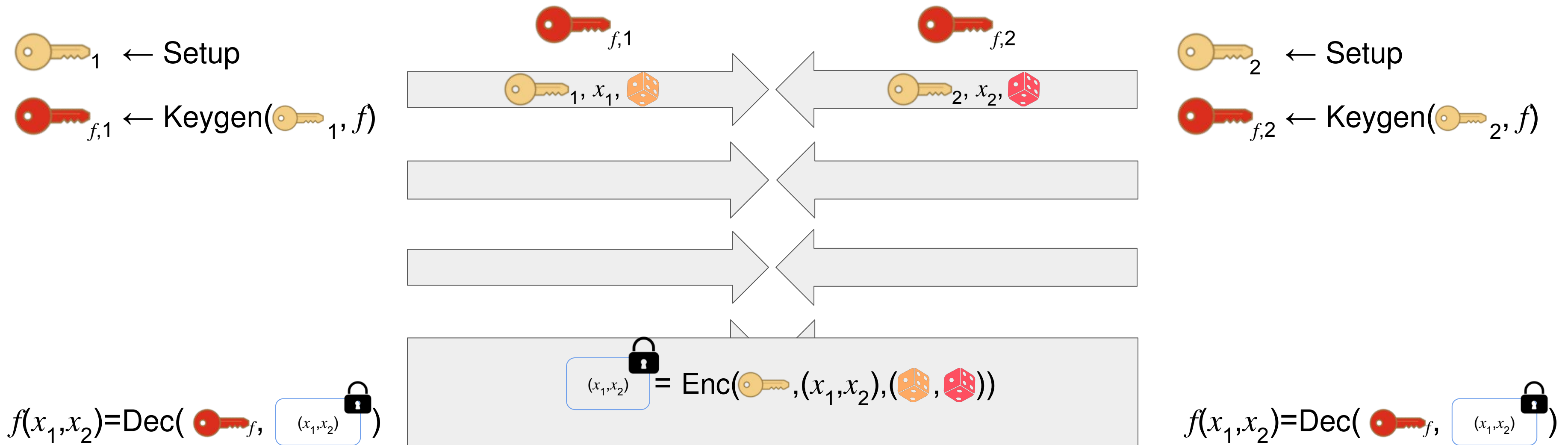
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First Approach



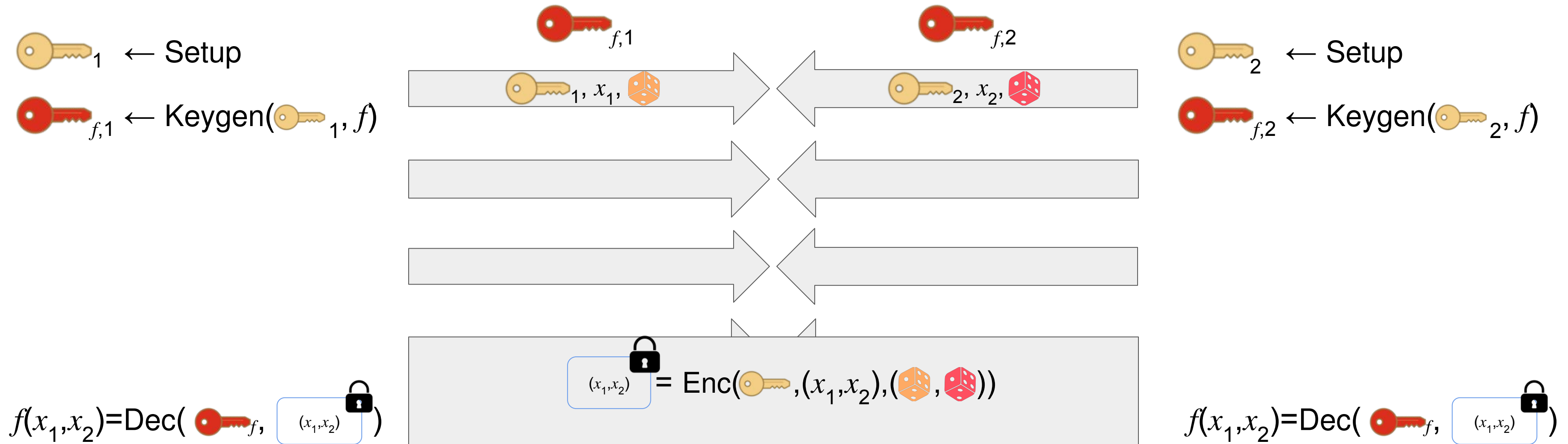
1. $\text{key}_{f,i}$ can be generated maliciously
 → Privacy with Knowledge of Outputs

First Approach



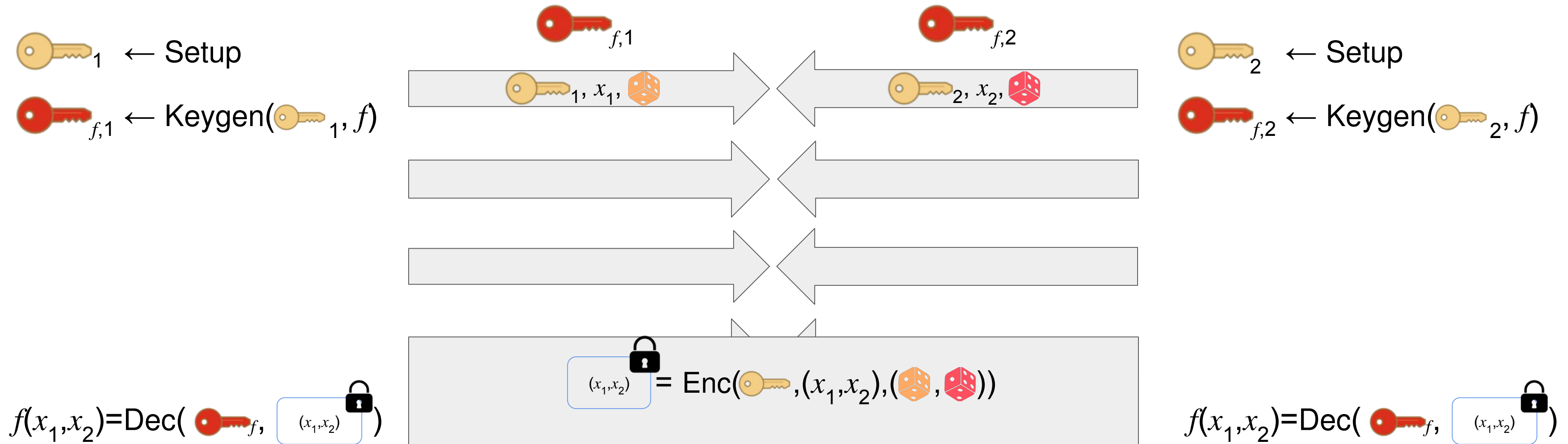
1. $k_{f,i}$ can be generated maliciously
 → Privacy with Knowledge of Outputs
 → Can be lifted using [IKP10, PC12]

First Approach



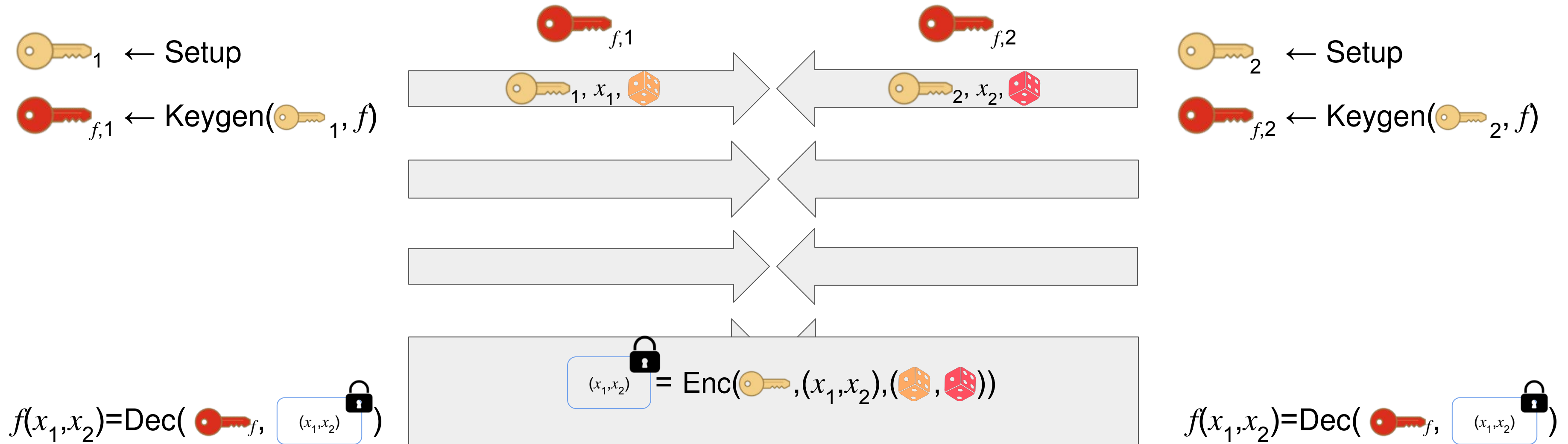
1. $k_{f,i}$ can be generated maliciously ✓
2. (orange die, red die) used for encryption can be "bad"
3. k_i can be generated arbitrarily

First Approach



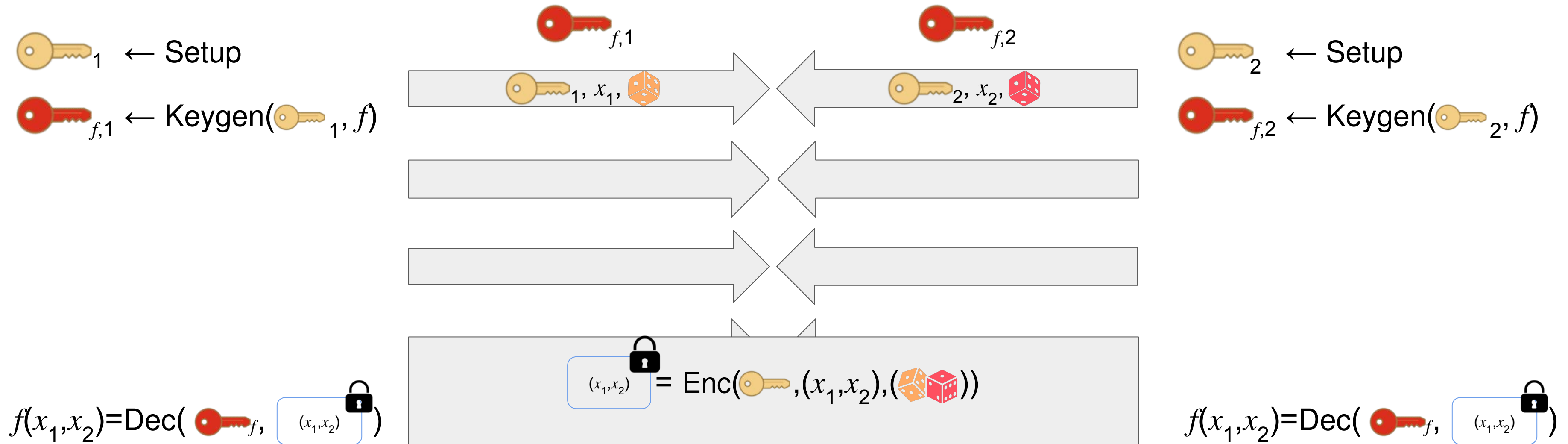
2. (orange die, red die) used for encryption can be “bad”

First Approach



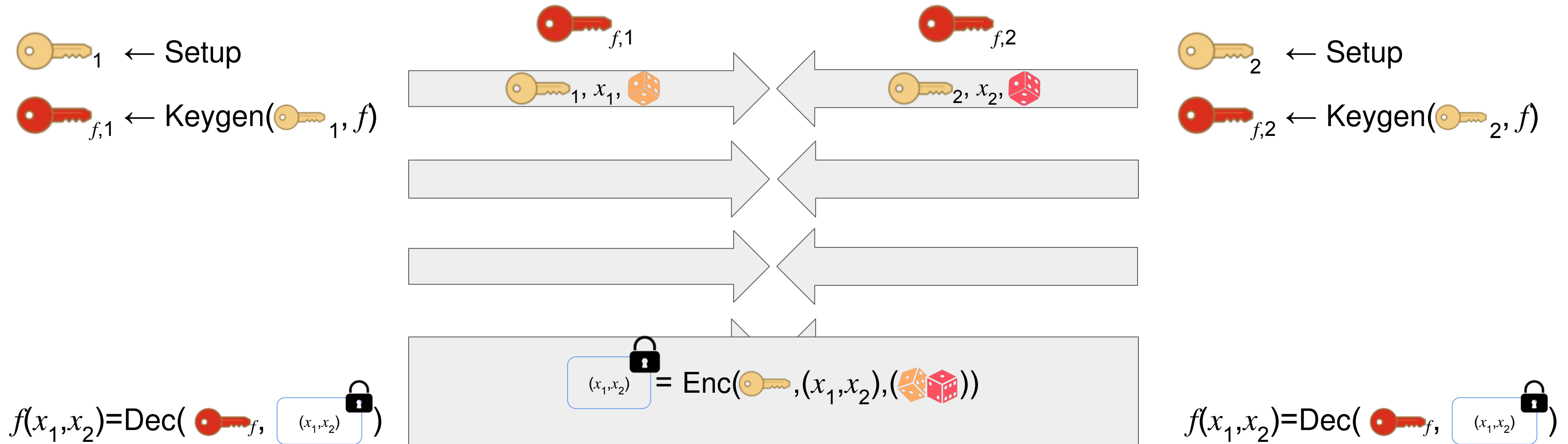
2. (orange die, red die) used for encryption can be “bad”
 → Use XOR instead of concatenation

First Approach



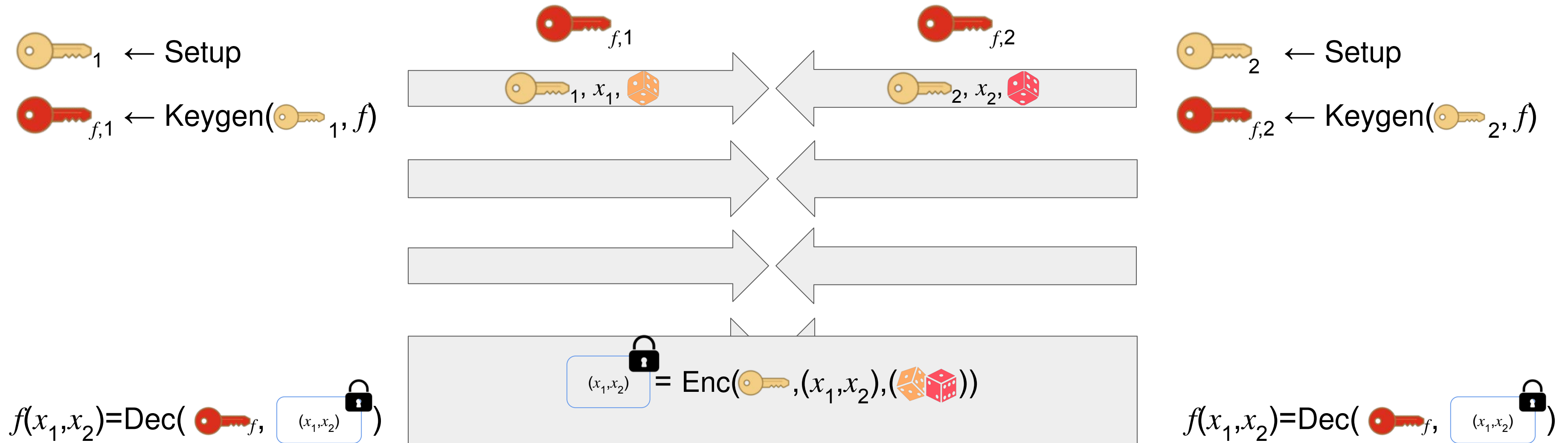
2. (🎲, 🎲) used for encryption can be “bad”
 → Use XOR instead of concatenation

First Approach



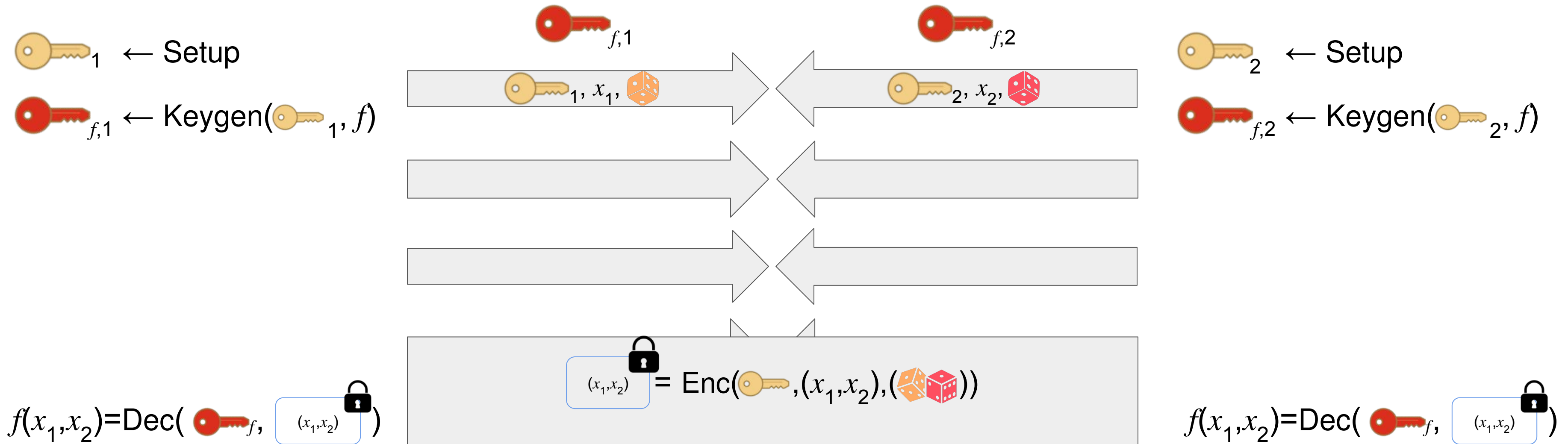
1. $k_{f,i}$ can be generated maliciously ✓
2. $(\text{die}_1, \text{die}_2)$ used for encryption can be “bad” ✓
3. k_i can be generated arbitrarily

First Approach



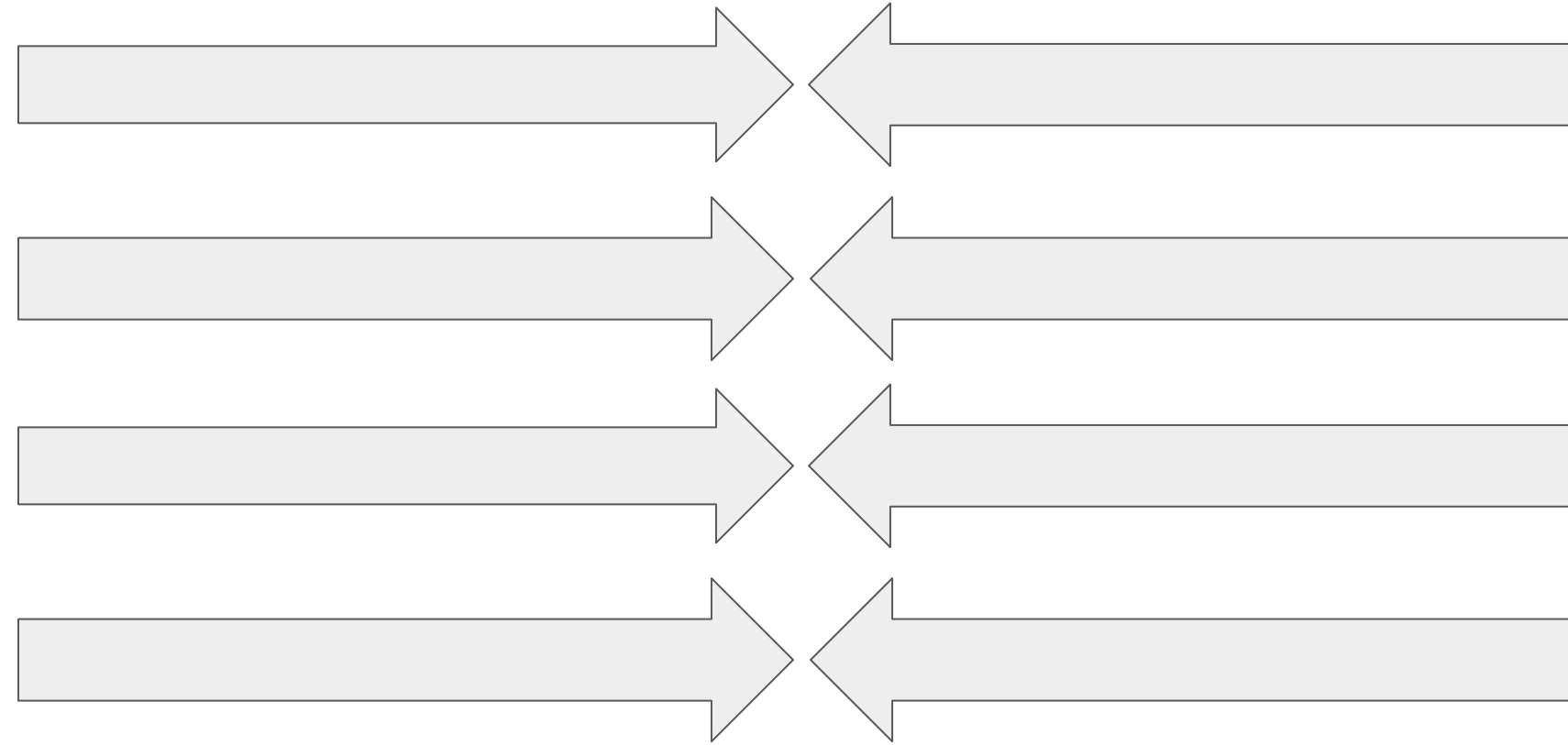
3. k_i can be generated arbitrarily

First Approach



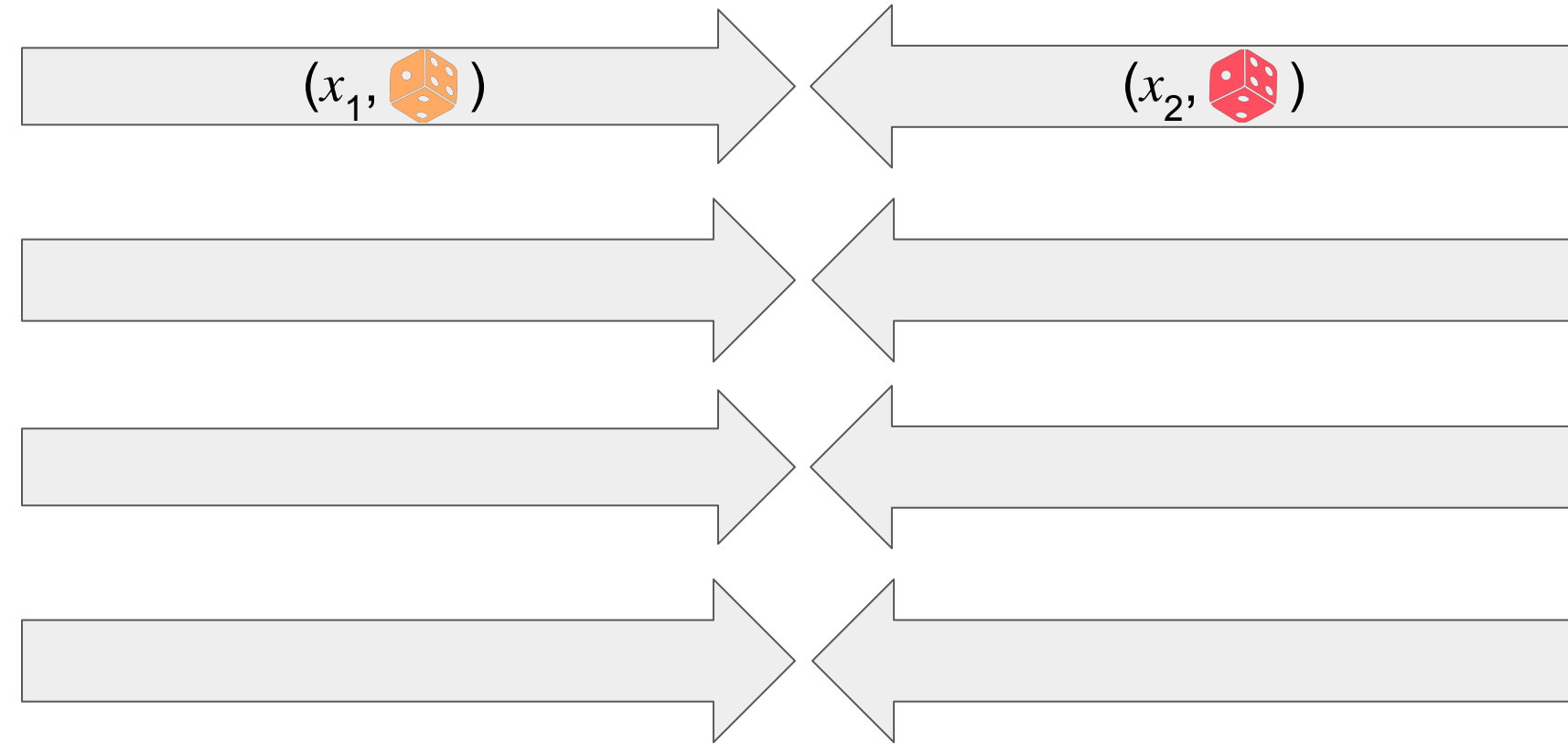
3. key_i can be generated arbitrarily
 → Solve as 2.

First Approach



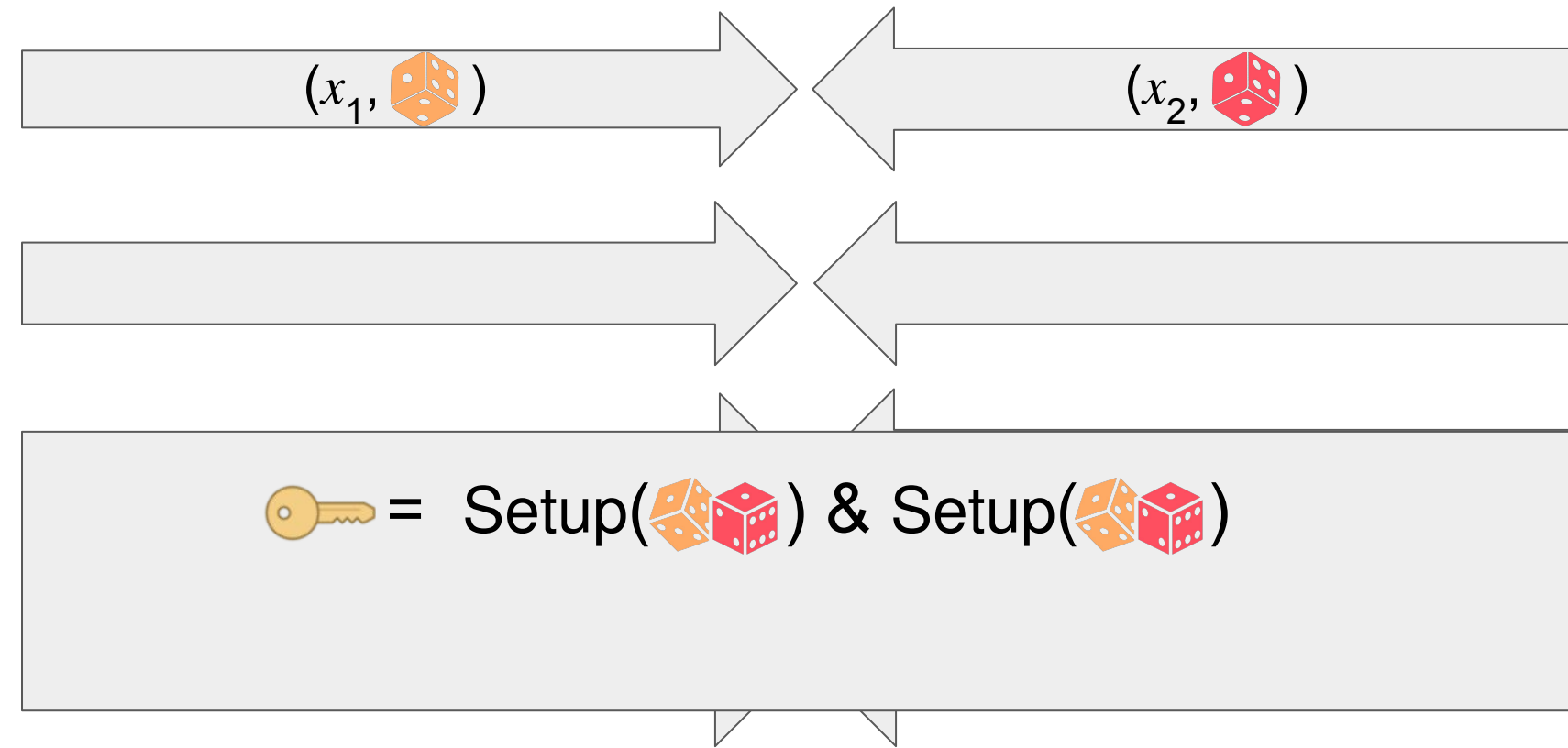
3. 🗝️_{*i*} can be generated arbitrarily
→ Solve as 2.

First Approach



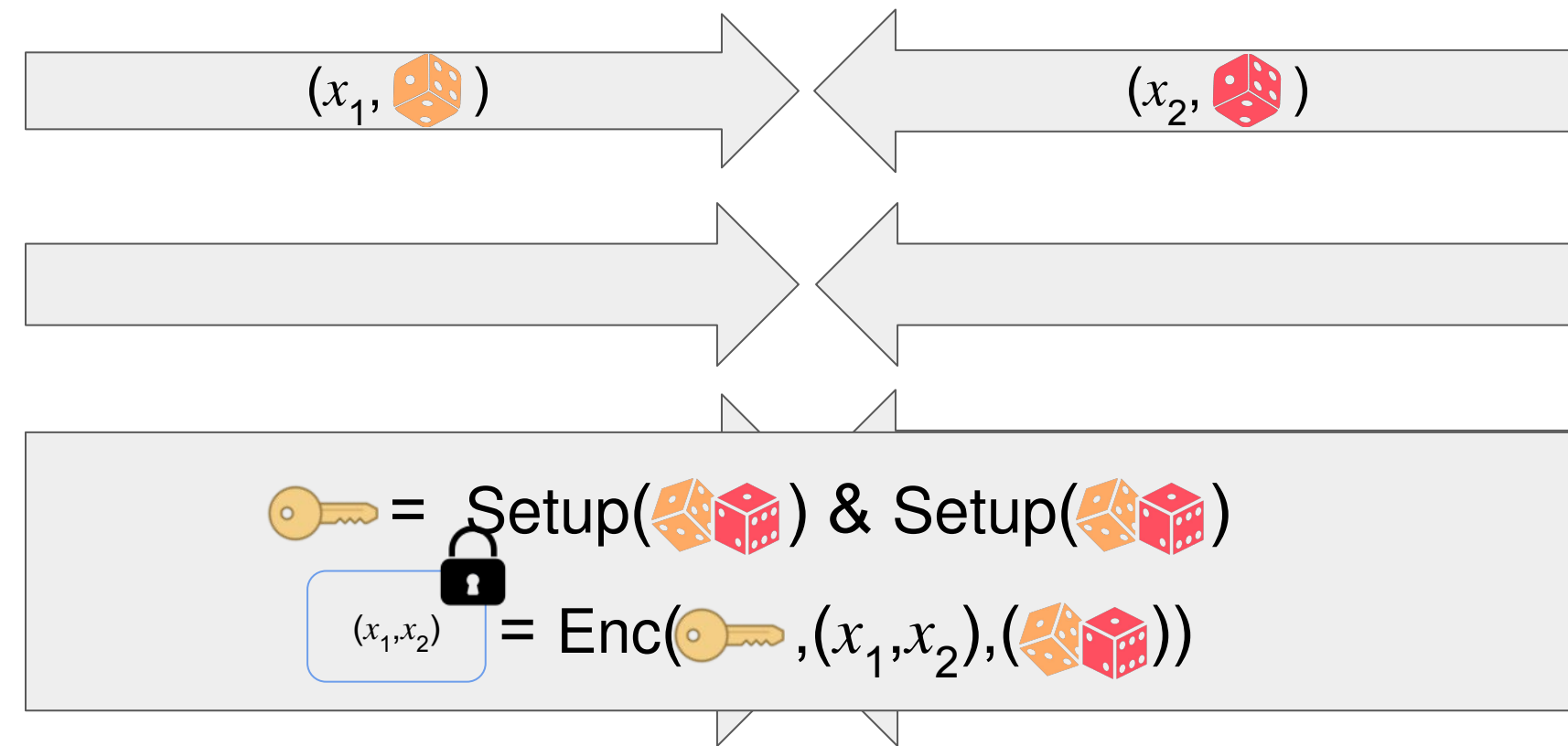
3. 🗝️_{*i*} can be generated arbitrarily
→ Solve as 2.

First Approach



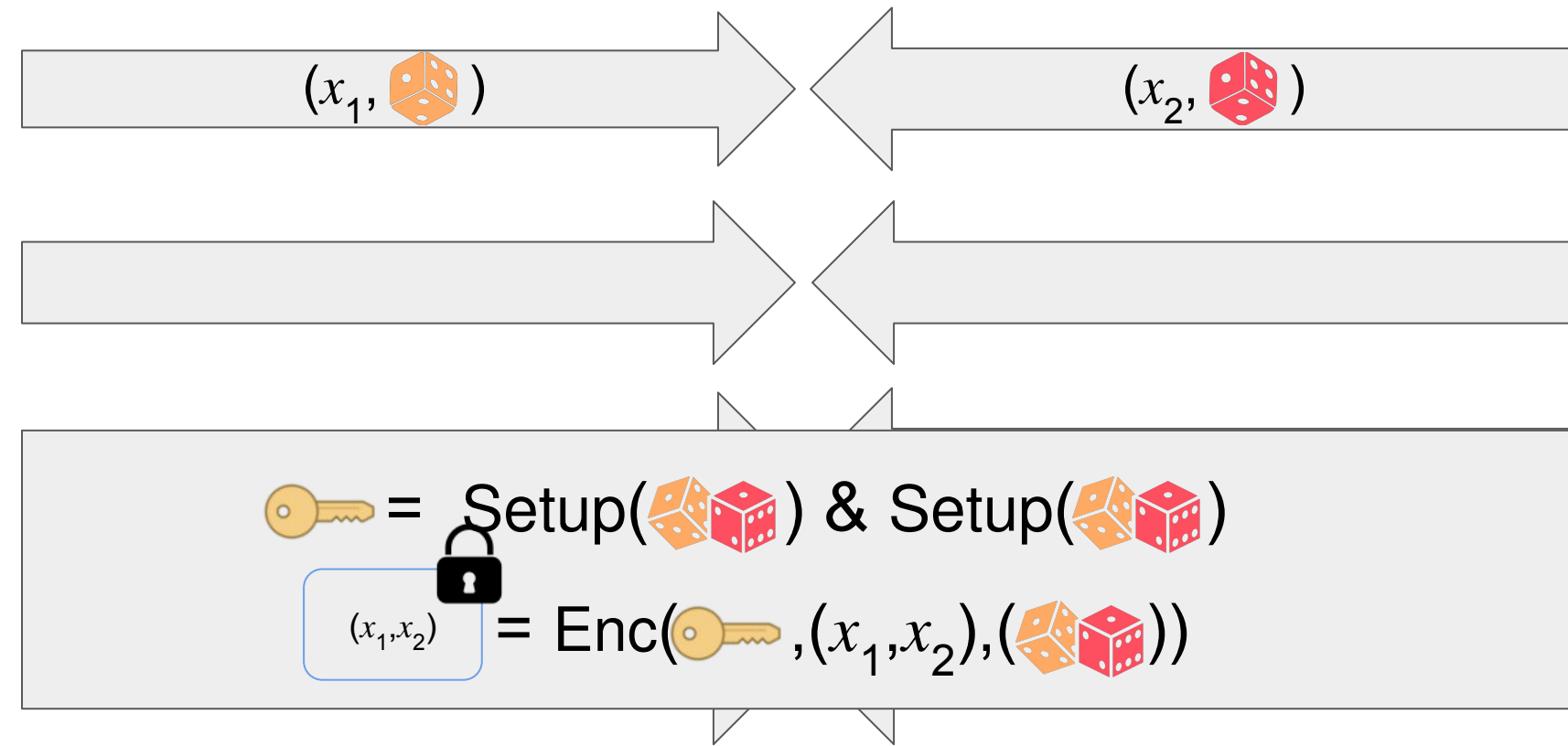
3. 🗝️_i can be generated arbitrarily
→ Solve as 2.

First Approach



3. key_i can be generated arbitrarily
→ Solve as 2.

First Approach

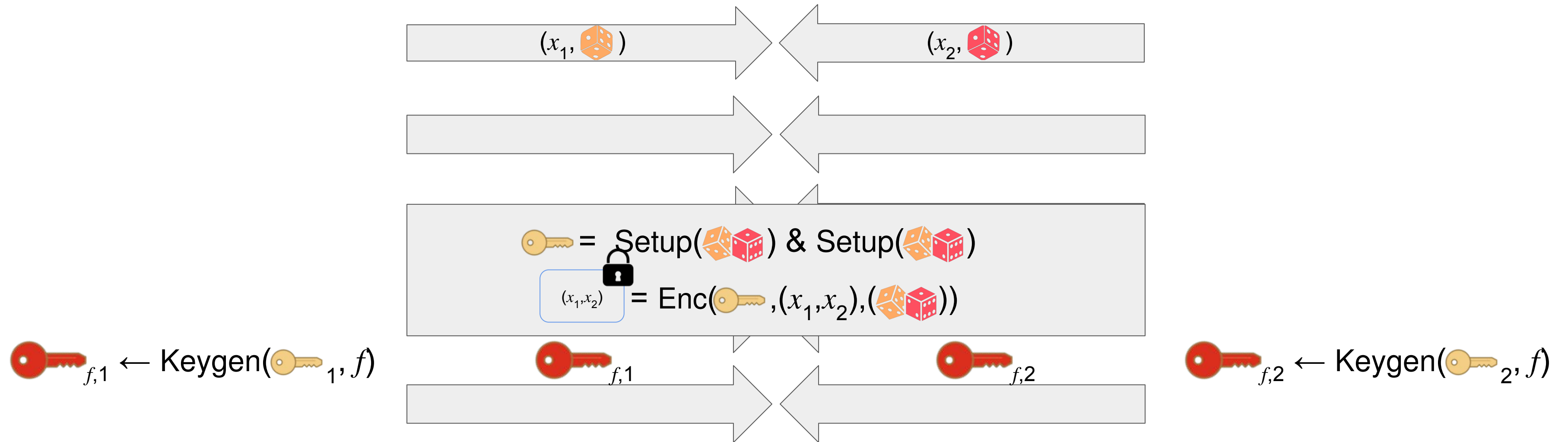


$$\text{key}_{f,1} \leftarrow \text{Keygen}(\text{key}_1, f)$$

$$\text{key}_{f,2} \leftarrow \text{Keygen}(\text{key}_2, f)$$

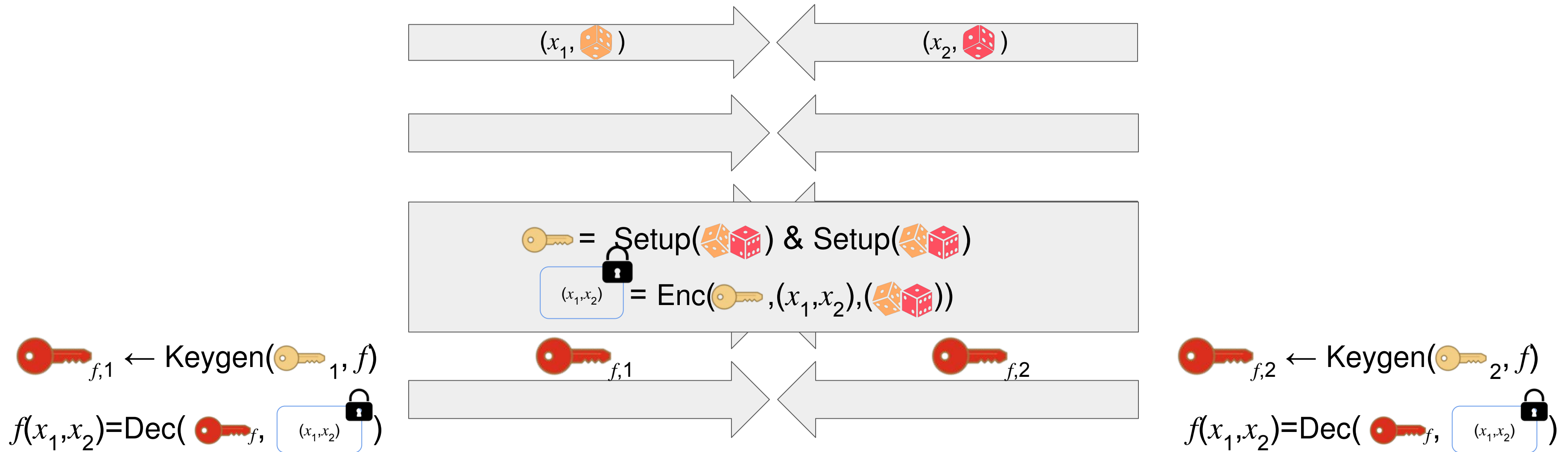
3. key_i can be generated arbitrarily
→ Solve as 2.

First Approach



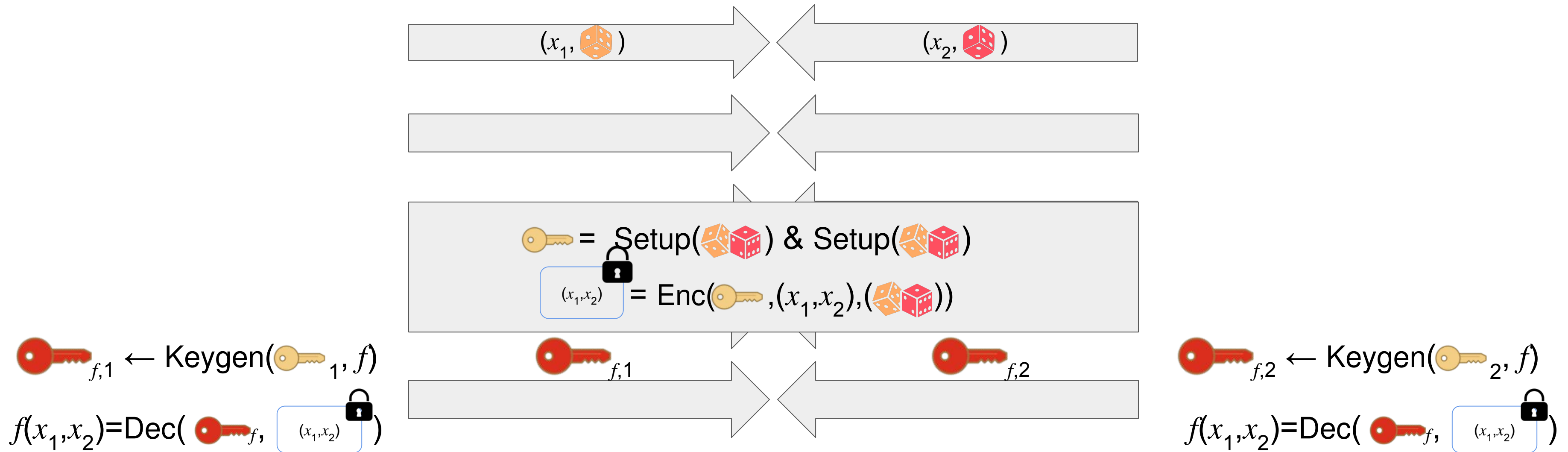
3. key_i can be generated arbitrarily
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First Approach



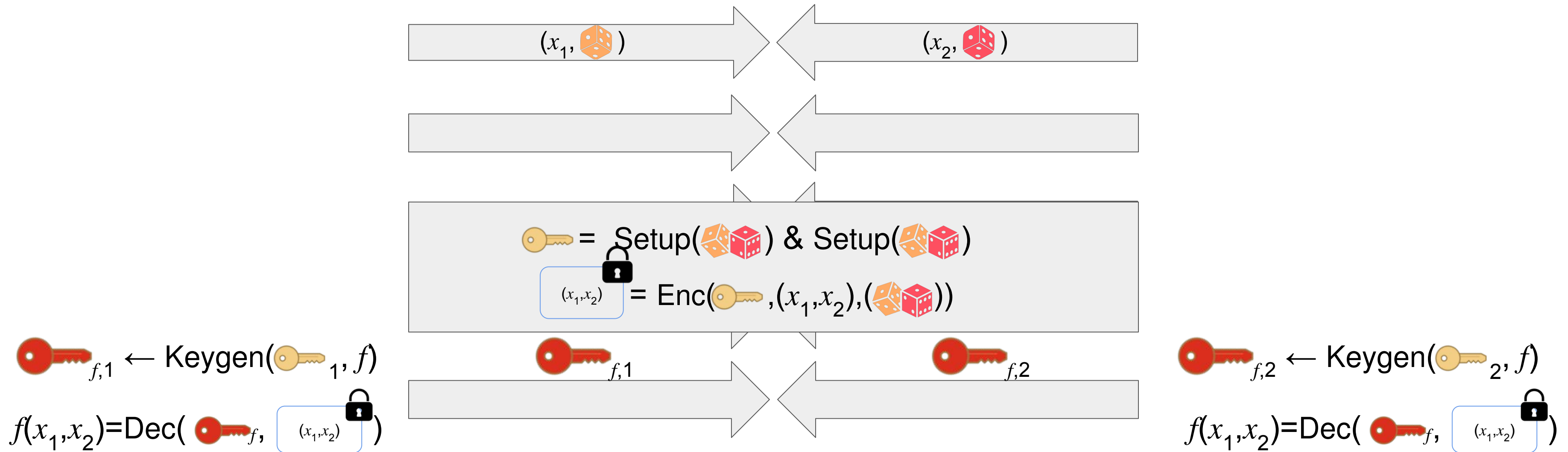
3. key_i can be generated arbitrarily
 → Solve as 2.

First Approach



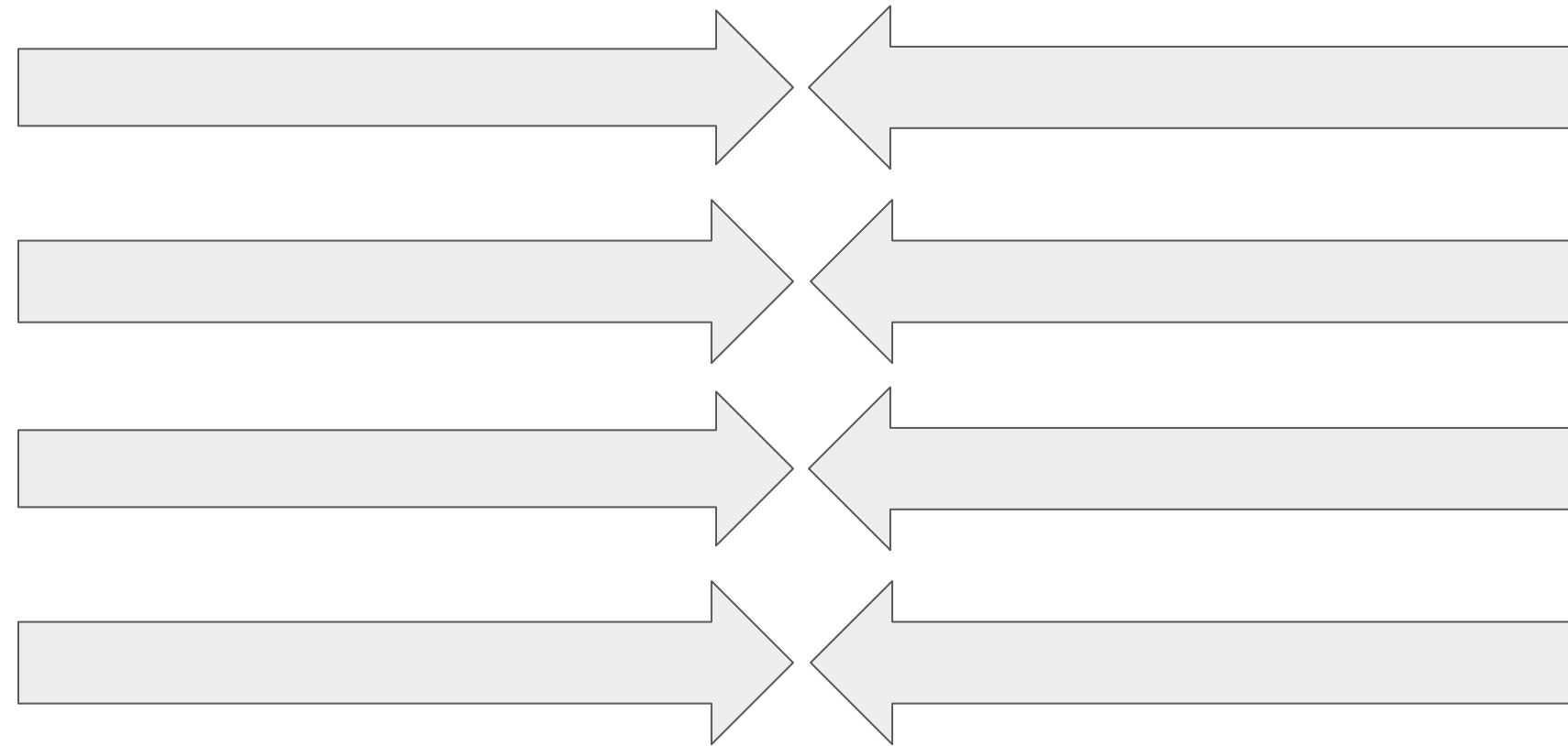
3. key_i can be generated arbitrarily
 → Solve as 2. ~~X~~ → Adds an additional round

First Approach



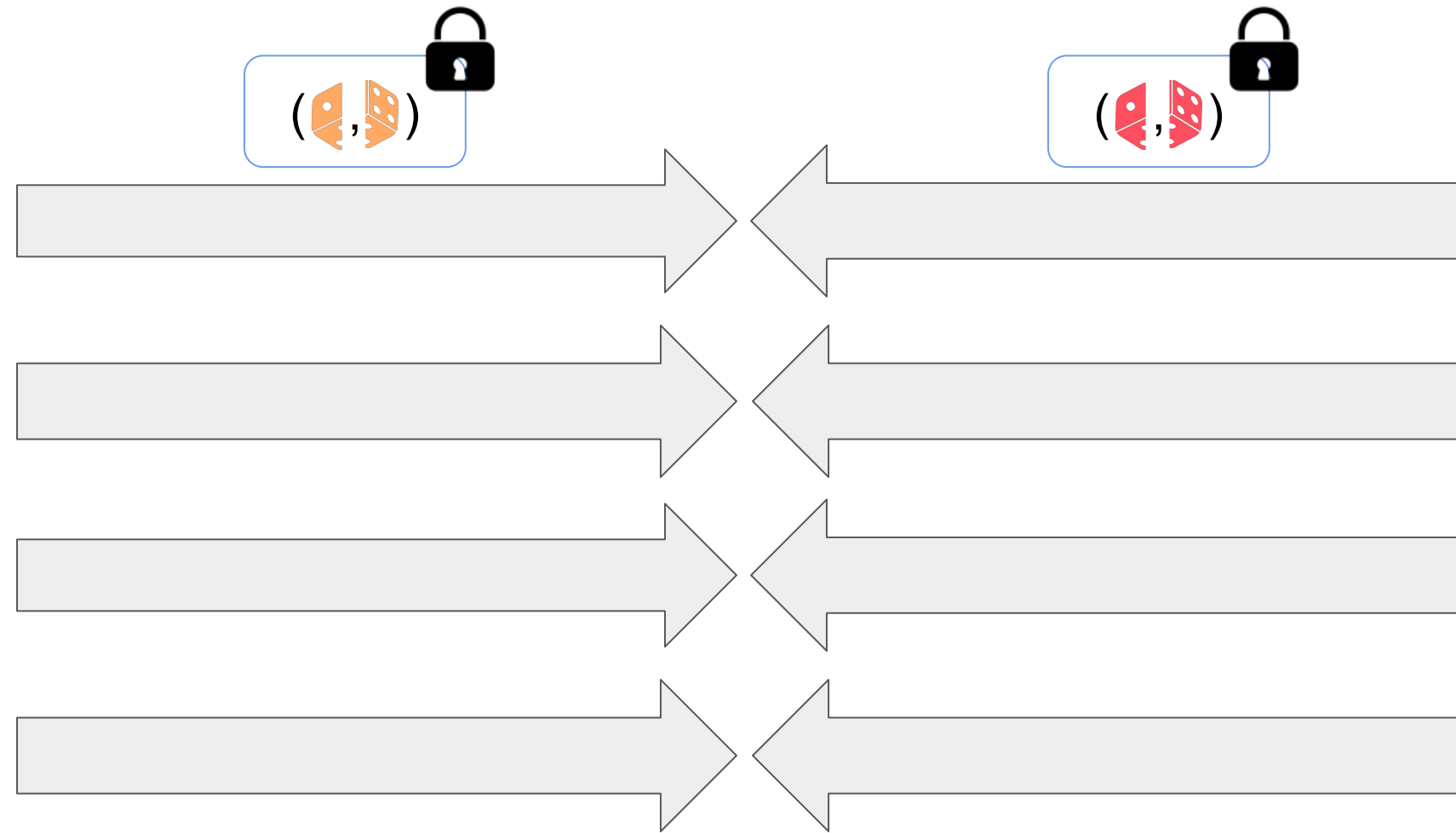
3. key_i can be generated arbitrarily
 - Solve as 2. ~~X~~ → Adds an additional round
 - Do Coin Flipping outside of protocol


Final Approach



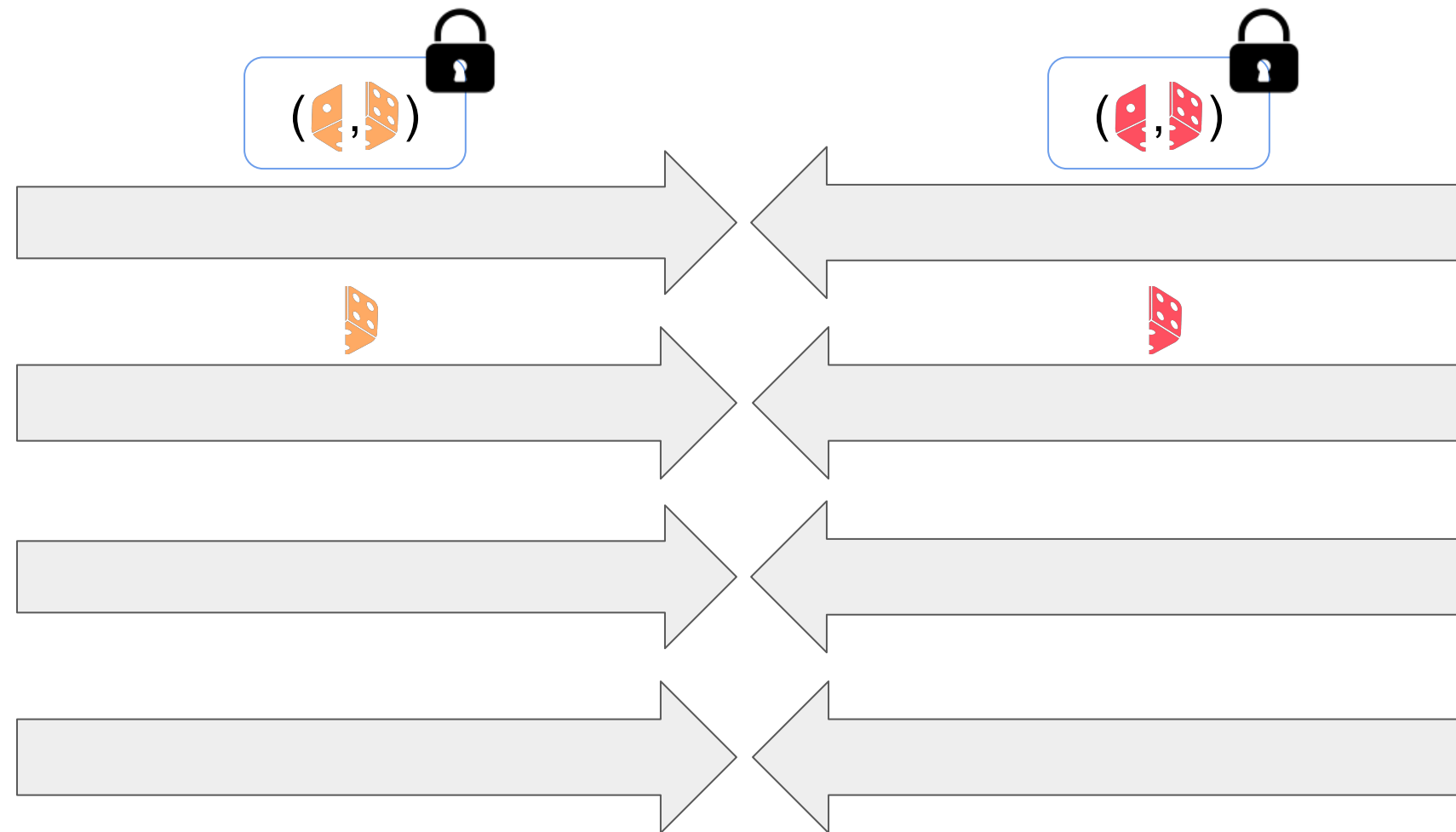
3. 🗝️_{*i*} can be generated arbitrarily
 - Solve as 2. ~~X~~ → Adds an additional round
 - Do Coin Flipping outside of protocol



Final Approach



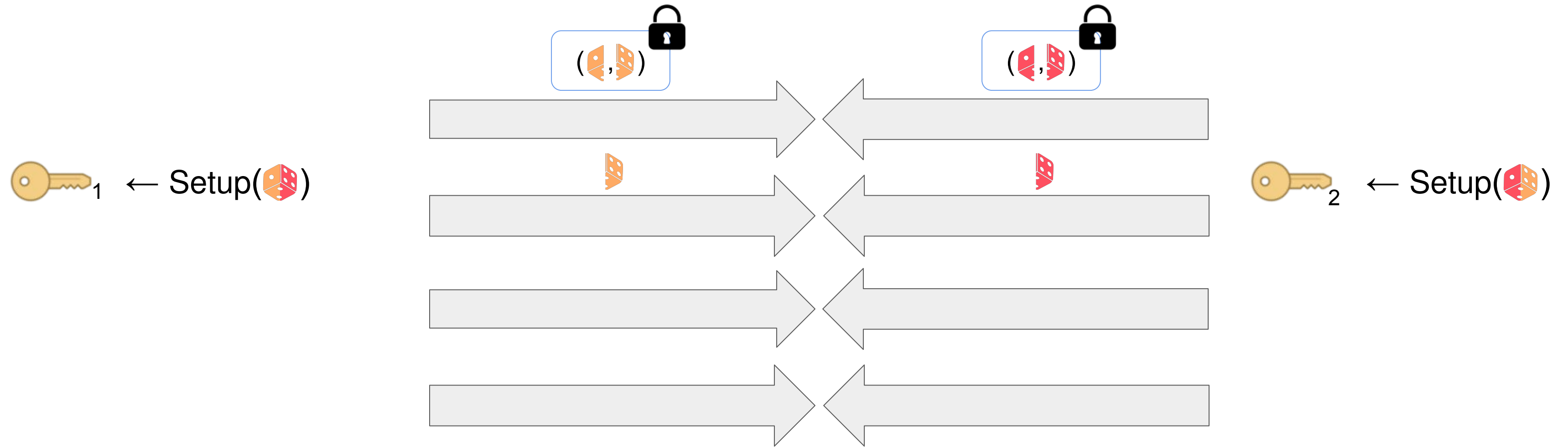
3.  i can be generated arbitrarily
 - Solve as 2. \times → Adds an additional round
 - Do Coin Flipping outside of protocol

Final Approach



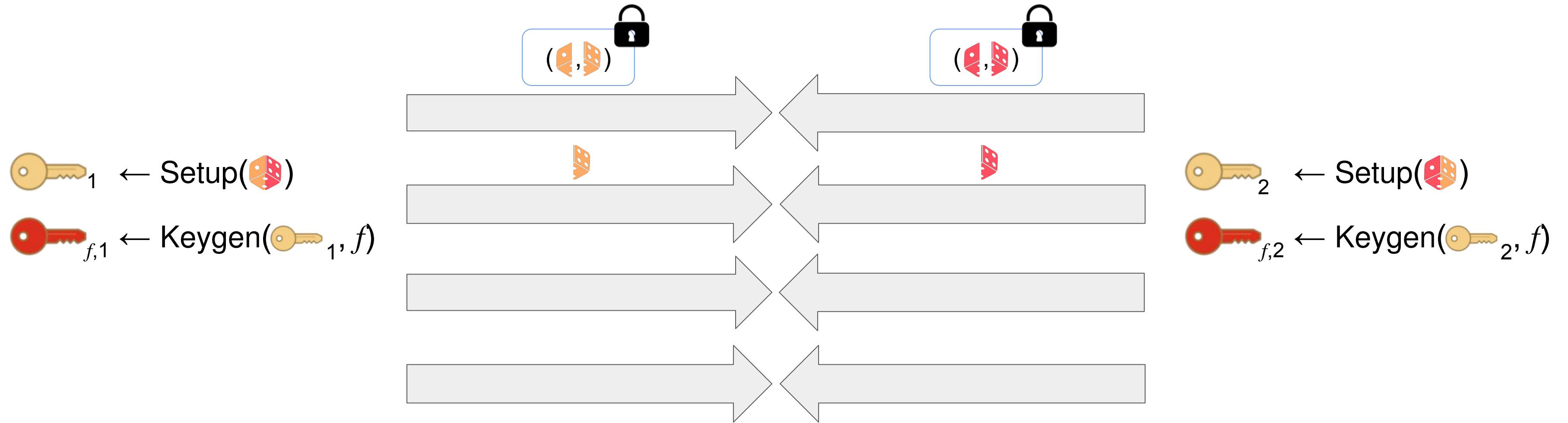
3.  i can be generated arbitrarily
 - Solve as 2.  → Adds an additional round
 - Do Coin Flipping outside of protocol

Final Approach



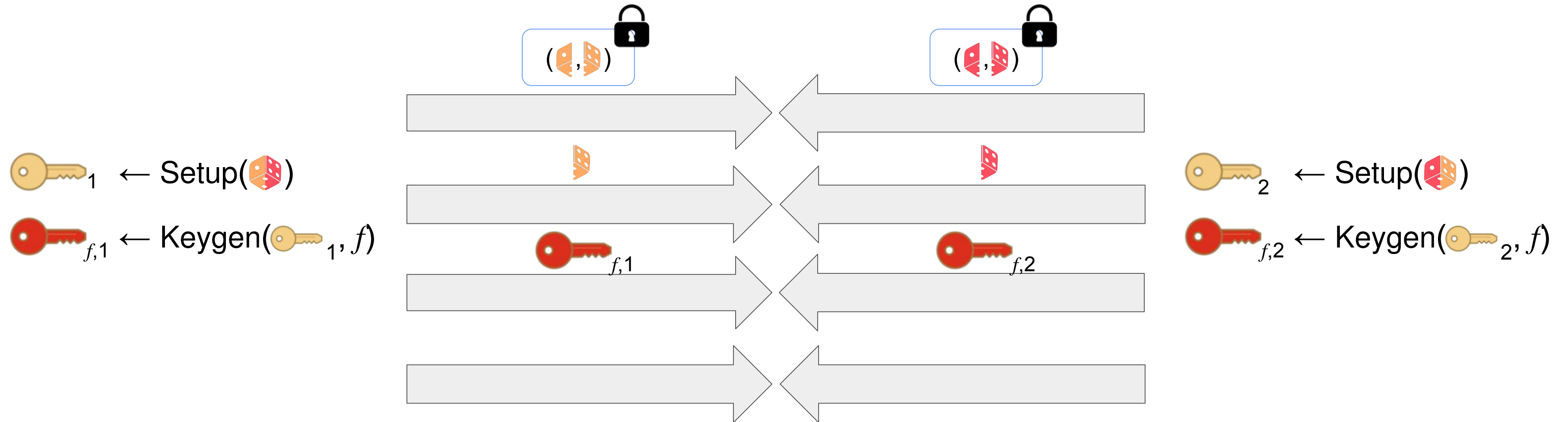
3. k_i can be generated arbitrarily
 - Solve as 2. \times → Adds an additional round
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Final Approach



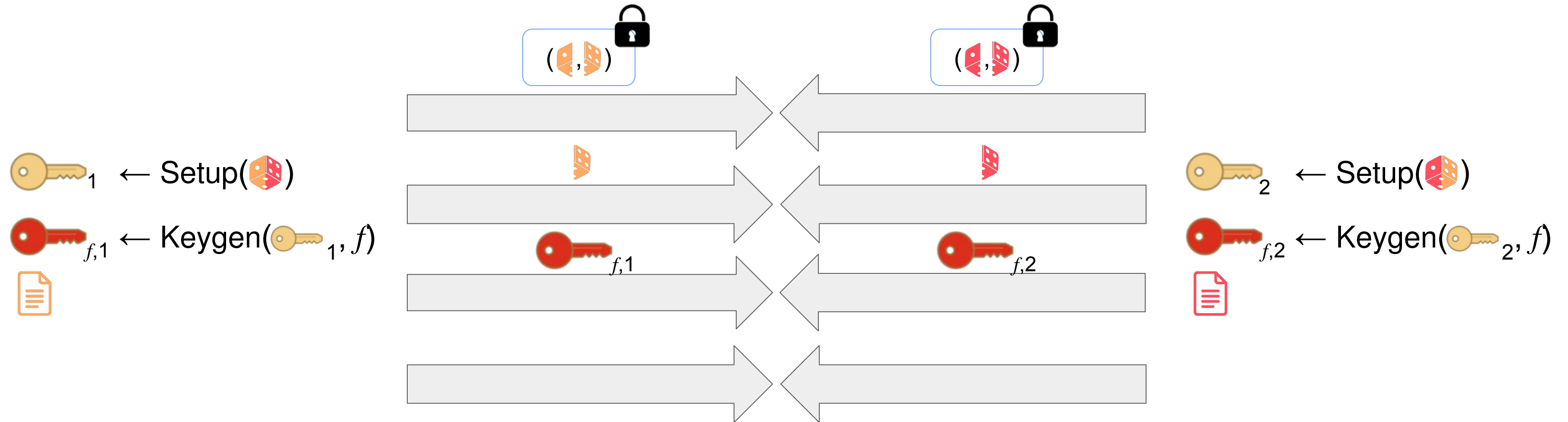
3. k_i can be generated arbitrarily
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Final Approach



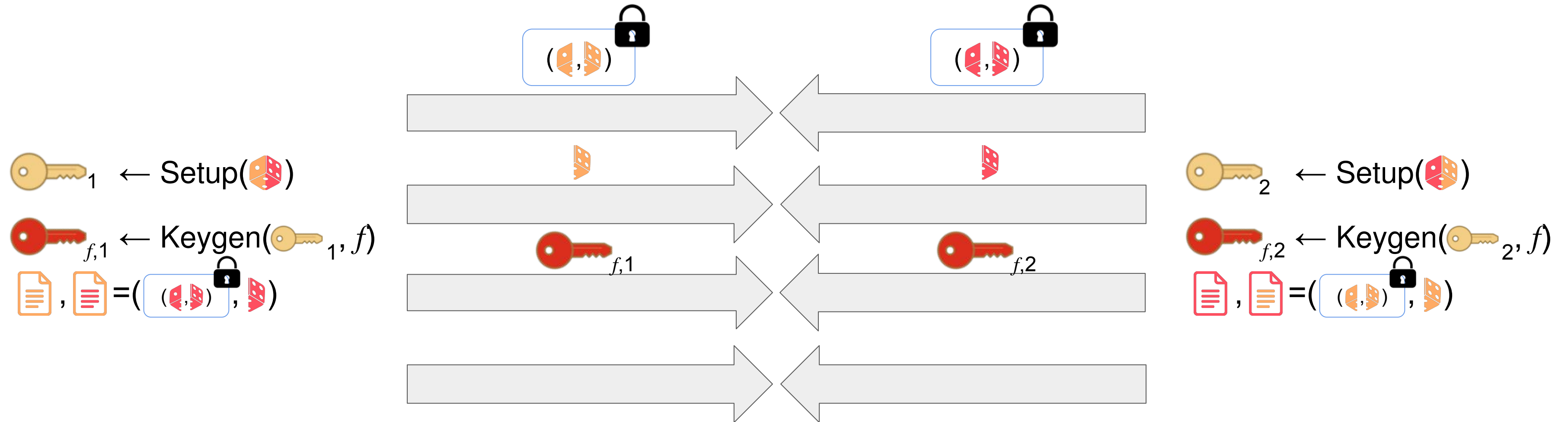
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Final Approach



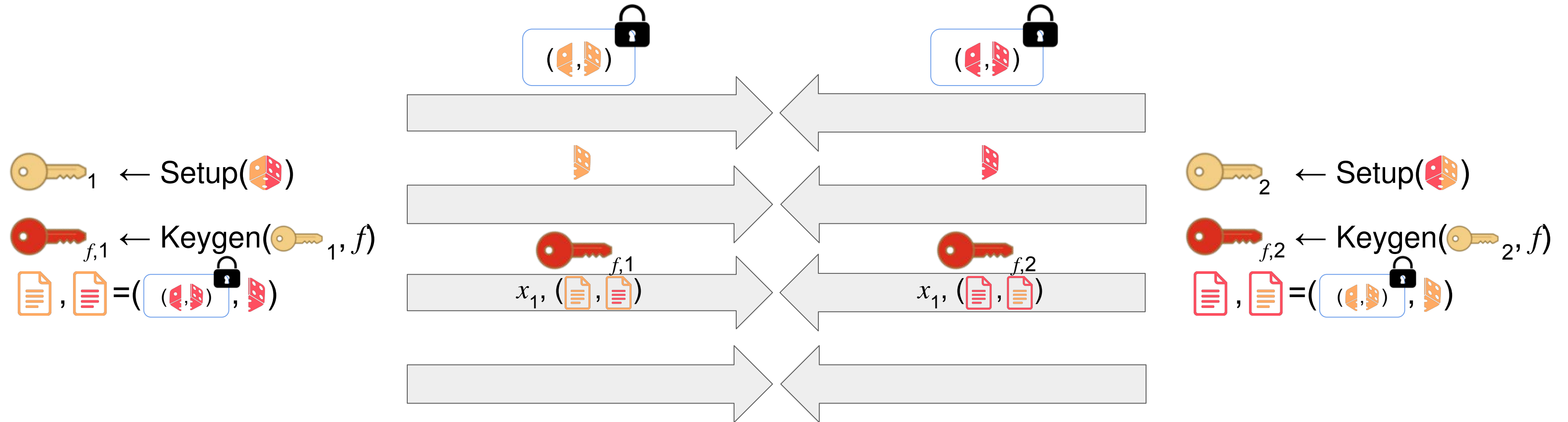
3. k_i can be generated arbitrarily
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Final Approach



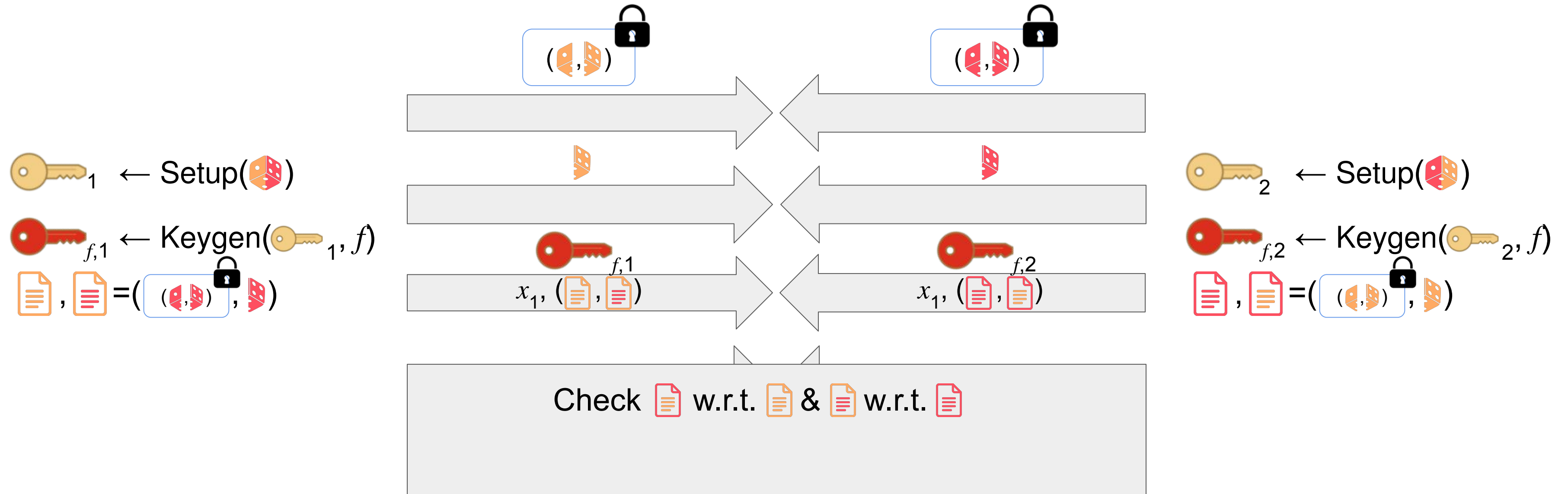
- k_i can be generated arbitrarily
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Final Approach



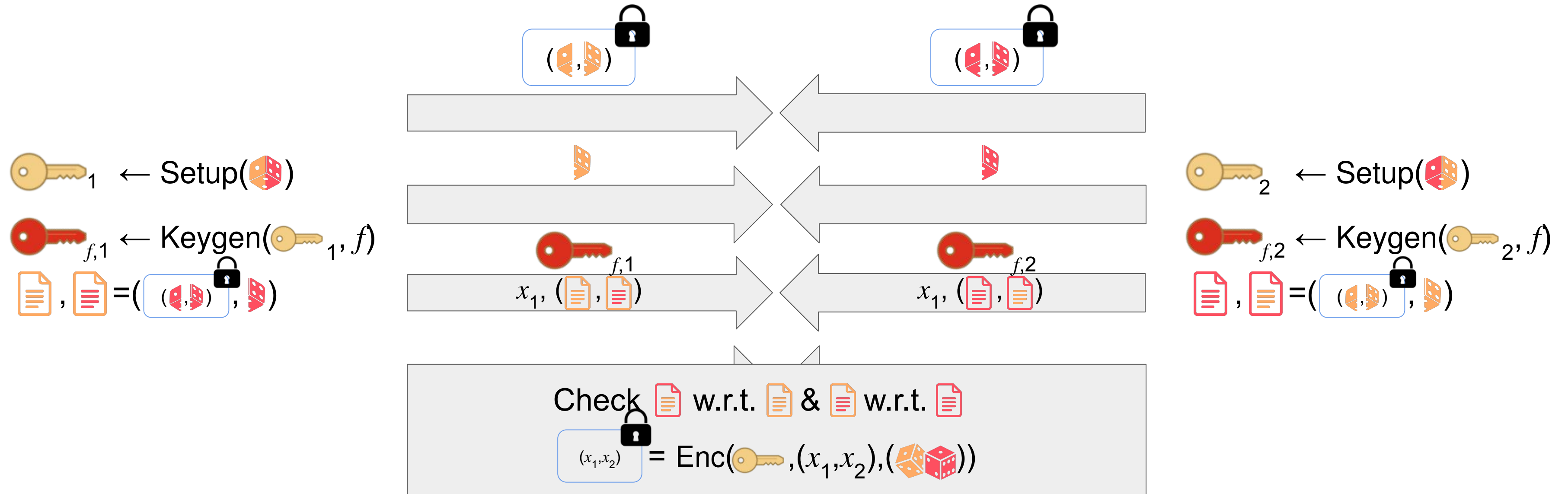
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Final Approach



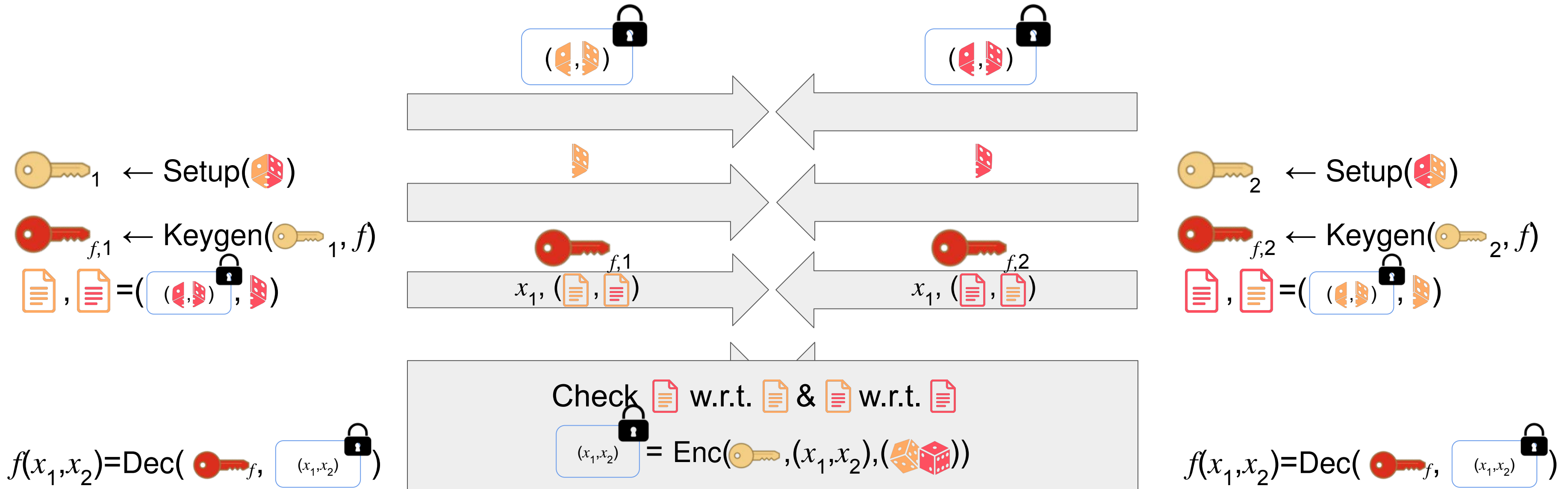
3. k_i can be generated arbitrarily
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Final Approach



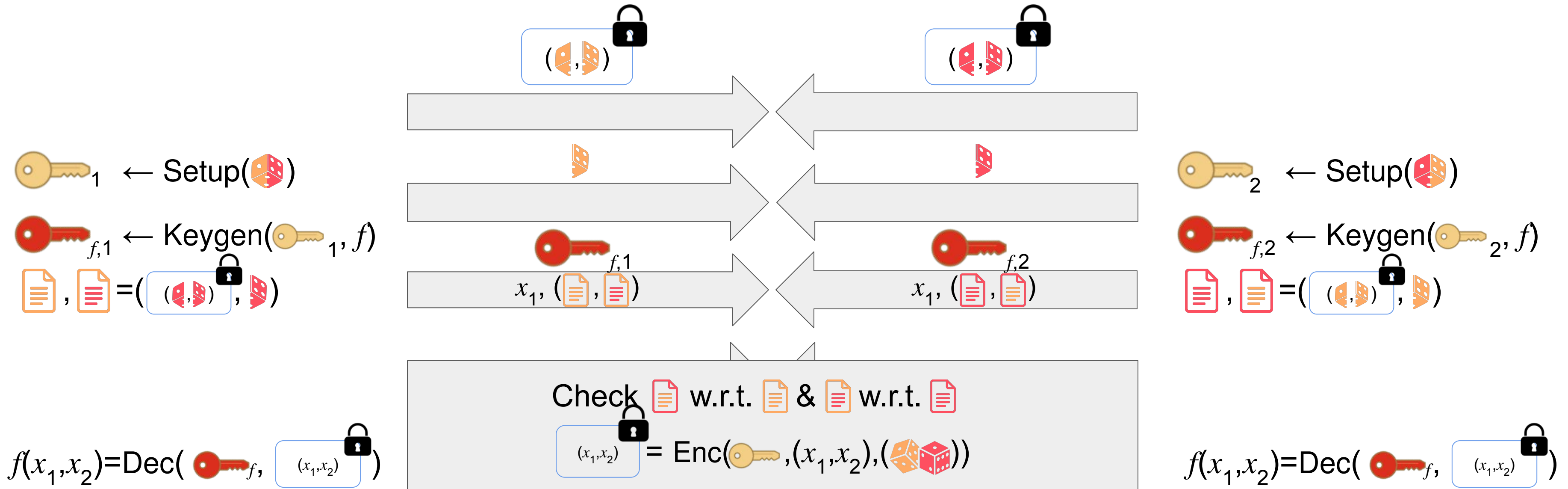
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Final Approach



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Final Approach

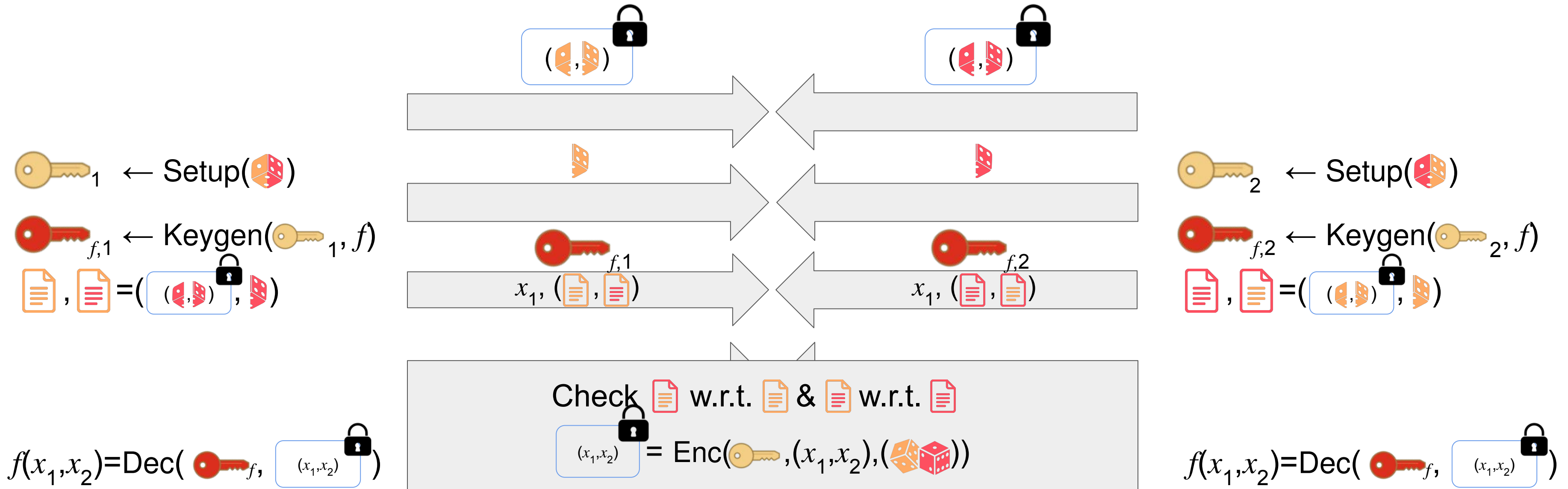


3. key_i can be generated arbitrarily

→ Solve as 2. \times → Adds an additional round

→ Do Coin Flipping outside of protocol → How to allow for honest behavior check?

Final Approach



3. k_i can be generated arbitrarily

→ Solve as 2. ~~X~~ → Adds an additional round

→ Do Coin Flipping outside of protocol → How to allow for honest behavior check?

⇒ k -Delayed-Input Function MPC

k -Delayed-Input vs. k -Delayed-Input Function MPC

k -Delayed-Input vs. k -Delayed-Input Function MPC

k -Delayed-Input MPC:

1. The input is needed in round k

k -Delayed-Input vs. k -Delayed-Input Function MPC

k -Delayed-Input MPC:

1. The input is needed in round k
2. but needs to be fixed before the protocol execution

k -Delayed-Input vs. k -Delayed-Input Function MPC

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k -Delayed-Input Function MPC:

1. The input is needed in round k

k -Delayed-Input vs. k -Delayed-Input Function MPC

k -Delayed-Input MPC:

1. The input is needed in round k
2. but needs to be fixed before the protocol execution

k -Delayed-Input Function MPC:

1. The input is needed in round k
2. and is partially decided during the protocol execution

k -Delayed-Input vs. k -Delayed-Input Function MPC

k -Delayed-Input MPC:

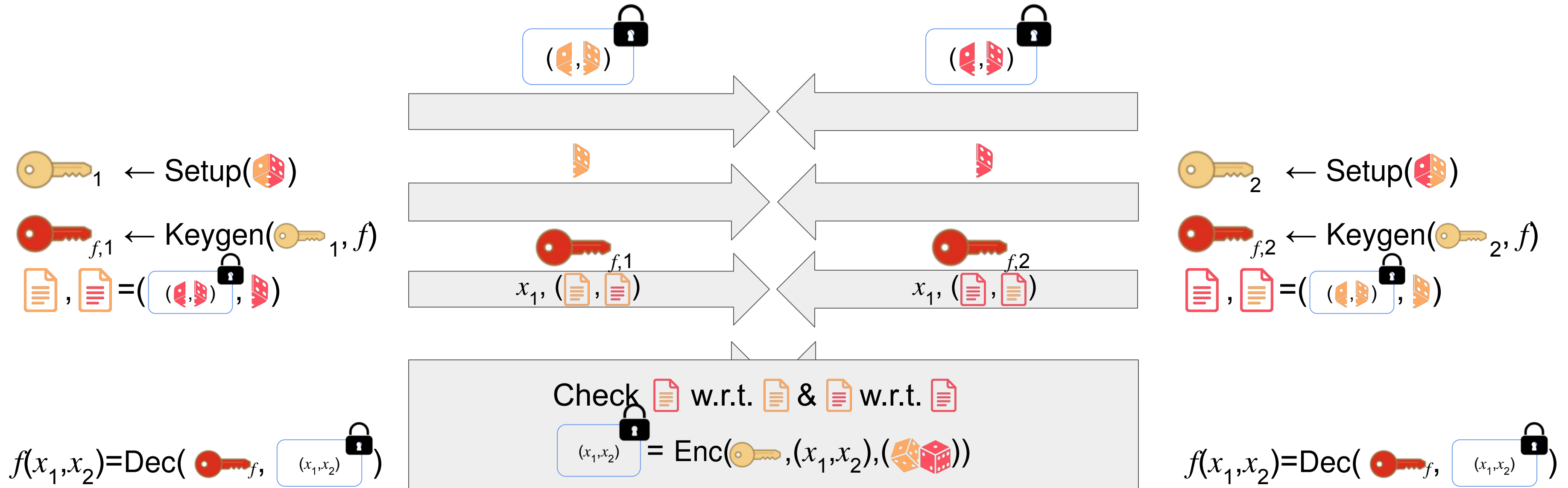
1. The input is needed in round k
2. but needs to be fixed before the protocol execution

k -Delayed-Input Function MPC:

1. The input is needed in round k
2. and is partially decided during the protocol execution

⇒ $2n$ -Party k -delayed-input MPC protocol + information-theoretic MAC

Final Approach

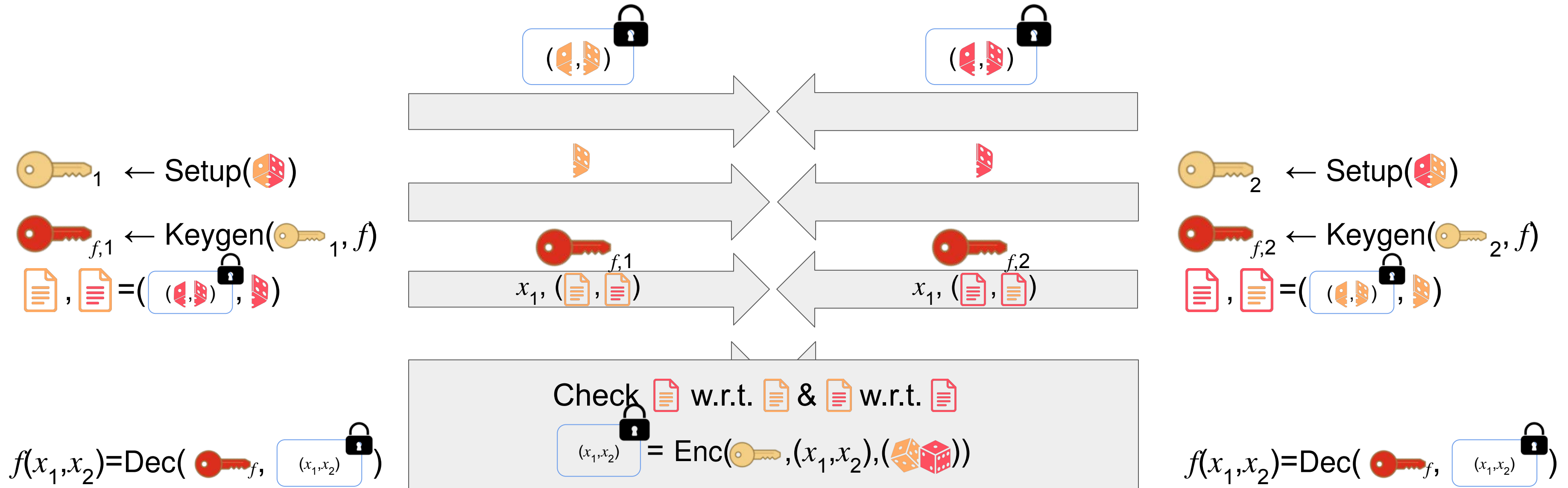


3. key_i can be generated arbitrarily

→ Solve as 2. \times → Adds an additional round

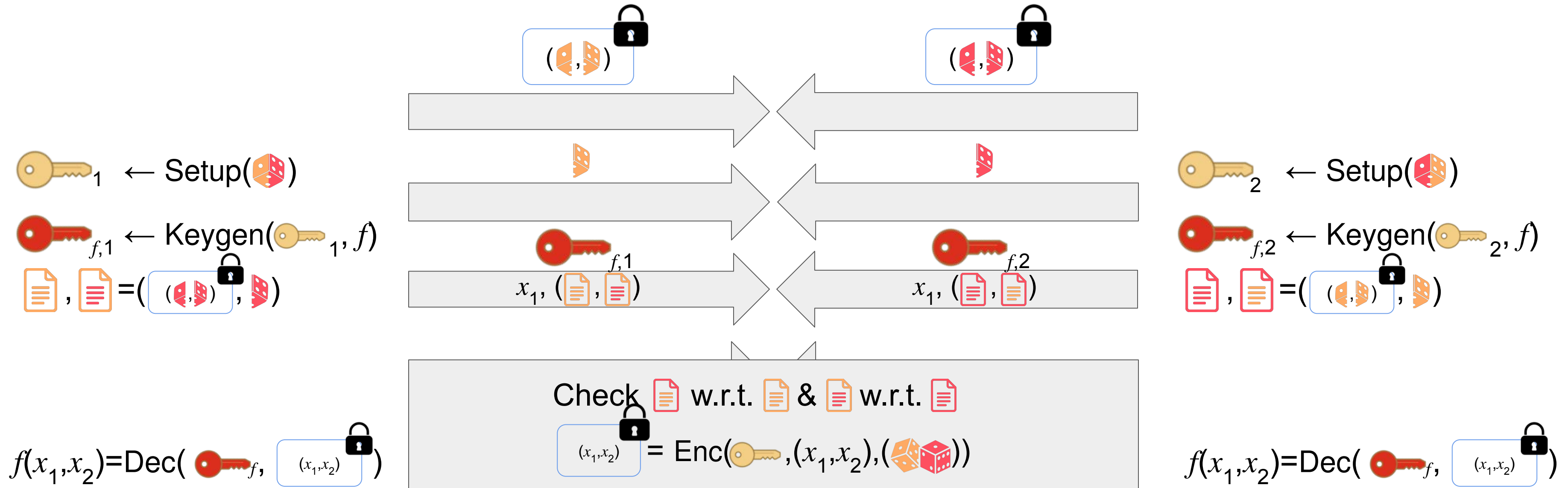
→ Do Coin Flipping outside of protocol → How to allow for honest behavior check?

Final Approach



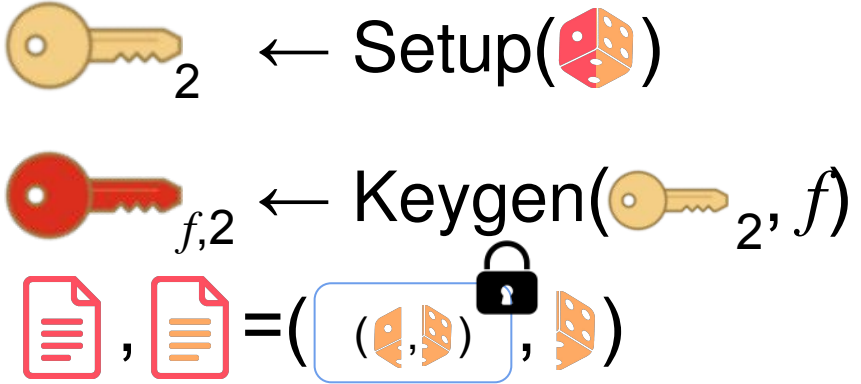
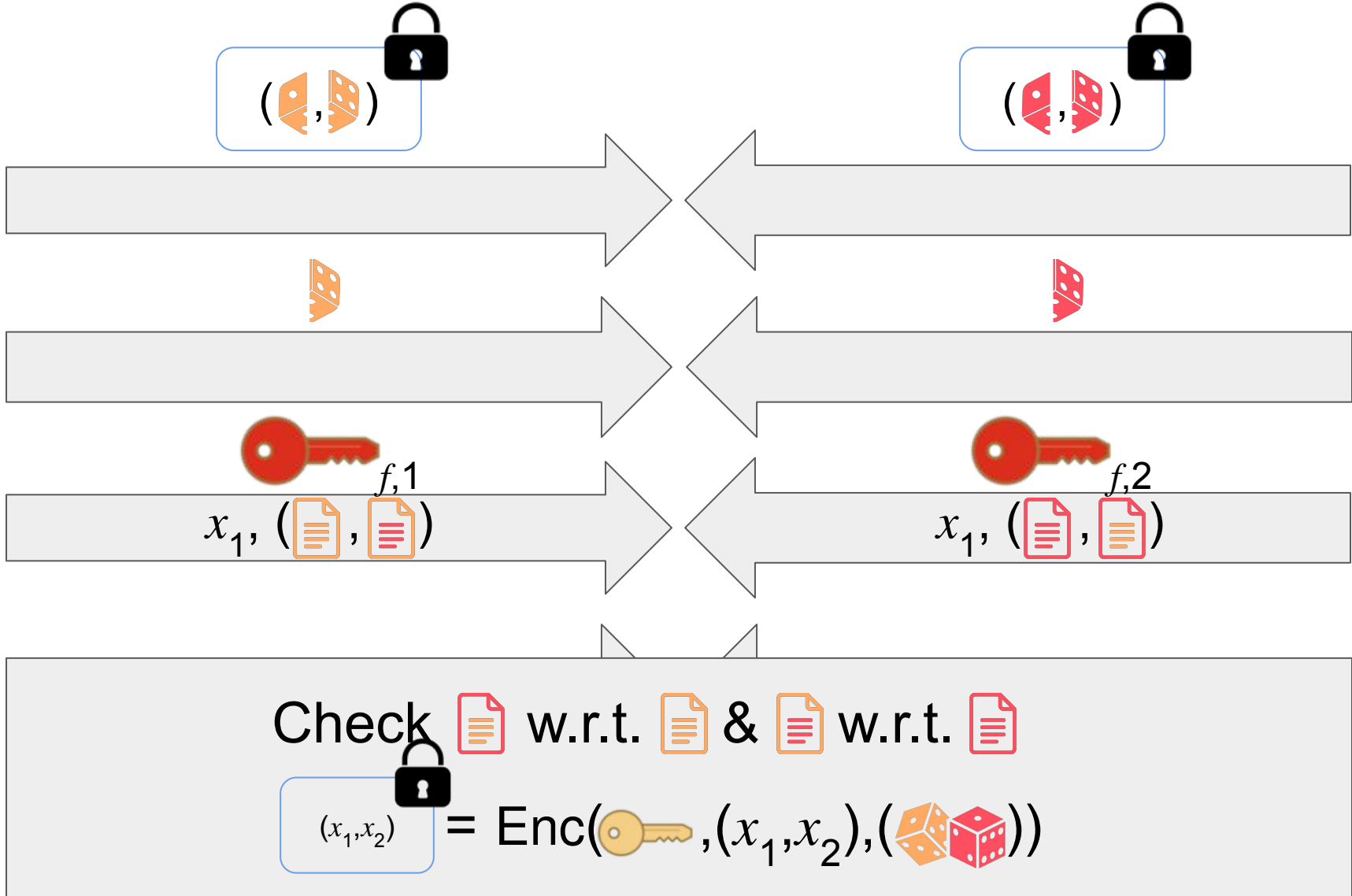
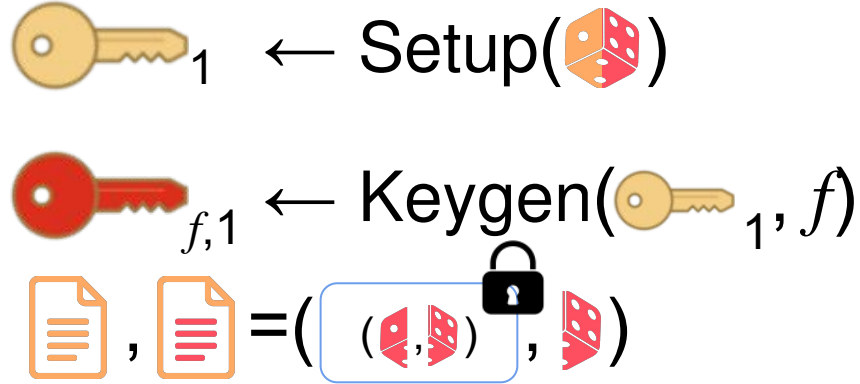
3. key_i can be generated arbitrarily
 - Solve as 2. \times → Adds an additional round
 - Do Coin Flipping outside of protocol → How to allow for honest behavior check? \checkmark

Final Approach



1. $\text{key}_{f,i}$ can be generated maliciously ✓
2. $(\text{seed}, \text{seed})$ used for encryption can be "bad" ✓
3. key_i can be generated arbitrarily ✓

Final Approach

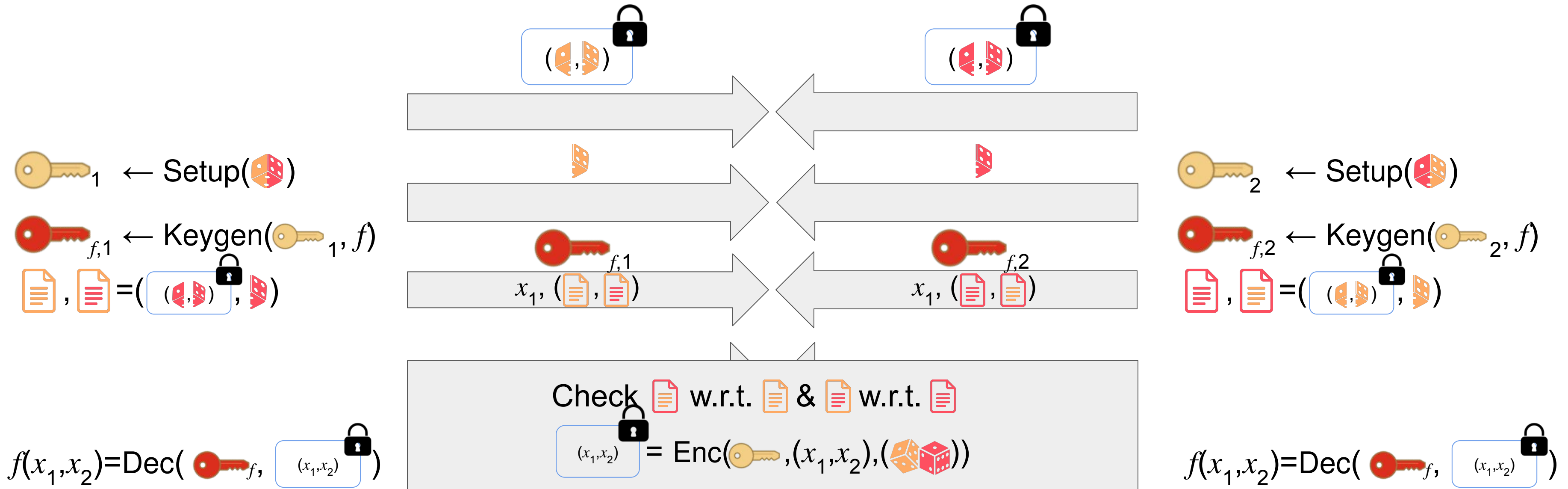


$f(x_1, x_2) = \text{Dec}(k_f, (x_1, x_2))$

$f(x_1, x_2) = \text{Dec}(k_f, (x_1, x_2))$

1. $k_{f,i}$ can be generated maliciously ✓
 2. (die, die) used for encryption can be “bad” ✓
 3. k_i can be generated arbitrarily ✓
- ⇒ Communication Complexity: $\text{depth}(f)$

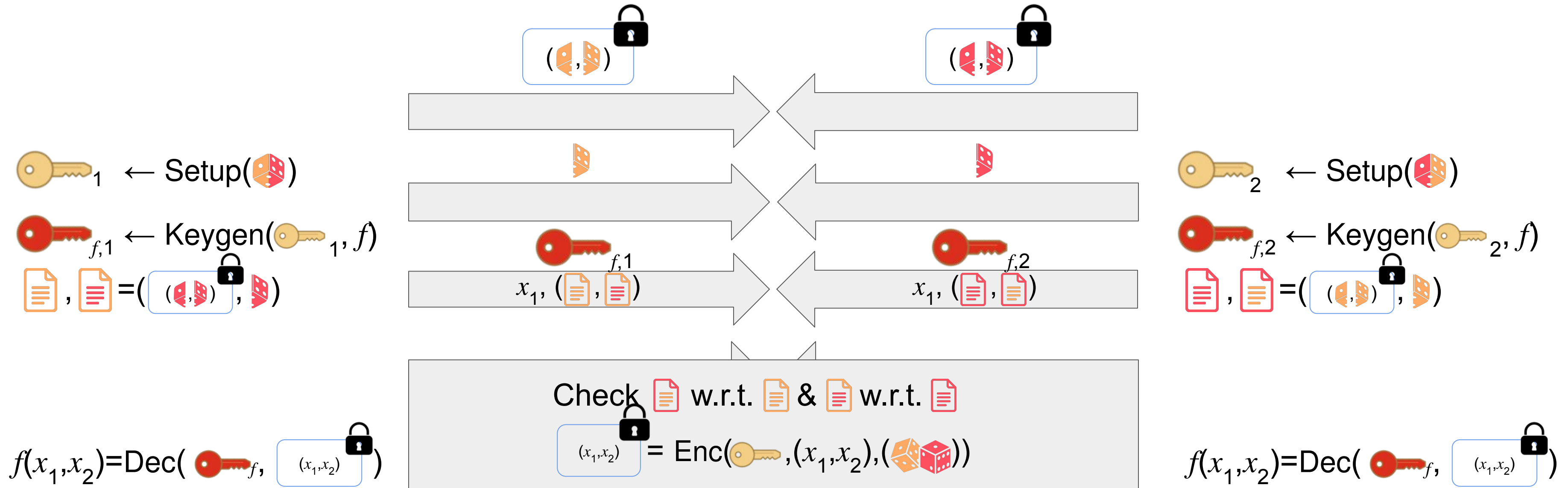
Final Approach



1. $\text{key}_{f,i}$ can be generated maliciously ✓
2. $(\text{dice}_1, \text{dice}_2)$ used for encryption can be “bad” ✓
3. key_i can be generated arbitrarily ✓

⇒ Communication Complexity: $\text{depth}(f)$ Can we do better?

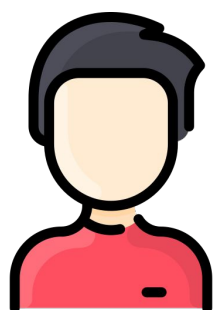
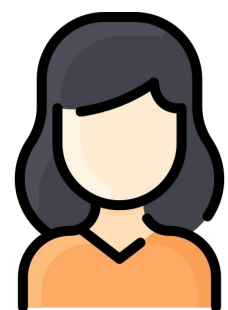
Final Approach



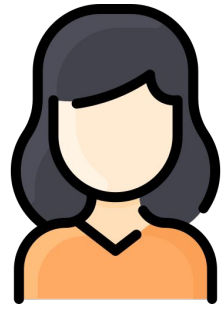
1. $\text{key}_{f,i}$ can be generated maliciously ✓
2. (die, die) used for encryption can be "bad" ✓
3. key_i can be generated arbitrarily ✓

⇒ Communication Complexity: $\text{depth}(f)$ Can we do better? Yes!

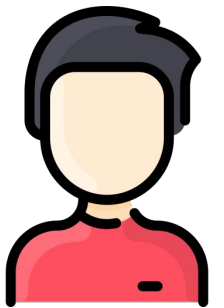
Multi-Key Fully Homomorphic Encryption [LTV12]



Multi-Key Fully Homomorphic Encryption [LTV12]

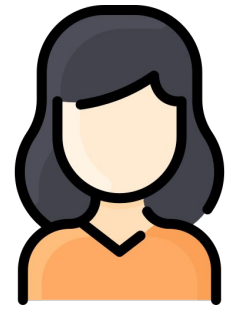


$(\text{key}_1, \text{key}'_1) \leftarrow \text{Setup}$



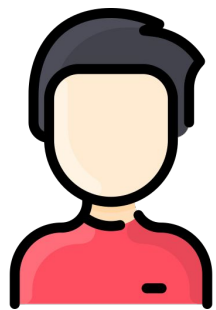
$(\text{key}_2, \text{key}'_2) \leftarrow \text{Setup}$

Multi-Key Fully Homomorphic Encryption [LTV12]



$(\text{key}_1, \text{priv}_1) \leftarrow \text{Setup}$

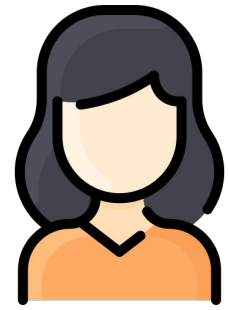
$x_1 \leftarrow \text{Enc}(\text{key}_1, x_1)$



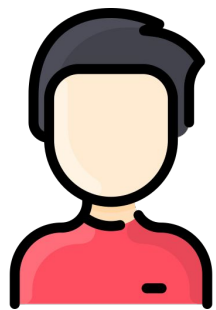
$(\text{key}_2, \text{priv}_2) \leftarrow \text{Setup}$

$x_2 \leftarrow \text{Enc}(\text{key}_2, x_2)$

Multi-Key Fully Homomorphic Encryption [LTV12]



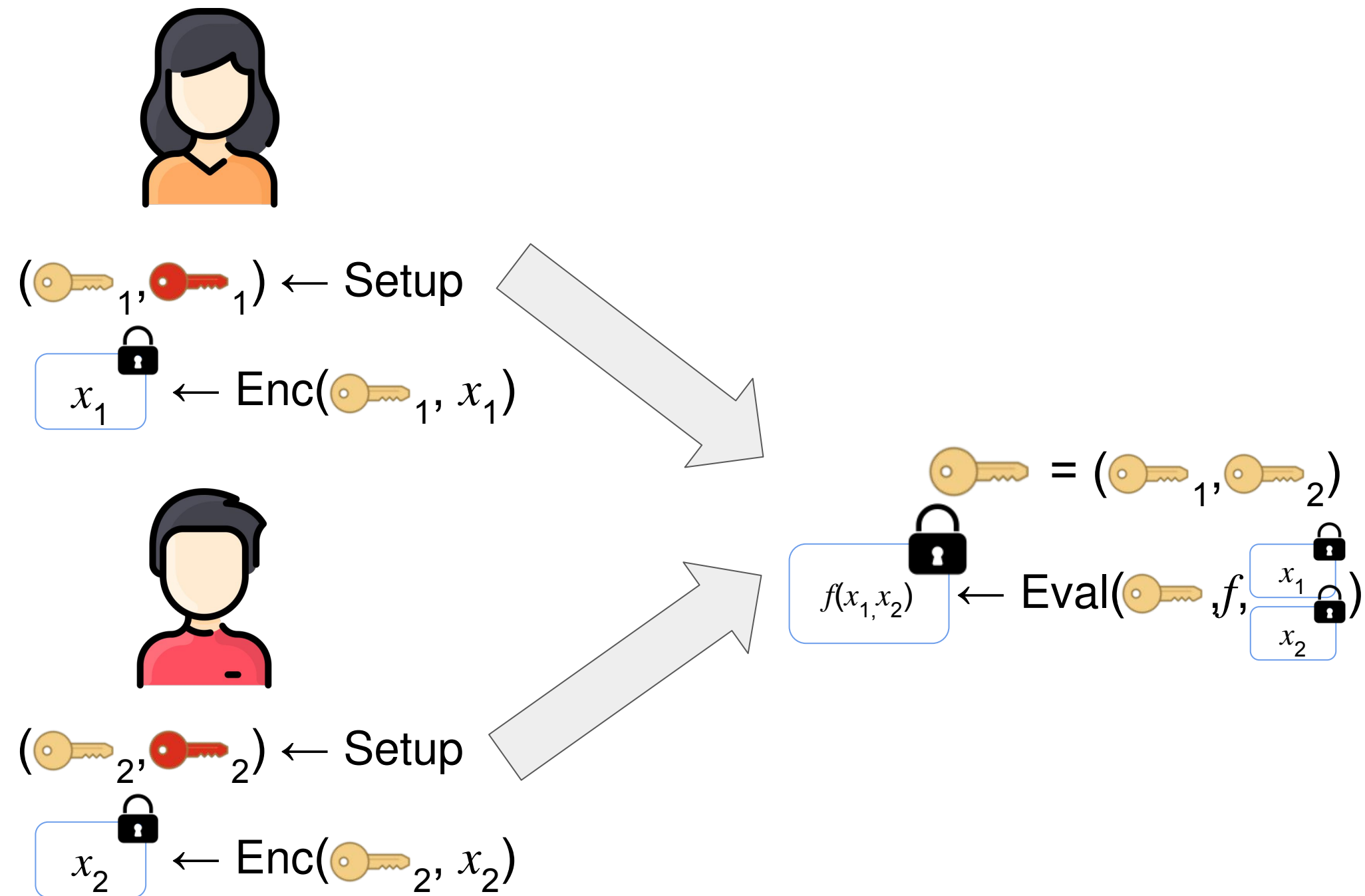
$(\text{key}_1, \text{sk}_1) \leftarrow \text{Setup}$
 $x_1 \leftarrow \text{Enc}(\text{key}_1, x_1)$



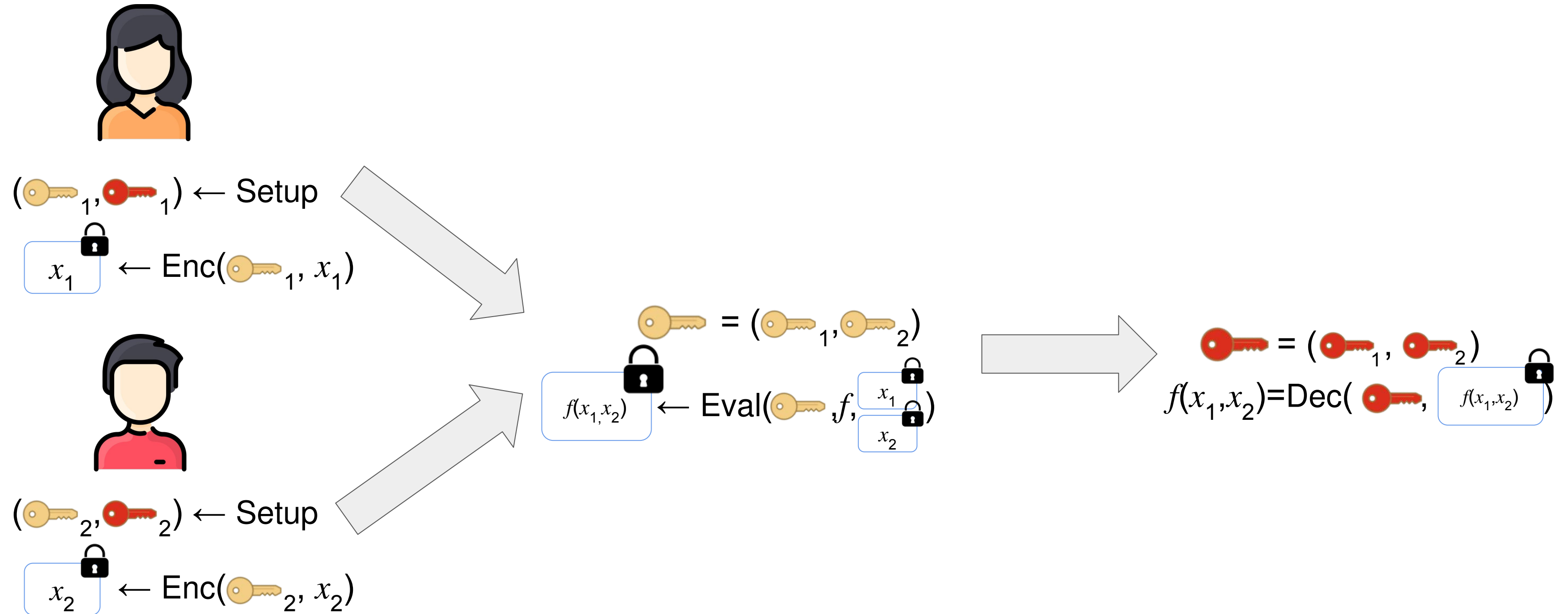
$(\text{key}_2, \text{sk}_2) \leftarrow \text{Setup}$
 $x_2 \leftarrow \text{Enc}(\text{key}_2, x_2)$

$$\text{key} = (\text{key}_1, \text{key}_2)$$

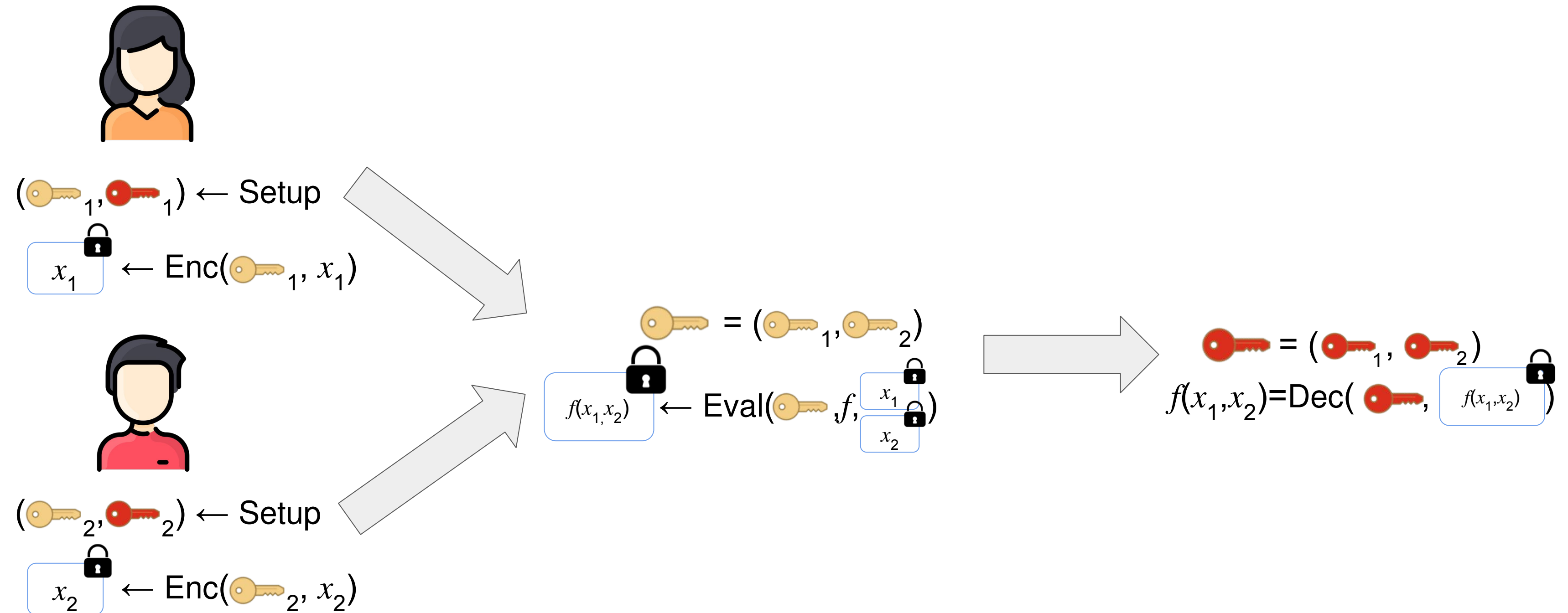
Multi-Key Fully Homomorphic Encryption [LTV12]



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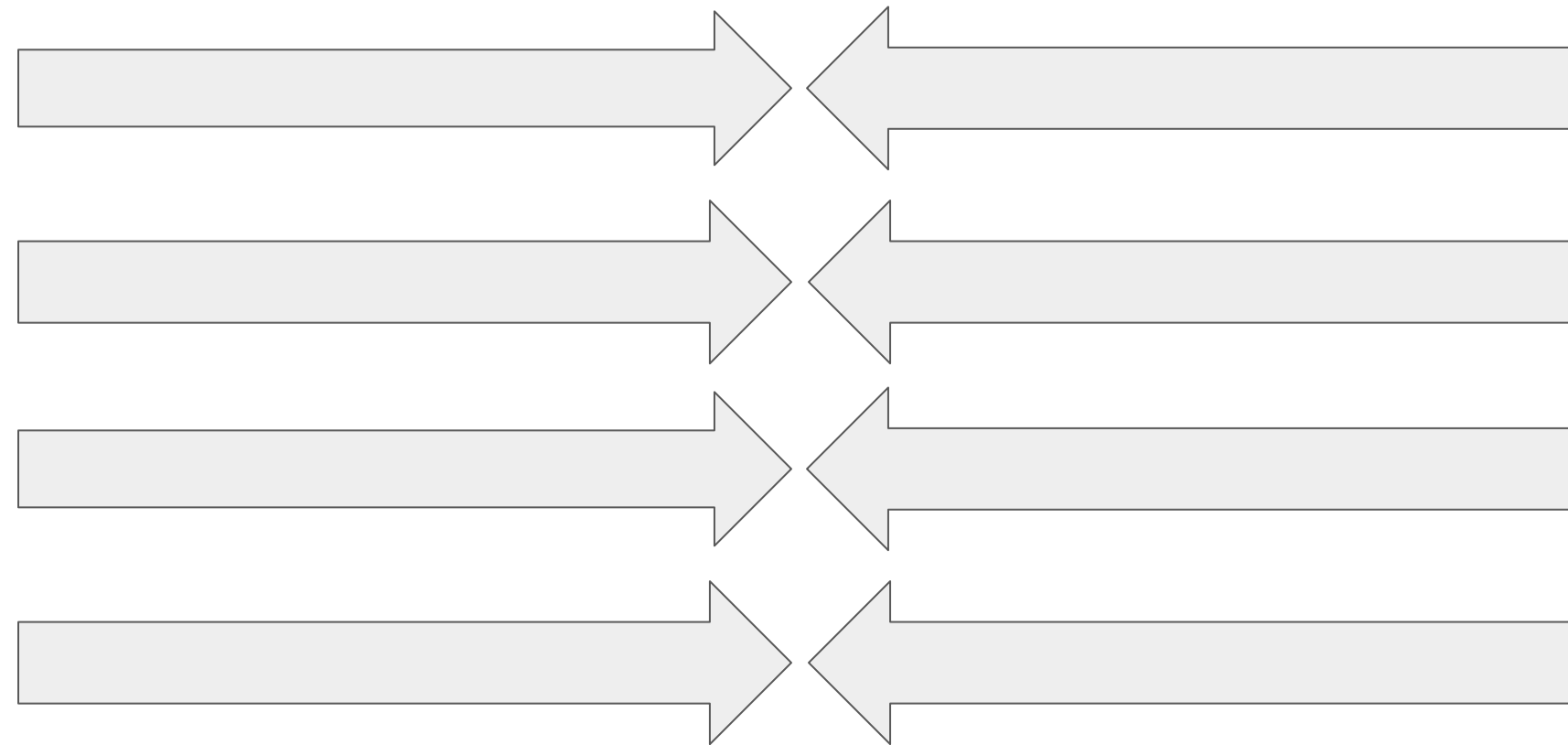
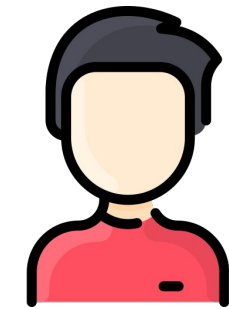
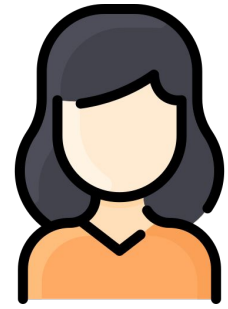


Multi-Key Fully Homomorphic Encryption [LTV12]

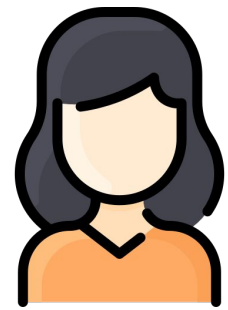


Compactness: $|E_{k_1}(x_1)|$ independent of f

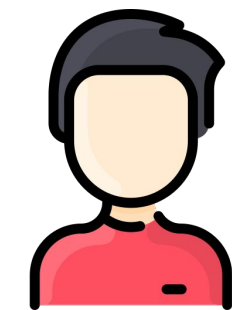
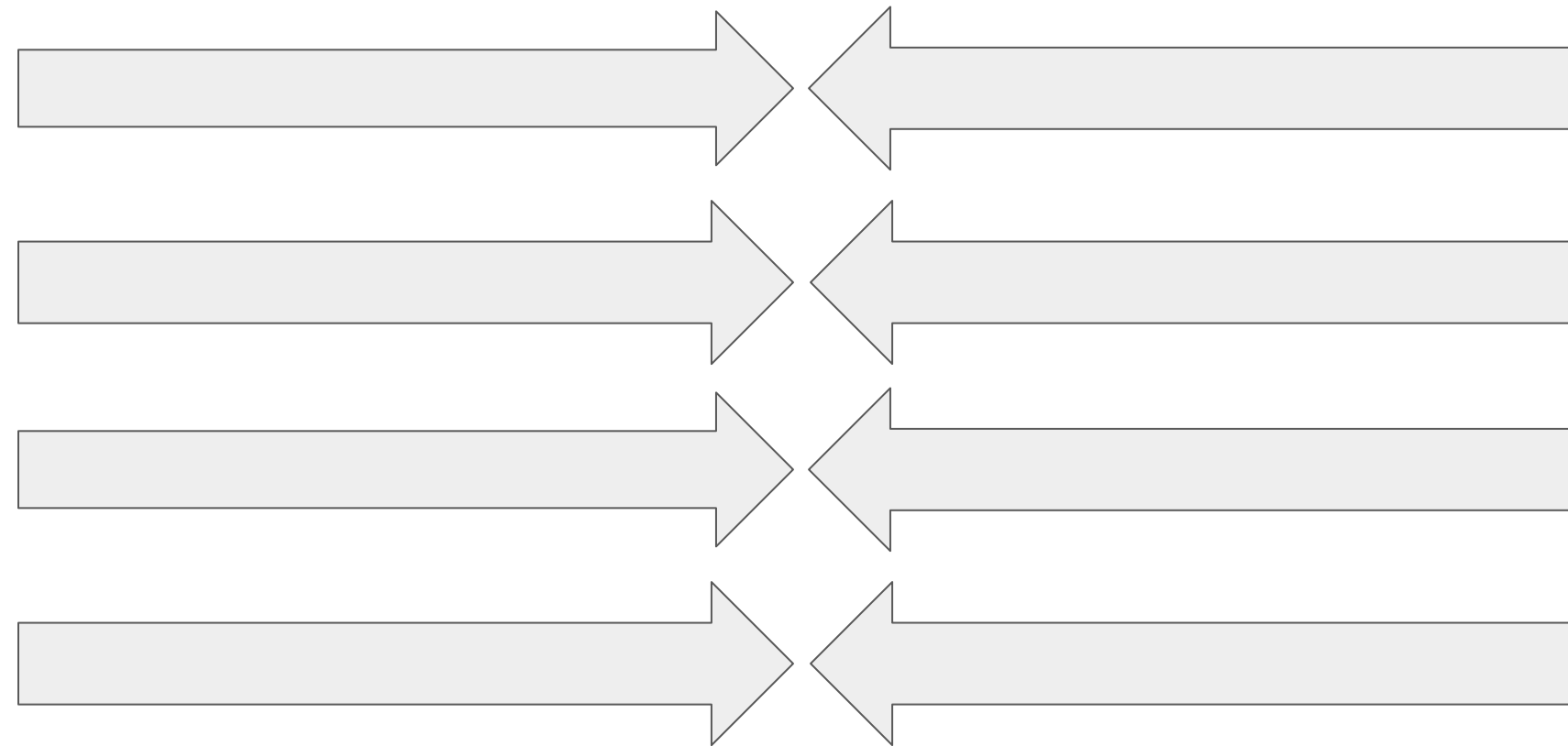
Round-Optimal and Communication-Efficient MPC



Round-Optimal and Communication-Efficient MPC

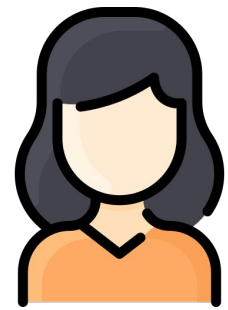


$(k_1, r_1) \leftarrow \text{Setup}$

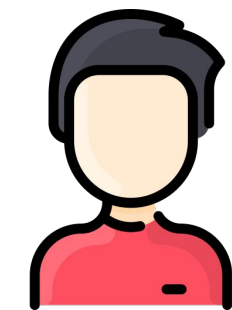
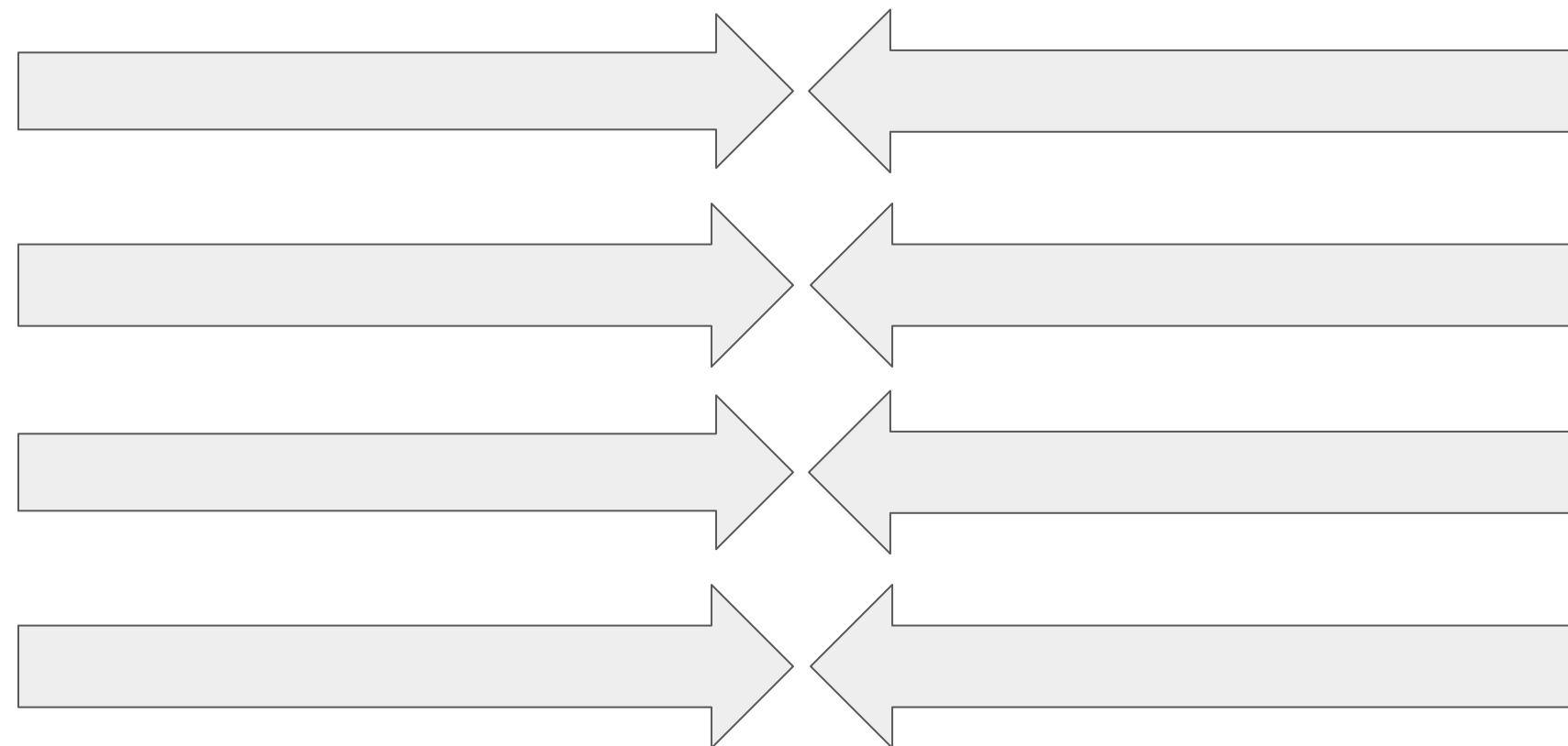


$(k_2, r_2) \leftarrow \text{Setup}$

Round-Optimal and Communication-Efficient MPC

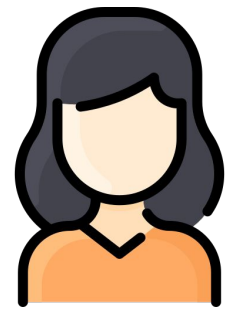


$(\text{key}_1, \text{key}_1) \leftarrow \text{Setup}$
 $x_1 \leftarrow \text{Enc}(\text{key}_1, x_1)$

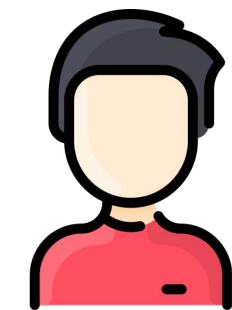
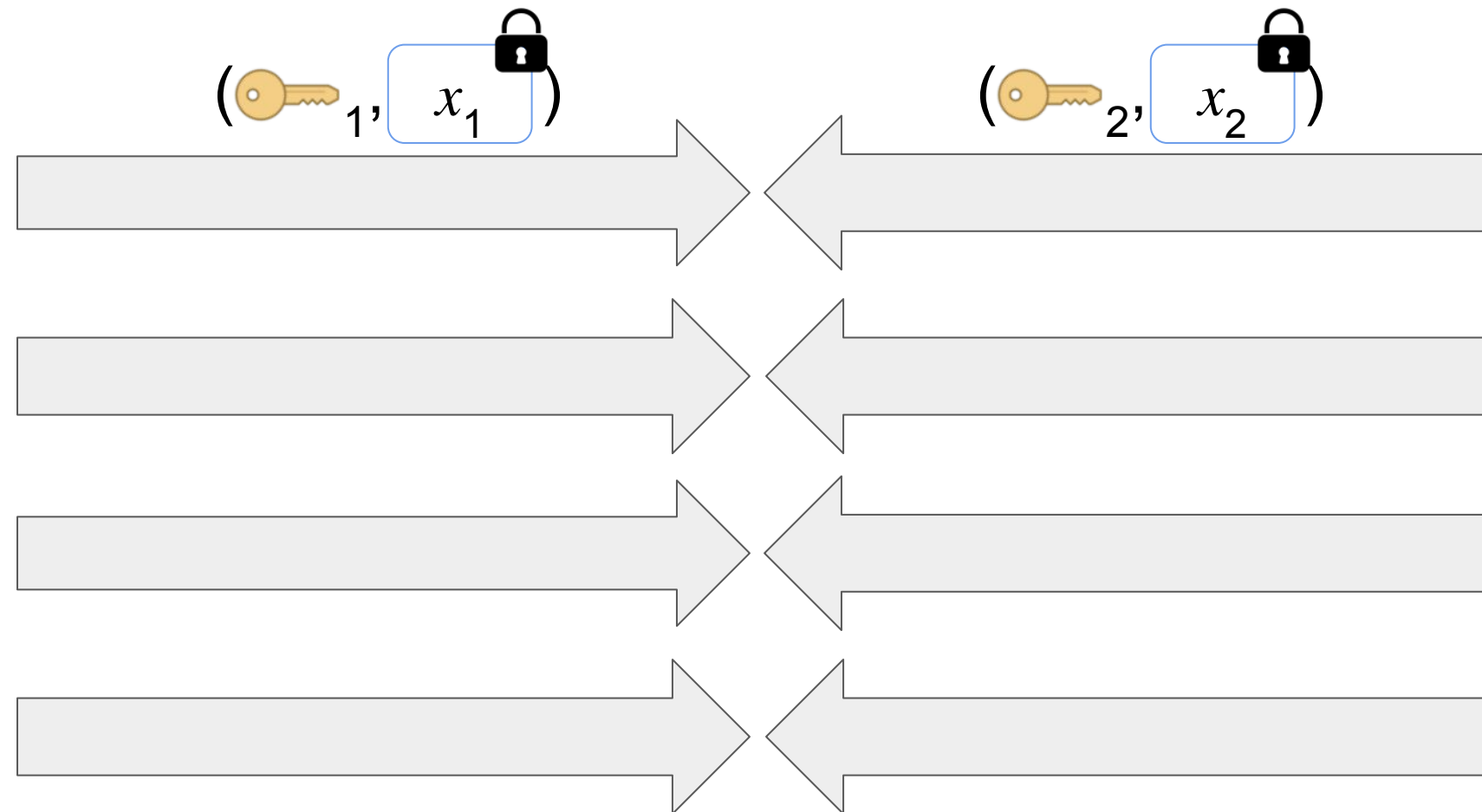


$(\text{key}_2, \text{key}_2) \leftarrow \text{Setup}$
 $x_2 \leftarrow \text{Enc}(\text{key}_2, x_2)$

Round-Optimal and Communication-Efficient MPC

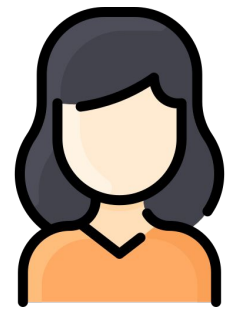


$(\text{key}_1, \text{red_key}_1) \leftarrow \text{Setup}$
 $x_1 \leftarrow \text{Enc}(\text{key}_1, x_1)$



$(\text{key}_2, \text{red_key}_2) \leftarrow \text{Setup}$
 $x_2 \leftarrow \text{Enc}(\text{key}_2, x_2)$

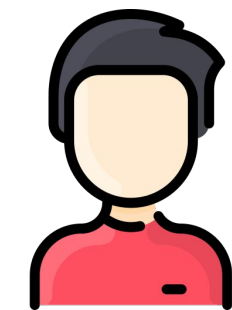
Round-Optimal and Communication-Efficient MPC



$(\text{key}_1, \text{red_key}_1) \leftarrow \text{Setup}$

$x_1 \leftarrow \text{Enc}(\text{key}_1, x_1)$

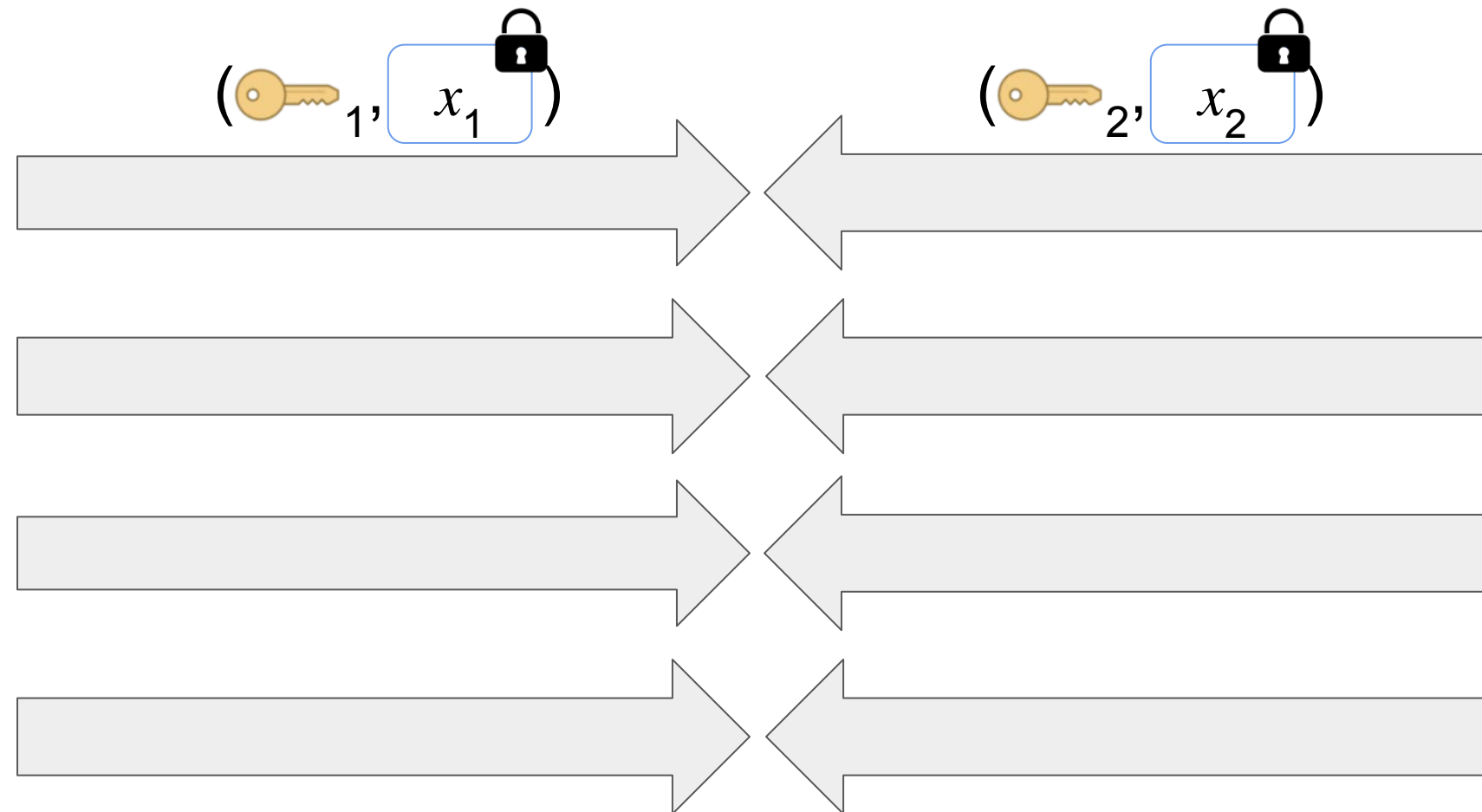
$f(x_1, x_2) \leftarrow \text{Eval}(\text{key}, f, \begin{matrix} x_1 \\ x_2 \end{matrix})$



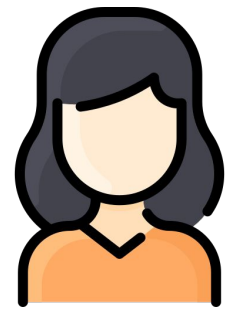
$(\text{key}_2, \text{red_key}_2) \leftarrow \text{Setup}$

$x_2 \leftarrow \text{Enc}(\text{key}_2, x_2)$

$f(x_1, x_2) \leftarrow \text{Eval}(\text{key}, f, \begin{matrix} x_1 \\ x_2 \end{matrix})$



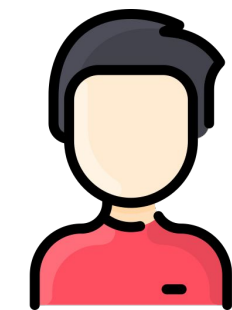
Round-Optimal and Communication-Efficient MPC



$(\text{key}_1, \text{red_key}_1) \leftarrow \text{Setup}$

$x_1 \leftarrow \text{Enc}(\text{key}_1, x_1)$

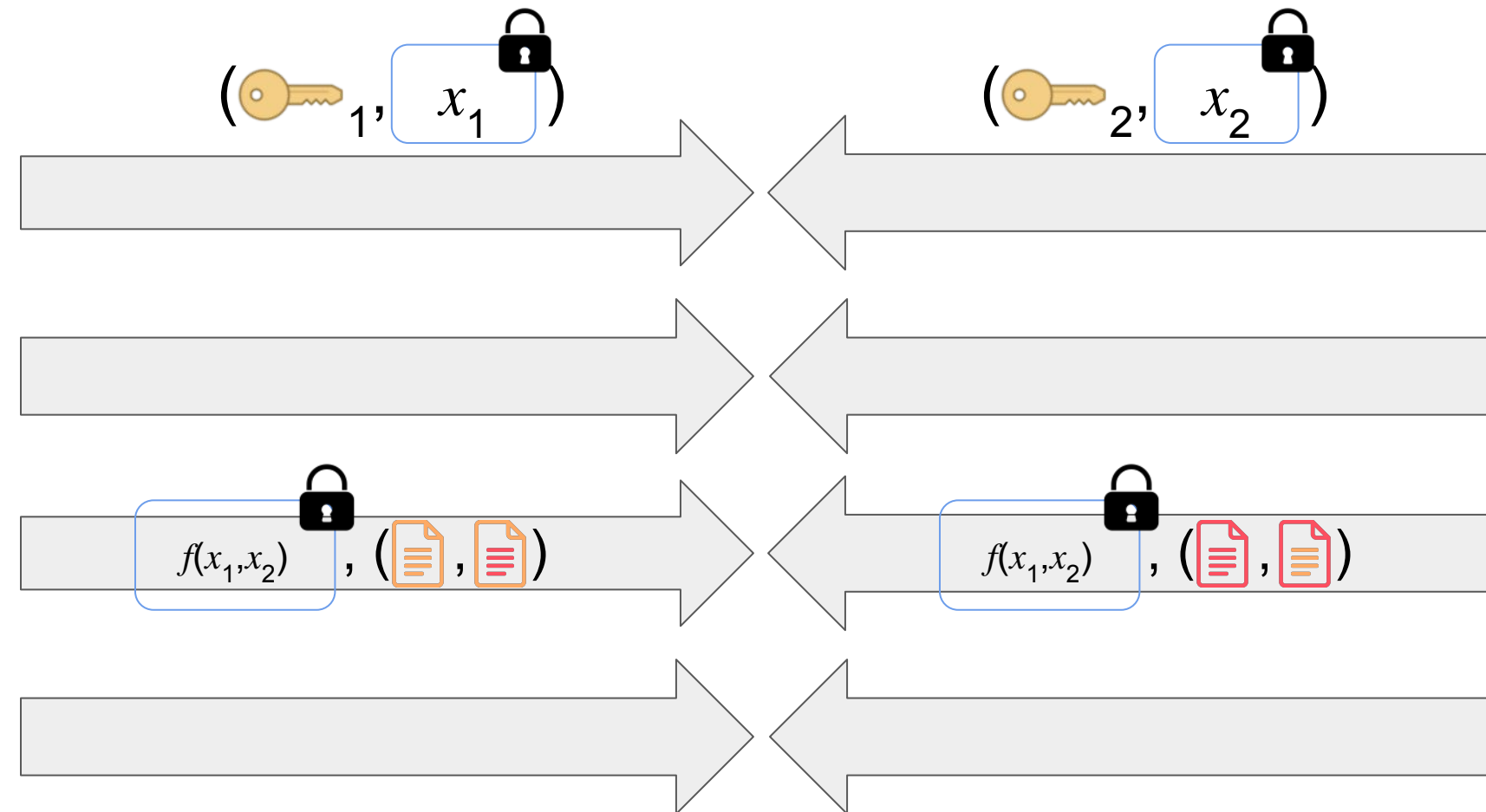
$f(x_1, x_2) \leftarrow \text{Eval}(\text{key}, f, \begin{matrix} x_1 \\ x_2 \end{matrix})$



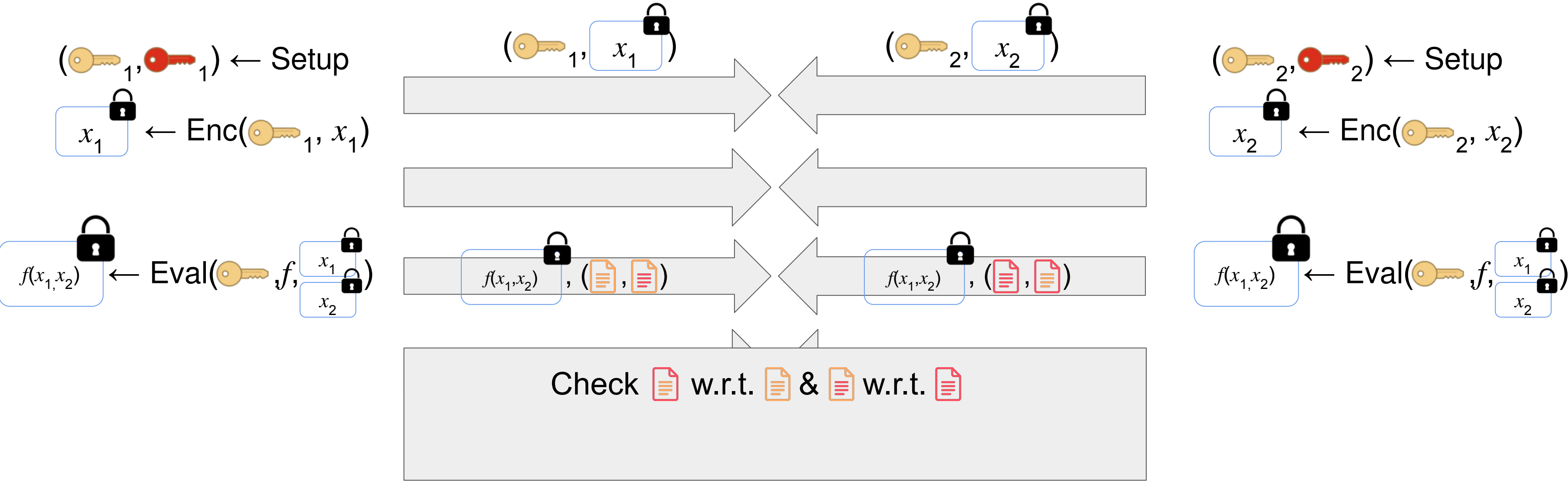
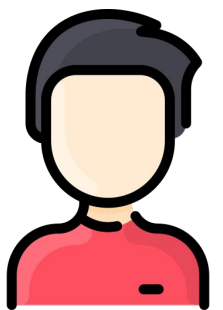
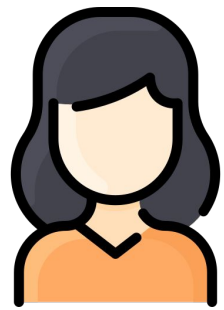
$(\text{key}_2, \text{red_key}_2) \leftarrow \text{Setup}$

$x_2 \leftarrow \text{Enc}(\text{key}_2, x_2)$

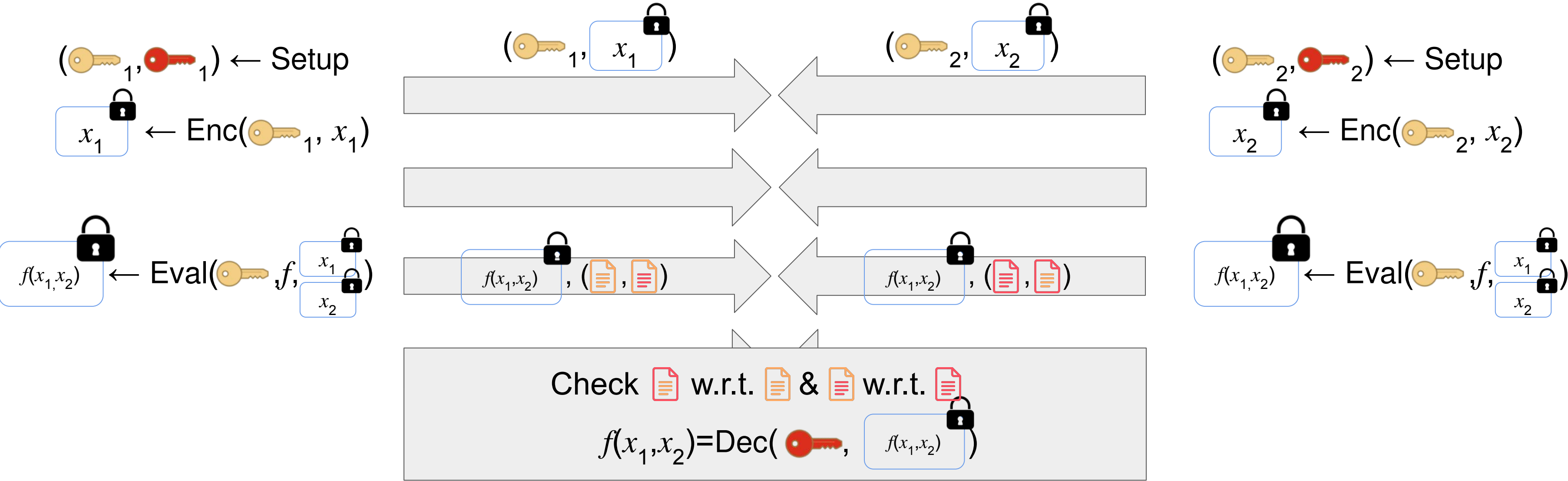
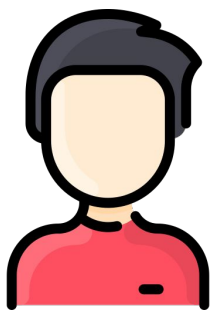
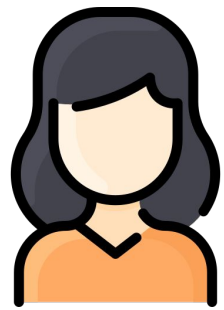
$f(x_1, x_2) \leftarrow \text{Eval}(\text{key}, f, \begin{matrix} x_1 \\ x_2 \end{matrix})$



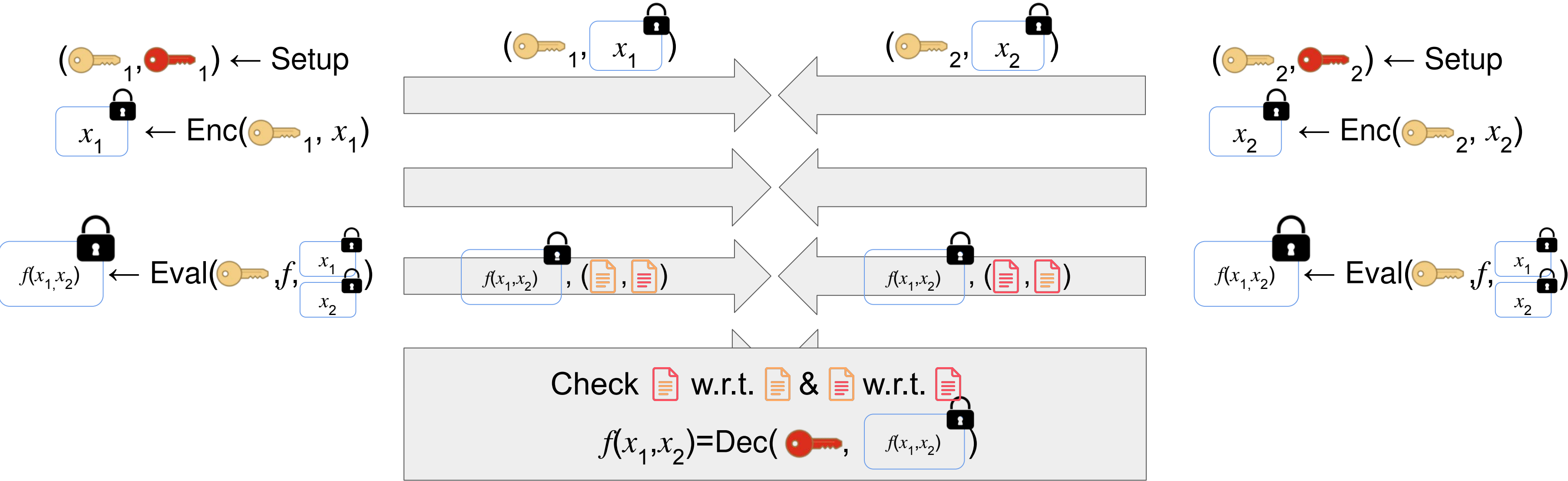
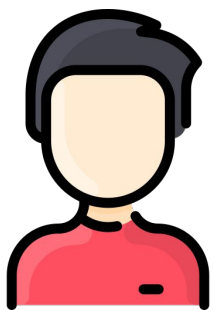
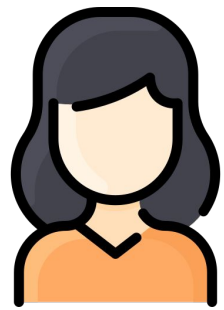
Round-Optimal and Communication-Efficient MPC



Round-Optimal and Communication-Efficient MPC



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⇒ Communication Complexity: L_{in} & L_{out} independent of f

Conclusion

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Thank You!