

Multi-Designated Receiver Signed Public Key Encryption

Ueli Maurer, Christopher Portmann, Guilherme Rito Eurocrypt 2022

Outline

1. Contributions

2. Public Key Encryption for Broadcast (PKEBC)

3. PKEBC Scheme Construction

4. Multi-Designated Receiver Signed Public Key Encryption (MDRS-PKE)

5. MDRS-PKE Scheme Construction



New types of Public Key Encryption (PKE) schemes:

- Multi-Designated Receiver Signed Public Key Encryption (MDRS-PKE);
- Public Key Encryption for Broadcast (PKEBC);

New types of Public Key Encryption (PKE) schemes:

- Multi-Designated Receiver Signed Public Key Encryption (MDRS-PKE);
- Public Key Encryption for Broadcast (PKEBC);

Constructions of both schemes from standard assumptions;

New types of Public Key Encryption (PKE) schemes:

- Multi-Designated Receiver Signed Public Key Encryption (MDRS-PKE);
- Public Key Encryption for Broadcast (PKEBC);

Constructions of both schemes from standard assumptions;

MDRS-PKE scheme yields Multi-Designated Verifier Signature scheme (MDVS) with Privacy of Identities;

The only prior construction is based on Verifiable Functional Encryption [1].

^[1] Damgård et al. (TCC '20).

Outline

1. Contributions

2. Public Key Encryption for Broadcast (PKEBC)

3. PKEBC Scheme Construction

4. Multi-Designated Receiver Signed Public Key Encryption (MDRS-PKE)

5. MDRS-PKE Scheme Construction

















 $({\tt rpk}_{{\sf B1}}, {\tt rsk}_{{\sf B1}})$

$({\tt rpk}_{{\sf B2}}, {\tt rsk}_{{\sf B2}})$

 $(\texttt{rpk}_{\texttt{B3}},\texttt{rsk}_{\texttt{B3}})$

 $({\tt rpk}_{{\sf B4}}, {\tt rsk}_{{\sf B4}})$

 $({\tt rpk_{B5}}, {\tt rsk_{B5}})$





2. PKEBC — Syntax







 $\textit{D}(\texttt{rsk}_{\texttt{B1}}, c) = ((\texttt{rpk}_{\texttt{B1}}, \texttt{rpk}_{\texttt{B4}}, \texttt{rpk}_{\texttt{B3}}), m)$



 $\textit{D}(\texttt{rsk}_{\texttt{B1}}, c) = ((\texttt{rpk}_{\texttt{B1}}, \texttt{rpk}_{\texttt{B4}}, \texttt{rpk}_{\texttt{B3}}), m)$

Syntax of decryption:

- Only rsk_{B1} is needed;
- Outputs vector of receivers' rpk.

2. PKEBC — Security Notions

Consistency

Robustness

Confidentiality (IND-CCA-2 security)



2. PKEBC — Security Notions

Consistency

Robustness

Confidentiality (IND-CCA-2 security)

+ Anonymity (IK-CCA-2 security)

















 $\textit{D}(\texttt{rsk}_{\texttt{B1}}, c) = ((\texttt{rpk}_{\texttt{B1}}, \texttt{rpk}_{\texttt{B3}}, \texttt{rpk}_{\texttt{B5}}), m)$



Eurocrypt' 22 6/35















 $\textit{D}(\texttt{rsk}_{\textsf{B1}}, c) = ((\texttt{rpk}_{\textsf{B1}}, \texttt{rpk}_{\textsf{B4}}), m)$



 $\textit{D}(\texttt{rsk}_{\textsf{B1}}, c) = ((\texttt{rpk}_{\textsf{B1}}, \texttt{rpk}_{\textsf{B4}}), m)$



2. PKEBC — Confidentiality (IND-CCA-2 Security)



2. PKEBC — Confidentiality (IND-CCA-2 Security)



2. PKEBC — Confidentiality (IND-CCA-2 Security)



2. PKEBC — Anonymity (IK-CCA-2 Security)



2. PKEBC — Anonymity (IK-CCA-2 Security)





2. PKEBC — Anonymity (IK-CCA-2 Security)





2. PKEBC — Anonymity (IK-CCA-2 Security)


2. PKEBC — Anonymity (IK-CCA-2 Security)





Outline

1. Contributions

2. Public Key Encryption for Broadcast (PKEBC)

3. PKEBC Scheme Construction

4. Multi-Designated Receiver Signed Public Key Encryption (MDRS-PKE)

5. MDRS-PKE Scheme Construction



3. PKEBC Scheme Construction Roadmap

Recall Naor-Yung's IND-CCA-1 PKE scheme construction [2];

Generalization to (non IK-CCA-2 secure) PKEBC scheme;

Making the PKEBC scheme IK-CCA-2 secure.

[2] Naor and Yung (STOC '90).

Building blocks:

- (IND-CPA secure) PKE scheme CPA = (*Gen*, *Enc*, *Dec*);
- NIZK = $(Gen_{CRS}, Prv, Vfy);$

Building blocks:

- (IND-CPA secure) PKE scheme CPA = (*Gen*, *Enc*, *Dec*);
- NIZK = $(Gen_{CRS}, Prv, Vfy);$

Construction of IND-CCA-1 secure PKE scheme $\Pi = (Gen, Enc, Dec)$:

П.*Gen*:

$$\begin{split} & \left((\mathtt{pk}_0, \mathtt{sk}_0), (\mathtt{pk}_1, \mathtt{sk}_1) \right) \leftarrow (\mathsf{CPA}.\textit{Gen}, \mathsf{CPA}.\textit{Gen}); \\ & \mathtt{crs} \leftarrow \mathsf{NIZK}.\textit{Gen}_{\textit{CRS}}; \\ & \mathsf{Output} \; \Big(\mathtt{pk} \coloneqq (\mathtt{crs}, \mathtt{pk}_0, \mathtt{pk}_1), \mathtt{sk} \coloneqq (\mathtt{sk}_0, \mathtt{pk}) \Big). \end{split}$$

Output (p, c_0, c_1) .

$$\begin{split} \Pi.\textit{Enc}(\texttt{pk} \coloneqq (\texttt{crs},\texttt{pk}_0,\texttt{pk}_1),m): \\ (c_0,c_1) \leftarrow (\texttt{CPA}.\textit{Enc}_{\texttt{pk}_0}(m),\texttt{CPA}.\textit{Enc}_{\texttt{pk}_1}(m)); \\ p \leftarrow \texttt{NIZK}.\textit{Prv}(\texttt{crs}, \\ \texttt{stmt} \coloneqq \texttt{``There is a message } m \texttt{ such that } c_0 \texttt{ and } c_1 \texttt{ are encryptions of } m \texttt{ under } \texttt{pk}_0 \\ \texttt{and } \texttt{pk}_1,\texttt{resp.''}, \\ \texttt{w} \coloneqq (m,\texttt{Encryption Randomness})); \end{split}$$

```
\begin{split} \Pi.\textit{Enc}(\mathtt{pk} \coloneqq (\mathtt{crs}, \mathtt{pk}_0, \mathtt{pk}_1), m) &: \\ (c_0, c_1) \leftarrow (\mathtt{CPA}.\textit{Enc}_{\mathtt{pk}_0}(m), \mathtt{CPA}.\textit{Enc}_{\mathtt{pk}_1}(m)); \\ p \leftarrow \mathtt{NIZK}.\textit{Prv}(\mathtt{crs}, \\ \texttt{stmt} \coloneqq \texttt{``There is a message } m \texttt{ such that } c_0 \texttt{ and } c_1 \texttt{ are encryptions of } m \texttt{ under } \mathtt{pk}_0 \\ \texttt{and } \mathtt{pk}_1, \texttt{resp.''}, \\ \texttt{w} \coloneqq (m, \mathtt{Encryption Randomness})); \\ \mathsf{Output} (p, c_0, c_1). \end{split}
```

```
\begin{split} \Pi.\textit{Dec}(\texttt{sk} \coloneqq (\texttt{sk}_0,\texttt{pk}), c \coloneqq (p, c_0, c_1)): \\ \text{Output} \perp \text{ if NIZK}.\textit{Vfy}(\texttt{crs},\texttt{stmt}, p) = \texttt{invalid}; \\ m \leftarrow \mathsf{CPA}.\textit{Dec}_{\texttt{sk}_0}(c_0); \\ \text{Output} \ m \text{ otherwise}. \end{split}
```

```
\begin{split} \Pi.\textit{Dec}(\texttt{sk} \coloneqq (\texttt{sk}_0,\texttt{pk}), c \coloneqq (p, c_0, c_1)): \\ \text{Output} \perp \text{if NIZK}.\textit{Vfy}(\texttt{crs},\texttt{stmt}, p) = \texttt{invalid}; \\ m \leftarrow \mathsf{CPA}.\textit{Dec}_{\texttt{sk}_0}(c_0); \\ \text{Output} \ m \text{ otherwise}. \end{split}
```

Simulation Sound NIZK \Rightarrow PKE scheme is IND-CCA-2 secure [3].

[3]: Sahai, FOCS '99

П.Setup:

 $crs \leftarrow NIZK.Gen_{CRS};$ Output crs;

П.**Gen**:

$$\begin{split} & \Big((\mathtt{pk}_0, \mathtt{sk}_0), (\mathtt{pk}_1, \mathtt{sk}_1) \Big) \leftarrow (\mathsf{CPA}.\textit{Gen}, \mathsf{CPA}.\textit{Gen}); \\ & \underbrace{\mathtt{crs} \leftarrow \mathsf{NIZK}.\textit{Gen}_{\mathit{CRS}};}_{\mathsf{Output}} \\ & \mathsf{Output} \ \Big(\mathtt{rpk} \coloneqq (\mathtt{pk}_0, \mathtt{pk}_1), \mathtt{rsk} \coloneqq (\mathtt{sk}_0, \mathtt{rpk}) \Big). \end{split}$$

$$\begin{split} \Pi.\textit{Enc}(\texttt{pp} \coloneqq \texttt{crs}, \vec{v} \coloneqq \big(\texttt{rpk}_1 \coloneqq (\texttt{pk}_{1,0},\texttt{pk}_{1,1}), \dots, \texttt{rpk}_{|\vec{v}|} \coloneqq (\texttt{pk}_{|\vec{v}|,0},\texttt{pk}_{|\vec{v}|,1})\big), m) &: \\ (c_{j,0}, c_{j,1}) \leftarrow (\mathsf{CPA}.\textit{Enc}_{\texttt{pk}_{j,0}}(m), \mathsf{CPA}.\textit{Enc}_{\texttt{pk}_{j,1}}(m)) \text{, for each } j \in \{1, \dots, |\vec{v}|\} \\ p \leftarrow \mathsf{NIZK}.\textit{Prv}\big(\texttt{crs}, \\ \texttt{stmt} \coloneqq \texttt{``There is a message } m \text{ such that for all } j \in \{1, \dots, |\vec{v}|\}, \text{ and all } b \in \{0, 1\}, \\ c_{j,b} \text{ is an encryption of } m \text{ under } v_{j,b}.", \\ \texttt{w} \coloneqq (m, \texttt{Encryption Randomness})\big); \\ \mathsf{Output} (p, \vec{c} \coloneqq \big((c_{1,0}, c_{1,1}), \dots, (c_{|\vec{v}|,0}, c_{|\vec{v}|,1})\big), \vec{v}). \end{split}$$

 $\Pi. Enc(\mathbf{pp} \coloneqq \mathbf{crs}, \vec{v} \coloneqq (\mathbf{rpk}_1 \coloneqq (\mathbf{pk}_{1,0}, \mathbf{pk}_{1,1}), \dots, \mathbf{rpk}_{|\vec{v}|} \coloneqq (\mathbf{pk}_{|\vec{v}|,0}, \mathbf{pk}_{|\vec{v}|,1})), m):$ $(c_{j,0}, c_{j,1}) \leftarrow (\mathsf{CPA}.\mathit{Enc}_{\mathsf{pk}_{i,0}}(m), \mathsf{CPA}.\mathit{Enc}_{\mathsf{pk}_{i,1}}(m))$, for each $j \in \{1, \ldots, |\vec{v}|\}$ $p \leftarrow \mathsf{NIZK}.Prv(\operatorname{crs.}$ stmt := "There is a message m such that for all $i \in \{1, \dots, |\vec{v}|\}$, and all $b \in \{0, 1\}$, $c_{i,b}$ is an encryption of m under $v_{i,b}$." $\mathbf{w} \coloneqq (m, \mathsf{Encryption Randomness})$; Output $(p, \vec{c} := ((c_{1,0}, c_{1,1}), \dots, (c_{|\vec{v}|,0}, c_{|\vec{v}|,1})), \vec{v}).$ $\Pi. Dec(pp \coloneqq crs, rsk \coloneqq (sk_0, rpk), c \coloneqq (p, \vec{c} \coloneqq ((c_{1,0}, c_{1,1}), \dots, (c_{|\vec{v}|, 0}, c_{|\vec{v}|, 1})), \vec{v})):$ Output \perp if NIZK. *Vfy*(crs, stmt, p) = invalid; Let $i \in \{1, \ldots, |\vec{v}|\}$ be (the least number) such that $v_i = rpk$; Output \perp if there is no such *i*:

 $m \leftarrow \mathsf{CPA}.\mathsf{Dec}_{\mathsf{sk}_0}(c_{i,0});$ Output (\vec{v}, m) otherwise.



Exists from Standard Assumptions





Exists from Standard Assumptions





Exists from Standard Assumptions







Exists from Standard Assumptions





Exists from Standard Assumptions

Does not exist from Standard Assumptions

Main idea:

Add a (Binding) Commitment to \vec{v} (vector of receivers' public keys) and m;

Main idea:

Add a (Binding) Commitment to \vec{v} (vector of receivers' public keys) and m;

Encrypt \vec{v} , m and commitment's randomness to each receiver;



Building Blocks:

- (Statistically Binding) Commitment scheme CS = (*Gen_{CRS}*, *Commit*, *Verify*);
- (IND-CPA and IK-CPA secure) PKE scheme CPA = (Gen, Enc, Dec);
- (Simulation Sound) NIZK = (Gen_{CRS}, Prv, Vfy) .

П.Setup:

 $\begin{array}{l} \mathtt{crs}_{\mathsf{NIZK}} \leftarrow \mathsf{NIZK}. \textit{Gen}_{\textit{CRS}};\\ \mathtt{crs}_{\mathsf{CS}} \leftarrow \mathsf{CS}. \textit{Gen}_{\textit{CRS}};\\ \mathsf{Output}\; (\mathtt{crs}_{\mathsf{NIZK}}, \mathtt{crs}_{\mathsf{CS}}); \end{array}$

П.*Gen*:

$$\begin{split} & \Big((\mathtt{pk}_0, \mathtt{sk}_0), (\mathtt{pk}_1, \mathtt{sk}_1) \Big) \leftarrow (\mathsf{CPA}.\textit{Gen}, \mathsf{CPA}.\textit{Gen}); \\ & \mathsf{Output} \; \Big(\mathtt{rpk} := (\mathtt{pk}_0, \mathtt{pk}_1), \mathtt{rsk} := (\mathtt{sk}_0, \mathtt{rpk}) \Big). \end{split}$$

$$\begin{split} \Pi.\textit{Enc}(\texttt{pp} \coloneqq (\texttt{crs}_{\mathsf{NIZK}},\texttt{crs}_{\mathsf{CS}}), \vec{v} \coloneqq \big(\texttt{rpk}_0 \coloneqq (\texttt{pk}_{1,0},\texttt{pk}_{1,1}), \dots, \texttt{rpk}_{|\vec{v}|} \coloneqq (\texttt{pk}_{|\vec{v}|,0},\texttt{pk}_{|\vec{v}|,1})\big), m):\\ \texttt{comm} \leftarrow \texttt{CS}.\textit{Commit}(\texttt{crs}_{\mathsf{CS}}, (\vec{v}, m); \rho);\\ (c_{j,0}, c_{j,1}) \leftarrow (\texttt{CPA}.\textit{Enc}_{\texttt{pk}_{j,0}}(\rho, \vec{v}, m), \texttt{CPA}.\textit{Enc}_{\texttt{pk}_{j,1}}(\rho, \vec{v}, m)), \text{ for each } i \in \{1, \dots, |\vec{v}|\};\\ p \leftarrow \mathsf{NIZK}.\textit{Prv}(\texttt{crs}_{\mathsf{NIZK}},\\ \texttt{stmt} \coloneqq \texttt{``There is a message } m, \texttt{a vector } \vec{v} \texttt{ and a sequence } \rho \texttt{ such that:}\\ \texttt{for all } i \in \{1, \dots, |\vec{v}|\}, b \in \{0, 1\}, c_{i,b} \texttt{ is an encryption of } (\rho, \vec{v}, m) \texttt{ under } v_{i,b},\\ \texttt{and } \texttt{comm} = \texttt{CS}.\textit{Commit}(\texttt{crs}_{\mathsf{CS}}, (\vec{v}, m); \rho).",\\ \texttt{w} \coloneqq (m, \vec{v}, \rho, \texttt{Encryption Randomness}));\\ \texttt{Output}(\texttt{comm}, p, \vec{c}). \end{split}$$

```
\begin{split} \Pi.\textit{Dec}(\texttt{pp} \coloneqq (\texttt{crs}_{\mathsf{NIZK}},\texttt{crs}_{\mathsf{CS}}),\texttt{rsk} \coloneqq (\texttt{sk}_0,\texttt{rpk}), c \coloneqq (\texttt{comm}, p, \vec{c})):\\ \texttt{Output} \perp \texttt{if NIZK}.\textit{Vfy}(\texttt{crs},\texttt{stmt}, p) = \texttt{invalid};\\ \texttt{Find the least} i \in \{1, \ldots, |\vec{v}|\} \texttt{ with }\\ (\rho, \vec{v}, m) \leftarrow \mathsf{CPA}.\textit{Dec}_{\texttt{sk}_0}(c_{i,0}) \texttt{ satisfying}:\\ (\rho, \vec{v}, m) \neq \bot;\\ v_i = \texttt{rpk};\\ c.\texttt{comm} = \mathsf{CS}.\textit{Commit}(\texttt{crs}_{\mathsf{CS}}, (\vec{v}, m); \rho);\\ \texttt{Output} \perp \texttt{if there is no such } i; \end{split}
```

Output (\vec{v}, m) otherwise.











Outline

1. Contributions

2. Public Key Encryption for Broadcast (PKEBC)

3. PKEBC Scheme Construction

4. Multi-Designated Receiver Signed Public Key Encryption (MDRS-PKE)

5. MDRS-PKE Scheme Construction















ETH zürich

Eurocrypt' 22 24/35





 $\textit{D}(\texttt{rsk}_{\texttt{B1}}, c) = (\texttt{spk}_{\texttt{A1}}, (\texttt{rpk}_{\texttt{B1}}, \texttt{rpk}_{\texttt{B2}}), m)$

Syntax of decryption:

- Only rsk_{B1} is needed;
- Outputs sender's spk and vector of receivers' rpk.

4. MDRS-PKE — Security Notions

Off-The-Record

Unforgeability

Consistency

Confidentiality (IND-CCA-2 security)

+ Anonymity (IK-CCA-2 security)




$$D(\texttt{rsk}_{\texttt{B5}}, c) = (\texttt{spk}_{\texttt{A2}}, (\texttt{rpk}_{\texttt{B4}}, \texttt{rpk}_{\texttt{B5}}, \texttt{rpk}_{\texttt{B2}}), m$$



$$D(\mathbf{rsk}_{\mathsf{B5}}, c) = (\mathbf{spk}_{\mathsf{A2}}, (\mathbf{rpk}_{\mathsf{B4}}, \mathbf{rpk}_{\mathsf{B5}}, \mathbf{rpk}_{\mathsf{B2}}), m)$$



 $\mathcal{D}(\texttt{rsk}_{\texttt{B5}}, c) = (\texttt{spk}_{\texttt{A2}}, (\texttt{rpk}_{\texttt{B4}}, \texttt{rpk}_{\texttt{B5}}, \texttt{rpk}_{\texttt{B2}}), m)$



 $c = \textit{Forge}(\texttt{rsk}_{\texttt{B5}}, \texttt{spk}_{\texttt{A2}}, (\texttt{rpk}_{\texttt{B4}}, \texttt{rpk}_{\texttt{B5}}, \texttt{rpk}_{\texttt{B2}}), m)$





4. MDRS-PKE — Authenticity (Existential Unforgeability)







4. MDRS-PKE — Authenticity (Existential Unforgeability)



4. MDRS-PKE — Authenticity (Existential Unforgeability)

















Eurocrypt' 22 28/35













ETH zürich

Eurocrypť 22 29/35









ETH zürich

Eurocrypť 22 30/35



Outline

1. Contributions

2. Public Key Encryption for Broadcast (PKEBC)

3. PKEBC Scheme Construction

4. Multi-Designated Receiver Signed Public Key Encryption (MDRS-PKE)

5. MDRS-PKE Scheme Construction



Building Blocks:

- MDVS = (*Setup*, *Gen*_{Sig}, *Gen*_{Vrf}, *Sign*, *Vfy*);
- (IK-CCA-2 secure) $PKEBC = (Setup, Gen_{Snd}, Gen_{Rcv}, Enc, Dec);$

Building Blocks:

- MDVS = (*Setup*, *Gen*_{Sig}, *Gen*_{Vrf}, *Sign*, *Vfy*);
- (IK-CCA-2 secure) $PKEBC = (Setup, Gen_{Snd}, Gen_{Rcv}, Enc, Dec);$

Main idea: "Sign-then-Encrypt"

Building Blocks:

- $MDVS = (Setup, Gen_{Sig}, Gen_{Vrf}, Sign, Vfy);$
- (IK-CCA-2 secure) $PKEBC = (Setup, Gen_{Snd}, Gen_{Rcv}, Enc, Dec);$

Main idea: "Sign-then-Encrypt"

Use MDVS to sign vector of receivers and message;

Then, use PKEBC to encrypt the sender's and all receivers' public keys, message and the signature.















Thank you!

Bibliography

[1] Ivan Damgård, Helene Haagh, Rebekah Mercer, Anca Nitulescu, Claudio Orlandi, and Sophia Yakoubov.

Stronger security and constructions of multi-designated verifier signatures.

In Rafael Pass and Krzysztof Pietrzak, editors, *TCC 2020, Part II*, volume 12551 of *LNCS*, pages 229–260. Springer, Heidelberg, November 2020.

[2] Moni Naor and Moti Yung.

Public-key cryptosystems provably secure against chosen ciphertext attacks. In 22nd ACM STOC, pages 427–437. ACM Press, May 1990.

[3] Amit Sahai.

Non-malleable non-interactive zero knowledge and adaptive chosen-ciphertext security. In *40th FOCS*, pages 543–553. IEEE Computer Society Press, October 1999.


Outline

6. Full MDRS-PKE construction



6. MDRS-PKE Scheme Construction

```
\begin{split} \text{MDRS-PKE}. & \text{Setup:} \\ & \text{pp}_{\text{MDVS}} \leftarrow \text{MDVS}. & \text{Setup}(1^k); \\ & \text{pp}_{\text{PKEBC}} \leftarrow \text{PKEBC}. & \text{Setup}(1^k); \\ & \text{Output pp} = (\text{pp}_{\text{MDVS}}, \text{pp}_{\text{PKEBC}}); \end{split}
```

```
\begin{split} & \mathsf{MDRS}\text{-}\mathsf{PKE}.\textit{Gen}_{\mathit{Snd}}(\mathtt{pp} = (\mathtt{pp}_{\mathsf{MDVS}}, \mathtt{pp}_{\mathsf{FKEBC}})): \\ & (\mathtt{spk}_{\mathsf{MDVS}}, \mathtt{ssk}_{\mathsf{MDVS}}) \leftarrow \mathsf{MDVS}.\textit{Gen}_{\mathit{Sig}}(\mathtt{pp}_{\mathsf{MDVS}}); \\ & \mathsf{Output}\; (\mathtt{spk} \coloneqq \mathtt{spk}_{\mathsf{MDVS}}, \mathtt{ssk} \coloneqq (\mathtt{spk}, \mathtt{ssk}_{\mathsf{MDVS}})); \end{split}
```

```
\begin{split} \text{MDRS-PKE}. & \textit{Gen}_{\textit{Rcv}}(\texttt{pp} = (\texttt{pp}_{\text{MDVS}}, \texttt{pp}_{\text{PKEBC}})): \\ & (\texttt{vpk}_{\text{MDVS}}, \texttt{vsk}_{\text{MDVS}}) \leftarrow \text{MDVS}. \textit{Gen}_{V}(\texttt{pp}_{\text{MDVS}}); \\ & (\texttt{rpk}_{\text{PKEBC}}, \texttt{rsk}_{\text{PKEBC}}) \leftarrow \text{PKEBC}. \textit{Gen}(\texttt{pp}_{\text{PKEBC}}); \\ & \text{Output} \left(\texttt{rpk} \coloneqq (\texttt{vpk}_{\text{MDVS}}, \texttt{rpk}_{\text{PKEBC}}), \texttt{rsk} \coloneqq (\texttt{rpk}, (\texttt{vsk}_{\text{MDVS}}, \texttt{rsk}_{\text{PKEBC}})\right) \right); \end{split}
```

ETH zürich

6. MDRS-PKE Scheme Construction

```
\begin{aligned} &\mathsf{MDRS}\mathsf{-PKE}.\textit{Enc}(\mathsf{ssk}_i \coloneqq (\mathsf{spk}_i, \mathsf{ssk}_{\mathsf{MDVS}i}), \vec{v} \coloneqq \big(\mathsf{rpk}_1, \dots, \mathsf{rpk}_{|\vec{v}|}\big), m) \\ & (\mathsf{where } \mathsf{rpk}_i \coloneqq (\mathsf{vpk}_{\mathsf{MDVS}i}, \mathsf{rpk}_{\mathsf{PKEBC}i})) \end{aligned} \\ & \mathsf{Let } \vec{v}_{\mathsf{PKEBC}} = (\mathsf{rpk}_{\mathsf{PKEBC}1}, \dots, \mathsf{rpk}_{\mathsf{PKEBC}|\vec{v}|}); \\ & \mathsf{Let } \vec{v}_{\mathsf{MDVS}} = (\mathsf{vpk}_{\mathsf{MDVS}1}, \dots, \mathsf{vpk}_{\mathsf{MDVS}|\vec{v}|}); \\ & \sigma \leftarrow \mathsf{MDVS}.\textit{Sign}(\mathsf{ssk}_{\mathsf{MDVS}i}, \vec{v}_{\mathsf{MDVS}}, (\vec{v}_{\mathsf{PKEBC}}, m)); \\ & \mathsf{Output } \mathsf{PKEBC}.\textit{Enc}(\vec{v}_{\mathsf{PKEBC}}, (\mathsf{spk}_i, \vec{v}_{\mathsf{MDVS}}, m, \sigma)); \end{aligned}
```

6. MDRS-PKE Scheme Construction

$$\begin{split} \mathsf{MDRS}\text{-}\mathsf{PKE}.\textit{Dec}(\mathbf{rsk}_j) &:= (\mathbf{rpk}_j, (\mathbf{vsk}_{\mathsf{MDVS}_j}, \mathbf{rsk}_{\mathsf{PKEBC}_j})), c):\\ (\mathsf{where } \mathbf{rpk}_j &:= (\mathbf{vpk}_{\mathsf{MDVS}_j}, \mathbf{rpk}_{\mathsf{PKEBC}_j})) \\ & \left(\vec{v}_{\mathsf{PKEBC}}, (\mathbf{spk}_i, \vec{v}_{\mathsf{MDVS}}, m, \sigma)\right) \leftarrow \mathsf{PKEBC}.\textit{D}(\mathbf{rsk}_{\mathsf{PKEBC}_j}, c);\\ \mathsf{Output} \perp \mathsf{if} \left(\vec{v}_{\mathsf{PKEBC}}, (\mathbf{spk}_i, \vec{v}_{\mathsf{MDVS}}, m, \sigma)\right) = \perp \mathsf{or} |\vec{v}_{\mathsf{PKEBC}}| \neq |\vec{v}_{\mathsf{MDVS}}|;\\ \mathsf{Let} \ \vec{v} = \left((v_{\mathsf{MDVS}_1}, v_{\mathsf{PKEBC}_1}), \dots, (v_{\mathsf{MDVS}}|\vec{v}_{\mathsf{PKEBC}}|, v_{\mathsf{PKEBC}}|\vec{v}_{\mathsf{PKEBC}}|)\right);\\ \mathsf{Output} \perp \mathsf{if} \ \mathsf{rpk}_j \notin \vec{v} \ \mathsf{or} \ \mathsf{MDVS}.\textit{Vfy}(\mathsf{spk}_i, \mathsf{vsk}_{\mathsf{MDVS}_j}, \vec{v}_{\mathsf{MDVS}}, (\vec{v}_{\mathsf{PKEBC}}, m), \sigma) \neq \mathsf{valid};\\ \mathsf{Output} (\mathsf{spk}_i, \vec{v}, m); \end{split}$$