A Fast and Simple Partially Oblivious PRF, with Applications

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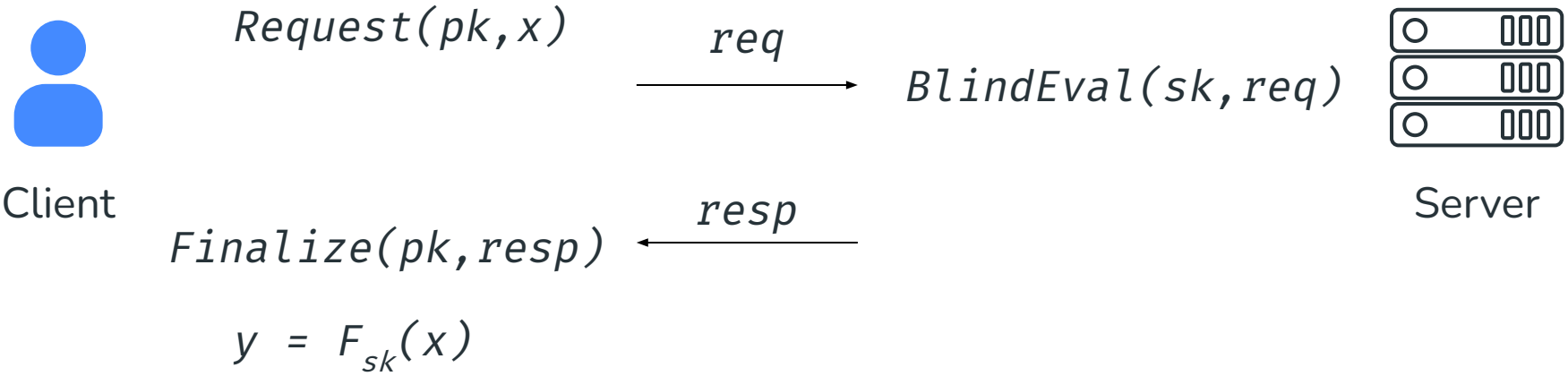
Stefano Tessaro Chris Wood

EUROCRYPT 2022

Background: Oblivious PRFs (OPRFs)

[FIPR TCC '05]
[JL TCC '09]

An OPRF allows for oblivious evaluation of a pseudorandom function (PRF)



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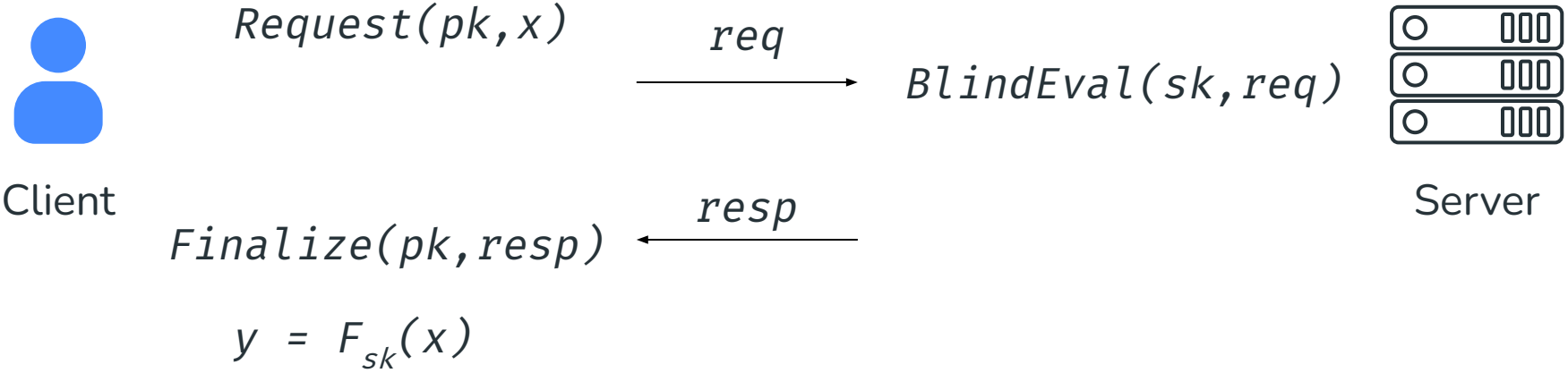

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Client

Used in many privacy-preserving applications:

- One-time-use authentication tokens [DGSTV PETS ‘18]
- Bucketized PSI for password breach alerting [LPASCR CCS ‘19]
- Password-authenticated key exchange [JKX EUROCRYPT ‘18]
- Online advertisement attribution [SHD ePrint ‘21]

$y = F_{sk}(x)$

Background: Standards-track 2HashDH OPRF

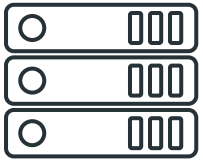
[JKK ASIACRYPT '14]

$F_{sk}(x) = H_2(x, H_1(x)^{sk})$

Client

$Request(pk=g^{sk}, x)$

$BlindEval(sk, req)$



Server

$Finalize(pk, resp)$

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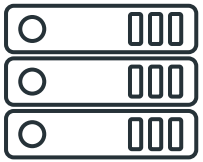

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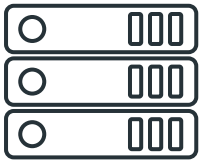

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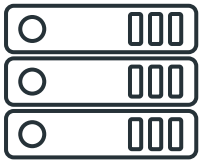

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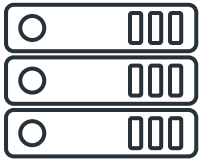

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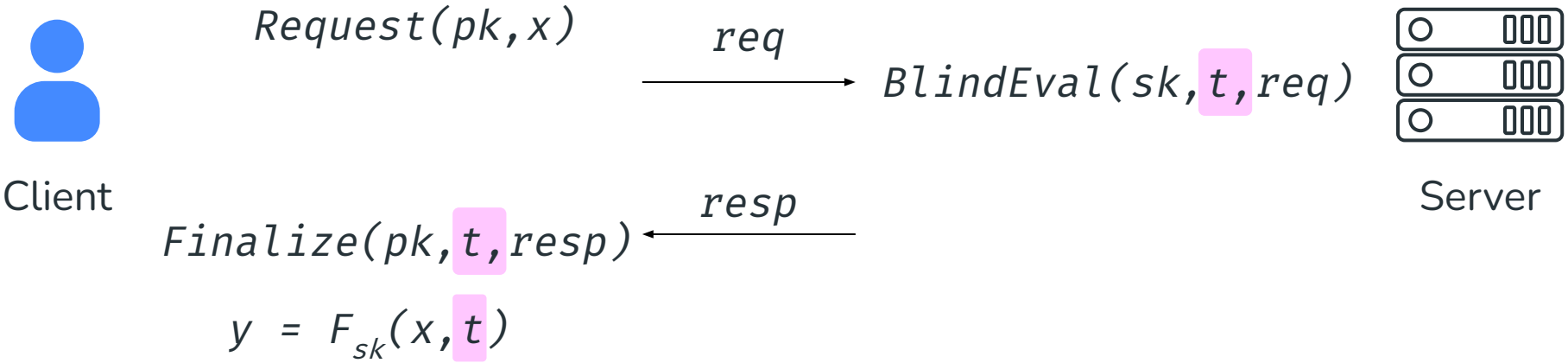
Background: Partially Oblivious PRFs (POPRFs)

[ECSJR USENIX Security ‘15]

A POPRF allows for oblivious evaluation of a pseudorandom function (PRF) on a hybrid input of secret data and public data (called a tag).



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  - POPRFs simplify use of OPRFs in existing applications
  - Previous candidate POPRFs rely on pairings
- 3HashSDHI: A new, simple POPRF
  - Shares similar structure to standards-track 2HashDH OPRF
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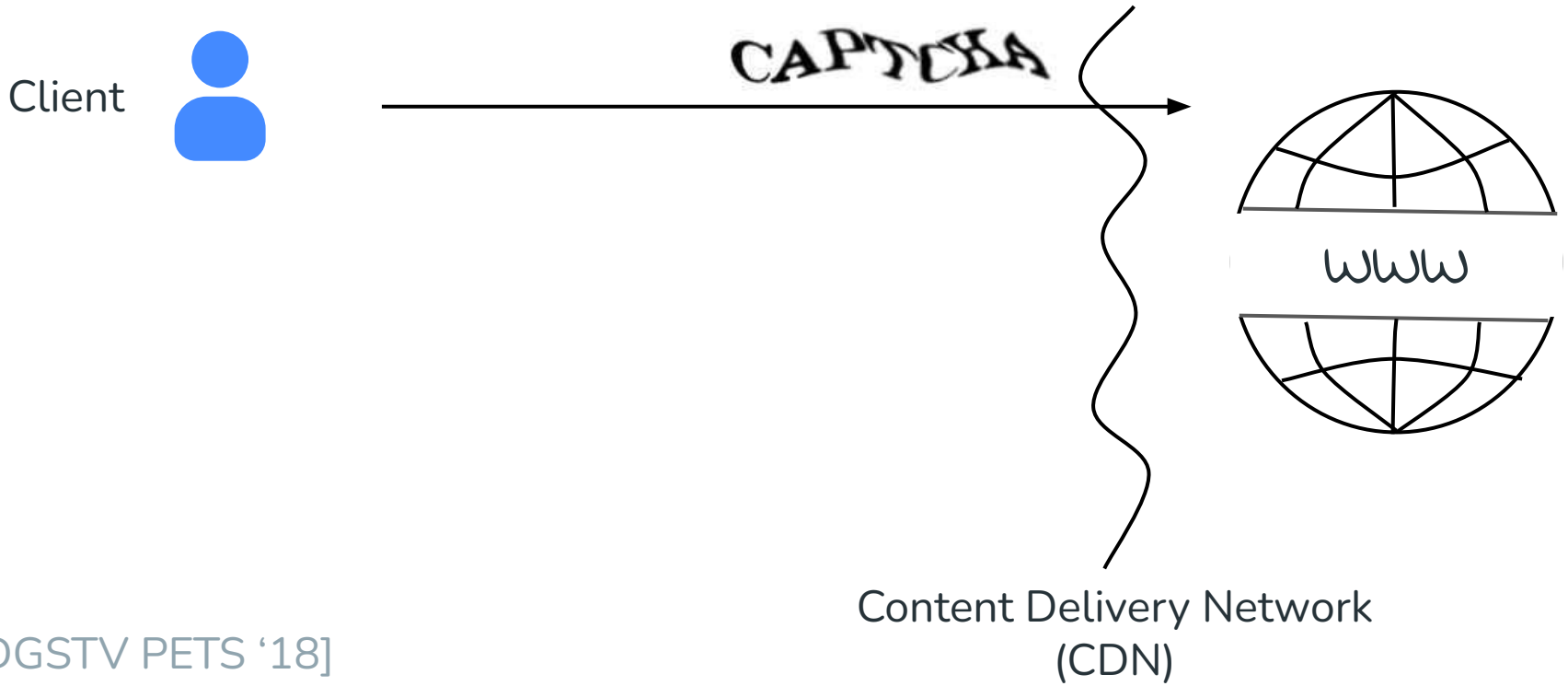
Application: PrivacyPass for bypassing CAPTCHAs

- CAPTCHAs are often used by CDNs to prevent botnets from attacking sites



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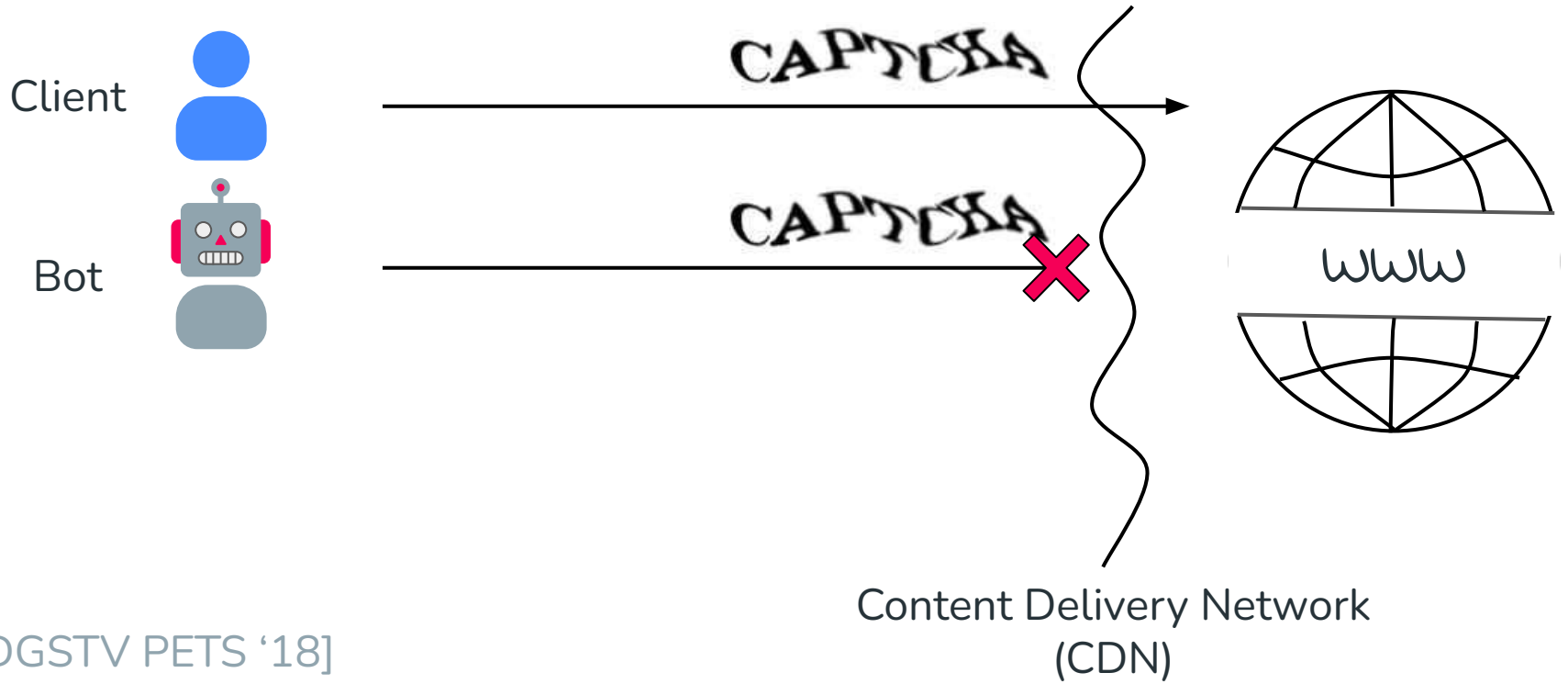

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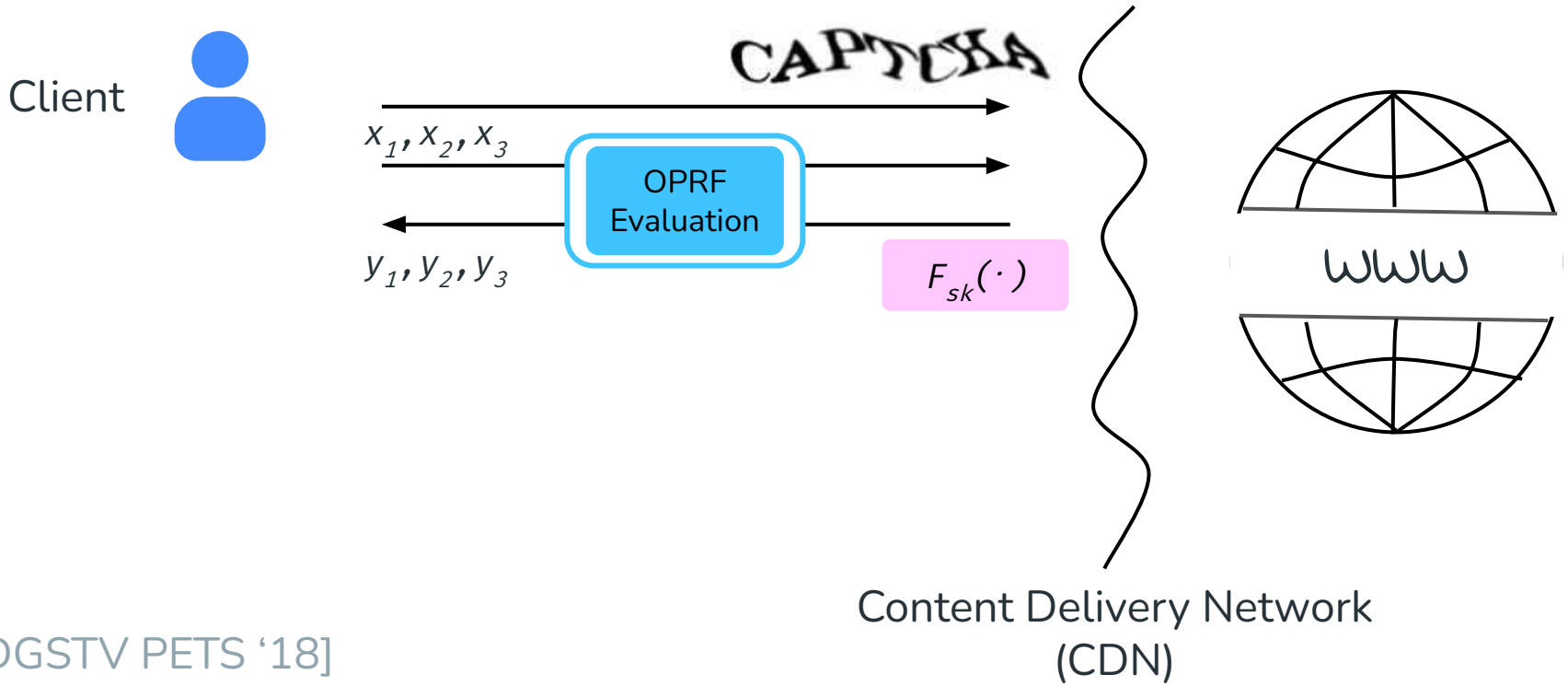

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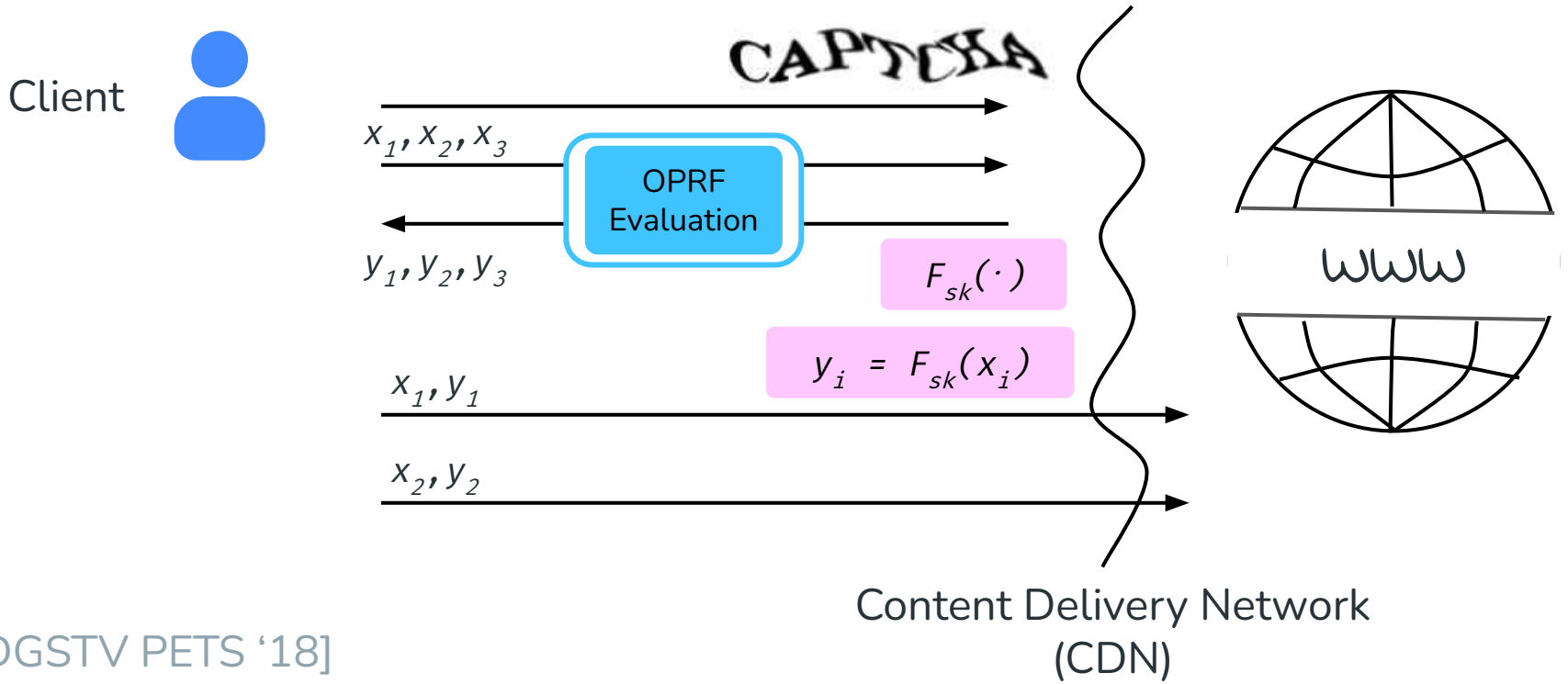

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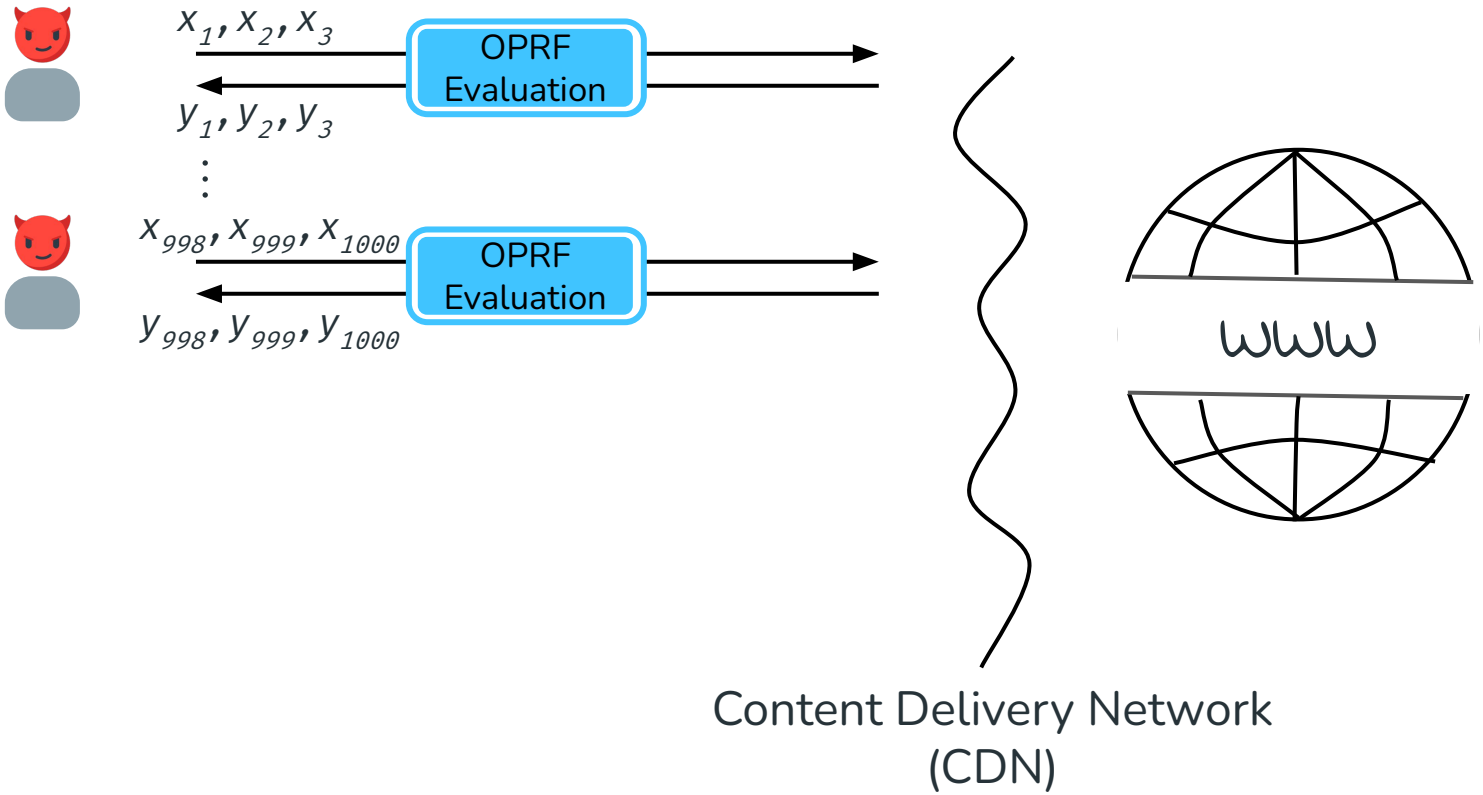
Attack against PrivacyPass: Token hoarding

- Human users collect enough evaluations over time to attack a site



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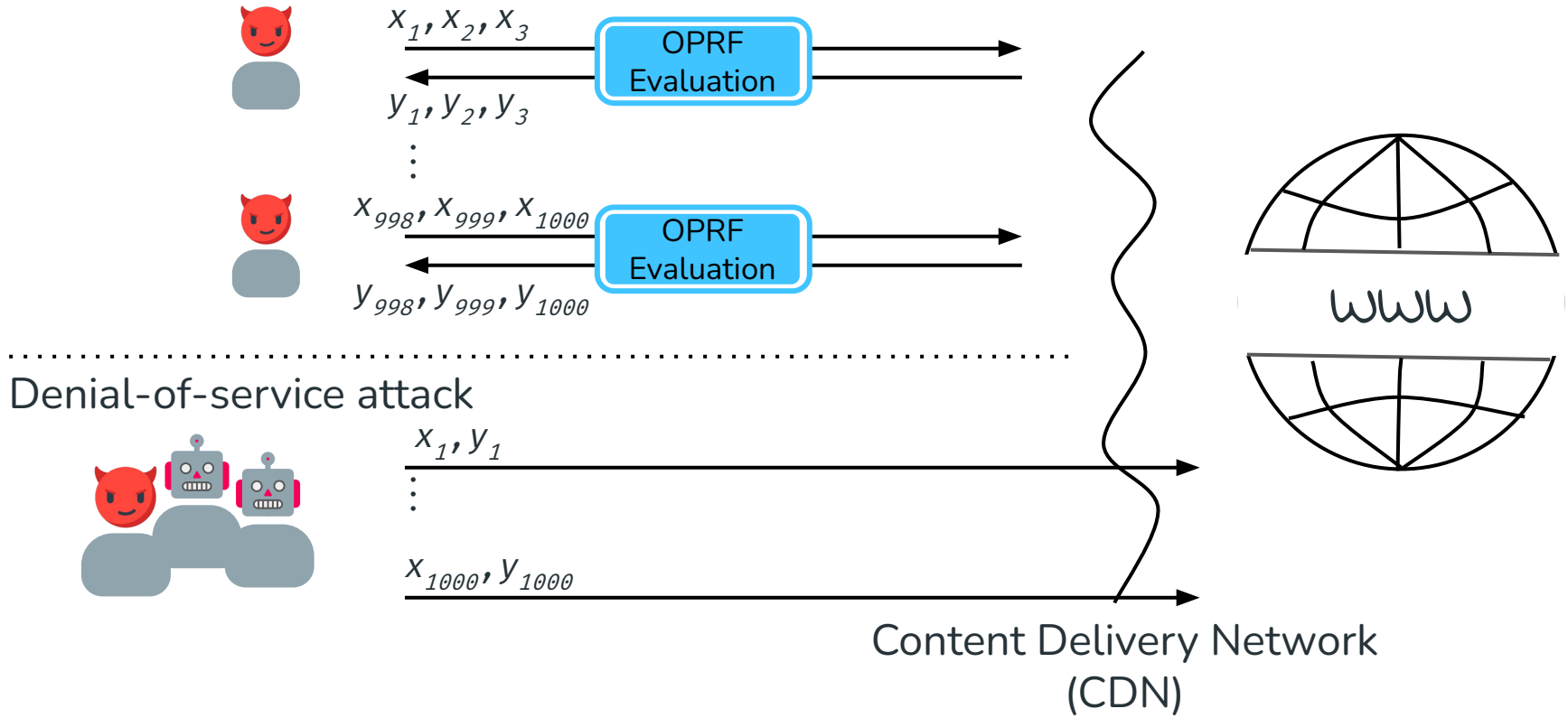
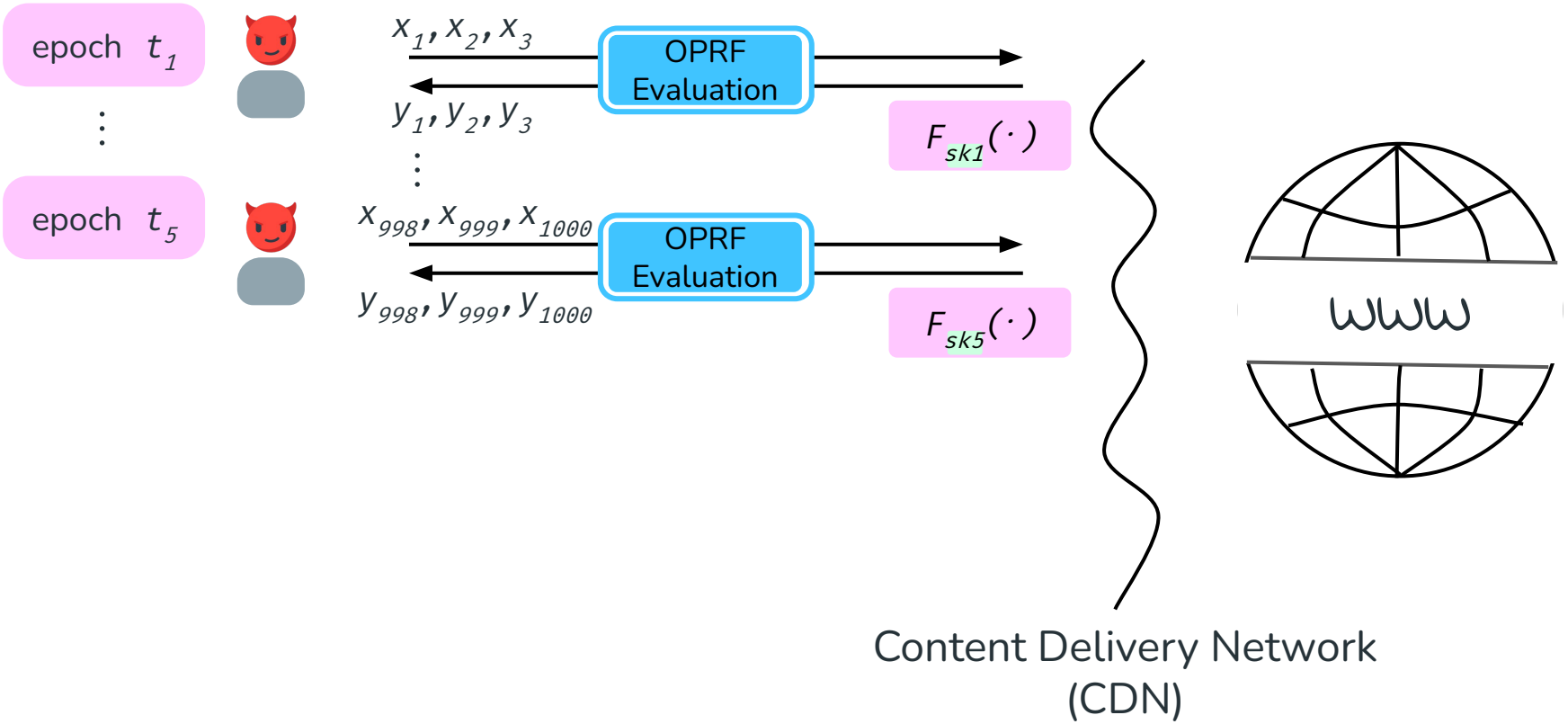


Token hoarding mitigation: OPRF per-epoch key

- Evaluations are provided using separate OPRF keys specified for a time epoch



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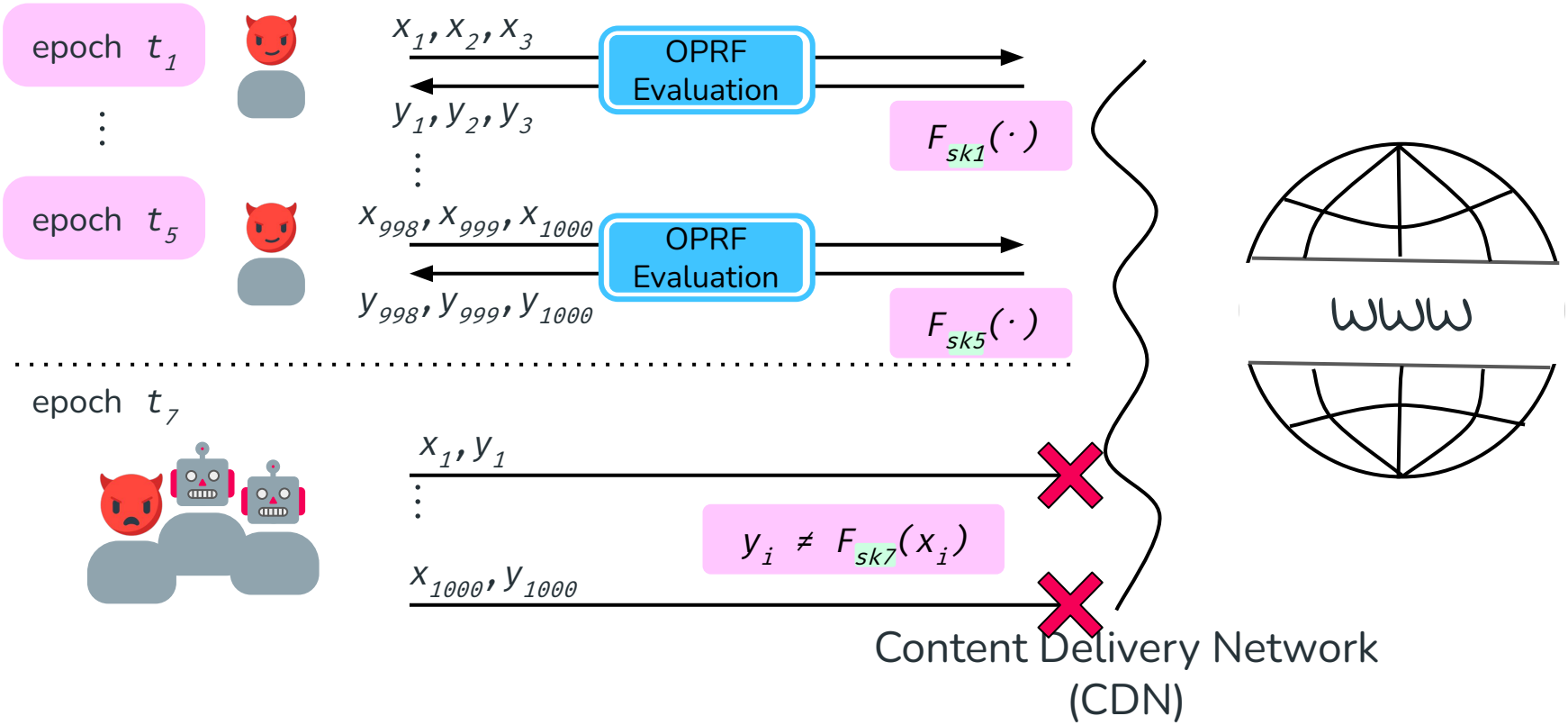

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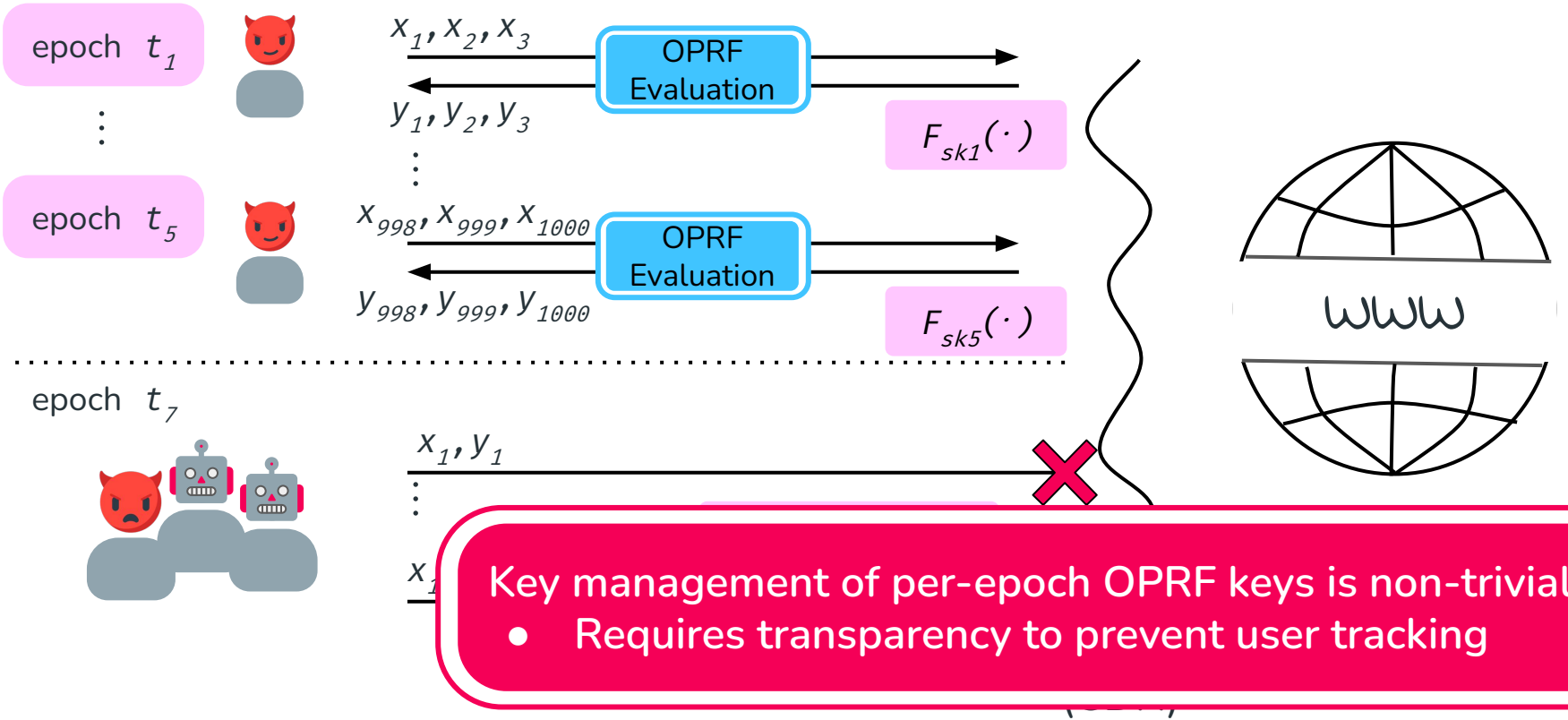
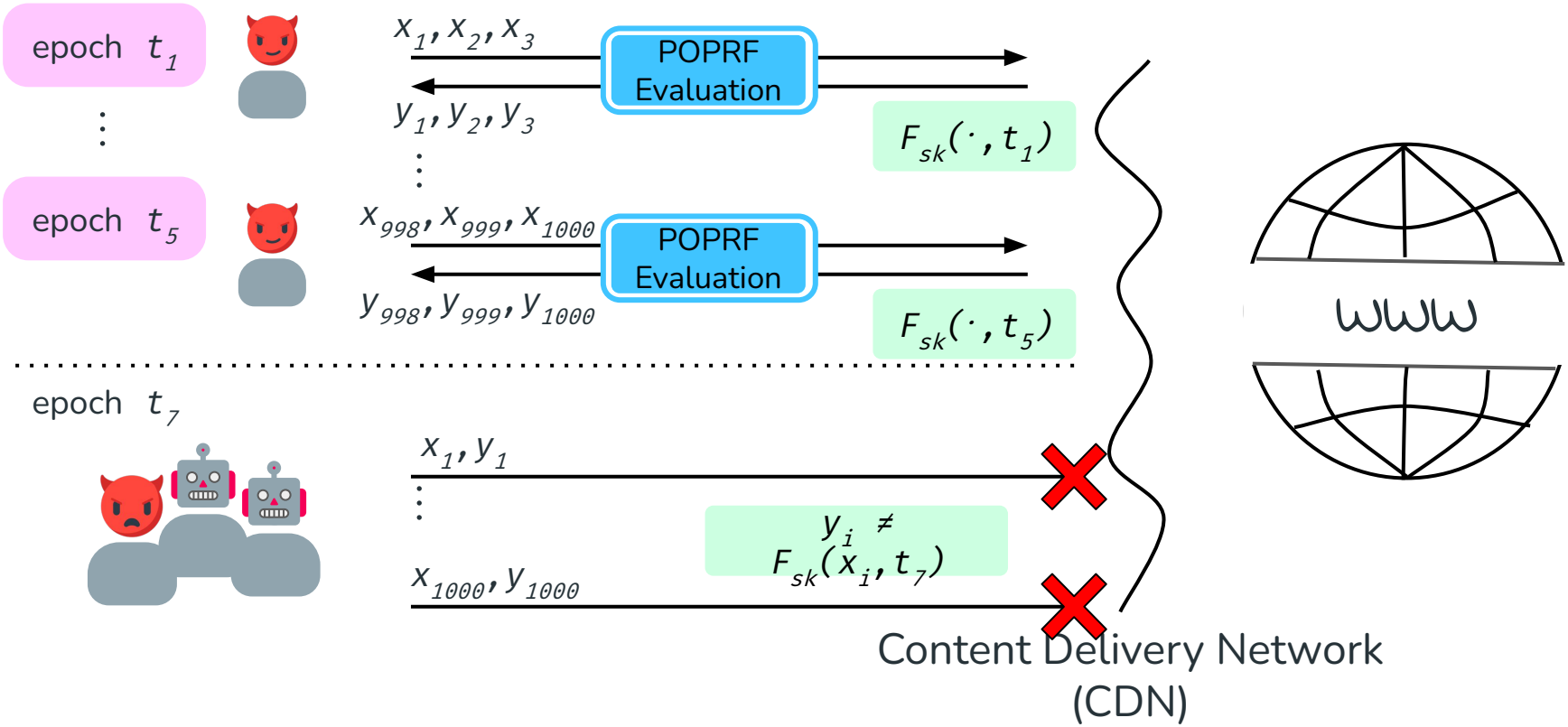


Token hoarding mitigation: POPRF w/ epoch tag

- Evaluations use a POPRF key and specify the epoch in the public tag



Motivation: POPRFs simplify many OPRF apps

Application	OPRF	POPRF
One-time-use authentication tokens [DGSTV PETS ‘18]	per-epoch key	epoch tag
Bucketized private set membership for password breach alerting [LPASCR CCS ‘19]	per-bucket key	bucket tag
Password-authenticated key exchange [JKX EUROCRYPT ‘18]	per-user key	user tag
Online advertisement attribution [SHD ePrint ‘21]	per-publisher key	publisher tag

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2HashDH-ext POPRF [JKR ePrint '18]

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- Generic transform OPRF to POPRF
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Can we build a verifiable POPRF from just a discrete-log hard group?

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Our construction

3HashSDHI POPRF [This work]

$$F_{sk}(x,t) = H_2(x,t,H_1(x)^{1/(sk+H_3(t))})$$

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DY PRF [DY PKC '05]

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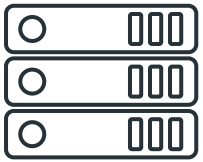

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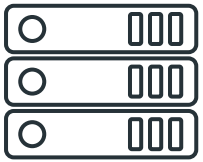

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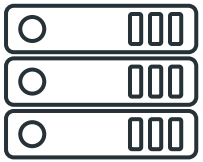

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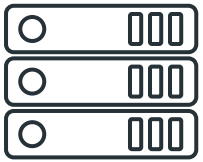

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3HashSDHI: Obli

Minimal delta from 2HashDH oblivious evaluation!

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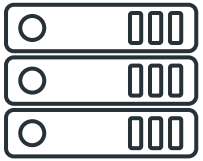

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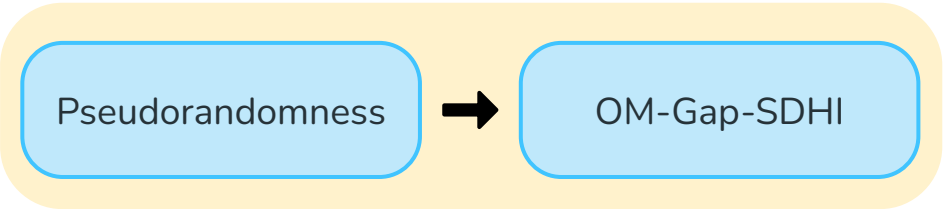

Standard model

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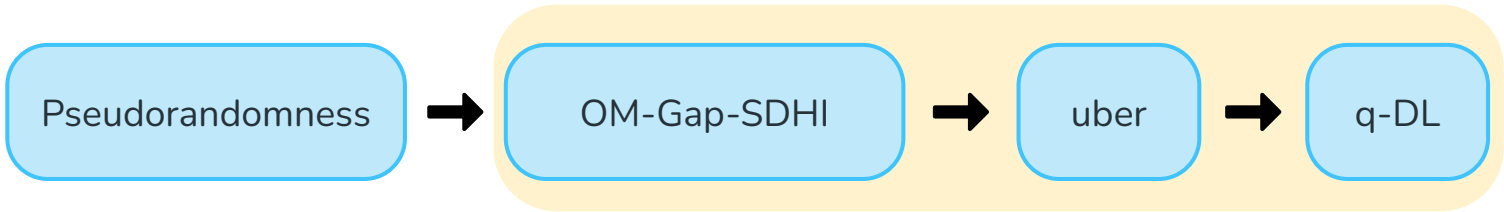

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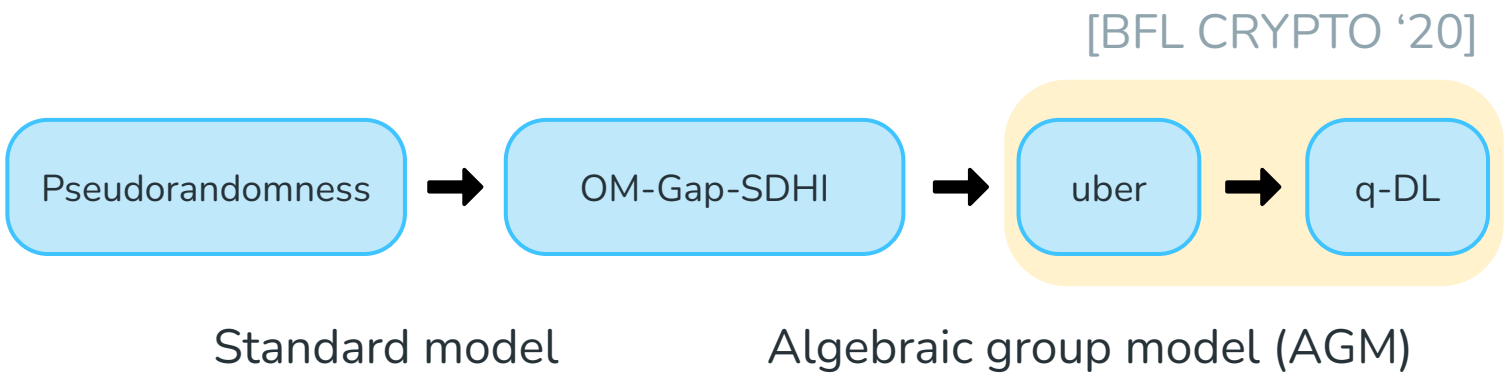

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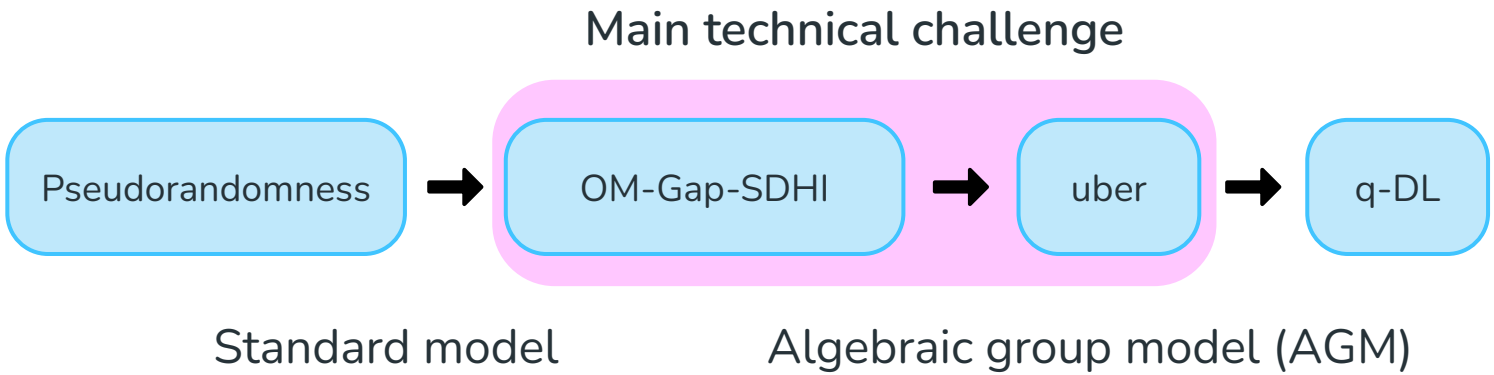

One-more (Gap) Strong Diffie-Hellman Inversion

OM-SDHI

$x \leftarrow_{\$} Z_p;\ \forall t,\ q_t \leftarrow 0$

$y_1,\ldots,y_m \leftarrow_{\$} (Z_p)^m$

$C_1,\ldots,C_l,t \leftarrow_{\$} A^{SDH}(g^x, g^{y_1},\ldots,g^{y_m})$

A *wins if* $q_t < l \wedge \forall i, \exists j, C_i = {}^{y_j}g^{1/(x+t)}$

Oracle SDH(B,t)

$q_t \leftarrow q_t + 1$

Return $B^{1/(x+t)}$

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Per-tag query counter!

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A *wins if* $q_t < l \wedge \forall i,\ \exists j,\ C_i = {}^{y_j}g^{1/(x+t)}$

Oracle SDH(B,t)

$q_t \leftarrow q_t + 1$

Return $B^{1/(x+t)}$

Per-tag query counter!

- Relation to POPRF security: Per-tag query counter ensures each blind evaluation results in the client learning at most one PRF evaluation restricted to the queried tag

One-more (Gap) Strong Diffie-Hellman Inversion

OM-SDHI

$x \leftarrow_{\$} Z_p$; $\forall t,\ q_t \leftarrow 0$

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- Challenge: Repeated queries to SDH that “mix” tags

Leads to non-trivial polynomial independence argument!

Crux: Handling “mixed” SDHI queries

Oracle SDH(B,t)

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Return $B^{1/(x+t)}$

	query 1	query 2	query 3
input element	$B_1 = g^{y_1}$		
input tag	t_1		
output	$R_1 = g^{\frac{y_1}{x+t_1}}$		

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input tag	t_1	t_2	t_1
output	$R_1 = g^{\frac{y_1}{x+t_1}}$	$R_2 = g^{\frac{y_2}{x+t_2}}$	

Repeated query to t_1 but does not mix tags!

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Repeated query to t_1 that mixes tags!

In reduction to uber assumption, need to show linear independence of exponents

Crux: Handling “mixed” SDHI queries

query 1 query 2 query 3

independence argument

t_1 t_2 t_3

Crux: Handling “mixed” SDHI queries

query 1 query 2 query 3

independence argument $t_1\ t_2\ t_3$

$$\frac{y_1}{x+t_1} \qquad \frac{y_2}{x+t_2} \qquad \frac{y_3}{x+t_1} + \frac{y_1}{(x+t_1)^2}$$

Crux: Handling “mixed” SDHI queries

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Idea: “Unmix” mixed query while preserving span

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$\frac{y_1}{x+t_1}$	$\frac{y_2}{x+t_2}$	$\frac{\alpha_1 y_1+\alpha_2 y_2+\alpha_3 y_3}{x+t_1} + \frac{y_1}{(x+t_1)^2}$		

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Same span!

Idea: “Unmix” mixed query while preserving span

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$\frac{y_1}{x+t_1}$	$\frac{y_2}{x+t_2}$	$\frac{y_3}{x+t_1} + \frac{y_1}{(x+t_1)^2}$	easy	2 1 0
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$\frac{y_1}{x+t_1}$	$\frac{y_2}{x+t_2}$	$\frac{\alpha_1 y_1+\alpha_2 y_2+\alpha_3 y_3}{x+t_1} + \frac{y_1}{(x+t_1)^2}$	easy	2 1 0

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$\frac{y_1}{x+t_1}$	$\frac{y_2}{x+t_2}$	$\frac{\alpha_1 y_1+\alpha_2 y_2+\alpha_3 y_3}{x+t_1} + \frac{y_1}{(x+t_1)^2}$	easy	2 1 0

Idea: “Unmix” mixed query while preserving span

Main Lemma (informal):

A transcript of $n+1$ queries where the first n queries are “unmixed” can be rewritten as a transcript of $n+1$ queries where the $n+1^{th}$ query q_{n+1} is replaced with “unmixed” query q such that the span of the two transcripts are the equivalent.

t_2 t_3

1 0

$$\frac{y_1}{x+t_1} \quad \frac{y_2}{x+t_2} \quad \frac{y_3}{x+t_1} + \frac{y_1}{(x+t_1)^2} + \frac{y_2}{(x+t_1)(x+t_2)}$$

unmixing!

2 1 0

$$\frac{y_1}{x+t_1} \quad \frac{y_2}{x+t_2} \quad \frac{\alpha_1 y_1+\alpha_2 y_2+\alpha_3 y_3}{x+t_1} + \frac{y_1}{(x+t_1)^2}$$

easy

2 1 0

Impact

- Implementation confirms practicality
  - 25% overhead over 2HashDH (~ 400µs vs 500µs)
- Interest by companies deploying OPRF applications
- Incorporated into OPRF standardization effort
  - https://datatracker.ietf.org/doc/html/draft-irtf-cfrg-voprf

Conclusion

- Motivation: POPRFs simplify use of OPRFs in existing applications
- 3HashSDHI: A new, simple POPRF
  - Previous candidate POPRFs rely on pairings or RSA groups
  - Shares similar structure to standards-track 2HashDH OPRF
- Non-trivial security analysis: OM-Gap-SDHI
  - Recovers security of closely related partially blind signature scheme

Standards track: `https://datatracker.ietf.org/doc/html/draft-irtf-cfrg-voprf`
Archive: `https://ia.cr/2021/864`

Back-up slides

Previous “not simple” approaches

Construction	Limitation
Pythia [ECSJR USENIX Security ‘15]	Pairings

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OPRFs from unique blind signature schemes [JKK14 ASIACRYPT ‘14]

→ POPRFs from unique partially-blind signature schemes [This work]

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Pythia [ECSJR USENIX Security ‘15]	Pairings
RSA partially-blind sig [AF ASIACRYPT ‘96]	RSA
Schnorr partially-blind sig [AO CRYPTO ‘00]	Insecure [BLLOR EUROCRYPT ‘21]
ZSS partially-blind sig [ZSS INDOCRYPT ‘03]	Pairings, and invalid proof

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	* Repaired [This work]

OPRFs from unique blind signature schemes [JKK14 ASIACRYPT ‘14]

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3HashSDHI: Obli

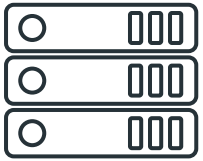
Minimal delta from 2HashDH oblivious evaluation!

Client

$Request(pk=g^{sk}, x)$

$r \leftarrow_{\$} Z_p$

$req = H_1(x)^r$



Server

$BlindEval(sk, req)$

$B \leftarrow req$

$\pi \leftarrow DLEQ\{pk = g^{sk} \wedge \quad ^{sk}\}$

$resp = (B, \pi)$

$Finalize(pk, resp)$

$Verify\ \pi: \exists \alpha,\ DLEQ\{pk = g^{\alpha} \wedge \quad = B^{\alpha}\}$

$B' \leftarrow B^{1/r}$

$y = F_{sk}(x,) \leftarrow H_2(x, B')$

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$x \leftarrow_{\$} Z_p$

$y_1, \ldots, y_m \leftarrow_{\$} (Z_p)^m$

$(C_1, t_1), \ldots, (C_l, t_l) \leftarrow_{\$} A^{SDH}(g^x, g^{y_1}, \ldots, g^{y_m})$

Oracle SDH(B,t)

Oracle SDDH(A,B,t)

Our construction

3HashSDHI POPRF [This work]

$$F_{sk}(x,t) = H_2(x,t,H_1(x)^{1/(sk+H_3(t))})$$

Combines aspects of

2HashDH OPRF [JKK ASIACRYPT '14]

$$F_{sk}(x) = H_2(x,H_1(x)^{sk})$$

&

DY PRF [DY PKC '05]

$$F_{sk}(x) = g^{1/(sk+x)}$$

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Repeated query to t_1 but does not mix tags!

In reduction to uber assumption, focus on linear independence of exponents