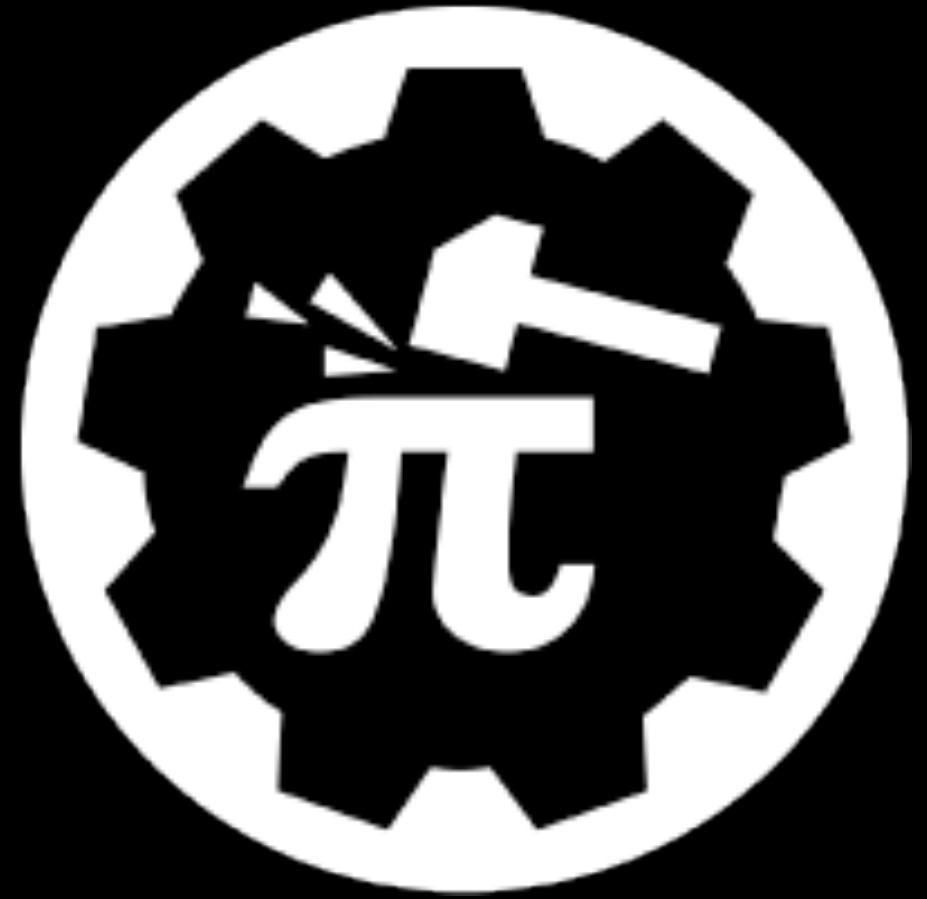


# Gemini

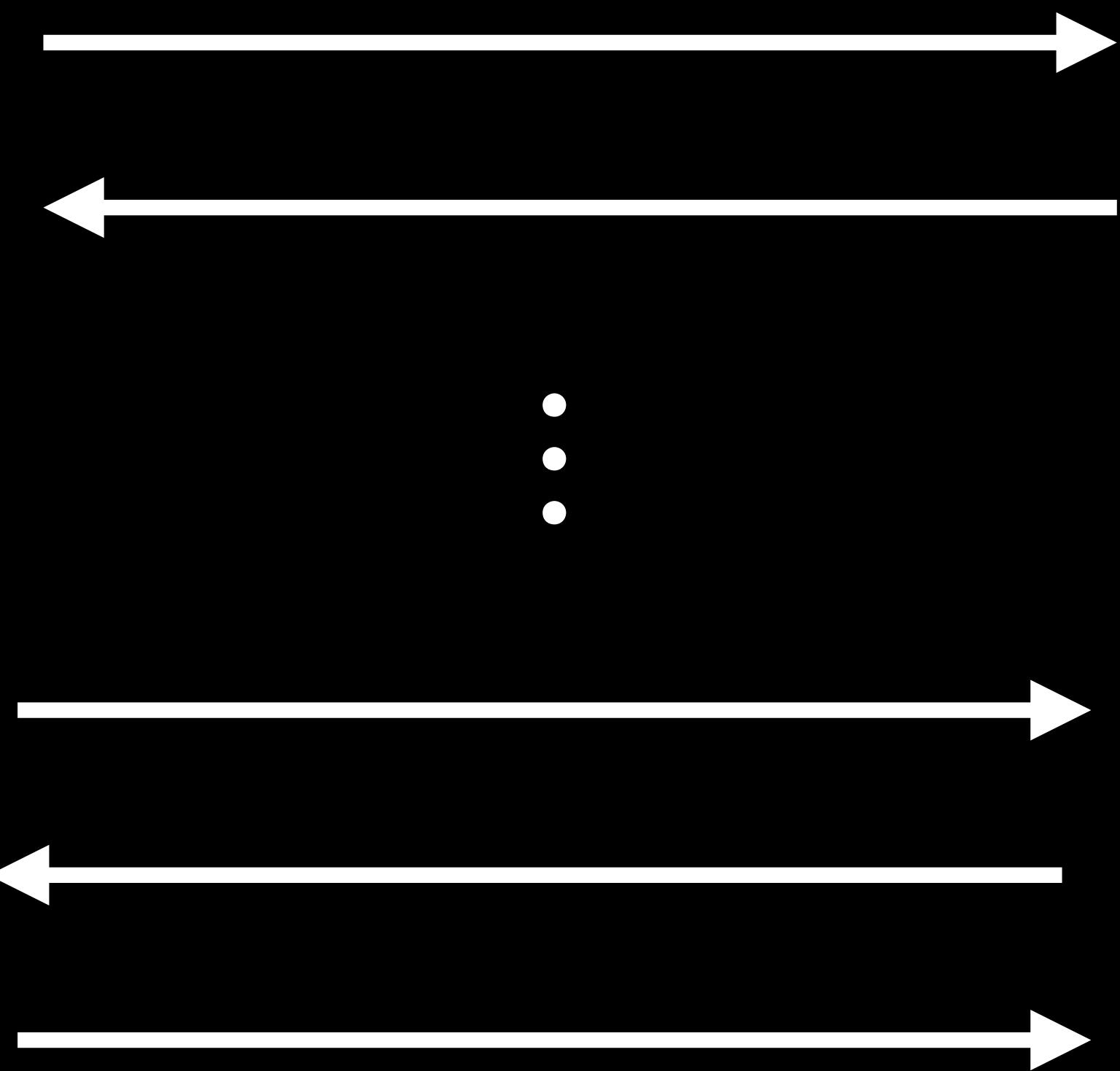
**Elastic proofs for diverse environments**

Joint work with Jonathan Bootle, Alessandro Chiesa, Yuncong Hu



# Succinct arguments

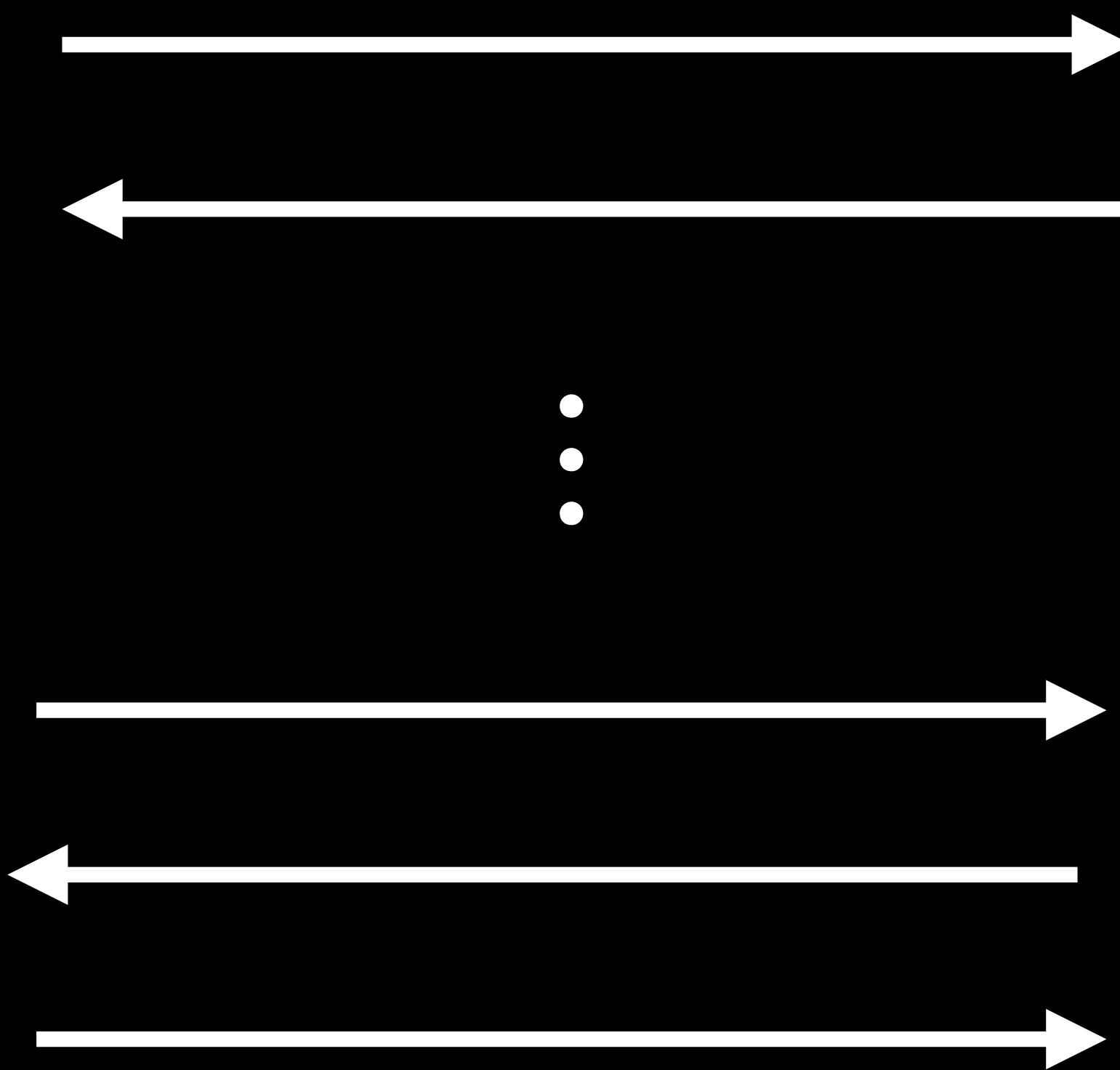
*P*



*V*

# Succinct arguments

$\mathcal{P}$



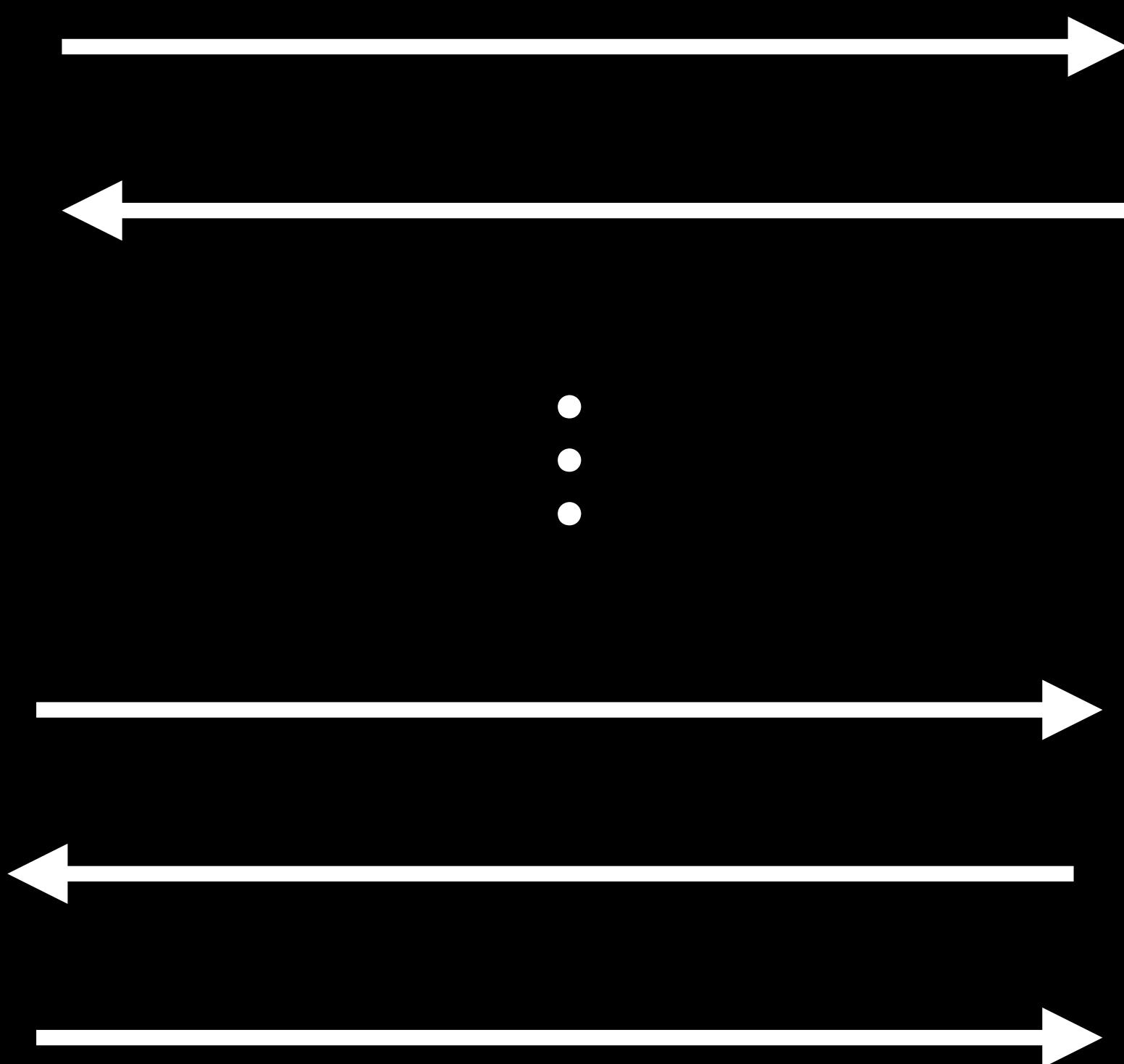
$\mathcal{V}$

$\exists w . (x, w) \in R$

# Succinct arguments

(with preprocessing)

$\mathcal{P}$



$\mathcal{V}$

$$\exists w . (x, w) \in R$$

# **Are we done?**

**Today, the biggest bottleneck is space.**

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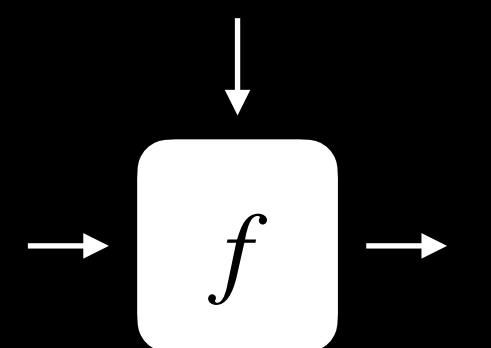
```
$ <prover> [billions of gate]  
Killed
```

- Proving time:  $\tilde{O}(n)$
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# An example

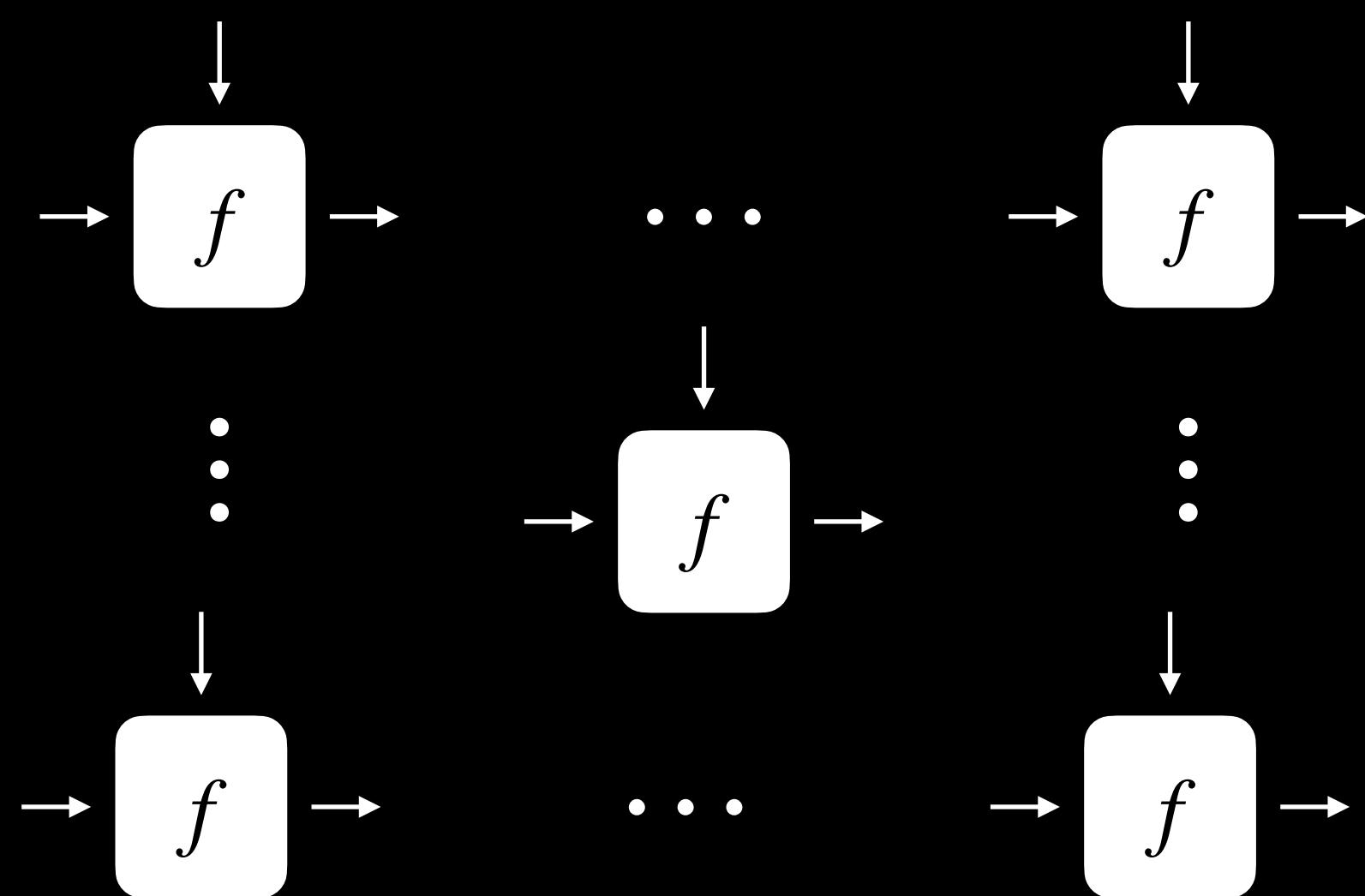
# An example

- SHA256 compression function is ~30K constraints



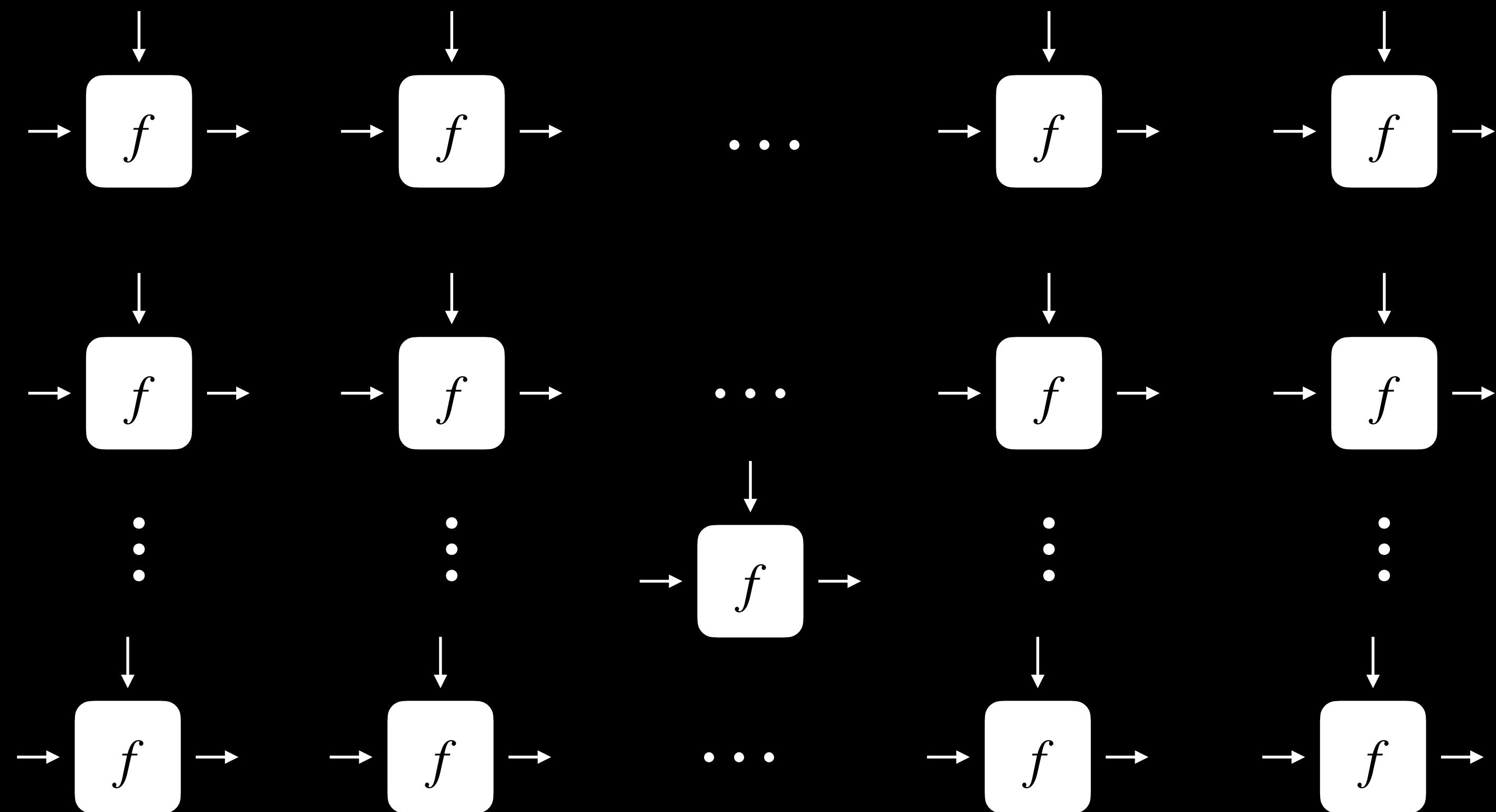
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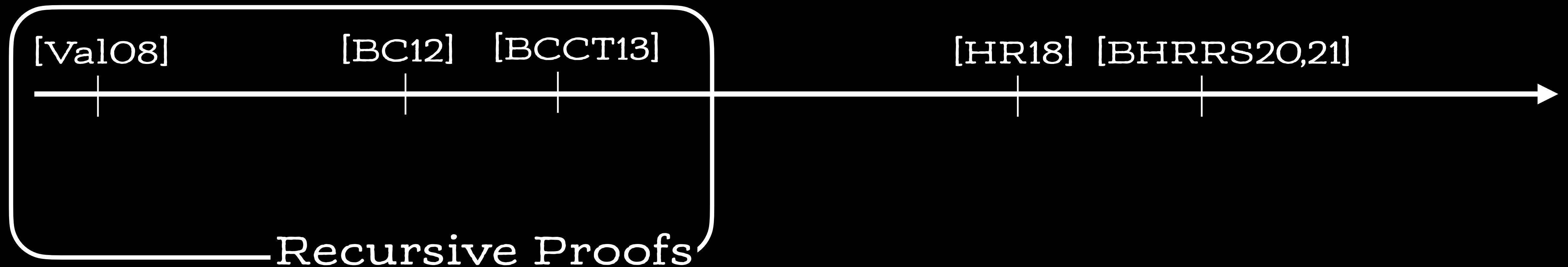
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# Space-efficient arguments



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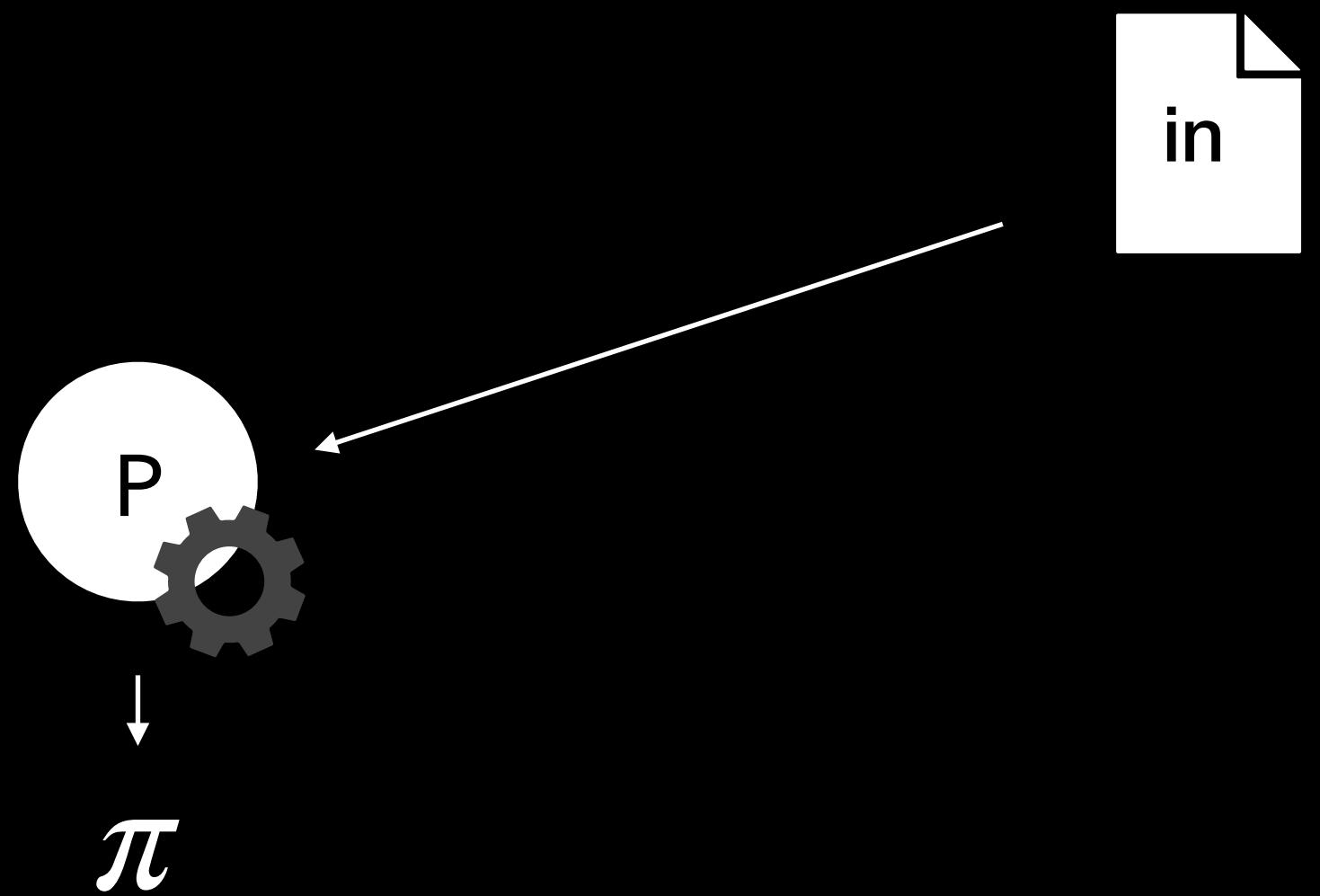
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In practice

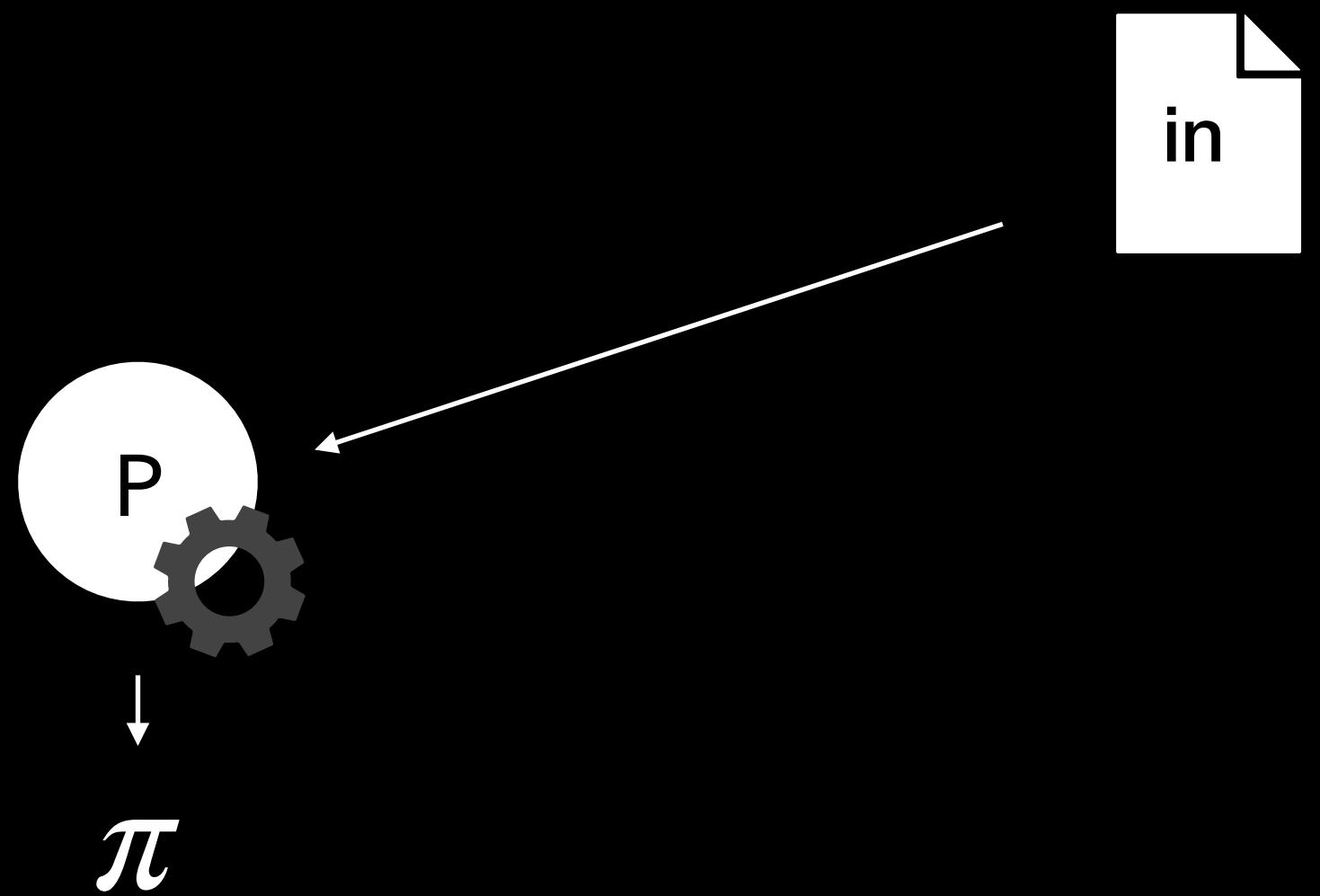
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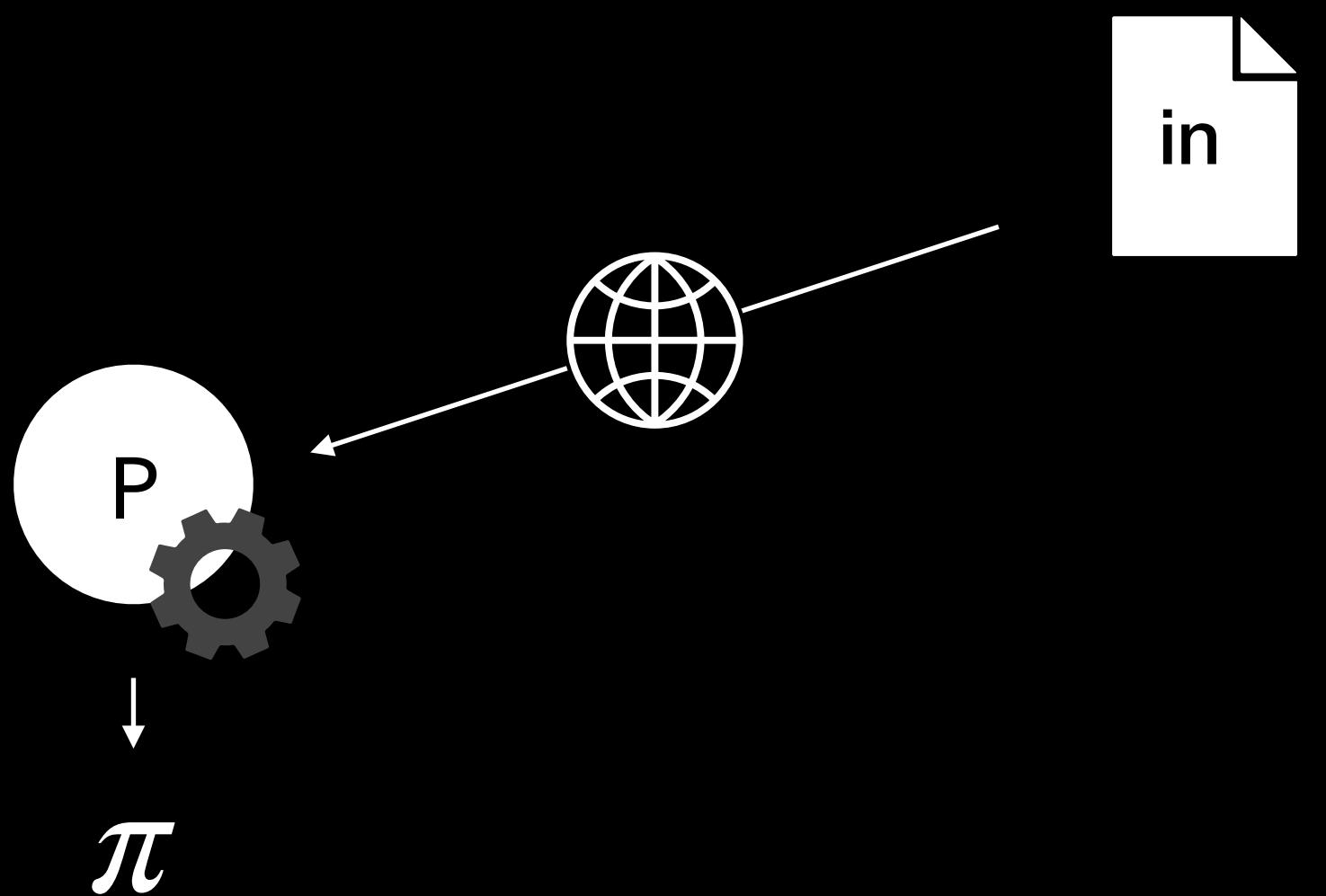
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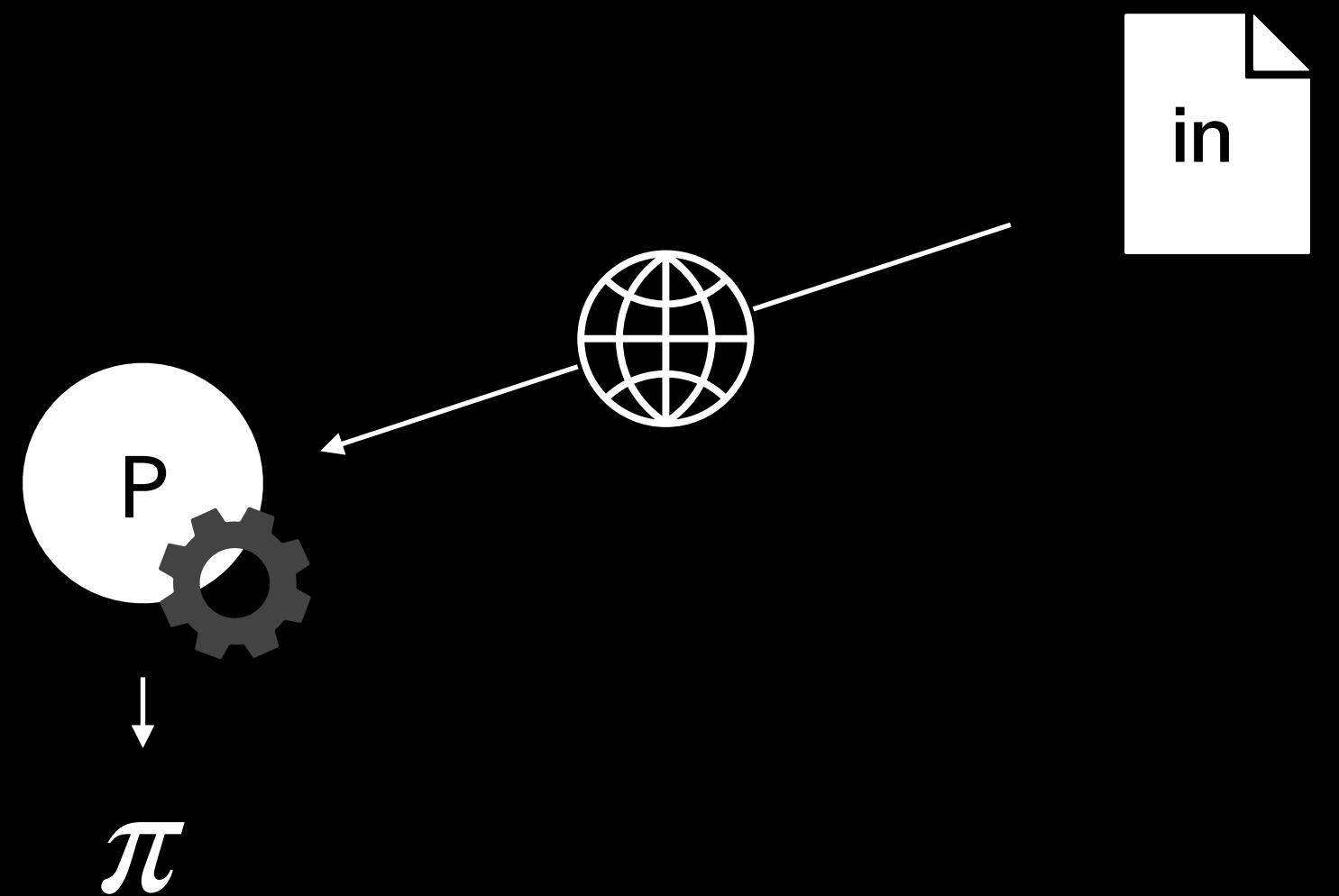
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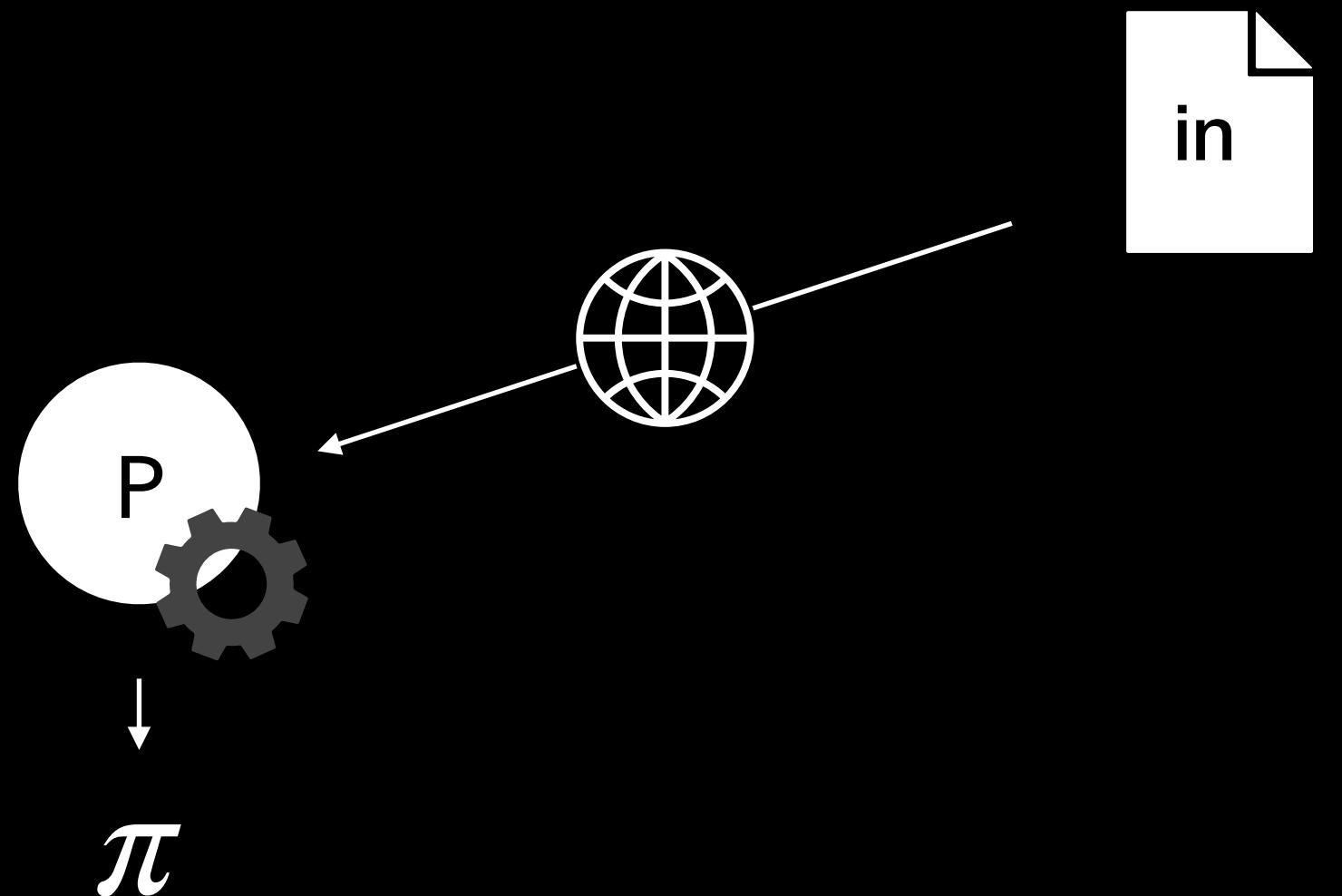
In practice



In theory

# Streaming provers

In practice



In theory

$$P^{\mathcal{S}(\text{in})} \rightarrow \pi$$

# Elasticity

## Our contribution

Two proving algorithms:

- Time-efficient 
- Space-efficient 

Same proof.

# Main theorem

## Our contribution

Thm. There exists an elastic (preprocessing) succinct argument for circuit satisfiability that admits two realizations:

- A time-efficient prover that runs in  $O(N)$  time and  $O(N)$  space,
- A space-efficient prover that runs in  $O(N \log^2 N)$  time and  $O(\log N)$  space.

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# Rank-1 Constraint Systems

$C$  circuit is satisfied



$$\exists z \quad \text{s.t.} \quad Az \circ Bz = Cz$$

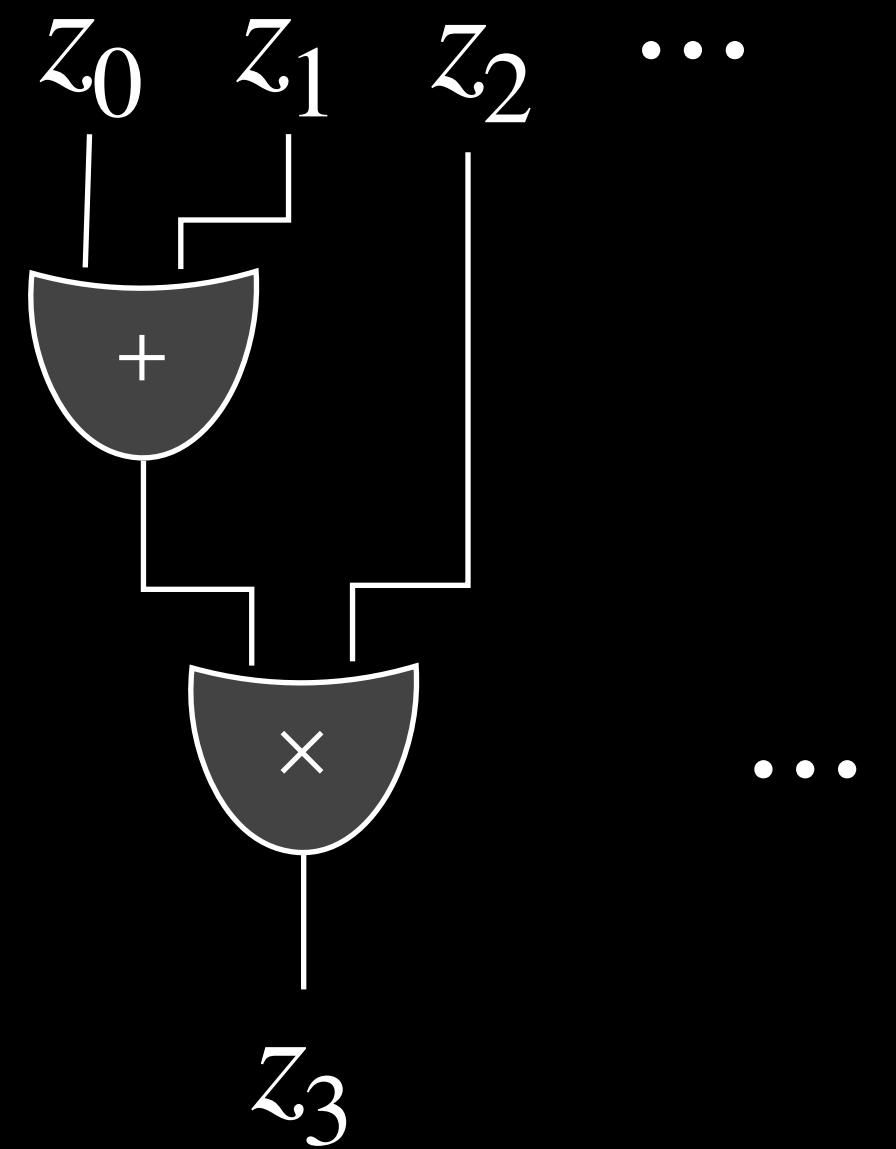
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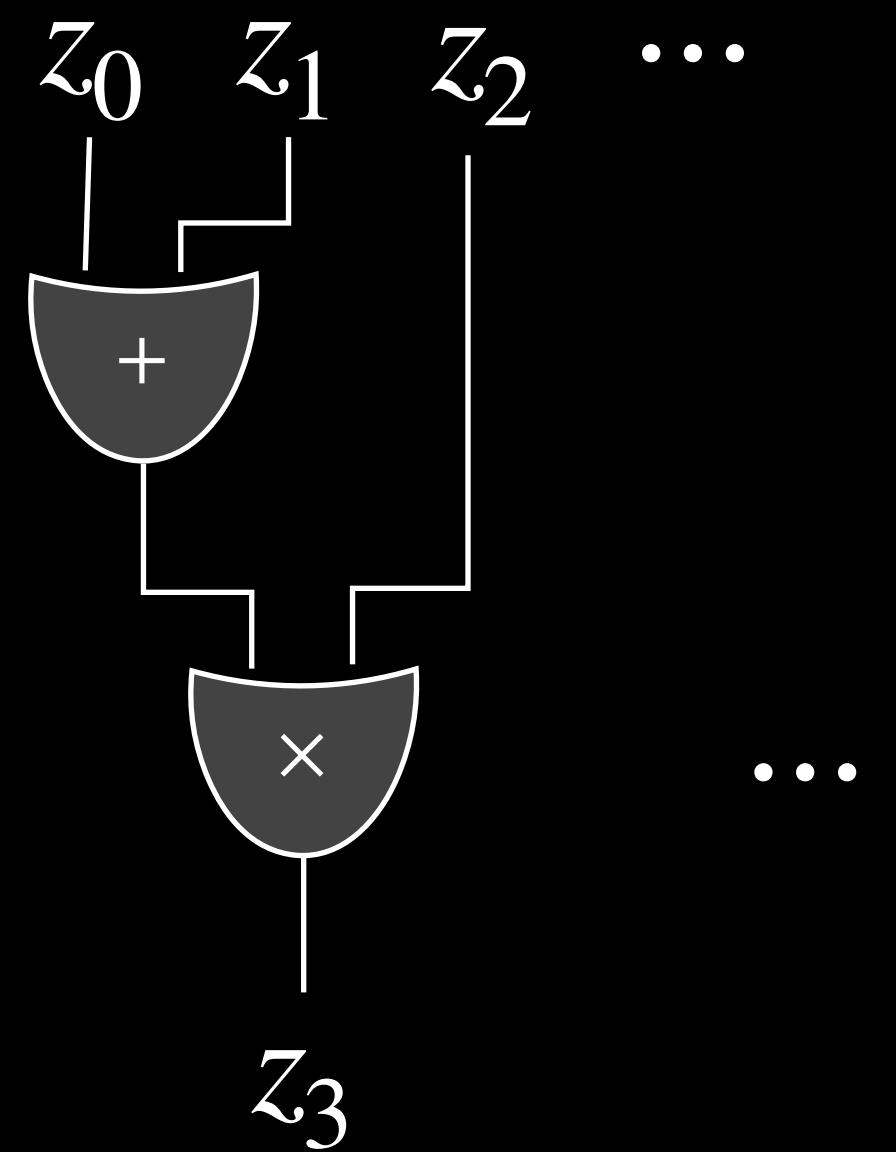
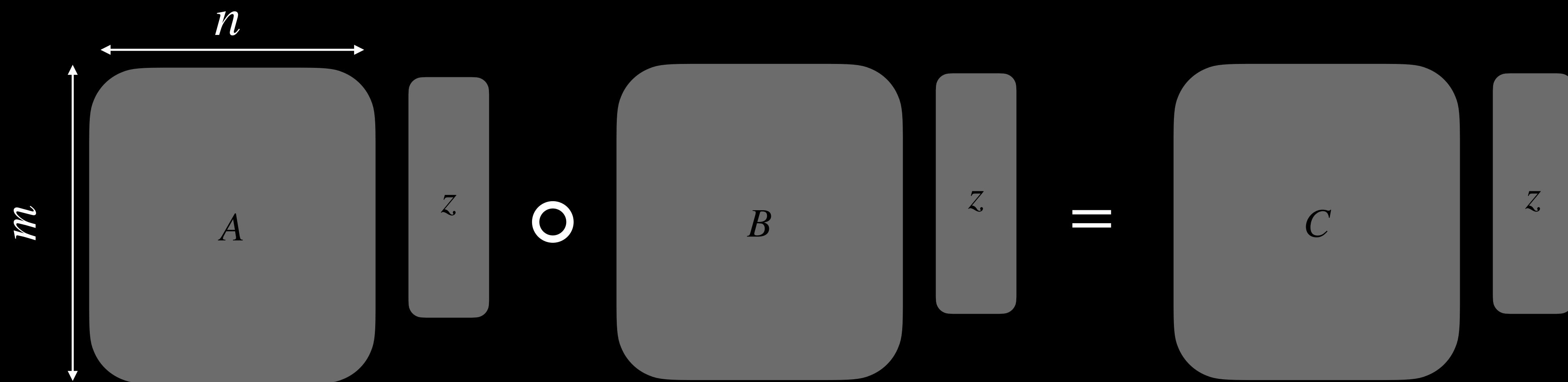
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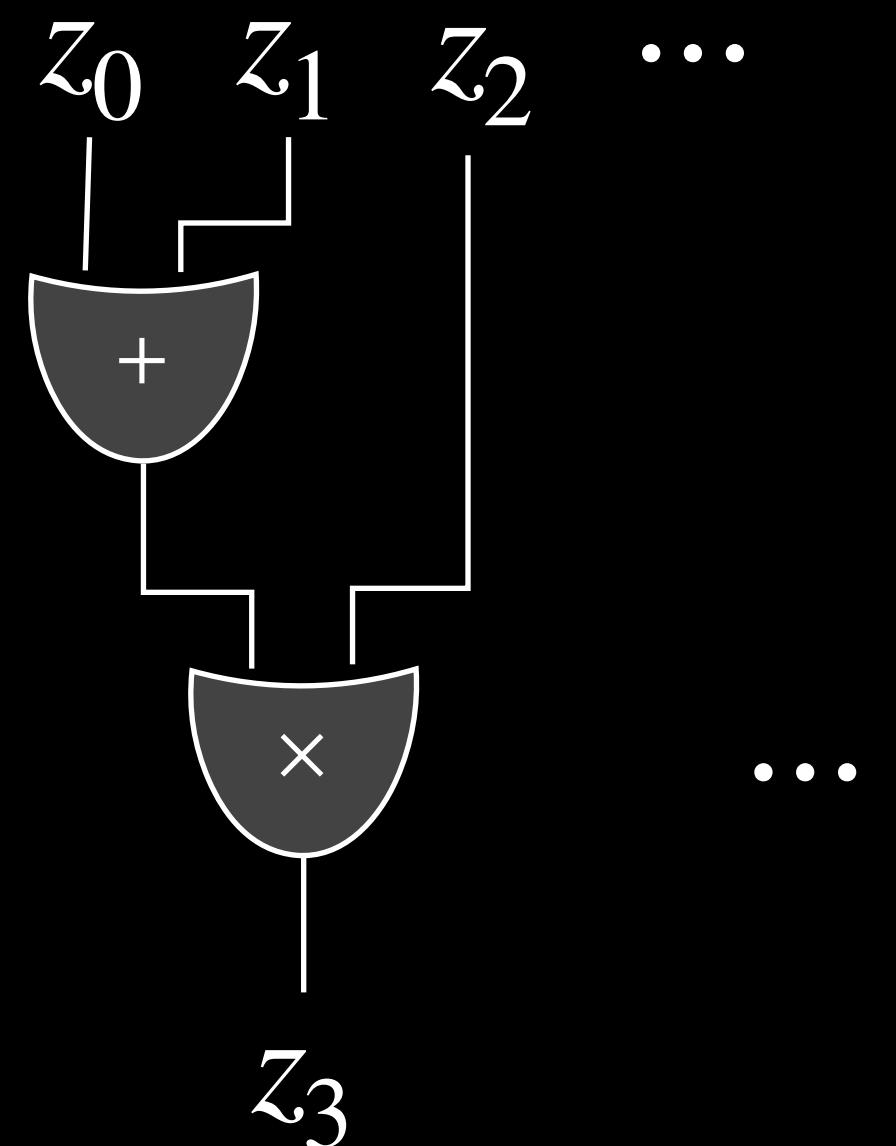
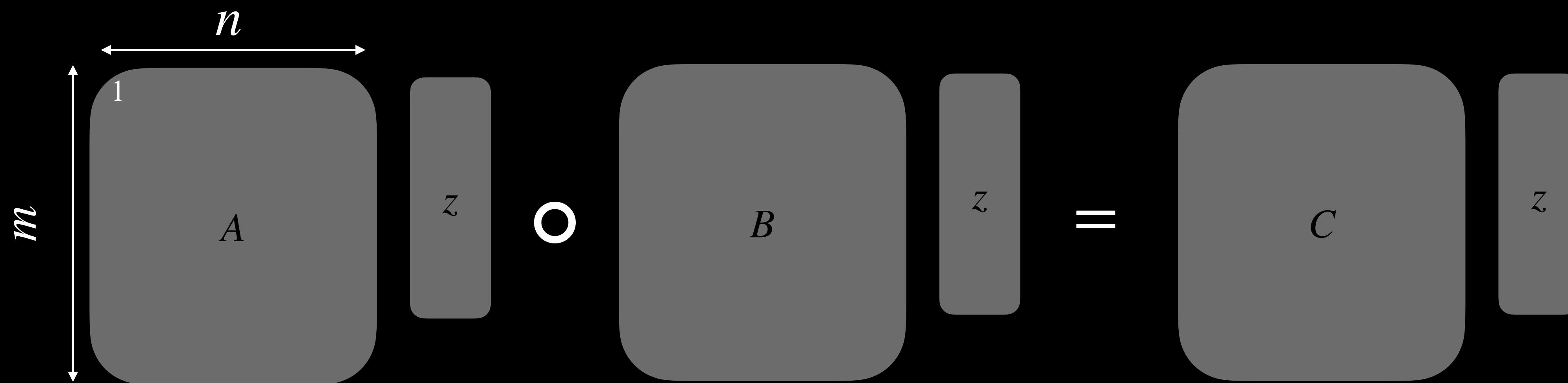
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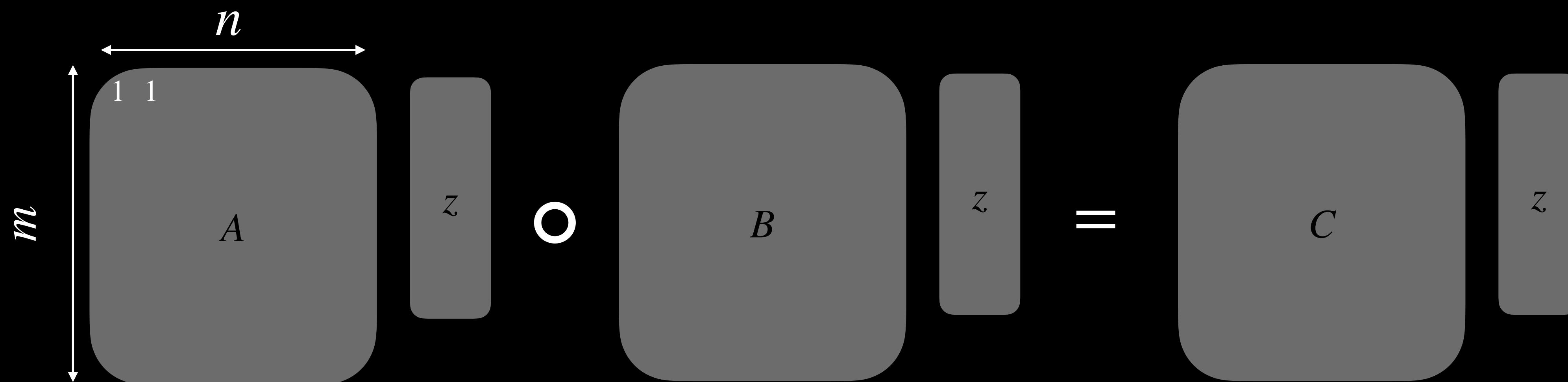
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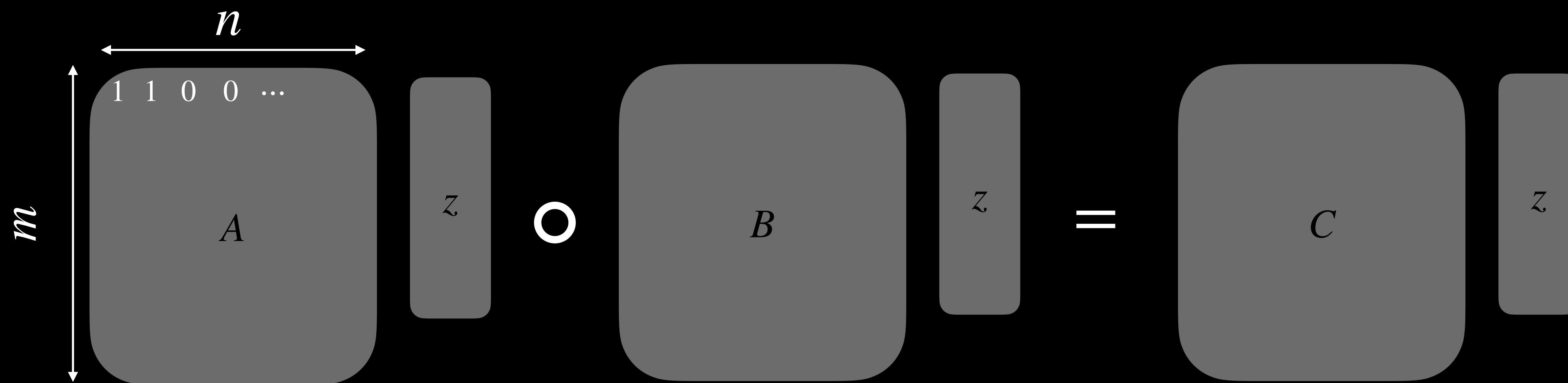
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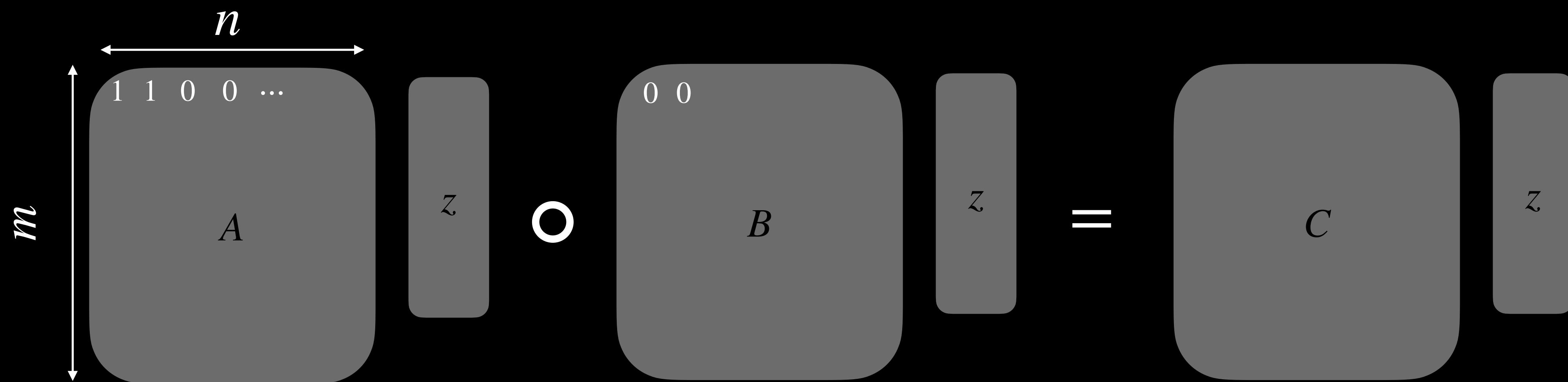
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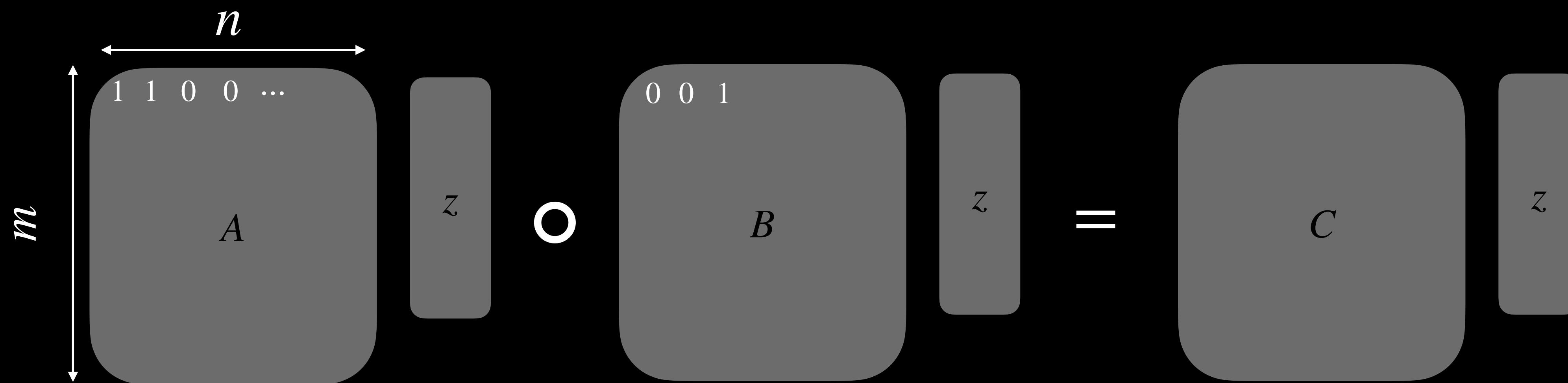
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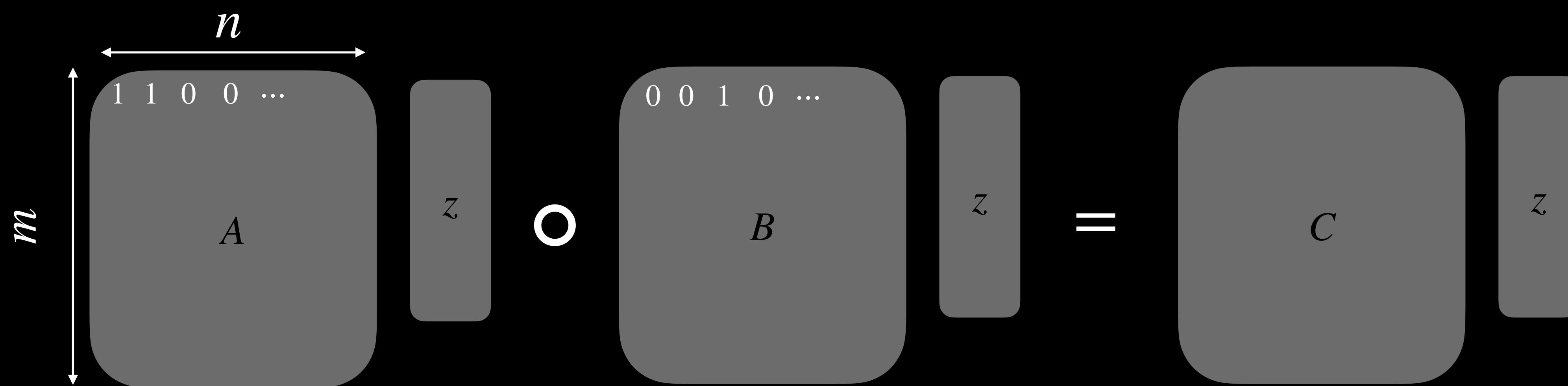
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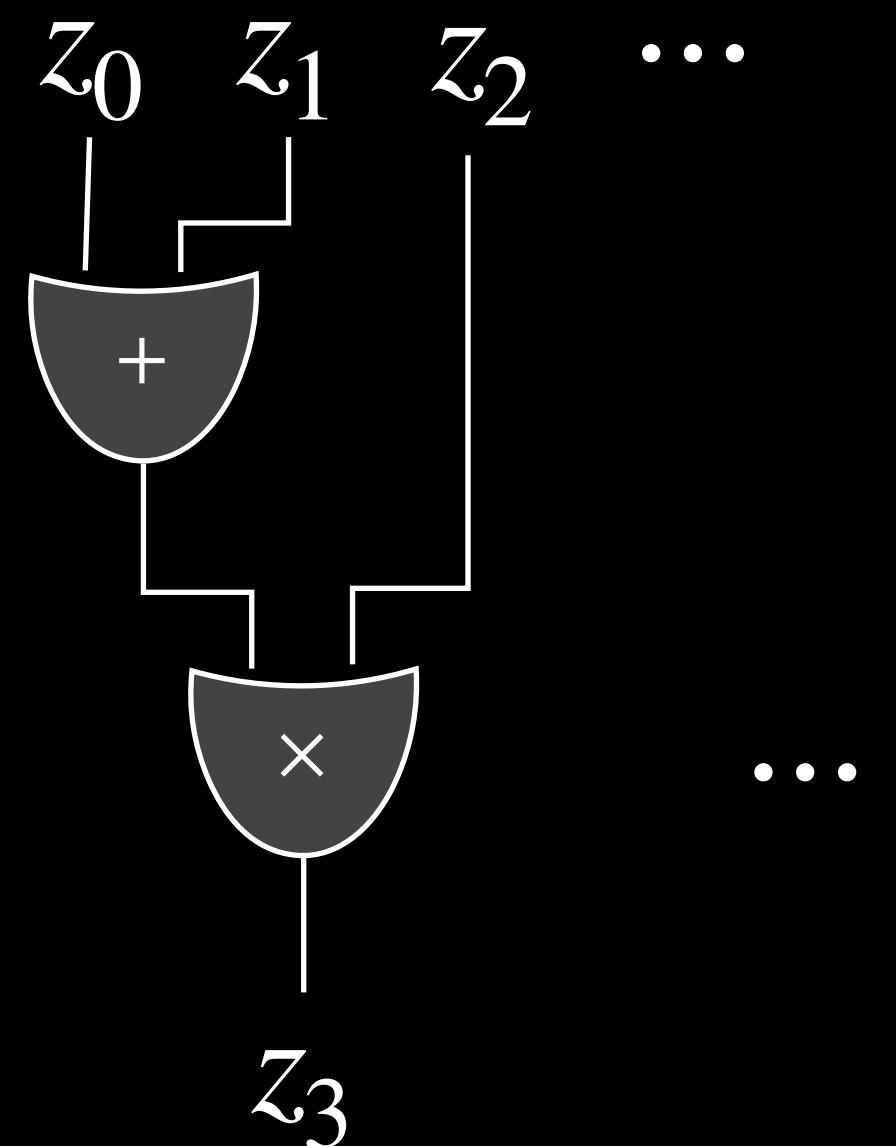
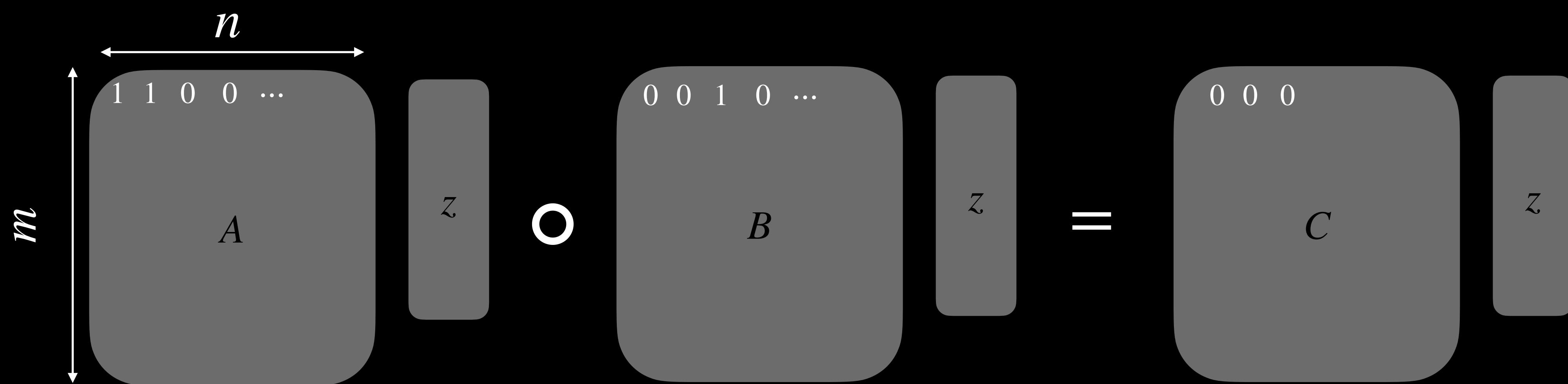
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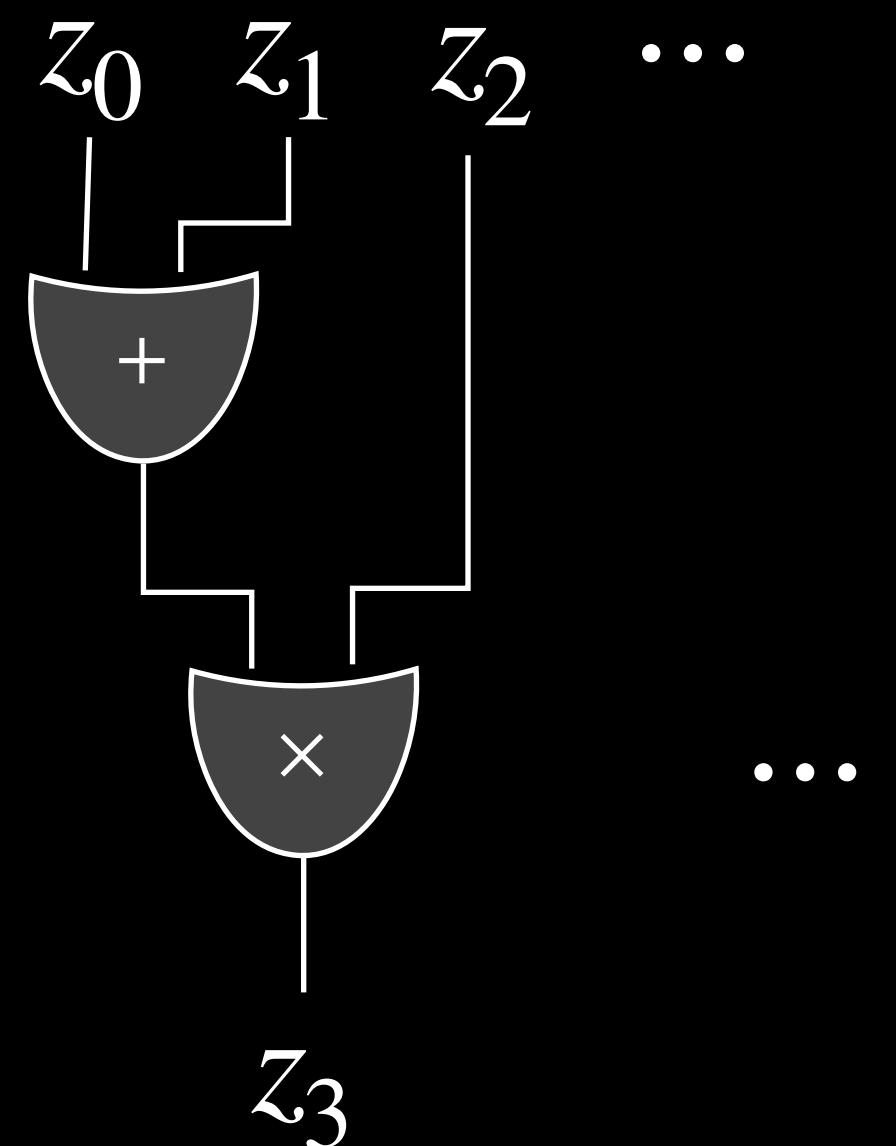
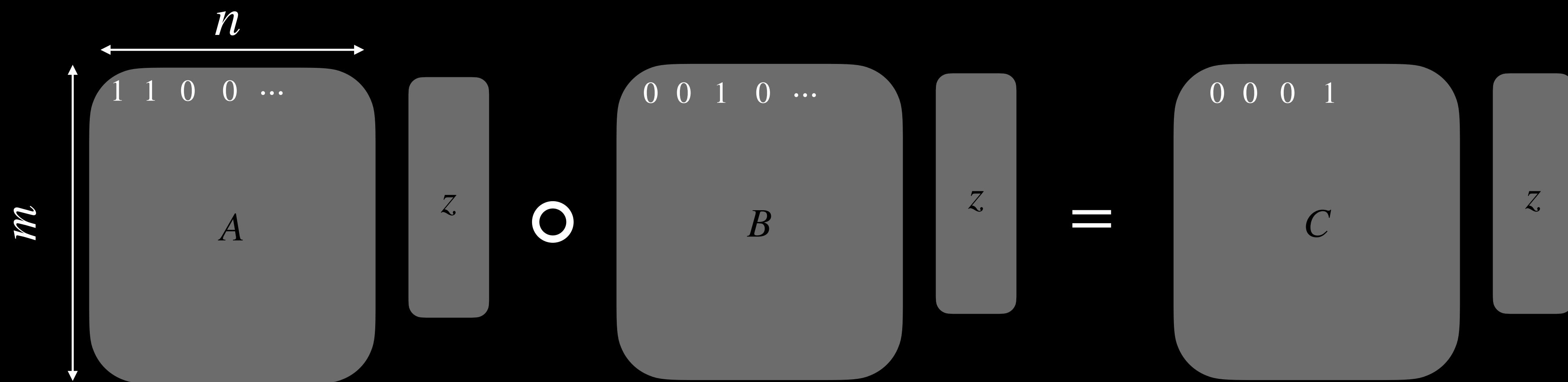
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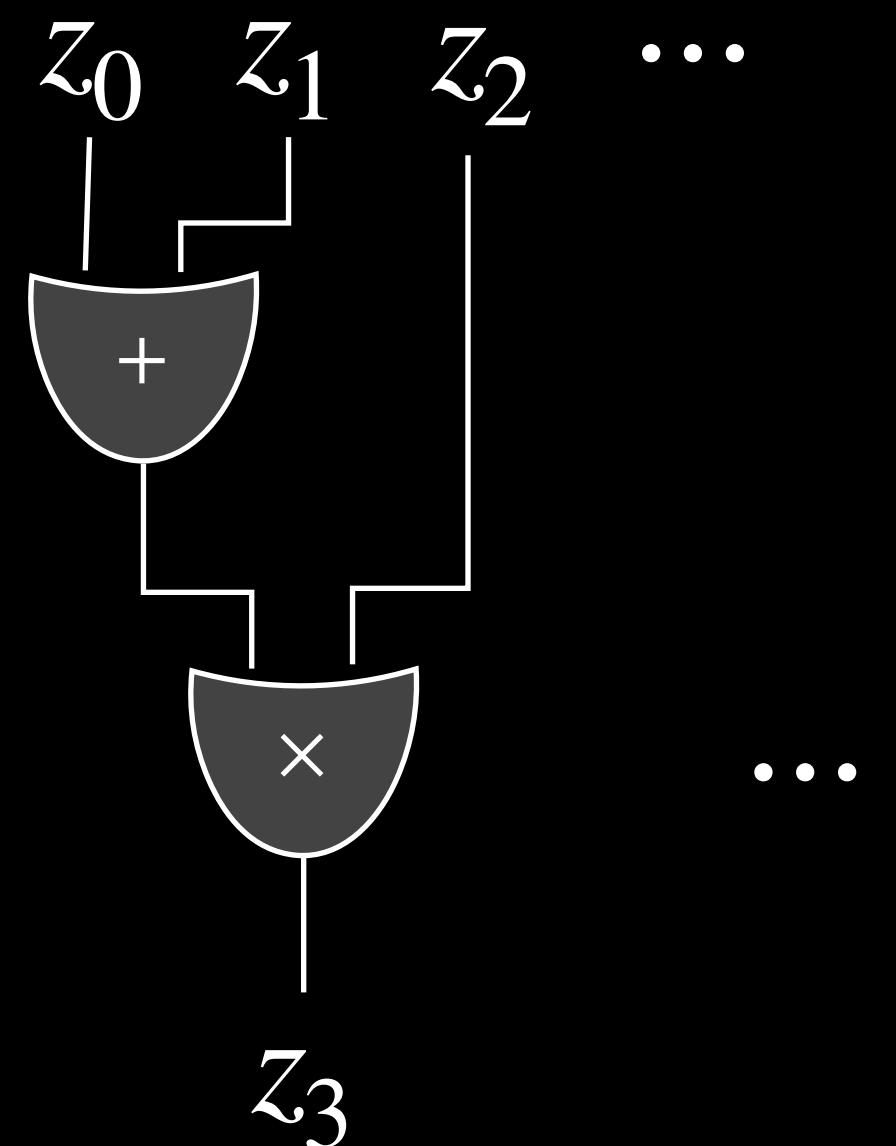
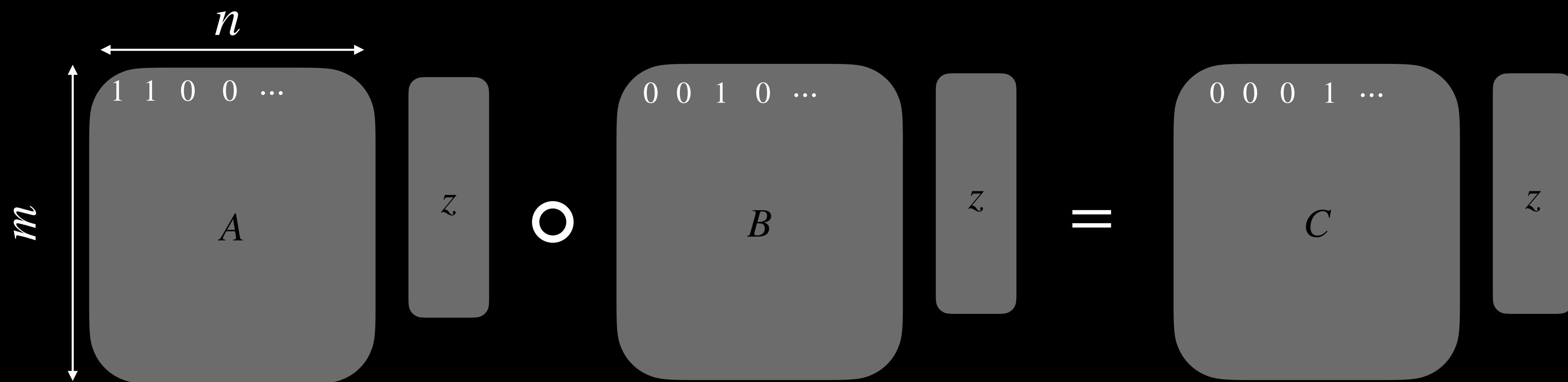
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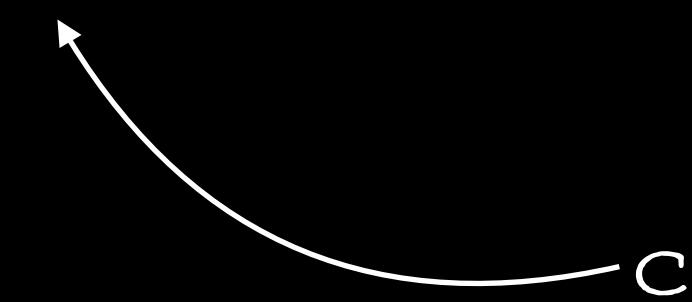
# Streaming R1CS

Our contribution

$$\exists z \text{ s.t. } Az \circ Bz = Cz$$

Def. The stream of a R1CS relation consists of:

- the full instance  $\mathcal{S}(z)$ ,
- the matrices  $\mathcal{S}_{\text{colmaj}}(A), \mathcal{S}_{\text{rowmaj}}(A)$ , etc.,
- the computation trace  $\mathcal{S}(Az), \mathcal{S}(Bz), \mathcal{S}(Cz)$ .



Can be generated composing streams!

# Idealized Protocol

# Idealized Protocol

An Elastic Polynomial Interactive Oracle Proof for Inner Product

# Inner product argument

# Inner product argument

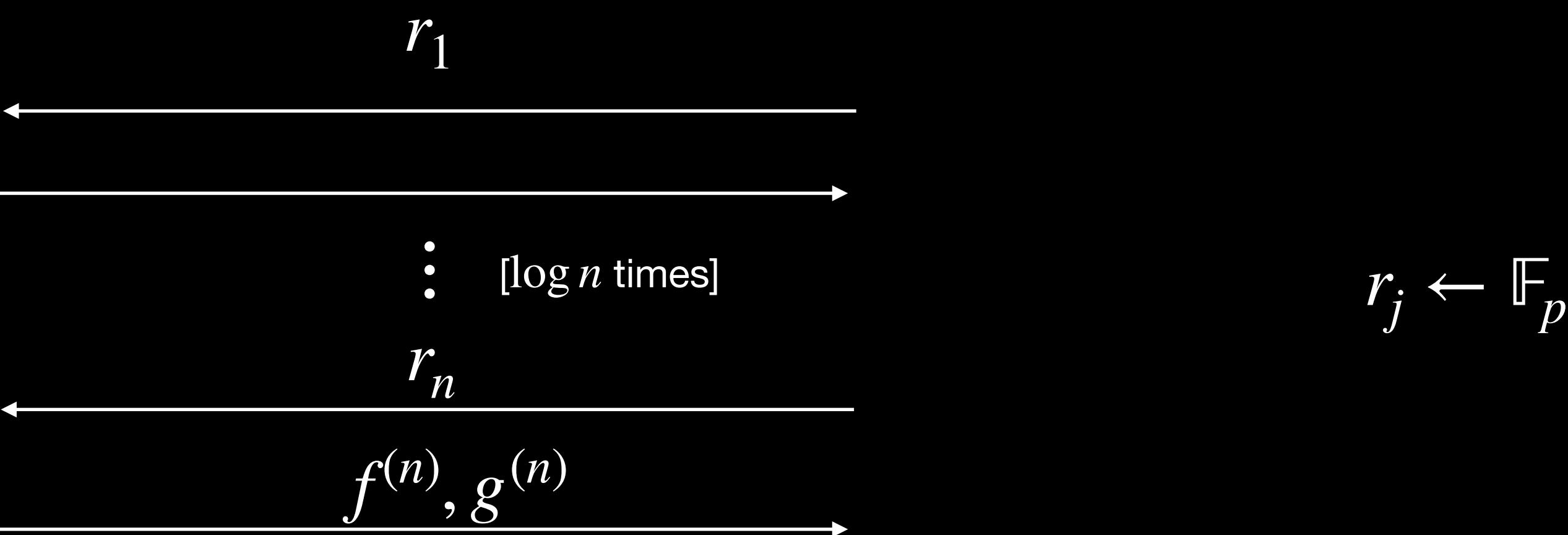
$$\langle \vec{f}, \vec{g} \rangle = y \text{ of size } n \text{ is valid} \xrightarrow{\text{whp}} \langle \vec{f}', \vec{g}' \rangle = y' \text{ of size } n/2 \text{ is valid}$$

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Folding:

$$\cdot \vec{f}' = \vec{f}_{\text{even}} + r_1 \cdot \vec{f}_{\text{odd}}$$

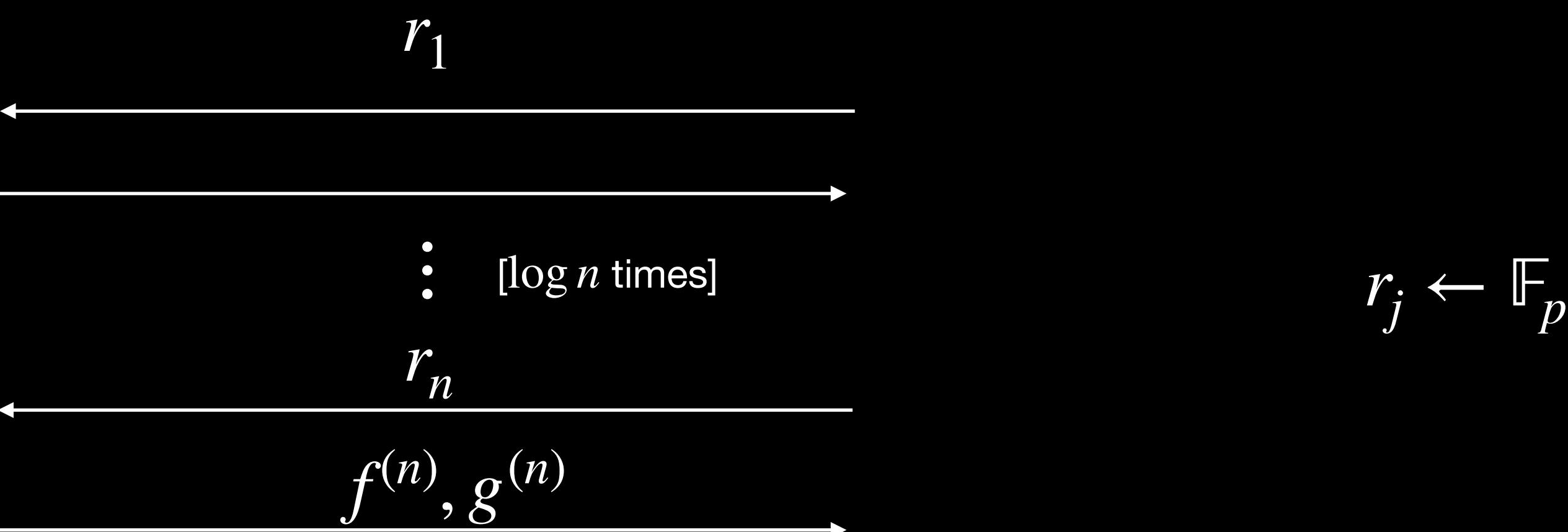


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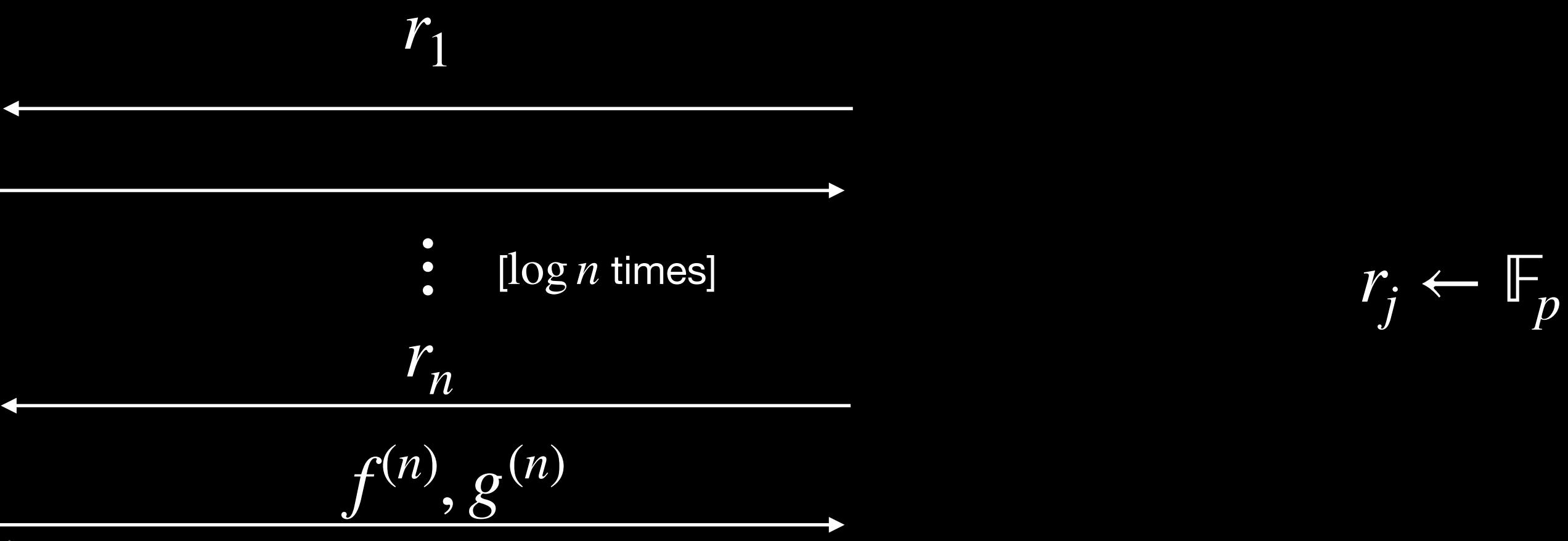
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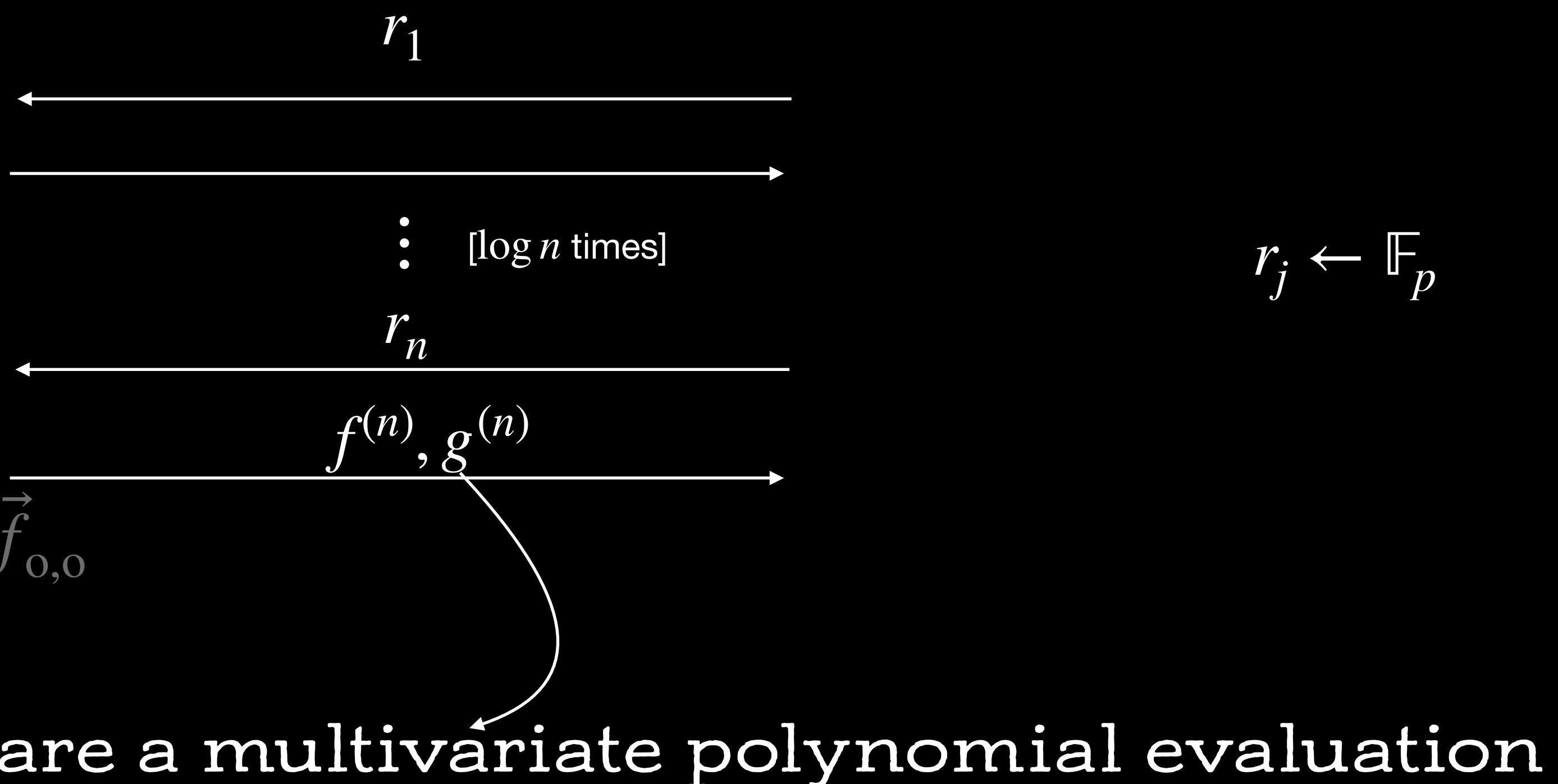
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# Streaming the prover's messages

Our contribution

Stream of  $f$ :

# Streaming the prover's messages

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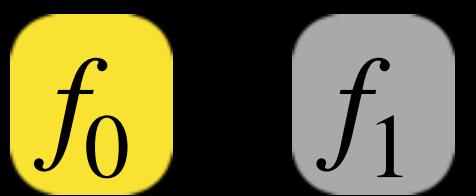
Stream of  $f$ :

$$f_0$$

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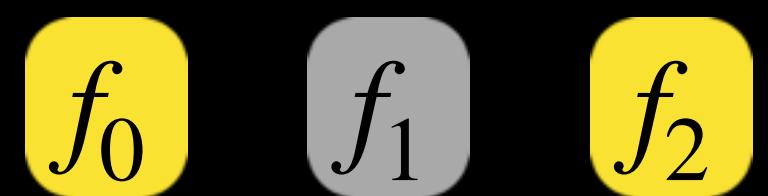
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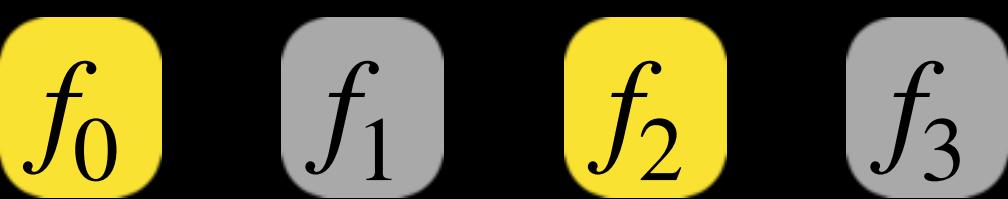
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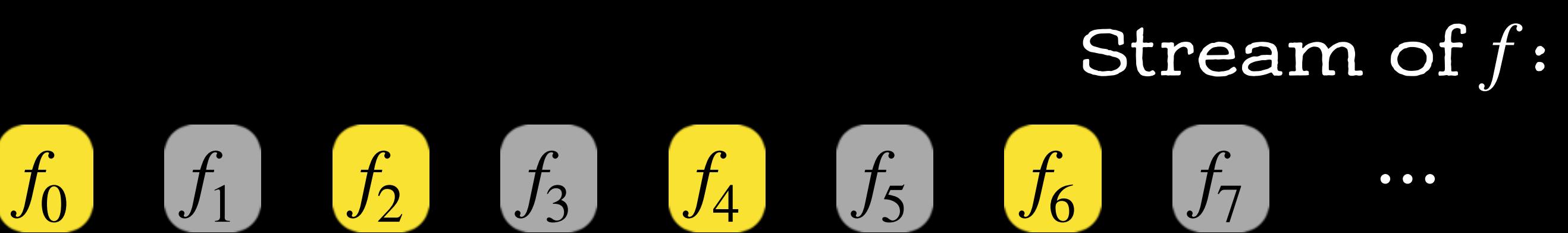
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Stack

$$f'_0 := f_0 + r_1 \cdot f_1$$

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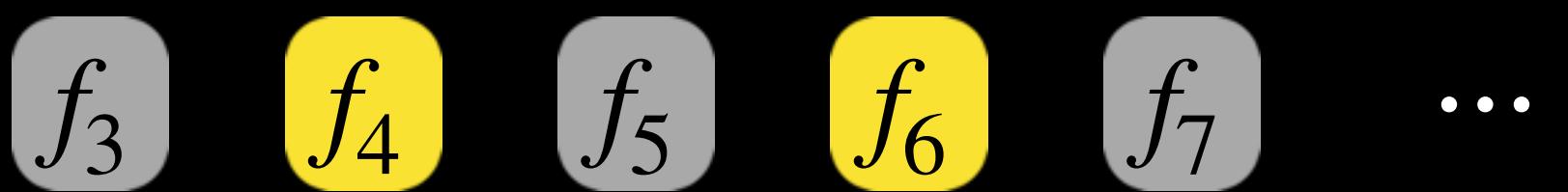
Stack



# Streaming the prover's messages

Our contribution

Stream of  $f$ :



Stack

$f'_0$

$f_2$

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Our contribution

Stream of  $f$ :



Stack

$f'_0$

$f_2$

$f_3$

# Streaming the prover's messages

Our contribution

Stream of  $f$ :



Stack

$$f'_0$$

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# Streaming the prover's messages

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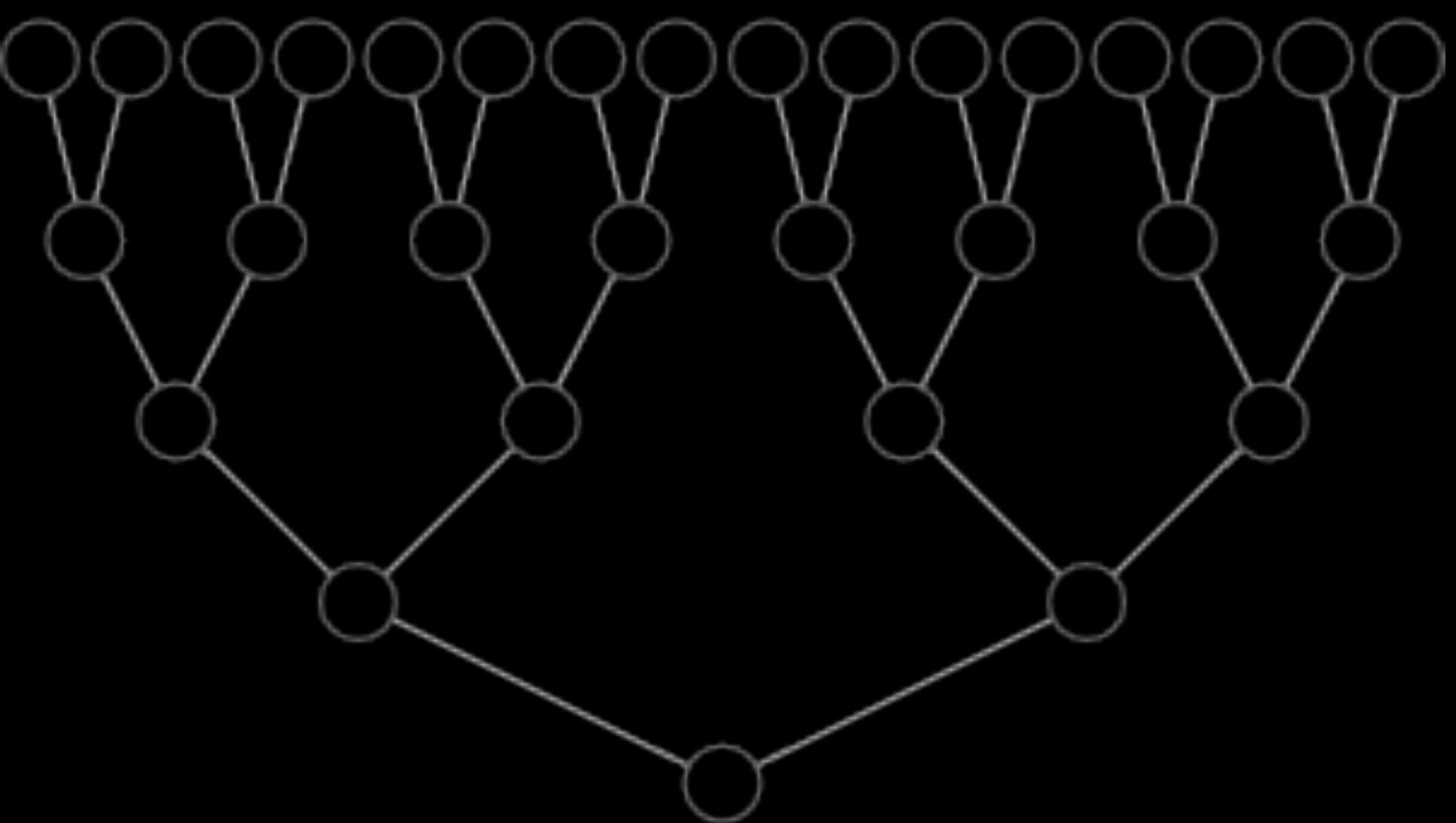
Stream of  $f$ :



## Stack

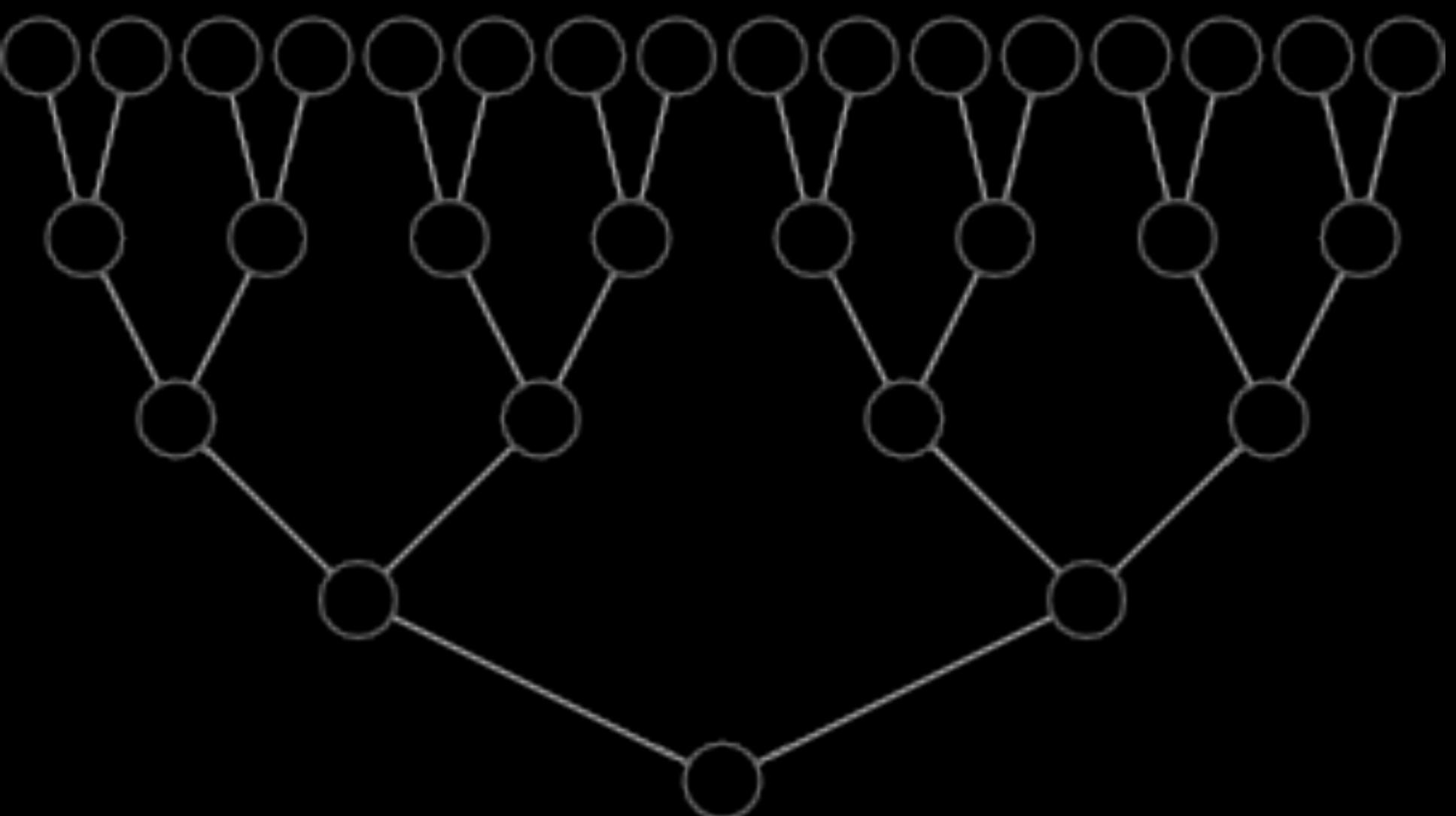
$$f''_0 := f'_0 + r_2 \cdot f'_1 \quad ; \text{ yield } f'_0 \quad [\text{the first coefficient!}]$$

# Inner product: complexity



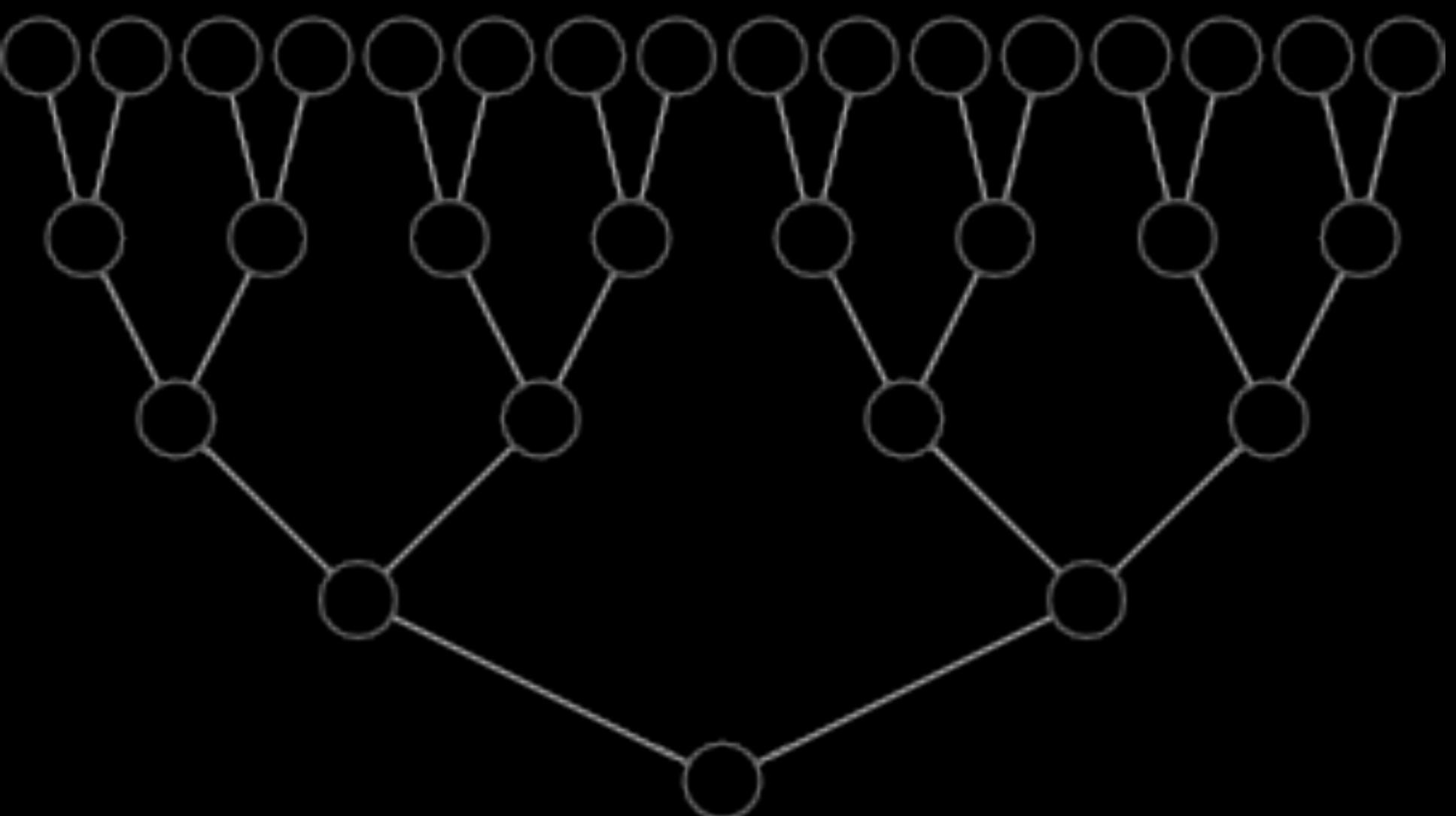
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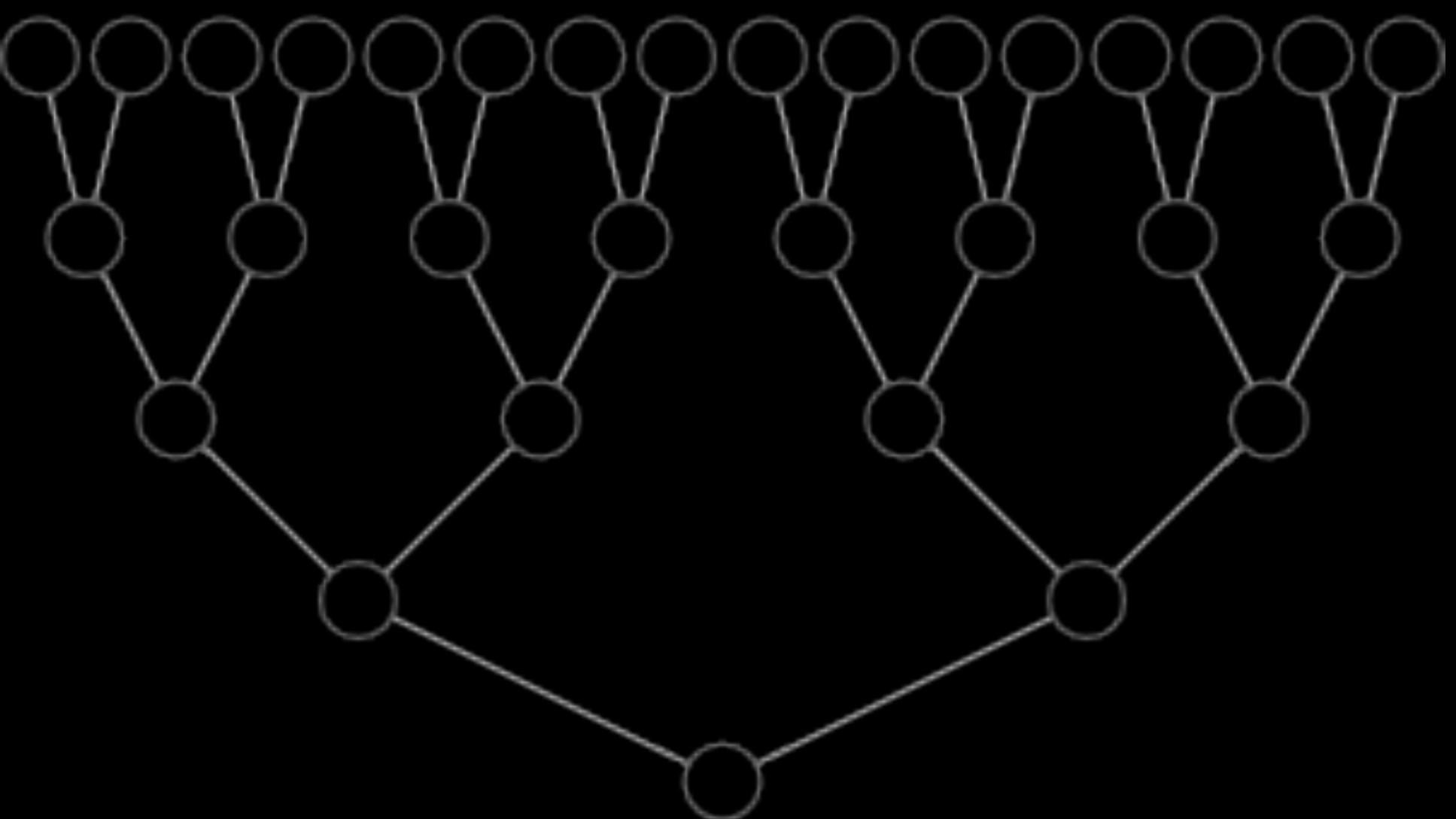
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Exploit divide and conquer structure:

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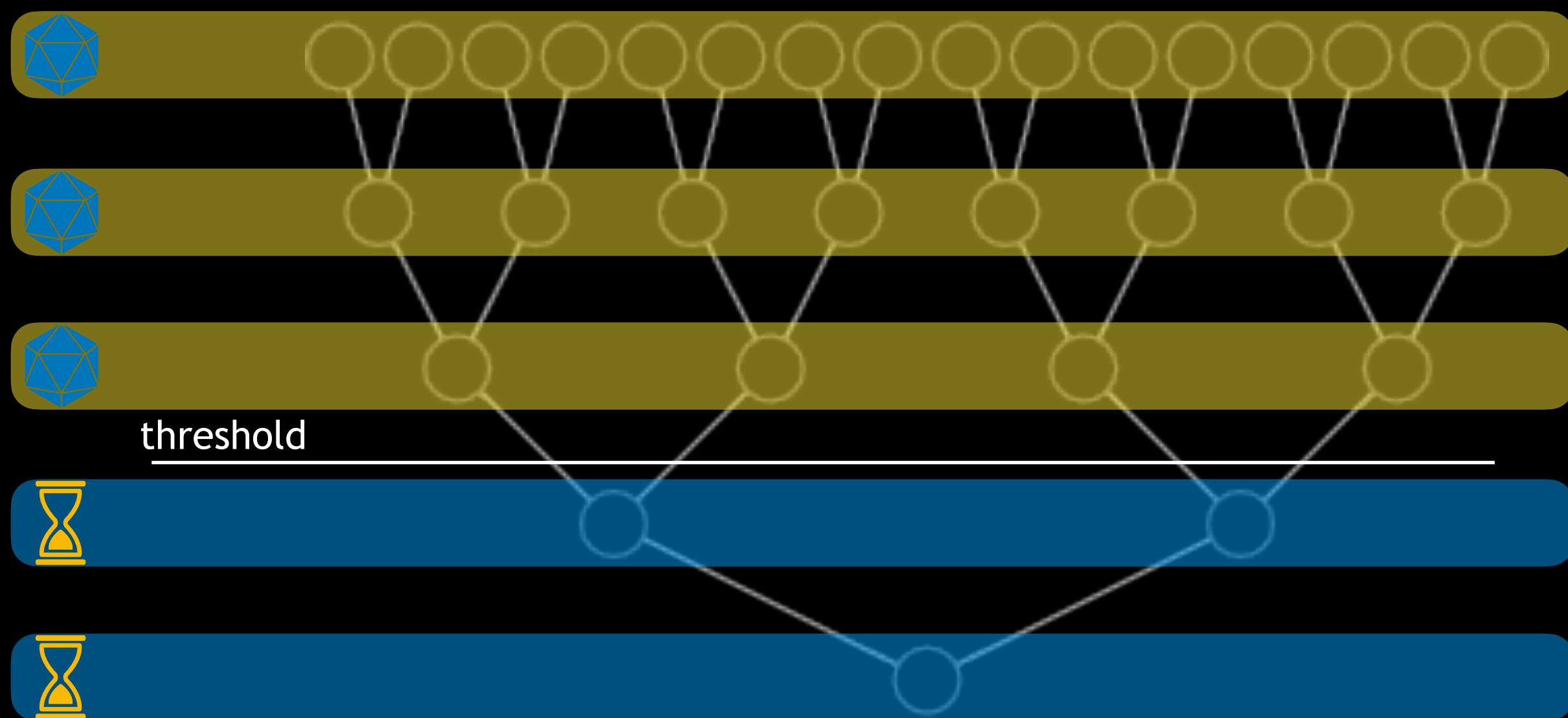


Exploit divide and conquer structure:

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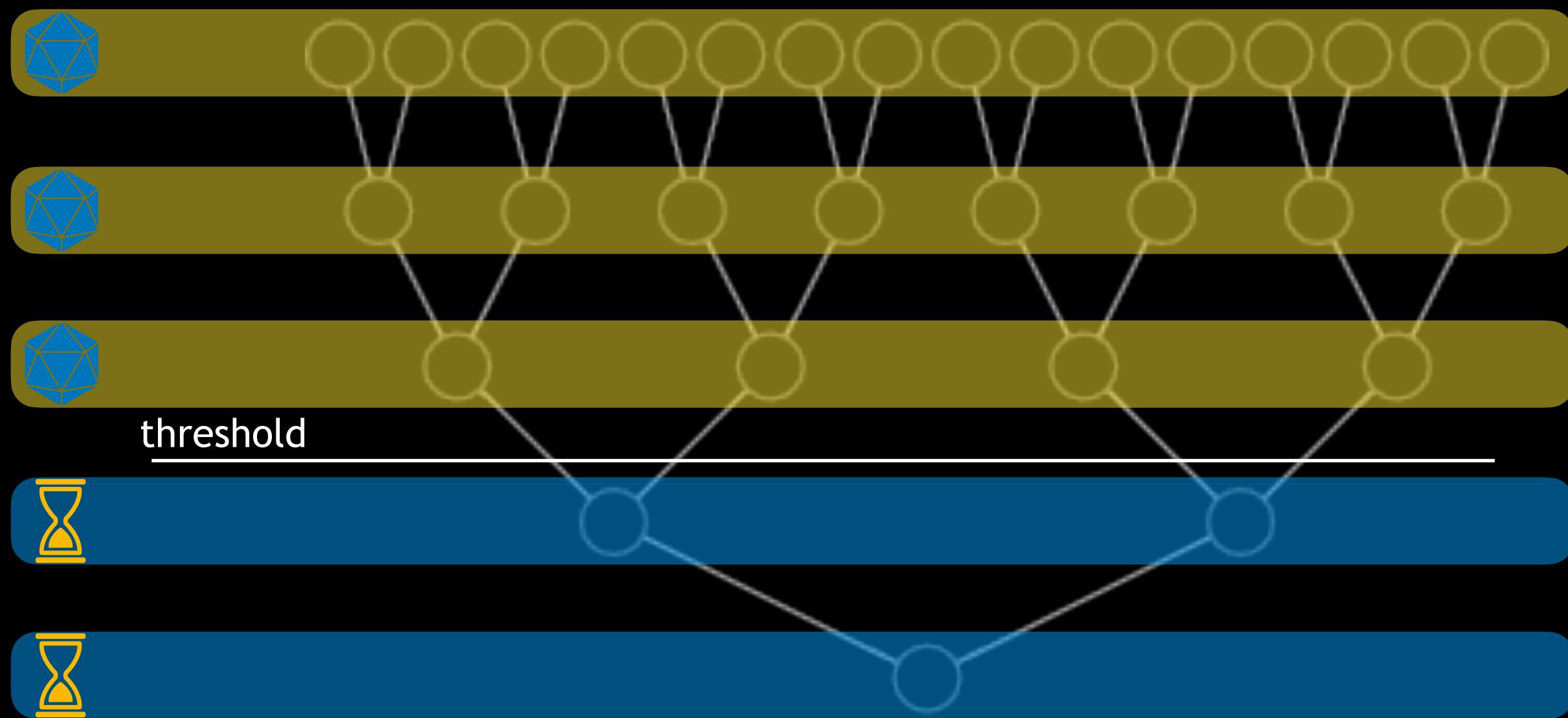


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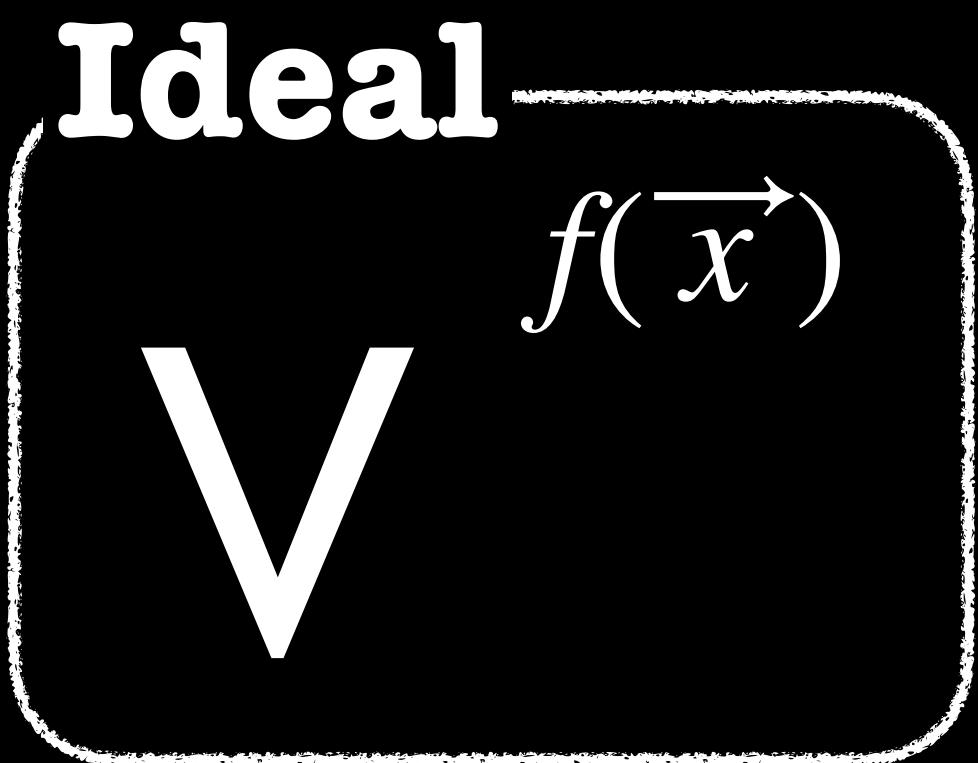


Exploit divide and conquer structure:

- Fix a memory budget
- Overall complexity  $O(n \log n)$  for arbitrary size,  $O(n)$  from threshold

# Tensorcheck

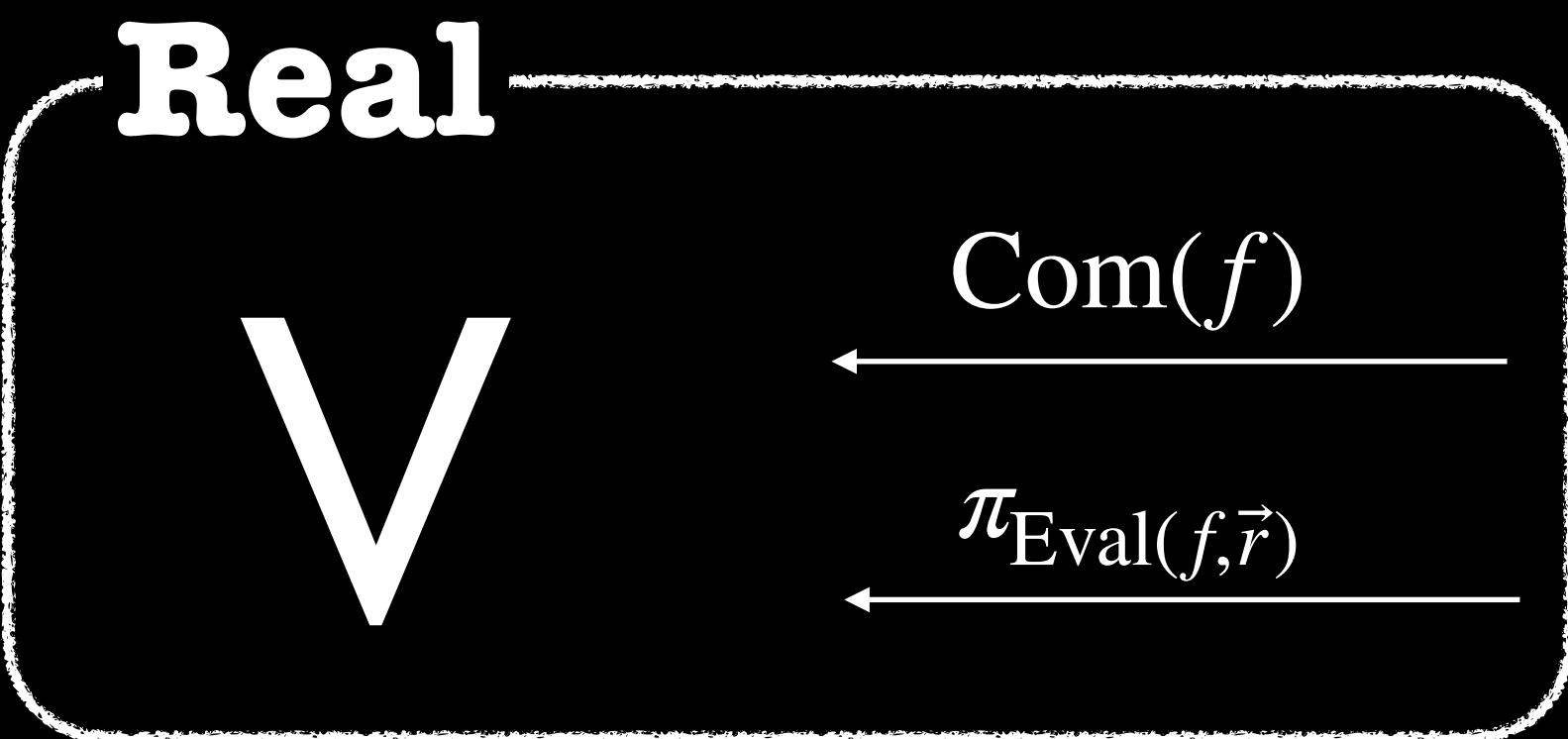
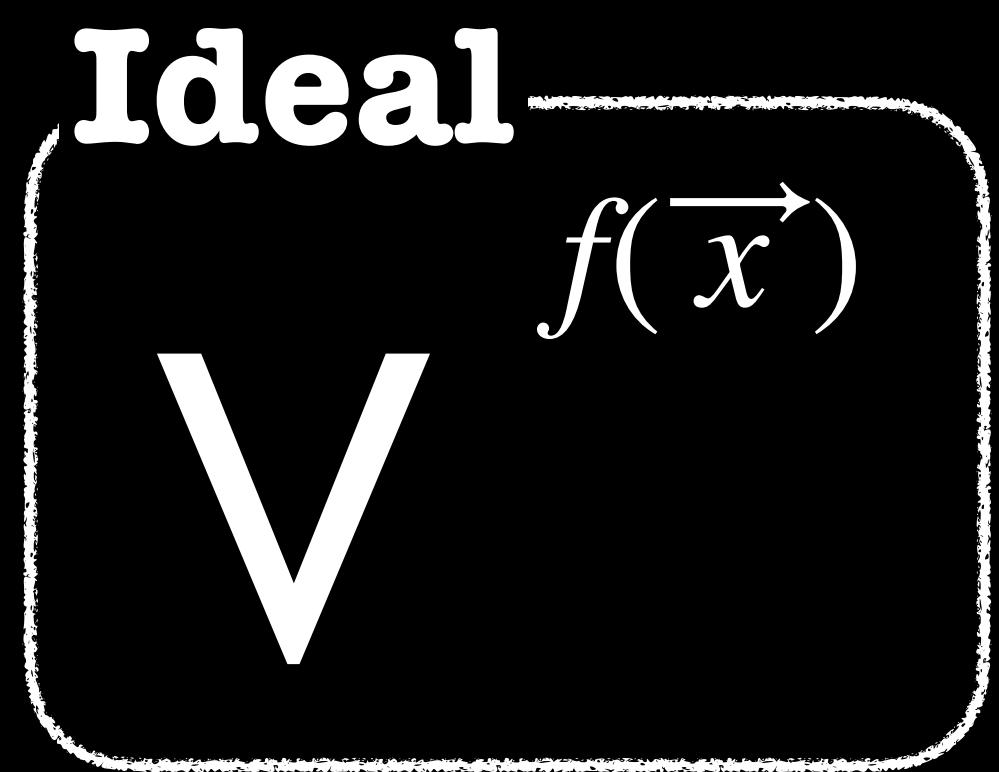
At the end of inner product:  $f(r_1, r_2, \dots, r_n) = f^{(n)}$



Probabilistic Interactive Oracle Proofs  
with Multivariate Polynomial queries

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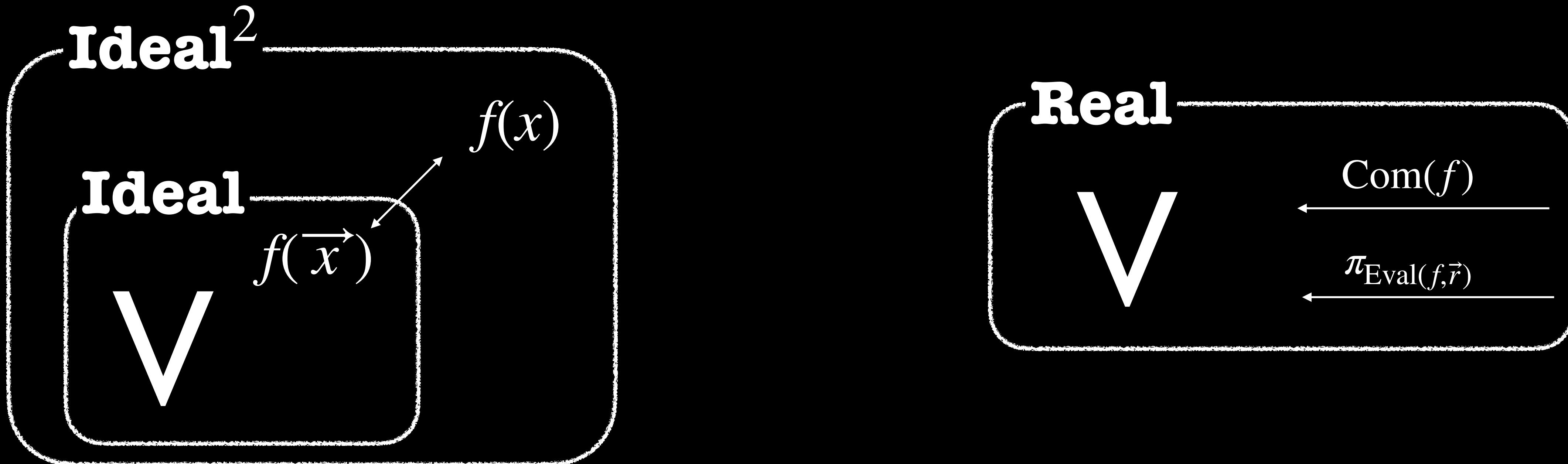


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Multivariate  
Polynomial Commitment Scheme

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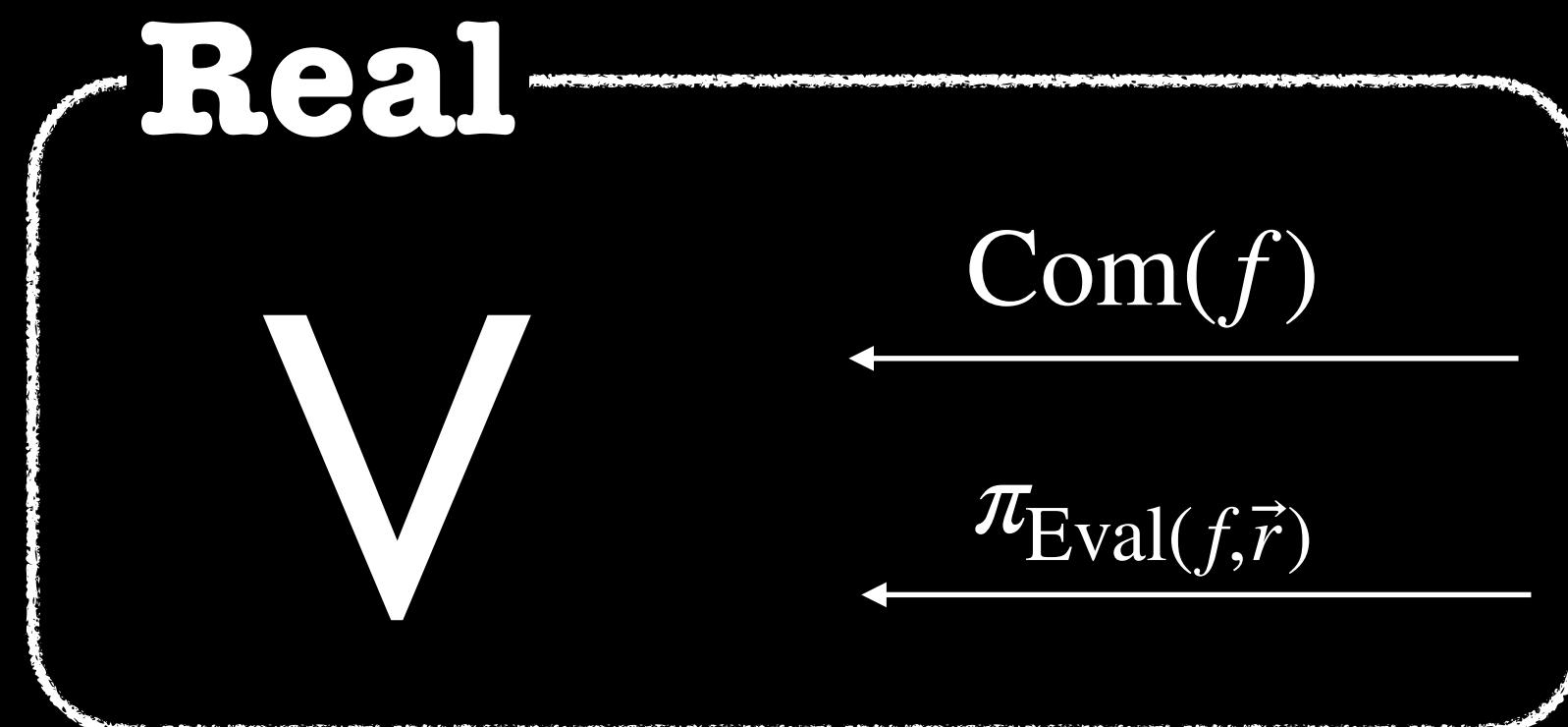
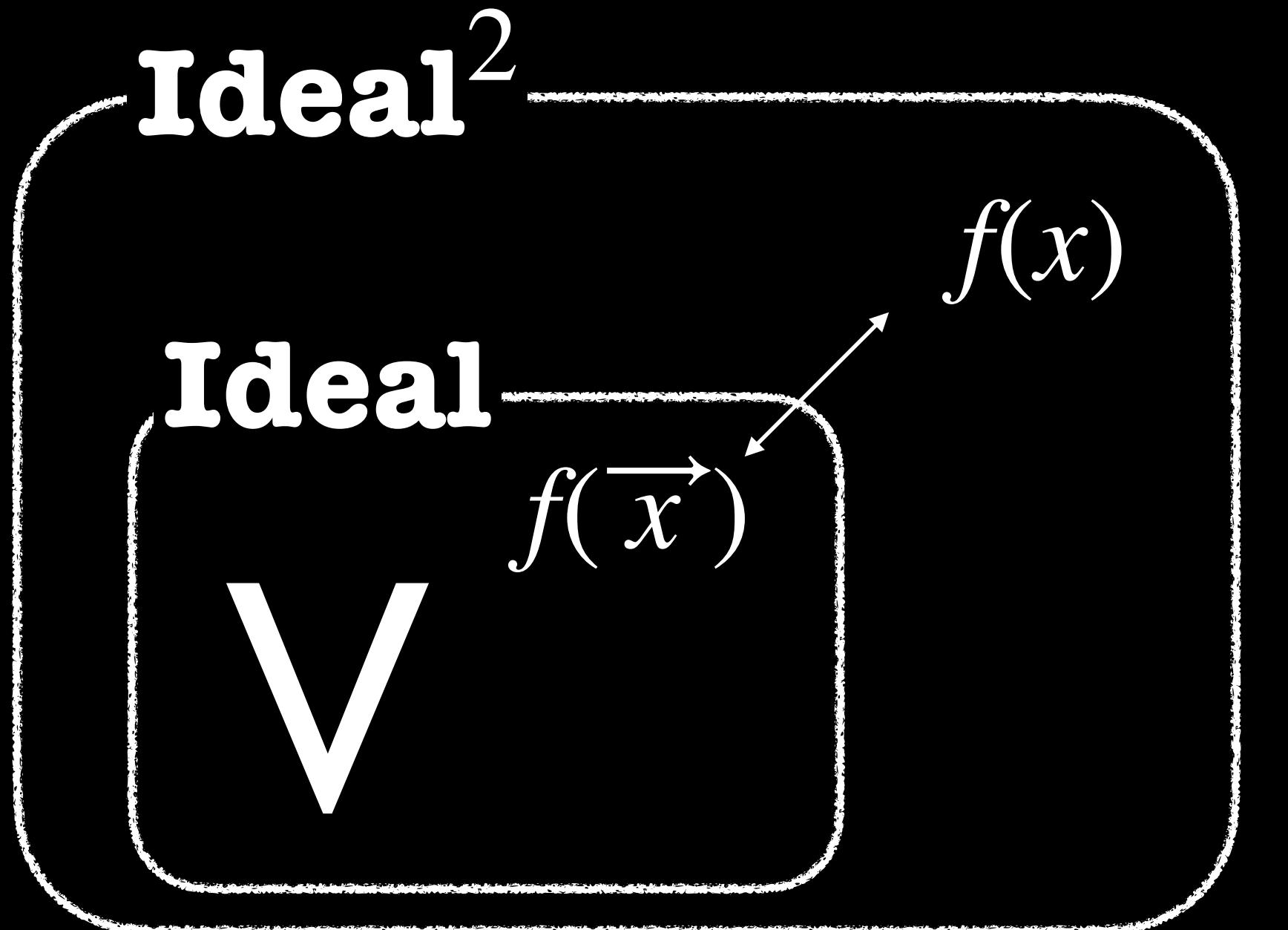


Probabilistic Interactive Oracle Proofs

Polynomial Commitment Scheme

# Tensorcheck

At the end of inner product:  $f(r_1, r_2, \dots, r_n) = f^{(n)}$



Probabilistic Interactive Oracle Proofs

Univariate  
Polynomial Commitment Scheme

# Cryptographic compiler

# Cryptographic compiler

An Elastic Polynomial Commitment Scheme

# Streaming KZG

## Overview

# Streaming KZG

## Overview

### Setup

Bilinear group  $(p, G, H, \mathbb{G}_1, \mathbb{G}_2, \mathbb{G}_T)$

$\text{ck} = (G, \tau G, \tau^2 G, \dots, \tau^n G),$

$\text{vk} = (H, \tau H)$

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Commit —

$C := \text{Com}_{\text{ck}}(f) = f(\tau) \cdot G$

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Stream of  $f$ :

$f_N$

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$f_{N-4}$

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Stream of  $f$ :

$f_N \quad f_{N-1} \quad f_{N-2} \quad f_{N-3} \quad f_{N-4} \quad \dots$

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Stream of  $\mathbf{ck}$ :

$\mathbf{ck}_N \quad \mathbf{ck}_{N-1} \quad \mathbf{ck}_{N-2} \quad \mathbf{ck}_{N-3} \quad \mathbf{ck}_{N-4} \quad \dots$

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Commit —

$C := \text{Com}_{\text{ck}}(f) = f(\tau) \cdot G$

**Accumulate**  $f_i \cdot \text{ck}_i$

Stream of  $f$ :

$f_N \quad f_{N-1} \quad f_{N-2} \quad f_{N-3} \quad f_{N-4} \quad \dots$

Stream of  $\text{ck}$ :

$\text{ck}_N \quad \text{ck}_{N-1} \quad \text{ck}_{N-2} \quad \text{ck}_{N-3} \quad \text{ck}_{N-4} \quad \dots$

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**Accumulate**  $f_i \cdot \text{ck}_i$

Stream of  $f$ :

$f_{N-1}$   $f_{N-2}$   $f_{N-3}$   $f_{N-4}$  ...

$C += f_N \text{ ck}_N$

Stream of  $\text{ck}$ :

$\text{ck}_{N-1}$   $\text{ck}_{N-2}$   $\text{ck}_{N-3}$   $\text{ck}_{N-4}$  ...

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$C +=$

Stream of  $\text{ck}$ :

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Commit —

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Accumulate  $f_i \cdot \mathbf{ck}_i$

Stream of  $f$ :

$f_{N-2}$   $f_{N-3}$   $f_{N-4}$  ...

$$C += f_{N-1} \quad \mathbf{ck}_{N-1}$$

Stream of  $\mathbf{ck}$ :

$\mathbf{ck}_{N-2}$   $\mathbf{ck}_{N-3}$   $\mathbf{ck}_{N-4}$  ...

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Eval

$\pi := \text{Eval}_{\text{ck}}(f, \alpha) = \text{Com}_{\text{ck}}(q)$

where  $f = (x - \alpha)q(x) + r(x)$

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Stream of  $f$ :

$f_N$

$f_{N-1}$

$f_{N-2}$

$f_{N-3}$

$f_{N-4}$

...

# Streaming KZG

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Stream of  $q$ :

Stream of  $f$ :

$f_N \quad f_{N-1} \quad f_{N-2} \quad f_{N-3} \quad f_{N-4} \quad \dots$

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Ruffini's law:  $q_i = f_{i+1} + q_{i+1}\alpha !$

Stream of  $q$ :

$q_N$

Stream of  $f$ :

$f_{N-1}$

$f_{N-2}$

$f_{N-3}$

$f_{N-4}$

...

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Bilinear group  $(p, G, H, \mathbb{G}_1, \mathbb{G}_2, \mathbb{G}_T)$

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Ruffini's law:  $q_i = f_{i+1} + q_{i+1}\alpha !$

Stream of  $q$ :

$q_N$

$q_{N-1} := q_N\alpha + f_{N-1}$

$f_{N-2}$

$f_{N-3}$

$f_{N-4}$

...

Stream of  $f$ :

# Streaming KZG

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Ruffini's law:  $q_i = f_{i+1} + q_{i+1}\alpha$  !

Stream of  $f$ :

Stream of  $q$ :

$q_{N-1}$

$f_{N-2}$

$f_{N-3}$

$f_{N-4}$

...

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where  $f = (x - \alpha)q(x) + r(x)$

Ruffini's law:  $q_i = f_{i+1} + q_{i+1}\alpha !$

Stream of  $q$ :

$q_{N-1}$

Stream of  $f$ :

$f_{N-2}$   $f_{N-3}$   $f_{N-4}$  ...

Coding elastic SNARKS

# Implementing elastic arguments

Gemini—

# Implementing elastic arguments

Gemini

Polynomial Commitment schemes

Tensorcheck

Elastic KZG

Elastic PST

# Implementing elastic arguments

Gemini

Elastic Inner-Product

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Tensorcheck

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# Implementing elastic arguments

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Elastic SNARG

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# Implementing elastic arguments

Gemini

Subprotocols

Elastic Lookup protocol

Elastic Memory Checking

Elastic SNARG

Elastic Grand Product

Elastic Inner-Product

Polynomial Commitment schemes

Tensorcheck

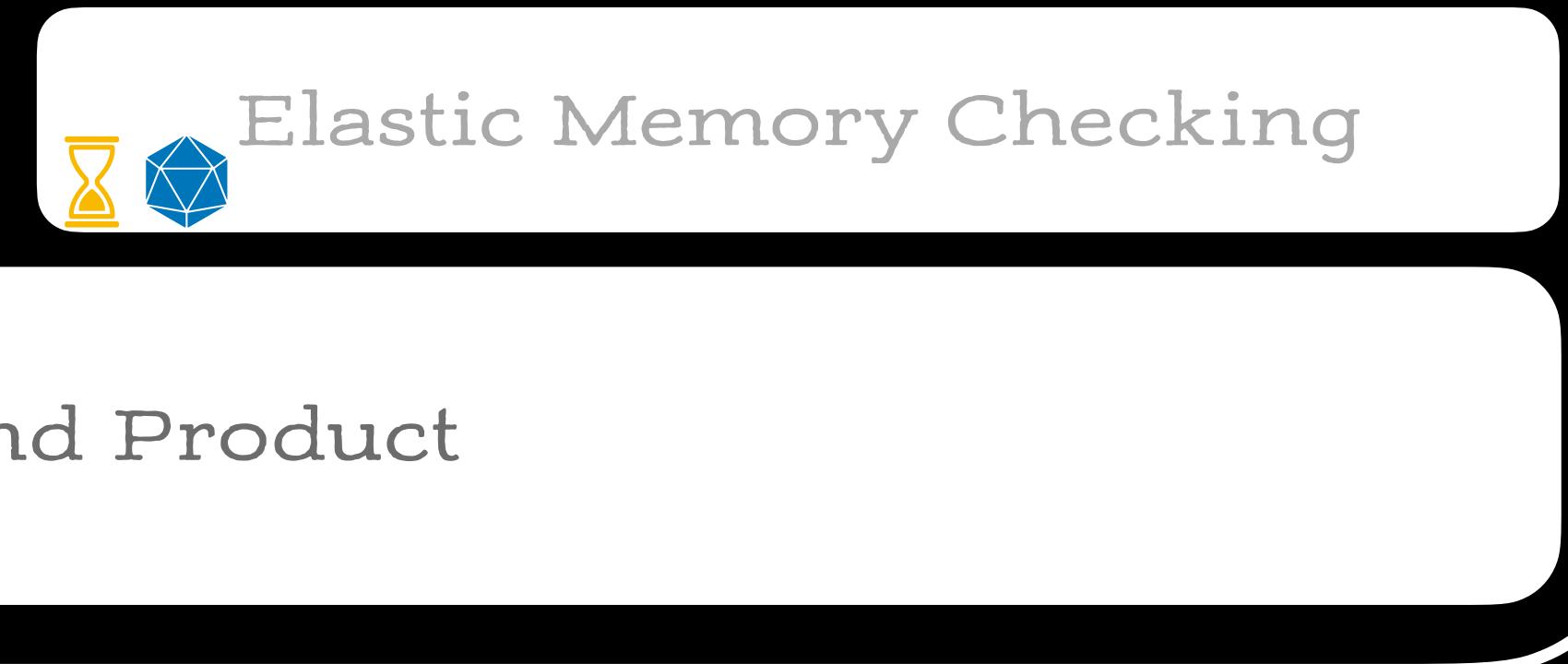
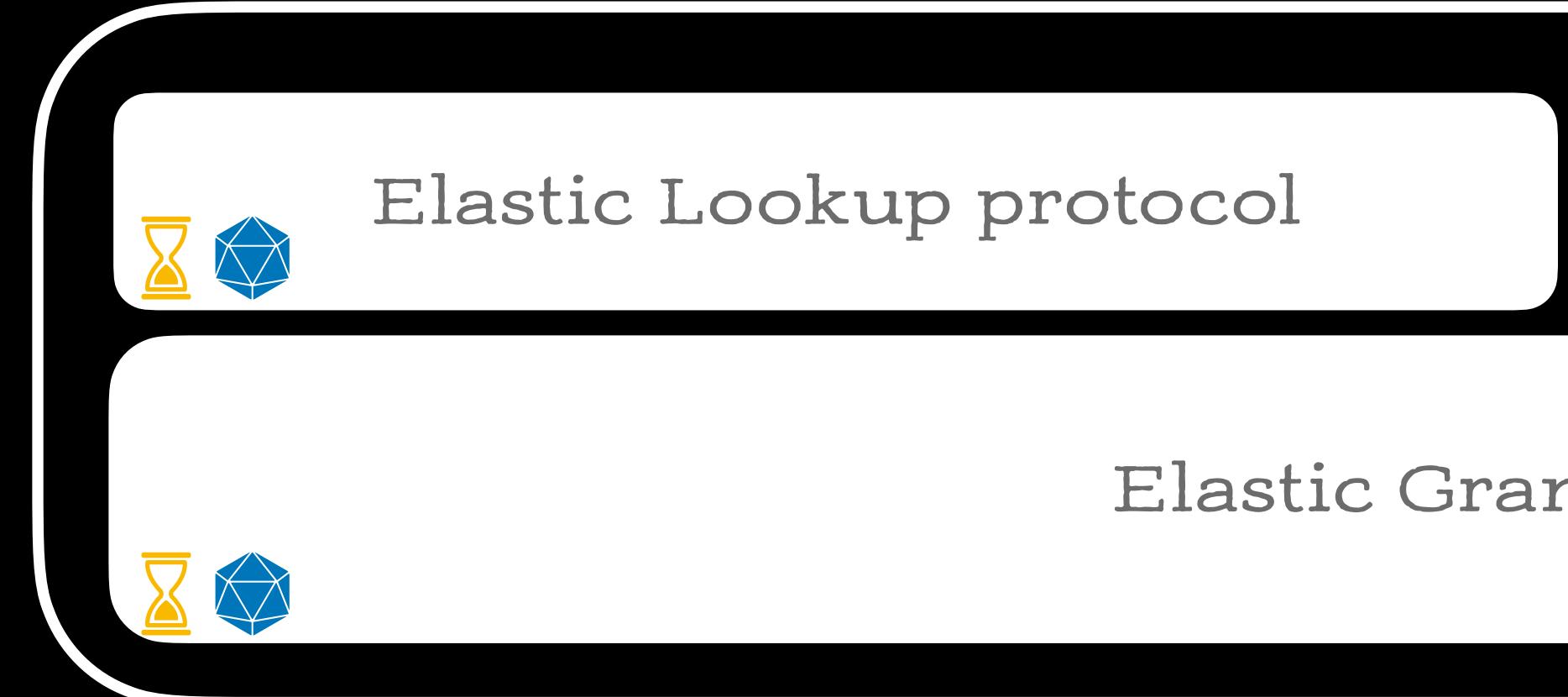
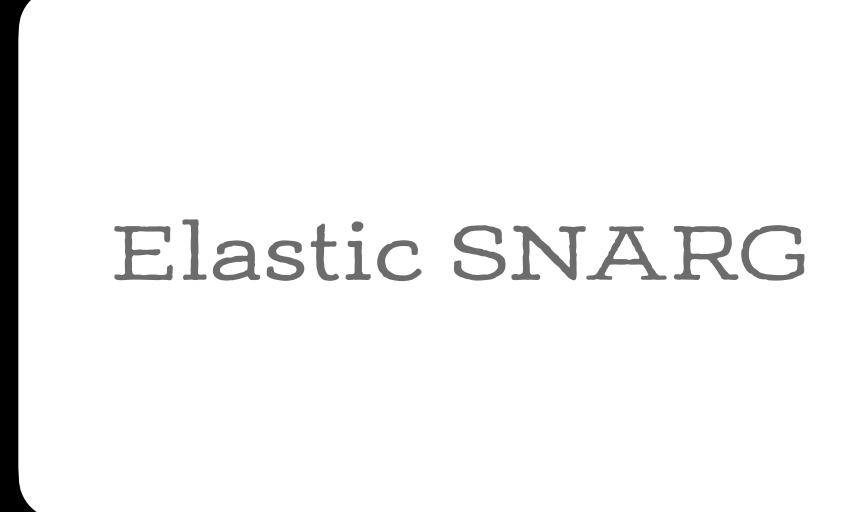
Elastic PST

Elastic KZG

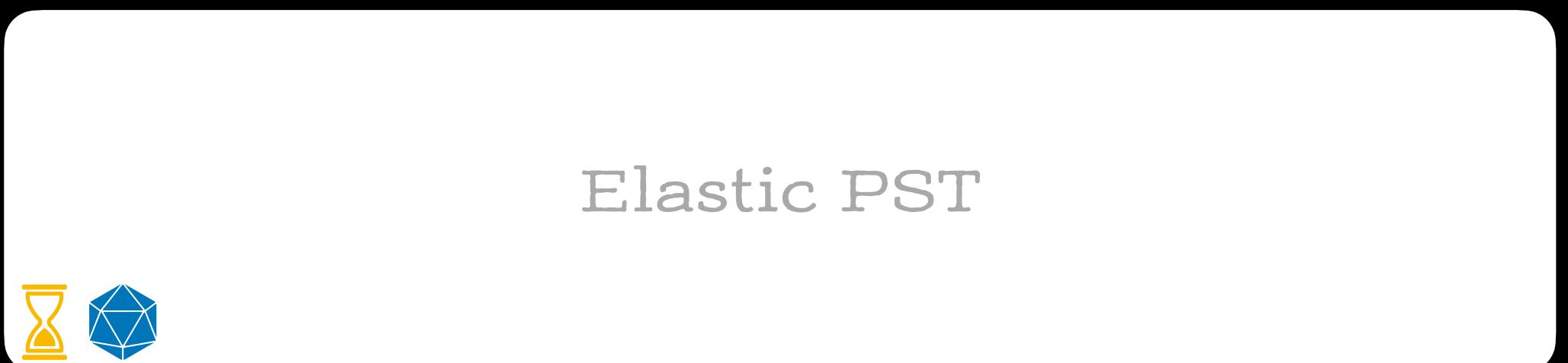
# Implementing elastic arguments

Gemini

Subprotocols



Polynomial Commitment schemes



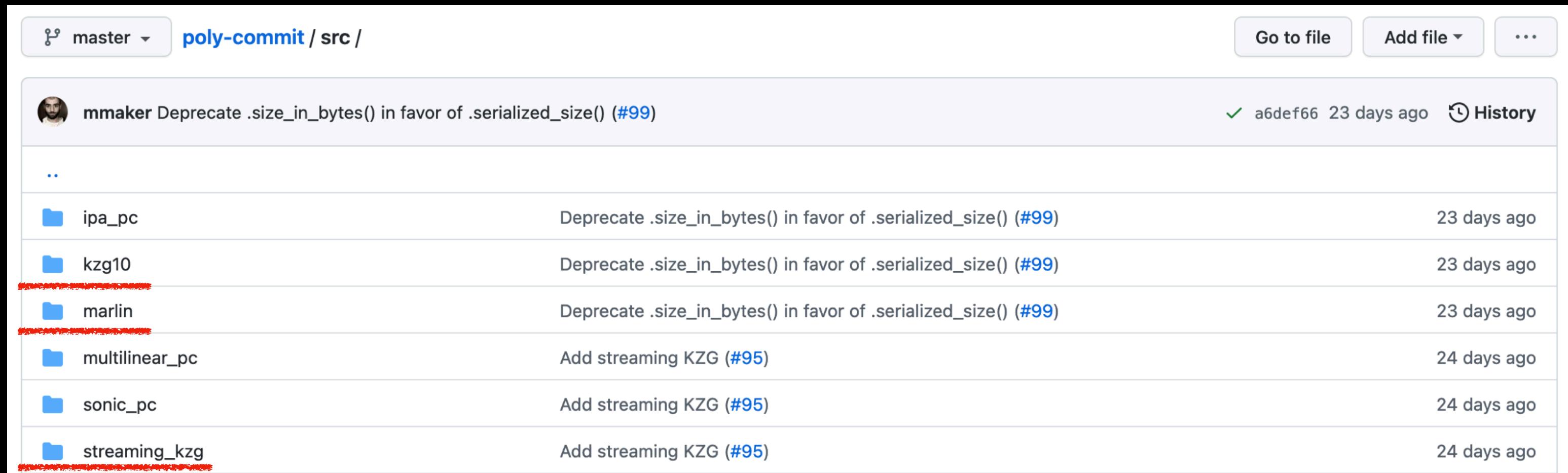
# Implementation overview

Contributing to arkworks-rs: streaming KZG and PST.

Streams are Iterators: embed computation tree in the type.

# Implementation overview

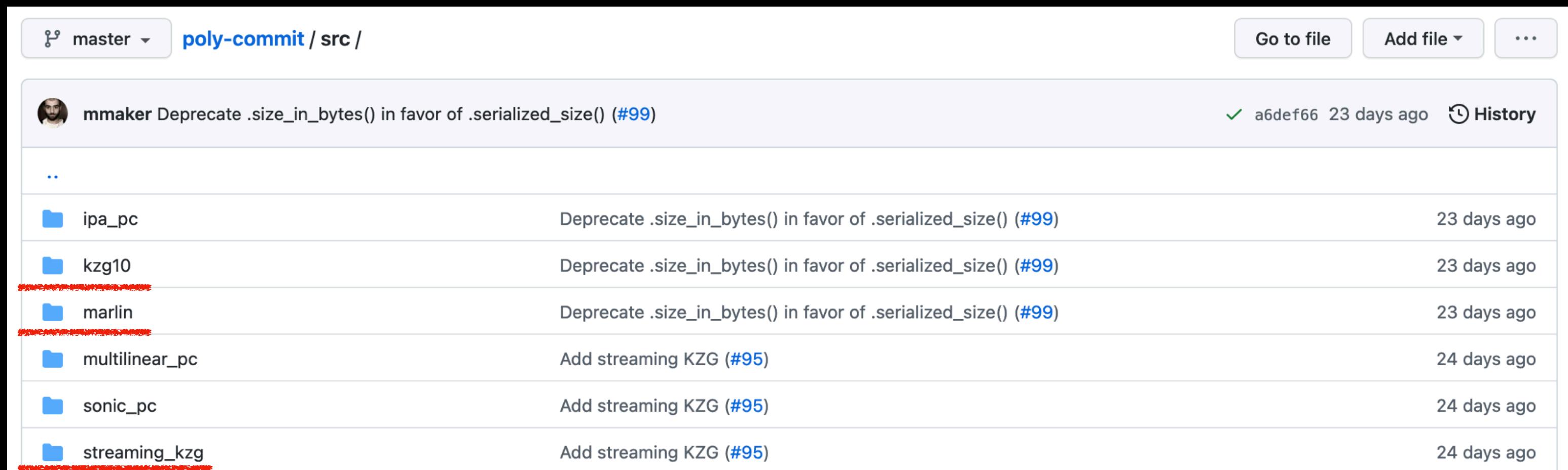
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# Implementation overview

Contributing to arkworks-rs: streaming KZG and PST.



Streams are Iterators: embed computation tree in the type.

```
SortedStreamer<'_>,  
AlgebraicHash<'_>, <E as PairingEngine>::Fr,  
Tensor<'_>, <E as PairingEngine>::Fr>, IterableRange>,  
JointRowStream<'_>, SM, SM, SM, <E as PairingEngine>::Fr>  
>  
>
```

Benchmark huge circuits

# Gemini in practice

Previous works: benchmarking stops at millions of gates.

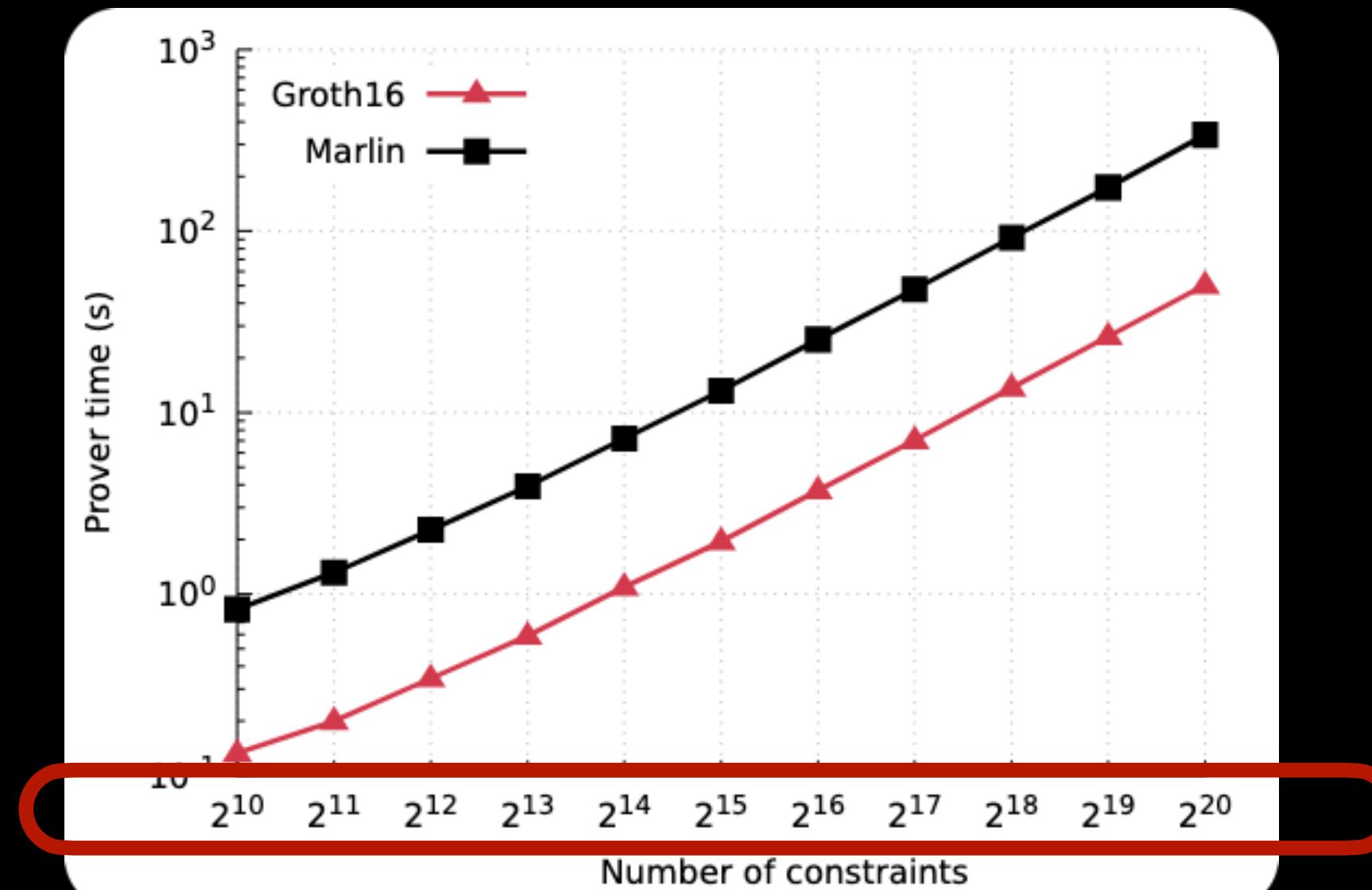
# Gemini in practice

Previous works: benchmarking stops at millions of gates.

	$2^{10}$	$2^{11}$	$2^{12}$	$2^{13}$	$2^{14}$	$2^{15}$	$2^{16}$	$2^{17}$	$2^{18}$	$2^{19}$	$2^{20}$
Groth16	0.17	0.26	0.46	0.85	1.5	2.8	5.4	10.1	23.2	44.7	76.2
Hyrax	1.2	1.7	2.8	4.1	7.2	13.9	22.5	44.6	90	181	447
Ligero	0.2	0.3	0.6	1.2	2.3	3.4	6.7	13.7	27.2	56.3	112
Ligero-heuristic	0.16	0.3	0.6	1.3	2.4	3.5	6.6	13	25	51.3	101
Aurora	0.7	1.2	2.5	4.6	9	17.3	36	69	140	282	688
Aurora-heuristic	0.4	0.7	1.4	3.2	5.8	12.2	24.8	52.2	108	224	509
Fractal	1	1.8	3.6	6.7	15	30	61	125	337	—	—
Fractal-heuristic	0.8	1.5	3	6.5	14	29	60	125	342	—	—
SpartanNIZK	0.02	0.03	0.04	0.06	0.1	0.17	0.33	0.57	1.1	2.14	4.5
SpartanSNARK	0.07	0.13	0.21	0.39	0.79	1.3	2.6	4.9	9.2	18.5	36.3

FIGURE 7—Prover’s performance (in seconds) for varying R1CS instance sizes under different schemes.

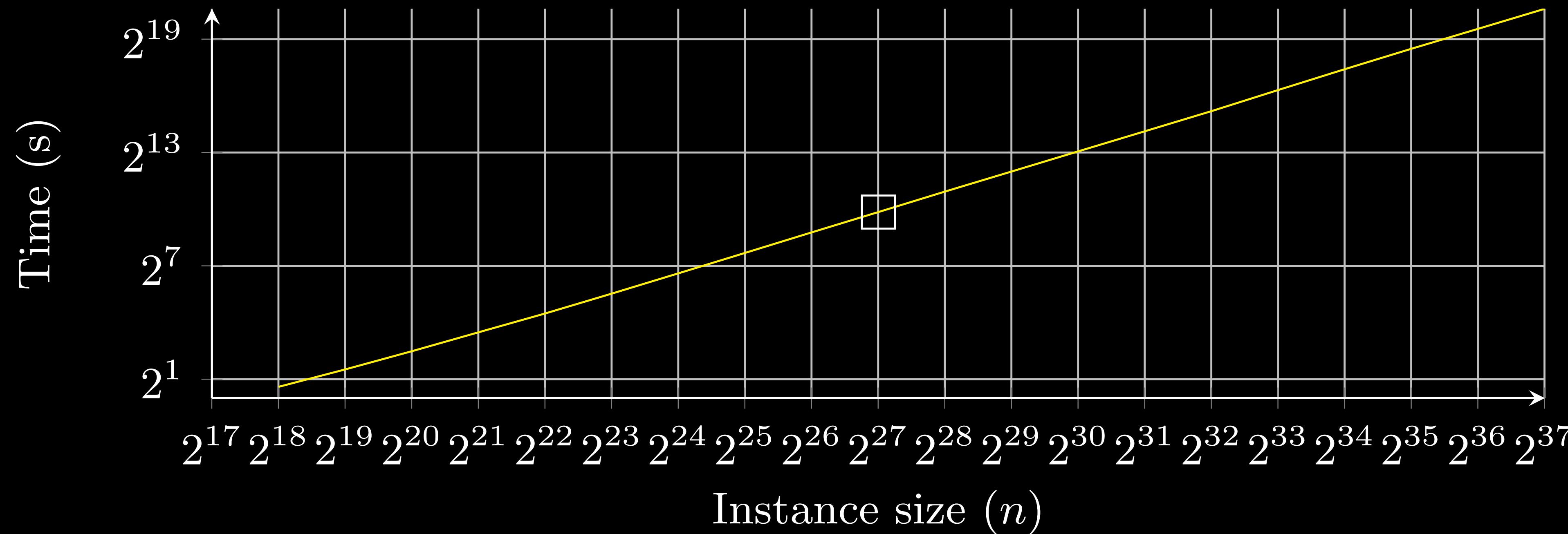
Srinath Setty, Spartan: Efficient and general-purpose zkSNARKs without trusted setup, 2019



Chiesa, Maller, Hu, Mishra, Vesely, Ward: Marlin: Preprocessing SNARKs with universal updatable CRS

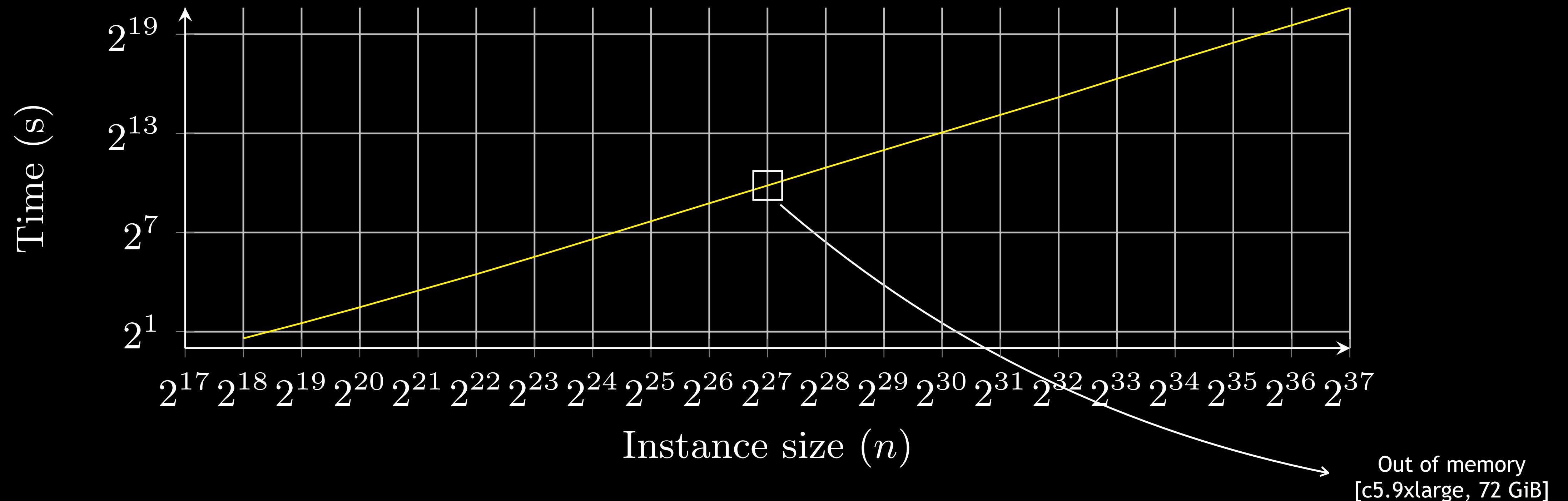
# Gemini in practice: SNARK

Hundreds of billions of gates



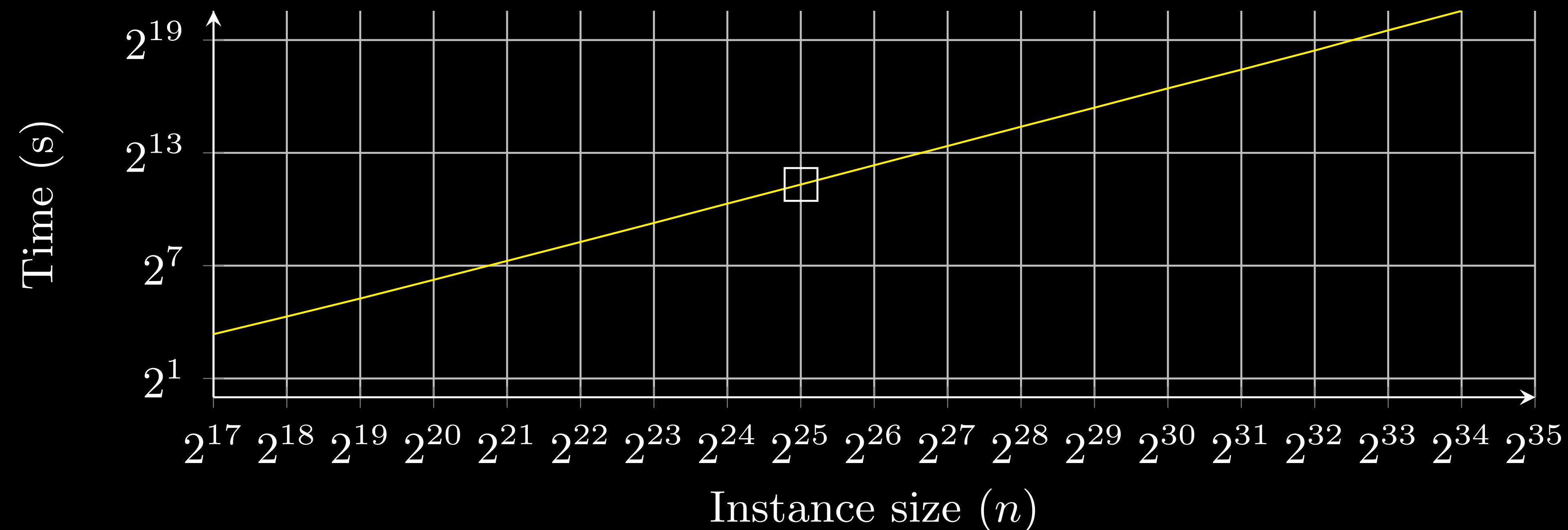
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# Gemini in practice: preprocessing SNARK

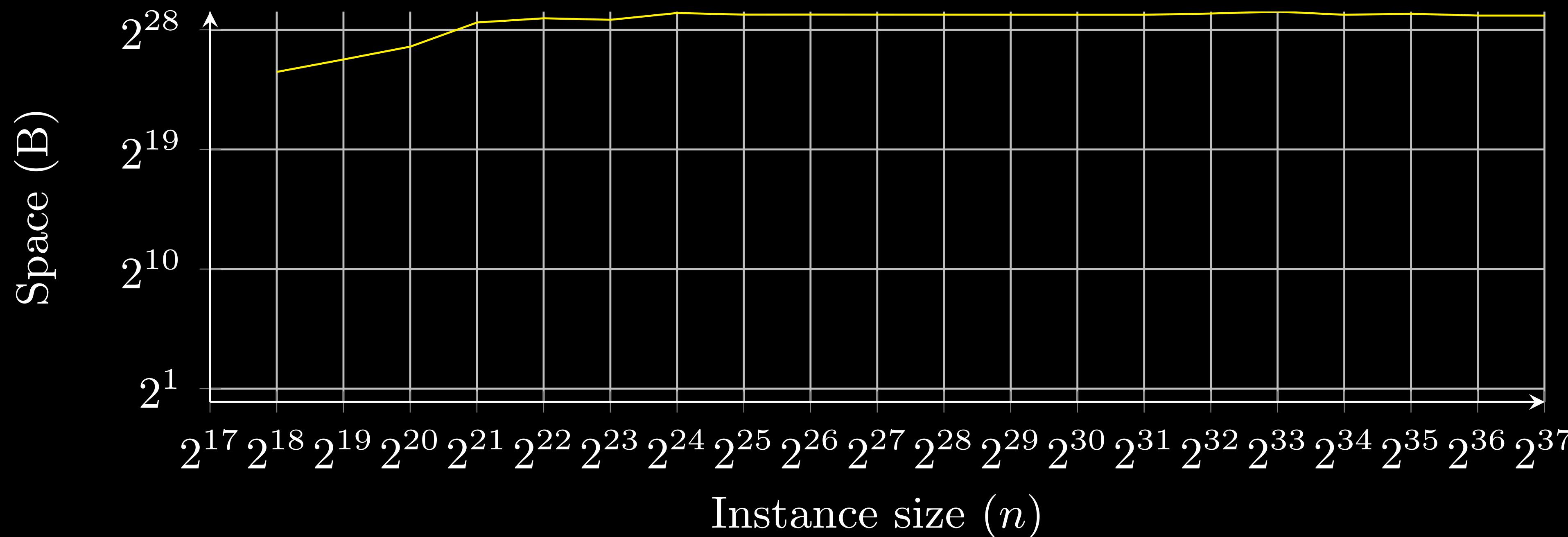
Tens of billions of gates



Proving time:  $O(n \log^2 n)$

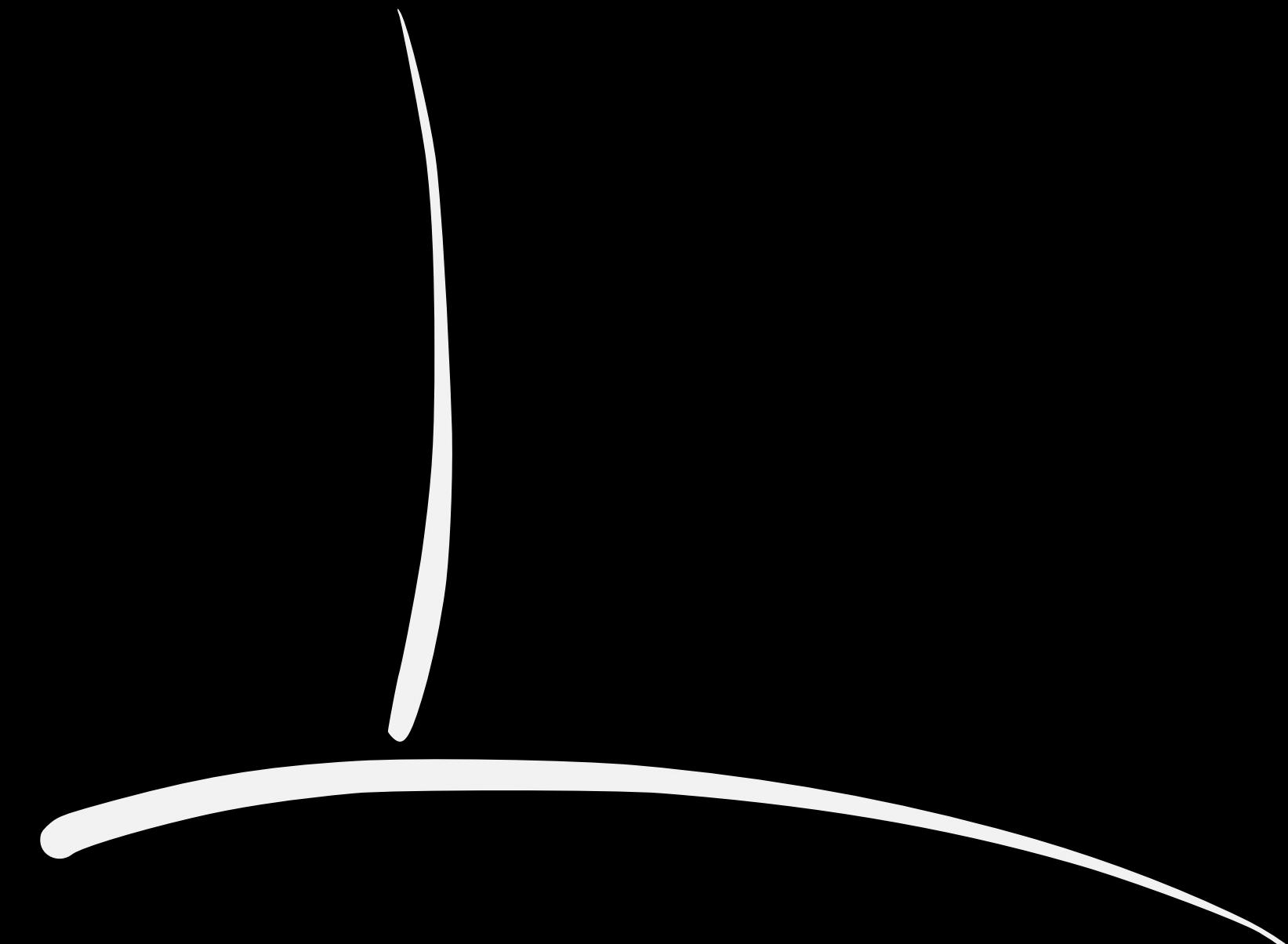
# Gemini in practice: SNARK

If you care about logarithmic factors, you care about constants

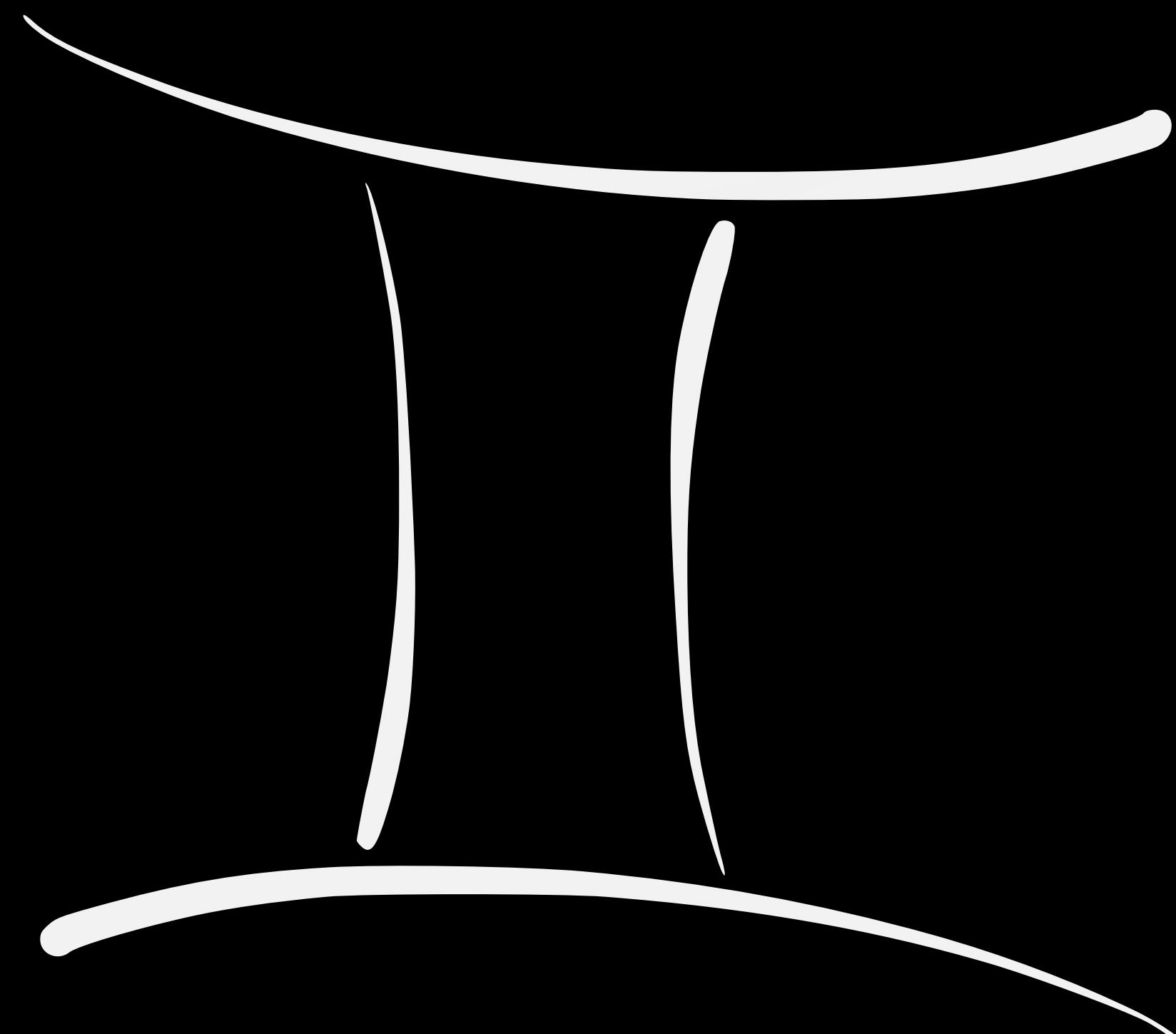


Proving space:  $O(\log n)$

[ia.cr/2022/420](https://ia.cr/2022/420)  
[arkworks.rs/gemini](https://arkworks.rs/gemini)



[ia.cr/2022/420](https://ia.cr/2022/420)  
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