Post-Quantum Security of the Even-Mansour Cipher

Gorjan Alagic, Chen Bai, Jonathan Katz and **Christian Majenz**

Eurocrypt 2022, Trondheim

Outline

- Motivation
- Introduction
- Result
- Techniques, Technical Contributions

Motivation

Quantum attacks

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Post-quantum attacks

Public-key cryptography: Shor's algorithm

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Post-quantum security for symmetric cryptography

Far less of a concern than for public-key crypto….

But we should prove post-quantum security where possible!

Introduction

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Security against quantum attacks…

Post-quantum security

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But what *are* realistic quantum attacks?

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Beyond post-quantum (Q2)

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Challenge: Mix of classical and quantum oracles!

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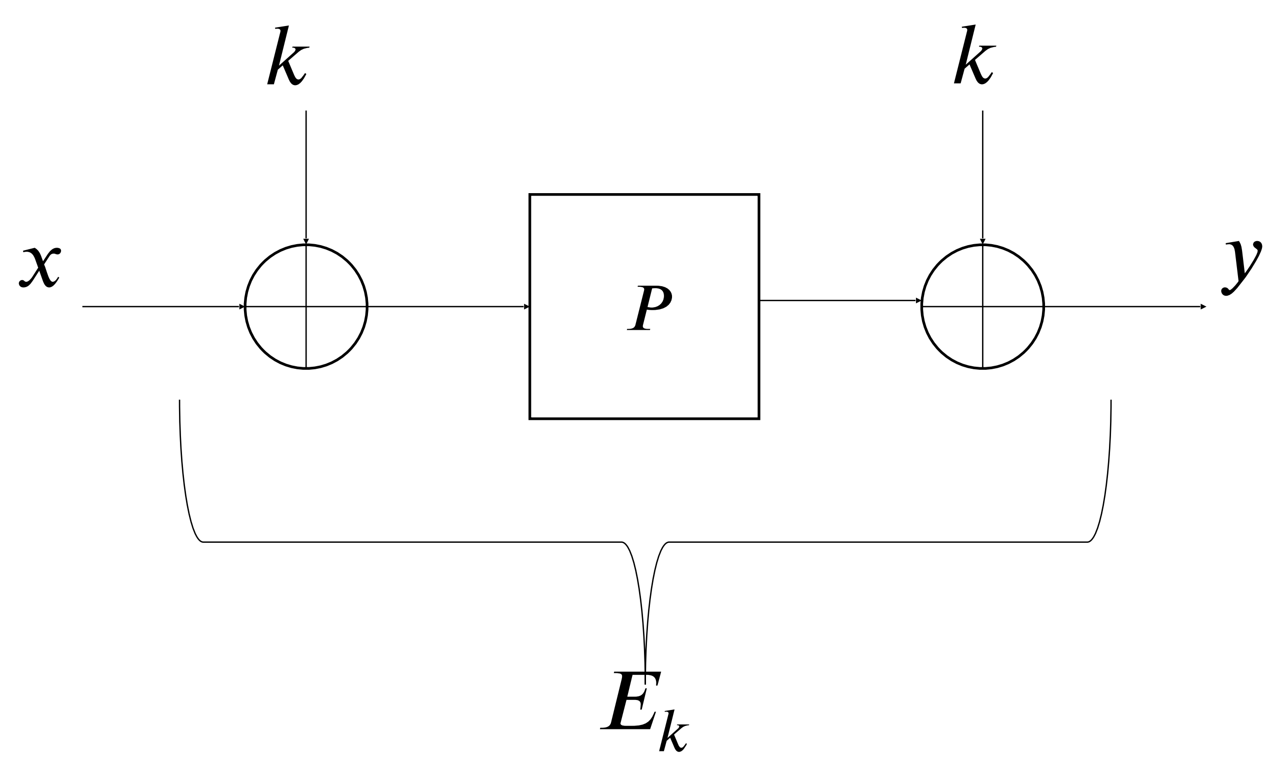
Unrealistic

The Even-Mansour Cipher

Given a public permutation $P: \{0,1\}^n \to \{0,1\}^n$ and a key $k \in \{0,1\}^n$, the cipher $E: \{0,1\}^n \times \{0,1\}^n \to \{0,1\}^n$ is defined as:

$E_k[P](x) = P(x \oplus k) \oplus k$



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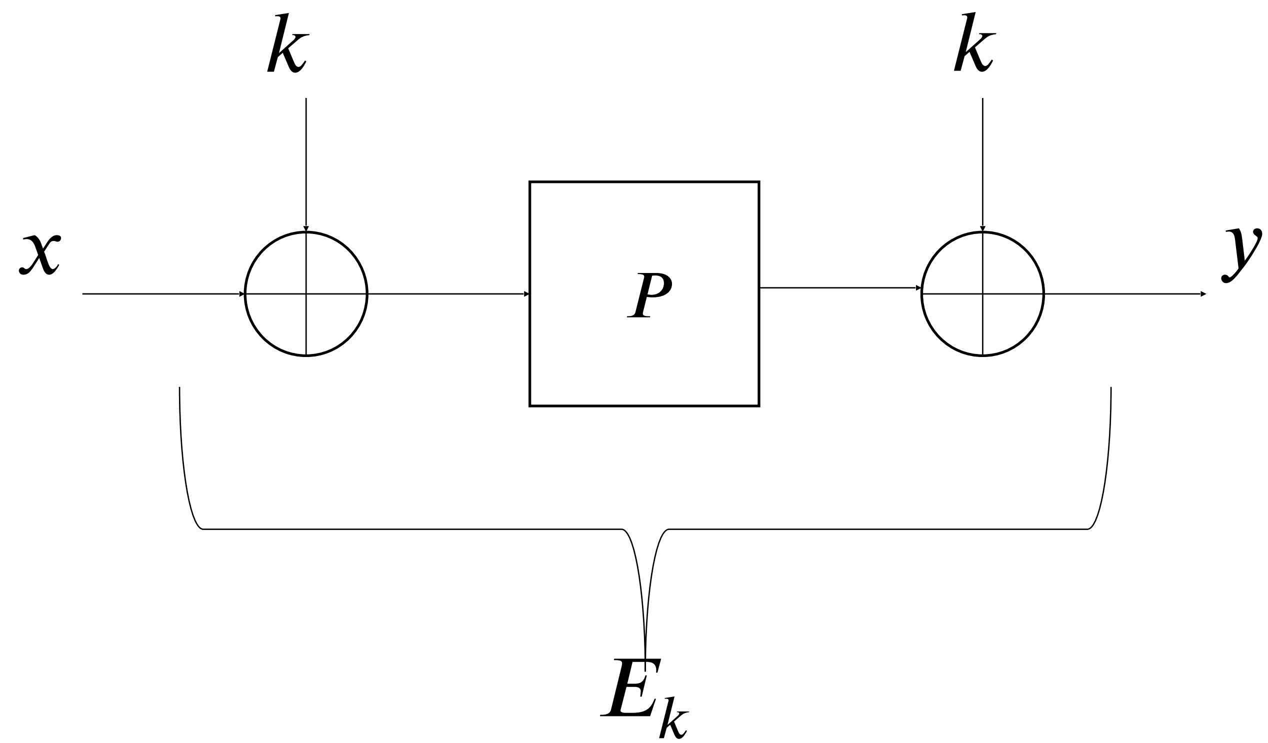

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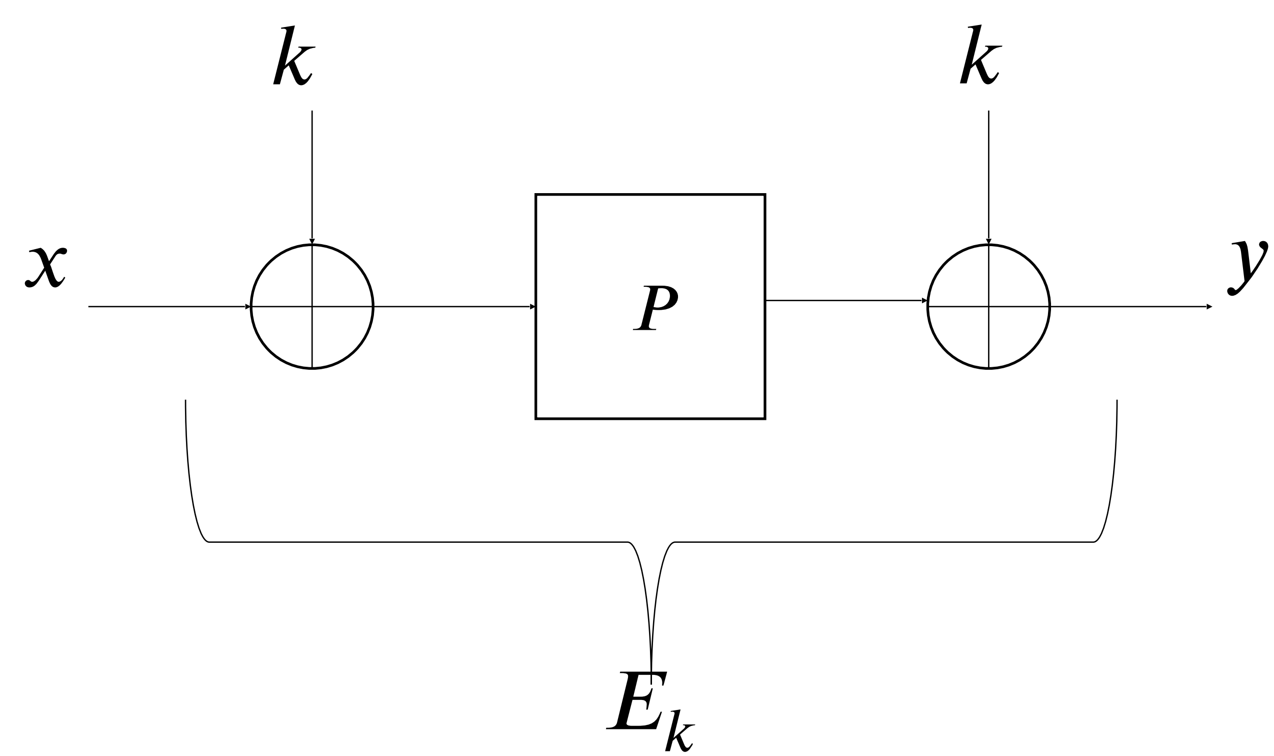
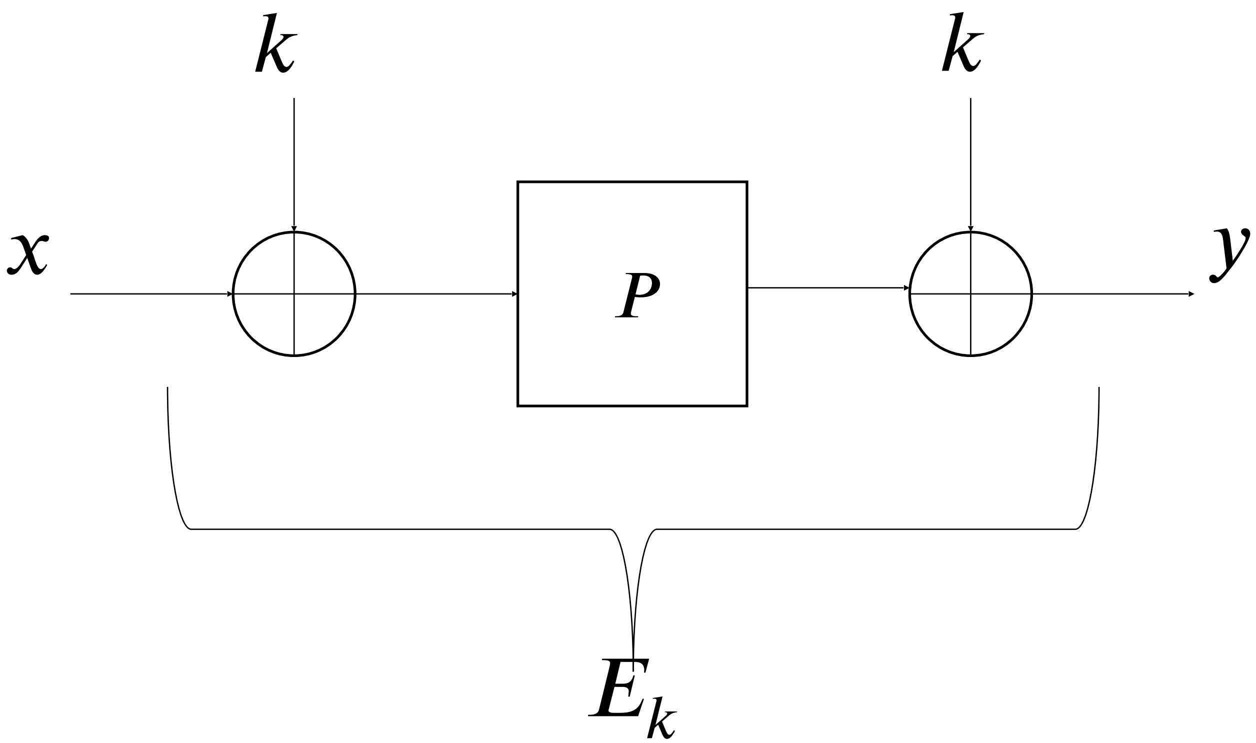
- Minimal Construction
- Key ingredient in many symmetric-key constructions, e.g. Elephant, Chaskey



Security of Even-Mansour

Suppose the adversary makes q_E queries to E_k and q_P queries to P:



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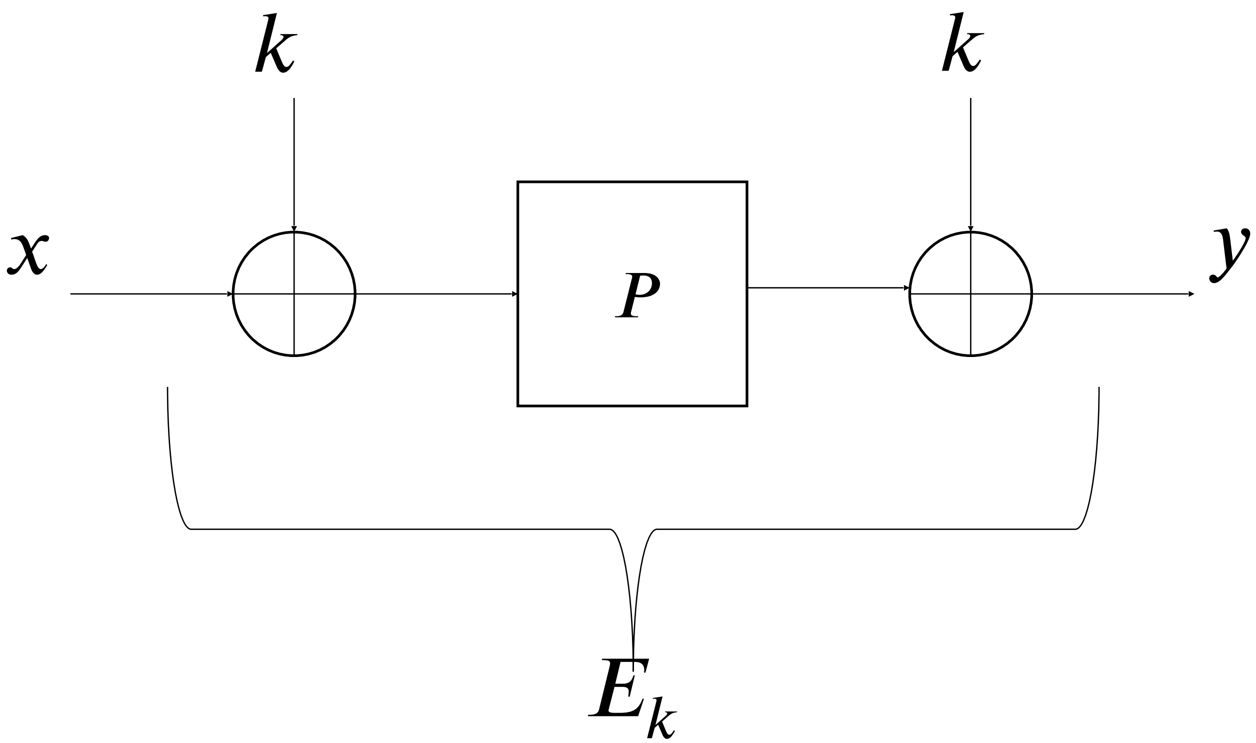

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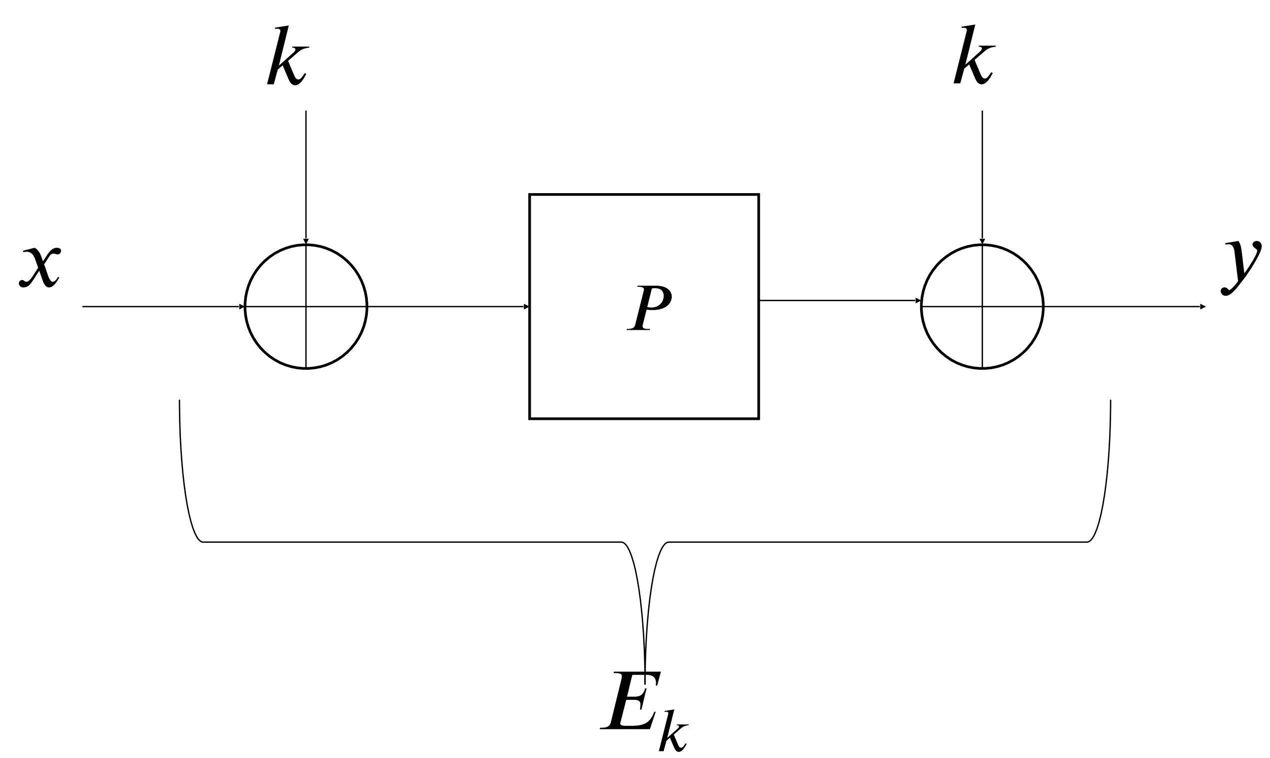

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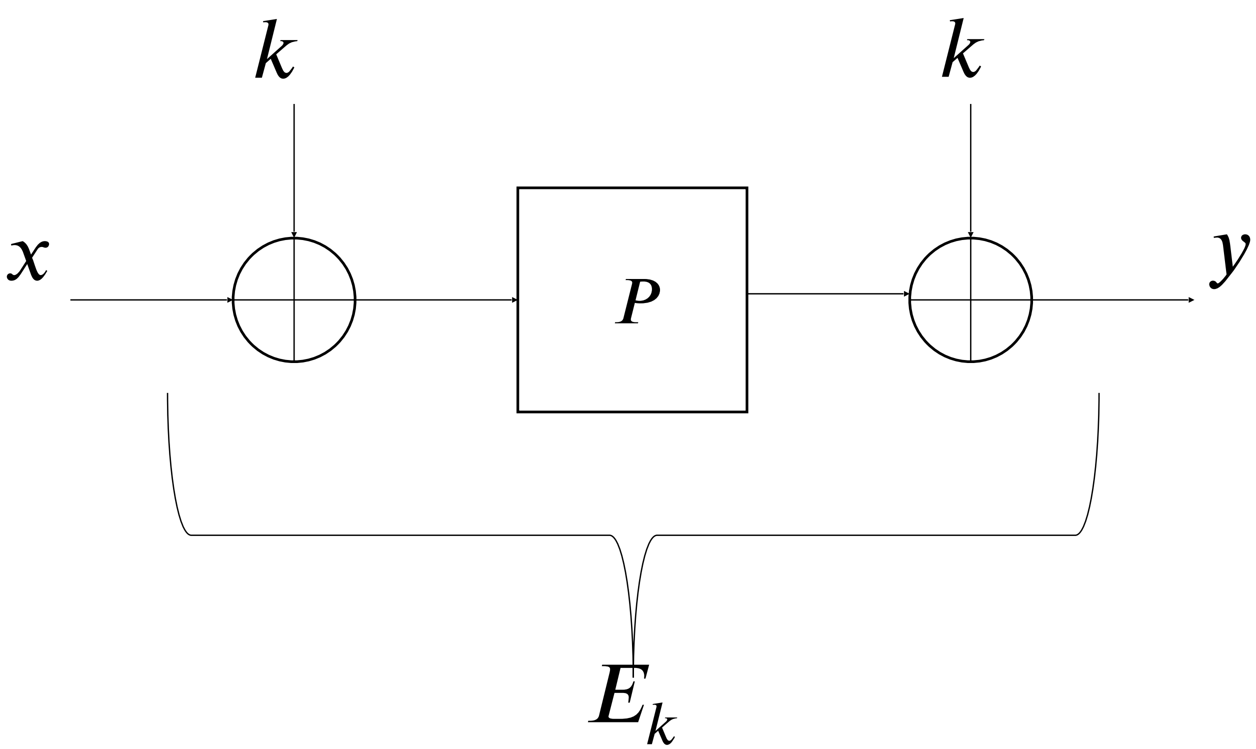
- **Q1 (post-quantum)?**



Result

Main Result

Theorem (Alagic, Bai, Katz, M):

Let P and R be a random permutations and E_k the Even-Mansour cipher using P. Any quantum adversary making at most q_P quantum queries to P and q_E classical queries to an oracle O has distinguishing advantage between $O = E_k$ and $O = R$ at most

$$10 \cdot 2^{-\frac{n}{2}} \left(q_P \sqrt{q_E} + q_E \sqrt{q_P} \right)$$

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- Tight characterization of query complexity assuming $q_E \ll q_P$ (matching attacks: BHT/offline Simon's)

Techniques, Technical Contributions

General approach

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- Quantum queries $\Rightarrow$ no transcript!
- Resort to “more primitive” technique: hybrid argument

The hybrid argument

- Think of adversary having $q_E + 1$ stages:

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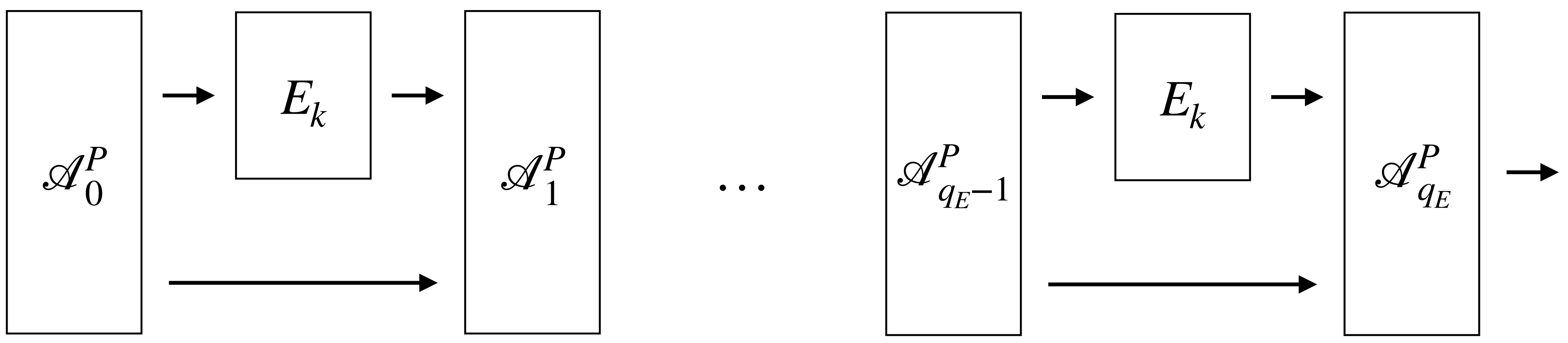

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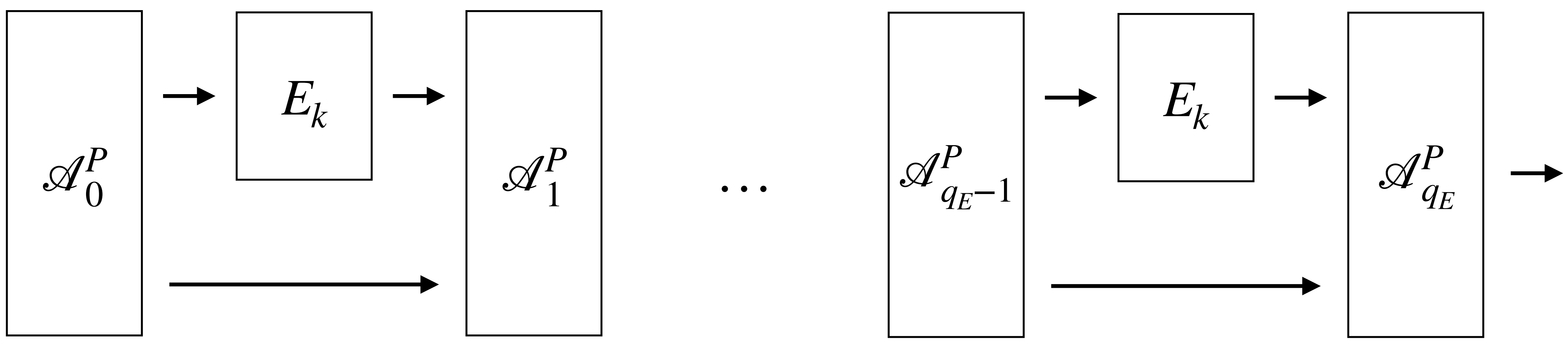

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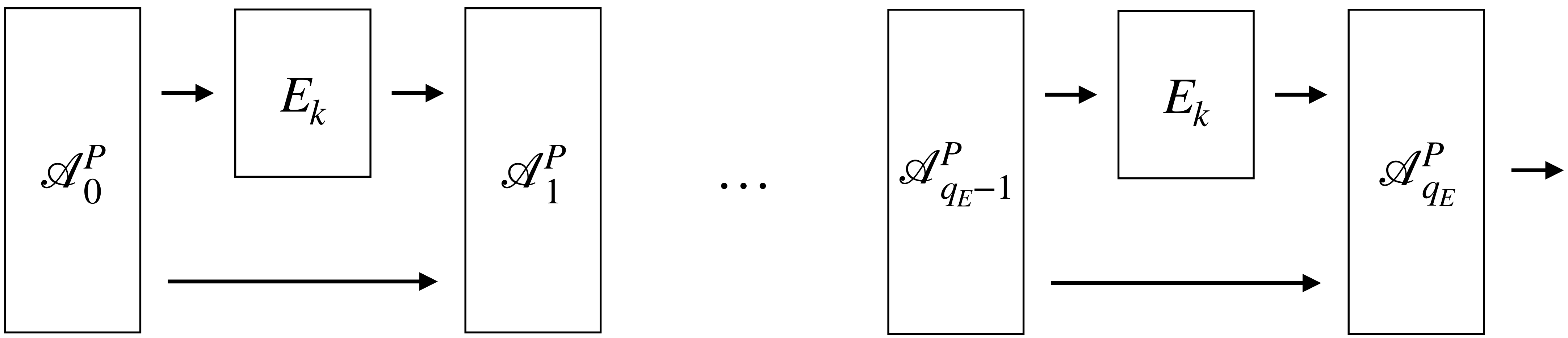
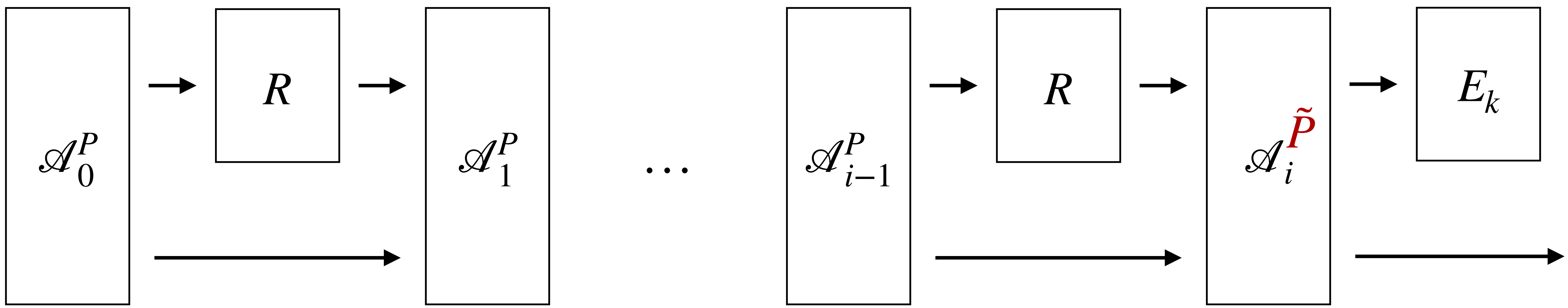
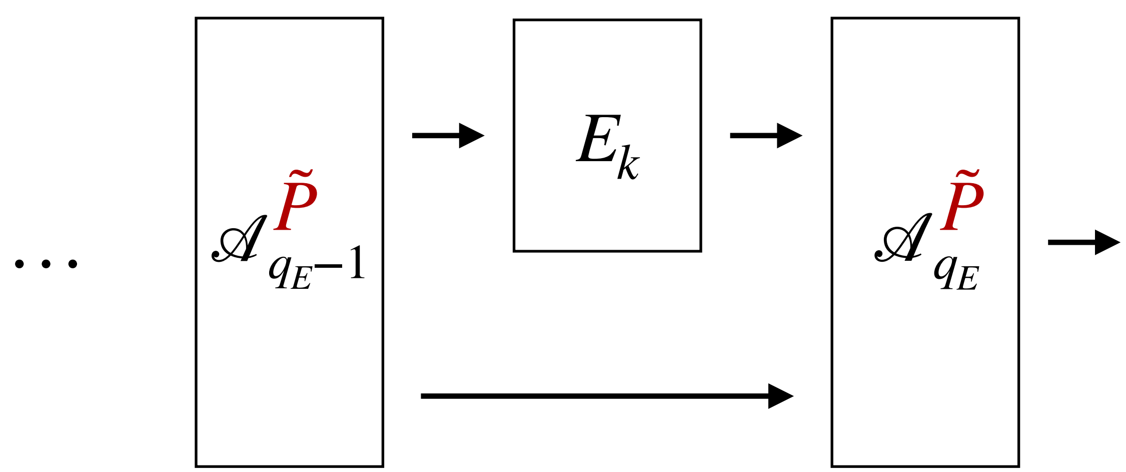


- Each $\mathscr{A}_i^P$ makes arbitrary number of quantum queries to P, sum $\leq q_P$
- Naive hybrids: replace E_k with R for first i calls
- Consistency of the oracles? Postpone problem until it goes away

The hybrids

H_i:





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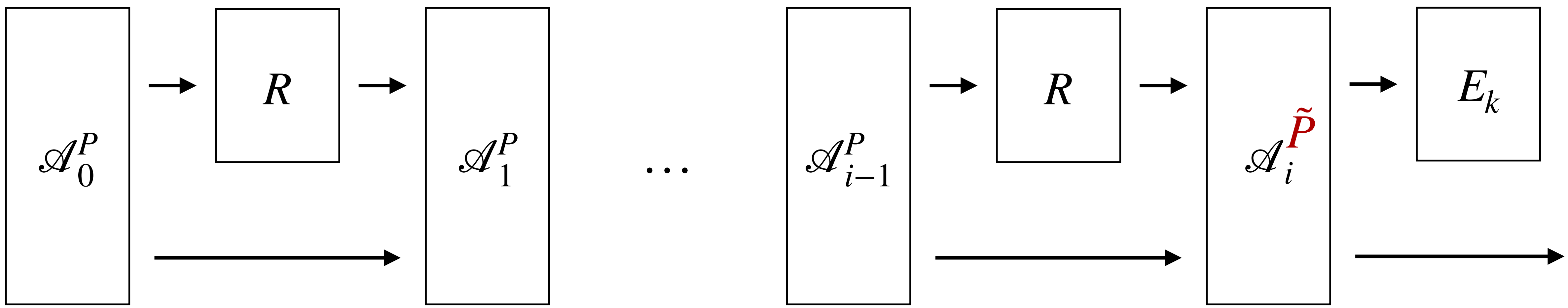

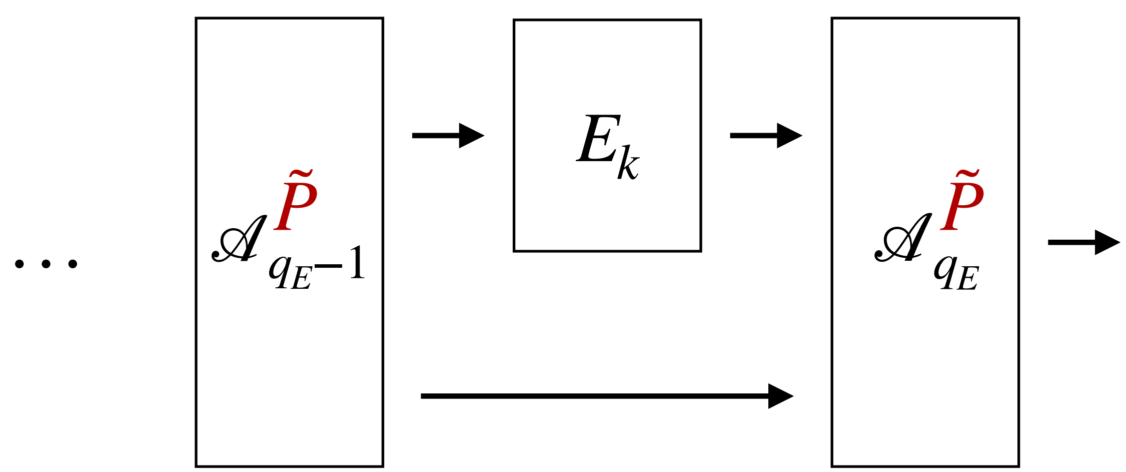

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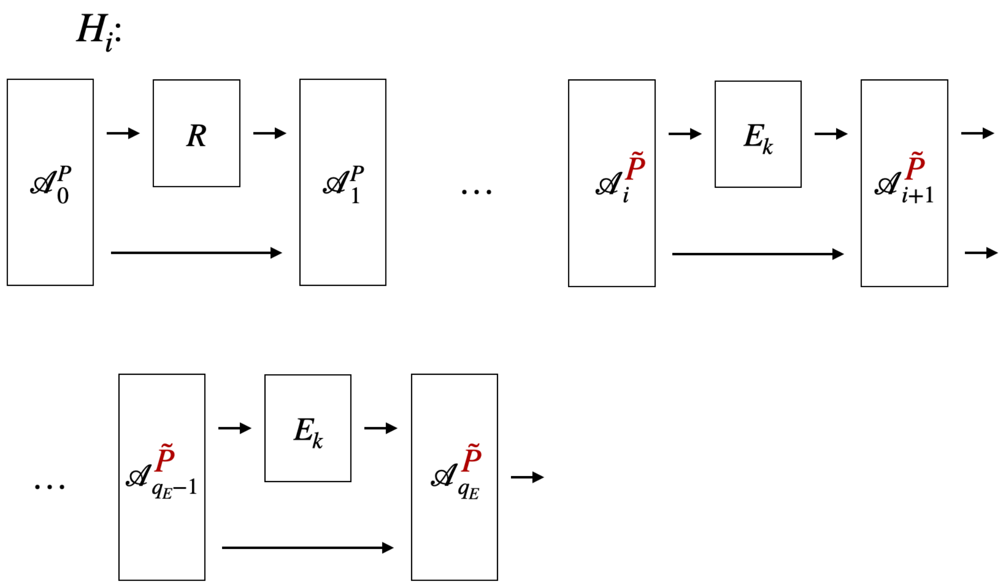
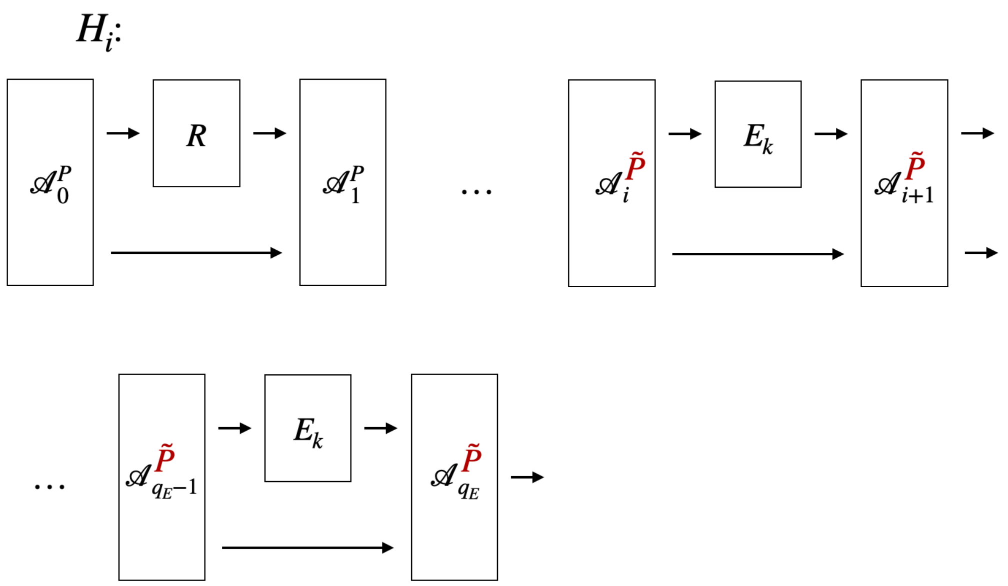
So $\tilde{P}$ gets messed up more and more as i grows… but in the end it's gone!

Technical Lemmas



- Two steps in hybrid transition:

Technical Lemmas



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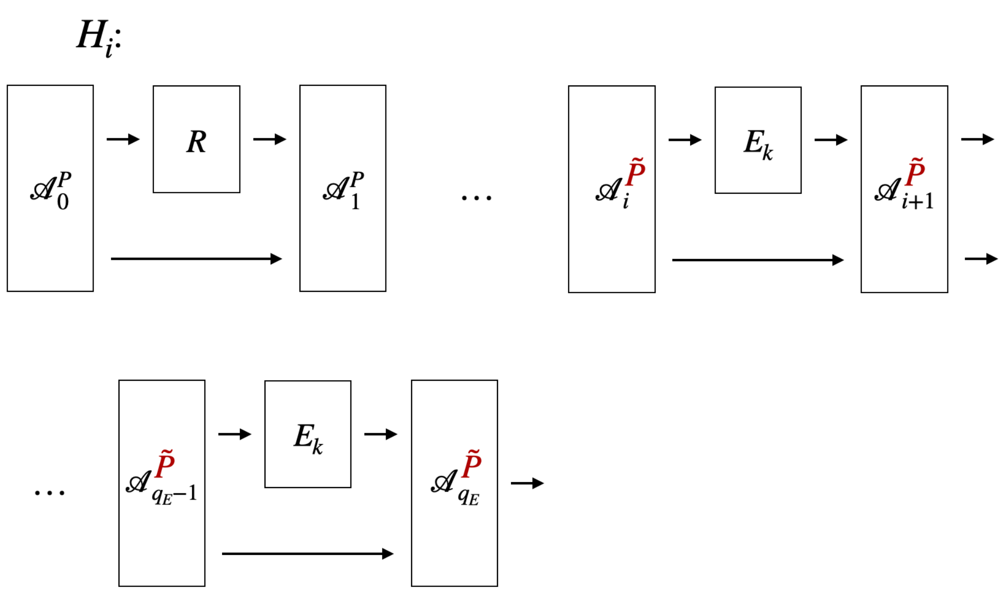

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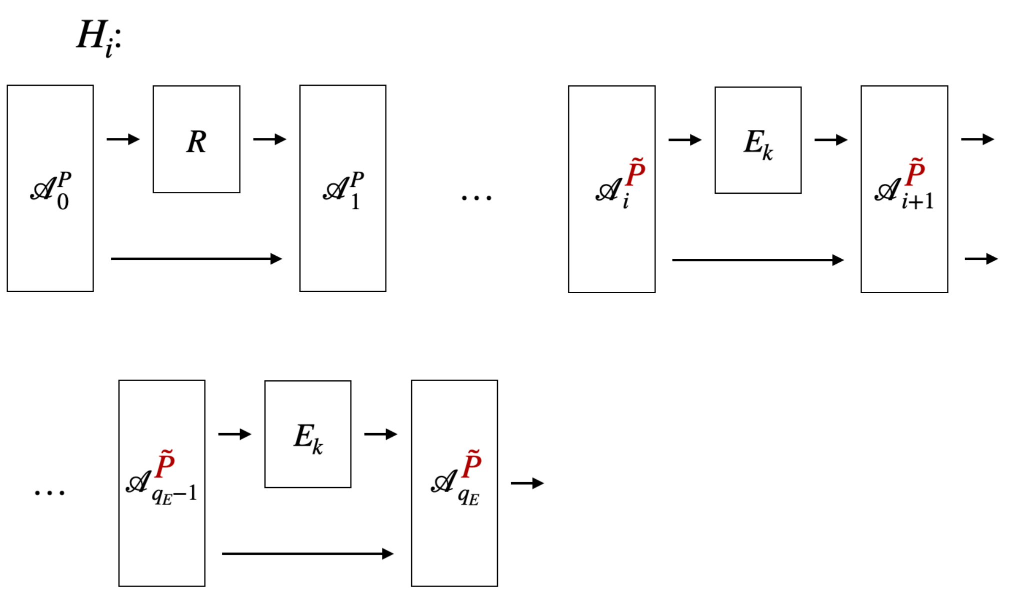

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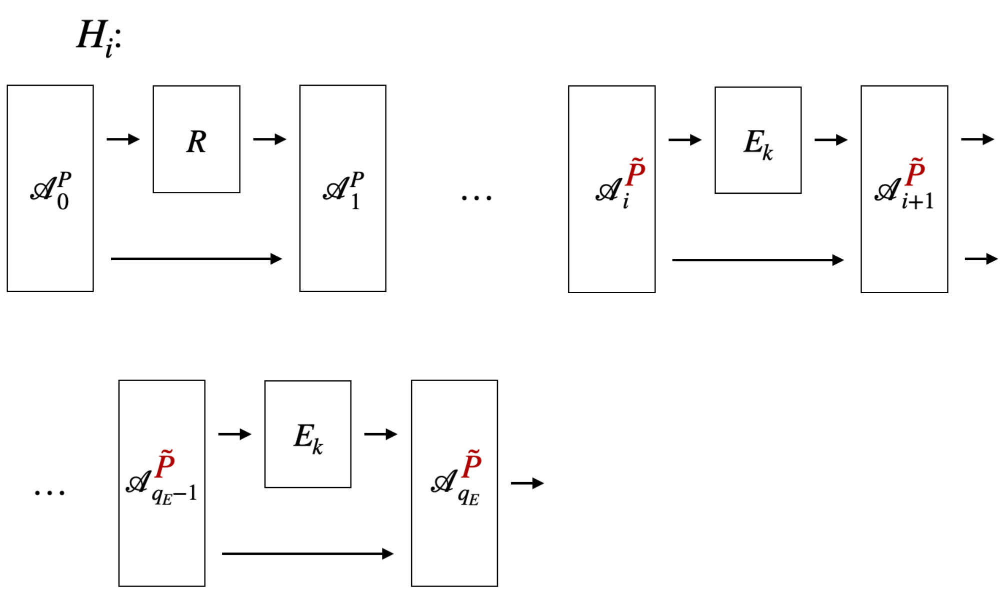

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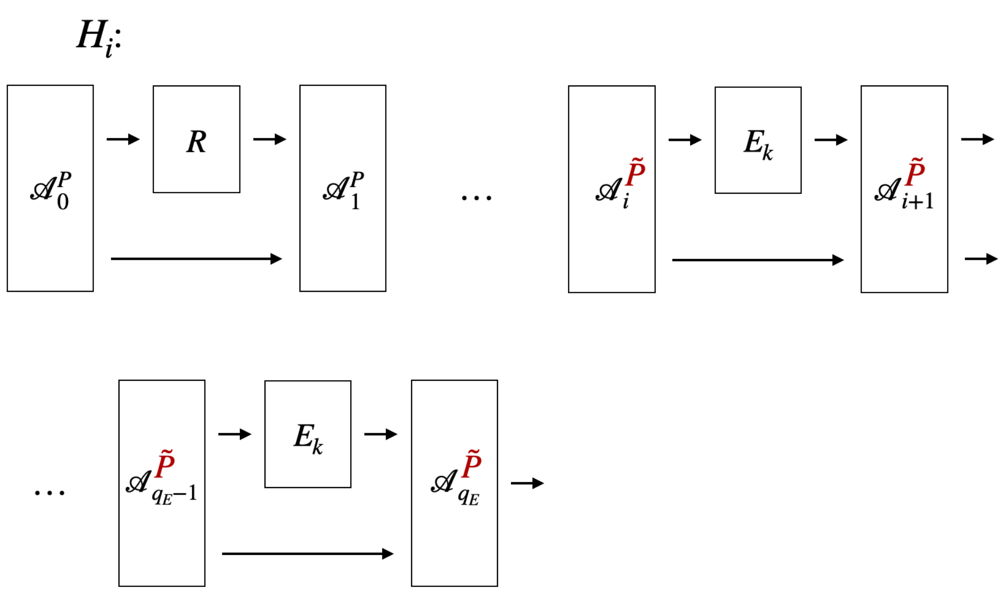

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  2. A reprogramming lemma in terms of expected number of queries

Resampling Lemma

Permutation version of “adaptive reprogramming lemma” (Grilo, Hövelmanns, Hülsing and Majenz, AC ’21)

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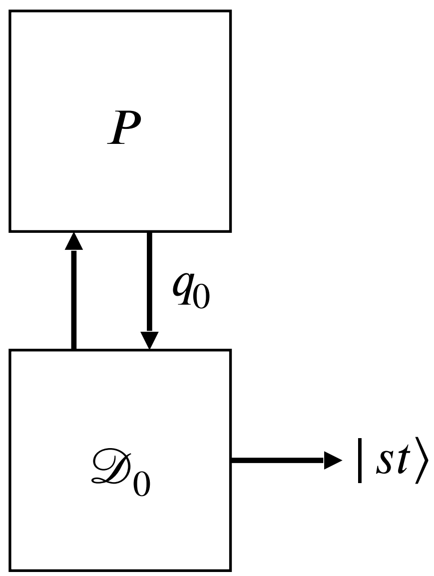

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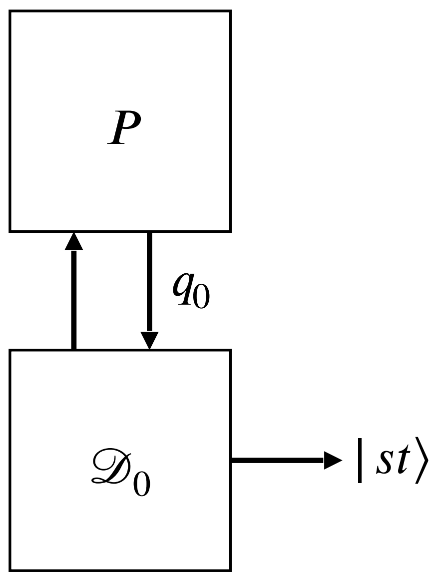

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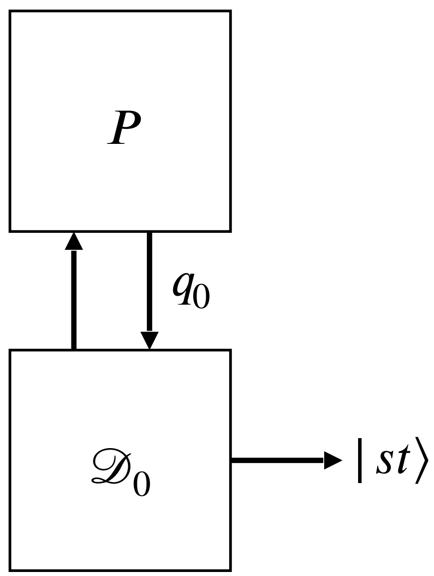

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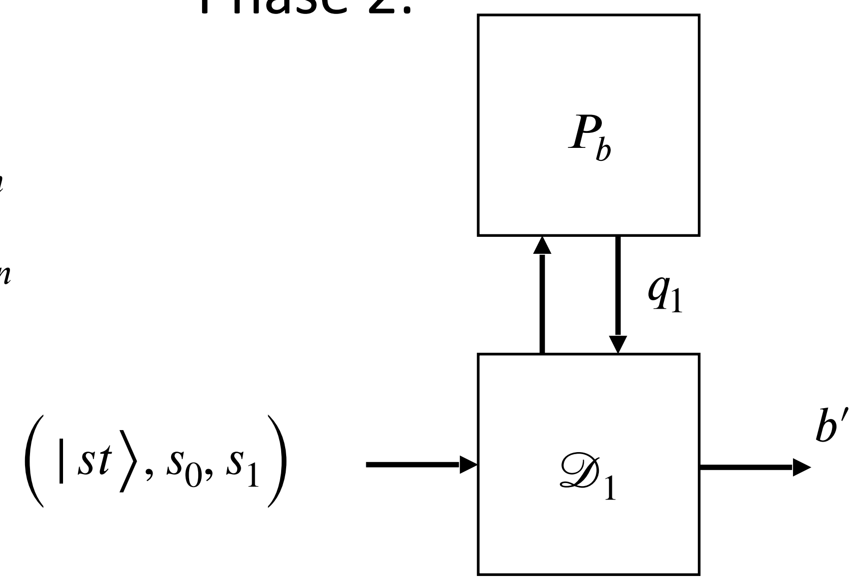

$P_0 = P$

$$P_1 = P \circ swap_{s_0,s_1} = \begin{cases} P(s_0) & if\ x = s_1 \\ P(s_1) & if\ x = s_0 \\ P(x) & otherwise \end{cases}$$

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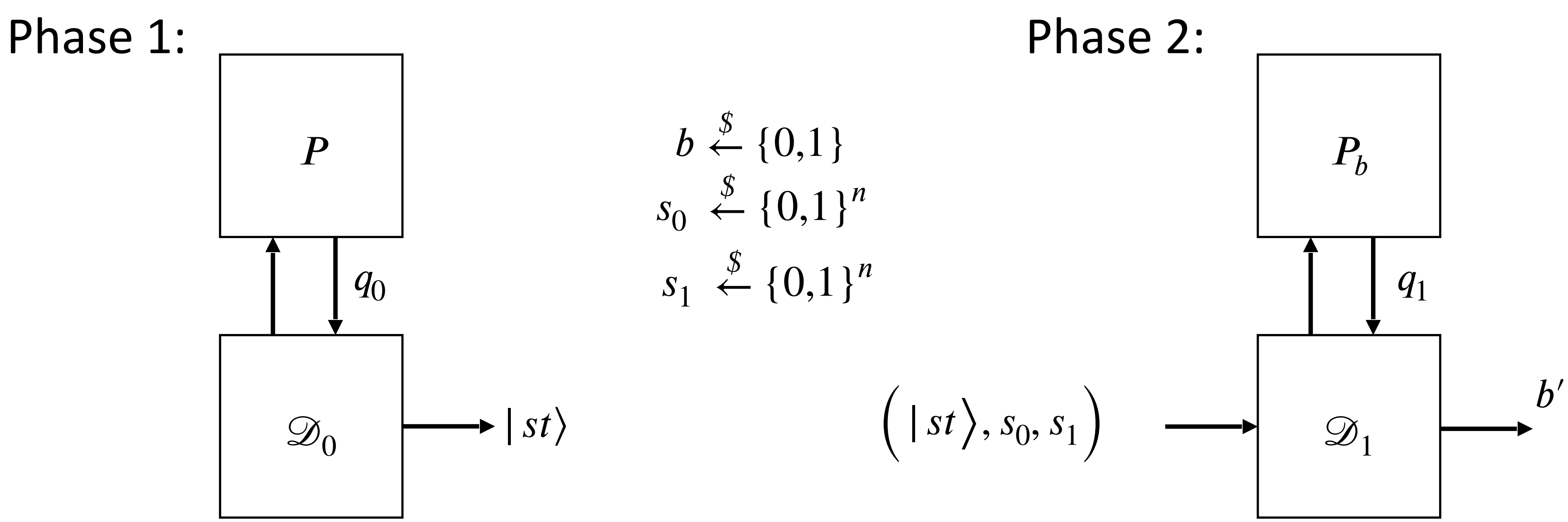

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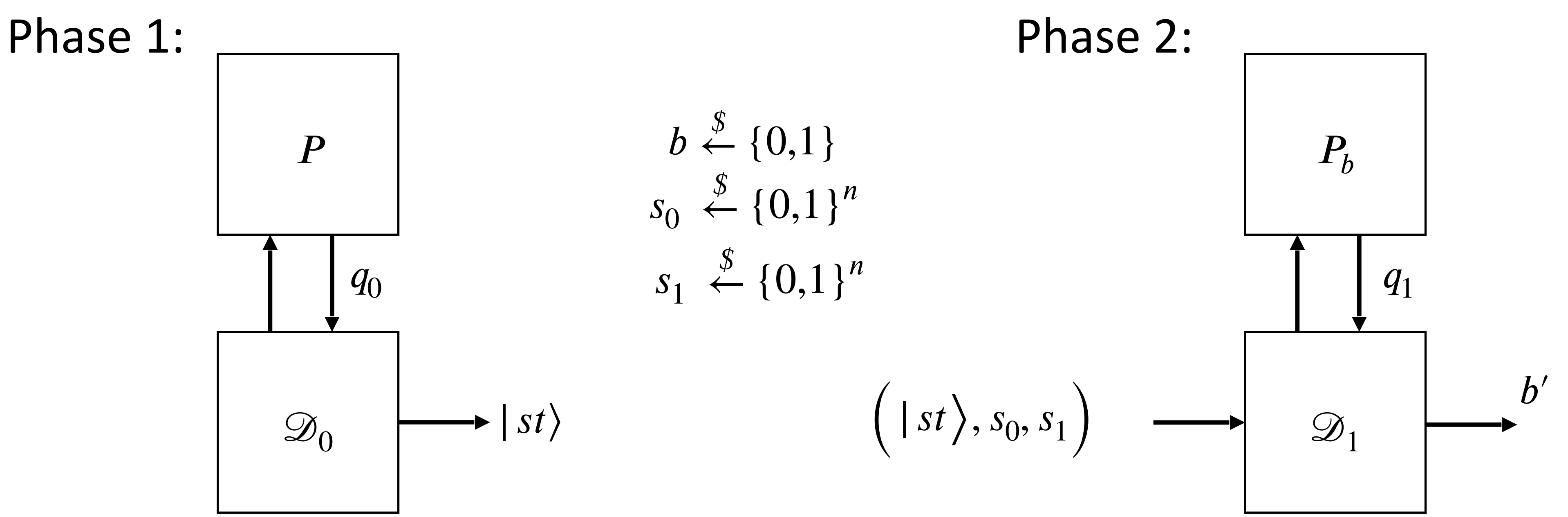

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Lemma: Advantage $\leq O\left(\sqrt{q \cdot 2^{-n}}\right)$

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Lemma: Advantage $\leq O\left(q\sqrt{2^{-n}}\right)$

Summary

- We proved post-quantum security of the Even-Mansour cipher
- Applications? Elephant and Chaskey use generlized versions of Even-Mansour. Also Jaeger et al. can actually do FX! $\Rightarrow$ Follow-up work (us+Patrick Struck), on eprint "soon" :)

Summary

- We proved post-quantum security of the Even-Mansour cipher
- Applications? Elephant and Chaskey use generlized versions of Even-Mansour. Also Jaeger et al. can actually do FX! ⇒ Follow-up work (us+Patrick Struck), on eprint "soon" :)

Coming soon: PhD position in provable post-quantum security @DTU (Copenhagen area)

Thank you for your attention!