Highly Efficient OT-Based Multiplication Protocols

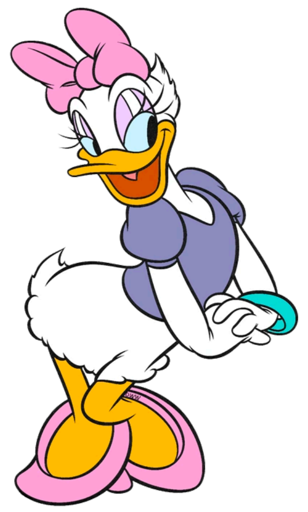
Nikolaos Makriyannis (Fireblocks)

Joint work w/ **Iftach Haitner**, **Samuel Ranellucci** & **Eliad Tsfadia**

Problem Statement (Two-Party Multiplication)

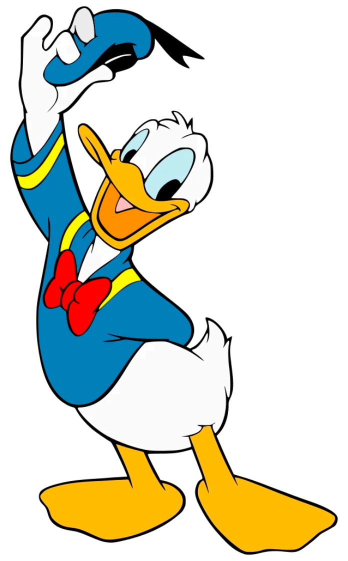
$\mathsf{Mult}(x, y) \mapsto (s_1, s_2 = x \cdot y - s_1)$

MULT



$\mathscr{P}_1$

$x \in \mathbb{Z}_q$



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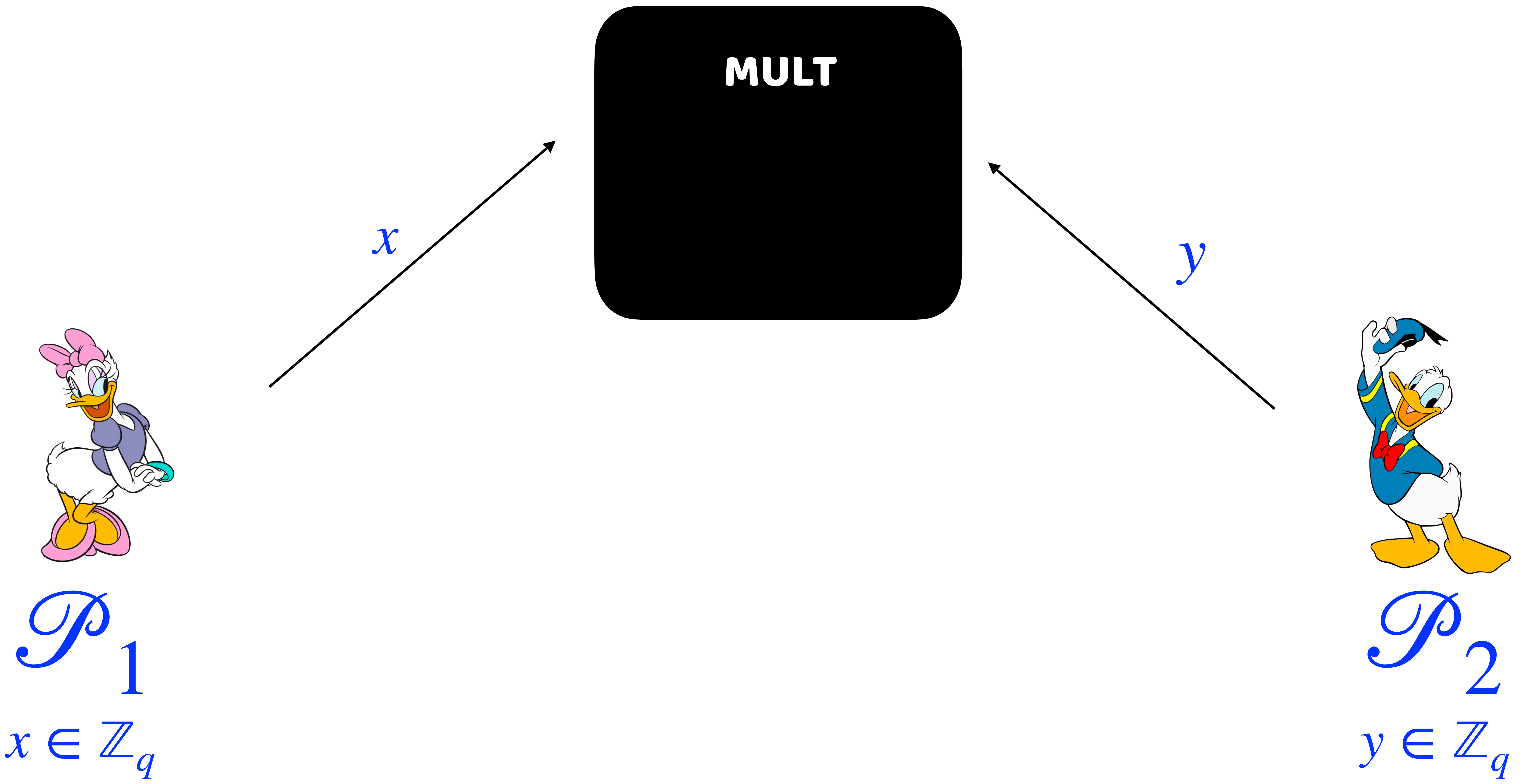

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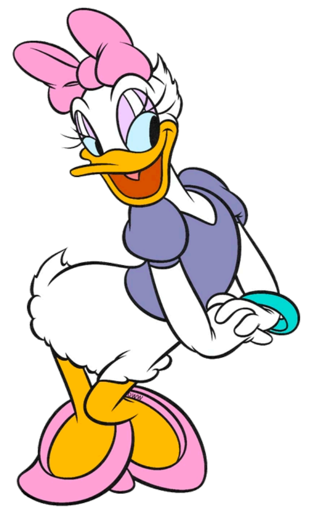
MULT

$s_2 = x \cdot y - s_1$

$s_1 \leftarrow \mathbb{Z}_q$

x

y



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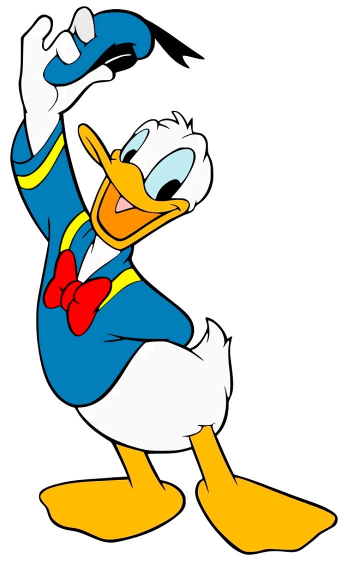

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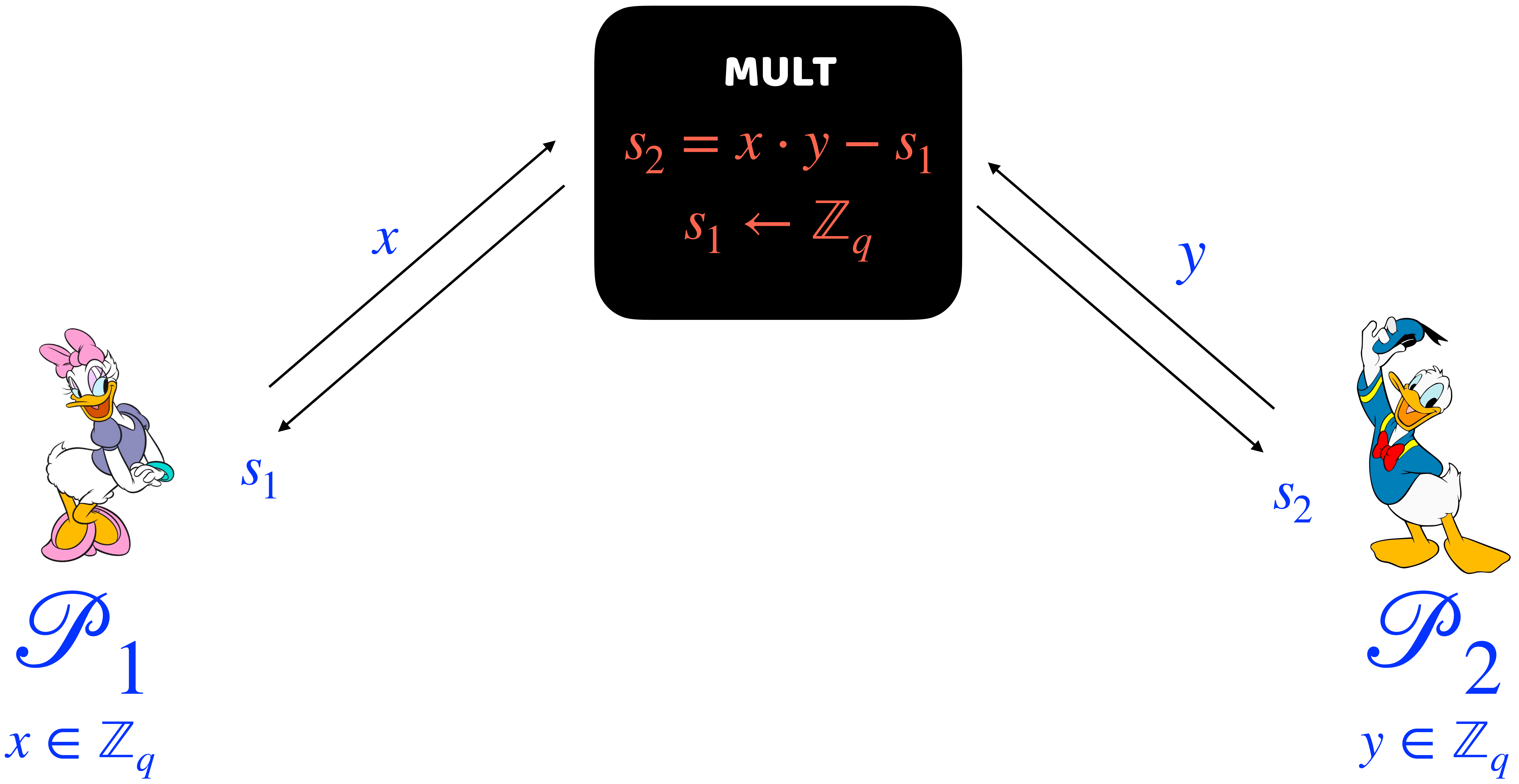

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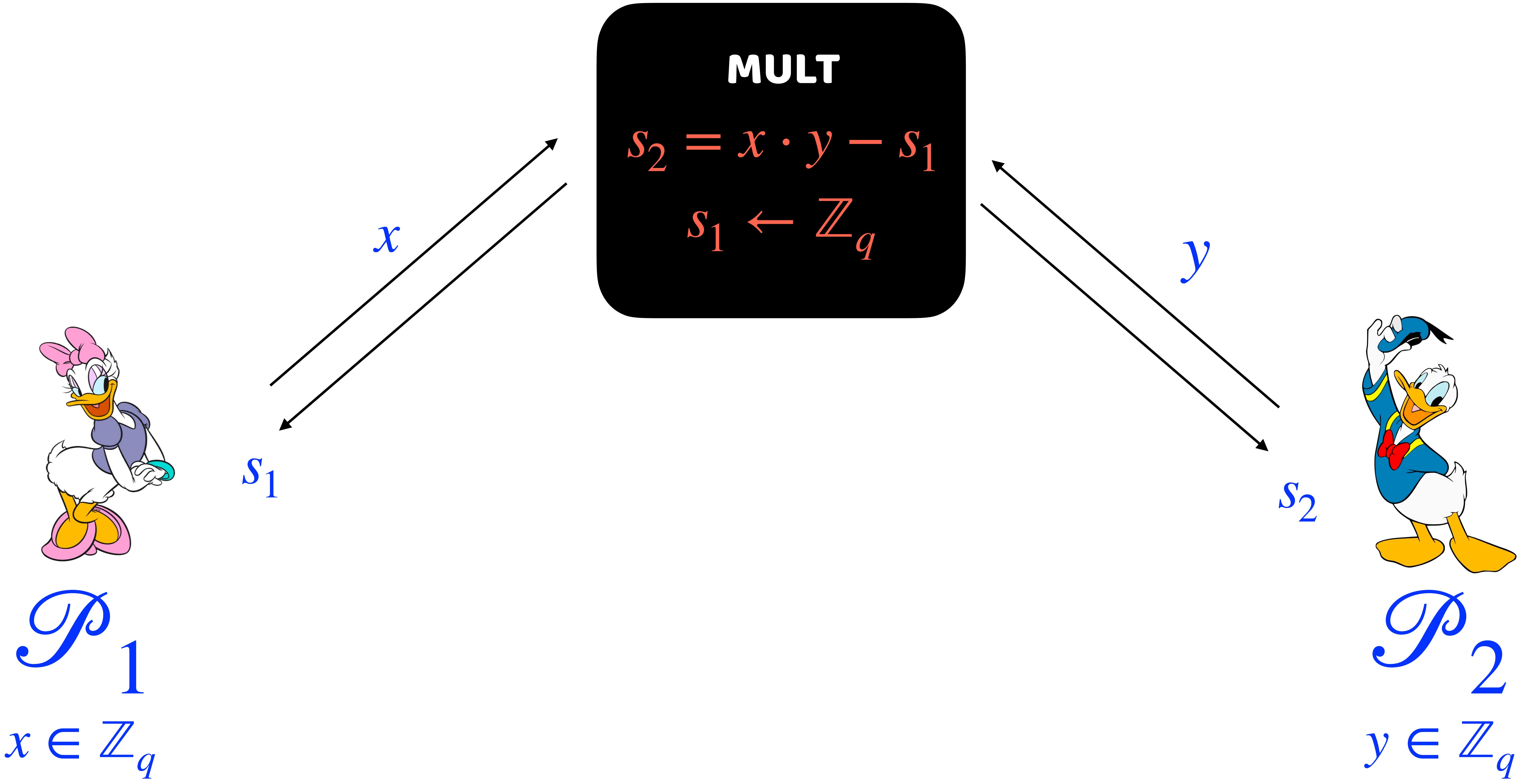
- q is a large prime



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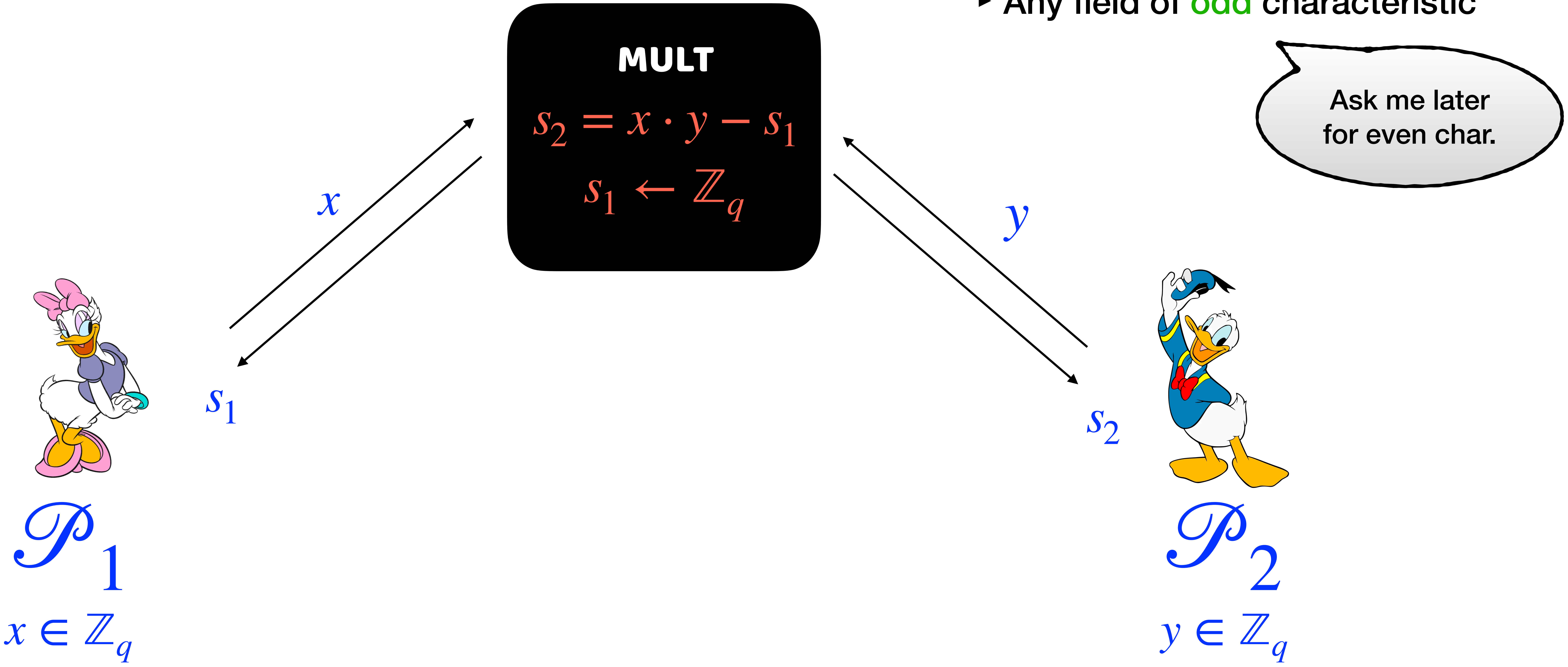
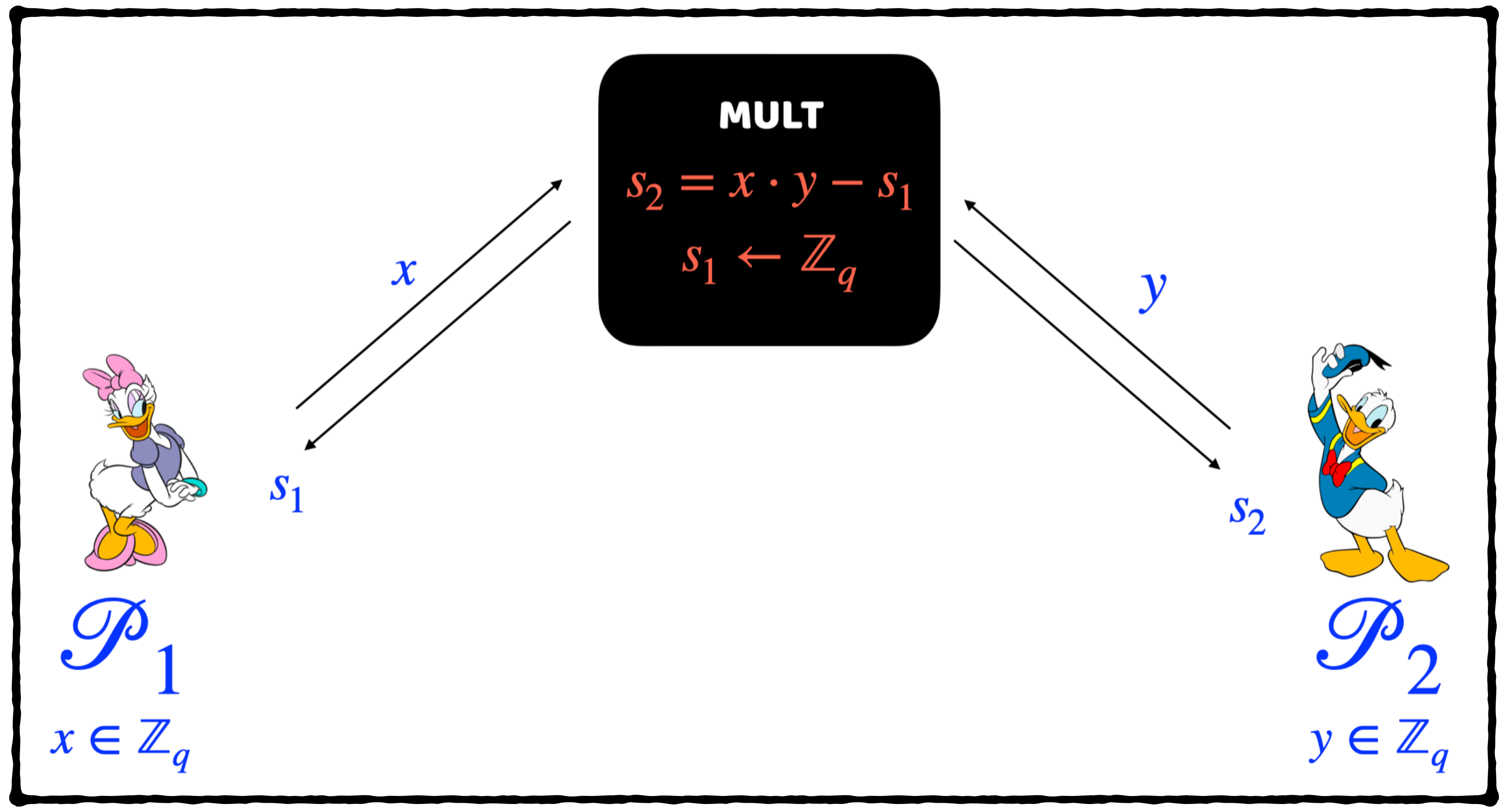
- q is a large prime
- Any field of odd characteristic



Motivation

Why Multiplication Protocols?



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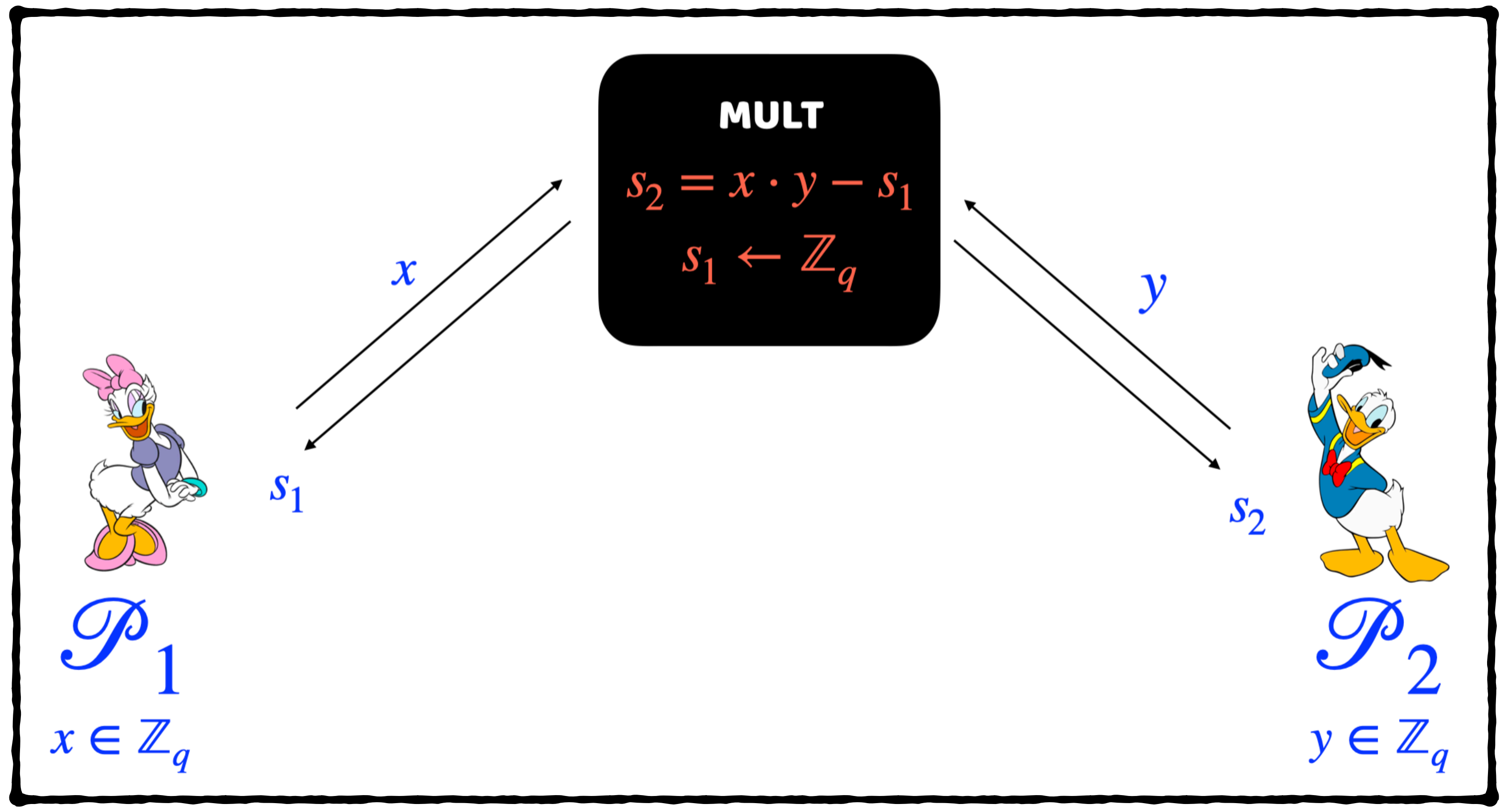
- Building block in *arithmetic* MPC



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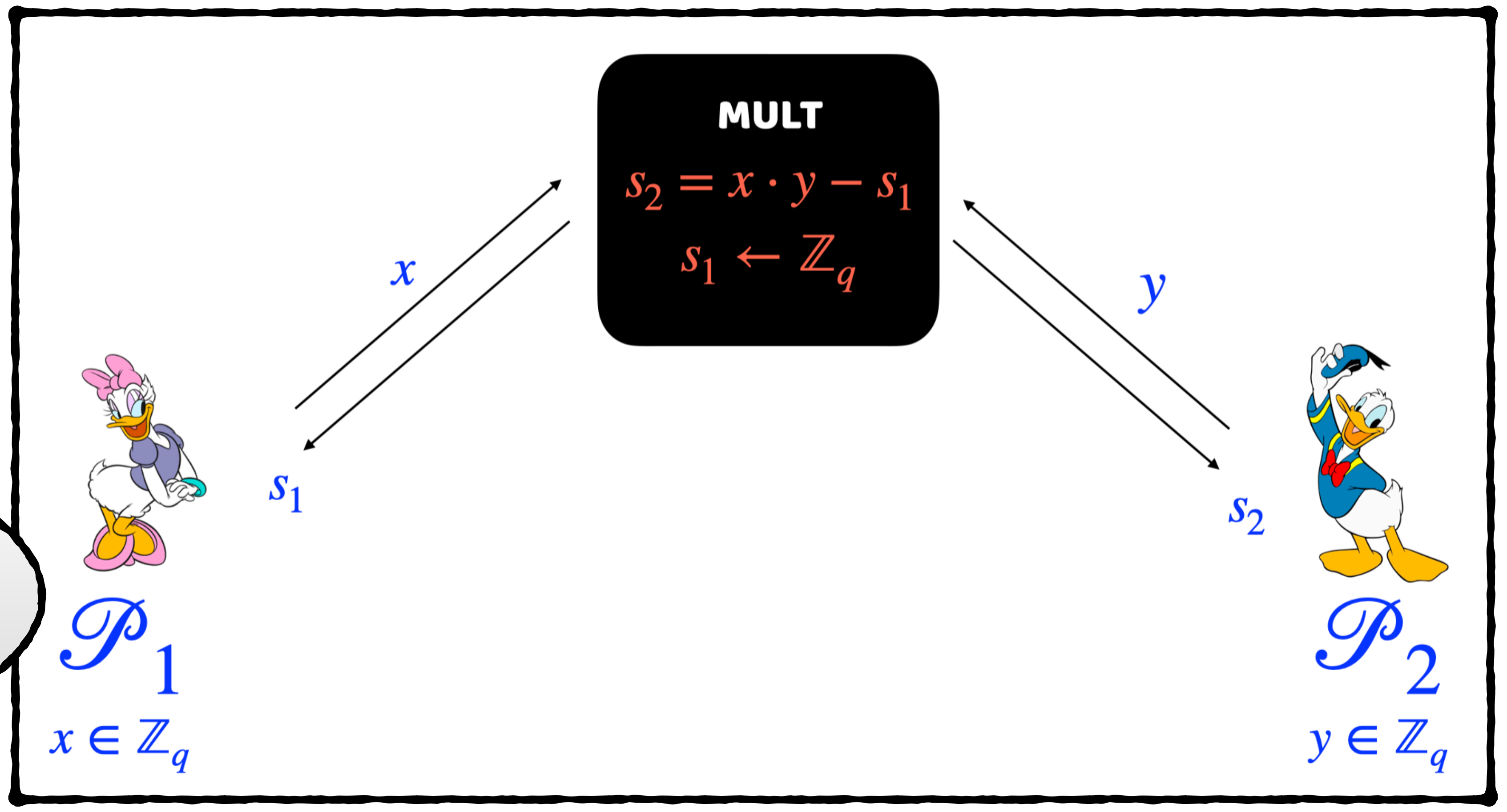

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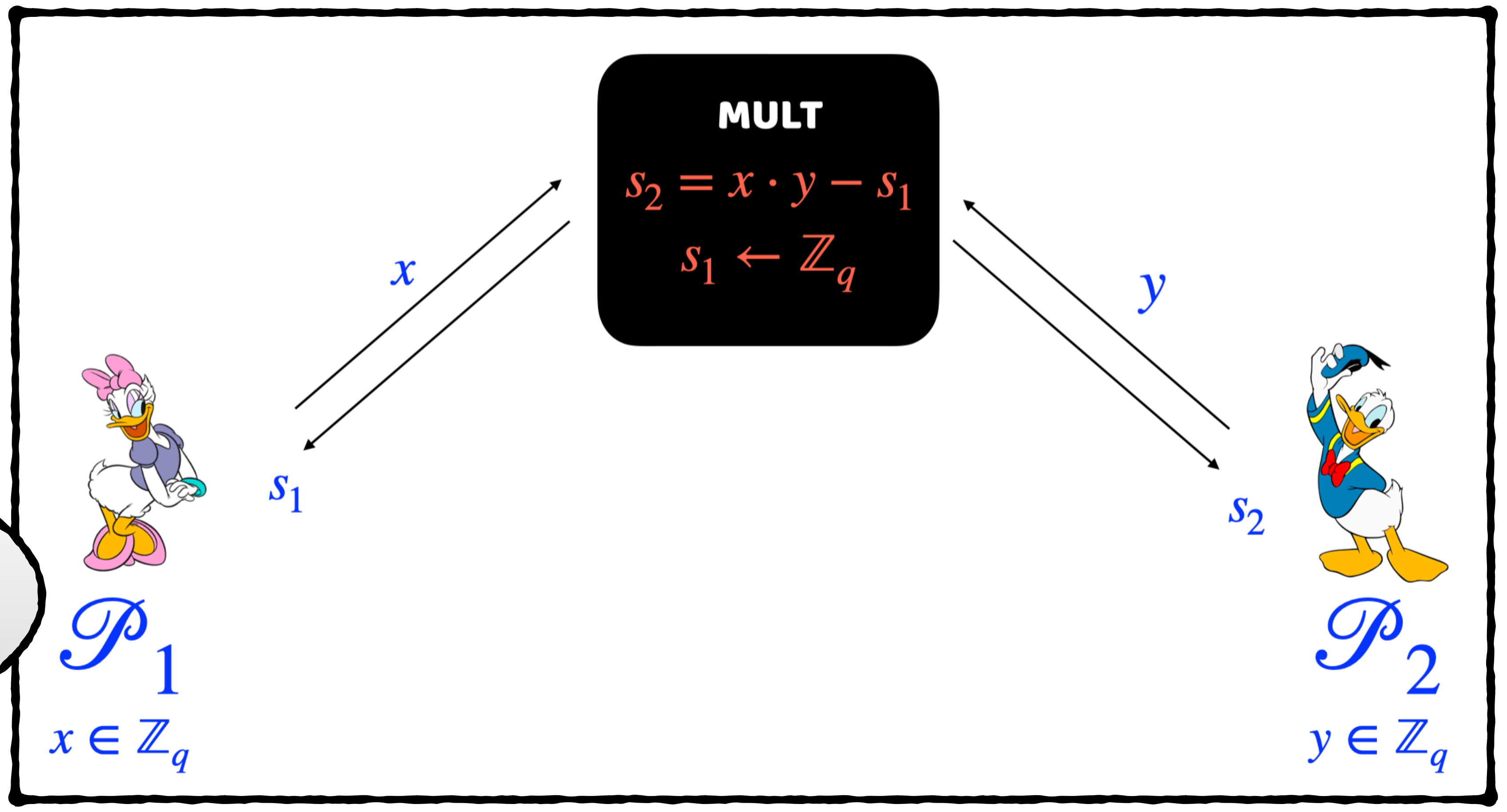

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- E.g., for generating tuples in preprocessing model
  - *Beaver triplets*: 2-out-of-2 shares of $(a, b, c = a \cdot b)$
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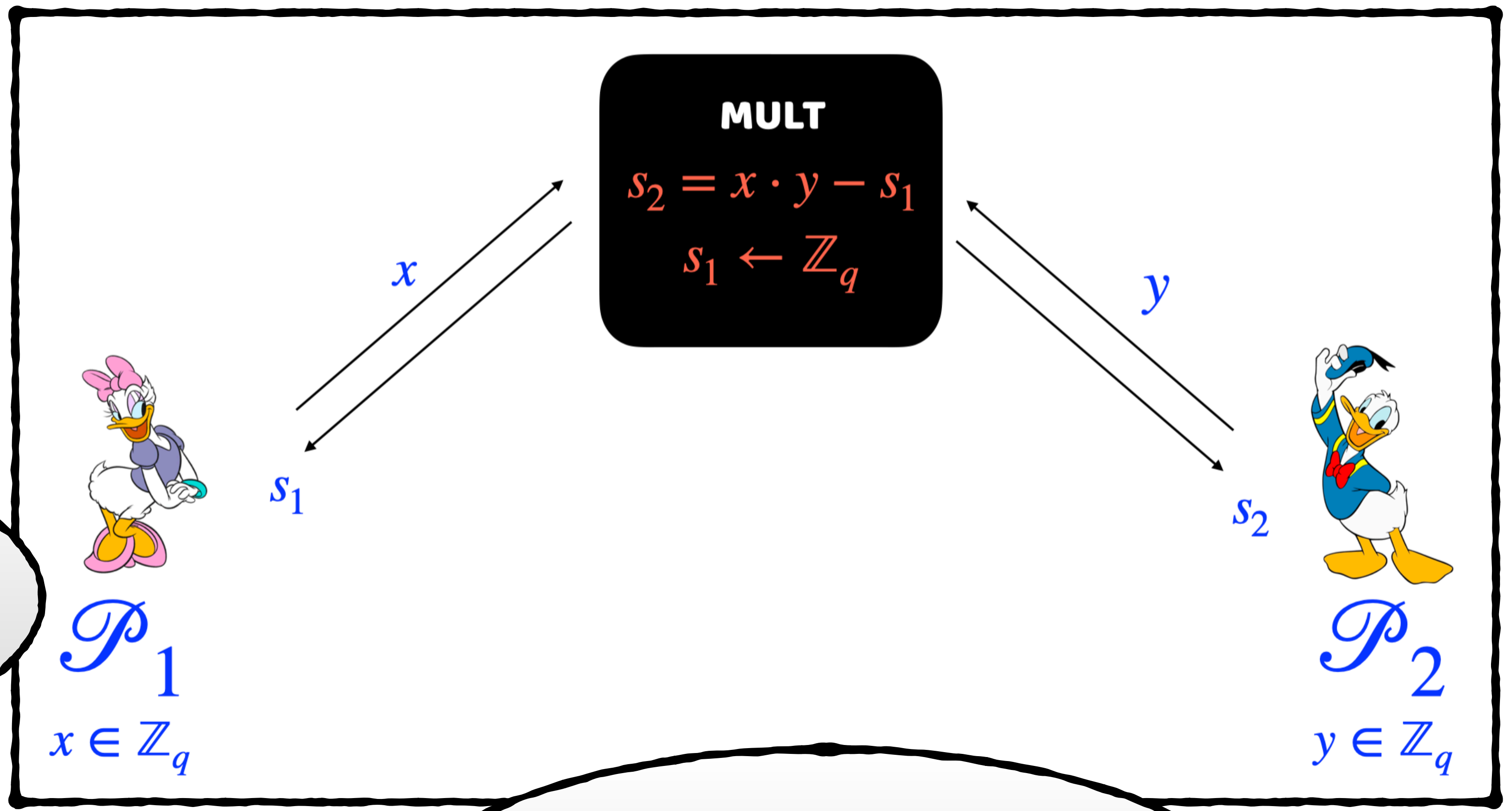

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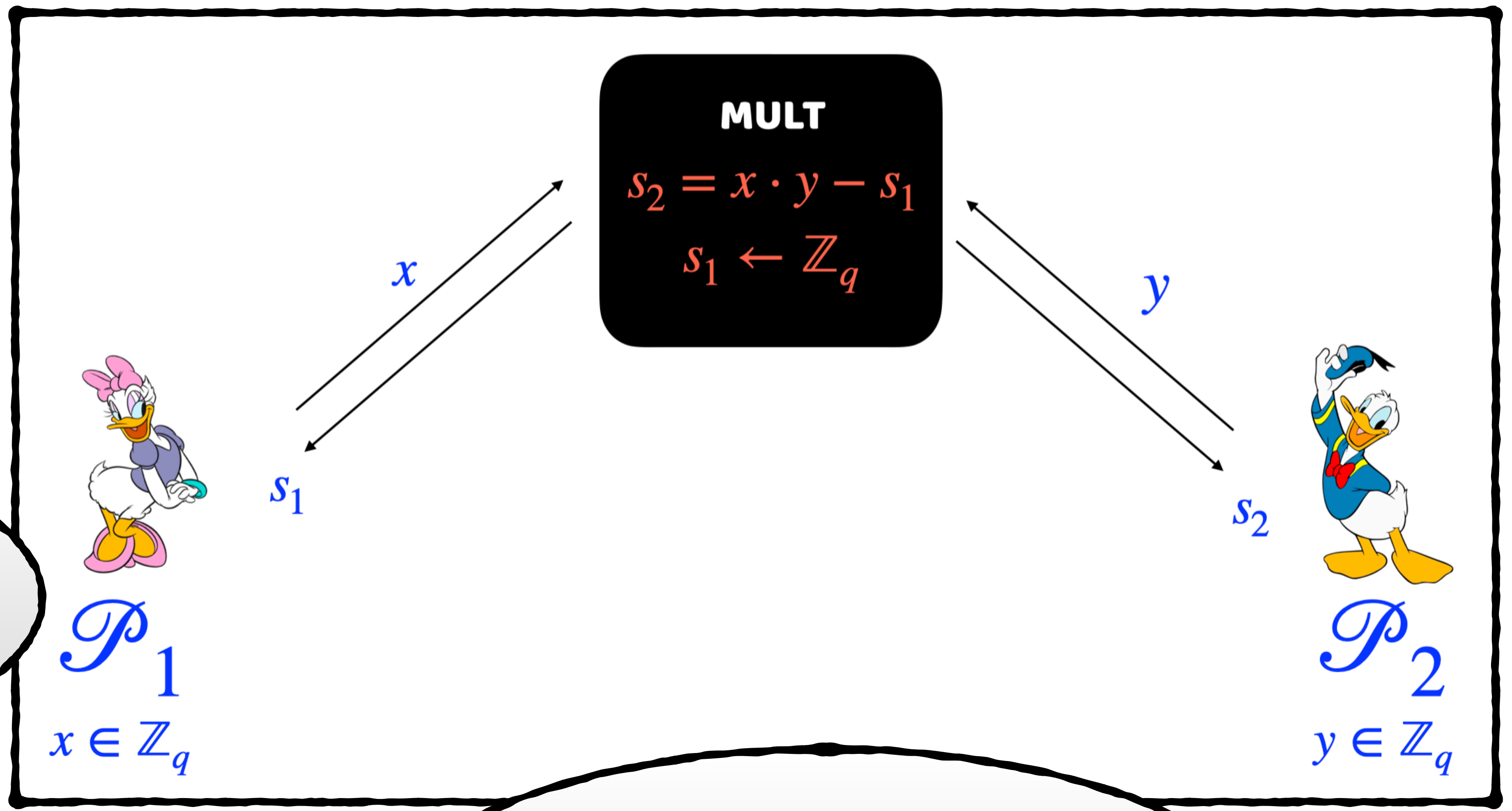

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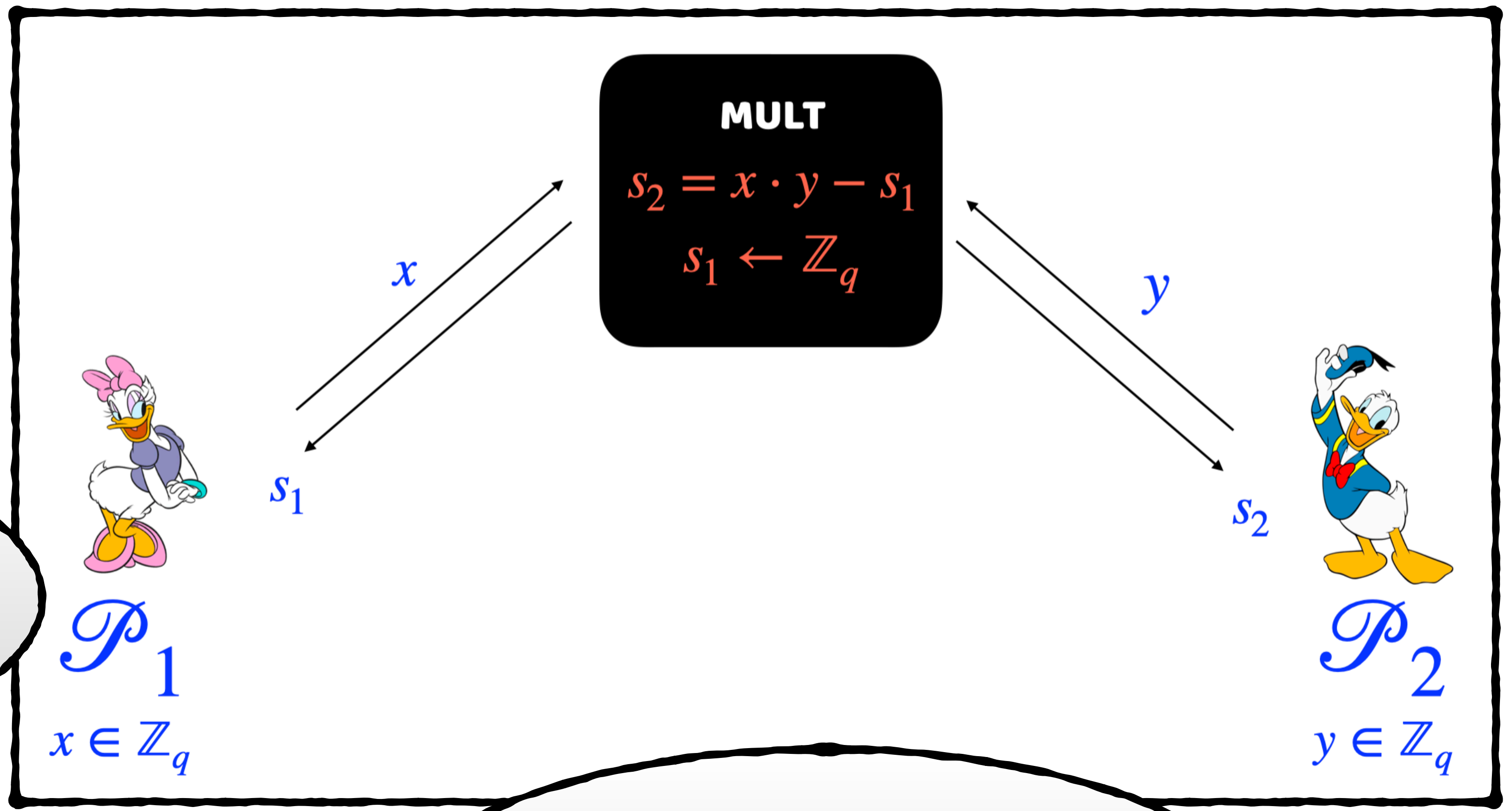

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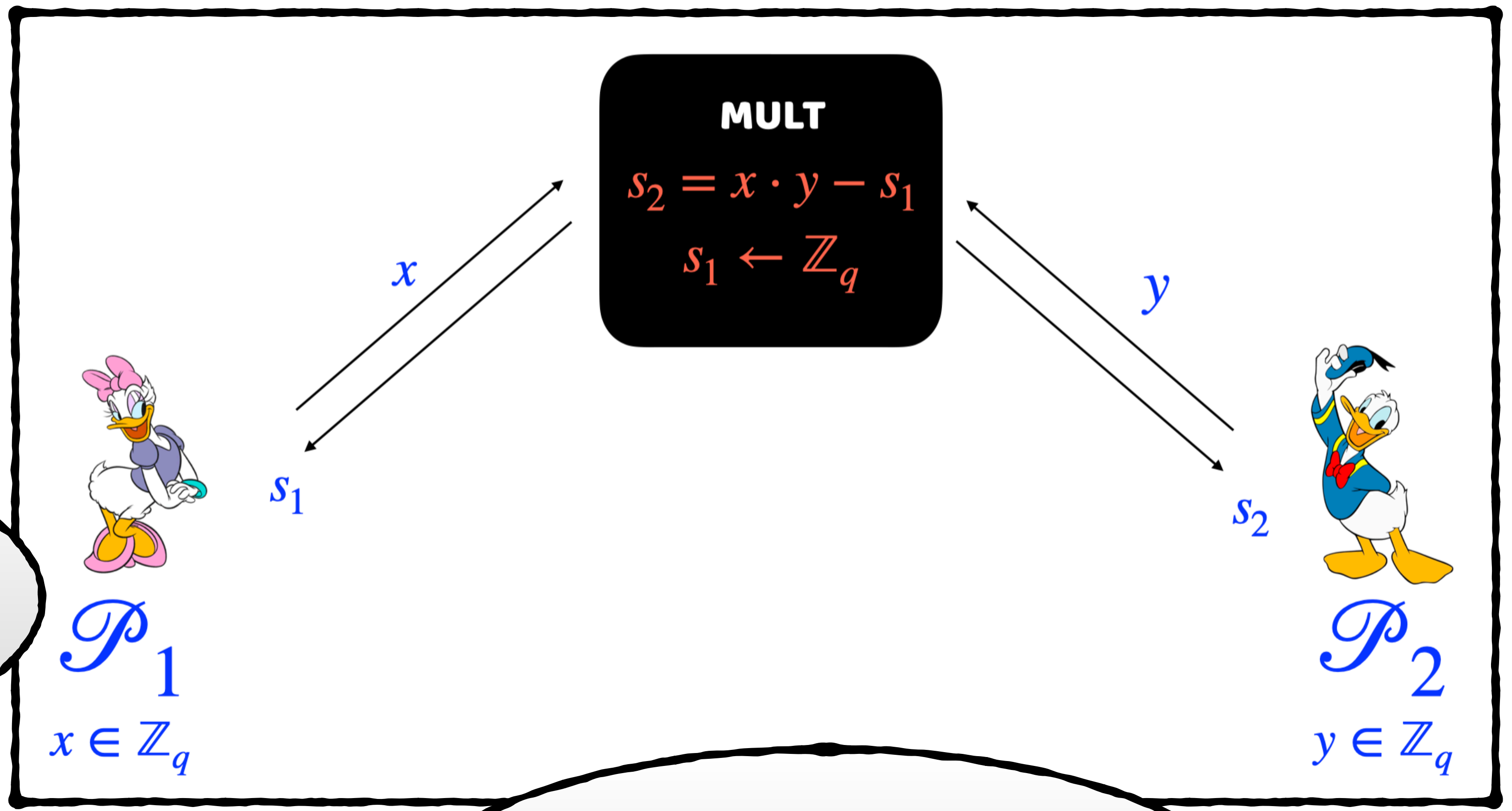

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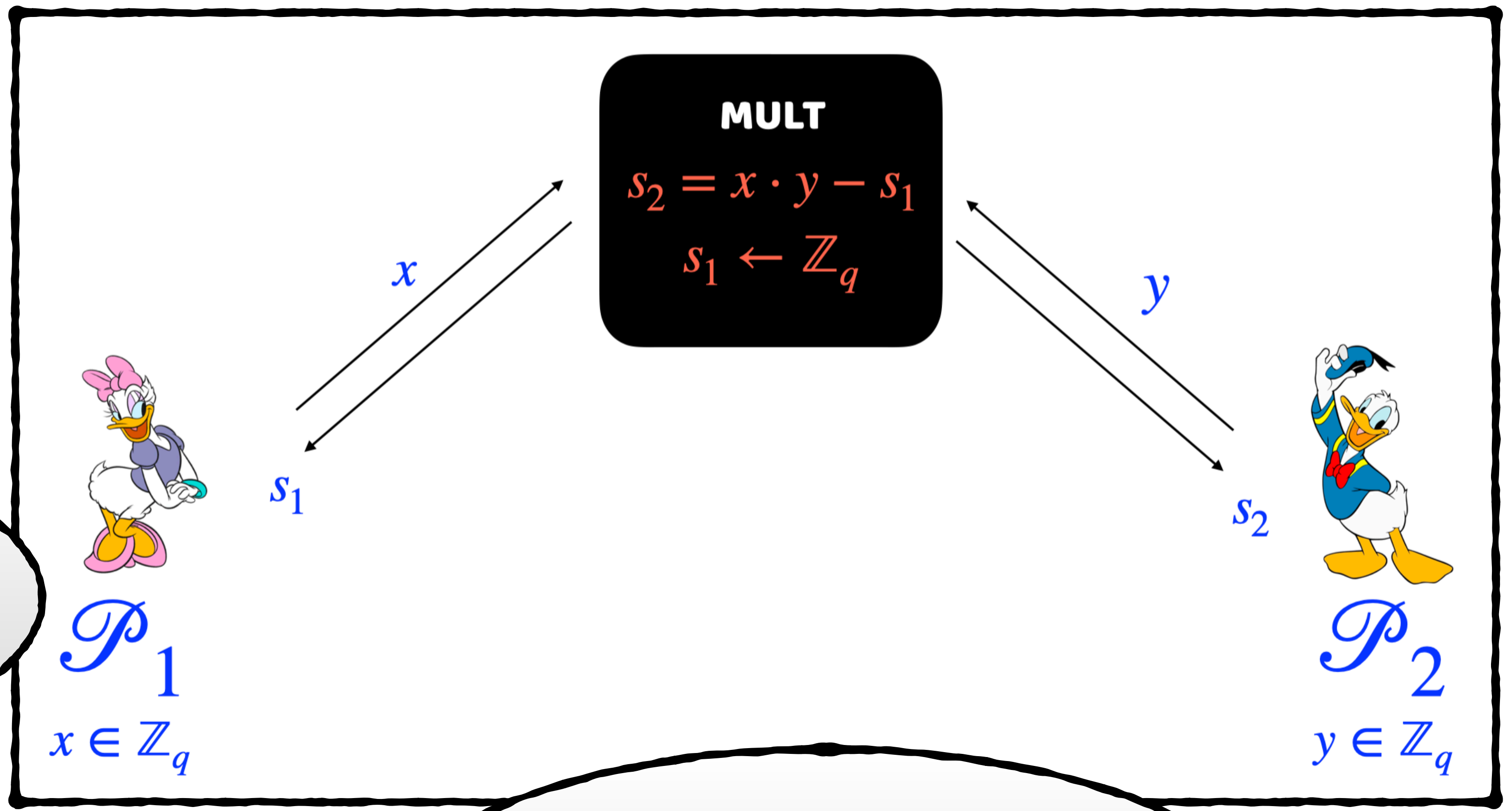

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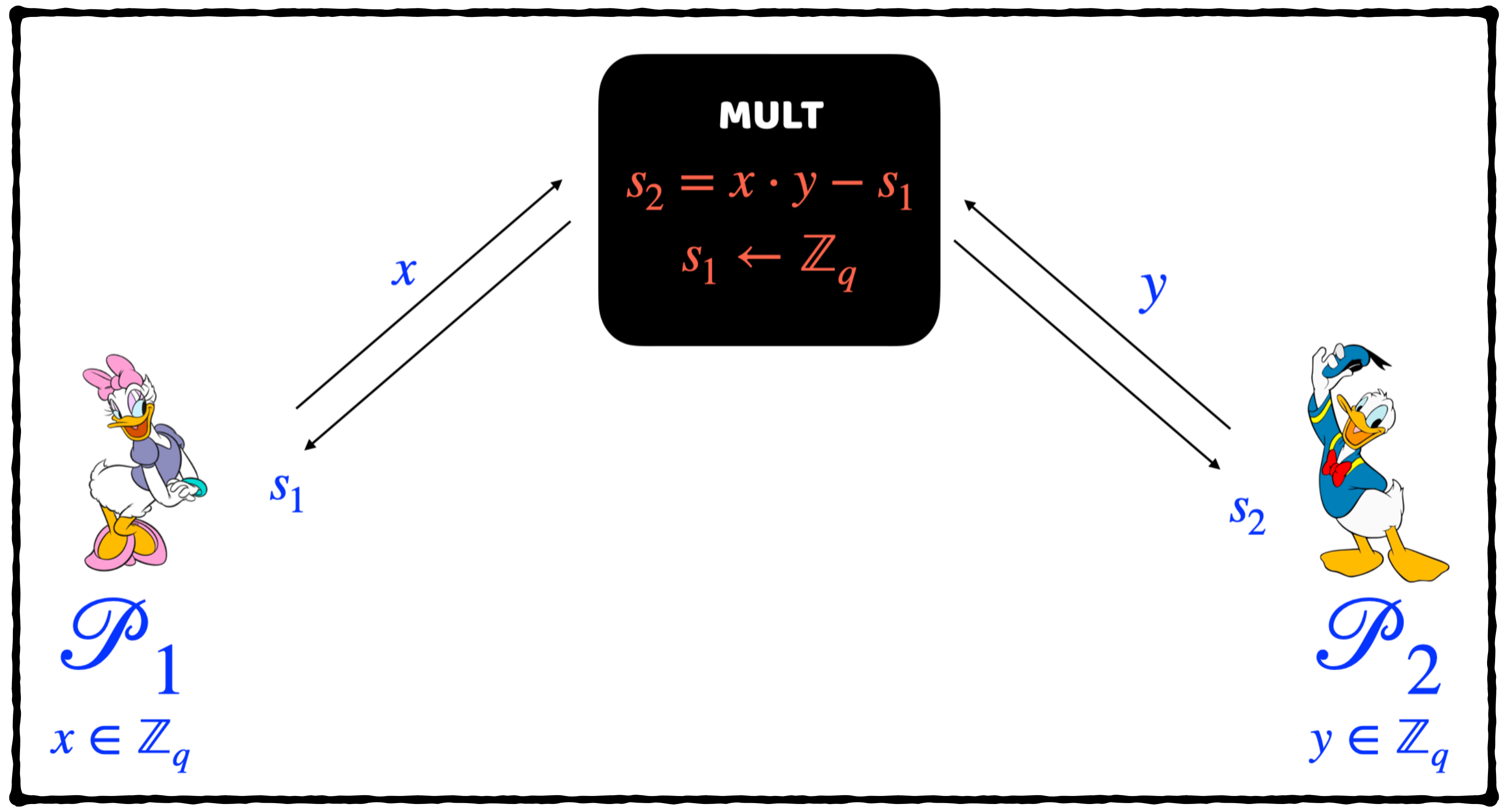
- Mult is "almost" the same functionality as *OLE* (Oblivious Linear Evaluation)

  $\text{OLE}(x, (y, \sigma)) \mapsto (x \cdot y + \sigma, \perp)$

Background

Background

Known Constructions/Paradigms



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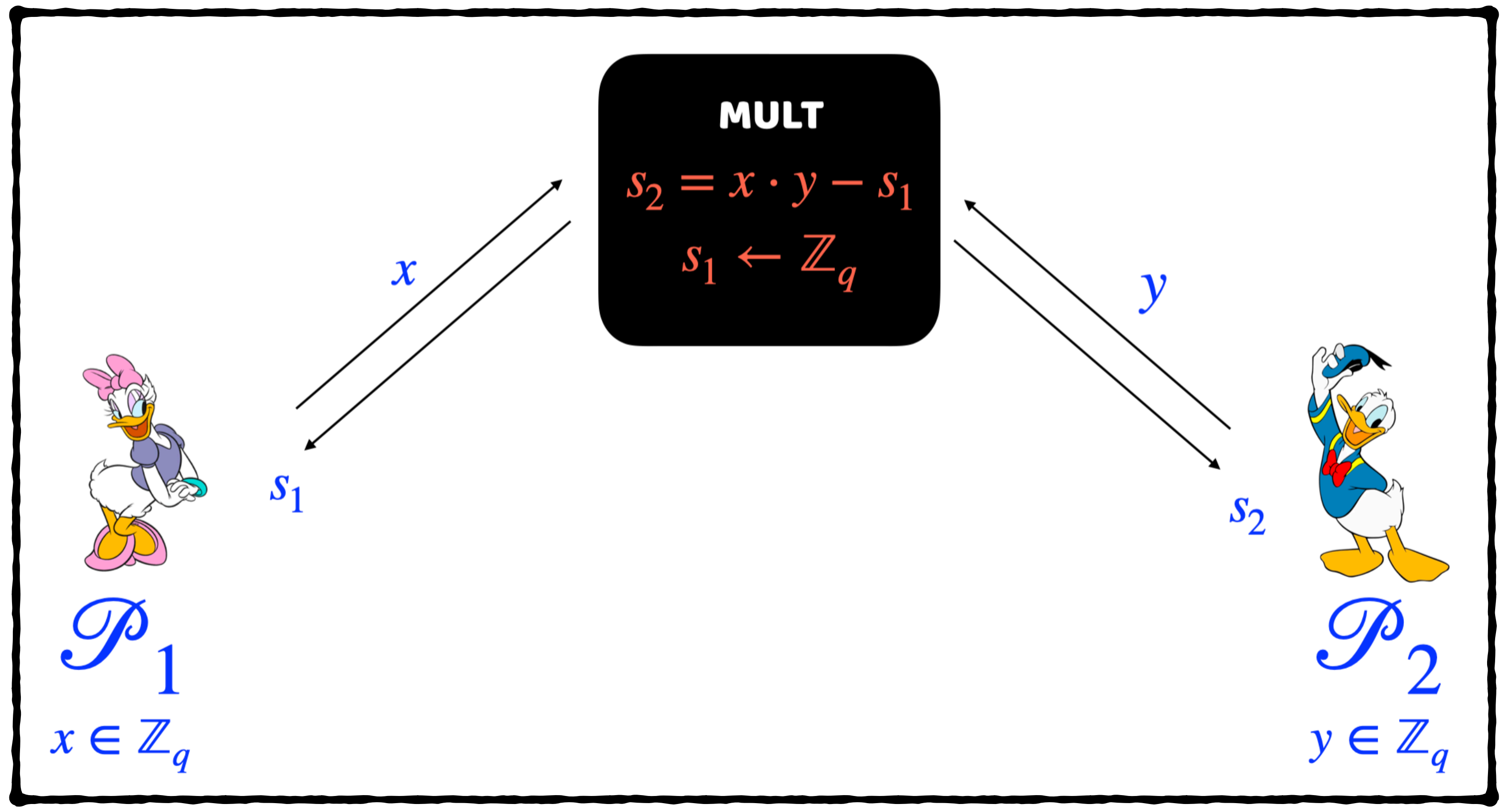

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  - 👍 Light Communication
  - 👎 Heavy Computation
  - 👎 Stronger Assumptions

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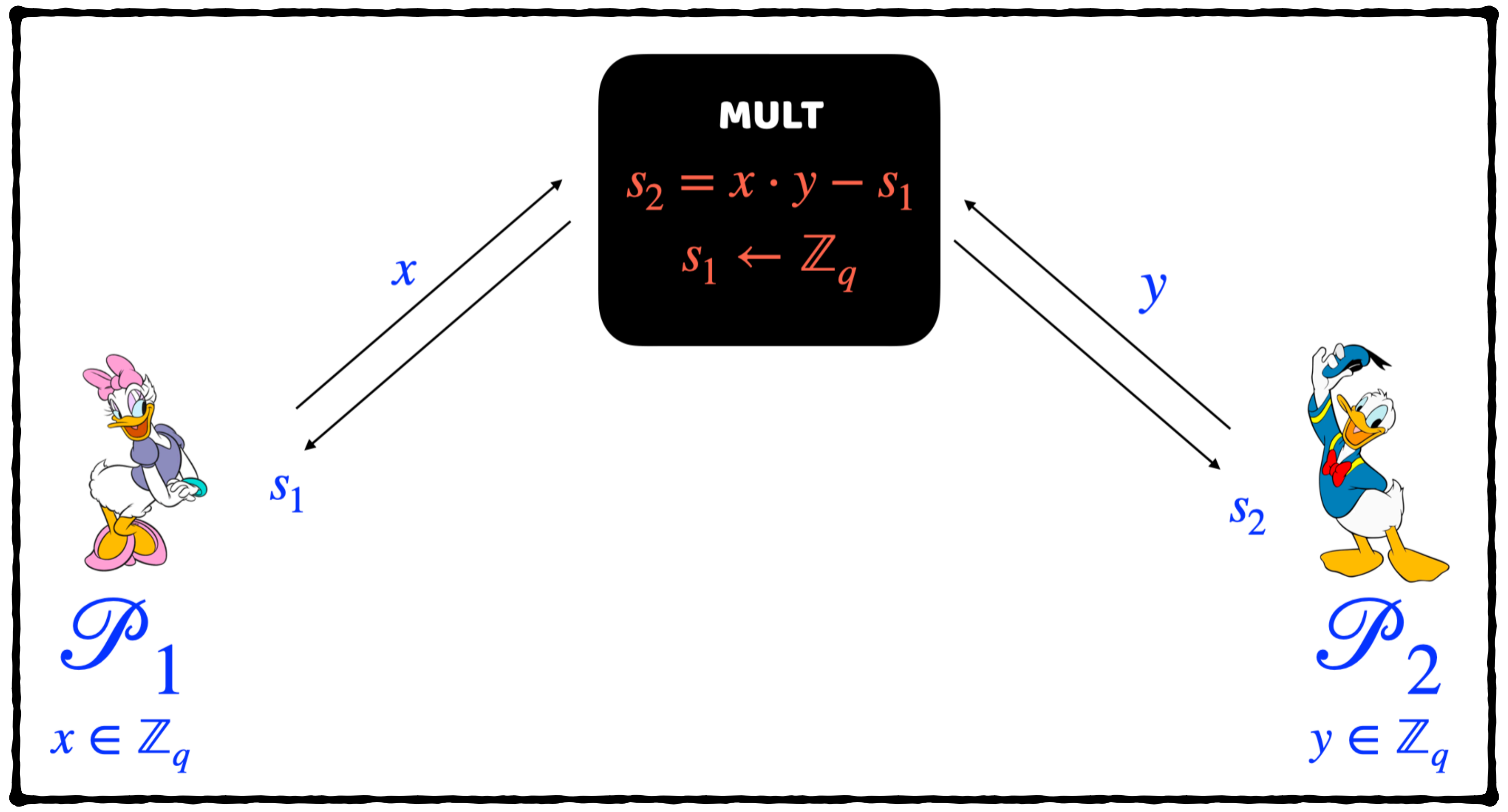

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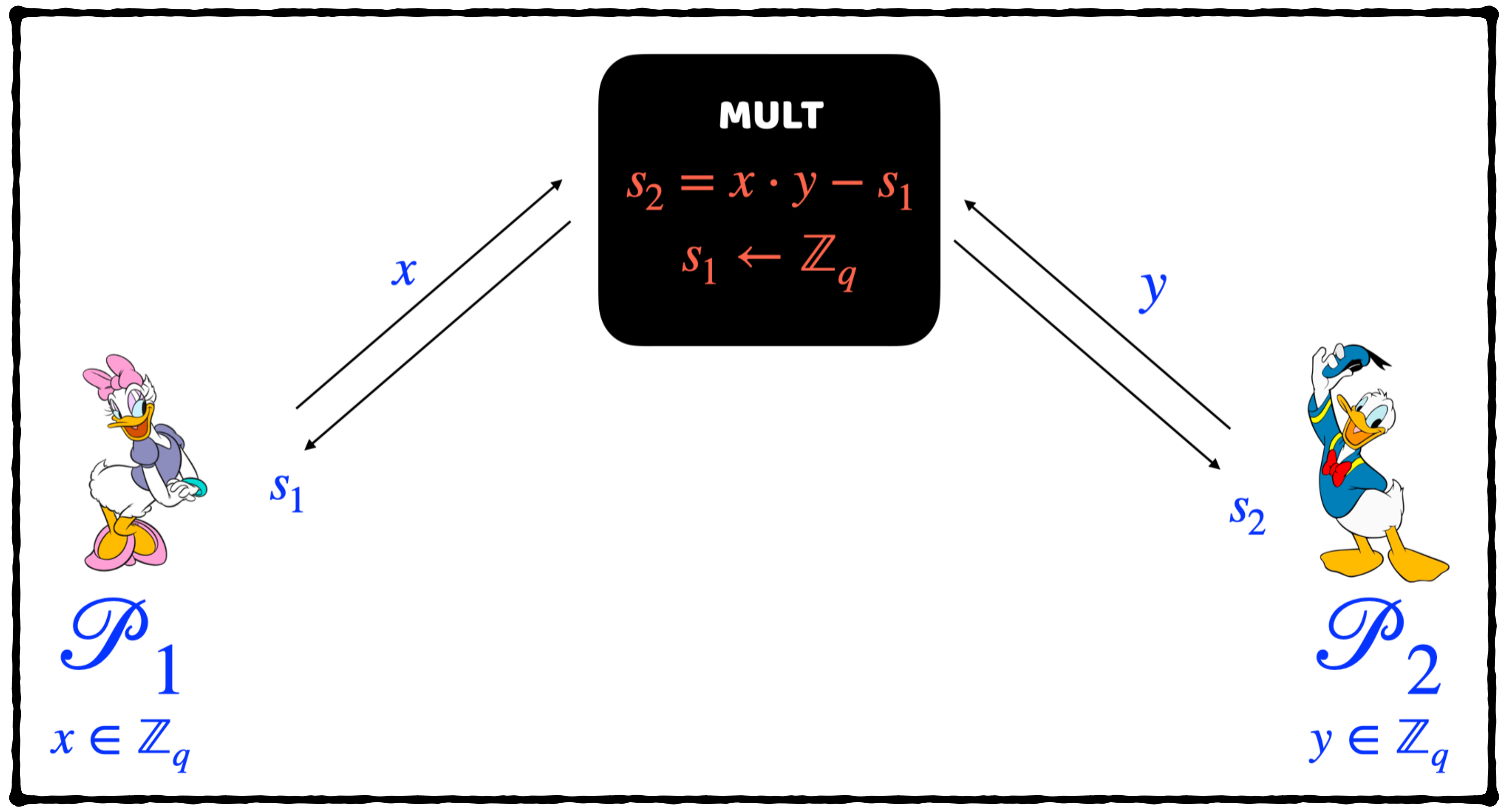

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Tens to *hundreds* of KiB
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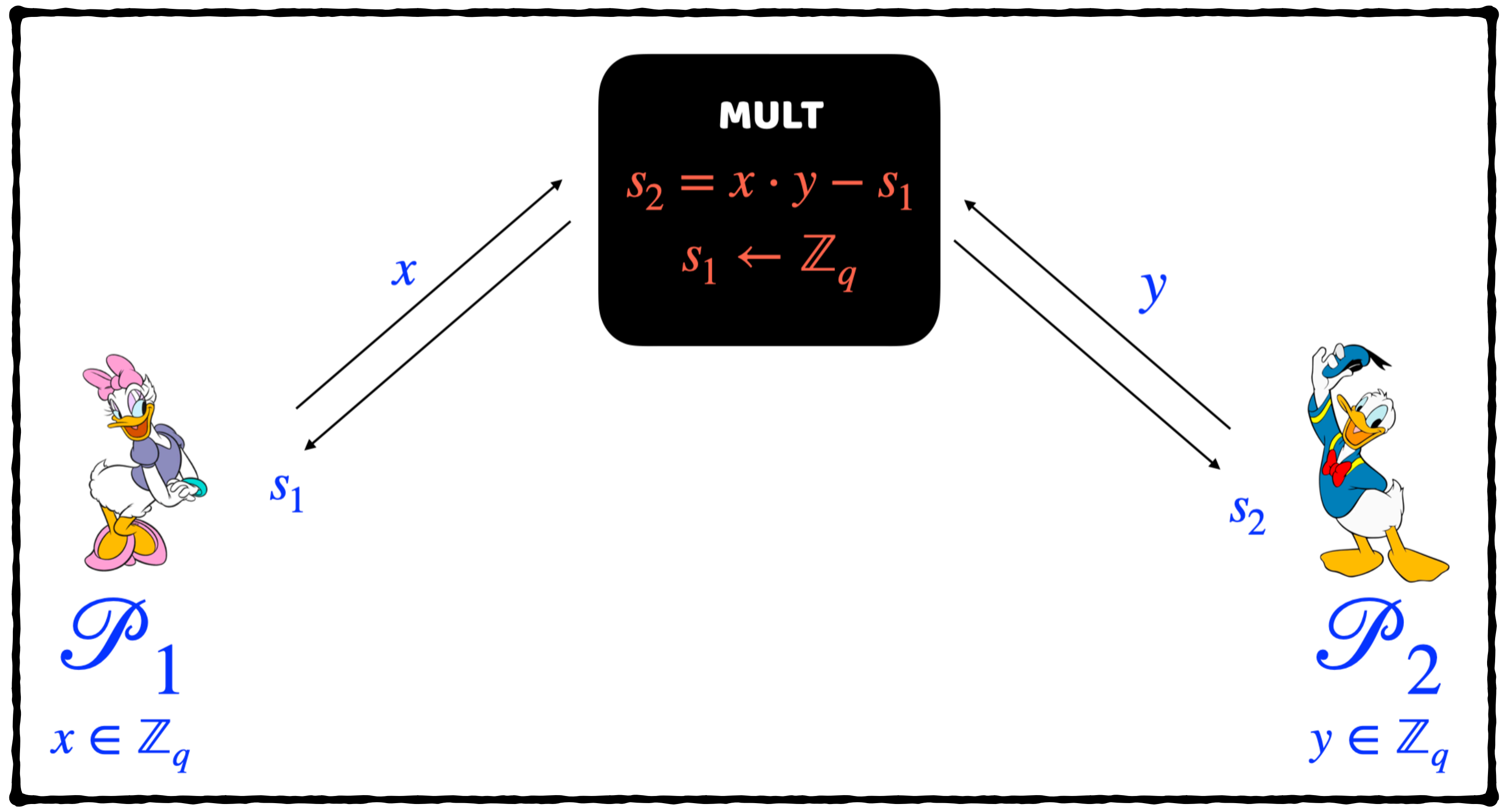

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This work: New OT-Based Multiplication protocol!

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OT-Hybrid Model

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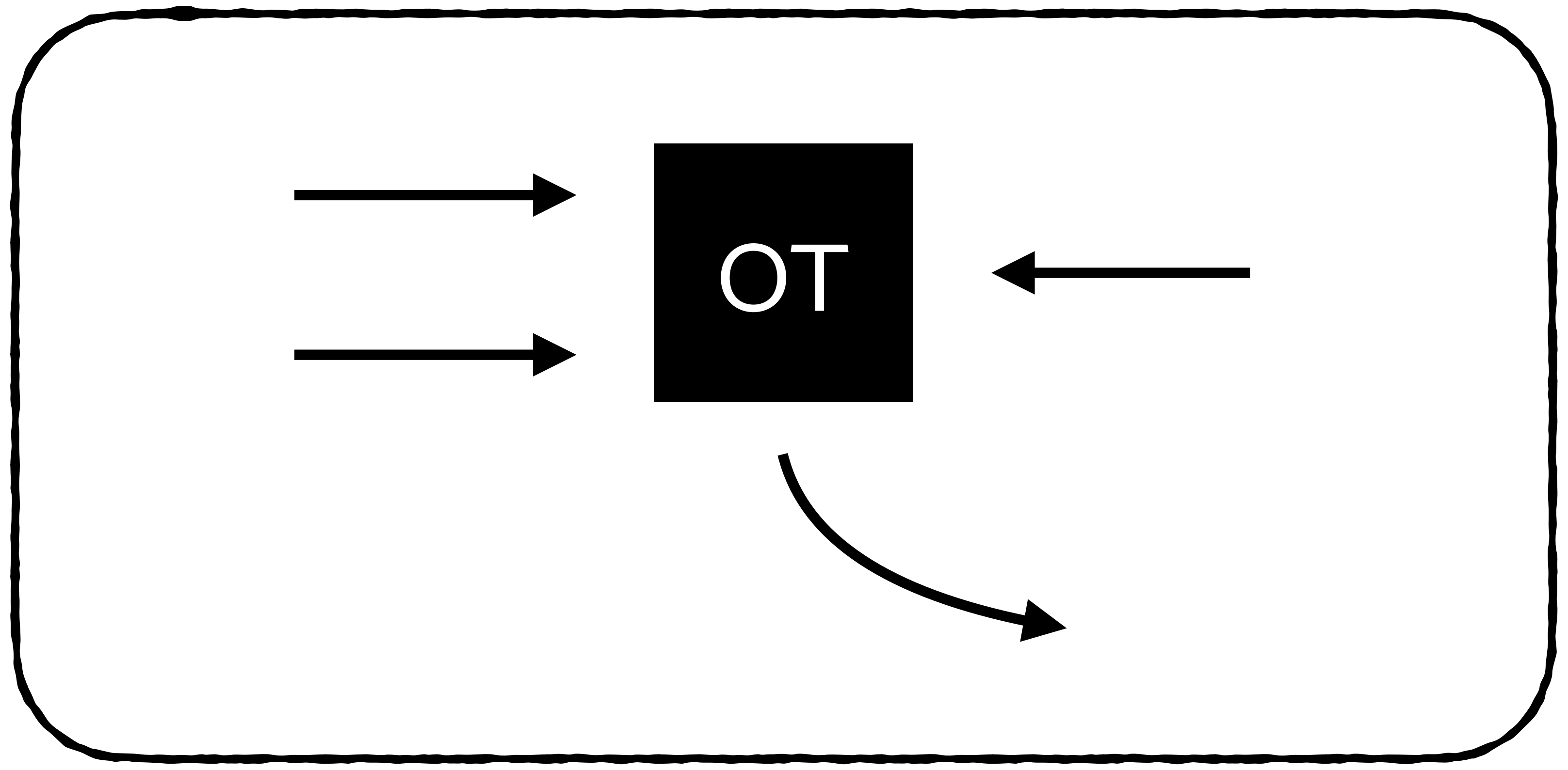

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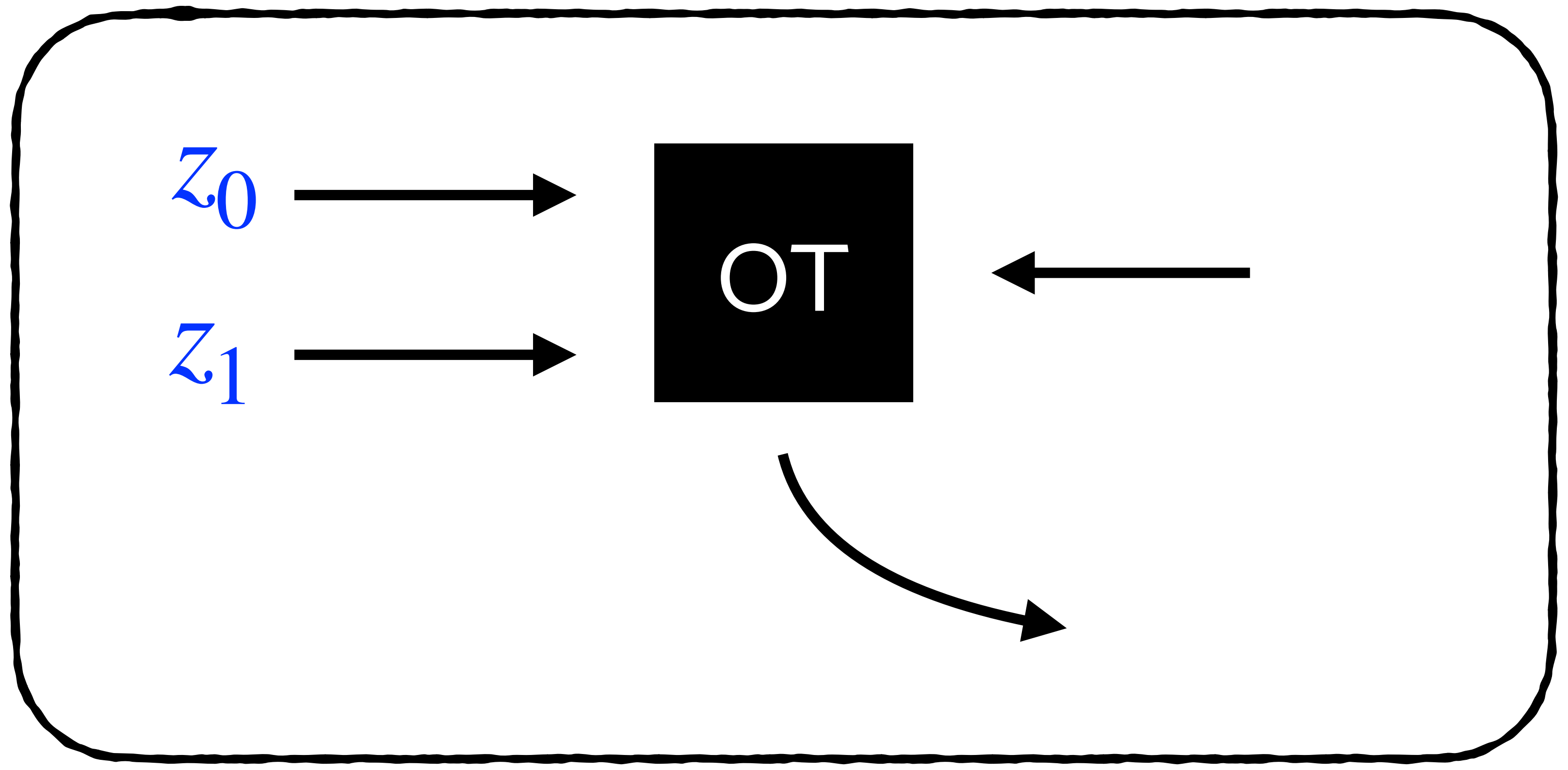

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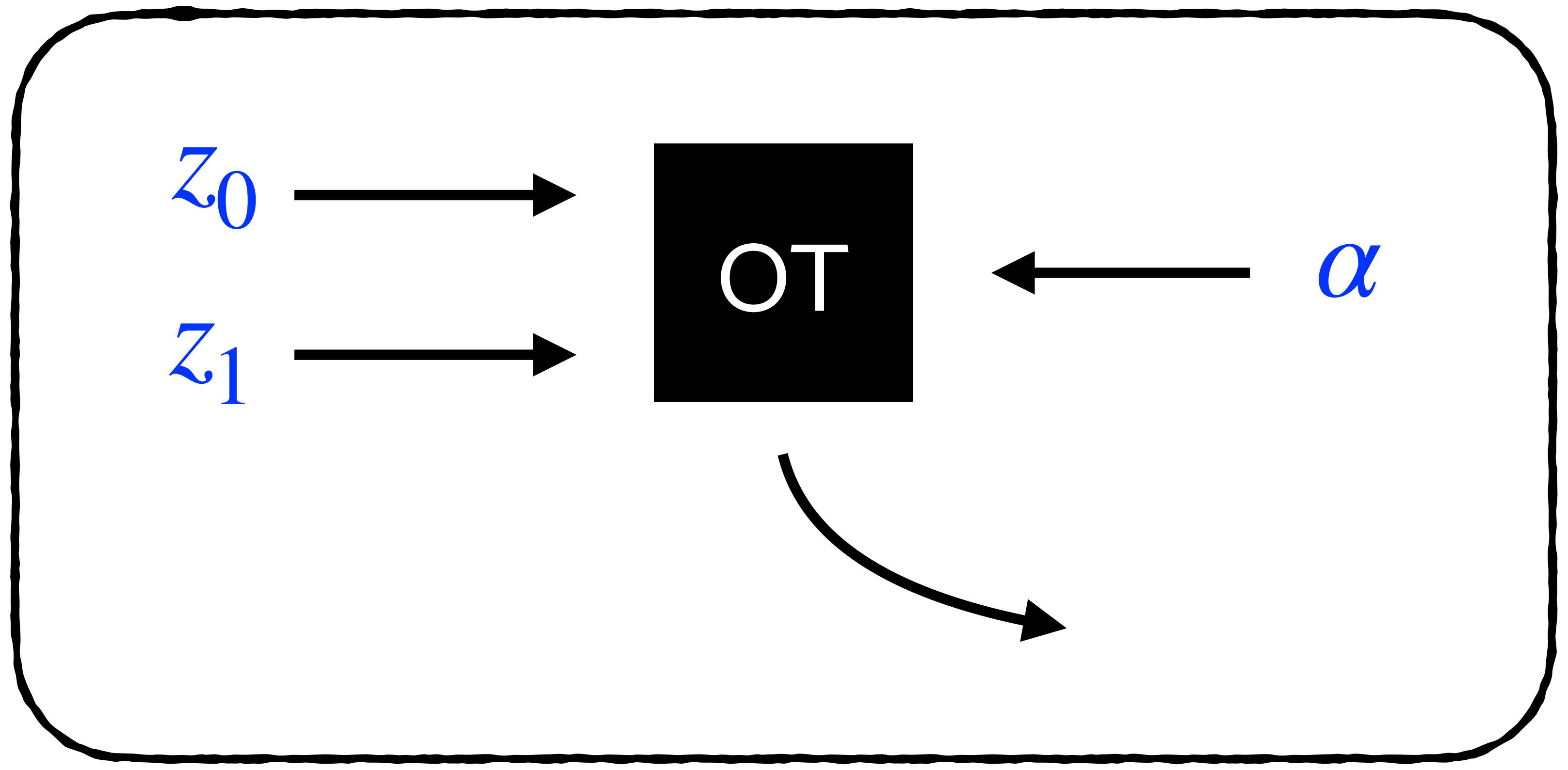

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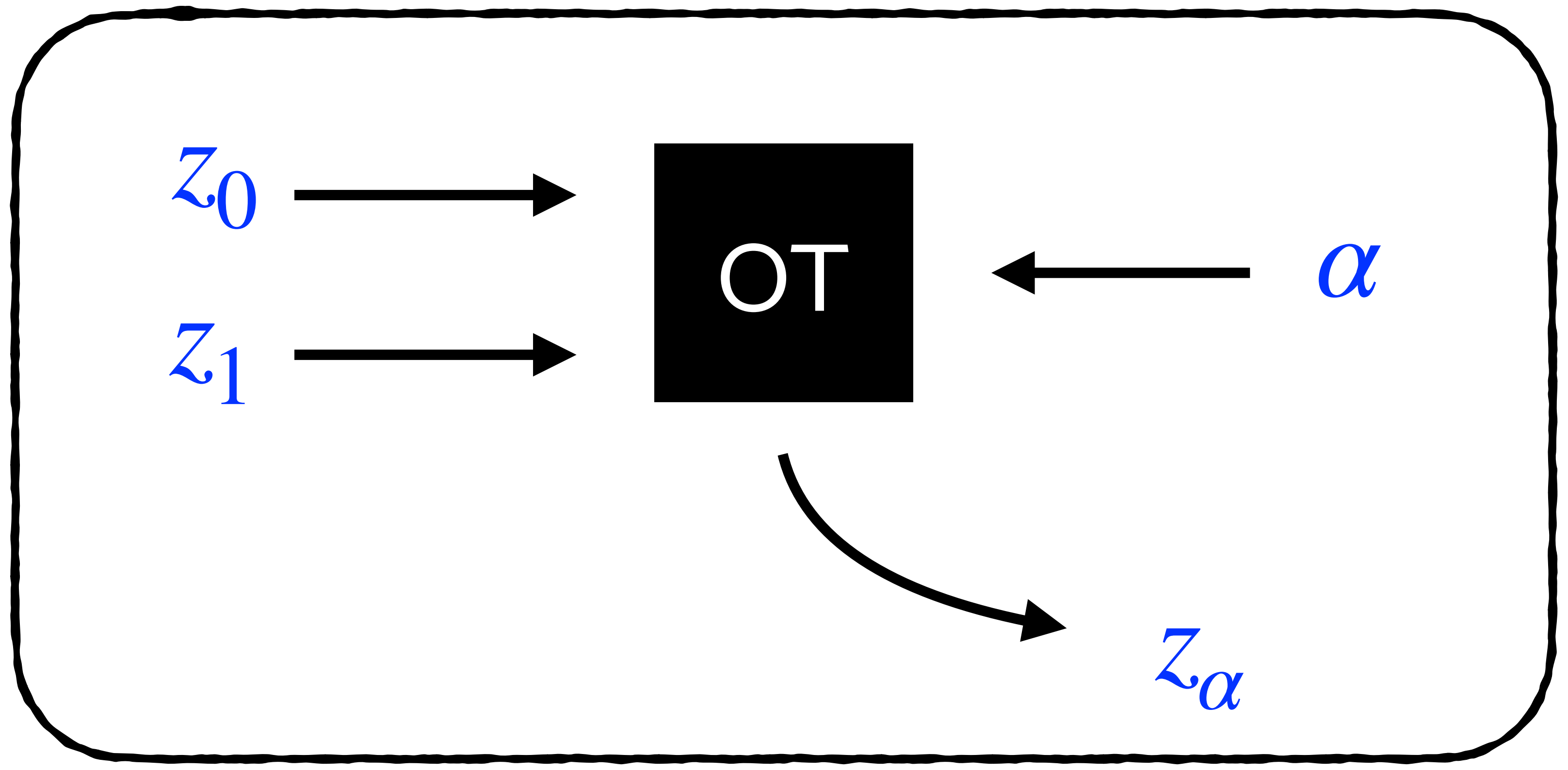

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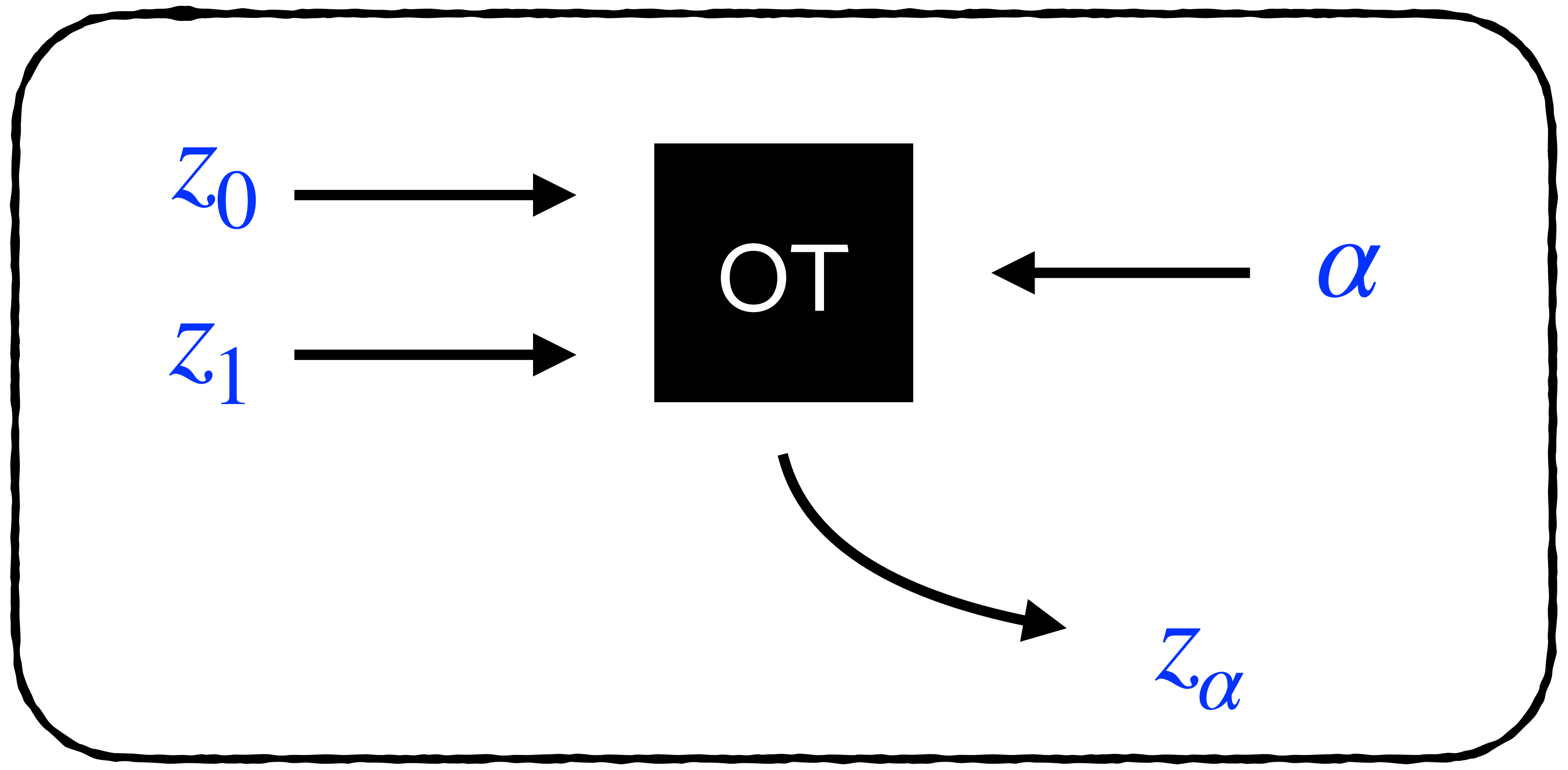

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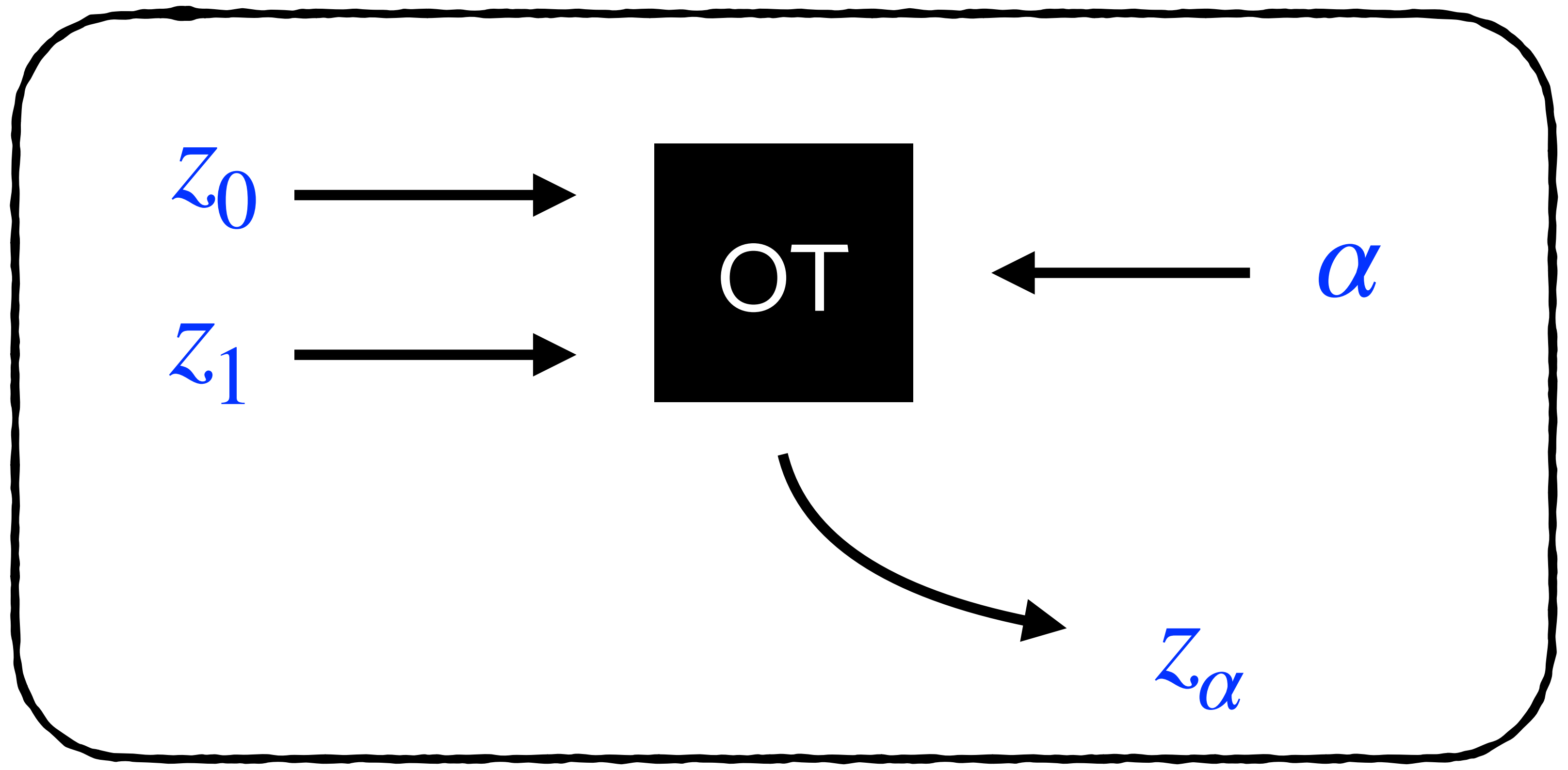

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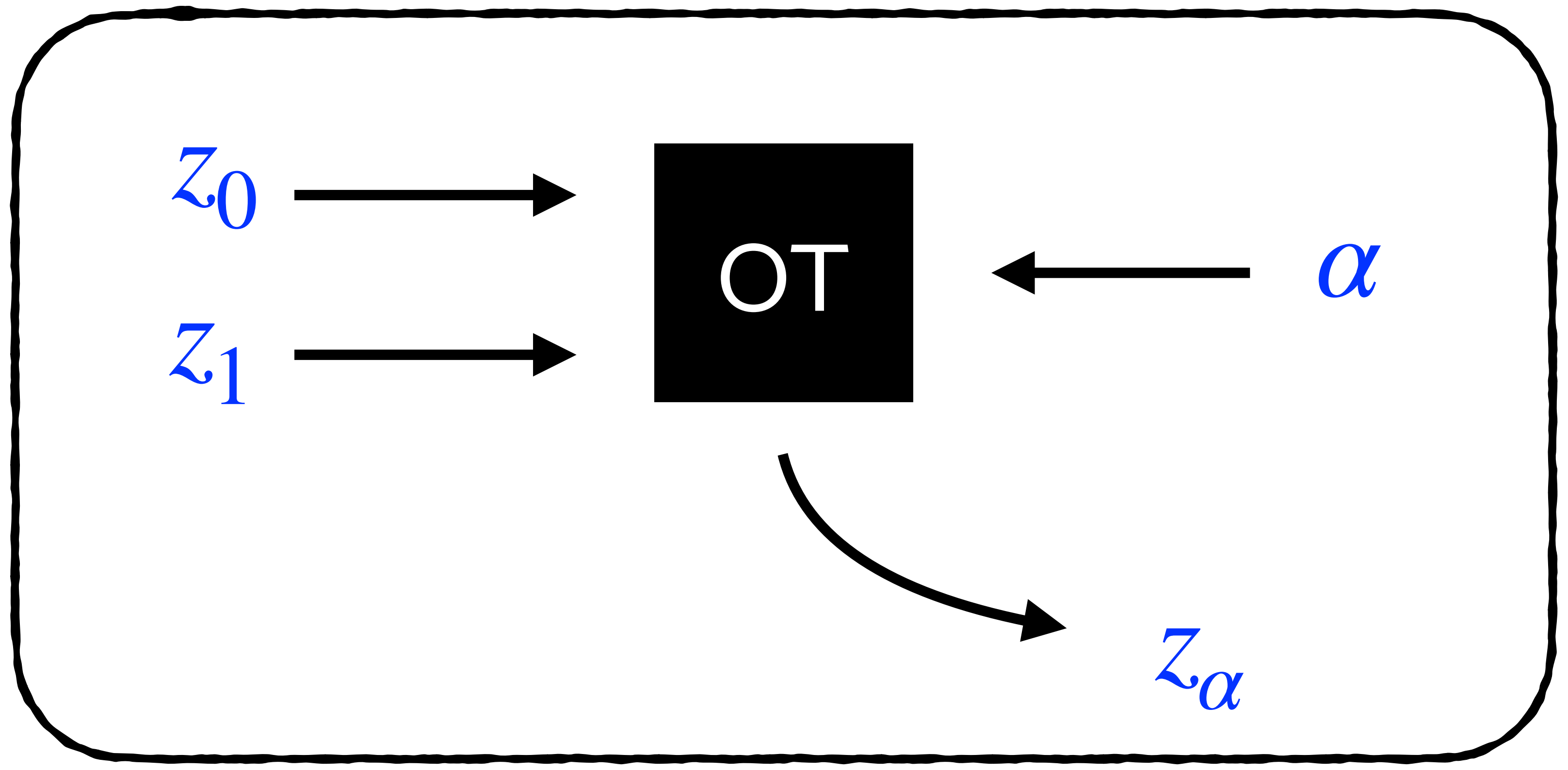

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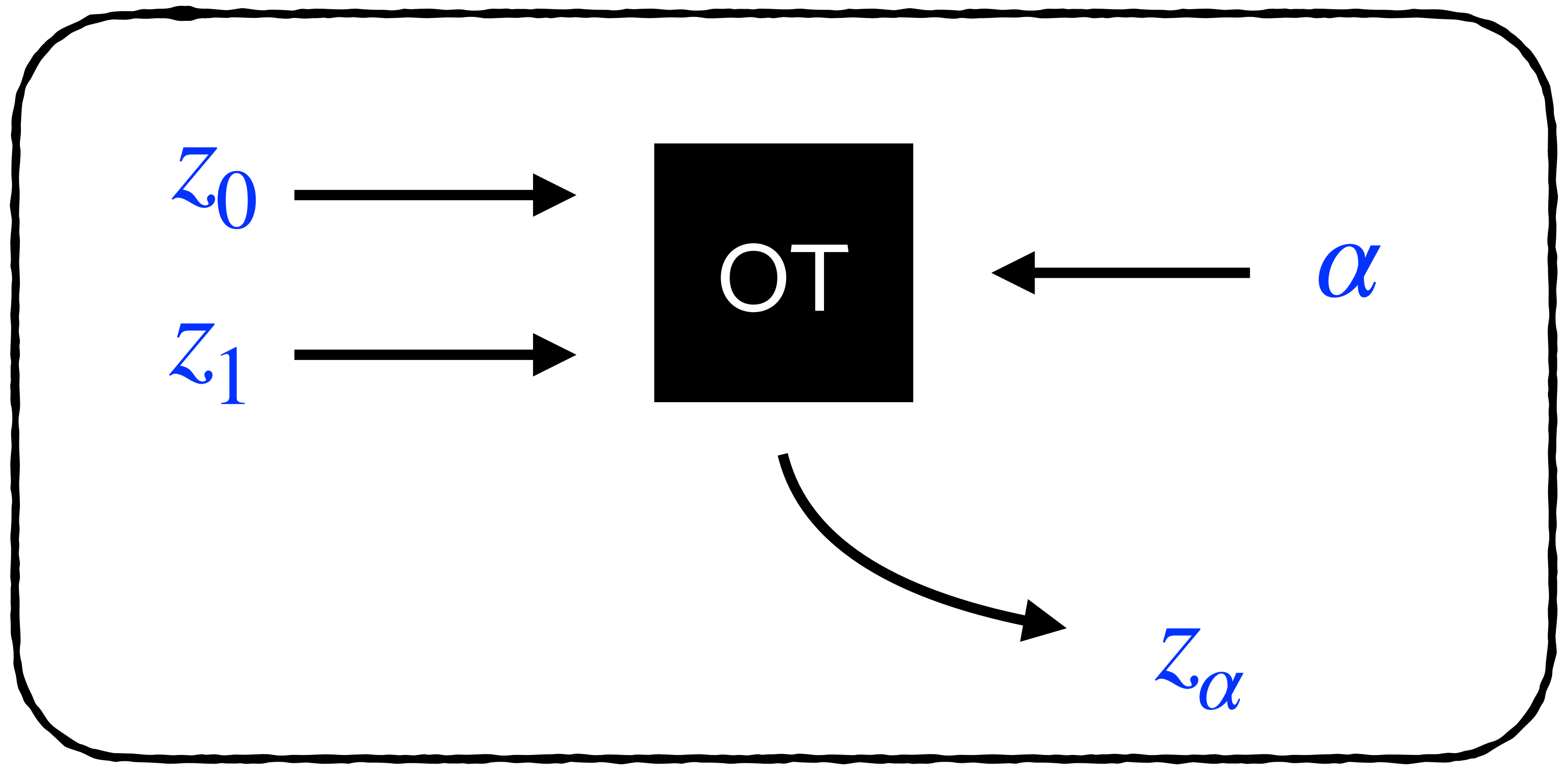

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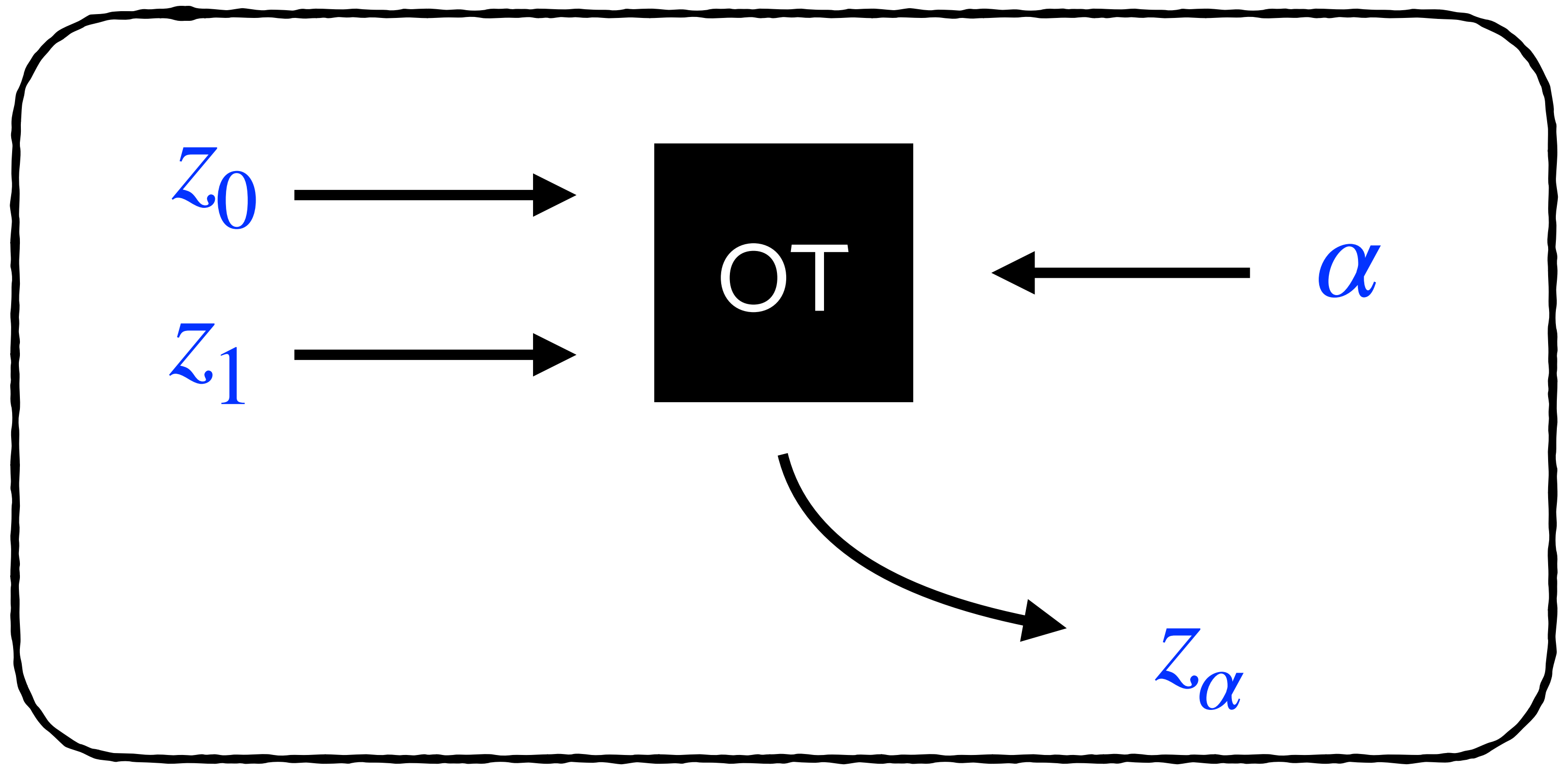

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Size of the z's

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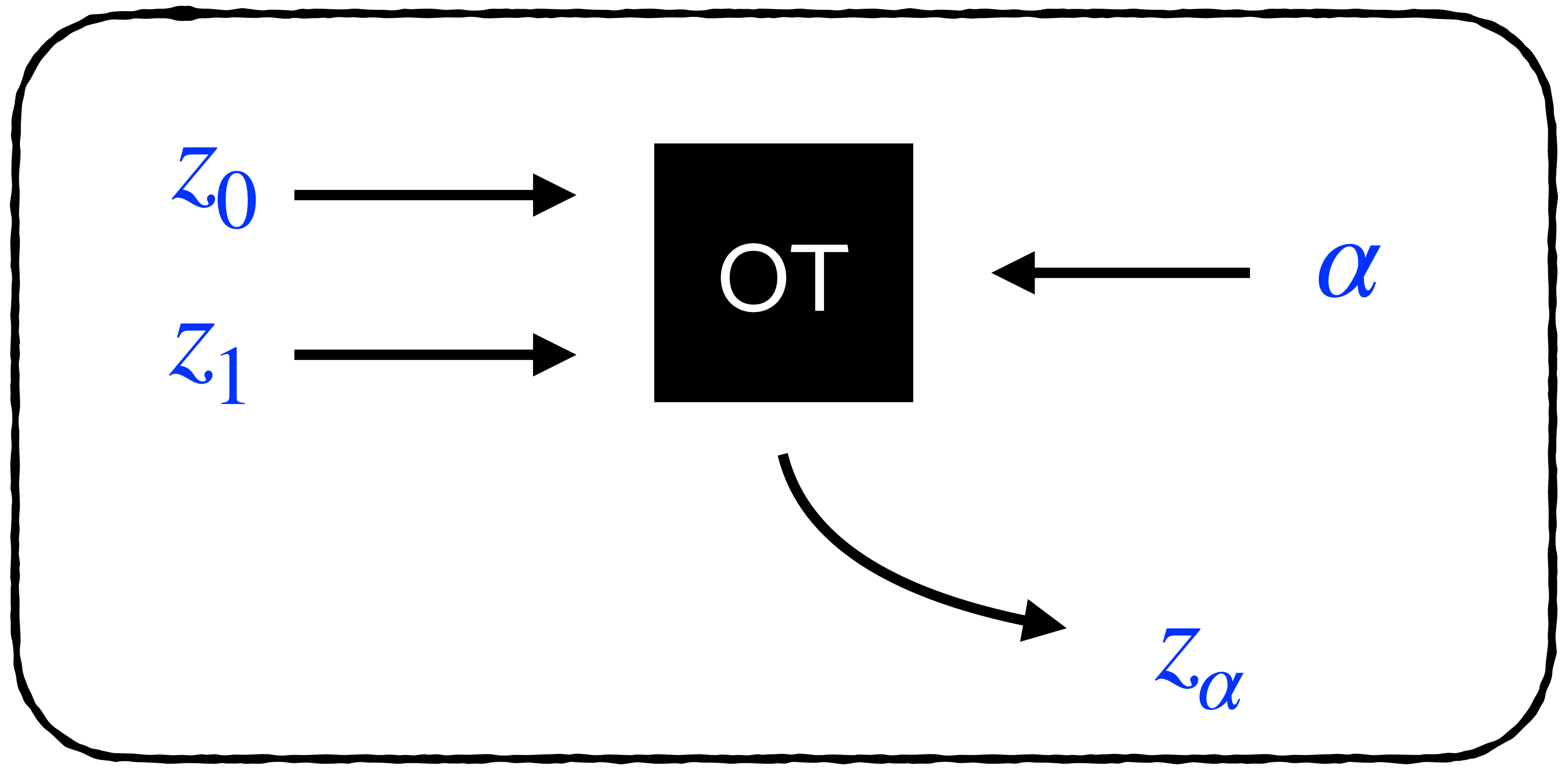

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$\implies$ communication costs in standard model

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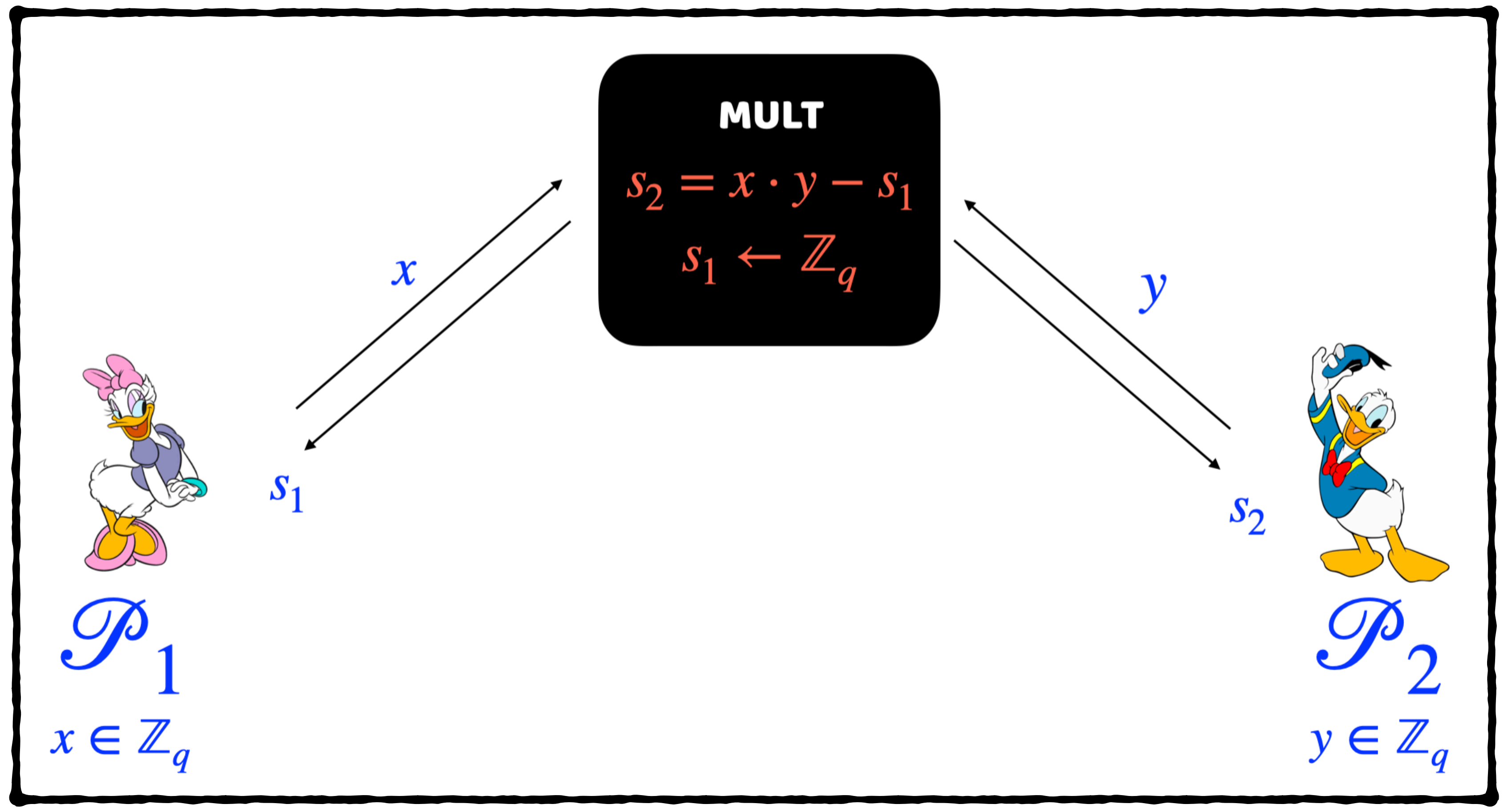
OT-Based Constructions



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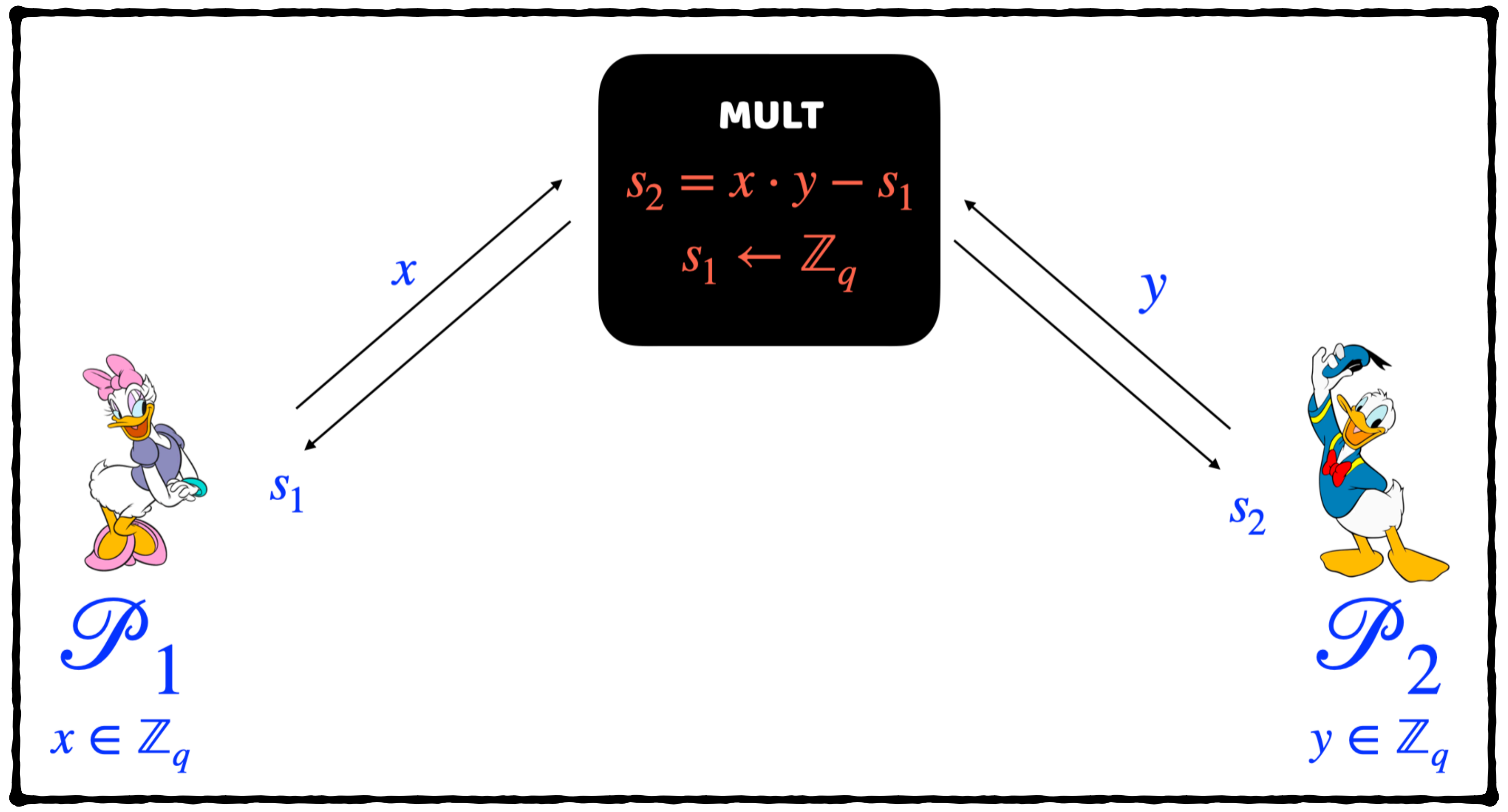

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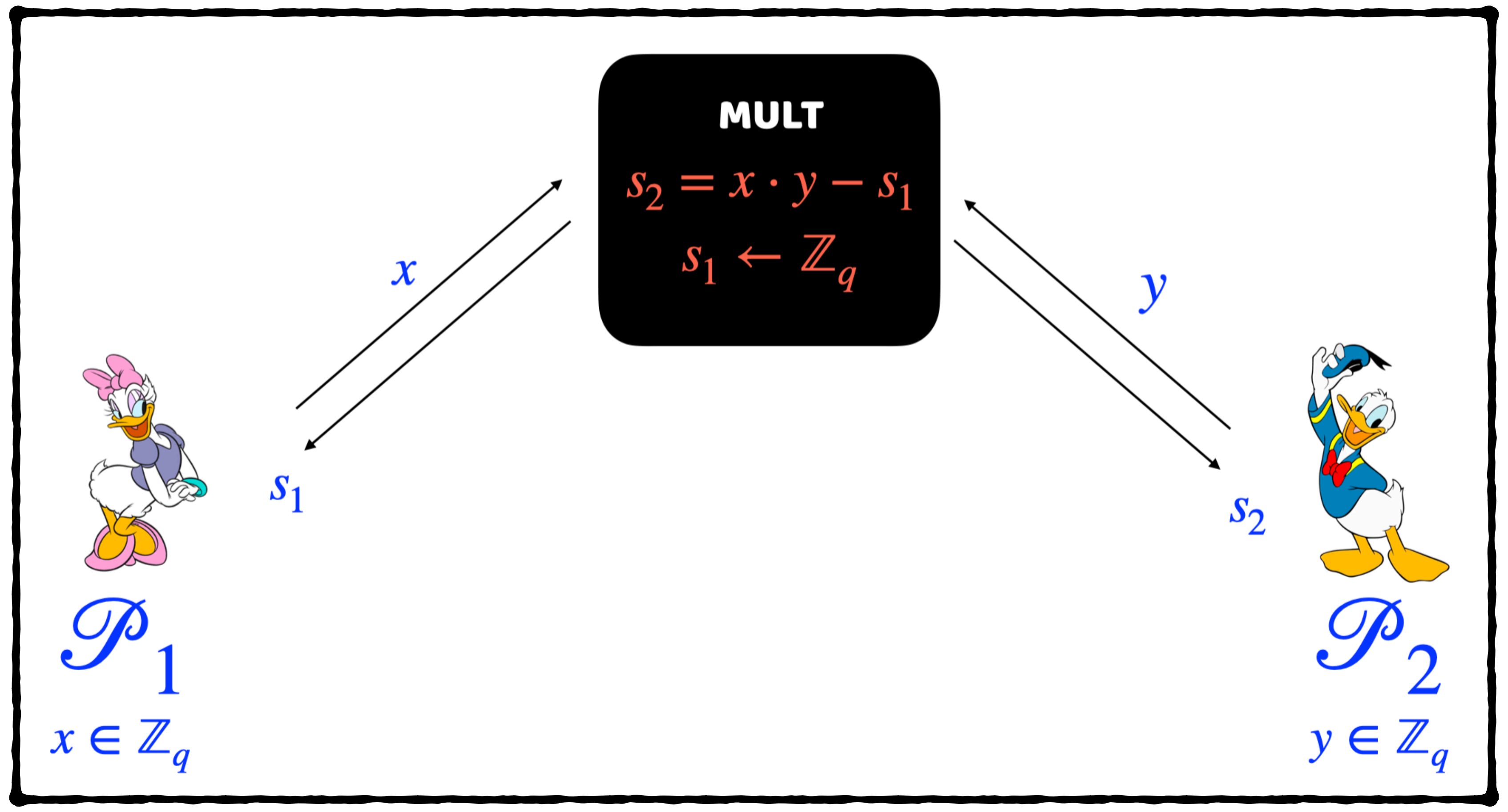

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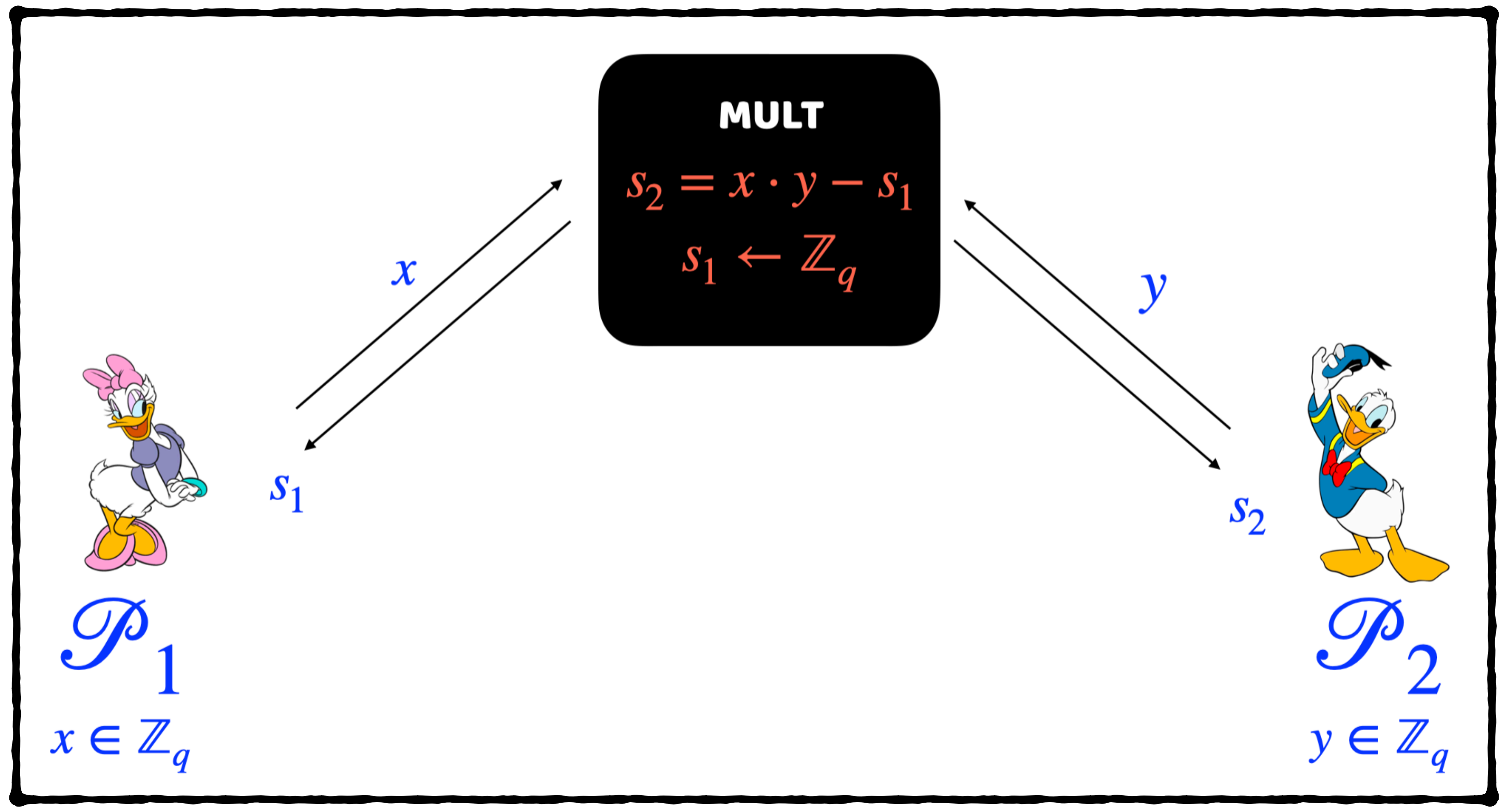

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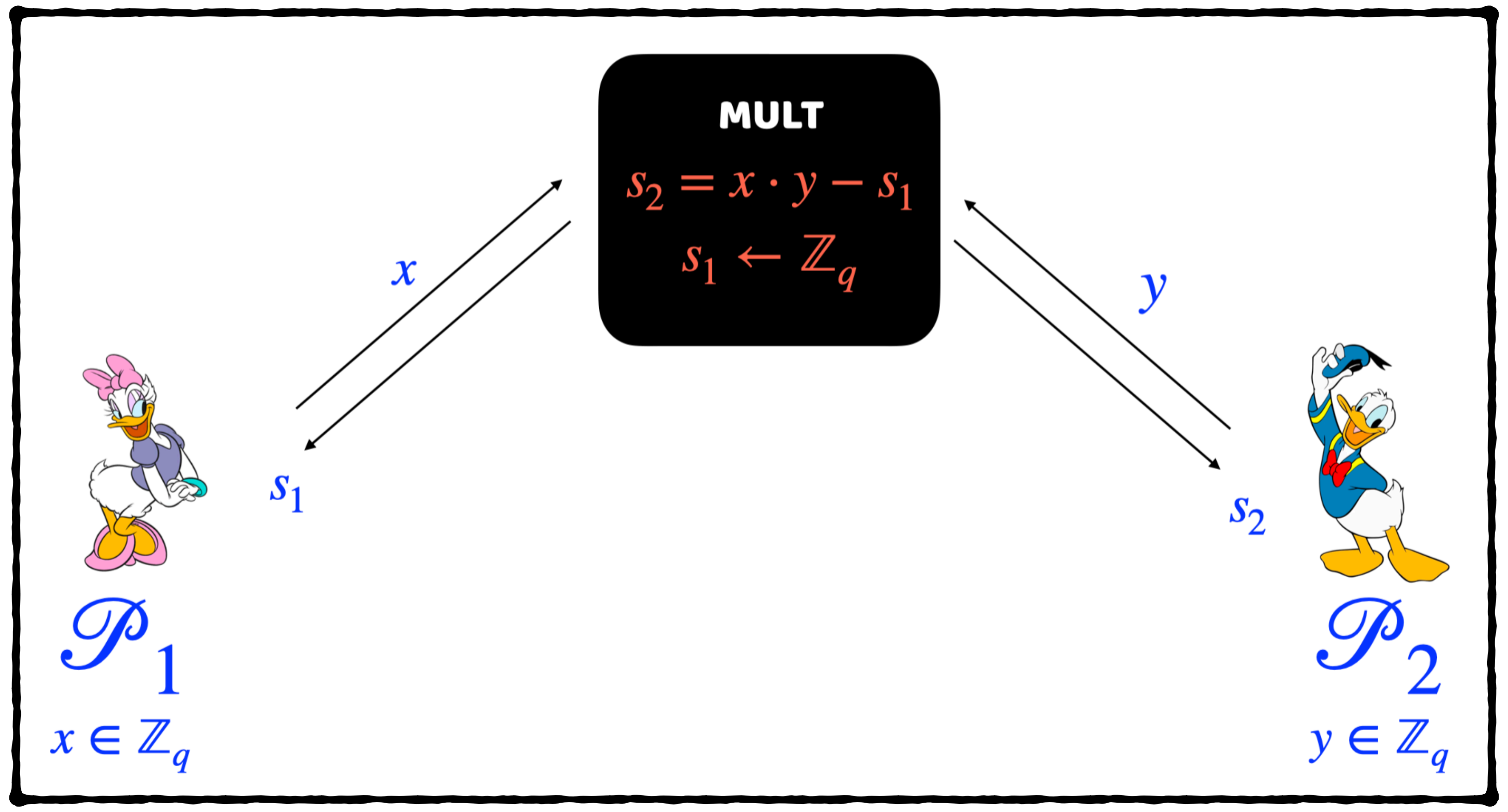

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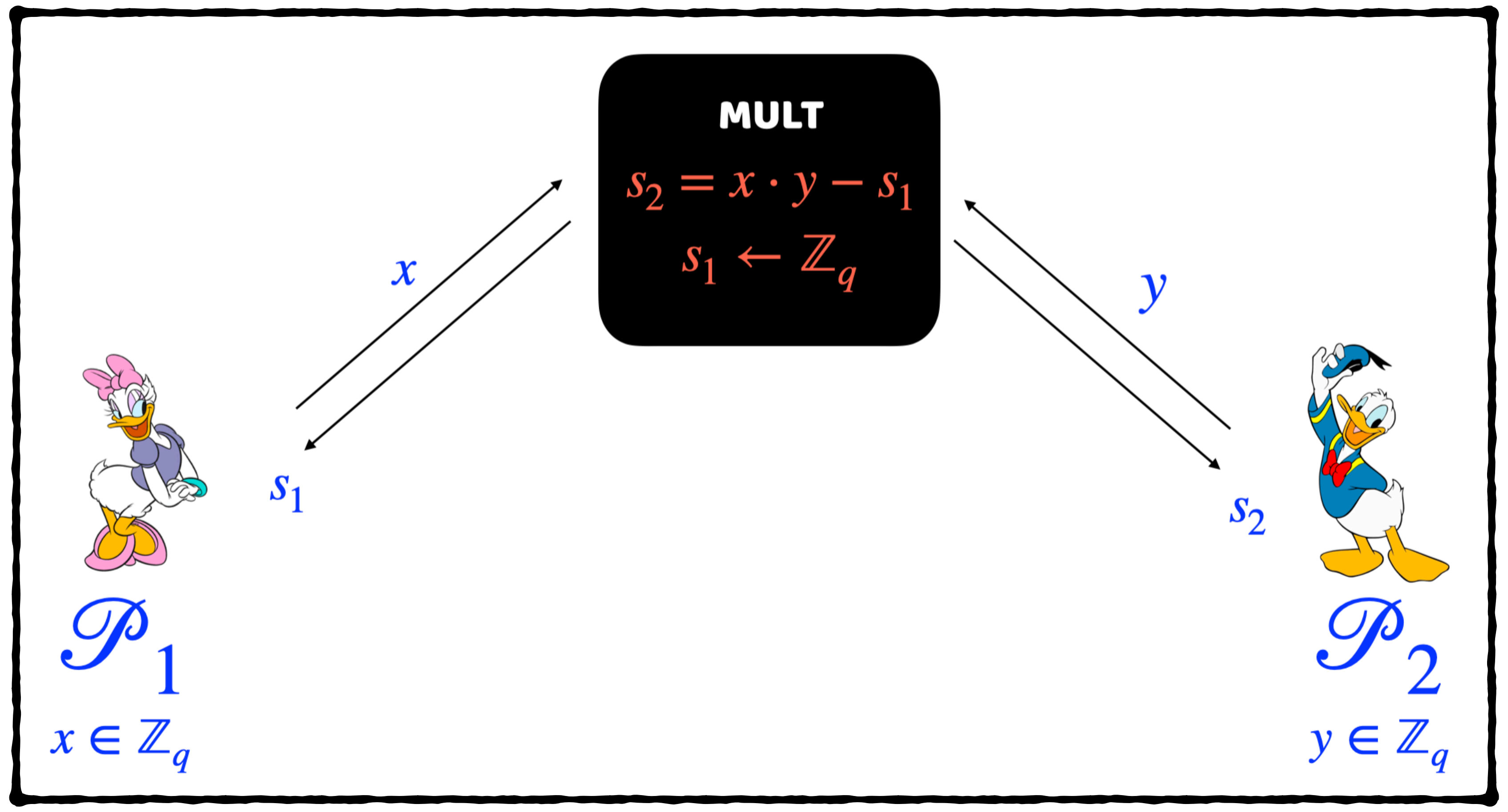

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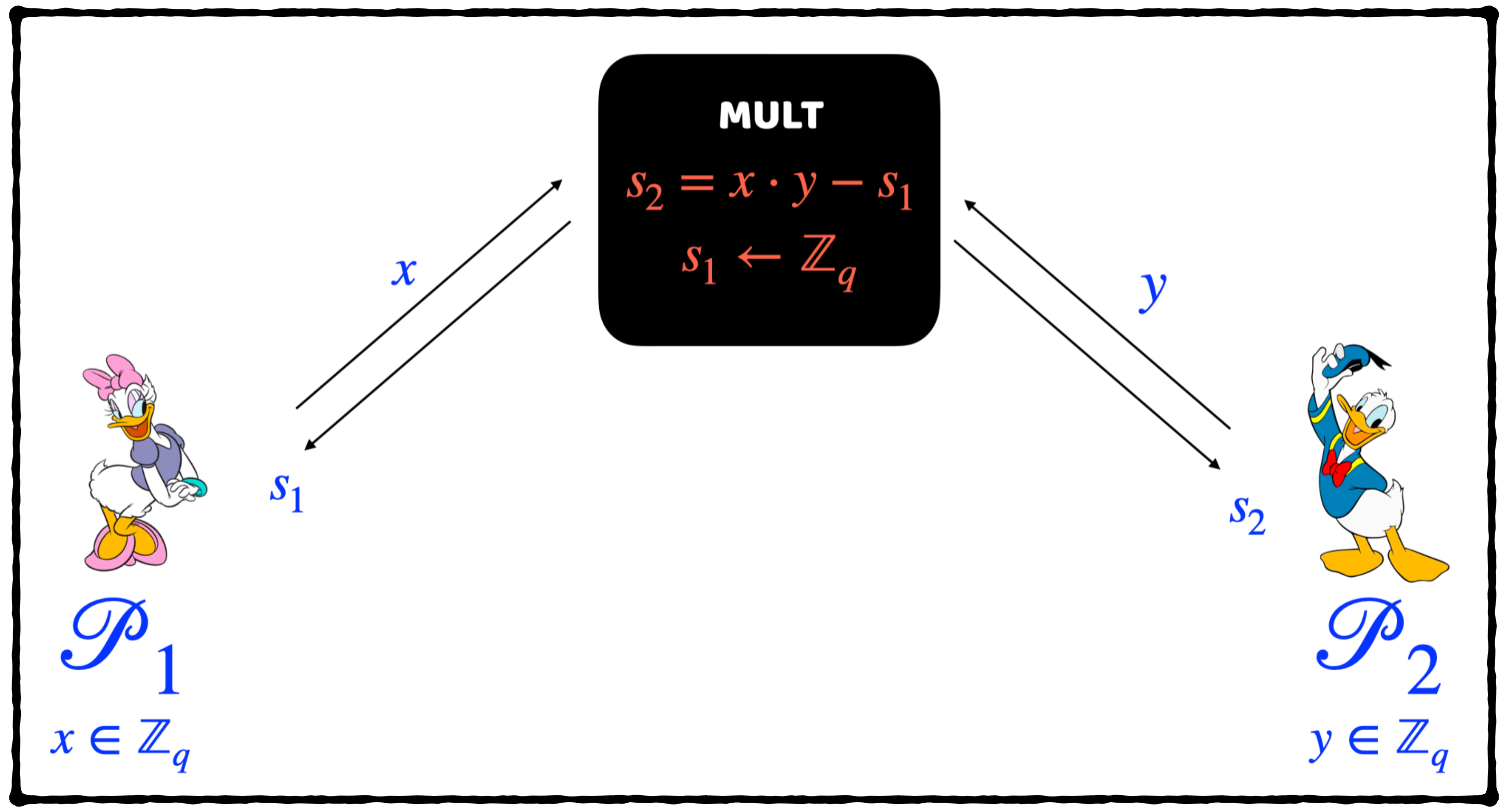

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  - ∃ malicious variant under non-standard assumption

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$\approx$ Vector-OLE

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4. x2 improvement in communication compared to SoA

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MinEntropy of honest output given
$\mathscr{A}$'s ***view and honest input***

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- Formal definition of "unpredictability" via functionality WeakMult

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- Batching variant is useful for generating triplets in preprocessing model.

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Comparison (in ROM)

Gilboa, IPS, MASCOT, DKLs

	Gilboa99	IPS09	MASCOT/ DKLs19	Our Work
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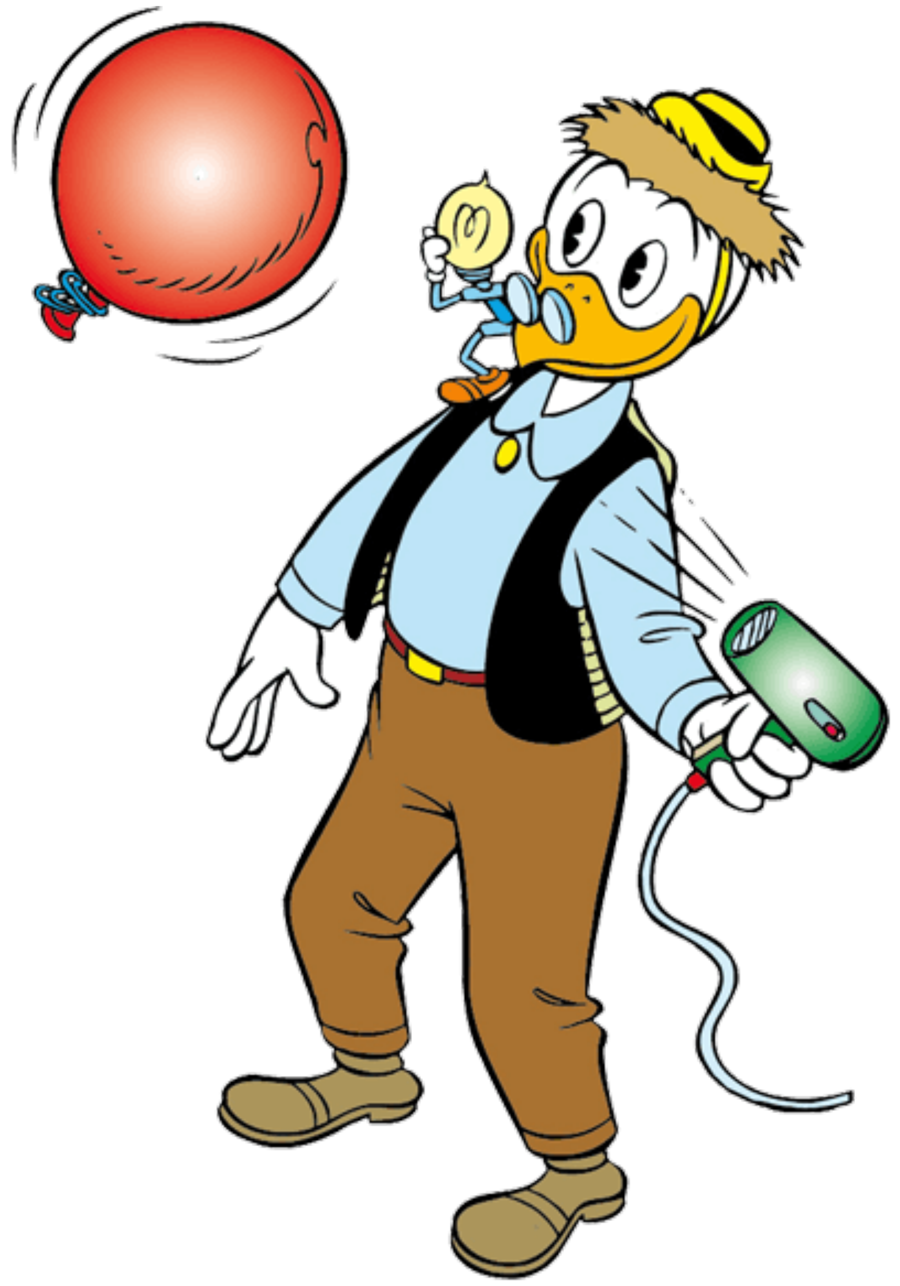
Batch Size $= \beta$

Technical Overview



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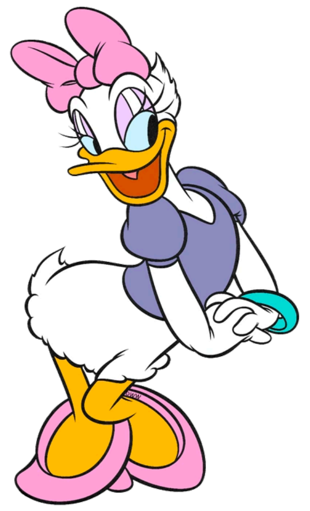
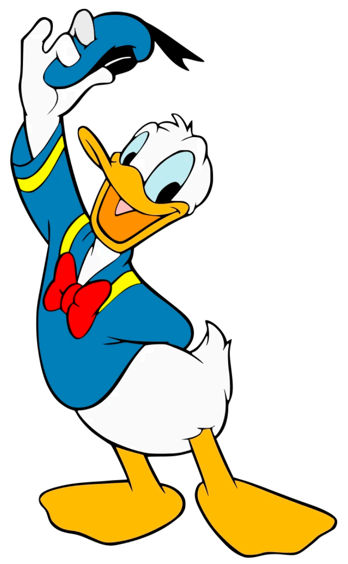
Our Protocol

$\mathscr{P}_1$ *and* $\mathscr{P}_2$ *hold inputs* x *and* $y \in \mathbb{Z}_q$ *respectively*





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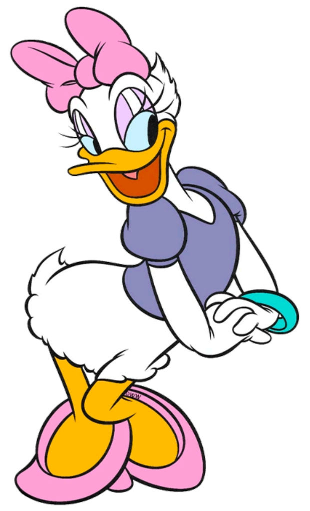

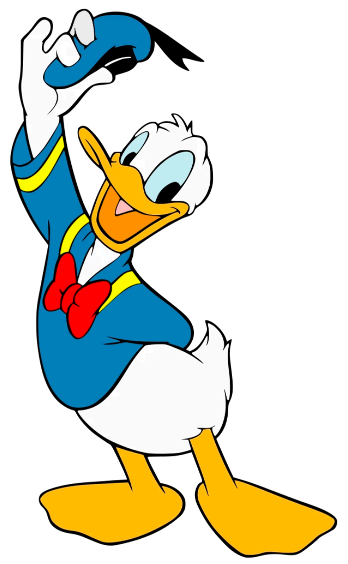

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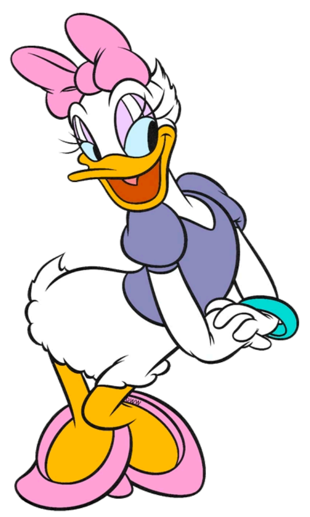

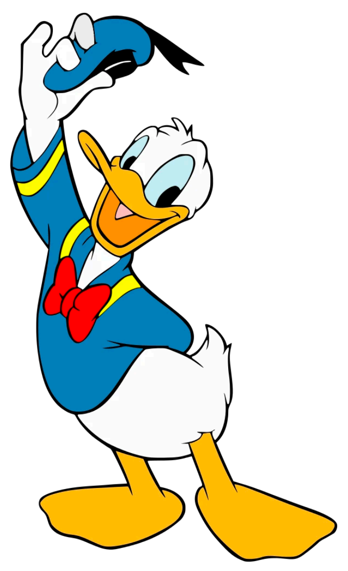

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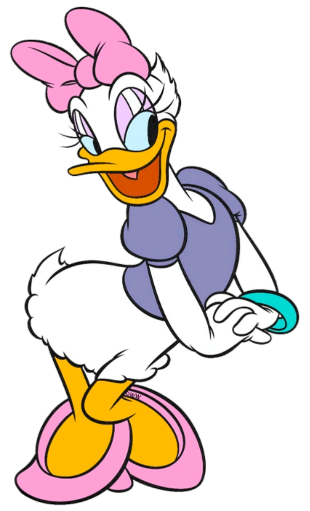

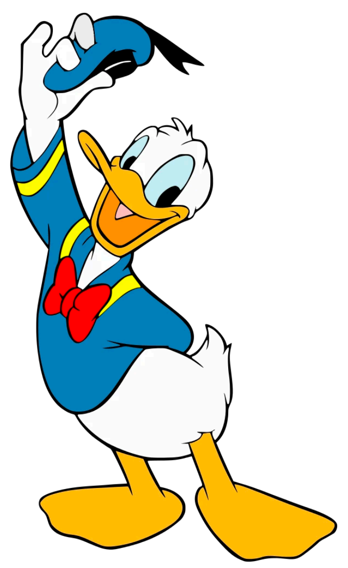

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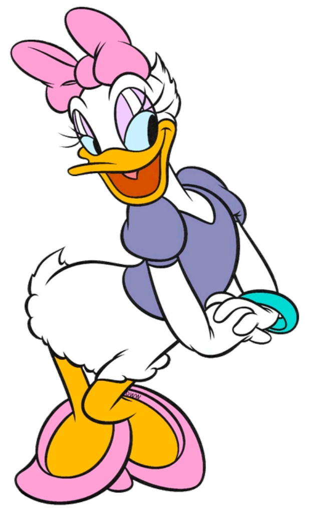

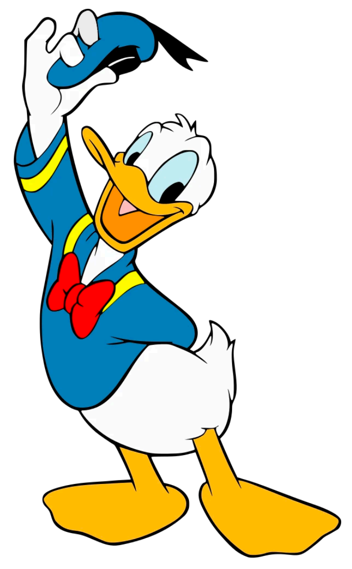

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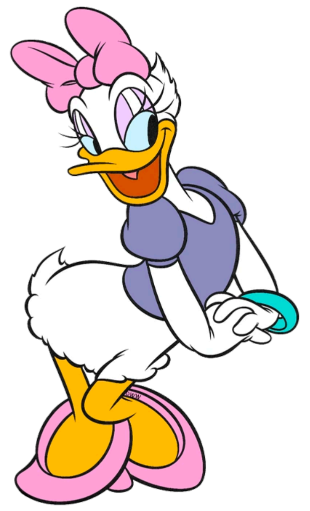

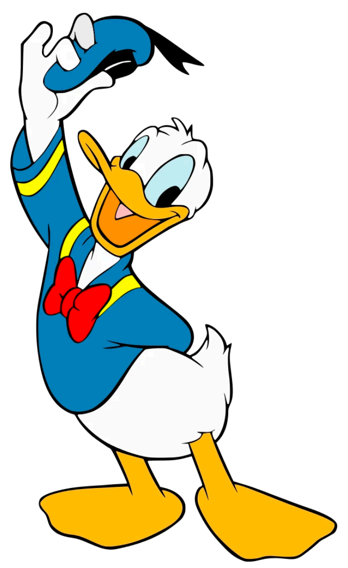

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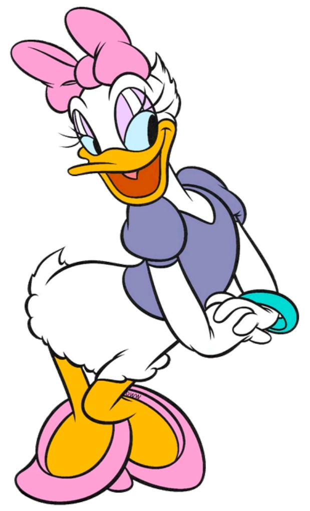

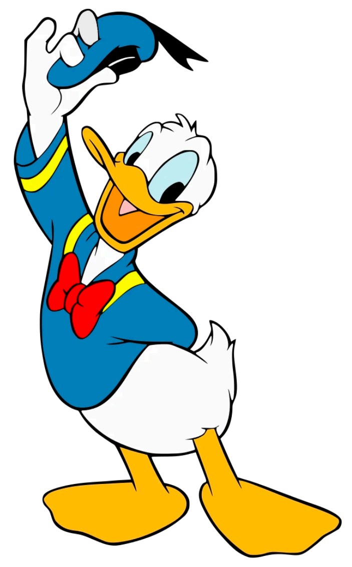

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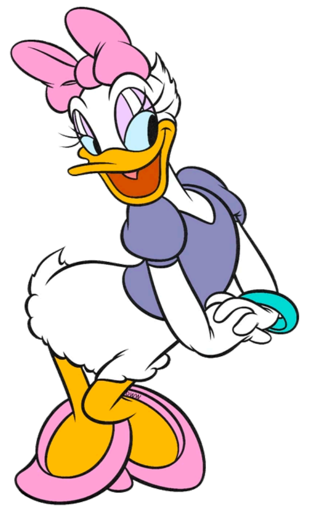

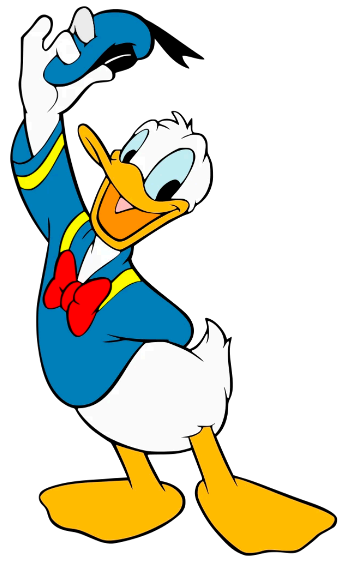

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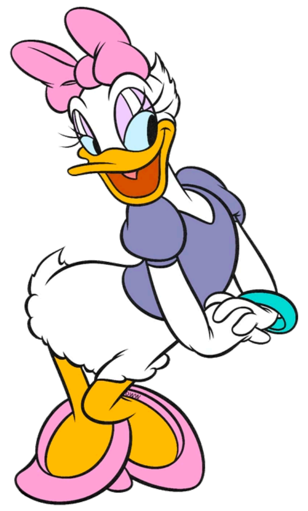

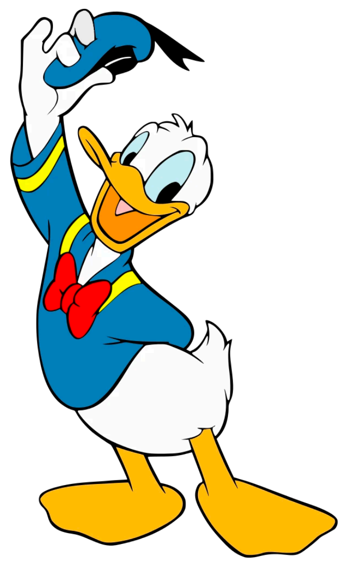

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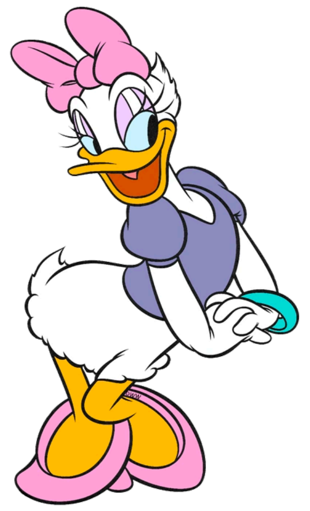

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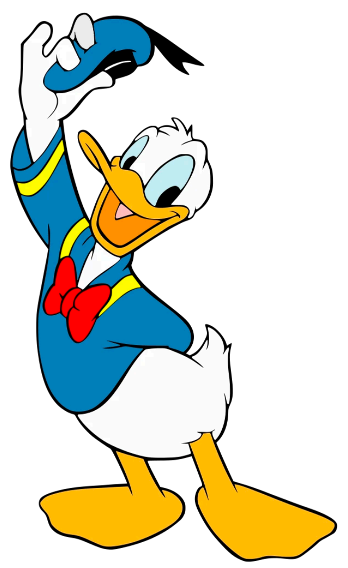

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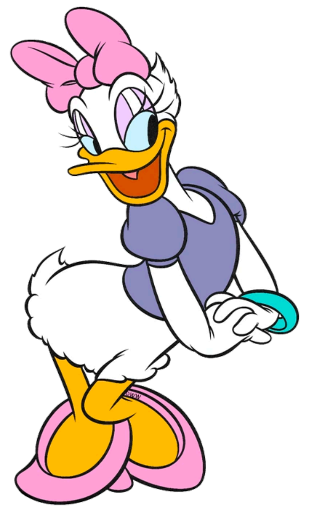

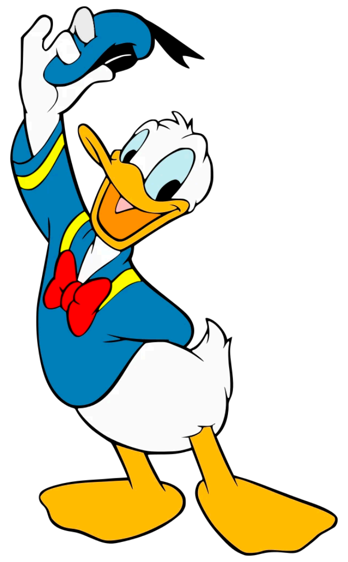

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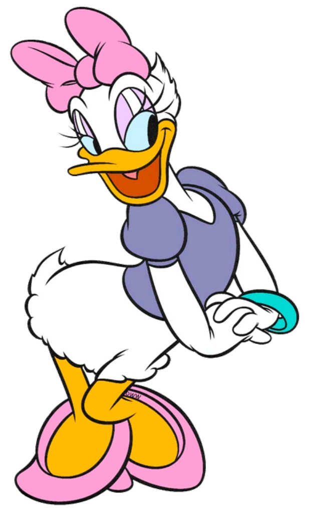

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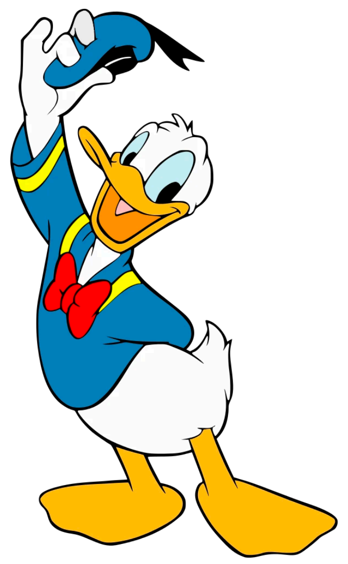

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$$\boxed{\langle -\boldsymbol{\delta} + \boldsymbol{z}, \boldsymbol{v} \rangle = \langle x \cdot \boldsymbol{t}, \boldsymbol{v} \rangle = x \cdot \langle \boldsymbol{t}, \boldsymbol{v} \rangle = x \cdot y}$$

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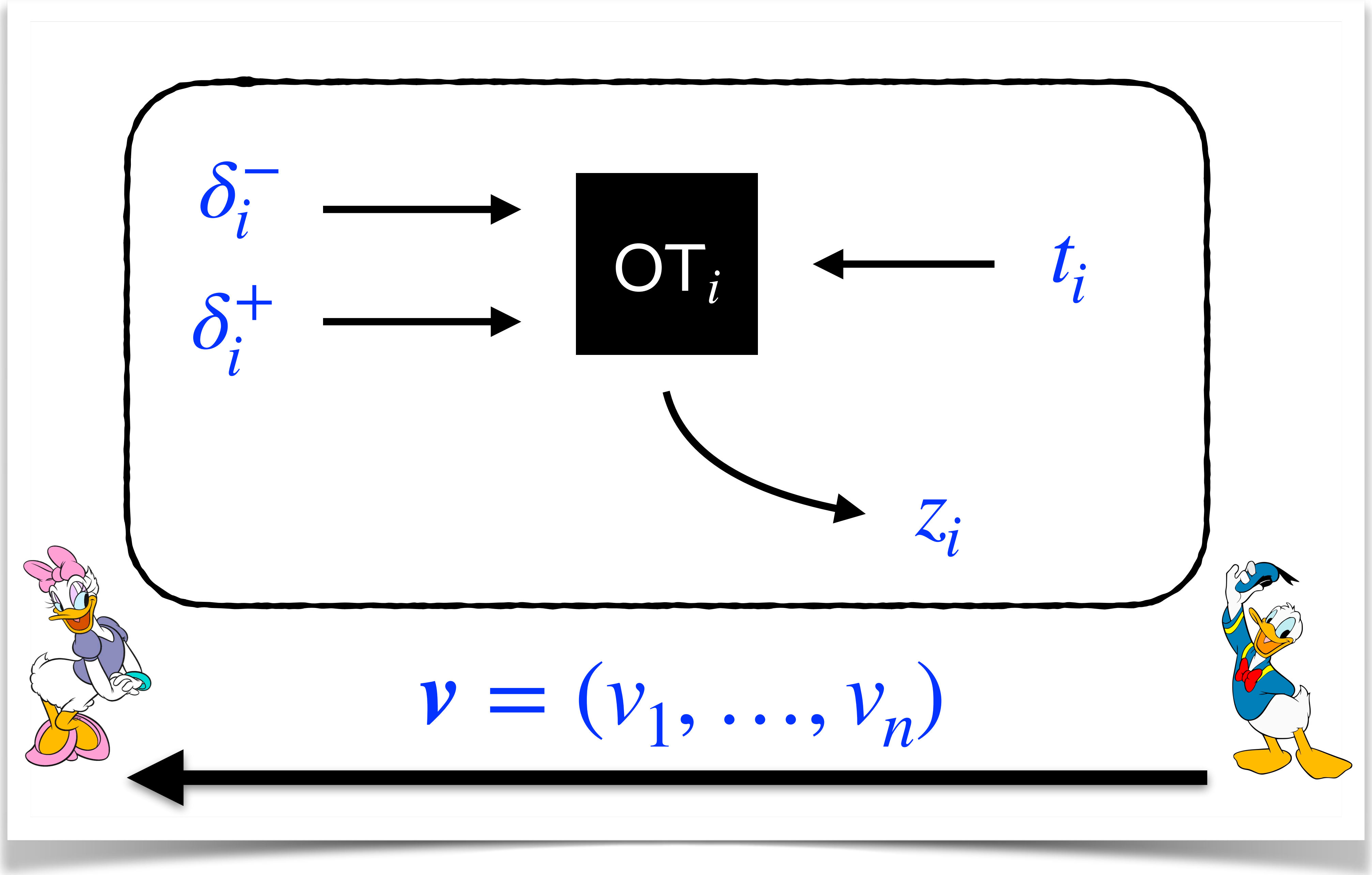

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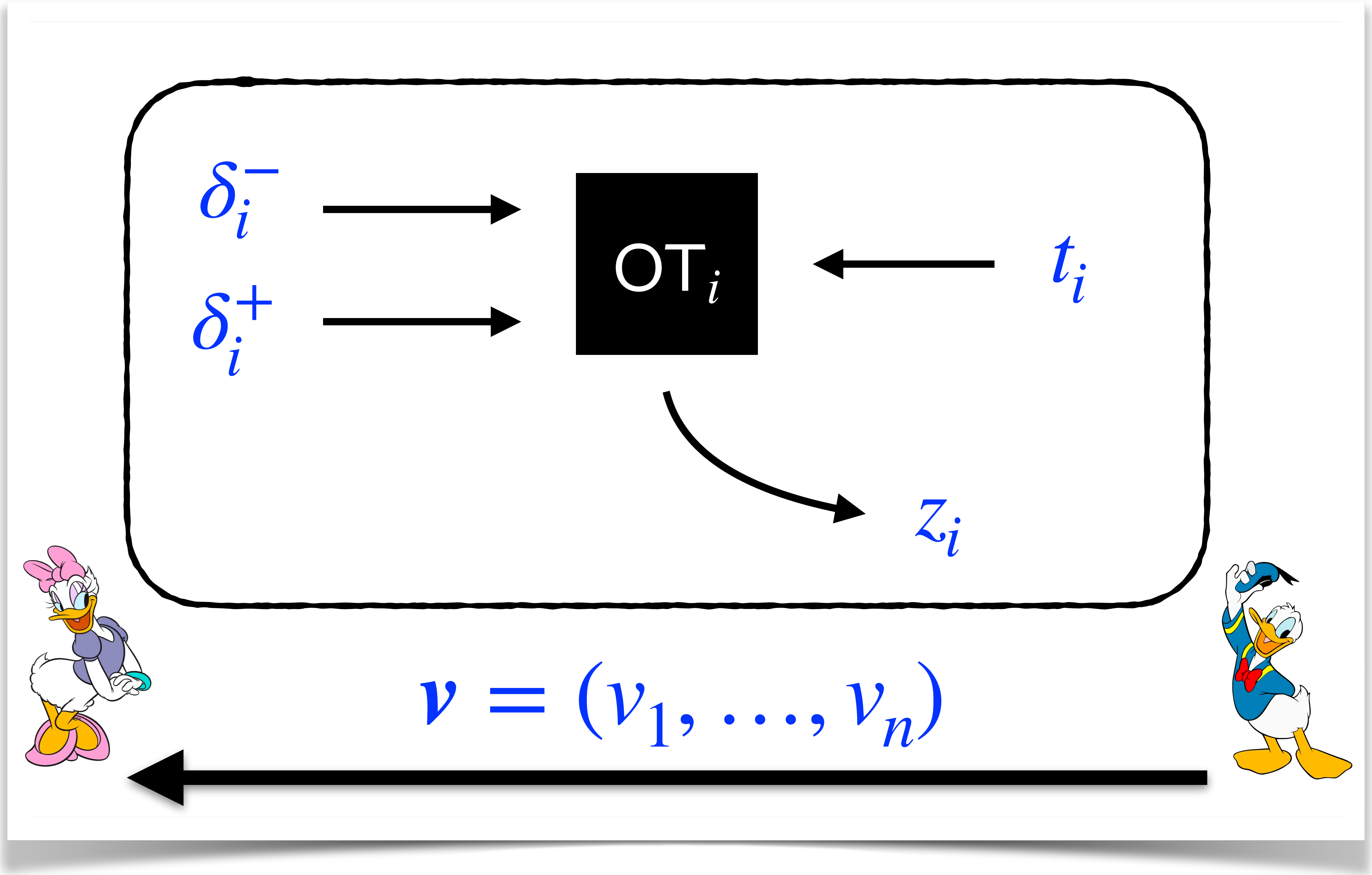

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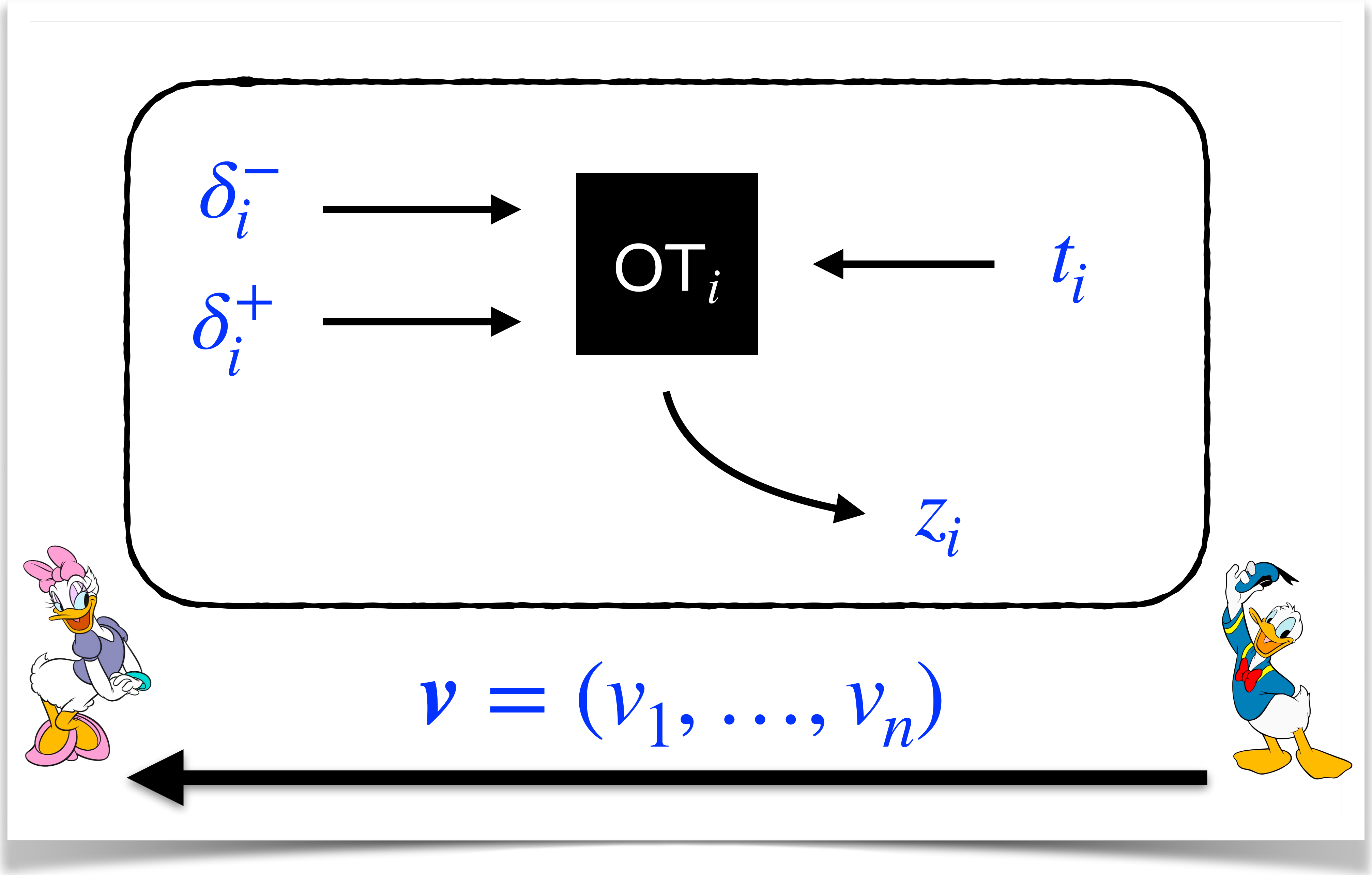

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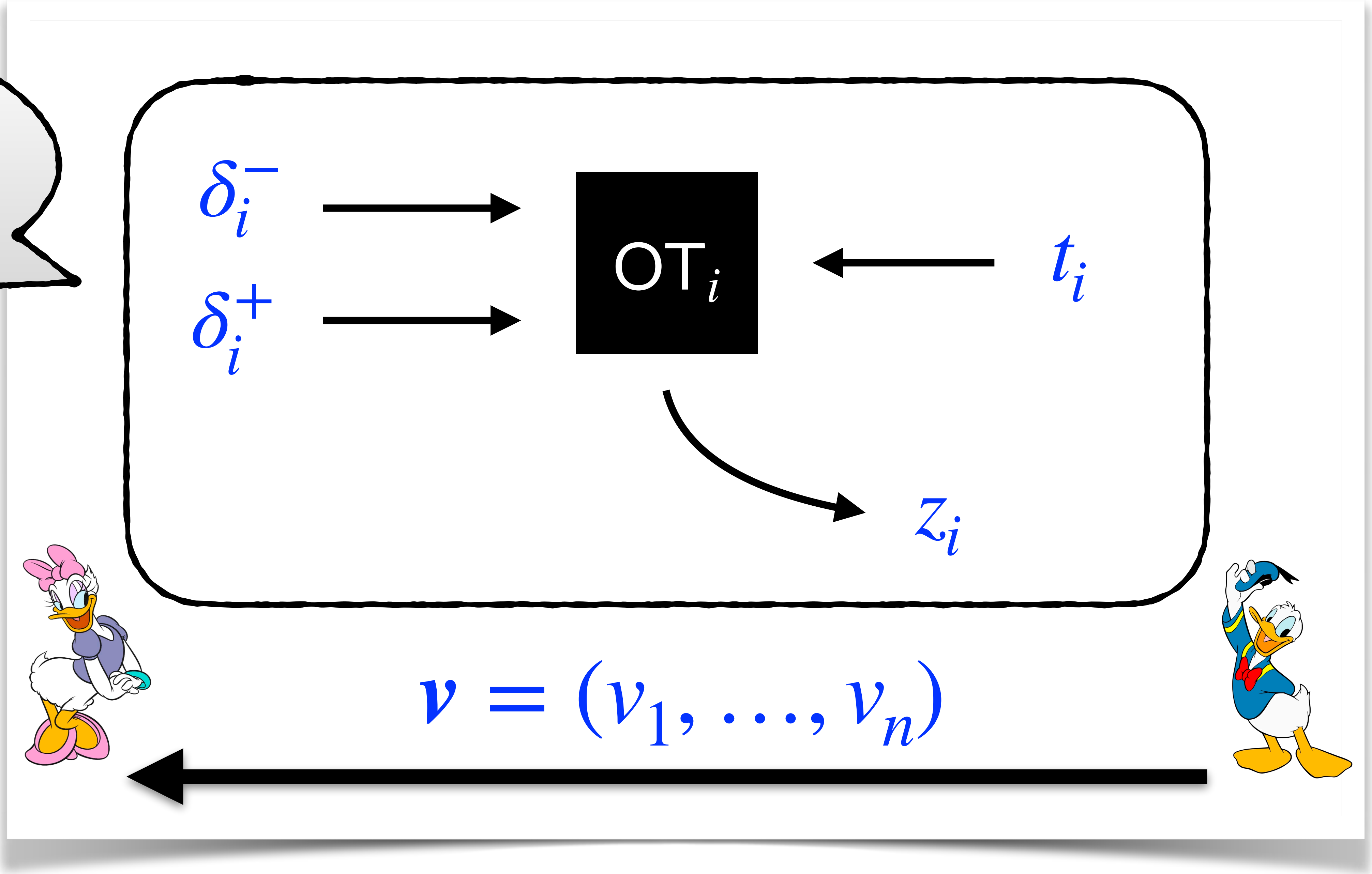

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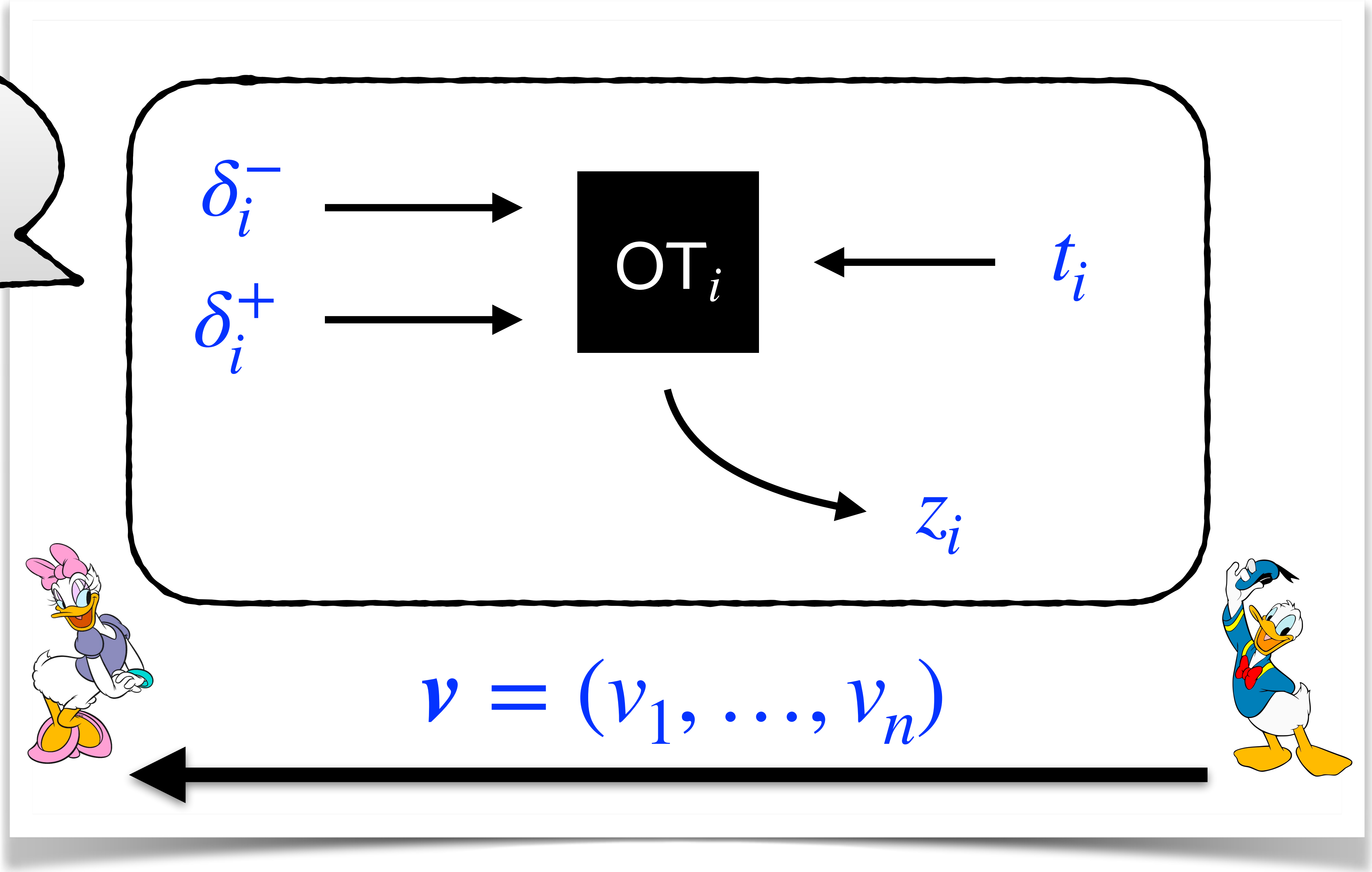

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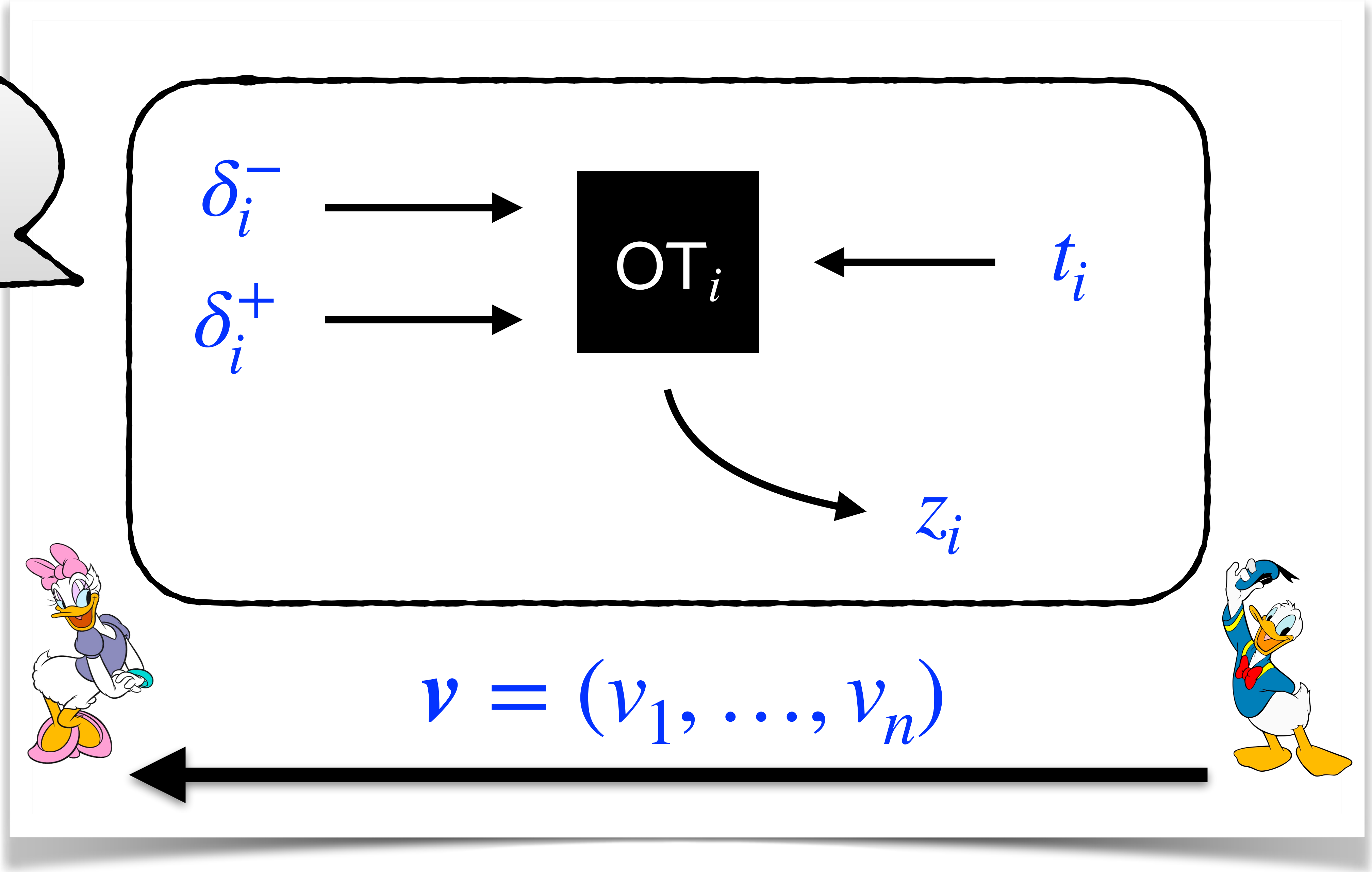

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Weight($\boldsymbol{d}$)
captures how much $\mathscr{A}$ deviates.

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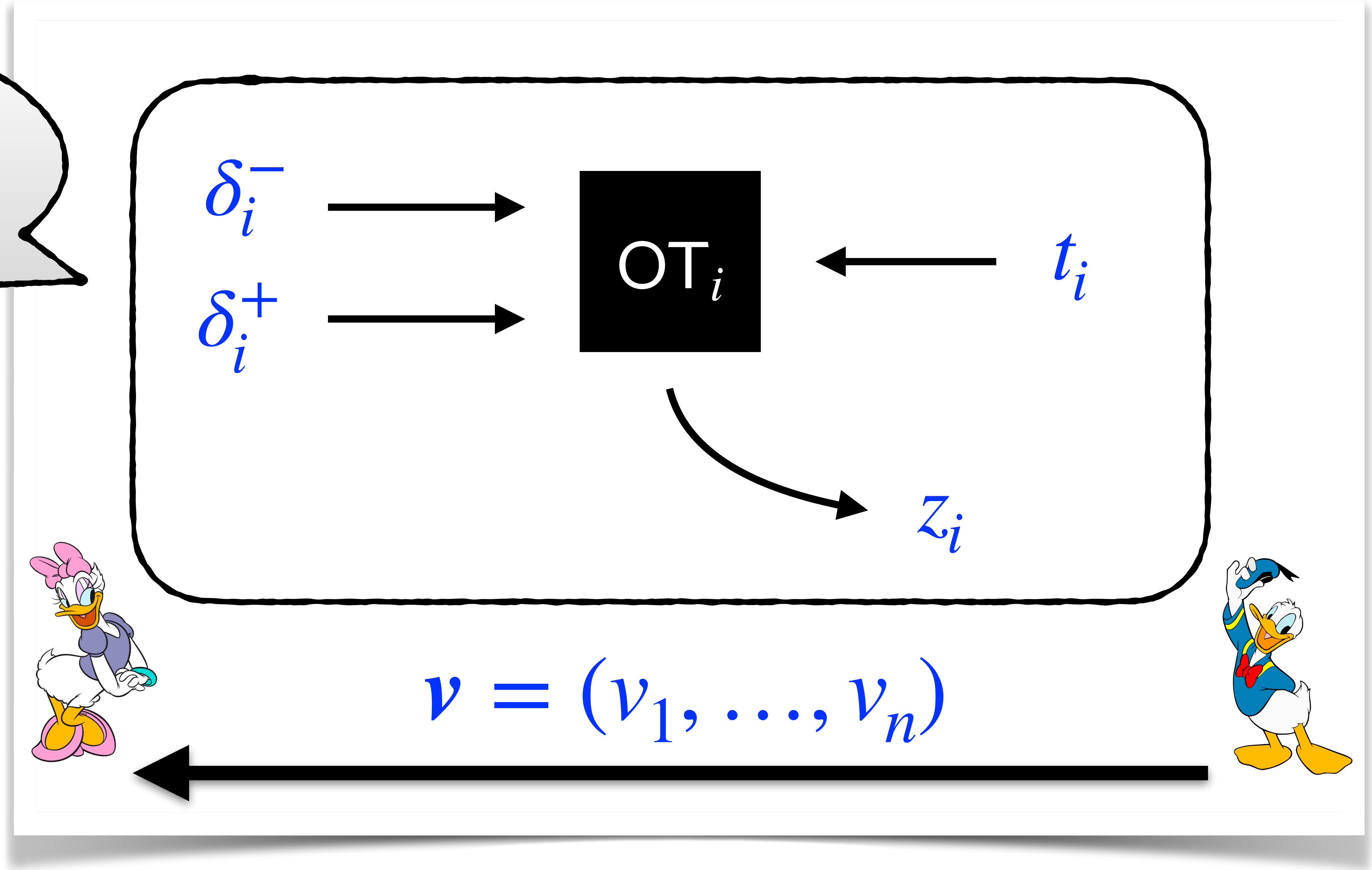

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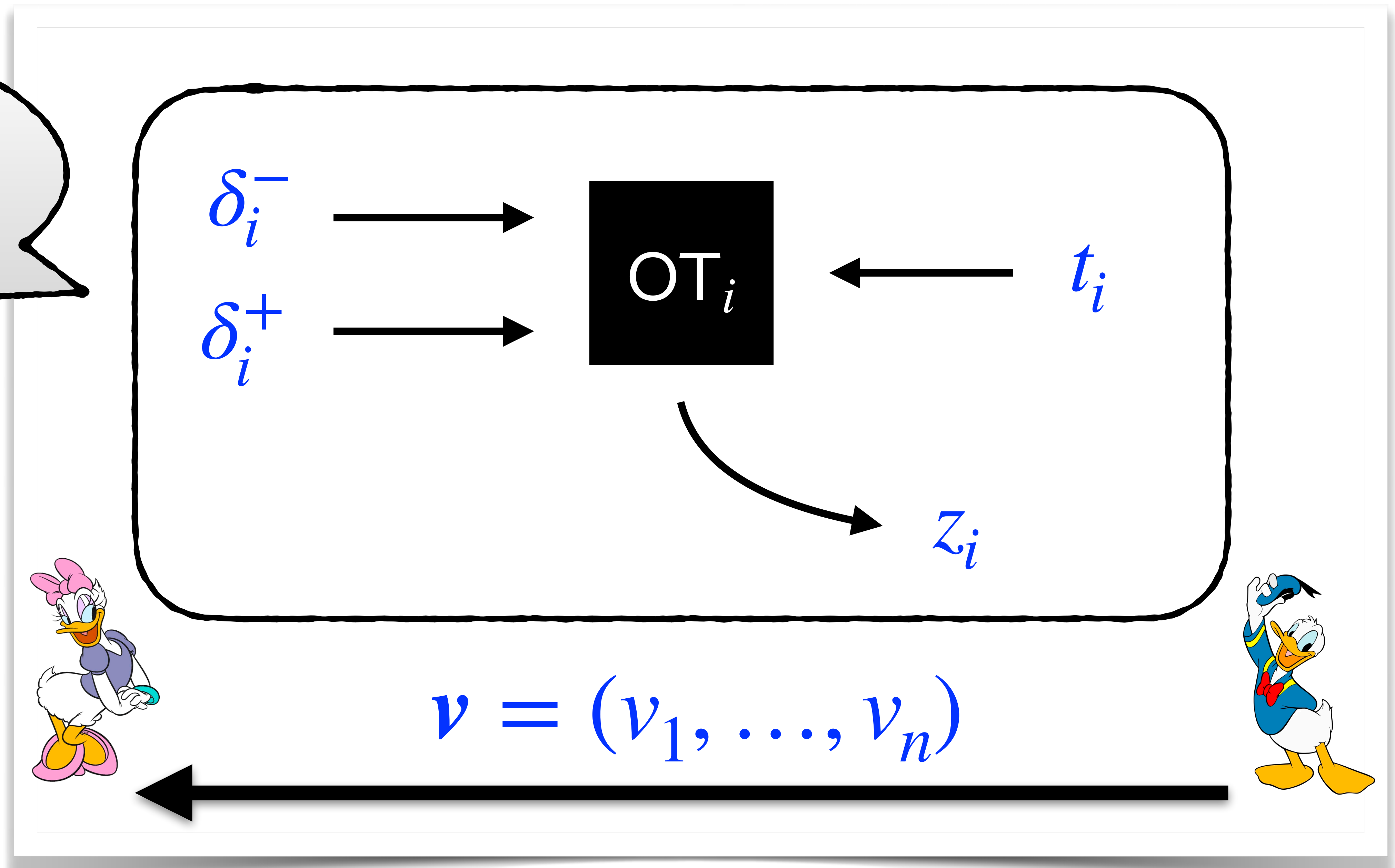
  - If $\boldsymbol{d}$ is close to zero then $\langle \boldsymbol{v} * \boldsymbol{t}, \boldsymbol{d} \rangle$ and $y = \langle \boldsymbol{t}, \boldsymbol{v} \rangle$ are $2^{-\kappa/4}$-close to ind.

$\mathsf{Weight}(\boldsymbol{d})$
captures how much $\mathscr{A}$ deviates.

Security

$\delta_j^+ - \delta_j^- \neq \delta_k^+ - \delta_k^-$
for $j \neq k$

$\mathscr{P}_1$ and $\mathscr{P}_2$ hold inputs x and $y \in \mathbb{Z}_q$ respectively



- Protocol is fully secure against $\mathscr{P}_2^{\mathscr{A}}$
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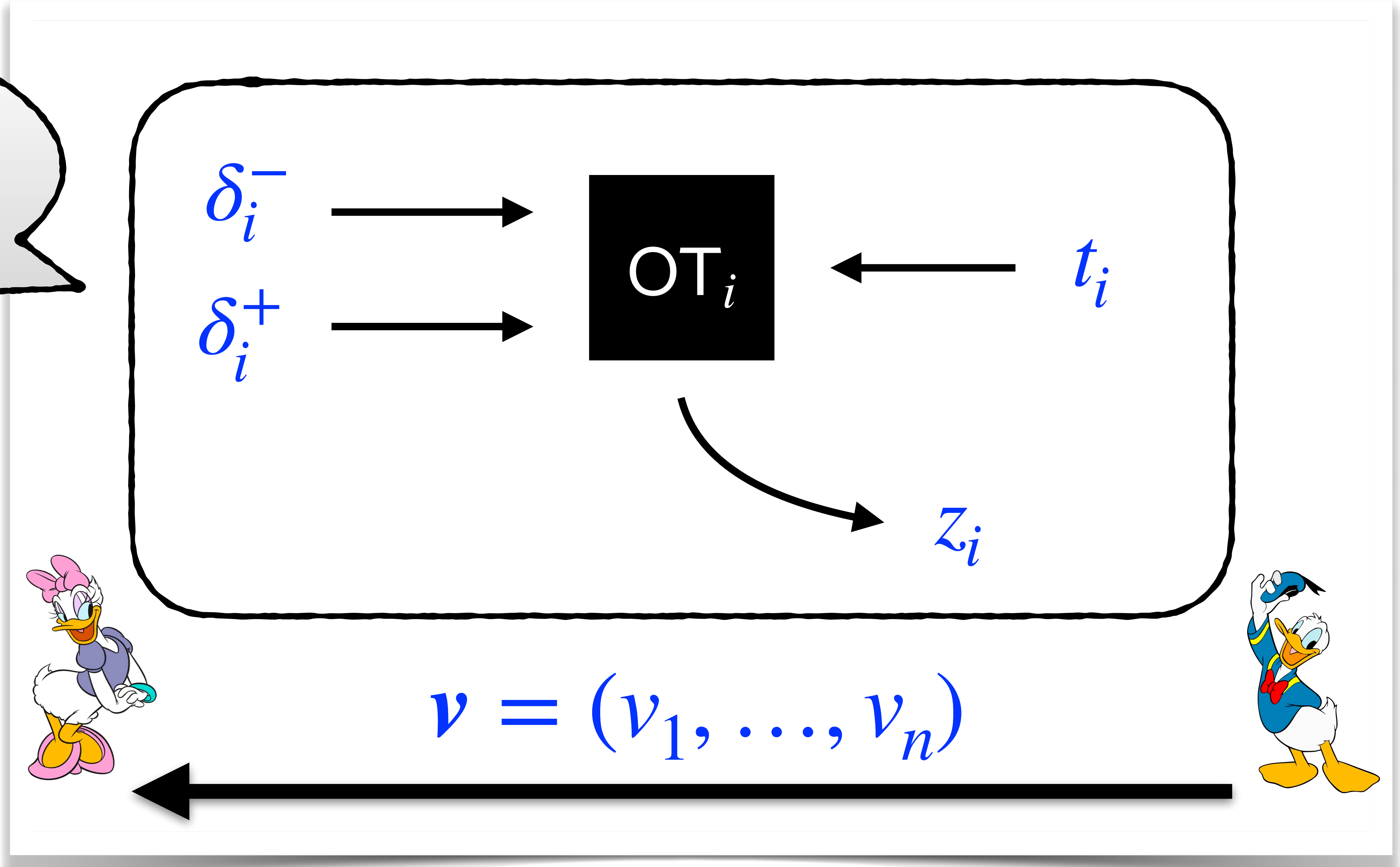

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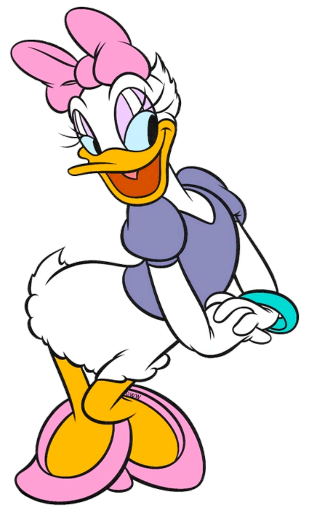
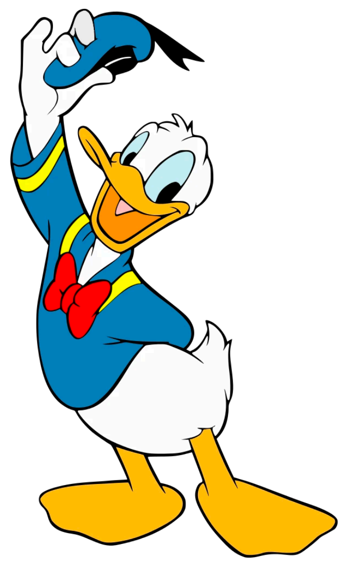
3rd moment concentration inequality

Batching Variant of our Protocol

$\mathscr{P}_1$ and $\mathscr{P}_2$ hold inputs x and $y_1, \ldots, y_\beta \in \mathbb{Z}_q$ respectively





1. Sample $\boldsymbol{\delta} \leftarrow \mathbb{Z}_q^n$ and set $\begin{cases} \delta_i^+ = \delta_i + x \\ \delta_i^- = \delta_i - x \end{cases}$

1. Sample $\boldsymbol{t} \leftarrow \{-1,1\}^n$.

Batching Variant of our Protoc

$n = \beta \cdot \log(q) + \kappa$
calls to OT (in parallel)

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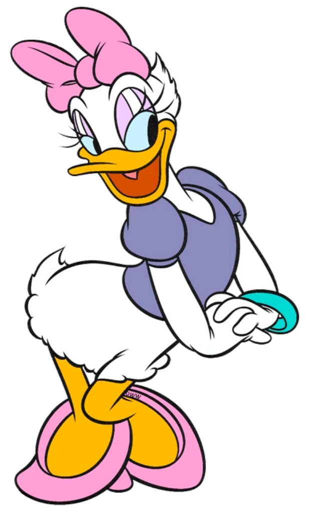

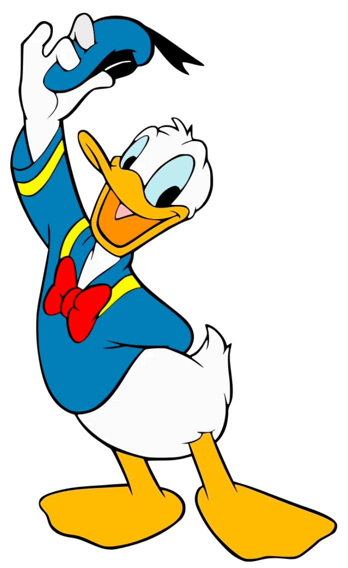

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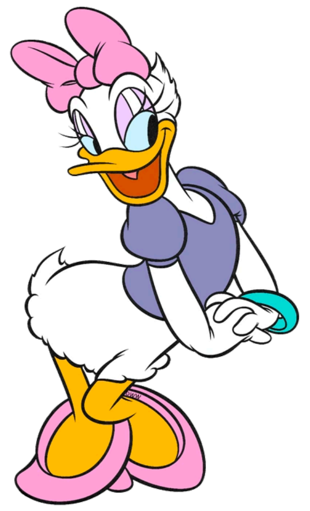

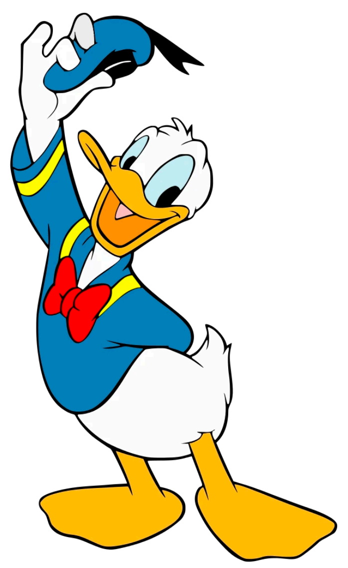

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2. $\forall j$, sample $\boldsymbol{v}_j \leftarrow \mathbb{Z}_q^n$ s.t. $\langle \boldsymbol{t}, \boldsymbol{v}_j \rangle = y_j$.

Batching Variant of our Protoc

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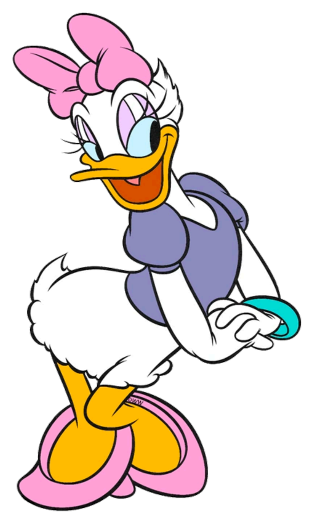
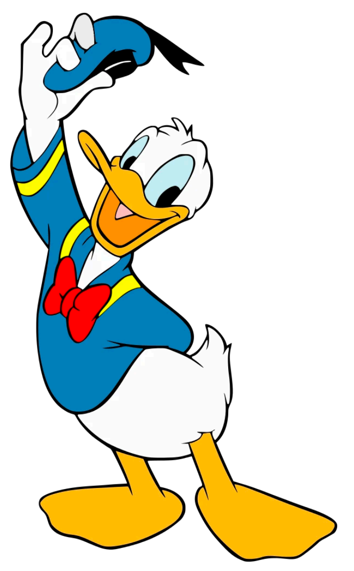


$v_1, v_2, \ldots, v_\beta$



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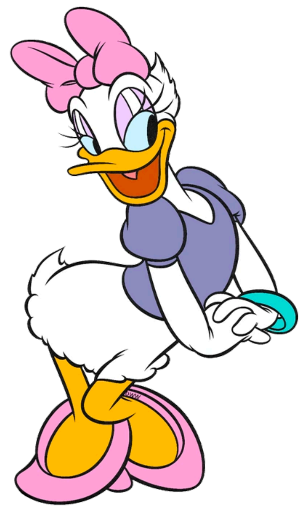
$n = \beta \cdot \log(q) + \kappa$
calls to OT (in parallel)

… OT_{i-1} …

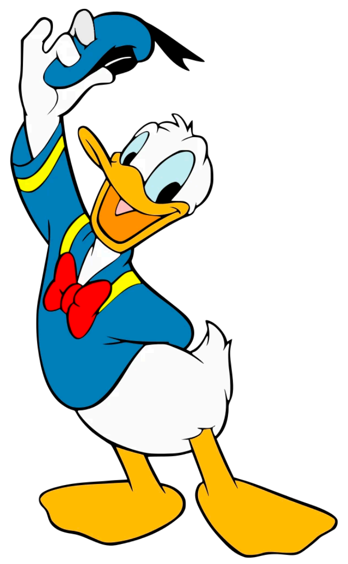
$\delta_i^- \longrightarrow \mathrm{OT}_i \longleftarrow t_i$

$\delta_i^+ \longrightarrow \mathrm{OT}_i \longrightarrow z_i$

OT_{i+1} …



$\boldsymbol{v}_1, \boldsymbol{v}_2, \ldots, \boldsymbol{v}_\beta$ ←



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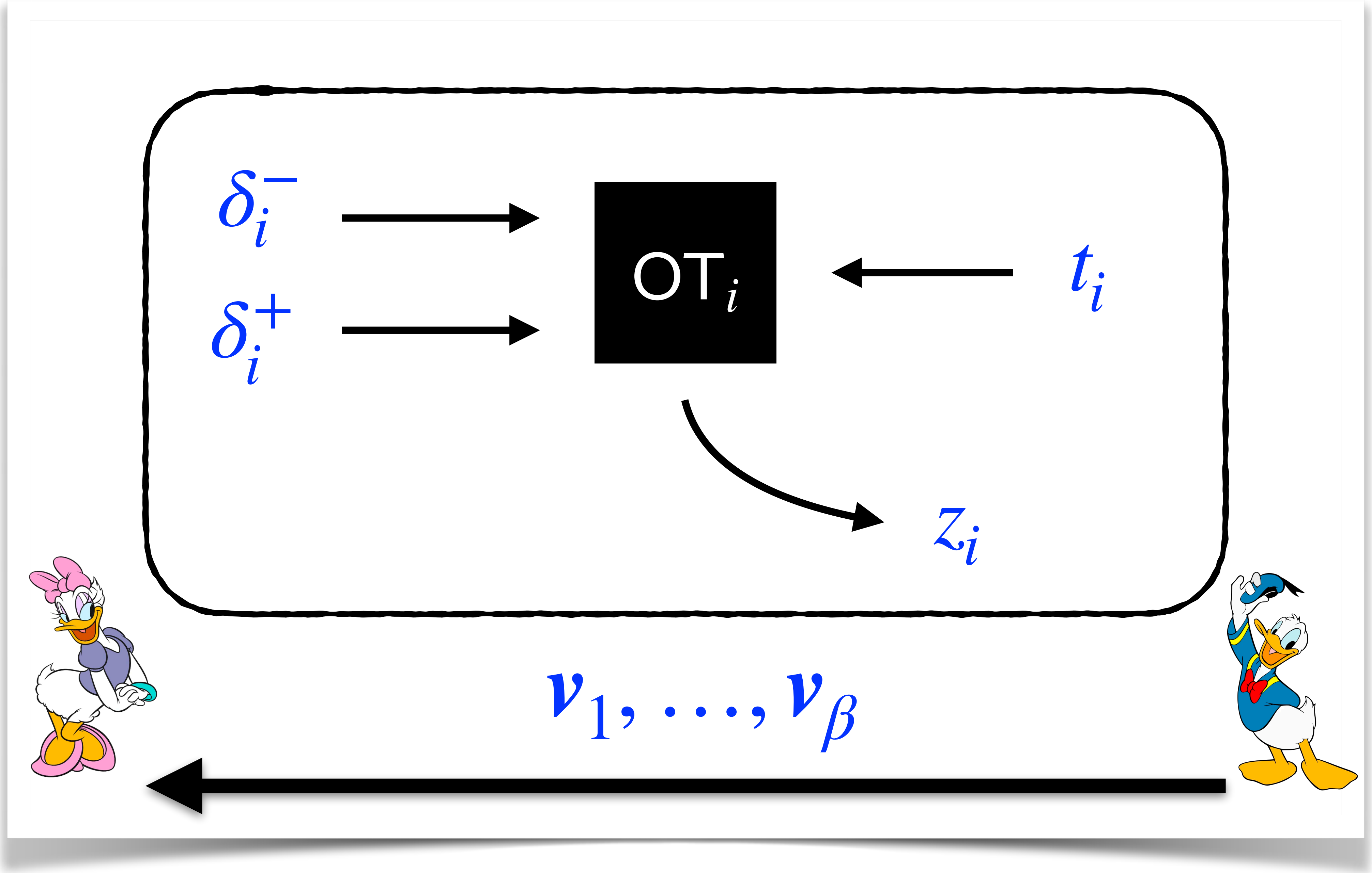
Output $\forall j, \langle -\boldsymbol{\delta}, \boldsymbol{v}_j \rangle$.

1. Sample $\boldsymbol{t} \leftarrow \{-1,1\}^n$.
2. $\forall j$, sample $\boldsymbol{v}_j \leftarrow \mathbb{Z}_q^n$ s.t. $\langle \boldsymbol{t}, \boldsymbol{v}_j \rangle = y_j$.

Output $\forall j, \langle \boldsymbol{z}, \boldsymbol{v}_j \rangle$.

Summary

New OT-based two-party mult. protocol



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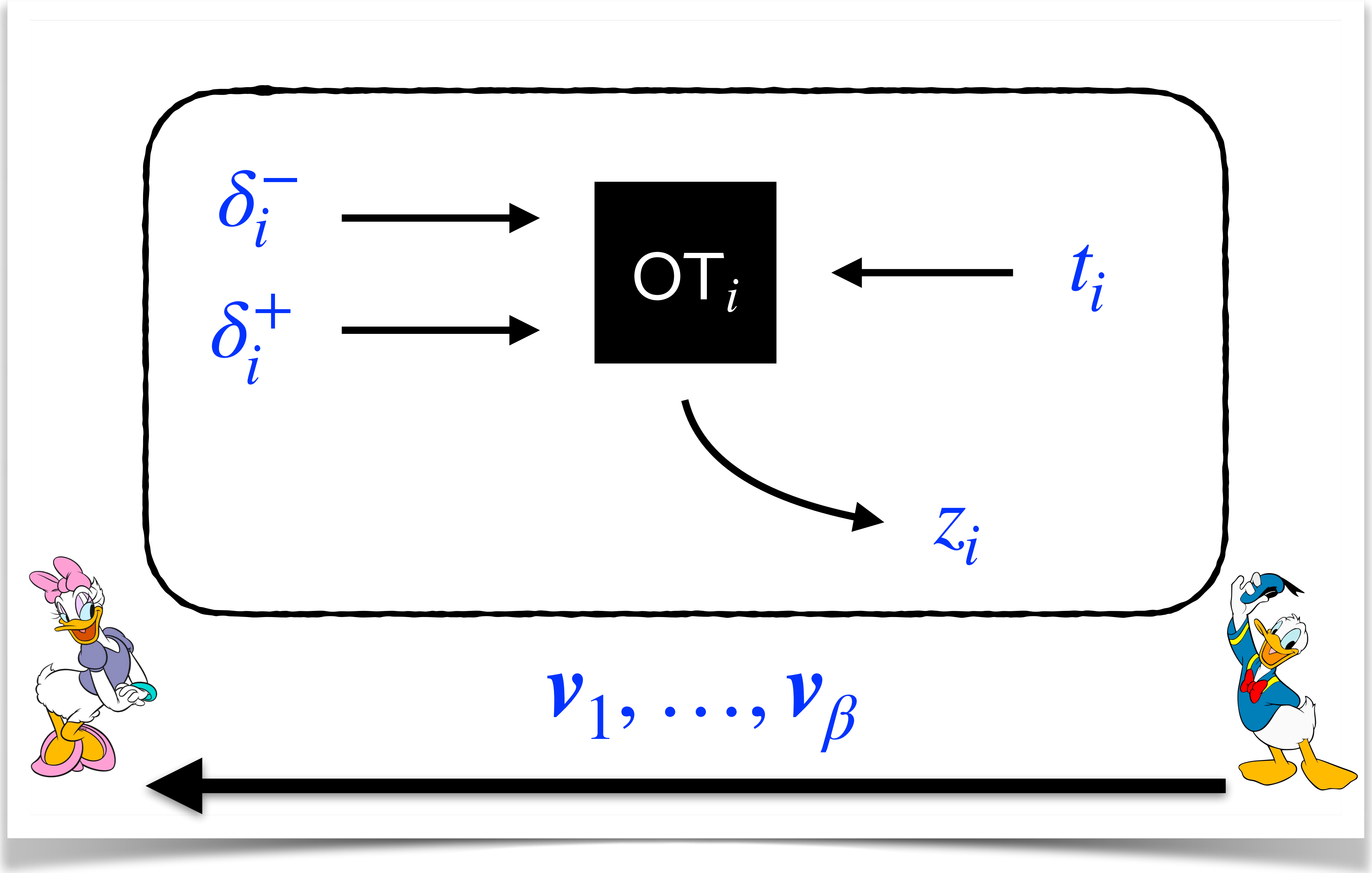

1. Our Protocol
    - "Sufficiently" secure for some applications
    - "Almost" as efficient as SoA semi-honest protocols
    - x2 more efficient than SoA in its fully malicious variant

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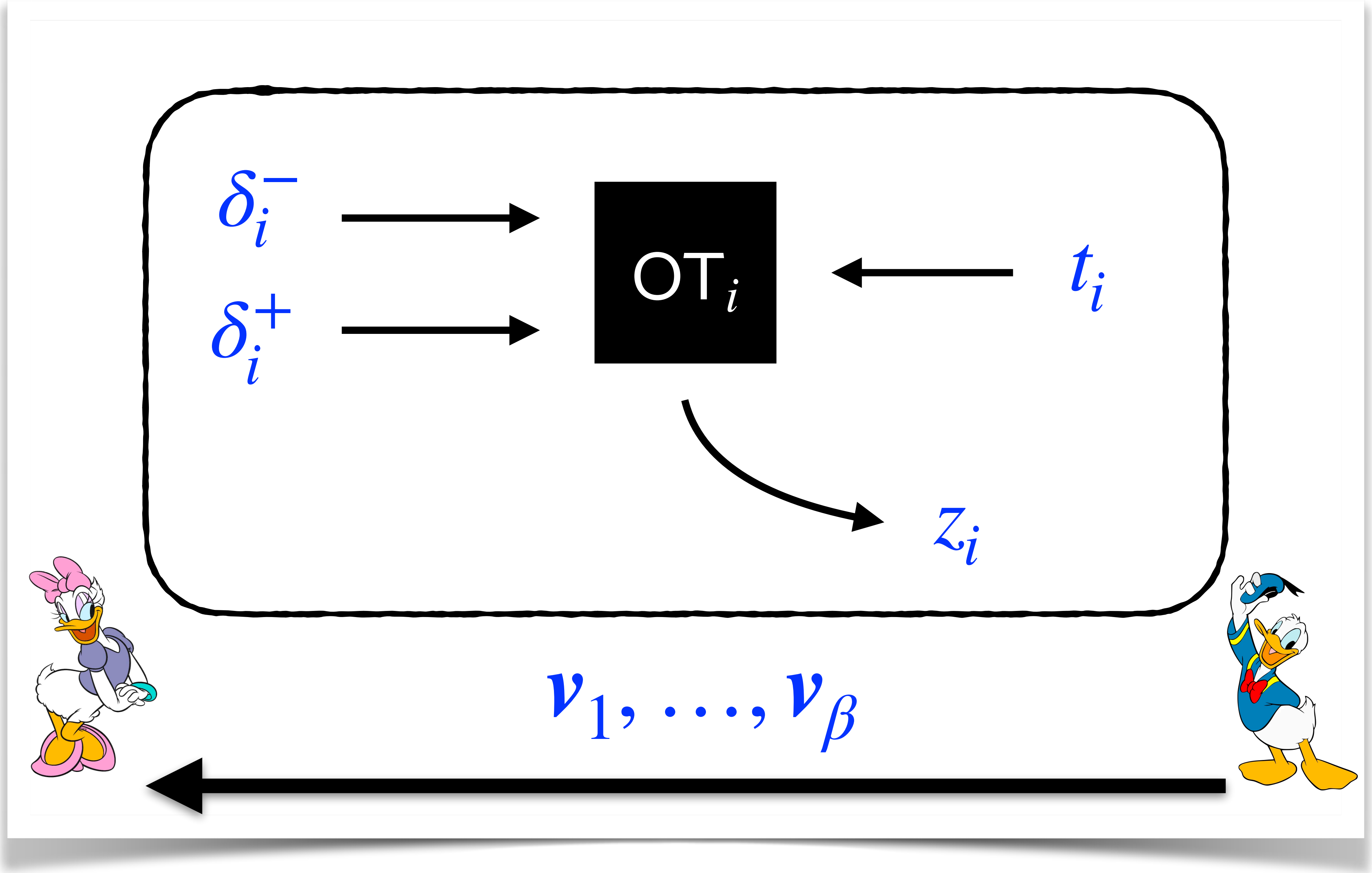

1. Our Protocol
   - "Sufficiently" secure for some applications
   - "Almost" as efficient as SoA semi-honest protocols
   - x2 more efficient than SoA in its fully malicious variant
2. Open Questions
   - Push efficiency further (go beyond Gilboa's $\log(q) \times \log(q)$ barrier)
   - Does IPS09 realize WeakMult?
   - Lower Bounds? *Is OT-mult. inherently wasteful communication-wise?*

Highly Efficient OT-Based Multiplication Protocols

Nikolaos Makriyannis (Fireblocks)

Joint work w/ **Iftach Haitner**, **Samuel Ranellucci** & **Eliad Tsfadia**