Property-Preserving Hash Functions for Hamming Distance from Standard Assumptions

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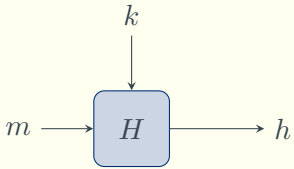
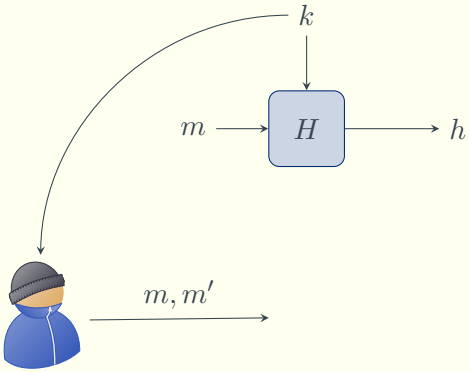
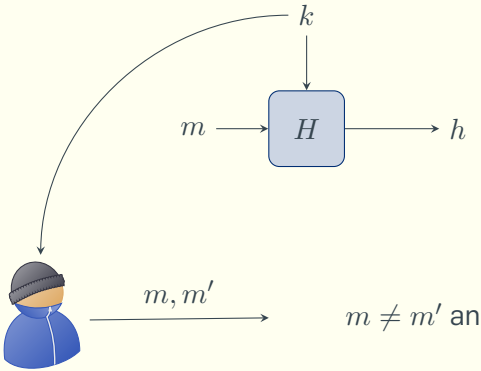
Property Preserving Hash-Functions [BLV19]



Property Preserving Hash-Functions [BLV19]

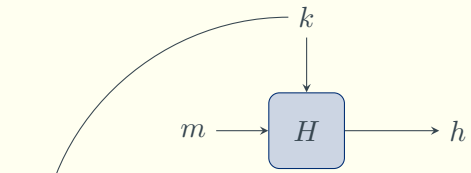


Property Preserving Hash-Functions [BLV19]



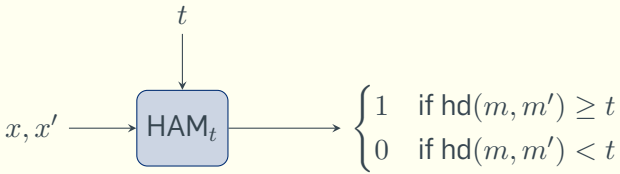
$m \neq m'$ and $H_k(m) = H_k(m')$

Property Preserving Hash-Functions [BLV19]

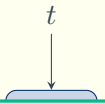


$$P(m, m') \neq \mathsf{Eval}(P, H_k(m), H_k(m'))$$

Hamming Distance Predicate



Hamming Distance Predicate



Observation [FS21]

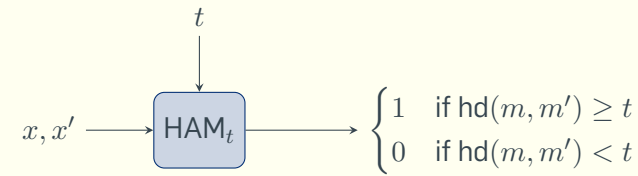
Let $x = x_1 x_2 \dots x_\ell$, $\quad X := \{2i - x_i \mid 1 \le 1 \le \ell\} \subset \{1, \dots, 2\ell\}$

and $x' = x'_1 x'_2 \dots x'_\ell$, $\quad X' := \{2i - x'_i \mid 1 \le 1 \le \ell\} \subset \{1, \dots, 2\ell\}$

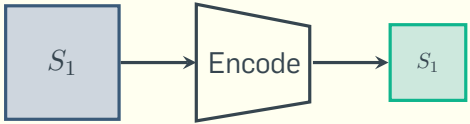
$$\mathsf{HAM}_t(x, x') = 1 \iff |S \triangle S'| \ge 2t$$

where $S \triangle S' = (S \cup S') \setminus (S \cap S')$

Hamming Distance Predicate



Robust Set Encodings



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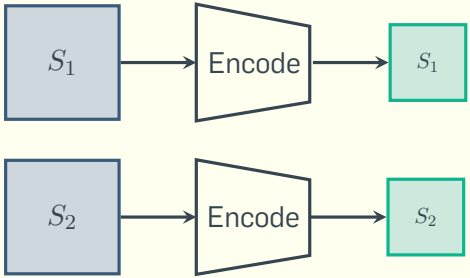

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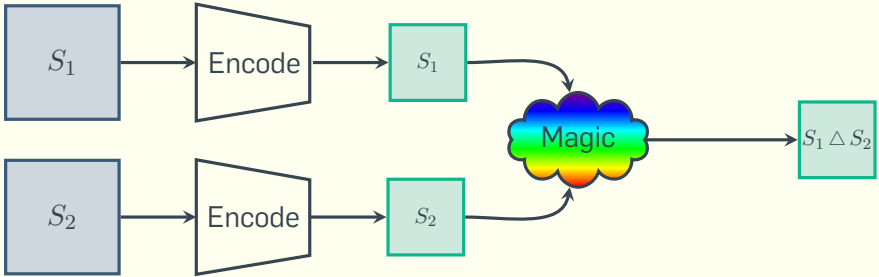

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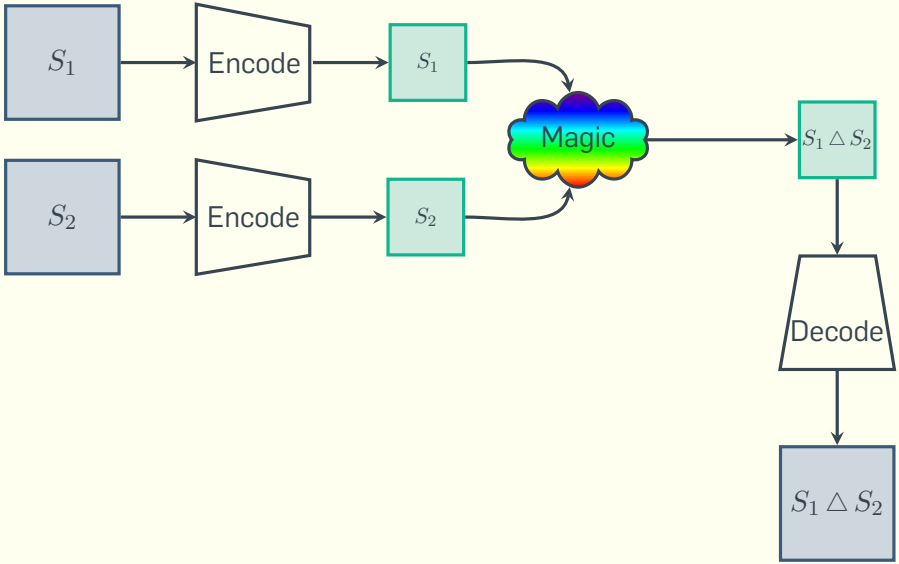

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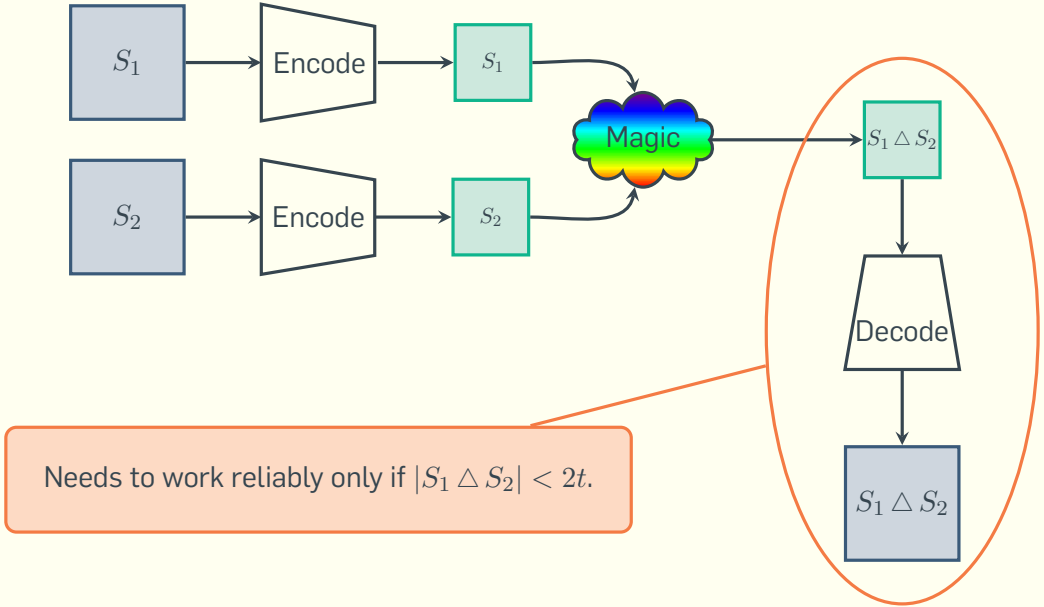


Needs to work reliably only if $|S_1 \triangle S_2| < 2t$.

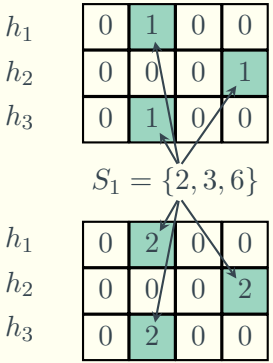
Invertible Bloom Lookup Tables [GM11]

h_1	0	0	0	0
h_2	0	0	0	0
h_3	0	0	0	0

$$S_1 = \{2, 3, 6\}$$

h_1	0	0	0	0
h_2	0	0	0	0
h_3	0	0	0	0

Invertible Bloom Lookup Tables [GM11]



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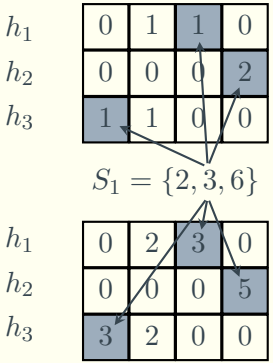

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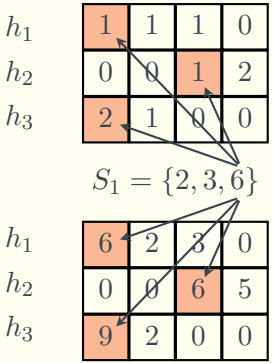


Invertible Bloom Lookup Tables [GM11]

h_1	1	1	1	0
h_2	0	0	1	2
h_3	2	1	0	0

$$S_1 = \{2, 3, 6\}$$

h_1	6	2	3	0
h_2	0	0	6	5
h_3	9	2	0	0

Invertible Bloom Lookup Tables [GM11]

h_1	1	1	1	0
h_2	0	0	1	2
h_3	2	1	0	0

	1	2	0	0
	0	1	1	1
	1	1	1	0

$S_1 = \{2, 3, 6\}$ $S_2 = \{2, 4, 6\}$

h_1	6	2	3	0
h_2	0	0	6	5
h_3	9	2	0	0

	6	6	0	0
	0	4	6	2
	6	2	4	0

Invertible Bloom Lookup Tables [GM11]

h_1	1	1	1	0		1	2	0	0		0	−1	1	0
h_2	0	0	1	2	−	0	1	1	1	=	0	−1	0	1
h_3	2	1	0	0		1	1	1	0		1	0	−1	0

$S_1 = \{2, 3, 6\}$ $S_2 = \{2, 4, 6\}$ $\approx S_1 \triangle S_2 = \{3, 4\}$

h_1	6	2	3	0		6	6	0	0		0	−4	3	0
h_2	0	0	6	5	−	0	4	6	2	=	0	−4	0	3
h_3	9	2	0	0		6	2	4	0		3	0	−4	0

Invertible Bloom Lookup Tables [GM11]

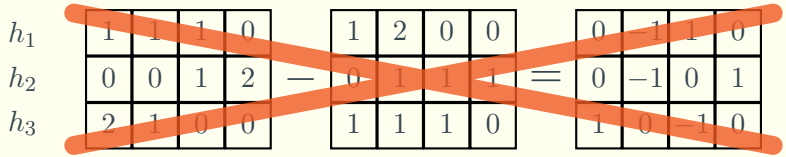
h_1	1	1	1	0		1	2	0	0		0	−1	1	0
h_2	0	0	1	2	−	0	1	1	1	=	0	−1	0	1
h_3	2	1	0	0		1	1	1	0		1	0	−1	0

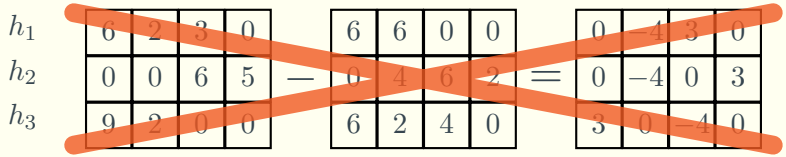
$S_1 = \{2, 3, 6\}$ $S_2 = \{2, 4, 6\}$ $\approx S_1 \triangle S_2 = \{3, 4\}$

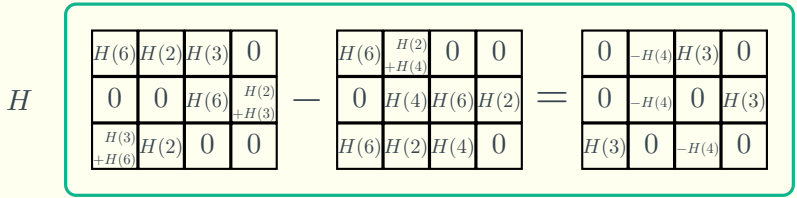
h_1	6	2	3	0		6	6	0	0		0	−4	3	0
h_2	0	0	6	5	−	0	4	6	2	=	0	−4	0	3
h_3	9	2	0	0		6	2	4	0		3	0	−4	0

H	$H(6)$	$H(2)$	$H(3)$	0		$H(6)$	$H(2)+H(4)$	0	0		0	$-H(4)$	$H(3)$	0
	0	0	$H(6)$	$H(2)+H(3)$	−	0	$H(4)$	$H(6)$	$H(2)$	=	0	$-H(4)$	0	$H(3)$
	$H(3)+H(6)$	$H(2)$	0	0		$H(6)$	$H(2)$	$H(4)$	0		$H(3)$	0	$-H(4)$	0

Invertible Bloom Lookup Tables [GM11]







Instantiation

We instantiate the hash function H with Ajtai's hash function [Ajt96] bases on SIS.

Theorem

If the $(n = n(\lambda), 2\ell, q, \sqrt{2\ell + 3}, 2)$-SIS problem is hard then there exists a robust set encoding for universe $[2\ell]$ with output length $\tilde{\mathcal{O}}(t\lambda^2 \log \ell)$.

Corollary

If the $(n = n(\lambda), 2\ell, q, \sqrt{2\ell + 3}, 2)$-SIS problem is hard, there exists a direct access robust PPH for exact Hamming distance for input length ℓ with output length $\tilde{\mathcal{O}}(t\lambda^2 \log \ell)$.

Comparison

	Predicate	Output Length	Assumption
[BLV19]	Gap Hamming	$c \cdot \ell, c < 1$	SSV
[FS21]	Exact Hamming	$\tilde{\mathcal{O}}(t\lambda)$	q-SBDL
This work	Exact Hamming	$\tilde{\mathcal{O}}(t\lambda^2 \log \ell)$	SIS
[Min22]	Exact Hamming	$\tilde{\mathcal{O}}(t^2\lambda^2 \log \ell)$	CRHF

t: threshold, ℓ: input length, λ: security parameter