

# optimal broadcast encryption and CP-ABE from evasive lattice assumptions



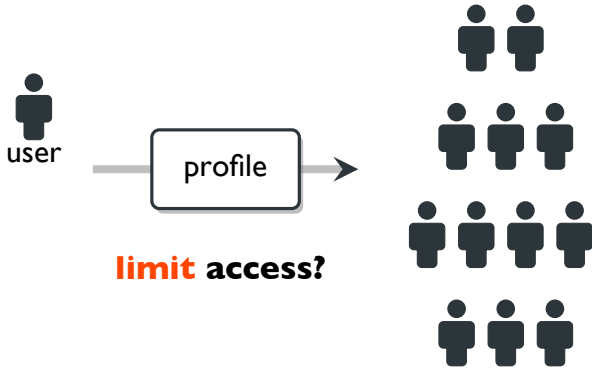
Hoeteck Wee

**NTT Research & ENS**

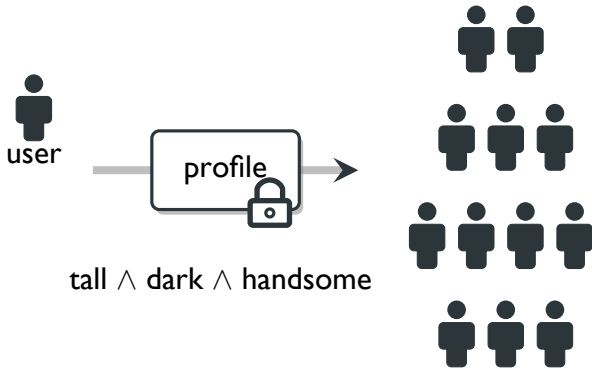
# dating + big data



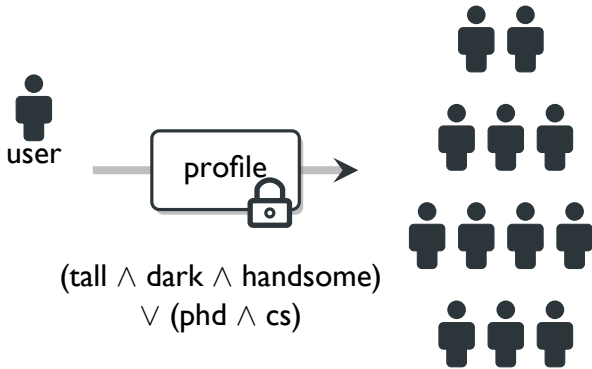
# dating + big data



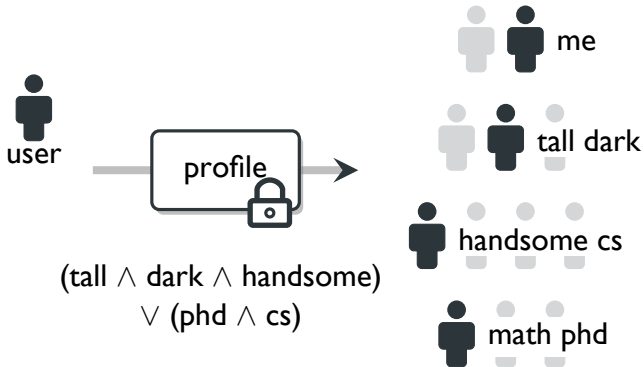
# dating + big data



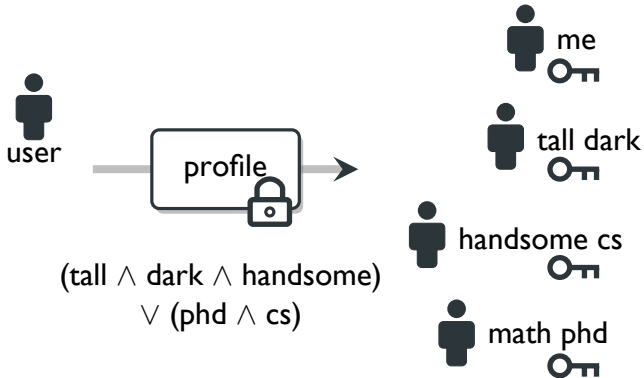
# dating + big data



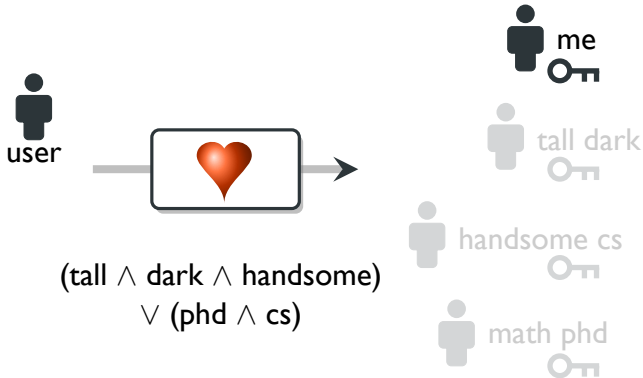
# dating + big data



# dating + big data

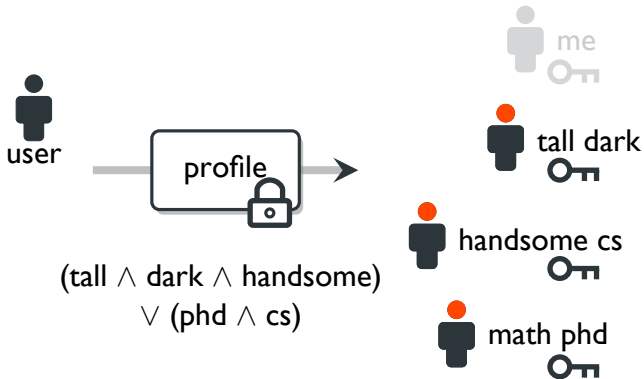


# dating + big data

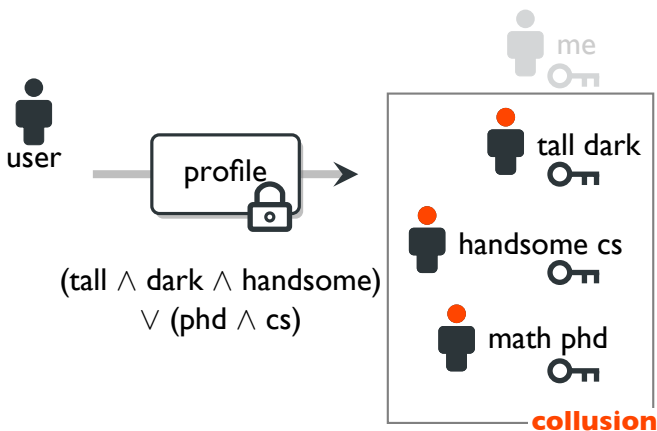




# dating + big data



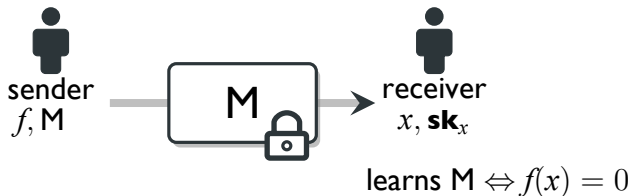
# dating + big data



# attribute-based encryption

ciphertext-policy

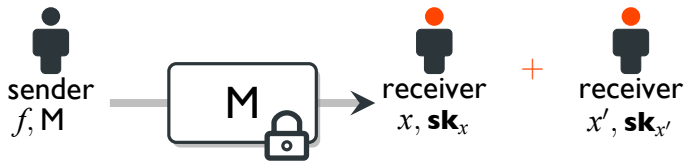
[GPSW06,SW05]



# attribute-based encryption

ciphertext-policy

[GPSW06,SW05]



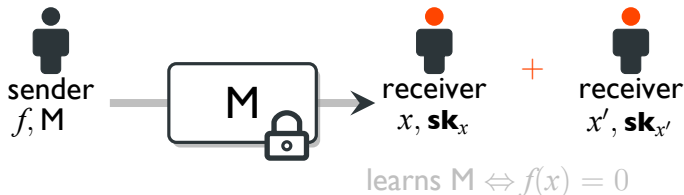
learns  $M \Leftrightarrow f(x) = 0$

**security** against **collusions**

# attribute-based encryption

ciphertext-policy

[GPSW06, SW05]



[GVW13, BGGHNSVV14]

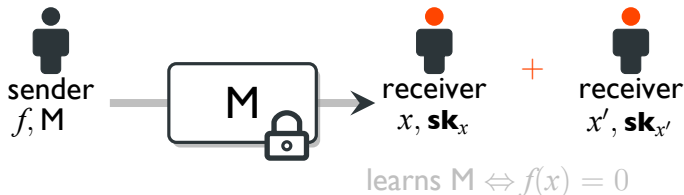
ABE for **circuits** from LWE

$$(\mathbf{A}, \mathbf{s}^\top \mathbf{A}) \approx_c \text{random}$$

# attribute-based encryption

ciphertext-policy

[GPSW06, SW05]



[GVW13, BGGHNSVV14]

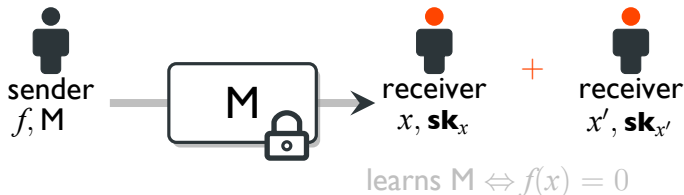
CP-ABE for **circuits** with  $|\mathbf{ct}| \approx \text{size}(f)$

$(\mathbf{A}, \mathbf{s}^\top \mathbf{A}) \approx_c \text{random}$

# attribute-based encryption

ciphertext-policy

[GPSW06,SW05]



[AY20, AWY20, BV22]

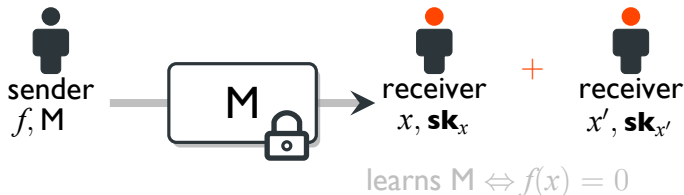
CP-ABE for **circuits** with  $|\mathbf{ct}| = \text{poly}(\text{depth}) \cdot |x|$

from LWE + ...

# attribute-based encryption

ciphertext-policy

[GPSW06, SW05]



[AY20, AWY20, BV22]

CP-ABE for **circuits** with  $|\mathbf{ct}| = \text{poly}(\text{depth}) \cdot |x|$

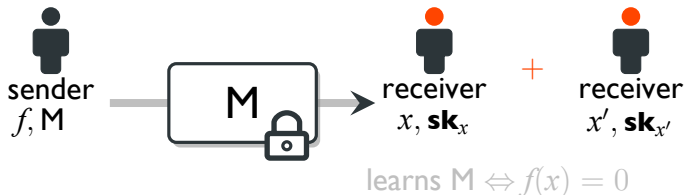
$\Rightarrow$  **optimal** broadcast enc // size  $\text{poly}(\log N)$



# attribute-based encryption

ciphertext-policy

[GPSW06, SW05]



[AY20, AWY20, BV22]

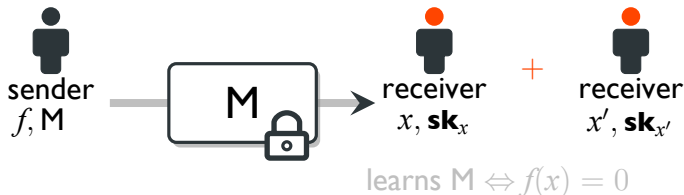
CP-ABE for **circuits** with  $|\mathbf{ct}| = \text{poly}(\text{depth}) \cdot |x|$

from LWE + pairings ( $\text{NC}^1$ )

# attribute-based encryption

ciphertext-policy

[GPSW06, SW05]



[AY20, AWY20, BV22]

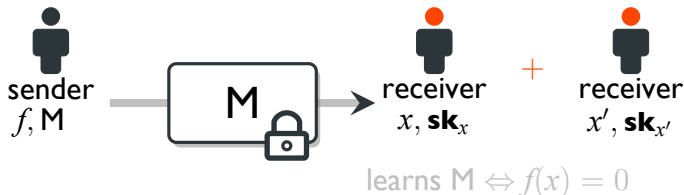
CP-ABE for **circuits** with  $|\mathbf{ct}| = \text{poly}(\text{depth}) \cdot |x|$

from lattice heuristics

# attribute-based encryption

ciphertext-policy

[GPSW06,SW05]



**our results** [W22]

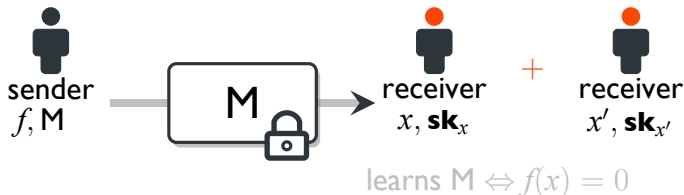
CP-ABE for **circuits** with  $|\mathbf{ct}| = \text{poly}(\text{depth})$

from **new** lattice assumptions

# attribute-based encryption

ciphertext-policy

[GPSW06,SW05]



**our results** [W22]

**optimal** broadcast enc // size  $\text{poly}(\log N)$

from **new** lattice assumptions

# warm-up

**fact.**  $[\mathbf{A} - \mathbf{x}^\top \otimes \mathbf{G}] \cdot \mathbf{H}_{f,\mathbf{x}} = \mathbf{A}_f - f(\mathbf{x})\mathbf{G}$  [BGGHNSVV14]

# warm-up

**fact.**  $[\mathbf{A} - \mathbf{x}^\top \otimes \mathbf{G}] \cdot \mathbf{H}_{f,\mathbf{x}} = \mathbf{A}_f - f(\mathbf{x})\mathbf{G}$  [BGGHNSVV14]

$$\text{mpk} = \mathbf{A}$$

$$\begin{array}{c} \text{kem} \\ \underbrace{\mathbf{s}^\top \mathbf{A}_f} \end{array}$$

$$\begin{array}{c} \text{ct}\cdot\text{sk} \\ \underbrace{\mathbf{s}^\top (\mathbf{A} - \mathbf{x}^\top \otimes \mathbf{G})} \end{array}$$

# warm-up

**fact.**  $[\mathbf{A} - \mathbf{x}^\top \otimes \mathbf{G}] \cdot \mathbf{H}_{f,\mathbf{x}} = \mathbf{A}_f - f(\mathbf{x})\mathbf{G}$  [BGGHNSVV14]

**mpk** =  $\mathbf{A}, \mathbf{B}$

$$\underbrace{\mathbf{s}^\top \mathbf{A}_f}_{\text{kem}} \cdot \underbrace{\mathbf{s}^\top \mathbf{B}}_{\text{ct}} \cdot \underbrace{\mathbf{B}^{-1}(\mathbf{A} - \mathbf{x}^\top \otimes \mathbf{G})}_{\text{sk}}$$

# warm-up

**fact.**  $[\mathbf{A} - \mathbf{x}^\top \otimes \mathbf{G}] \cdot \mathbf{H}_{f,\mathbf{x}} = \mathbf{A}_f - f(\mathbf{x})\mathbf{G}$  [BGGHNSVV14]

**mpk** =  $\mathbf{A}, \mathbf{B}$

$$\underbrace{\mathbf{s}^\top \mathbf{A}_f}_{\text{kem}} \approx \underbrace{\mathbf{s}^\top \mathbf{B}}_{\text{ct}} \cdot \underbrace{\mathbf{B}^{-1}(\mathbf{A} - \mathbf{x}^\top \otimes \mathbf{G})}_{\text{sk}} \cdot \mathbf{H}_{f,\mathbf{x}} \quad \text{if } f(\mathbf{x}) = 0$$



# warm-up

**fact.**  $[\mathbf{A} - \mathbf{x}^\top \otimes \mathbf{G}] \cdot \mathbf{H}_{f,\mathbf{x}} = \mathbf{A}_f - f(\mathbf{x})\mathbf{G}$  [BGGHNSVV14]

**mpk** =  $\mathbf{A}, \mathbf{B}$

$$\underbrace{\mathbf{s}^\top \mathbf{A}_f}_{\text{kem}} \quad \underbrace{\mathbf{s}^\top \mathbf{B}}_{\text{ct}} \quad \underbrace{\mathbf{B}^{-1}(\mathbf{A} - \mathbf{x}^\top \otimes \mathbf{G})}_{\text{sk}}$$

**secure** for **one** key [AY20, BV22, BK20]

# warm-up

**fact.**  $[\mathbf{A} - \mathbf{x}^\top \otimes \mathbf{G}] \cdot \mathbf{H}_{f,\mathbf{x}} = \mathbf{A}_f - f(\mathbf{x})\mathbf{G}$  [BGGHNSVV14]

mpk =  $\mathbf{A}, \mathbf{B}$

$$\underbrace{\mathbf{s}^\top \mathbf{A}_f}_{\text{kem}} \quad \underbrace{\mathbf{s}^\top \mathbf{B}}_{\text{ct}} \quad \underbrace{\mathbf{B}^{-1}(\mathbf{A} - \mathbf{x}^\top \otimes \mathbf{G})}_{\text{sk}}$$

**insecure** for **two** keys:

$$\text{ct} \cdot \text{sk}_i = \boxed{\mathbf{s}^\top} (\mathbf{A} - \mathbf{x}_i^\top \otimes \mathbf{G}), i=1,2 \mapsto \mathbf{s}^\top \mathbf{G}$$

# warm-up

**fact.**  $[A - \mathbf{x}^\top \otimes G] \cdot H_{f,\mathbf{x}} = A_f - f(\mathbf{x})G$  [BGGHNSVV14]

mpk = A, B

$$\underbrace{s^\top A_f}_{\text{kem}} \quad \underbrace{s^\top B}_{\text{ct}} \quad \underbrace{B^{-1}(A - \mathbf{x}^\top \otimes G)}_{\text{sk}}$$

**insecure** for **two** keys:

$$\text{ct} \cdot \text{sk}_i = \boxed{s_i^\top} \underbrace{(A - \mathbf{x}_i^\top \otimes G)}_{\text{sk}_i}, i=1,2 \mapsto \underbrace{s^\top G}_{\text{secret}}$$

**fix.** [AY20, BV22]  $S^\top \mapsto S_i^\top$

# warm-up

**fact.**  $[A - \mathbf{x}^\top \otimes G] \cdot H_{f,\mathbf{x}} = A_f - f(\mathbf{x})G$  [BGGHNSVV14]

$$\text{mpk} = A, B$$

$$\underbrace{\mathbf{s}^\top A_f}_{\text{kem}}$$

$$\underbrace{\mathbf{s}^\top B}_{\text{ct}}$$

$$\underbrace{B^{-1}(A - \mathbf{x}^\top \otimes G)}_{\text{sk}}$$

**insecure** for **two** keys:

$$\text{ct} \cdot \text{sk}_i = \boxed{\mathbf{s}_i^\top} \underbrace{(A - \mathbf{x}_i^\top \otimes G)}_{\text{sk}}, i=1,2 \mapsto \underbrace{\mathbf{s}^\top G}_{\text{ct}}$$

**fix.** [AY20, BV22]  $\mathbf{s}^\top \mapsto \mathbf{s}_i^\top \mapsto \underbrace{\mathbf{r}_i^\top}_{\text{sk}} \cdot \underbrace{\mathbf{S}}_{\text{ct}}$

# candidate cp-abe for circuits

$$\text{want } \mathbf{ct} \cdot \mathbf{sk} = \overbrace{\mathbf{r}^\top}^{\mathbf{sk}} \cdot \overbrace{\mathbf{S}}^{\mathbf{ct}} \cdot (\mathbf{A} - \overbrace{\mathbf{x}^\top}^{\mathbf{sk}} \otimes \mathbf{G})$$

# candidate cp-abe for circuits

$$\text{want } \mathbf{ct} \cdot \mathbf{sk} = \overbrace{\mathbf{r}^\top}^{\text{sk}} \cdot \overbrace{\mathbf{S}}^{\text{ct}} \cdot (\mathbf{A} - \overbrace{\mathbf{x}^\top}^{\text{sk}} \otimes \mathbf{G})$$

[BV22]  $\mathbf{r}$  IBE key for  $\mathbf{x}$

# candidate cp-abe for circuits

$$\begin{aligned} \text{want } \mathbf{ct} \cdot \mathbf{sk} &= \overbrace{\mathbf{r}^\top}^{\mathbf{sk}} \cdot \overbrace{\mathbf{S}}^{\mathbf{ct}} \cdot (\mathbf{A} - \overbrace{\mathbf{x}^\top}^{\mathbf{sk}} \otimes \mathbf{G}) \\ &= \text{flat}(\mathbf{S})((\mathbf{A} - \mathbf{x}^\top \otimes \mathbf{G}) \otimes \mathbf{r}) \end{aligned}$$



# candidate cp-abe for circuits

$$\begin{aligned} \text{want } \mathbf{ct} \cdot \mathbf{sk} &= \overbrace{\mathbf{r}^\top}^{\text{sk}} \cdot \overbrace{\mathbf{S}}^{\text{ct}} \cdot (\mathbf{A} - \overbrace{\mathbf{x}^\top}^{\text{sk}} \otimes \mathbf{G}) \\ &\approx \underbrace{\text{flat}(\mathbf{S})\mathbf{B}}_{\text{ct}} \cdot \overbrace{\mathbf{B}^{-1}((\mathbf{A} - \mathbf{x}^\top \otimes \mathbf{G}) \otimes \mathbf{r})}^{\text{sk}} \end{aligned}$$



# candidate cp-abe for circuits

$$\begin{aligned} \text{want } \mathbf{ct} \cdot \mathbf{sk} &= \overbrace{\mathbf{r}^\top}^{\mathbf{sk}} \cdot \overbrace{\mathbf{S}}^{\mathbf{ct}} \cdot (\mathbf{A} - \overbrace{\mathbf{x}^\top}^{\mathbf{sk}} \otimes \mathbf{G}) \\ &\approx \underbrace{\text{flat}(\mathbf{S})\mathbf{B}}_{\text{ct}} \cdot \overbrace{\mathbf{B}^{-1}((\mathbf{A} - \mathbf{x}^\top \otimes \mathbf{G}) \otimes \mathbf{r})}^{\mathbf{sk}} \end{aligned}$$

**candidate** CP-ABE for circuits

$$\mathbf{ct} = \underbrace{\text{flat}(\mathbf{S})\mathbf{B}}_{\text{ct}}, \quad \mathbf{kem} = \underbrace{\mathbf{S}\mathbf{A}_f}_{\text{kem}}$$

$$\mathbf{sk} = \mathbf{B}^{-1}((\mathbf{A} - \mathbf{x}^\top \otimes \mathbf{G}) \otimes \mathbf{r}), \mathbf{r} \quad // \mathbf{r} \text{ Gaussian}$$

# candidate cp-abe for circuits

$$\begin{aligned} \text{want } \mathbf{ct} \cdot \mathbf{sk} &= \overbrace{\mathbf{r}^\top}^{\mathbf{sk}} \cdot \overbrace{\mathbf{S}}^{\mathbf{ct}} \cdot (\mathbf{A} - \overbrace{\mathbf{x}^\top}^{\mathbf{sk}} \otimes \mathbf{G}) \\ &\approx \underbrace{\text{flat}(\mathbf{S})\mathbf{B}}_{\text{ct}} \cdot \overbrace{\mathbf{B}^{-1}((\mathbf{A} - \mathbf{x}^\top \otimes \mathbf{G}) \otimes \mathbf{r})}^{\mathbf{sk}} \end{aligned}$$

**candidate** CP-ABE for circuits      *proof?*

$$\mathbf{ct} = \underbrace{\text{flat}(\mathbf{S})\mathbf{B}}_{\text{ct}}, \quad \mathbf{kem} = \underbrace{\mathbf{S}\mathbf{A}_f}_{\text{kem}}$$

$$\mathbf{sk} = \mathbf{B}^{-1}((\mathbf{A} - \mathbf{x}^\top \otimes \mathbf{G}) \otimes \mathbf{r}), \quad \mathbf{r} \quad // \mathbf{r} \text{ Gaussian}$$

# candidate cp-abe for circuits

$$\begin{aligned} \text{want } \mathbf{ct} \cdot \mathbf{sk} &= \overbrace{\mathbf{r}_i^\top \cdot \mathbf{S} \cdot (\mathbf{A} - \mathbf{x}_i^\top \otimes \mathbf{G})}^{\mathbf{ct} \cdot \mathbf{sk}_i} \\ &\approx \underbrace{\text{flat}(\mathbf{S})\mathbf{B}}_{\text{ct}} \cdot \underbrace{\mathbf{B}^{-1}((\mathbf{A} - \mathbf{x}^\top \otimes \mathbf{G}) \otimes \mathbf{r})}_{\text{sk}} \end{aligned}$$

**candidate CP-ABE**    **step 1.**  $\mathbf{ct} \cdot \mathbf{sk}_i \approx_c \text{random}$

$$\mathbf{ct} = \underbrace{\text{flat}(\mathbf{S})\mathbf{B}}_{\text{ct}}, \quad \mathbf{kem} = \underbrace{\mathbf{S}\mathbf{A}_f}_{\text{kem}}$$

$$\mathbf{sk} = \mathbf{B}^{-1}((\mathbf{A} - \mathbf{x}^\top \otimes \mathbf{G}) \otimes \mathbf{r}), \mathbf{r} \quad // \mathbf{r} \text{ Gaussian}$$

# proof (attempt)

**fact 1.** by LWE

$$\underbrace{\mathbf{r}_i^\top \mathbf{S}} \approx_c \mathbf{s}_i^\top$$

# proof (attempt)

**fact 1.** by LWE, if  $\mathbf{A}$  has low-norm:

$$\underbrace{\mathbf{r}_i^\top \mathbf{S}(\mathbf{A} - \mathbf{x}_i^\top \otimes \mathbf{I})}_{\text{wavy line}} \approx_c \underbrace{\mathbf{s}_i^\top (\mathbf{A} - \mathbf{x}_i^\top \otimes \mathbf{I})}_{\text{wavy line}}$$

# proof (attempt)

**fact 1.** by LWE, if  $\mathbf{A}$  has low-norm:

$$\underbrace{\mathbf{r}_i^\top \mathbf{S}(\mathbf{A} - \mathbf{x}_i^\top \otimes \mathbf{I})}_{\text{wavy line}} \approx_c \underbrace{\mathbf{s}_i^\top (\mathbf{A} - \mathbf{x}_i^\top \otimes \mathbf{I})}_{\text{wavy line}} \approx_c \text{random}$$

# proof (attempt)

**fact 1.** by LWE, if  $\mathbf{A}$  has low-norm:

$$\underbrace{\mathbf{r}_i^\top \mathbf{S}(\mathbf{A} - \mathbf{x}_i^\top \otimes \mathbf{I})}_{\text{LWE}} \approx_c \underbrace{\mathbf{s}_i^\top (\mathbf{A} - \mathbf{x}_i^\top \otimes \mathbf{I})}_{\text{LWE}} \approx_c \text{random}$$

**fact 2.**  $[\mathbf{A} - \mathbf{x}^\top \otimes \mathbf{I}] \cdot \mathbf{H}_{f,\mathbf{x}} = \mathbf{A}_f - f(\mathbf{x})\mathbf{I}$

$$|\mathbf{H}_{f,\mathbf{x}}| = \lambda^{O(2^{\text{depth}})} \cdot \text{size}, |\mathbf{ct}|, |\mathbf{sk}| \approx 2^{\text{depth}}$$

# proof (attempt)

**fact 1.** by LWE, if  $\mathbf{A}$  has low-norm:

$$\underbrace{\mathbf{r}_i^\top \mathbf{S}(\mathbf{A} - \mathbf{x}_i^\top \otimes \mathbf{I})}_{\text{LWE}} \approx_c \underbrace{\mathbf{s}_i^\top (\mathbf{A} - \mathbf{x}_i^\top \otimes \mathbf{I})}_{\text{LWE}} \approx_c \text{random}$$

**fact 2.**  $[\mathbf{A} - \mathbf{x}^\top \otimes \mathbf{I}] \cdot \mathbf{H}_{f,\mathbf{x}} = \mathbf{A}_f - f(\mathbf{x})\mathbf{I}$

$$|\mathbf{H}_{f,\mathbf{x}}| = \lambda^{O(2^{\text{depth}})} \cdot \text{size}, |\mathbf{ct}|, |\mathbf{sk}| \approx 2^{\text{depth}}$$

$$\times \text{circuits} : 2^{\text{depth}} \approx \text{size}$$

$$\checkmark \text{ broadcast} : \text{depth} = O(\log \log N), \text{size} = O(N)$$



# proof (attempt)

**fact 1.** by LWE, if  $\mathbf{A}$  has low-norm:

$$\underbrace{\mathbf{r}_i^\top \mathbf{S} (\mathbf{A} - \mathbf{x}_i^\top \otimes \mathbf{I})}_{\mathbf{ct} \cdot \mathbf{sk}_i} \approx_c \underbrace{\mathbf{s}_i^\top (\mathbf{A} - \mathbf{x}_i^\top \otimes \mathbf{I})}_{\mathbf{sk}_i} \approx_c \text{random}$$

$$\underbrace{\text{flat}(\mathbf{S}) \mathbf{B}}_{\mathbf{ct}}, \quad \underbrace{\mathbf{B}^{-1} ((\mathbf{A} - \mathbf{x}_i^\top \otimes \mathbf{I}) \otimes \mathbf{r}_i)}_{\mathbf{sk}_i}$$

# proof (attempt)

**fact 1.** by LWE, if  $\mathbf{A}$  has low-norm:

$$\underbrace{\mathbf{r}_i^\top \mathbf{S} (\mathbf{A} - \mathbf{x}_i^\top \otimes \mathbf{I})}_{\mathbf{ct} \cdot \mathbf{sk}_i} \approx_c \underbrace{\mathbf{s}_i^\top (\mathbf{A} - \mathbf{x}_i^\top \otimes \mathbf{I})}_{\text{random}} \approx_c \text{random}$$

**step 2.**  $(\mathbf{ct}, \mathbf{sk}_i) \approx_c (\text{random}, \mathbf{sk}_i)$

$$\underbrace{\text{flat}(\mathbf{S})\mathbf{B}}_{\mathbf{ct}}, \underbrace{\mathbf{B}^{-1}((\mathbf{A} - \mathbf{x}_i^\top \otimes \mathbf{I}) \otimes \mathbf{r}_i)}_{\mathbf{sk}_i}$$

# proof (attempt)

**fact 1.** by LWE, if  $\mathbf{A}$  has low-norm:

$$\underbrace{\mathbf{r}_i^\top \mathbf{S} (\mathbf{A} - \mathbf{x}_i^\top \otimes \mathbf{I})}_{\mathbf{ct} \cdot \mathbf{sk}_i} \approx_c \underbrace{\mathbf{s}_i^\top (\mathbf{A} - \mathbf{x}_i^\top \otimes \mathbf{I})}_{\text{random}} \approx_c \text{random}$$

**step 2.**  $(\mathbf{ct}, \mathbf{sk}_i) \approx_c (\text{random}, \mathbf{sk}_i)$

$$\underbrace{\mathbf{ct}}_{\text{flat}(\mathbf{S})\mathbf{B}}, \underbrace{\mathbf{sk}_i}_{\mathbf{B}^{-1}((\mathbf{A} - \mathbf{x}_i^\top \otimes \mathbf{I}) \otimes \mathbf{r}_i)}$$

**intuition.**  $\mathbf{s}^\top \mathbf{B}, \mathbf{B}^{-1}(\mathbf{P})$  only “leaks”  $\mathbf{s}^\top \mathbf{P}$

# **evasive** LWE [W22, T22]

# evasive LWE [W22, T22]

assumption. fix distribution  $\mathbf{P}$

if  $s^\top [\mathbf{B} \mid \mathbf{P}] \approx_c \text{random}$

then  $s^\top [\mathbf{B}] \approx_c \text{random given } \mathbf{B}^{-1}(\mathbf{P})$

# evasive LWE [W22, T22]

assumption. fix distribution  $\mathbf{P}$

if  $\underbrace{s^\top[\mathbf{B} \mid \mathbf{P}]} \approx_c$  random given  $\mathbf{B}, \mathbf{P}$

then  $\underbrace{s^\top[\mathbf{B}]} \approx_c$  random given  $\mathbf{B}^{-1}(\mathbf{P}), \mathbf{B}$

# evasive LWE [W22, T22]

assumption. fix distribution  $\mathbf{P}$

if  $\mathbf{s}^\top [\mathbf{B} \mid \mathbf{P}] \approx_c$  random given  $\mathbf{B}, \mathbf{P}$

then  $\mathbf{s}^\top [\mathbf{B}] \approx_c$  random given  $\mathbf{B}^{-1}(\mathbf{P}), \mathbf{B}$

**examples.**  $\mathbf{P}$  uniform (both true via LWE)

# evasive LWE [W22, T22]

assumption. fix distribution  $\mathbf{P}$

if  $s^\top [\mathbf{B} \mid \mathbf{P}] \approx_c$  random given  $\mathbf{B}, \mathbf{P}$

then  $s^\top [\mathbf{B}] \approx_c$  random given  $\mathbf{B}^{-1}(\mathbf{P}), \mathbf{B}$

**examples.**  $\mathbf{P} = \mathbf{G}$  (both false)



# evasive LWE [W22, T22]

assumption. fix distribution  $\mathbf{P}$

if  $\underbrace{s^\top[\mathbf{B} \mid \mathbf{P}]} \approx_c$  random given  $\mathbf{B}, \mathbf{P}$

then  $\underbrace{s^\top[\mathbf{B}]} \approx_c$  random given  $\mathbf{B}^{-1}(\mathbf{P}), \mathbf{B}$

**avoids** zeroizing attacks [CHLRS15, CLLT16, MSZ16, ...]

# evasive LWE [W22, T22]

assumption. fix distributions  $\mathbf{P}, \mathbf{A}'$

if  $\underbrace{s^\top[\mathbf{B} \mid \mathbf{P} \mid \mathbf{A}']}_{\text{wavy line}} \approx_c \text{random given } \mathbf{B}, \mathbf{P}, \mathbf{A}'$

then  $\underbrace{s^\top[\mathbf{B} \mid \mathbf{A}']}_{\text{wavy line}} \approx_c \text{random given } \mathbf{B}^{-1}(\mathbf{P}), \mathbf{B}, \mathbf{A}'$

# proof (almost)

$$\mathbf{mpk} = \mathbf{A}, \mathbf{B}$$

$$\mathbf{ct} = \underbrace{\text{flat}(\mathbf{S})\mathbf{B}},$$

$$\mathbf{kem} = \underbrace{\mathbf{S}\mathbf{A}_f}$$

$$\mathbf{sk} = \mathbf{B}^{-1}((\mathbf{A} - \mathbf{x}^\top \otimes \mathbf{I}) \otimes \mathbf{r}), \mathbf{r}$$

**summary.**  $(\mathbf{ct}, \mathbf{sk}_i) \approx_c (\text{random}, \mathbf{sk}_i)$

# proof (almost)

$$\text{mpk} = \mathbf{A}, \mathbf{B}$$

$$\text{ct} = \underbrace{\text{flat}(\mathbf{S})\mathbf{B}},$$

$$\text{kem} = \underbrace{\mathbf{S}\mathbf{A}_f}$$

$$\text{sk} = \mathbf{B}^{-1}((\mathbf{A} - \mathbf{x}^\top \otimes \mathbf{I}) \otimes \mathbf{r}), \mathbf{r}$$

**summary.**  $(\text{ct}, \text{sk}_i) \not\approx_c (\text{random}, \text{sk}_i)$  given **kem**

# proof (almost)

$$\text{mpk} = \mathbf{A}, \mathbf{B}$$

$$\text{ct} = \text{flat}(\mathbf{S})\mathbf{B},$$

$$\text{kem} = \mathbf{S}\mathbf{A}_f + \mathbf{S}_0\mathbf{A}_0$$

$$\text{sk} = \mathbf{B}^{-1}((\mathbf{A} - \mathbf{x}^\top \otimes \mathbf{I}) \otimes \mathbf{r}), \mathbf{r}$$

**fix.** kem leaks  $\mathbf{r}_i\mathbf{S}\mathbf{A}_f$  instead of  $\mathbf{S}\mathbf{A}_f$ .

# proof (final)

$$\mathbf{mpk} = \mathbf{A}, \mathbf{B}, \mathbf{A}_0, \mathbf{B}_0$$

$$\mathbf{ct} = \underbrace{\text{flat}(\mathbf{S})\mathbf{B}}_{\text{wavy}}, \underbrace{\text{flat}(\mathbf{S}_0)\mathbf{B}_0}_{\text{wavy}}, \quad \mathbf{kem} = \underbrace{\mathbf{S}\mathbf{A}_f + \mathbf{S}_0\mathbf{A}_0}_{\text{wavy}}$$

$$\mathbf{sk} = \mathbf{B}^{-1}((\mathbf{A} - \mathbf{x}^\top \otimes \mathbf{I}) \otimes \mathbf{r}), \mathbf{r}, \mathbf{B}_0^{-1}(\mathbf{A}_0 \otimes \mathbf{r})$$

# proof (final)

$$\mathbf{mpk} = \mathbf{A}, \mathbf{B}, \mathbf{A}_0, \mathbf{B}_0$$

$$\mathbf{ct} = \underbrace{\text{flat}(\mathbf{S})\mathbf{B}}_{\text{flat}(\mathbf{S})\mathbf{B}}, \underbrace{\text{flat}(\mathbf{S}_0)\mathbf{B}_0}_{\text{flat}(\mathbf{S}_0)\mathbf{B}_0}, \quad \mathbf{kem} = \underbrace{\mathbf{S}\mathbf{A}_f + \mathbf{S}_0\mathbf{A}_0}_{\mathbf{S}\mathbf{A}_f + \mathbf{S}_0\mathbf{A}_0}$$

$$\mathbf{sk} = \mathbf{B}^{-1}((\mathbf{A} - \mathbf{x}^\top \otimes \mathbf{I}) \otimes \mathbf{r}), \mathbf{r}, \mathbf{B}_0^{-1}(\mathbf{A}_0 \otimes \mathbf{r})$$

**thm.** assuming sub-exp evasive LWE (+ LWE)

1 **optimal** broadcast enc // size  $\text{poly}(\log N)$

# proof (final)

$$\mathbf{mpk} = \mathbf{A}, \mathbf{B}, \mathbf{A}_0, \mathbf{B}_0$$

$$\mathbf{ct} = \underbrace{\text{flat}(\mathbf{S})\mathbf{B}}_{\text{flat}(\mathbf{S})\mathbf{B}}, \underbrace{\text{flat}(\mathbf{S}_0)\mathbf{B}_0}_{\text{flat}(\mathbf{S}_0)\mathbf{B}_0}, \quad \mathbf{kem} = \underbrace{\mathbf{S}\mathbf{A}_f + \mathbf{S}_0\mathbf{A}_0}_{\mathbf{S}\mathbf{A}_f + \mathbf{S}_0\mathbf{A}_0}$$

$$\mathbf{sk} = \mathbf{B}^{-1}((\mathbf{A} - \mathbf{x}^\top \otimes \mathbf{G}) \otimes \mathbf{r}), \mathbf{r}, \mathbf{B}_0^{-1}(\mathbf{A}_0 \otimes \mathbf{r})$$

**thm.** assuming sub-exp evasive LWE (+ LWE)

1 **optimal** broadcast enc // size  $\text{poly}(\log N)$

2 **compact** CP-ABE for circuits, assuming

$$\mathbf{A}, \mathbf{r}_i, \underbrace{\mathbf{r}_i \mathbf{S}(\mathbf{A} - \mathbf{x}_i^\top \otimes \mathbf{G})}_{\mathbf{r}_i \mathbf{S}(\mathbf{A} - \mathbf{x}_i^\top \otimes \mathbf{G})} \approx_c \text{random}$$



# conclusion

**this.** broadcast enc, CP-ABE, evasive LWE

# conclusion

**this.** broadcast enc, CP-ABE, evasive LWE

**open.**

- evasive LWE: crypt-analysis? reductions?
- applications? witness encryption [T22, VW22]
- CP-ABE for circuits from *just* evasive LWE

# conclusion

**this.** broadcast enc, CP-ABE, evasive LWE

**open.**

- evasive LWE: crypt-analysis? reductions?
- applications? witness encryption [T22, VW22]
- CP-ABE for circuits from *just* evasive LWE

// merci !