Single-Server Private Information Retrieval with Sublinear Amortized Time

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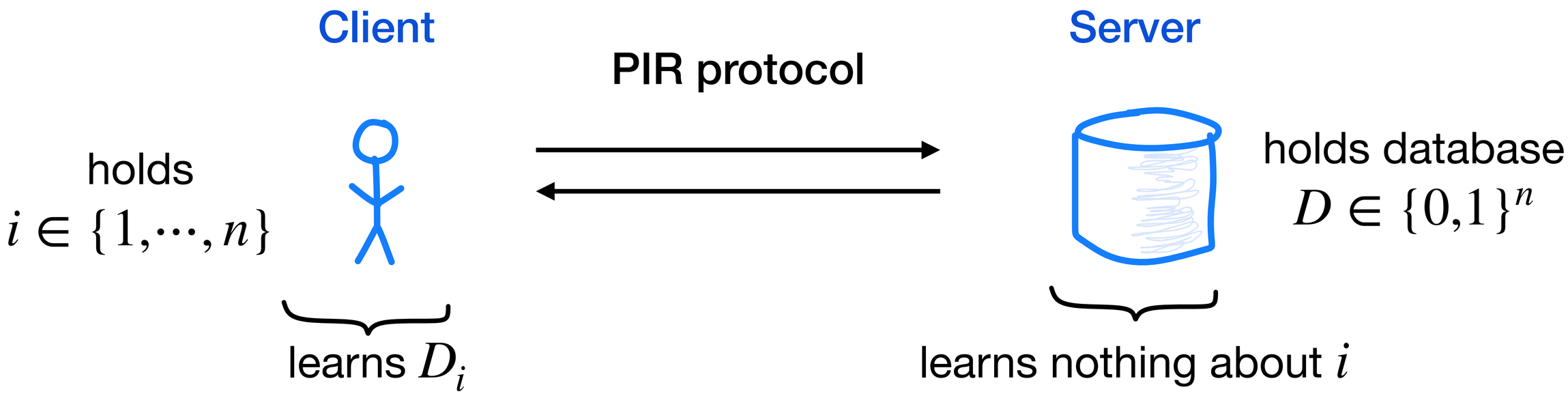
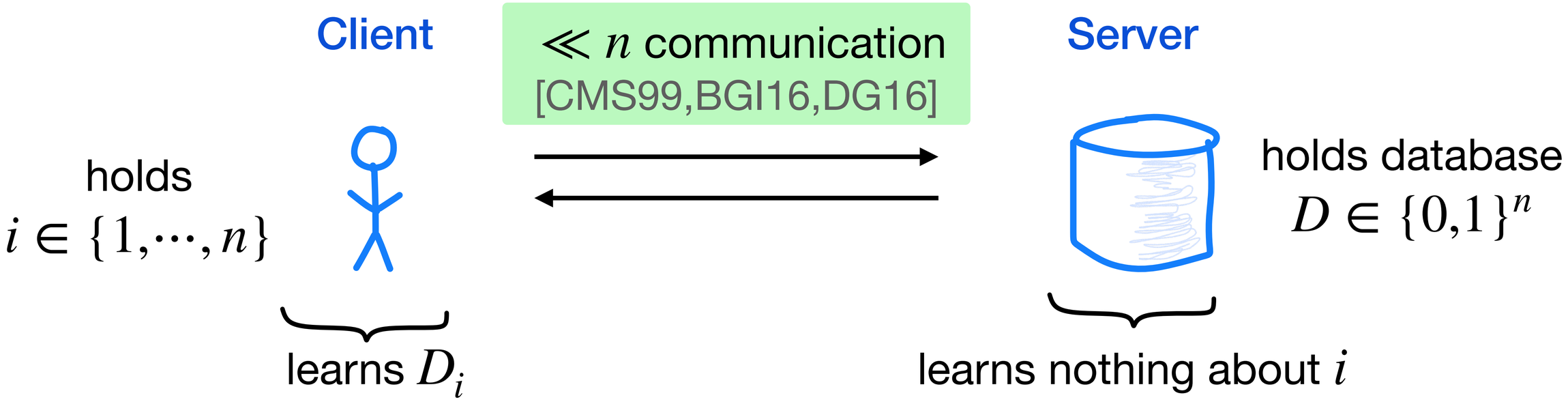
Eurocrypt ’22

Private information retrieval [CGKS95,KO97]



Applications: private media [GCMSAW16], private e-commerce [HOG11], private ads [J01…], private web browsing [KC21], metadata-hiding messaging [AS16…], …

Private information retrieval [CGKS95,KO97]



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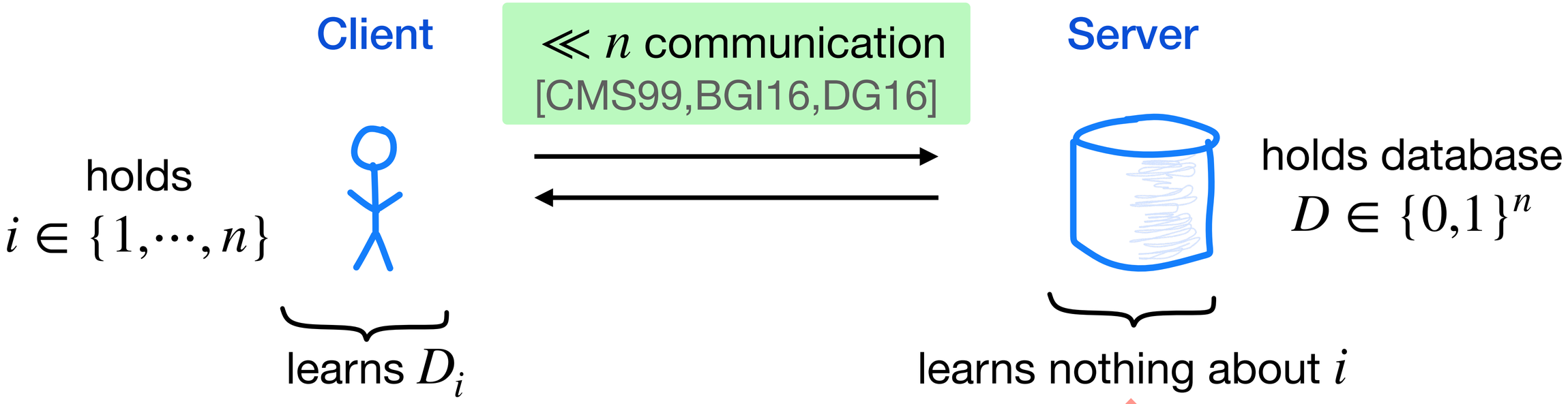

PIR inherently has **high server-side computation** costs [BIM04,PY22]:
To answer a single query, the server(s) must run in time n.

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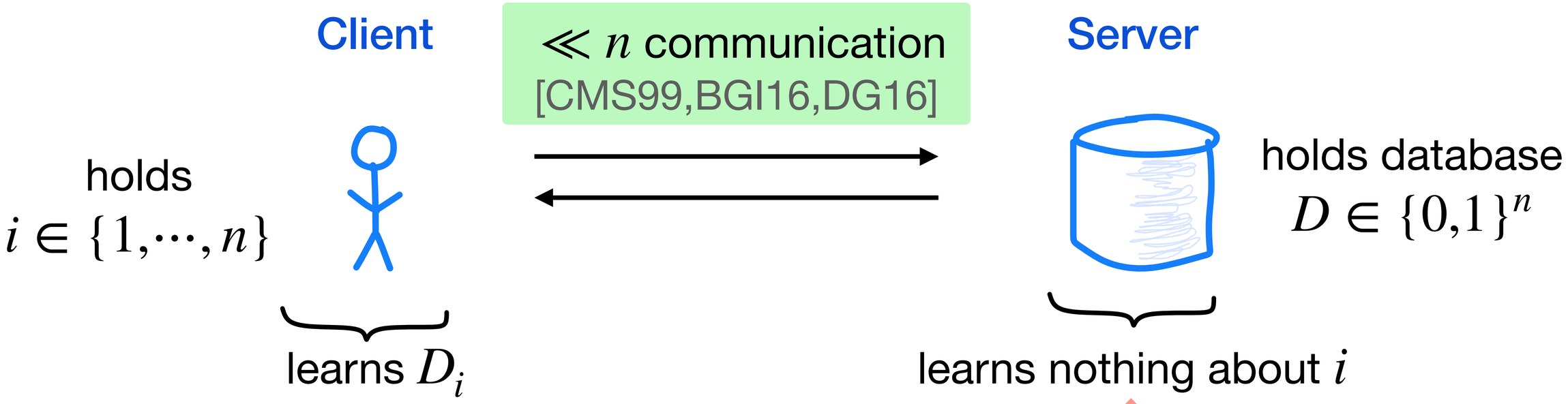

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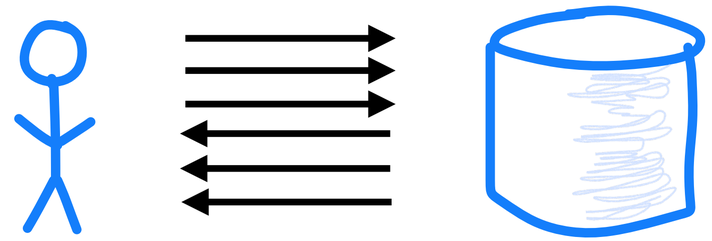
Idea: Amortize the server time over <u>many</u> queries [BIM04,IKOS04]

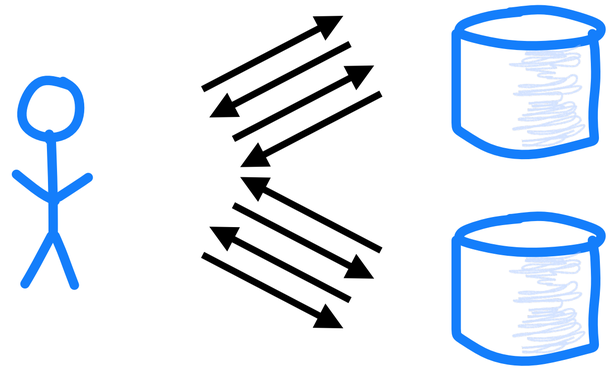
Existing PIR with sublinear time

Batch PIR with non-adaptive queries



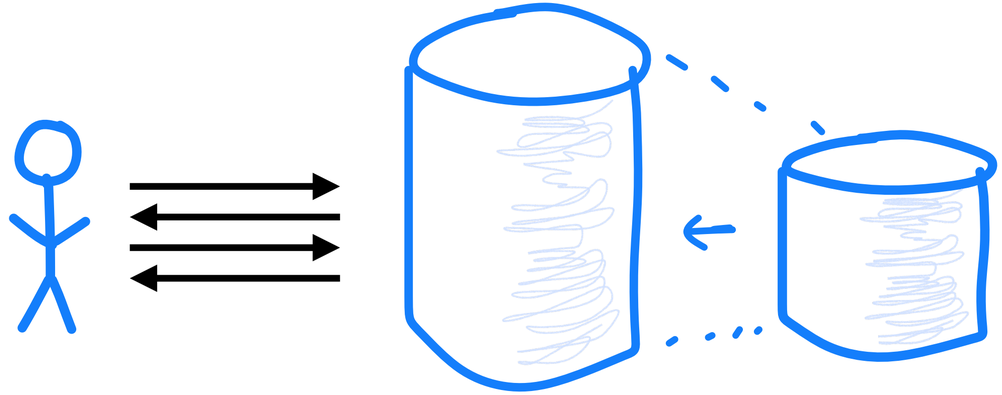
[IKOS04,HHG13,GKL10,AS16,H16,ACLS18,CHLR18]

Offline/online PIR with 2 servers



[CK20,SACM21,KC21]

PIR with preprocessing



[BIPW17,CHR17,HOWW18]

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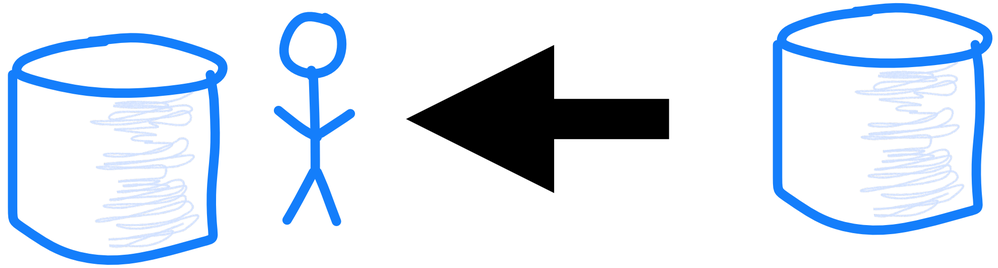

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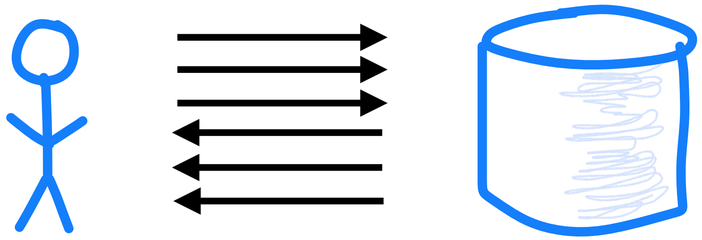

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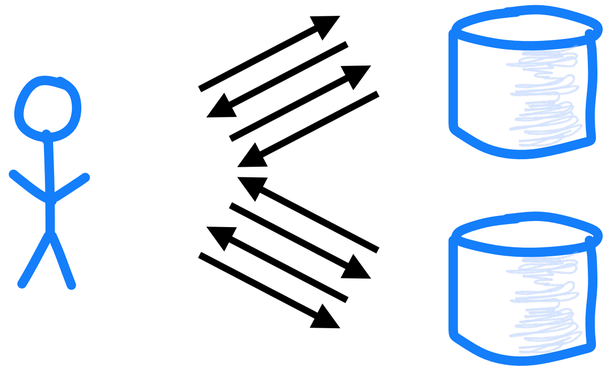

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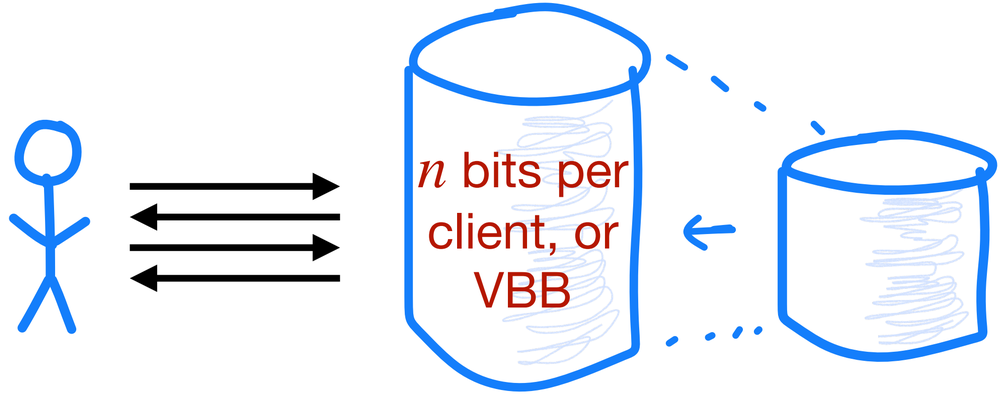

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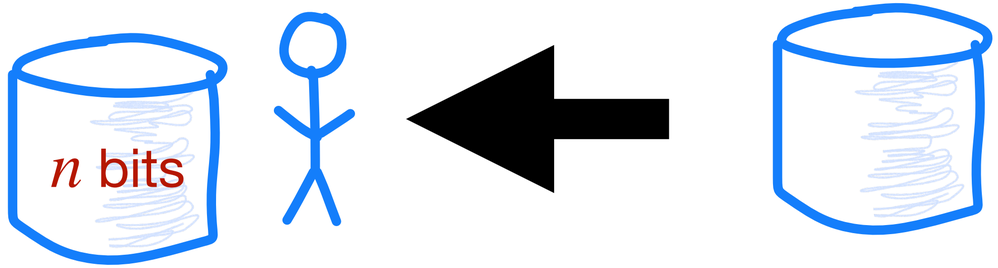

This work

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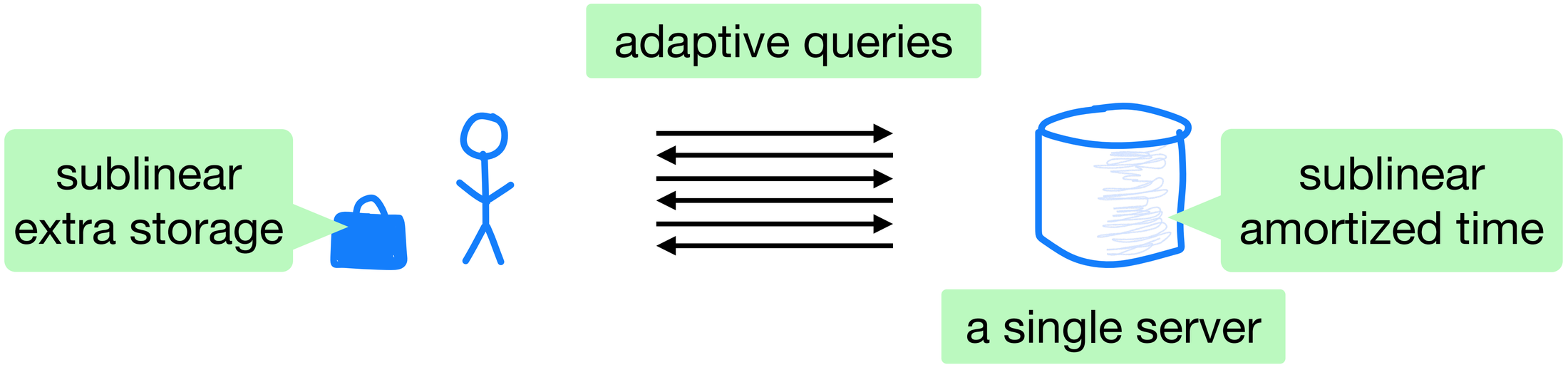

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Results preview:
$n^{3/4}$ time + storage from DDH
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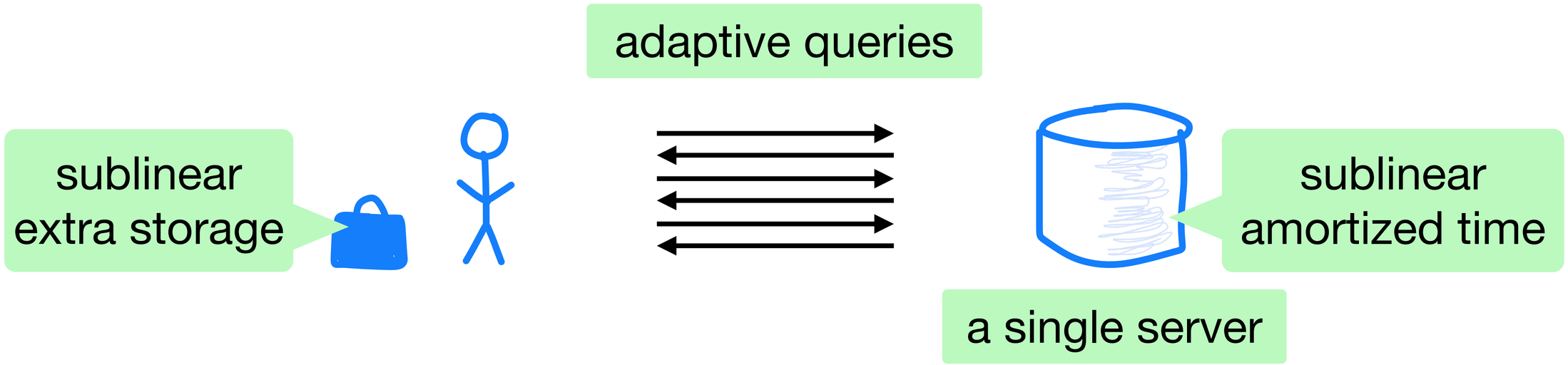

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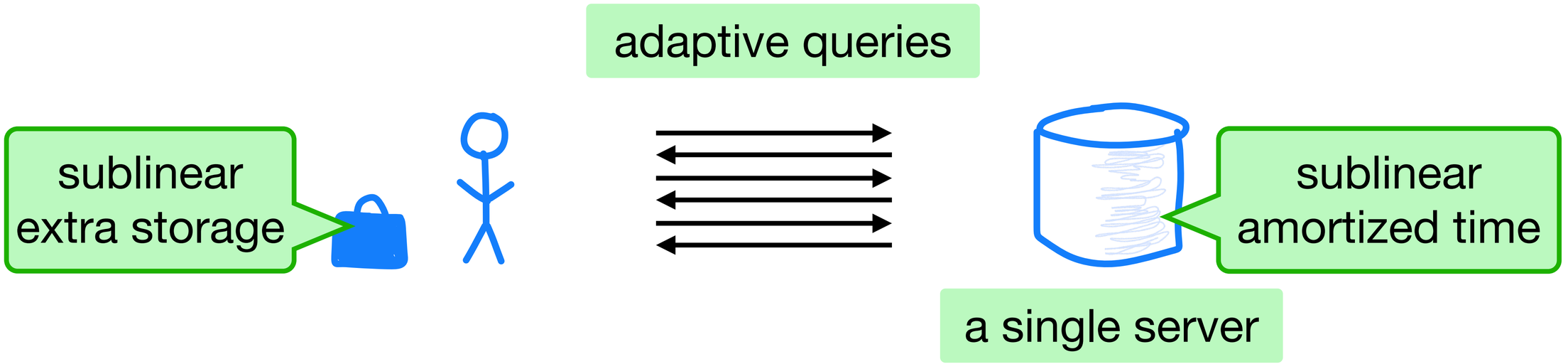

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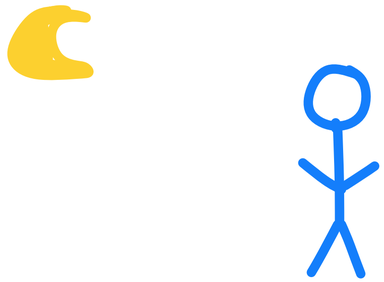
Goal: build PIR for Q adaptive queries, with **sublinear amortized time**

Our approach: build PIR with two phases

1. Once, run a linear-time "offline" phase.

2. For each of the Q queries, run a sublinear-time "online" phase.

Many-query offline/online PIR



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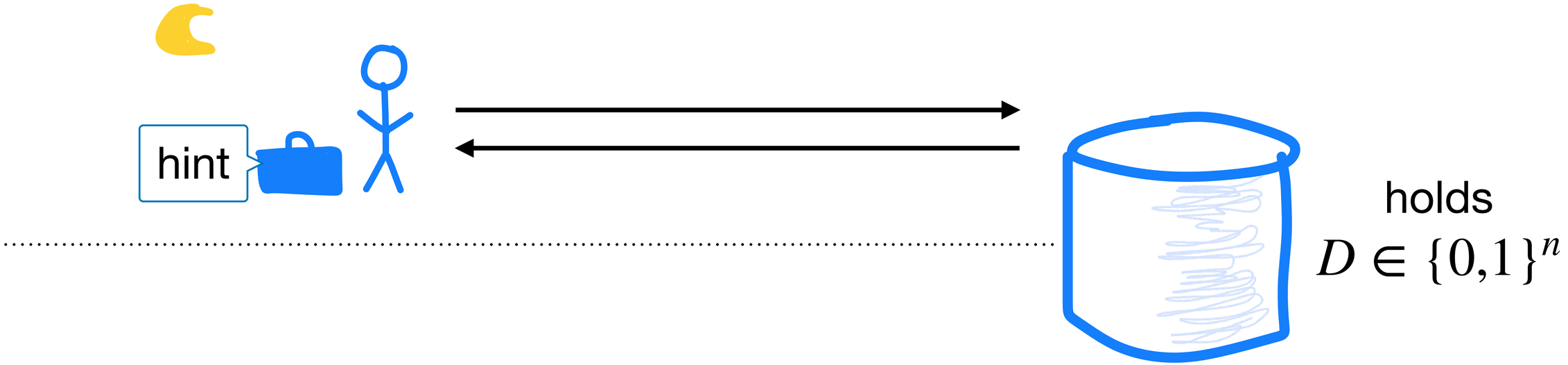

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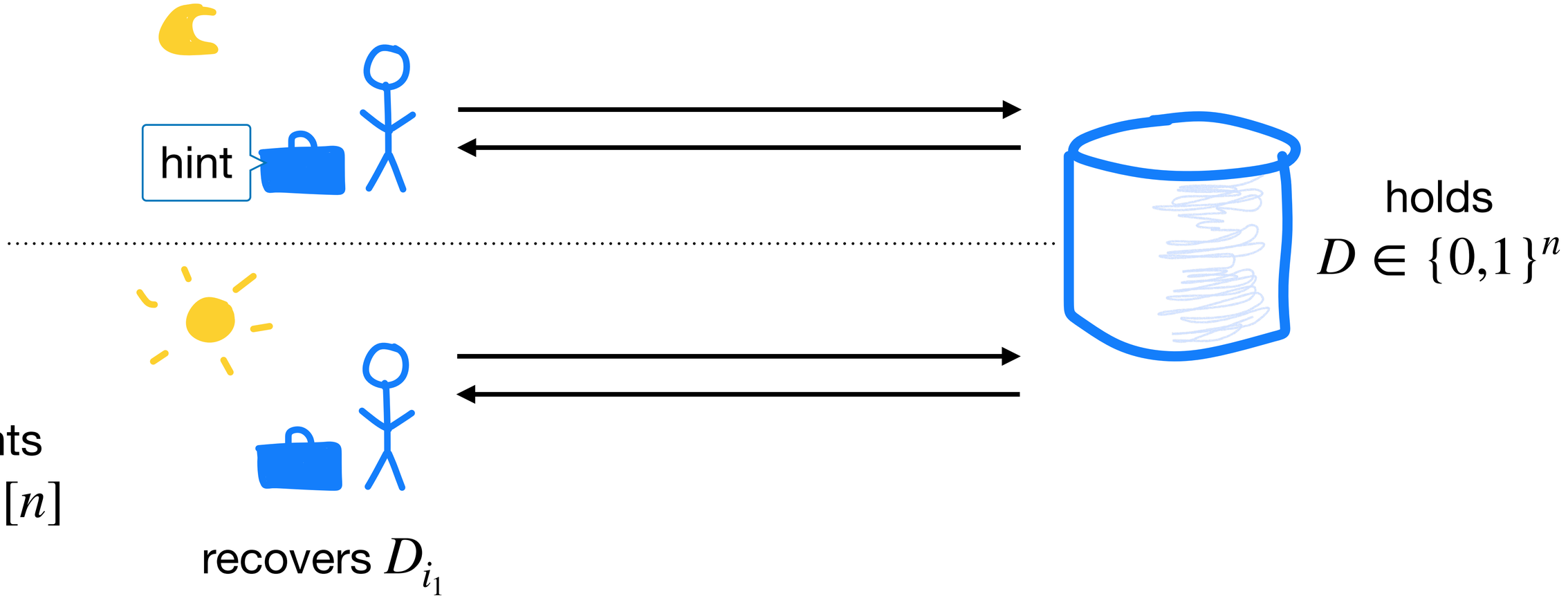

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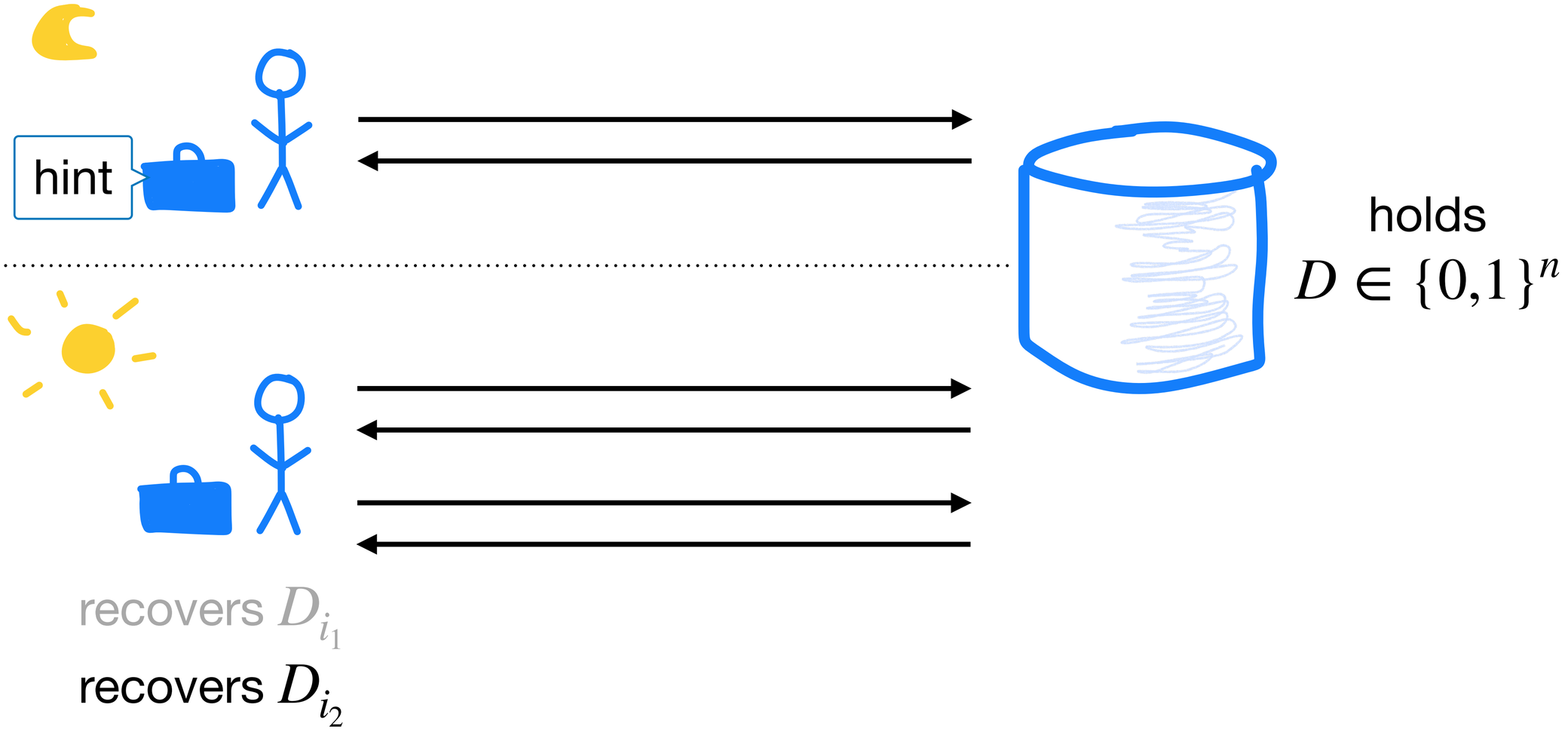

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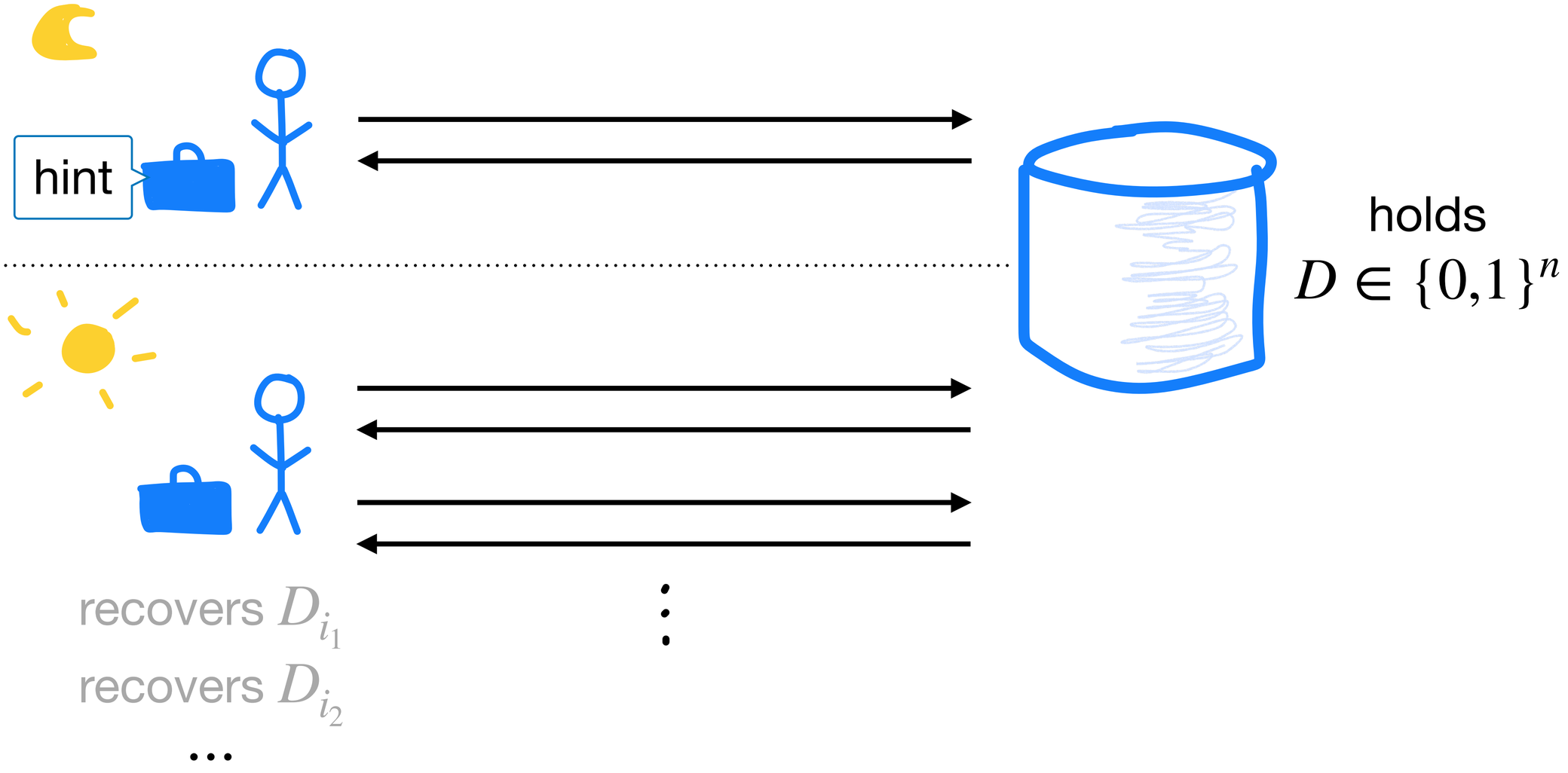

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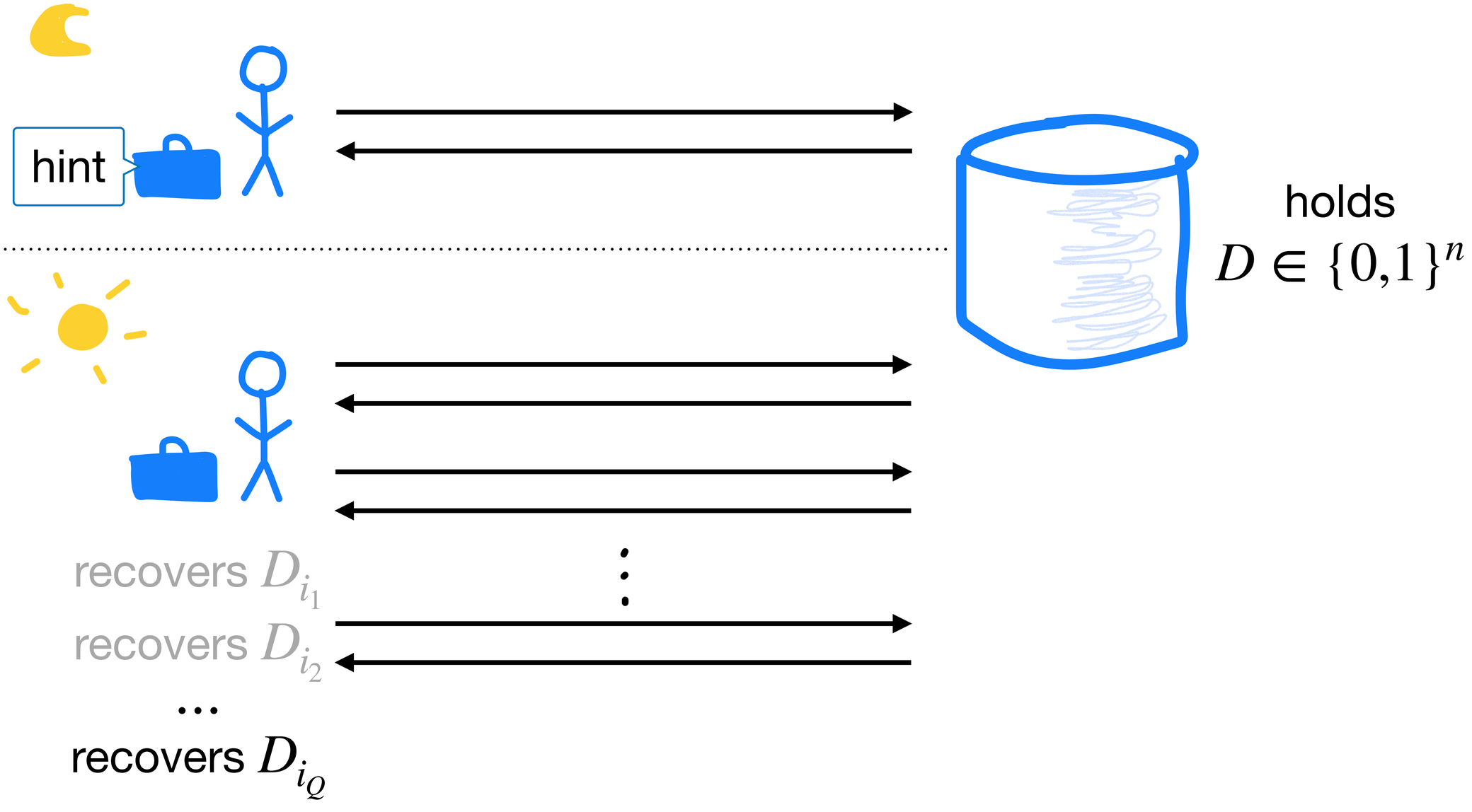

Many-query PIR requirements

- **Correctness**: If the client and server execute the protocol faithfully, for any D, for any $i_1, \cdots, i_Q \in [n]$, the client correctly recovers $D_{i_1}, \cdots, D_{i_Q}$, with overwhelming probability.

- **Malicious security**: Even if the server does not follow the protocol, the server learns nothing about $i_1, \cdots, i_Q$.

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In our schemes, the queries are independent of the server's past answers.

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Goal: Minimize communication, computation, and storage costs

This talk

1. Background: The offline/online PIR model
2. Our results: New PIR schemes with sublinear time
3. Open questions

Theorem 1: From linearly homomorphic encryption.

Under DDH, QR, DCR, or LWE, there is a single-server PIR scheme that, on database size n, when the client makes $n^{1/4}$ adaptive queries, has:

- amortized server time $n^{3/4}$,
- client storage $n^{3/4}$ and no extra server storage,
- amortized client time $n^{1/2}$, and
- amortized communication $n^{1/2}$.

Throughout this talk, we omit $\log(n)$ and $\mathrm{poly}(\lambda)$ factors.

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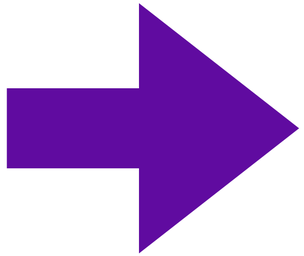
Proof sketch for Theorem 1

Single-query PIR with sublinear online time
[CK20]



New: Many-query PIR with sublinear amortized time

New: generic compiler,
applying ideas from batch codes
[IKOS04]

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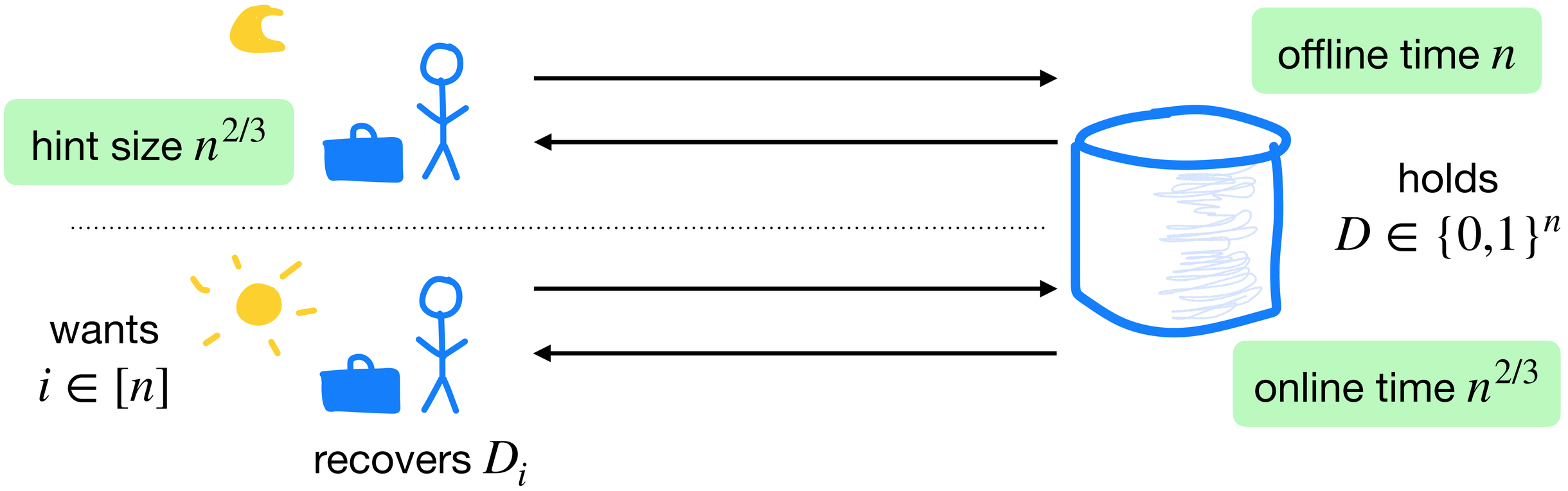

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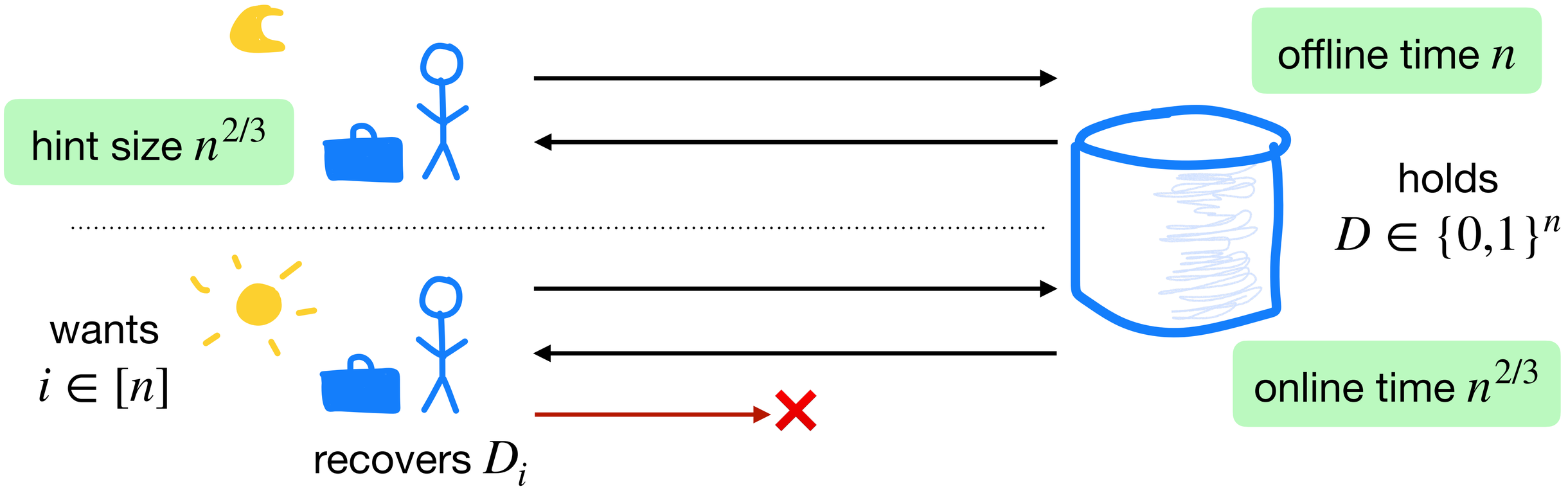

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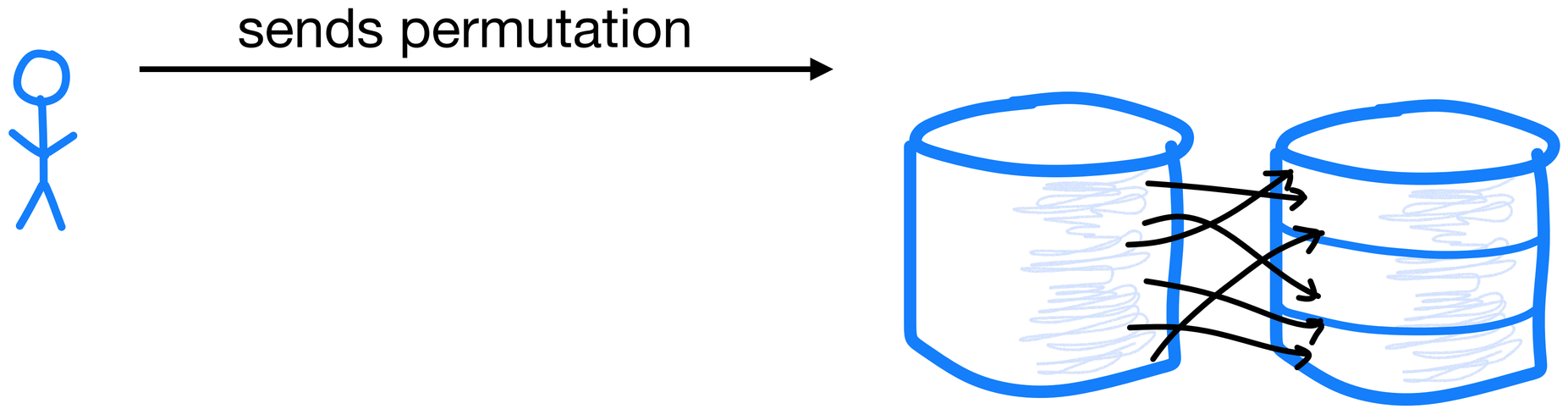
Our compiler: To handle Q adaptive queries, split the database into Q random chunks.



holds $D \in \{0,1\}^n$,
in Q chunks of size n/Q

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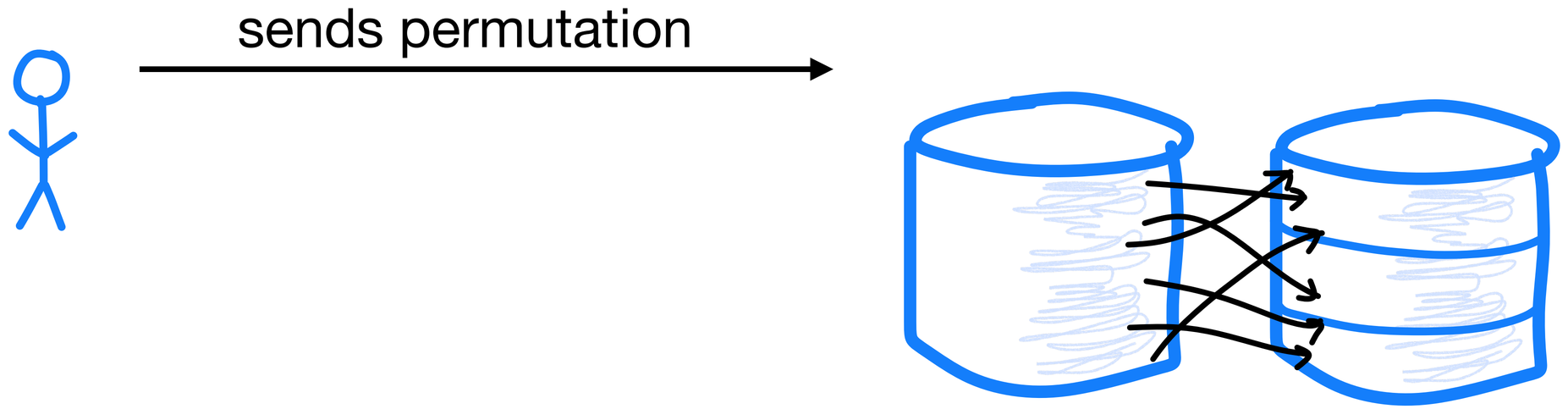

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Observation: When the client makes Q adaptive queries,
at most λ distinct queries fall in any one chunk,
with probability $1 - \mathrm{negl}(\lambda)$.

Offline: Permute + partition the database, then run λ offline phases on each chunk.

Online: Run an online phase on each chunk, using a hint matching the index.

The client caches all recovered bits, to never re-query for the same index.

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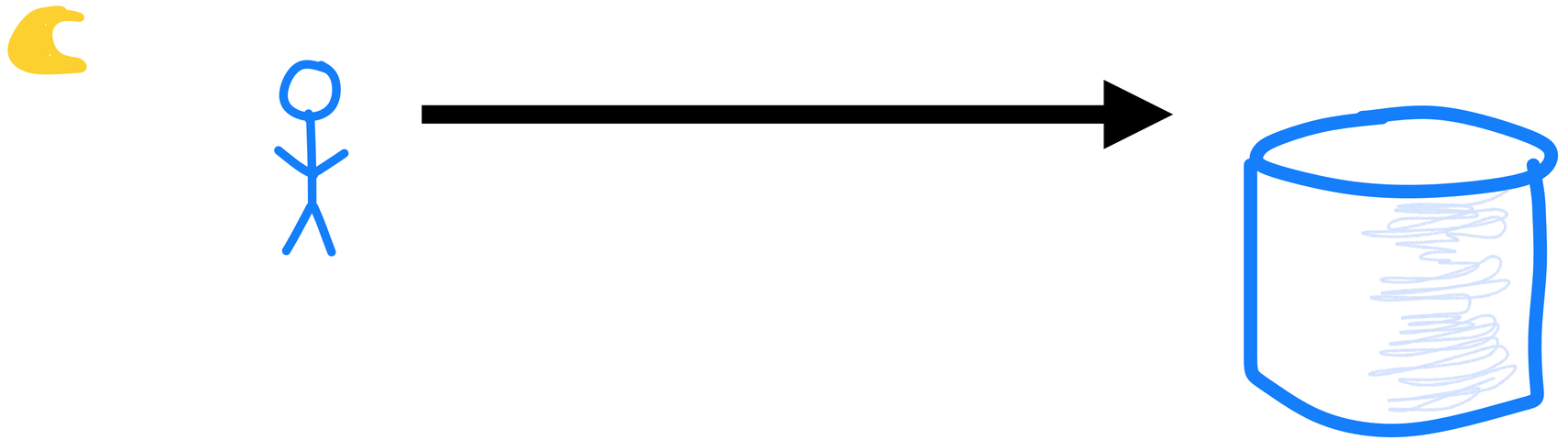

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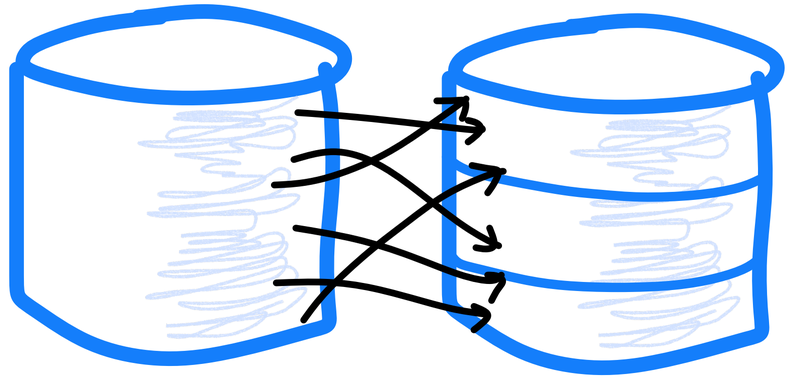

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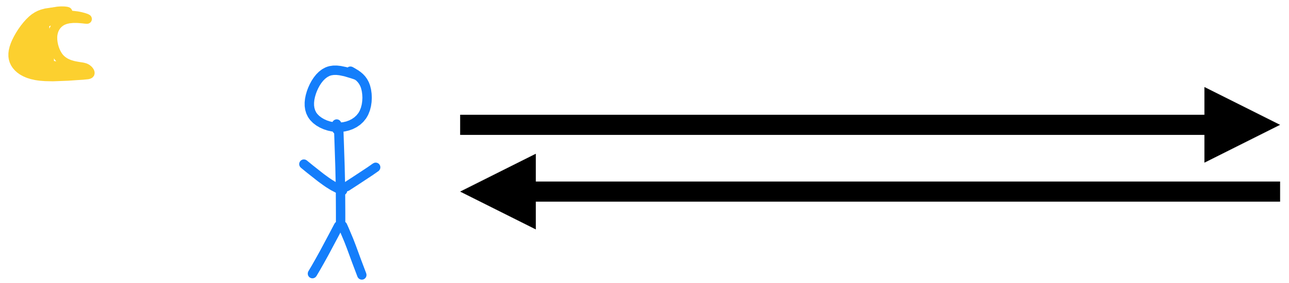

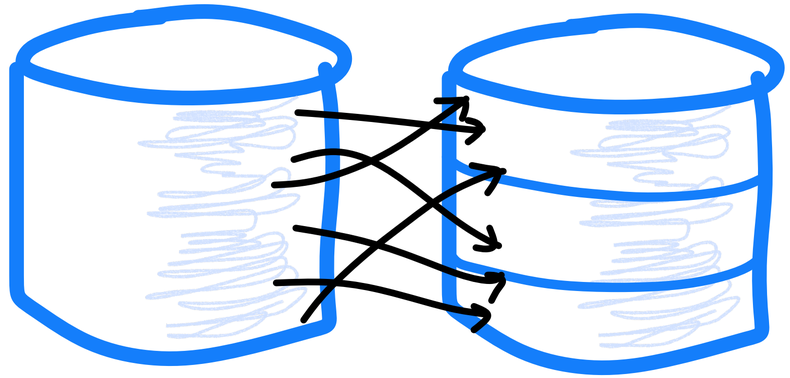

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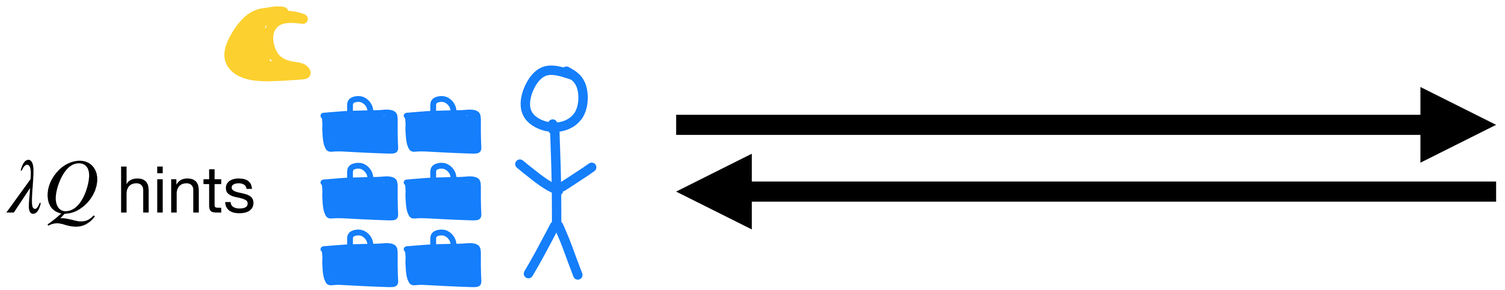

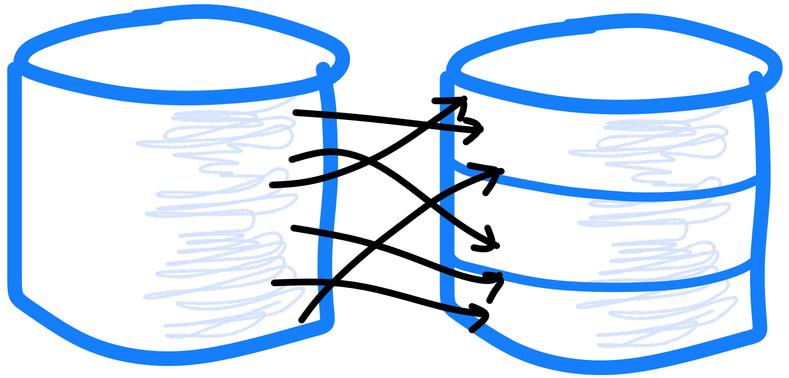

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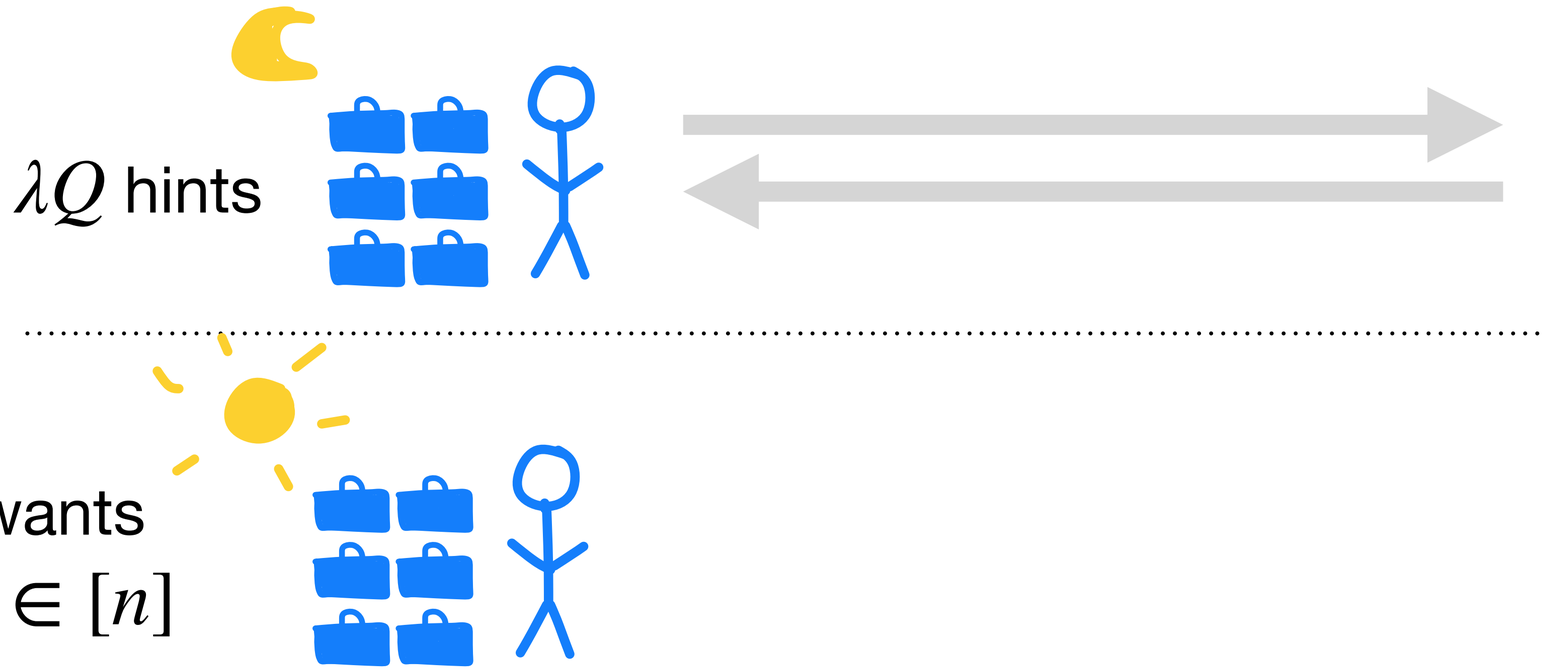

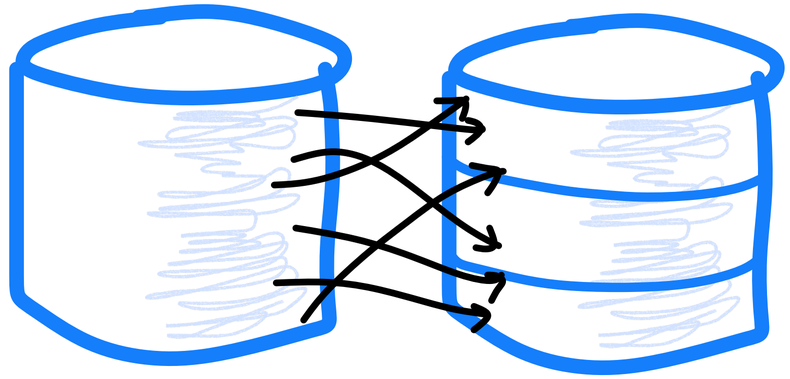

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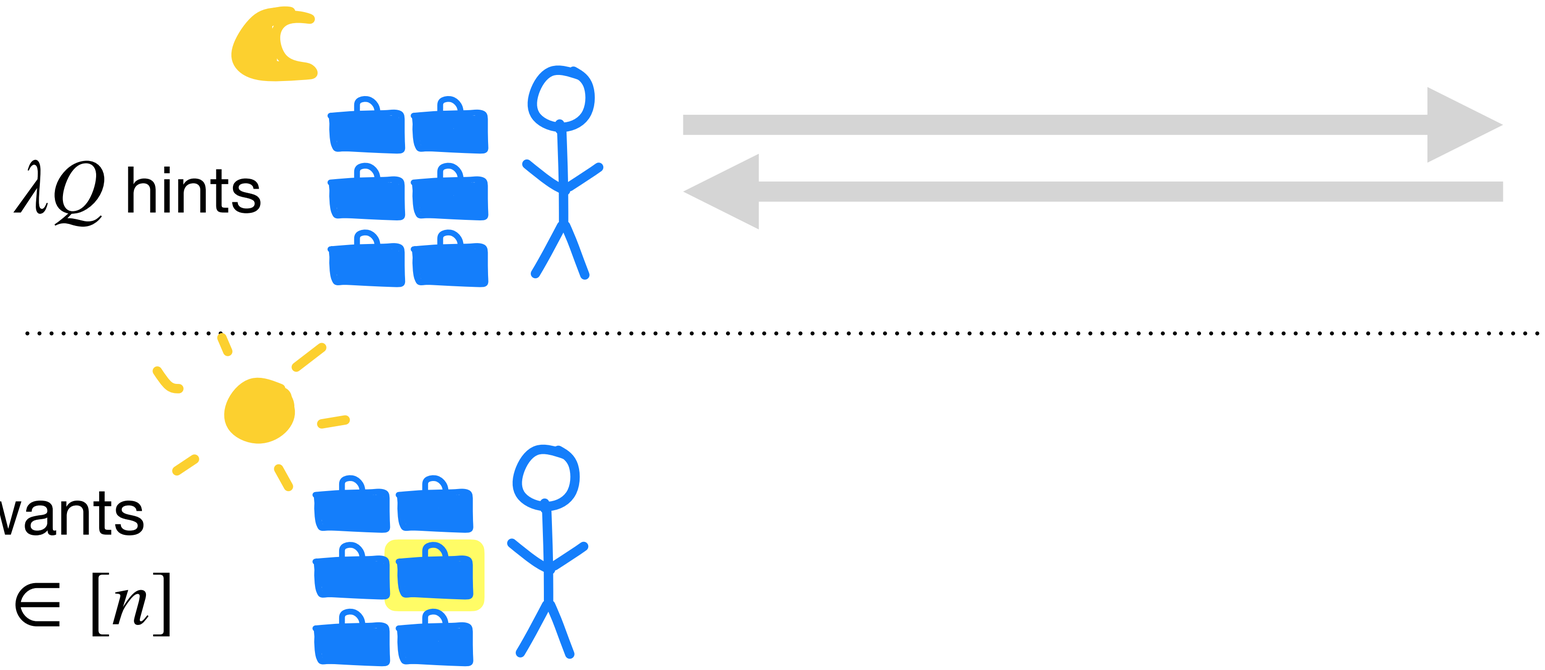

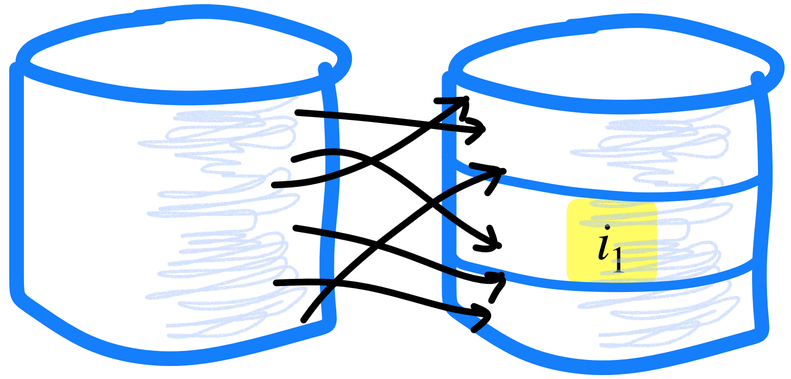

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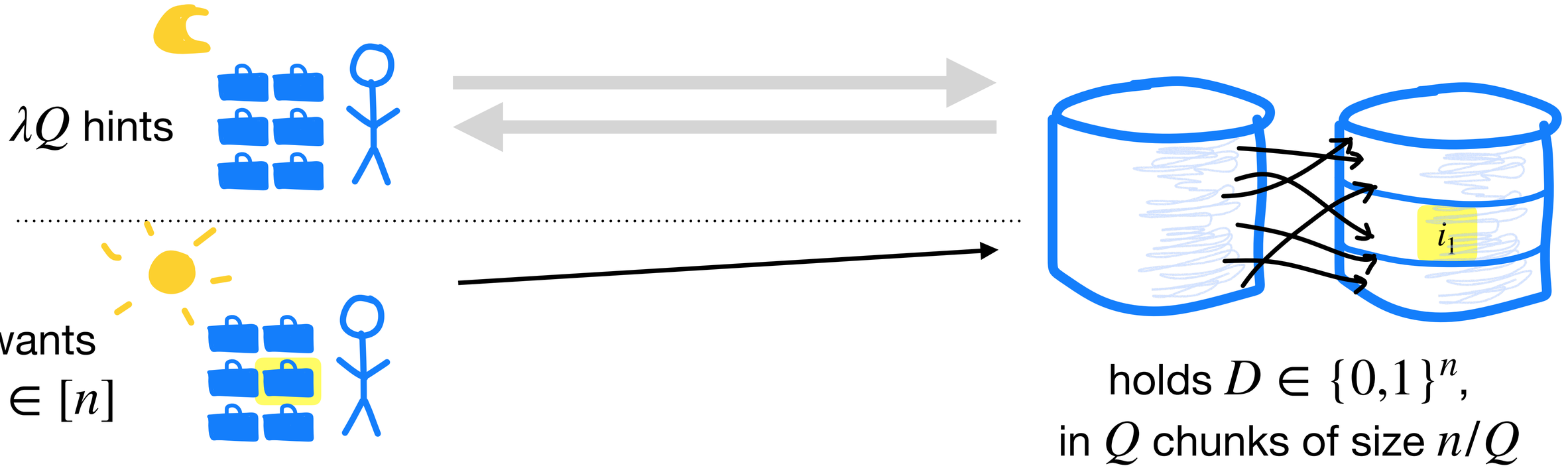

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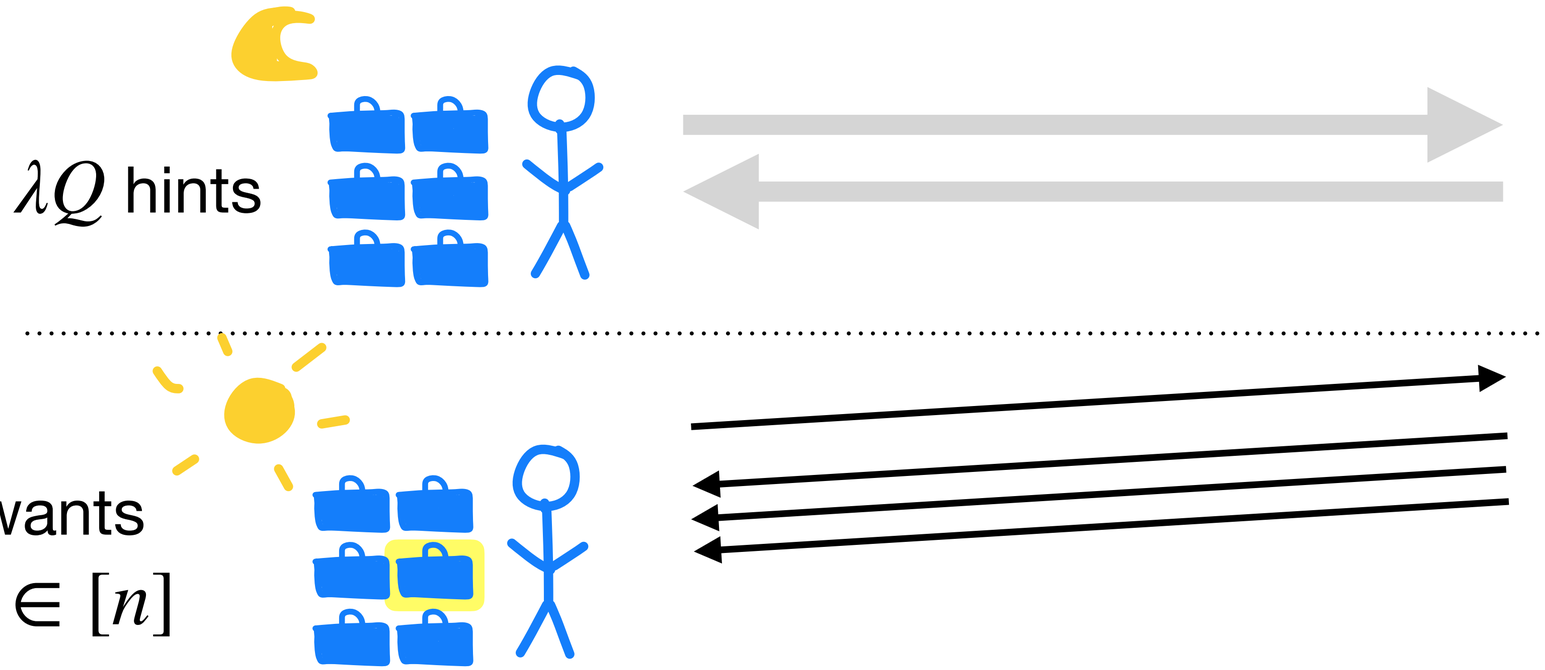

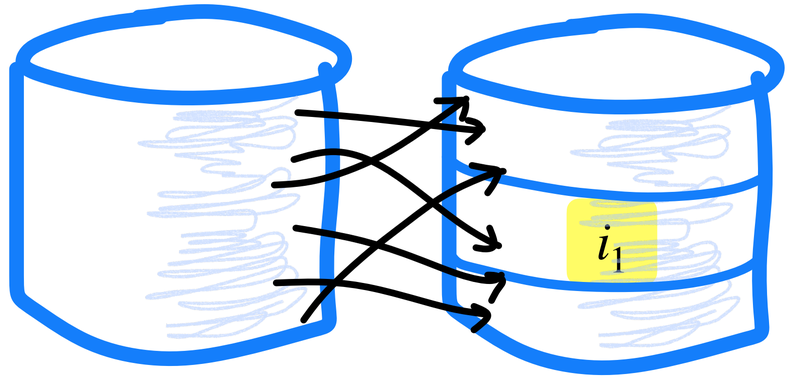

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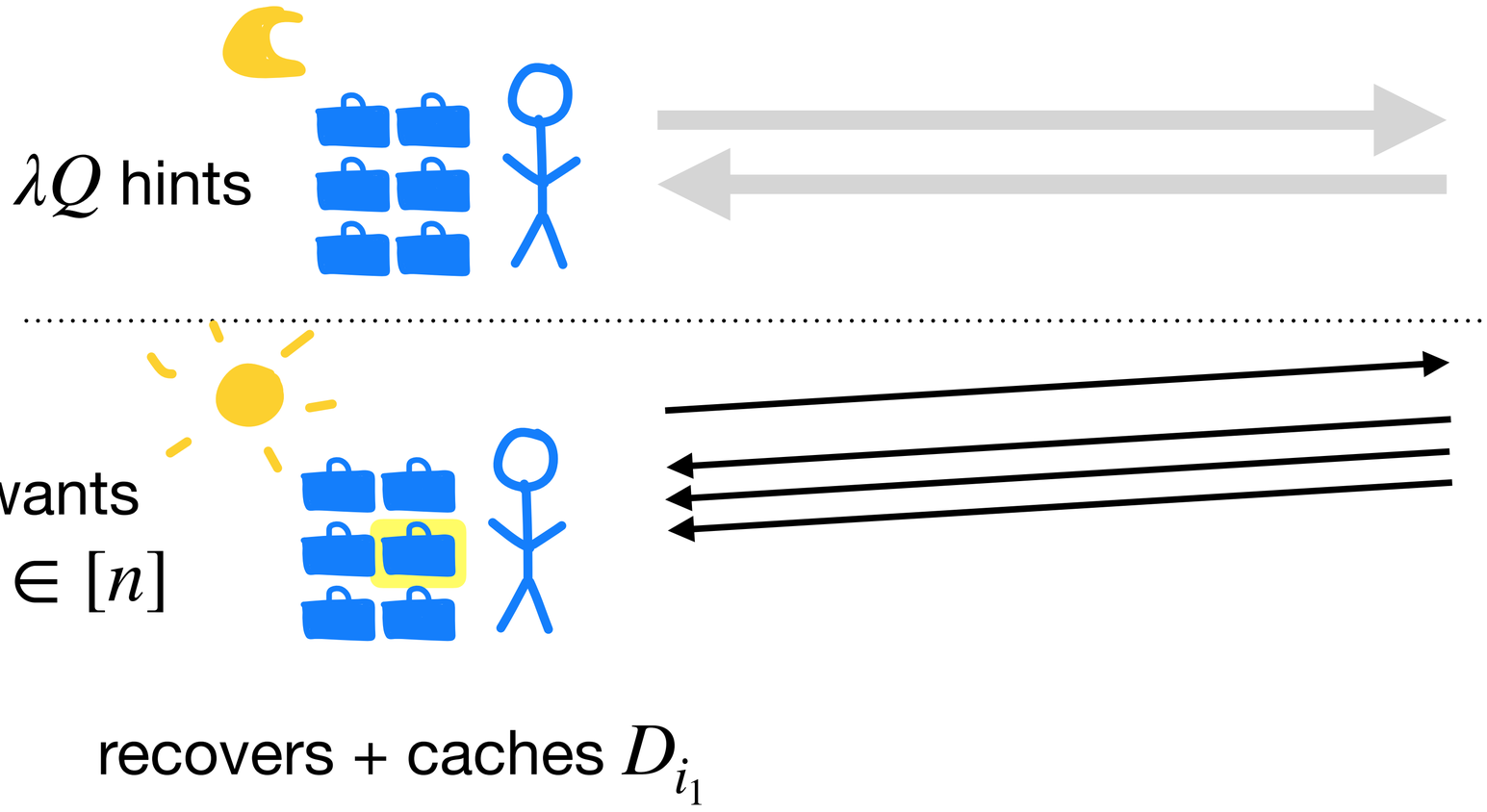

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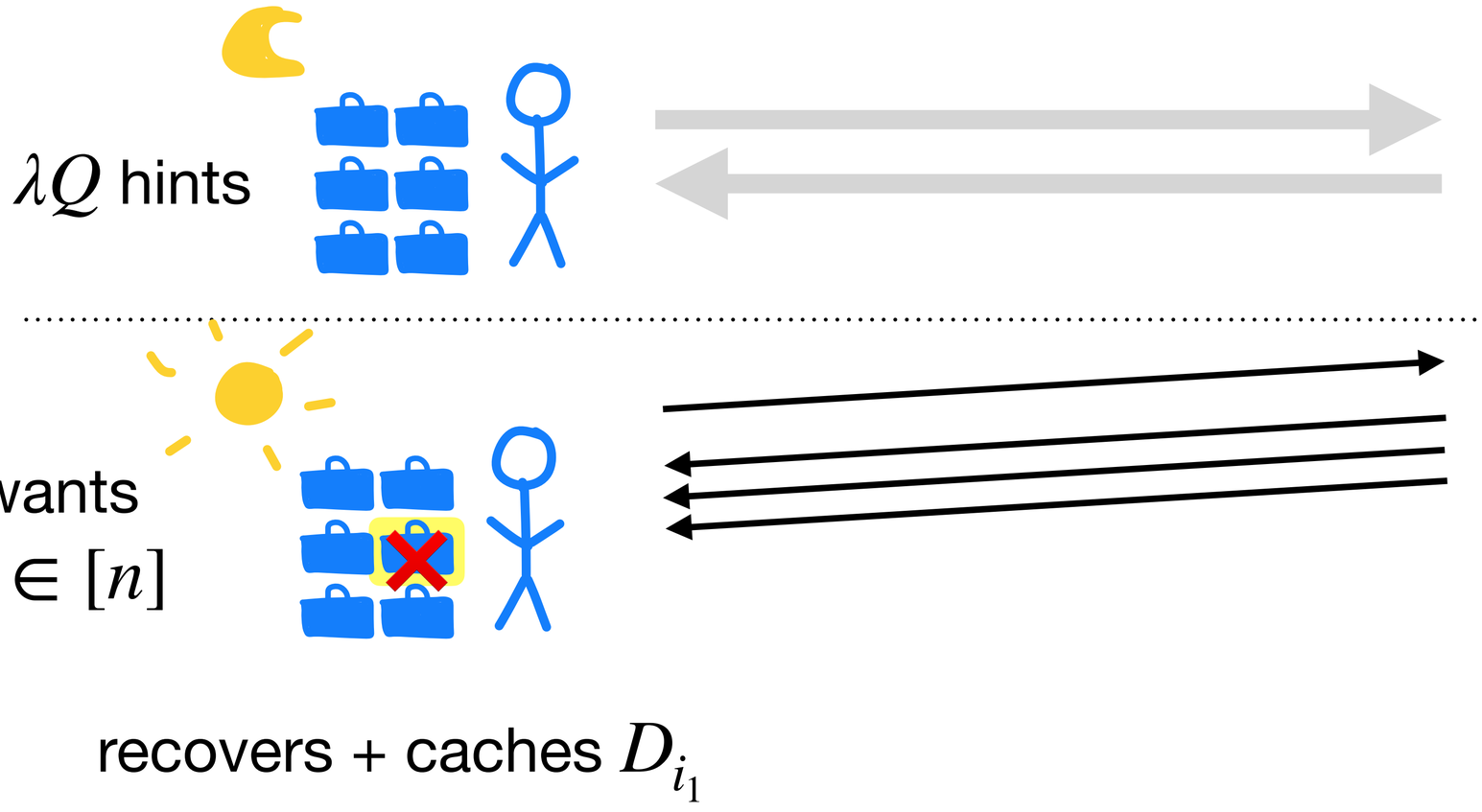

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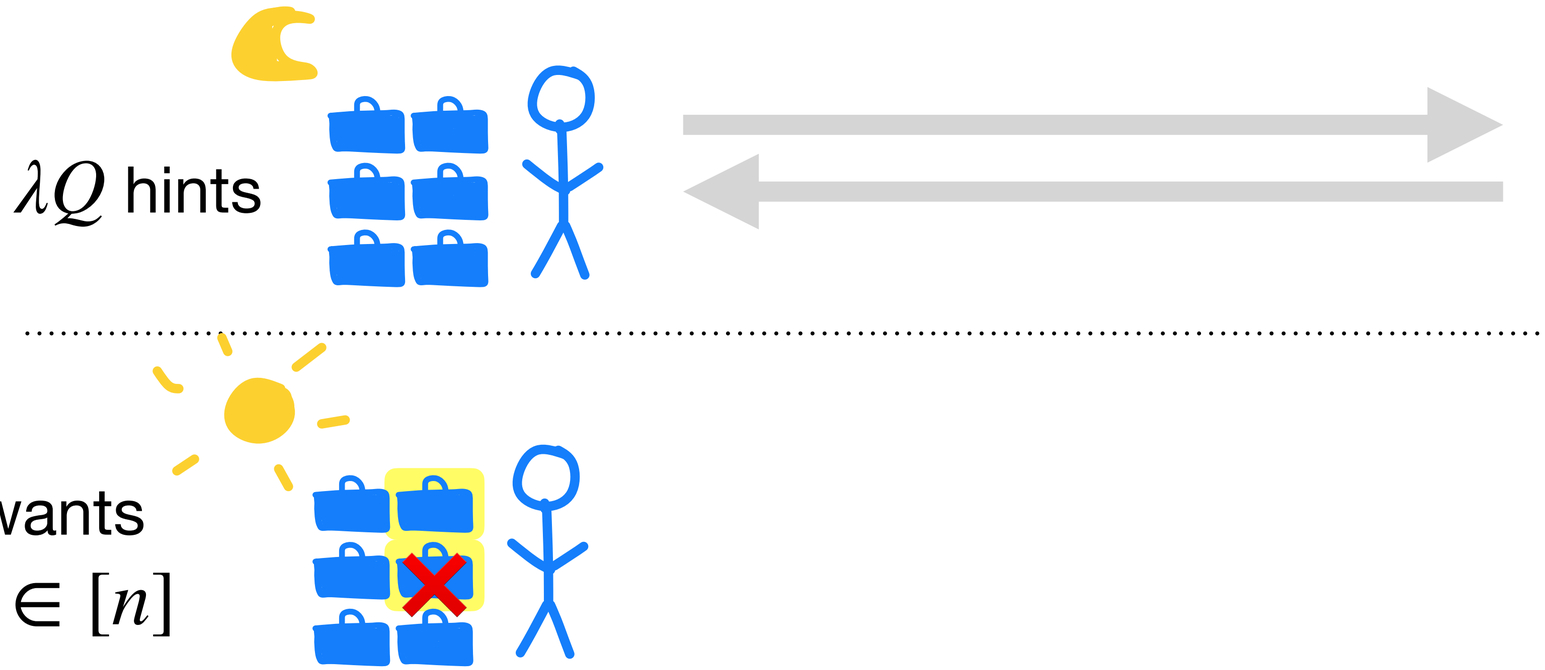

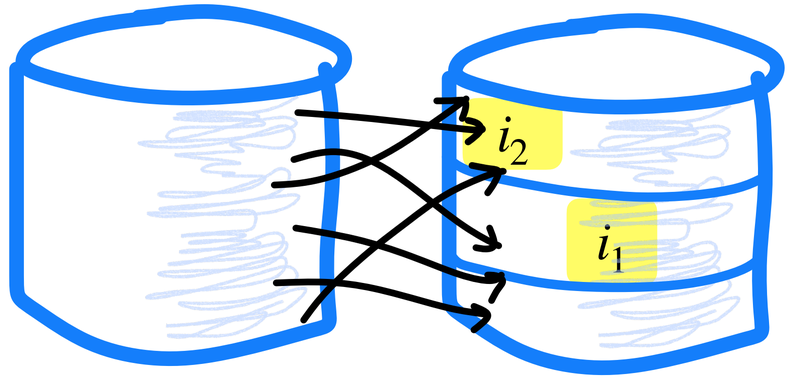

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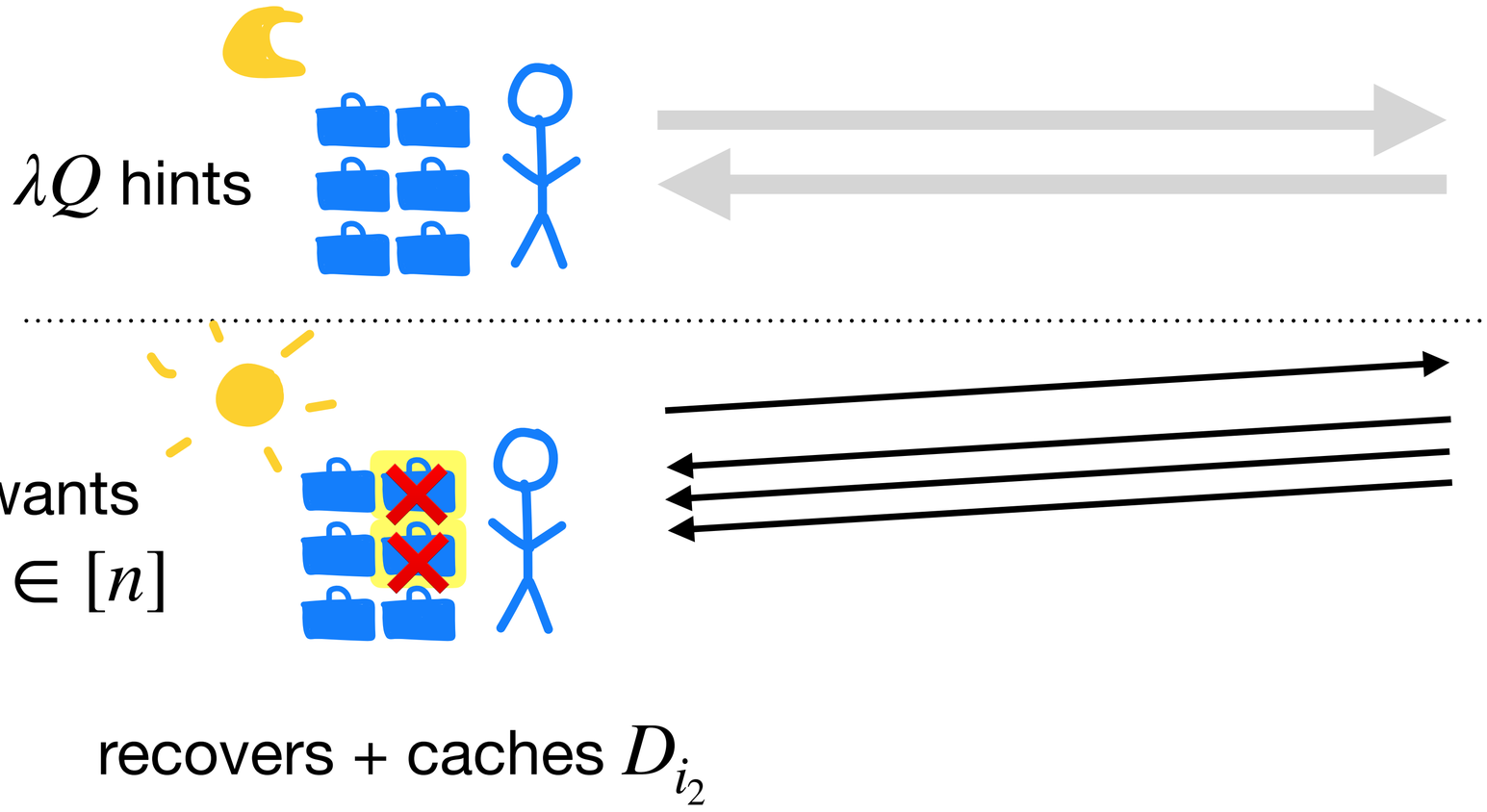

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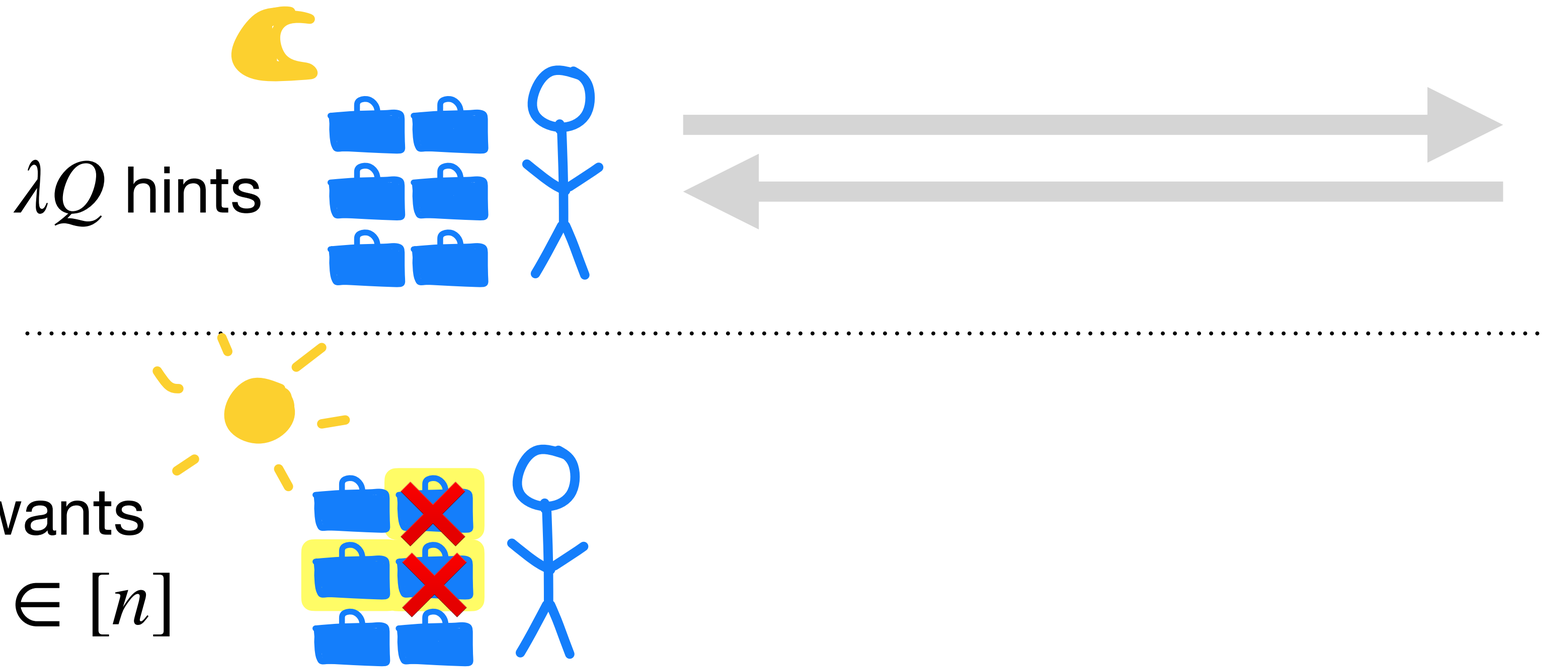

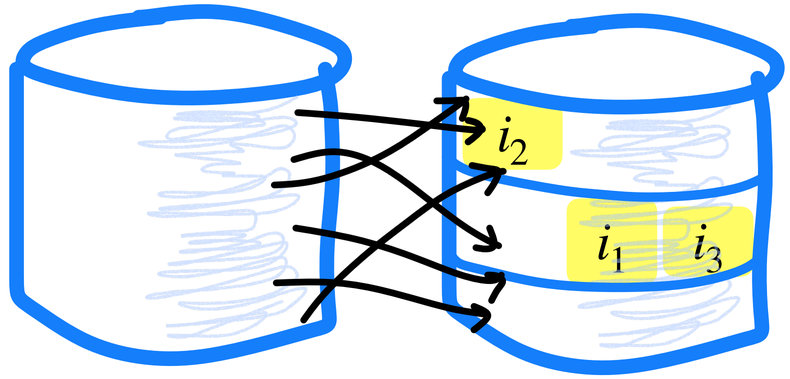

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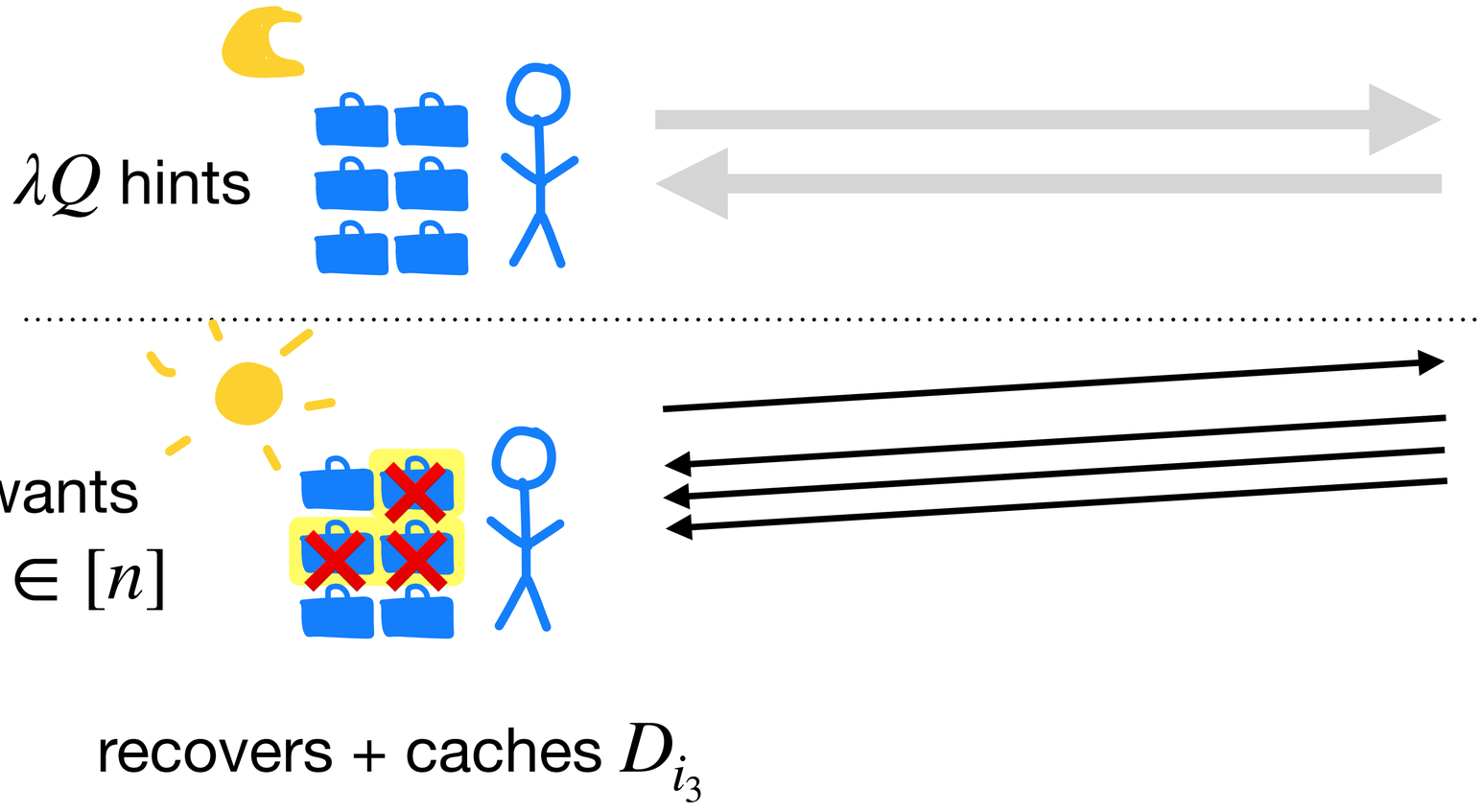

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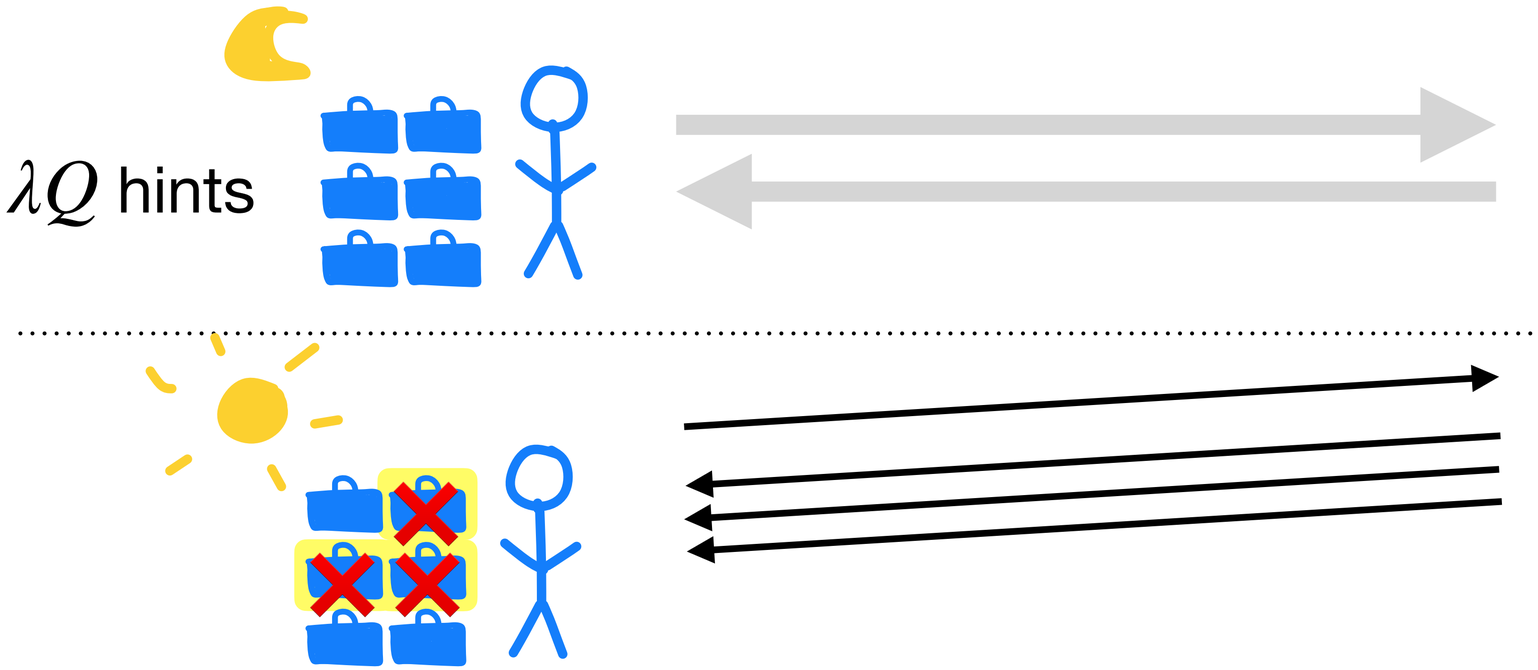

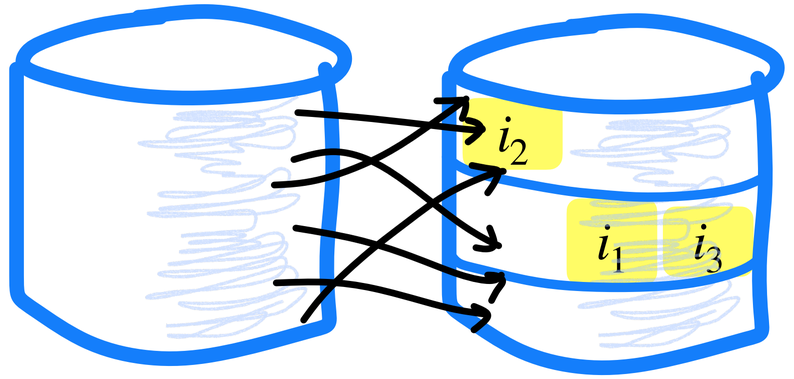

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Correctness: for any query sequence, the client does not run out of fresh hints (with overwhelming probability over the choice of permutation).

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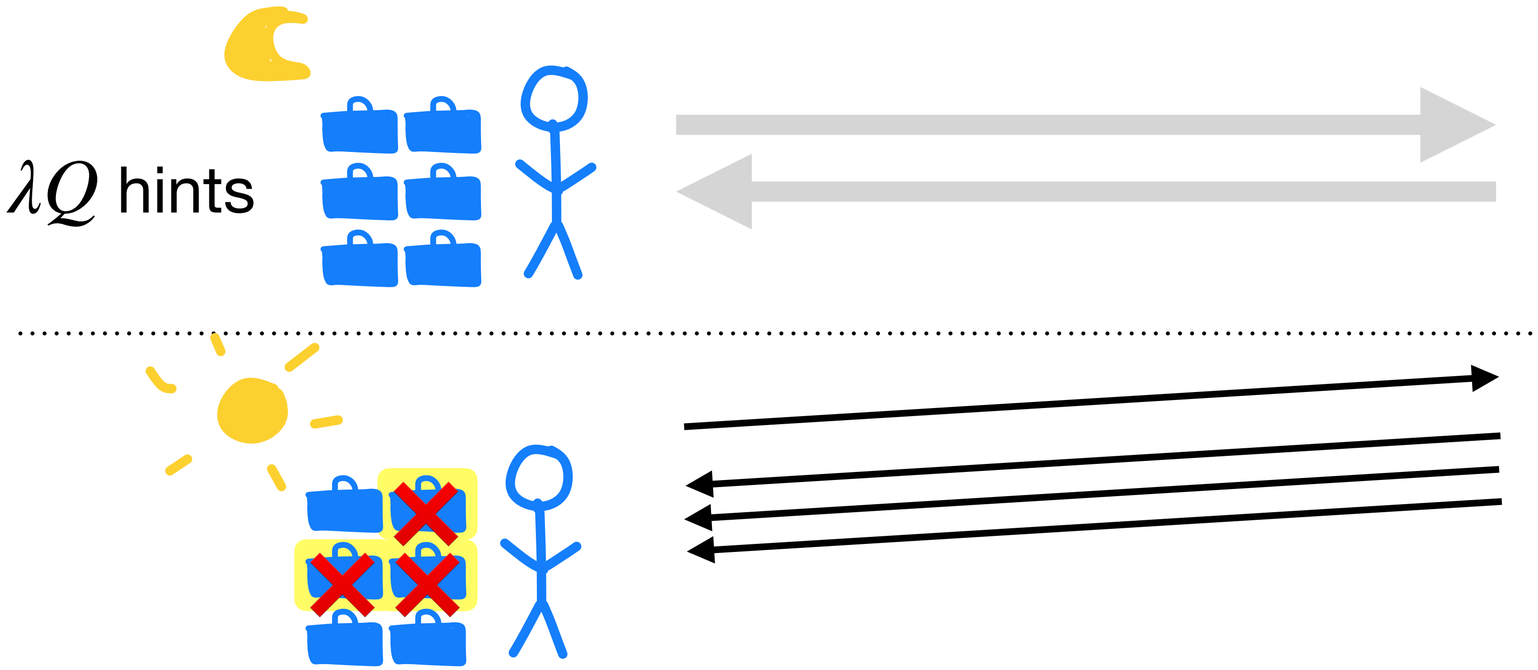

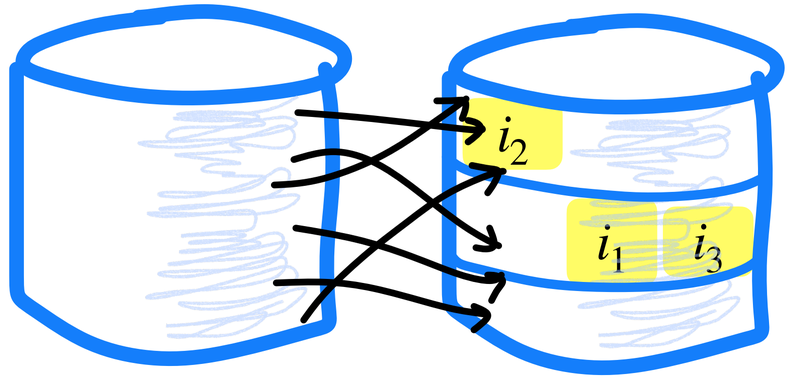

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Security: The client's query does not reveal which chunk it is reading.

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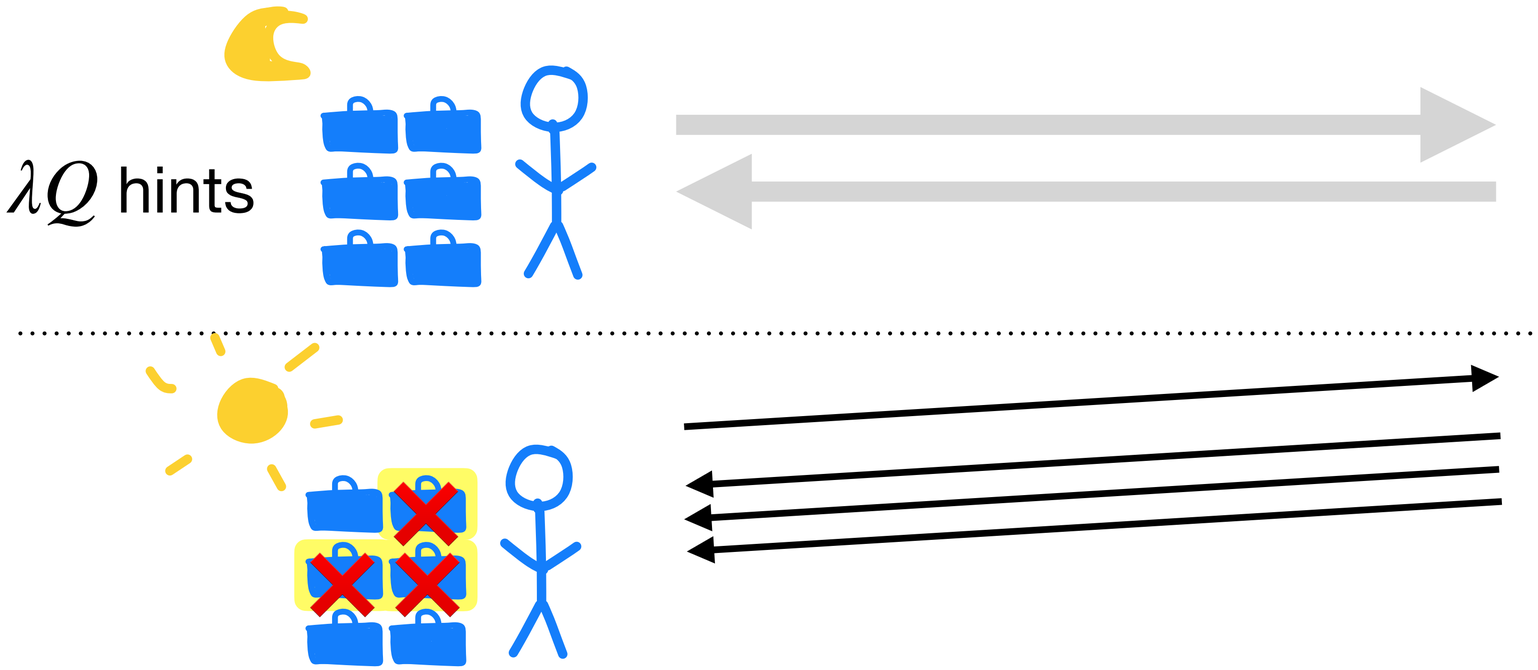

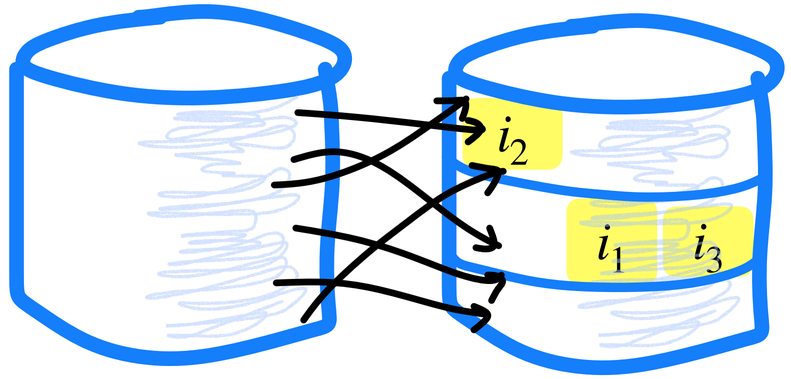

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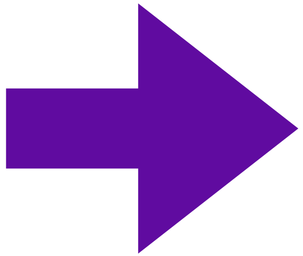
Cost: We ran the underlying PIR λQ times, on database size n/Q.

Proof sketch for Theorem 1

Input: Single-query PIR with sublinear online time [CK20]

- hint size $n^{2/3}$
- offline time n
- online time $n^{2/3}$



Generic compiler, with $Q = n^{1/4}$ queries

Output: Many-query PIR with sublinear amortized time

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Throughout this talk, we omit $\log(n)$ and $\mathrm{poly}(\lambda)$ factors.

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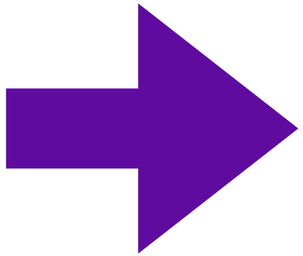

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Theorem 1: From linearly homomorphic encryption.

Under DDH, QR, DCR, or LWE, there is a single-server PIR scheme that, on database size n, when the client makes $\geq n^{1/4}$ adaptive queries, has:

- amortized server time $n^{3/4}$,
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Assuming FHE, we improve the amortized server time and client storage to $n^{1/2}$, if the client makes $n^{1/2}$ adaptive queries.

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Informal claim 1.

Given the parities of $O(Q)$ random, size-n/Q subsets of the database, the client can make Q adaptive queries with online time n/Q.

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Informal claim 2.

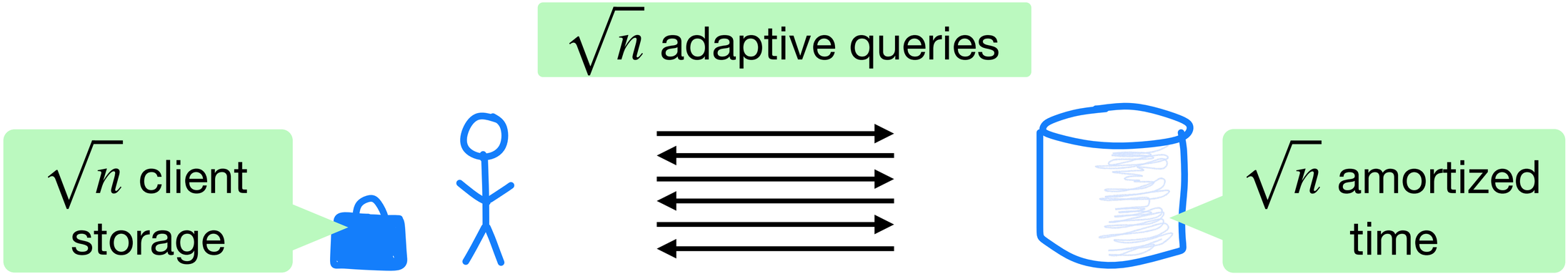
We give a Boolean circuit for retrieving the parities of $O(Q)$ subsets of the database, each of size n/Q, in $O(n)$ gates.

In the offline phase, the server runs the circuit under FHE in linear time.

This talk

1. Background: The offline/online PIR model
2. Our results: New PIR schemes with sublinear time
3. Open questions

Adaptive single-server PIR with sublinear time + storage is feasible:



But, these schemes are not yet efficient enough for use in practice.

- Follow-up work [ZLTS22] improves the communication to $\tilde{O}_\lambda(1)$.
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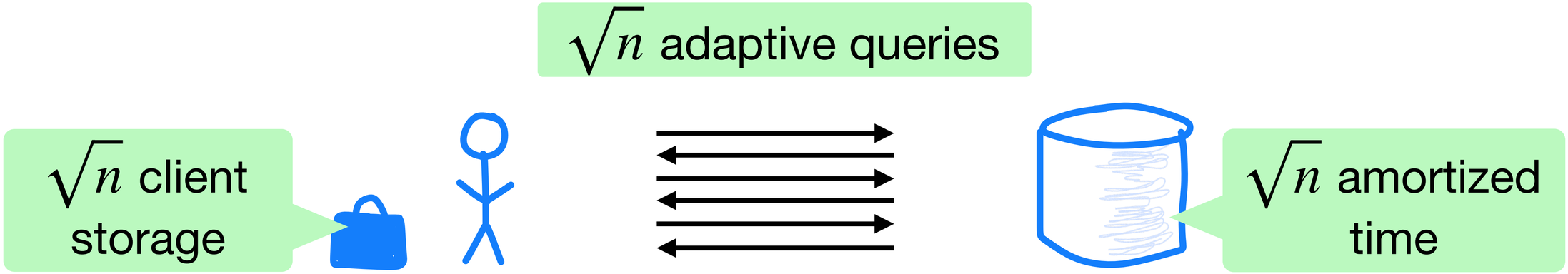

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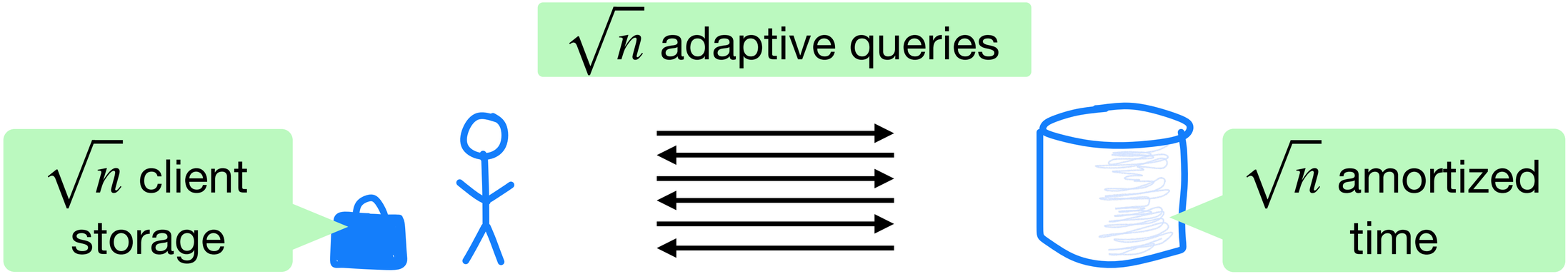

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