

# Key Guessing Strategies for Linear Key-Schedule Algorithms in Rectangle Attacks

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# Outline

- 1 Background and Motivations
- 2 Key-Guessing Strategies in the Rectangle Attack
- 3 Automatic Model For SKINNY
- 4 Improved Rectangle Attacks on SKINNY
- 5 Conclusion and Further Discussion

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# Boomerang and Rectangle Attack

- Boomerang Attack proposed by Wagner [Wag99]

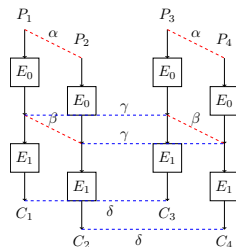
$$E_d = \underbrace{E_1}_{\gamma \xrightarrow{q} \delta} \circ \underbrace{E_0}_{\alpha \xrightarrow{p} \beta}$$

$$\Pr[E_d^{-1}(E_d(x) \oplus \delta) \oplus E_d^{-1}(E_d(x \oplus \alpha) \oplus \delta) = \alpha] = p^2 q^2$$

- Adaptive chosen plaintext and ciphertext  
→ chosen-plaintext attack:

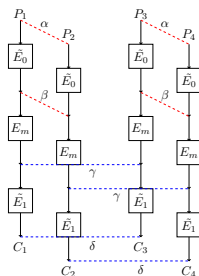
- amplified boomerang attack [KKS00]
- rectangle attack [BDK01]
- probability  $2^{-n} \hat{p}^2 \hat{q}^2$

$$\hat{p} = \sqrt{\sum_i \Pr^2(\alpha \rightarrow \beta_i)}, \hat{q} = \sqrt{\sum_i \Pr^2(\gamma_i \rightarrow \delta)}$$

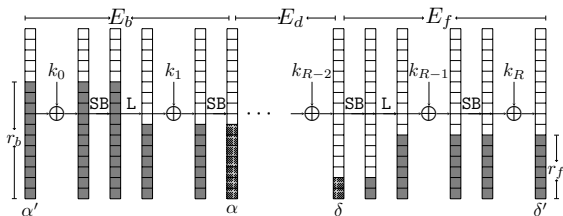


# Boomerang and Retangle Attack

- Improvements based on the dependence between two differentials
  - Boomerang switch [Mur11, BK09, BDK05]
  - Sandwich attack [DKS14]
    - $$E_d = \underbrace{\tilde{E}_1}_{\gamma \xrightarrow{\tilde{q}} \delta} \circ \underbrace{E_M}_{\beta \xrightarrow{t} \gamma} \circ \underbrace{\tilde{E}_0}_{\alpha \xrightarrow{\tilde{p}} \beta}$$
    - probability  $2^{-n} \tilde{p}^2 \tilde{q}^2 t$
  - Boomerang connectivity Table [CHP<sup>+</sup>18]
  - BCT in multiple rounds [WP19, SQH19]



## Generalized Key-Recovery Algorithms for Rectangle Attack



- Attack I: Biham-Dunkelman-Keller's Attack at EC01 [BDK01]
- Attack II: Biham-Dunkelman-Keller's Attack at FSE02 [BDK02]
- Attack III: Zhao *et al.*'s Related-Key Attack at ToSC19 and DCC20 [ZDJ19, ZDM<sup>+</sup>20]

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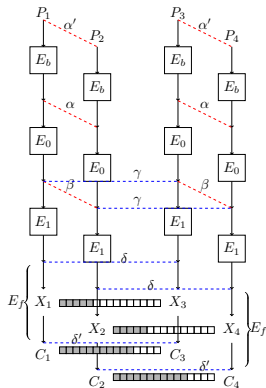
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# Related-key Rectangle Attack with Linear Key Schedule

- Invalid quartet:  $(P_1, P_2, P_3, P_4)$  meet  $(\alpha', \delta')$  but never suggest any key candidates. Guess  $k_{1b}$ , other  $k_{2b}, k_{3b}, k_{4b}$ , are fixed

$$\begin{cases} S(k_{1b} \oplus P_1) \oplus S(k_{2b} \oplus P_2) = \alpha, \\ S(k_{3b} \oplus P_3) \oplus S(k_{4b} \oplus P_4) = \alpha \end{cases}$$

- A tradeoff of the rectangle attack framework with linear key schedule
  - guess  $k_b$  in  $E_b$  and part of  $k_f$  in  $E_f$
  - gain more inactive bits (or fixed differences) from the internal state





# Algorithm 1 (Attack IV)

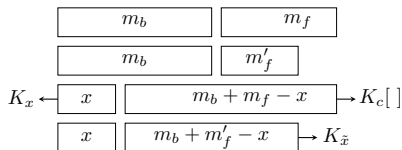
```

1 Construct  $y$  structures of  $2^{r_b}$  plaintexts each. For structure  $i$  ( $1 \leq i \leq y$ ), query the  $2^{r_b}$  plaintexts by
  encryption under  $K_1, K_2, K_3$  and  $K_4$  and store them in  $L_1[i], L_2[i], L_3[i]$  and  $L_4[i]$ 
2 for each of the  $x$ -bit key  $K_x$ , which is a part of  $(m_b + m'_f)$ -bit  $K_1$  do
3    $K_c \leftarrow []$ 
4   for each of  $(m_b + m'_f - x)$ -bit  $K_{\bar{x}}$  of  $K_1$  involved in  $E_b$  and  $E_f$  do
5     for  $i$  from 1 to  $y$  do
6       for  $(P_1, C_1) \in L_1[i]$  do
7          $P_2 = E_{b[K_1 \oplus \Delta K]}^{-1}(E_{bK_1}(P_1) \oplus \alpha), S_1 \leftarrow (P_1, C_1, P_2, C_2)$ 
8       end
9       for  $(P_3, C_3) \in L_3[i]$  do
10         $P_4 = E_{b[K_1 \oplus \Delta K \oplus \nabla K]}^{-1}(E_{b[K_1 \oplus \nabla K]}(P_3) \oplus \alpha), S_2 \leftarrow (P_3, C_3, P_4, C_4)$ 
11      end
12    end
13    for  $(P_1, C_1, P_2, C_2) \in S_1$  do
14       $X_1[1, \dots, h_f] = E_{f[K_1]}^{-1}(C_1), X_2[1, \dots, h_f] = E_{f[K_1 \oplus \Delta K]}^{-1}(C_2)$ 
15       $\tau = (X_1[1, \dots, h_f], X_2[1, \dots, h_f], C_1[1, \dots, n - r_f], C_2[1, \dots, n - r_f]),$ 
16       $H[\tau] \leftarrow (P_1, C_1, P_2, C_2)$ 
17    end
18    for  $(P_3, C_3, P_4, C_4) \in S_2$  do
19       $X_3[1, \dots, h_f] = E_{f[K_1 \oplus \nabla K]}^{-1}(C_3), X_4[1, \dots, h_f] = E_{f[K_1 \oplus \Delta K \oplus \nabla K]}^{-1}(C_4)$ 
20       $\tau' = (X_3[1, \dots, h_f], X_4[1, \dots, h_f], C_3[1, \dots, n - r_f], C_4[1, \dots, n - r_f])$ 
21      Access  $H[\tau']$  to find  $(P_1, C_1, P_2, C_2)$  to generate quartet  $(C_1, C_2, C_3, C_4)$ 
22      for each generated quartet do
23        Determine other  $(m_f - m'_f)$ -bit key  $k_f'', K_c[K_{\bar{x}} \| k_f''] \leftarrow K_c[K_{\bar{x}} \| k_f''] + 1$ 
24      end
25    end
26  end
27 end
  
```

Select top  $2^{m_b + m'_f - x - h}$  hits in  $K_c$ . Guess remaining  $k - (m_b + m'_f)$ -bit key to check the full key

# Related-key Rectangle Attack with Linear Key Schedule

- The guessed keys in Algorithm 1



- $m_b$ -bit  $k_b$  and  $m_f$ -bit  $k_f$  in total
- guess  $m_b$ -bit  $k_b$  and  $m'_f$ -bit  $k'_f$  before generating quartets
- guess  $x$ -bit  $K_x$  before initializing the key counter  $K_c[ ]$

# Complexity

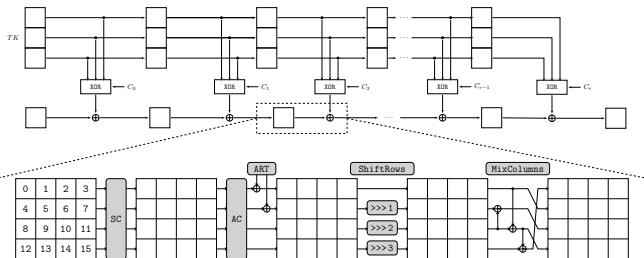
- set  $(y \cdot 2^{2r_b})^2 \cdot 2^{-2r_b} \cdot 2^{-n} \hat{p}^2 \hat{q}^2 = s$ ,  $s$  is the expected number of right quartets
- Data complexity:  $4y \cdot 2^{r_b} = \sqrt{s} \cdot 2^{n/2+2} / \hat{p} \hat{q}$
- Time complexity:
  - **Time I** :  $T_1 = 2^{x+m_b+m'_f-x} \cdot y \cdot 2^{r_b} \cdot 2 = \sqrt{s} \cdot 2^{m_b+m'_f+n/2+1} / \hat{p} \hat{q}$
  - **Time II** :  $T_2 = (s \cdot 2^{m_b+m'_f-n+2r_f-2h_f} / \hat{p}^2 \hat{q}^2) \cdot \varepsilon$
  - **Time III** :  $T_3 = 2^x \cdot 2^{m_b+m_f-x-h} \cdot 2^{k-(m_b+m_f)} = 2^{k-h}$
- Memory complexity of the key counters and data structures :  
 $2^{m_b+m_f-x} + 4y \cdot 2^{r_b} = 2^{m_b+m_f-x} + \sqrt{s} \cdot 2^{n/2+2} / \hat{p} \hat{q}$
- Success Probability:  $P_s = \Phi\left(\frac{\sqrt{sS_N} - \Phi^{-1}(1-2^{-h})}{\sqrt{S_N+1}}\right)$  [Sel08]
  - $h < m_b + m_f - x$ ,  $x \leq m_b + m'_f$

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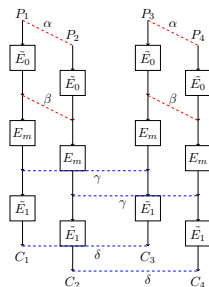
# Introduction to SKINNY

- Proposed by Beierle et al. [BJK<sup>+</sup>16]
- SKINNY- $n$ - $t$ : block size  $n \in \{64, 128\}$ , tweak size  $t = n, 2n$  or  $3n$



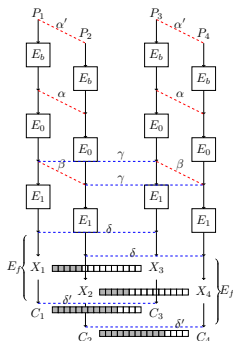
# Previous Automatic Search Models for Boomerang Distinguishers on SKINNY

- **MILP**, Beierle et al. [BJK<sup>+</sup>16]
- **MILP**, Liu et al. [LGS17]
- **MILP+SAT**, Hadipour et al. [HBS21]
  - considering the switching effect in multiple rounds
- **MILP+CP**, Delaune et al. [DDV20]
  - automatically handling the middle rounds
- **MILP+CP**, Qin et al. [QDW<sup>+</sup>21]
  - search the entire  $(N_b + N_d + N_f)$ -round attack

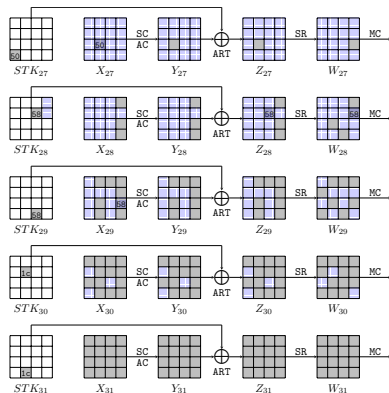


# Our Model to Determine the Optimal Distinguisher

- search the entire  $(N_b + N_d + N_f)$ -round attack for our new rectangle framework
  - based on [HBS21, DDV20, QDW<sup>+</sup>21]
  - previous constraints:  $m_b, r_f, \tilde{p}^2 \tilde{q}^2 t$
  - new constraints :  $m_f, m'_f, h_f, h$
  - objective :  $\min\{T_1, T_2, T_3\}$



# Modelling Propagation of Cells with Known Differences in $E_f$



- $DXFixed(DWFixed)$ : cells with known differences in  $X_r(W_r)$ , marked by  $\square$  and  $\blacksquare$
- the impact of SC and SR :  
 $DWFixed[r][i] = \neg DXL[r_m + r_1 + r][P_{SR}[i]]$
- the impact of MC :  
 $DXFixed[r+1][i] = DWFixed[r][i] \wedge DWFixed[r][i+8] \wedge DWFixed[r][i+12]$

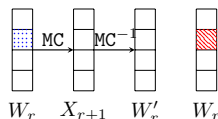
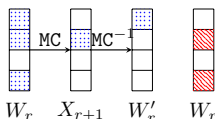


Modeling Cells that could be Used to Filter Quartets in  $E_f$ 

- DXFilter: cells can be used as filters in  $X_r$ 
  - cells with nonzero fixed difference:  $\blacksquare \xrightarrow{SC} \blacksquare$

DXFixed[r][i]	DXL[r <sub>m</sub> + r <sub>1</sub> + r][i]	DXFilter[r][i]
0	1	0
1	0	0
1	1	1

- DWFilter: cells can be used as filters in  $W_r$ 
  - $r = N_f - 1$ , DWFilter[ $N_f - 1$ ][i] = DWFixed[ $N_f - 1$ ][i]
  - $0 \leq r \leq N_f - 2$ , extra cells with fixed differences in  $W_r$  than  $W'_r$  (deduced from  $X_{r+1}$ ) can act as filters



# Modeling $m'_f$ -bit Subtweakeys in $E_f$ for Generating Quartets

- DXGuess (DWGuess): cells need to know in  $X_r(W_r)$  in decryption from ciphertexts to the filters
- $0 \leq r \leq N_f - 1$ , the cells in  $W_r$  need to know involve two types
  - cells to be known from  $X_r$  over the SR operation
  - cells used as filters in  $W_r$

$$\text{DWGuess}[r][i] = \text{DWisFilter}[r][i] \vee \text{DXGuess}[r][P_{\text{SR}}[i]]$$

- $0 \leq r \leq N_f - 2$ , the cells in  $X_{r+1}$  need to know involve two types:
  - cells need to know from  $W_r$  over the MC operation
  - cells used as filters in  $X_{r+1}$

# Modeling the Advantage $h$ in the Key-recovery Attack

- Adv: the advantage  $h$  determines the exhaustive search time  $T_3$
- KnownDec: the cells in  $Y_r$  need to know in the decryption from ciphertext to  $\delta$
- $h \leq m_b + m_f - x$  and  $x \leq m_b + m'_f$ :

$$\begin{cases} x \leq \sum_{0 \leq r \leq N_b - 2, 0 \leq i \leq 7} \text{KnownEnc}[r][i] + \sum_{0 \leq r \leq N_f - 1, 0 \leq i \leq 7} \text{DXGuess}[r][i] \\ \text{Adv} + x \leq \sum_{0 \leq r \leq N_b - 2, 0 \leq i \leq 7} \text{KnownEnc}[r][i] + \sum_{0 \leq r \leq N_f - 1, 0 \leq i \leq 7} \text{KnownDec}[r][i] \end{cases}$$

# The Objective Function

- Time complexities: **Time I** ( $T_1$ ), **Time II** ( $T_2$ ) and **Time III** ( $T_3$ )

- $$\text{DXU} \xrightarrow[w_0]{\text{upper differential}} \tilde{p}, \text{DXL} \xrightarrow[w_1]{\text{lower differential}} \tilde{q}, \text{DXU} \wedge \text{DXL} \xrightarrow[w_m]{\text{middle part}} t$$

- $$\text{KnownEnc} \xrightarrow[w_{m_b}]{} m_b, \text{DXGuess} \xrightarrow[w_{m_f}]{} m'_f$$

- $$\text{DXisFilter} + \text{DWisFilter} \xrightarrow[w_{h_f}]{} h_f$$

- $$\text{Adv} \rightarrow h$$

- The uniformed objective:

Minimize  $obj$ ,  $obj \geq T_1$ ,  $obj \geq T_2$ ,  $obj \geq T_2$ .

# New Distinguishers of SKINNY

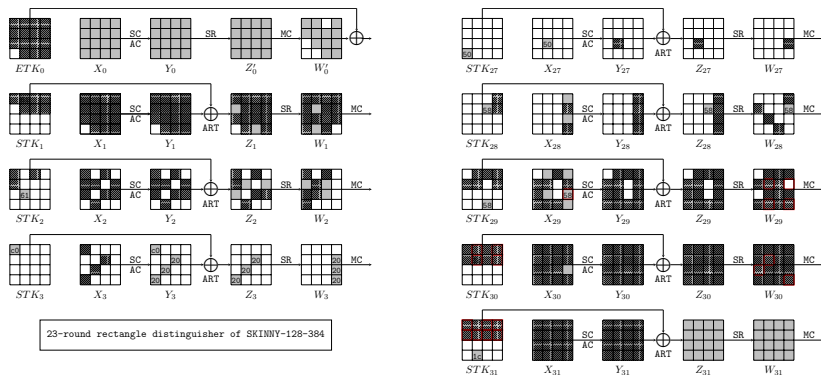
- MILP+CP model (based on [DDV20, HBS21, QDW<sup>+</sup>21])

Version	$N_d$	Probability $\bar{p}^2 \bar{q}^2 t$	$N_b + N_d + N_f$	Ref.
64-128	17	$2^{-29.78}$	-	[SQH19]
	17	$2^{-48.72}$	21	[LGS17]
	19	$2^{-51.08}$	23	[HBS21]
	19	$2^{-54.36}$	-	[DDV20]
	18	$2^{-55.34}$	24	[QDW <sup>+</sup> 21]
	<b>18</b>	$2^{-55.34}$	<b>25</b>	Ours
64-192	22	$2^{-42.98}$	-	[SQH19]
	22	$2^{-54.94}$	26	[LGS17]
	23	$2^{-55.85}$	29	[HBS21]
	23	$2^{-57.93}$	-	[DDV20]
	22	$2^{-57.73}$	30	[QDW <sup>+</sup> 21]
	<b>22</b>	$2^{-57.56}$	<b>31</b>	Ours
128-256	18	$2^{-77.83}$	-	[SQH19]
	18	$2^{-103.84}$	22	[LGS17]
	20	$2^{-85.77}$	-	[DDV20]
	21	$2^{-116.43}$	24	[HBS21]
	19	$2^{-116.97}$	25	[QDW <sup>+</sup> 21]
	18	$2^{-108.51}$	25	Ours
<b>19</b>	$2^{-121.07}$	<b>26</b>	Ours	
128-384	22	$2^{-48.30}$	-	[SQH19]
	23	$2^{-112}$	27	[LGS17]
	23	$2^{-112}$	28	[ZDM <sup>+</sup> 20]
	24	$2^{-86.09}$	-	[DDV20]
	25	$2^{-116.59}$	30	[HBS21]
	22	$2^{-101.49}$	30	[QDW <sup>+</sup> 21]
	<b>23</b>	$2^{-115.09}$	<b>32</b>	Ours

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## Improved Attack on 32-round SKINNY-128-384



- 4-round  $E_b$  + 23-round distinguisher + 5-round  $E_f$ 
  - $W'_0 \rightarrow$  plaintext,  $Z_{31} \rightarrow$  ciphertext
  - $r_b = 12c$ ,  $m_b = 18c$ ,  $r_f = 16c$ ,  $m_f = 24c$

## Internal State Used for Filtering and Involved Subtweakeys

	Round	Filter	Involved subtweakeys
1	30	$\Delta W_{30}[5] = 0$	$STK_{31}[5]$
2		$\Delta W_{30}[8] = 0$	$STK_{31}[4]$
3		$\Delta W_{30}[15] = 0$	$STK_{31}[3]$
4	29	$\Delta W_{29}[5] = 0$	$STK_{30}[5], STK_{31}[0, 6, 7]$
5		$\Delta W_{29}[7] = 0$	$STK_{30}[7], STK_{31}[2, 4, 5]$
6		$\Delta W_{29}[10] = 0$	$STK_{30}[6], STK_{31}[1, 7]$
7		$\Delta W_{29}[13] = 0$	$STK_{30}[1], STK_{31}[0, 5]$
8		$\Delta W_{29}[15] = 0$	$STK_{30}[3], STK_{31}[2, 7]$
9		$\Delta X_{29}[11] = 0x58$	$STK_{30}[5], STK_{31}[0, 6]$
10	28	$\Delta W_{28}[5] = 0$	$STK_{29}[5], STK_{30}[0, 6, 7], STK_{31}[1, 2, 3, 4, 7]$
11		$\Delta W_{28}[11] = 0$	$STK_{29}[7], STK_{30}[2, 4], STK_{31}[1, 3, 5, 6]$
12		$\Delta W_{28}[13] = 0$	$STK_{29}[1], STK_{30}[0, 5], STK_{31}[3, 4, 6]$
13		$\Delta W_{28}[15] = 0$	$STK_{29}[3], STK_{30}[2, 7], STK_{31}[1, 4, 6]$
14	27	$\Delta W_{27}[7] = 0$	$STK_{28}[7], STK_{29}[2, 4, 5], STK_{30}[0, 1, 3, 5, 6], STK_{31}[0, 1, 2, 3, 4, 5, 6, 7]$
15		$\Delta W_{27}[15] = 0$	$STK_{28}[3], STK_{29}[2, 7], STK_{30}[1, 4, 6], STK_{31}[0, 3, 5, 6, 7]$
16		$\Delta X_{27}[9] = 0x50$	$STK_{28}[7], STK_{29}[2, 4], STK_{30}[1, 3, 5, 6], STK_{31}[0, 1, 2, 5, 6, 7]$

- $k'_f = \{STK_{30}[1, 3, 5, 7], STK_{31}[0, 2, 3, 4, 5, 6, 7]\}$  ( $m'_f = 11c$ )
- $h_f = \{X_{29}[11], W_{29}[5, 7, 13, 15], W_{30}[5, 8, 15]\}$  ( $h_f = 8c$ )



## Tweakey Recovery for 32-round SKINNY-128-384

Step	Internal state	Involved subweakeys
A	$Z_{30}[6]$ $Z_{30}[14]$	$STK_{31}[7]$ $STK_{31}[1]$
B	$Z_{29}[1]$ $Z_{29}[9]$ $Z_{29}[13]$	$STK_{30}[5], STK_{31}[6]$ $STK_{30}[7], STK_{31}[2, 4]$ $STK_{30}[0], STK_{31}[3, 4]$
C	$Z_{29}[3]$ $Z_{29}[7]$ $Z_{29}[15]$	$STK_{30}[7], STK_{31}[4]$ $STK_{30}[4], STK_{31}[3, 5, 6]$ $STK_{30}[2], STK_{31}[1, 6]$
D	$Z_{28}[3]$ $Z_{28}[11]$ $Z_{28}[15]$	$STK_{29}[7], STK_{30}[4], STK_{31}[3, 5, 6]$ $STK_{29}[5], STK_{30}[0, 6], STK_{31}[1, 3, 4, 7]$ $STK_{29}[2], STK_{30}[1, 6], STK_{31}[0, 5, 7]$
E	$X_{27}[9]$	$STK_{28}[7], STK_{29}[2, 4], STK_{30}[1, 3, 5, 6], STK_{31}[0, 1, 2, 5, 6, 7]$

- set  $s = 1$ ,  $h = 40$ ,  $x = 208$
- **Time complexity:**  $2^{354.99}$ 
  - $T_1 = \sqrt{s} \cdot 2^{m_b+m'_f+n/2+1} / \sqrt{\tilde{p}^2 t \tilde{q}^2} = 2^{354.54}$
  - $T_3 = s \cdot 2^{m_b+m'_f-2h_f-n+2r_f} / (\tilde{p}^2 t \tilde{q}^2) \cdot \varepsilon = 2^{353.1}$
  - $T_3 = 2^{k-h} = 2^{344}$
- Success probability [Sel08]:  $P_s = \Phi\left(\frac{\sqrt{sS_N} - \Phi^{-1}(1-2^{-h})}{\sqrt{S_N+1}}\right) = 82.1\%$

## Summary of Attacks on Skinny in Related-tweakey Setting

Version	Rounds	Data	Time	Memory	Approach	Setting	Ref.
64-128	22	$2^{63.5}$	$2^{110.9}$	$2^{63.5}$	Rectangle	RK	[LGS17]
	23	$2^{62.47}$	$2^{125.91}$	$2^{124}$	ID	RK	[LGS17]
	23	$2^{62.47}$	$2^{124}$	$2^{77.47}$	ID	RK	[SMB18]
	23	$2^{71.4}$	$2^{79}$	$2^{64.0}$	ID	RK	[ABC <sup>+</sup> ]
	23	$2^{60.54}$	$2^{120.7}$	$2^{60.9}$	Rectangle	RK	[HBS21]
	24	$2^{61.67}$	$2^{96.83}$	$2^{84}$	Rectangle	RK	[QDW <sup>+</sup> 21]
	<b>25</b>	$2^{61.67}$	$2^{118.43}$	$2^{64.26}$	Rectangle	RK	Ours
64-192	27	$2^{63.5}$	$2^{165.5}$	$2^{80}$	Rectangle	RK	[LGS17]
	29	$2^{62.92}$	$2^{181.7}$	$2^{80}$	Rectangle	RK	[HBS21]
	30	$2^{62.87}$	$2^{163.11}$	$2^{68.05}$	Rectangle	RK	[QDW <sup>+</sup> 21]
	<b>31</b>	$2^{62.78}$	$2^{182.07}$	$2^{62.79}$	Rectangle	RK	Ours
128-256	22	$2^{127}$	$2^{235.6}$	$2^{127}$	Rectangle	RK	[LGS17]
	23	$2^{124.47}$	$2^{251.47}$	$2^{248}$	ID	RK	[LGS17]
	23	$2^{124.41}$	$2^{243.41}$	$2^{155.41}$	ID	RK	[SMB18]
	24	$2^{125.21}$	$2^{209.85}$	$2^{125.54}$	Rectangle	RK	[HBS21]
	25	$2^{124.48}$	$2^{226.38}$	$2^{168}$	Rectangle	RK	[QDW <sup>+</sup> 21]
	<b>25</b>	$2^{120.25}$	$2^{193.91}$	$2^{136}$	Rectangle	RK	Ours
	<b>26</b>	$2^{126.53}$	$2^{254.4}$	$2^{128.44}$	Rectangle	RK	Ours
128-384	27	$2^{123}$	$2^{331}$	$2^{155}$	Rectangle	RK	[LGS17]
	28	$2^{122}$	$2^{315.25}$	$2^{122.32}$	Rectangle	RK	[ZDM <sup>+</sup> 20]
	30	$2^{125.29}$	$2^{361.68}$	$2^{125.8}$	Rectangle	RK	[HBS21]
	30	$2^{122}$	$2^{341.11}$	$2^{128.02}$	Rectangle	RK	[QDW <sup>+</sup> 21]
	<b>32</b>	$2^{123.54}$	$2^{354.99}$	$2^{123.54}$	Rectangle	RK	Ours

# Outline

- 1 Background and Motivations
- 2 Key-Guessing Strategies in the Rectangle Attack
- 3 Automatic Model For SKINNY
- 4 Improved Rectangle Attacks on SKINNY
- 5 Conclusion and Further Discussion**

## Application to ForkSkinny, Deoxys-BC and GIFT

Version	Rounds	Data	Time	Memory	Approach	Setting	Ref.
<b>ForkSkinny</b>							
128-256 (256-bit key)	26	$2^{125}$	$2^{254.6}$	$2^{160}$	ID	RK	[BDL20]
	26	$2^{127}$	$2^{250.3}$	$2^{160}$	ID	RK	[BDL20]
	28	$2^{118.88}$	$2^{246.98}$	$2^{136}$	Rectangle	RK	[QDW <sup>+</sup> 21]
	<b>28</b>	$2^{118.88}$	<b><math>2^{224.76}</math></b>	$2^{118.88}$	Rectangle	RK	Ours
<b>Deoxys-BC</b>							
128-384	13	$2^{127}$	$2^{270}$	$2^{144}$	Rectangle	RK	[CHP <sup>+</sup> 17]
	14	$2^{127}$	$2^{286.2}$	$2^{136}$	Rectangle	RK	[ZDJ19]
	14	$2^{125.2}$	$2^{282.7}$	$2^{136}$	Rectangle	RK	[ZDJM19]
	<b>14</b>	$2^{125.2}$	<b><math>2^{260}</math></b>	$2^{140}$	Rectangle	RK	Ours
<b>GIFT</b>							
64-128	25	$2^{63.78}$	$2^{120.92}$	$2^{64.1}$	Rectangle	RK	[JZZD20]
	26	$2^{60.96}$	$2^{123.23}$	$2^{102.86}$	Differential	RK	[SWW21]
	<b>26</b>	$2^{63.78}$	<b><math>2^{122.78}</math></b>	$2^{63.78}$	Rectangle	RK	Ours

# Application in Single-key Setting

- Rectangle attack in single-key model: set differences of keys to 0
  - set  $y = \sqrt{s} \cdot 2^{n/2-r_b+1.5} / \hat{p}\hat{q}$ , data complexity  $y \cdot 2^{r_b}$
  - $T_1 = 2^{m_b+m'_f} \cdot y \cdot 2^{r_b} = \sqrt{s} \cdot 2^{m_b+m'_f+\frac{n}{2}+1.5} / \hat{p}\hat{q}$
  - $T_2 = (s \cdot 2^{m_b+m'_f-n+2r_f-2h_f} / \hat{p}^2\hat{q}^2) \cdot \varepsilon$
- Application to Serpent:
  - reuse the rectangle distinguisher by Biham, Dunkelman and Keller [BDK01]

Key size	Time	Data	Memory	Approach	Ref.
256	$2^{173.8}$	$2^{126.3}$	$2^{126.3}$	rectangle	[BDK02]
256	$2^{164}$	$2^{126.3}$	$2^{126.3}$	rectangle	ours

# Overall Analysis of the Four Attack Models

- For generating quartets:
  - Attack I: guesses all the  $(m_b + m_f)$ -bit key at once
  - Attack II: does not guess the key involved in  $E_b$  and  $E_f$
  - Attack III: guesses  $m_b$ -bit key in  $E_b$
  - Our new attack: guesses  $m_b$ -bit key in  $E_b$  and  $m'_f$ -bit key in  $E_f$
- Time complexities:
  - Attack I:  $T_I = 2^{m_b+m_f+n/2+t+2}$
  - Attack II:  $T_{II} = 2^{m_b+r_b+2r_f-n+2t} + 2^{m_f+2r_b+r_f-n+2t}$
  - Attack III:  $T_{III} = 2^{m_b+2r_f-n+2t} \cdot \varepsilon$
  - Attack IV:  $T_{IV} = 2^{m_b+2r_f-n+m'_f-2h_f+2t} \cdot \varepsilon$

**Thanks for Your Attention!**

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