

# Efficient Lattice-Based Blind Signatures via Gaussian One-Time Signatures

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# Notations

- $q$  a prime modulus for the blind signature ( $\simeq 64$  bits)
- $q'$  a prime modulus for the encryption scheme ( $\simeq 128$  bits)
- $d$  is a power of two ( $= 128$ )
- $\mathbb{Z}_q$  is the field of integers modulo  $q$
- $\mathcal{R}_q$  is the cyclotomic ring  $\mathbb{Z}_q[X]/(X^d + 1)$
- $S_\gamma$  is the set of ring elements of infinity norm less than  $\gamma$
- $n, m, k, l$  are dimensions over  $\mathcal{R}_q$

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The scheme is round-optimal (2 rounds of communication) and the signature size is  $\simeq 150$  KB.

## Lattice-Based One-Time Signatures (OTS).



# Lattice-Based Construction from [LM18]

## Setup

- Uniformly random matrix  $\mathbf{A} \leftarrow \mathcal{R}_q^{n \times m}$
- Secret key :  $(\mathbf{s}, \mathbf{y}) \leftarrow \{(\mathbf{s}, \mathbf{y}) \in \mathcal{R}_q^m \times \mathcal{R}_q^m / \|(\mathbf{s}, \mathbf{y})\|_\infty \leq b\}$
- Public key :  $(\mathbf{v}, \mathbf{w}) := (\mathbf{As}, \mathbf{Ay})$

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## Verify

To verify a signature  $\mathbf{z}$  : check that  $\mathbf{A}\mathbf{z} \stackrel{?}{=} \mu\mathbf{v} + \mathbf{w}$ , and  $\|\mathbf{z}\| \leq kb$ .

# Overview of the Security

Consider an adversary  $\mathcal{A}$  against the unforgeability game of the OTS that sees a signature  $\sigma(= \mathbf{y} + \mu\mathbf{s})$  of some message  $\mu$  of his choice.

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The parameters of the scheme are chosen so there exists at least another pair  $(\mathbf{s}^*, \mathbf{y}^*)$  such that

$$\mathbf{As}^* = \mathbf{v}, \mathbf{Ay}^* = \mathbf{w}, \sigma = \mathbf{y}^* + \mu\mathbf{s}^*, \text{ and } \|(\mathbf{s}^*, \mathbf{y}^*)\|_\infty \leq b.$$

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- From the adversary's perspective, both worlds are information-theoretically indistinguishable, therefore with probability at least  $1/2$ ,  $\mathbf{z} \neq \mathbf{z}'$ .

# Gaussian version of [LM18]

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- Secret key :  $(\mathbf{s}, \mathbf{y}) \leftarrow D_{\sigma_s}^m \times D_{\sigma_y}^m$
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Signature of a message  $\mu \in \{0, 1\}^d$  :  $\mathbf{z}(\mu) := \mu\mathbf{s} + \mathbf{y}$

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# Security of the Gaussian version

- For the original uniform distribution, one takes  $n, m, b$  such that there exists at least two solutions to

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- For the Gaussian version, we take  $n, m, \sigma_s, \sigma_y$  such that

$$\max_{(\mathbf{s}^*, \mathbf{y}^*)} D_{\Lambda, \sigma}(\mathbf{s}^*, \mathbf{y}^*) \leq 1/2,$$

where  $\Lambda = \{(\mathbf{s}^*, \mathbf{y}^*) \in \mathcal{R}_q^m \times \mathcal{R}_q^m / \mathbf{A}\mathbf{s}^* = \mathbf{v}, \mathbf{A}\mathbf{y}^* = \mathbf{w}, \mathbf{z} = \mathbf{y}^* + \mu\mathbf{s}^*\}$  is a coset of a lattice and  $\sigma = (\sigma_s, \dots, \sigma_s, \sigma_y, \dots, \sigma_y)$ .

Blind signatures.



# Blind Signature Definition

Two parties : the user and the signer



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## Security notions

- *Blindness*. The signer cannot learn anything about  $\mu$ , nor during which interaction the signature  $\sigma$  was produced.
- *One-More Unforgeability*. After some number  $\ell$  of interactions with the signer, the user cannot produce  $\ell + 1$  valid signatures for the public key of the signer.

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- The public key is a collection of  $N$  OTS public keys
- The user sends his message to the signer
- The signer sends the OTS of the message for the  $i$ -th public key
- The blind signature is a zero-knowledge proof of knowledge of some  $\mathbf{z}$  such that

$$\mathbf{Az} \in \{\mu\mathbf{v}_i + \mathbf{w}_i, i \in [N]\}.$$

# Intuition of our construction

- The public key is a collection of  $N$  OTS public keys
- The user **encrypts his message**, and sends the ciphertext together with a well-formedness zero-knowledge proof to the signer
- The signer **homomorphically computes an encryption of the OTS** of the ciphertext for the  $i$ -th public key
- The user **decrypts the signer's response**. The blind signature is a zero-knowledge proof of knowledge of some  $\mathbf{z}$  such that

$$\mathbf{Az} \in \{\mu\mathbf{v}_i + \mathbf{w}_i, i \in [N]\}.$$

# Blind signature scheme : setup

## Server setup

- 1 Sample a trapdoor :  $\mathbf{R} \leftarrow (S_1^{n \times n})^{2 \times 3}$
- 2  $\mathbf{A}' \leftarrow \mathcal{R}_q^{n \times 2n}$ ,  $\mathbf{A} = [\mathbf{A}' | \mathbf{A}'\mathbf{R} - \mathbf{G}] \in \mathcal{R}_q^{n \times m}$
- 3 For  $i \in [N]$ ,  $(\mathbf{v}_i, \mathbf{w}_i) \leftarrow \mathcal{R}_q^m \times \mathcal{R}_q^m$

Public key :  $(\mathbf{A}, (\mathbf{v}_i, \mathbf{w}_i)_{i \in [N]})$ , Secret key :  $\mathbf{R}$

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For each signature, the server will sample a secret key  $(\mathbf{s}_i, \mathbf{y}_i)$  for the OTS public key  $(\mathbf{v}_i, \mathbf{w}_i)$  using the trapdoor  $\mathbf{R}$

For each signature, the user generates a key pair  $(pk, sk)$  for an encryption enc and runs the setup of a set membership proof.

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  - >  $\mathbf{z} = \text{dec}(\text{sk}, \mathbf{F}, \mathbf{f}')$
  - > Verify that  $\exists j \in [N]$  such that  $\mathbf{A}\mathbf{z} = \mu\mathbf{v}_j + \mathbf{w}_j$
  - > Return the signature  $\pi_{\in}(\mathbf{A}, (\mathbf{w}_j + \mu\mathbf{v}_j)_{j \in [N]}, \mathbf{z})$

# Encryption scheme I

## Key generation

$$\rightarrow \mathbf{B} \leftarrow \mathcal{R}_{q'}^{k \times l}$$

$$\rightarrow \mathbf{x} \leftarrow S_y^k$$

$$\rightarrow \mathbf{b}^T = \mathbf{x}^T \mathbf{B} \pmod{q'}$$

$$\rightarrow \text{pk} = (\mathbf{B}, \mathbf{b}), \text{sk} = \mathbf{x}$$

# Encryption scheme II

$\text{enc}(\text{pk}, \mu)$ :

- $(\mathbf{r}, \mathbf{e}, e') \leftarrow S_\gamma^l \times S_\gamma^k \times S_{\text{gamma}}$
- $\mathbf{t} = p\mathbf{B}\mathbf{r} + p\mathbf{e} \pmod{q'}$
- $t' = p\mathbf{b}^T\mathbf{r} + pe' + \mu \pmod{q'}$
- Return  $(\mathbf{t}, t')$

$\text{dec}(\text{sk}, \mathbf{t}, t')$

- $z = t' - \mathbf{x}^T\mathbf{t} \pmod{q'}$
- Return  $z \pmod{p}$

# Blind computation on the ciphertext

The encryption scheme is a Regev-type encryption. The signer can homomorphically compute an encryption of  $\mathbf{z} := \mathbf{y} + \mu\mathbf{s}$  as  $\mathbf{F} = \mathbf{t}\mathbf{s}_i^T$ ,  $\mathbf{f}' = \mathbf{y}_i + t'\mathbf{s}_i$ .

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- $\mathbf{m}' = p \mathbf{b}^T \mathbf{Y} + p \mathbf{y}' + \mu \pmod{q'}$  for  $t'$
- Set  $\mathbf{F} = \mathbf{t} \mathbf{s}_i^T + \mathbf{M}$ ,  $\mathbf{f}' = \mathbf{y}_i + t' \mathbf{s}_i + \mathbf{m}'$

# Parameters

Parameter	Definition	Instantiation
$N$	maximum number of signing queries	$2^{18}$
$d$	dimension of the ring $\mathcal{R}$	128
$q$	modulus for the blind signature	$\approx 2^{64}$
$q'$	modulus for the encryption	$\approx 2^{128}$
$\alpha$	height of the matrix $\mathbf{A} = [\mathbf{A}'   \mathbf{A}'\mathbf{R} - \mathbf{G}]$	21
$\sigma_y$	standard deviation for sampling $\mathbf{y}$	$\approx 2^{30}$
$\sigma_s$	standard deviation for sampling $\mathbf{s}$	$\approx 2^{30}$
$n$	height of the encryption public key matrix $\mathbf{B}$	80
$m$	width of the encryption public key matrix $\mathbf{B}$	40
$\gamma$	maximum coefficient of the $\mathbf{x}$ , $\mathbf{r}$ and errors $\mathbf{e}$ , $\mathbf{e}'$	4
$p$	additional prime number, less than $q'$ , used for encryption	$\approx 2^{43}$
$\sigma'$	standard deviation used to sample maskings $\mathbf{Y}$ , $\mathbf{Y}'$ and $\mathbf{y}''$	$\approx 2^{26}$

**Fig. 3.** Definition and concrete numbers for parameters used in the blind signature construction.

Sizes :

Public key : 1.3 MB, Secret key : 75 KB, Signature 150 KB,  
Communication 16 MB

# Bibliography I



Vadim Lyubashevsky and Daniele Micciancio.  
Asymptotically efficient lattice-based digital signatures.  
*Journal of Cryptology*, 31(3):774–797, 2018.