On Pairing-Free Blind Signature Schemes in the Algebraic Group Model

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CISPA

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Algorand

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Three-Move Blind Signature Schemes

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sk

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pk, m

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$\sigma \xleftarrow{\$} \mathsf{User}_2(\mathsf{pk}, R, s, m)$

Security Notions - Blindness

sk

pk

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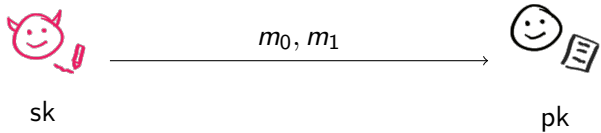

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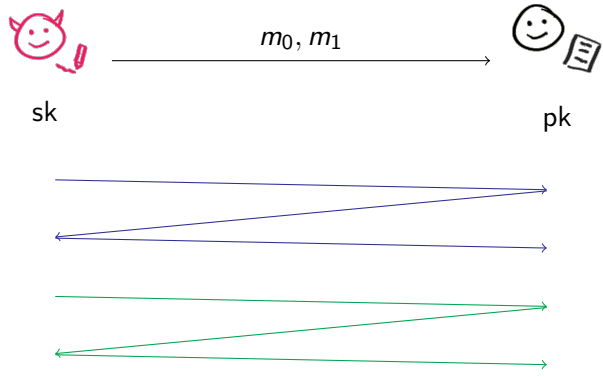

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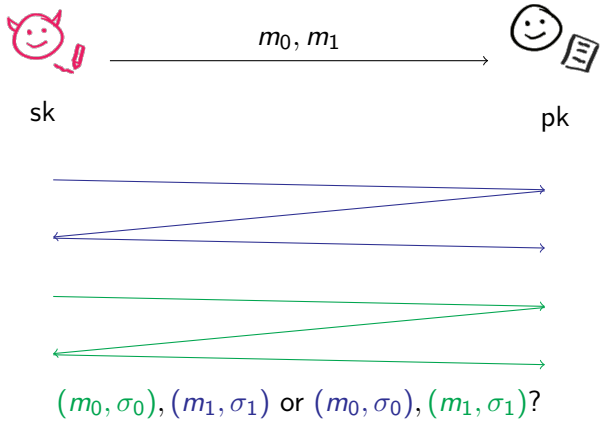

Security Notions - One-more-unforgeability

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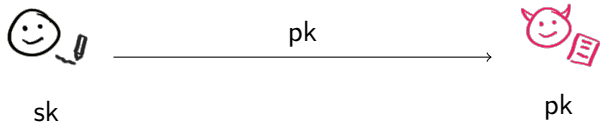

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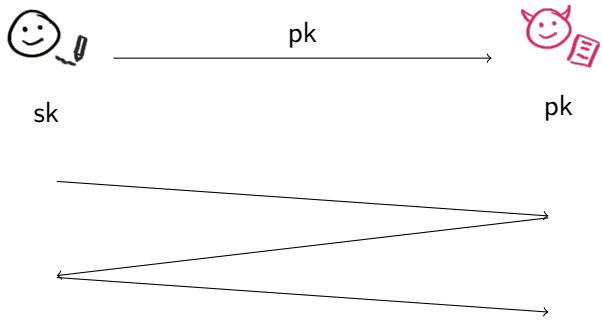

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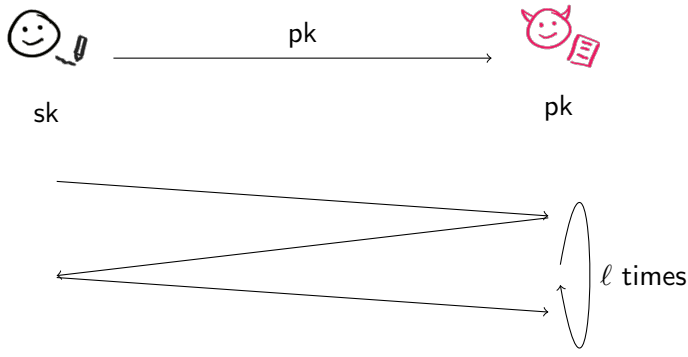

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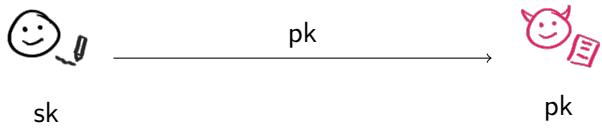

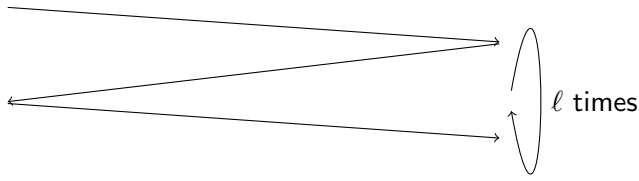

$\Downarrow$

$(m_1, \sigma_1), \ldots, (m_{\ell+1}, \sigma_{\ell+1})$

Motivation - eCash [Cha82]

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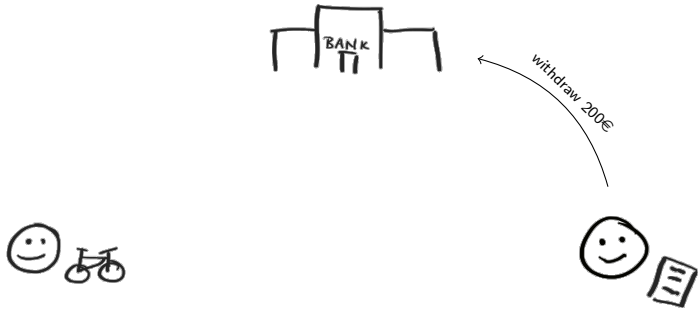

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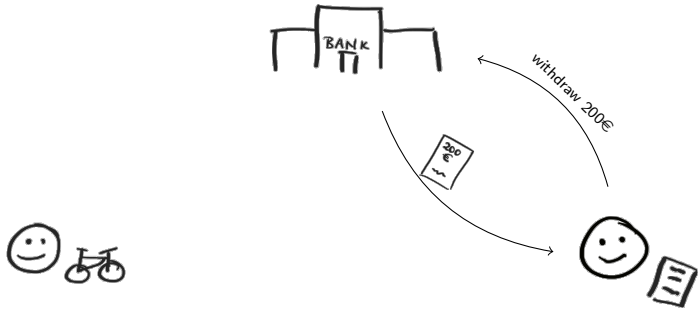

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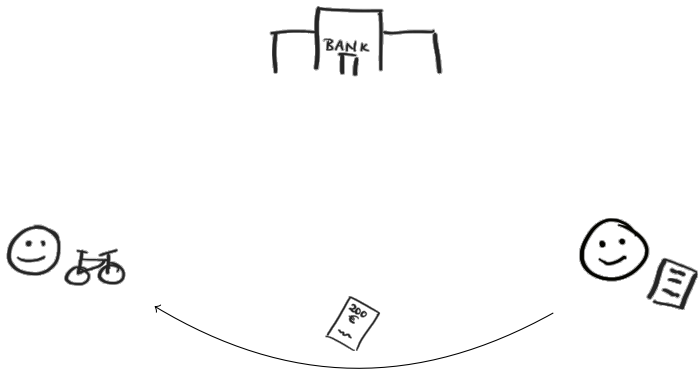

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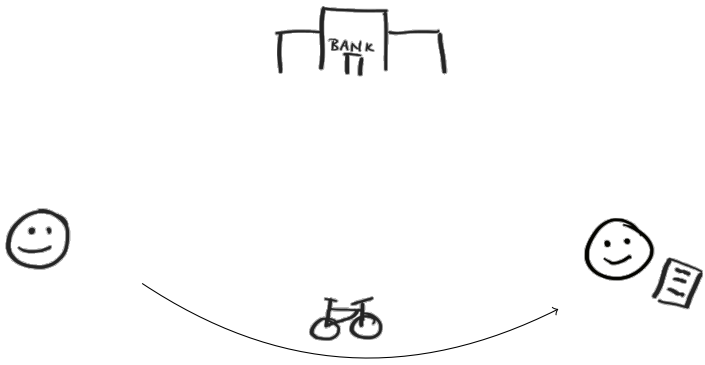

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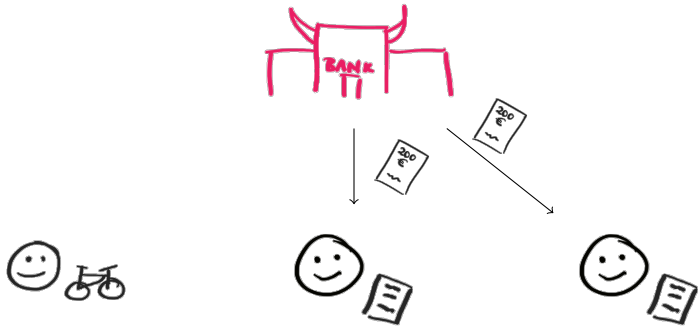

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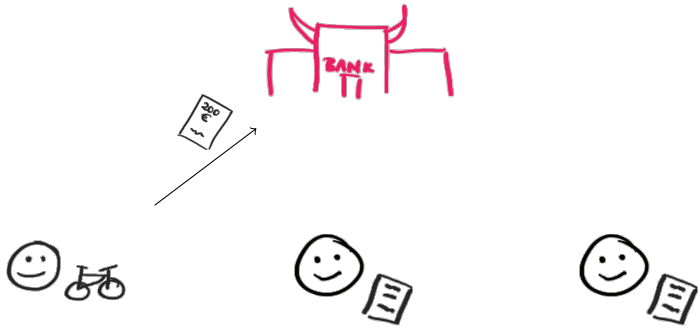

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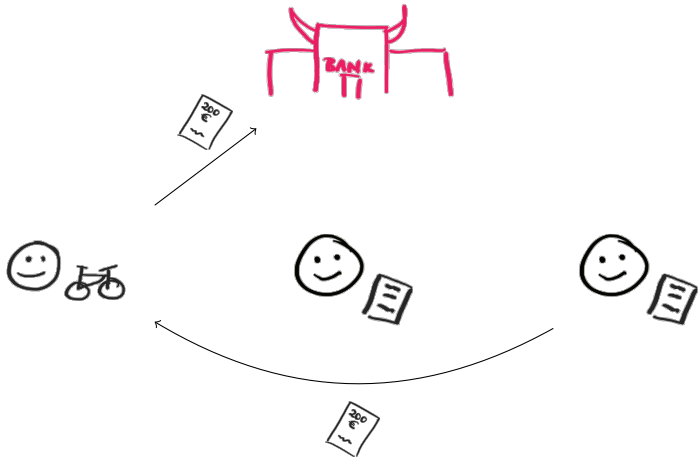

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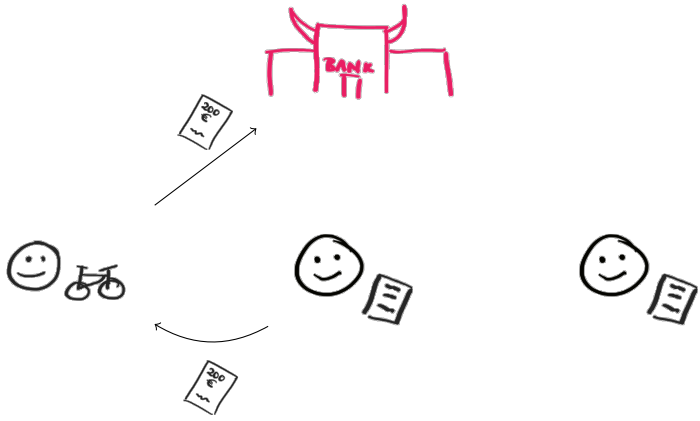

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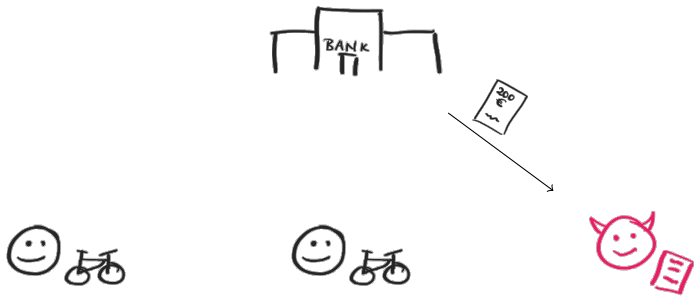

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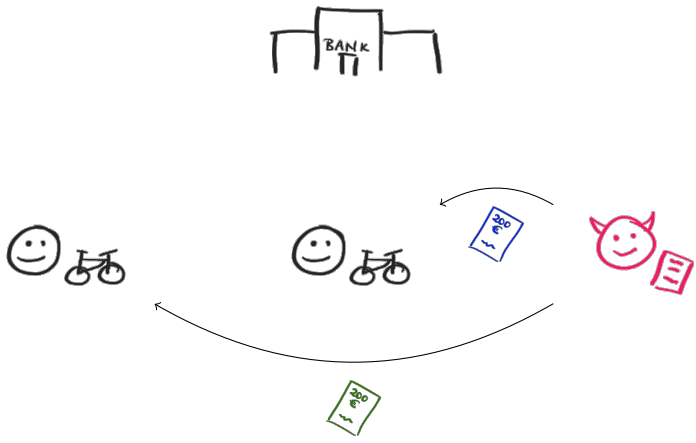

Partial Blindness [AF96]

Additional information info

- Date/Time
- Expiration Date
- Value 50€, 100€, 200€, ...

State of the Art

- Blind Schnorr:

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  - GGM-proof [OA03]

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The Algebraic Group Model [FKL18]

$$(y, \quad) \xleftarrow{\$} A(x_1, \ldots, x_n)$$

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$$(y, \overrightarrow{z}) \xleftarrow{\$} A(x_1, \ldots, x_n)$$

$$y = \prod_{j=1}^{n} x_j^{z_j}$$

Blind Schnorr Signatures

$\mathsf{sk} = x$

m

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$s \coloneqq c \cdot x + r$

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$\mathsf{g}^s \stackrel{?}{=} R \cdot \mathsf{y}^c$

$c' = c - \beta, s' \coloneqq s + \alpha$

$(m, \sigma = (c', s'))$

Reduction

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$\mathsf{Y} = \mathsf{chal}$

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Reduction

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$c := H(R\mathsf{g}^{\alpha}\mathsf{y}^{\beta}, m) + \beta$

$s := \mathsf{dlog}(R \cdot Y^c)$

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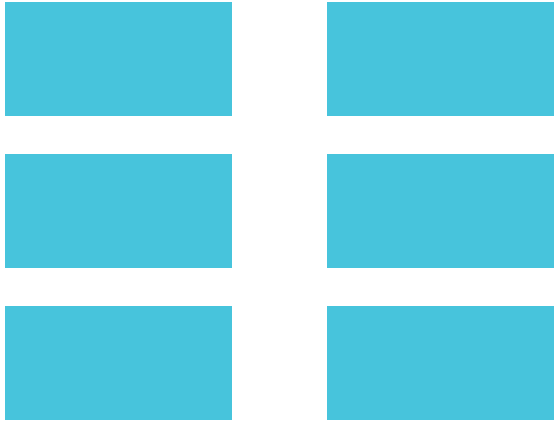
Sequential security of Blind Schnorr Signatures



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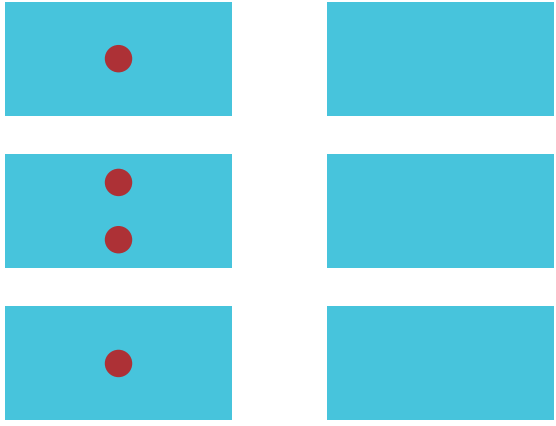

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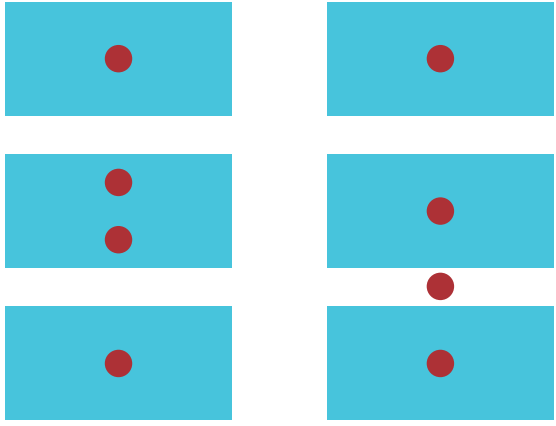

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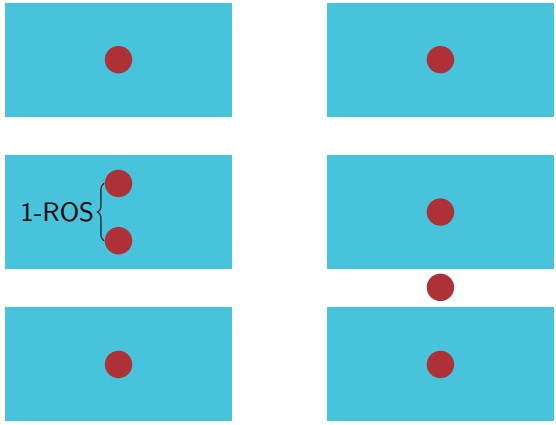

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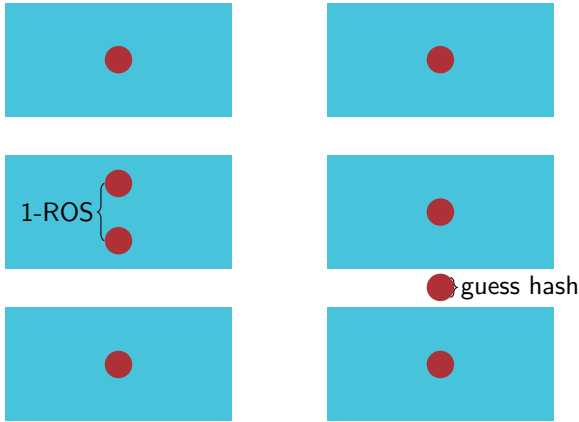
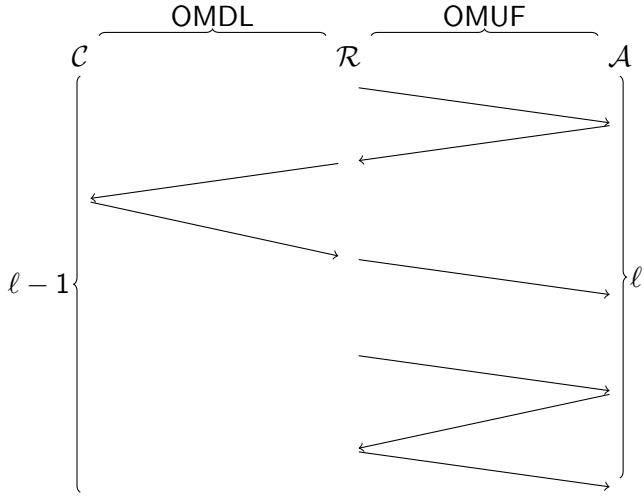


Optimality of the result



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x | h, z

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$y = g^x$ | h, z

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R= a $, z_1, b_1, b_2$

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R

Abe's blind signature scheme [Abe01]

x h, z

$\mathsf{y} = \mathsf{g}^x$ h, z

$\mathsf{R}=$ $a, \mathsf{z}_1, b_1, b_2$

R

$\varepsilon \xleftarrow{\$} H_3(\zeta, \zeta_1, \alpha, \beta_1, \beta_2, \eta, \mathsf{m})$

$\mathsf{e} = \varepsilon +$ $+$

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$e = c + d$

r, s_1, s_2

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$e = c + d$

r, s_1, s_2

c, r, d, s_1, s_2

$\varepsilon = \omega + \delta$

$\rho, \zeta, \zeta_1, \sigma_1, \sigma_2, \mu$

Abe's blind signature scheme [Abe01]

Y $h = g^w, z = g^{w_0}$

Y h, z

R= a, z_1, b_1, b_2

R

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e

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c, r, d, s_1, s_2

$\varepsilon = \omega + \delta$

$\rho, \zeta, \zeta_1, \sigma_1, \sigma_2, \mu$

Abe's blind signature scheme [Abe01]

x h Y

x h Y

R= a, z_1, b_1, b_2

R

$\varepsilon \xleftarrow{\$} H_3(\zeta, \zeta_1, \alpha, \beta_1, \beta_2, \eta, \mathrm{m})$

$\mathrm{e} = \varepsilon +$ $+$

e

$\mathrm{e} = c + d$

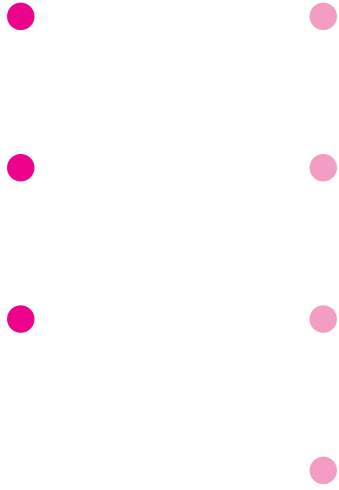
r, s_1, s_2

c, r, d, s_1, s_2

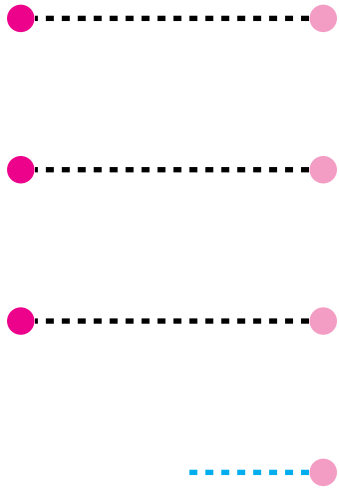
$\varepsilon = \omega + \delta$

$\rho, \zeta, \zeta_1, \sigma_1, \sigma_2, \mu$

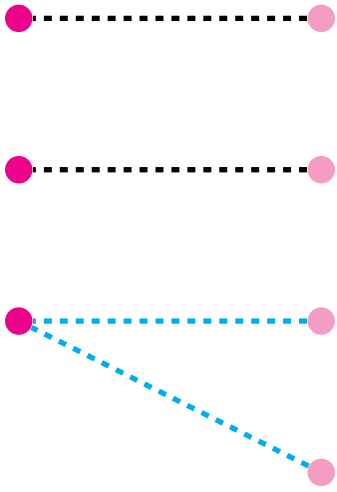
Linking Components



Linking Components



Linking Components



Preliminary Components

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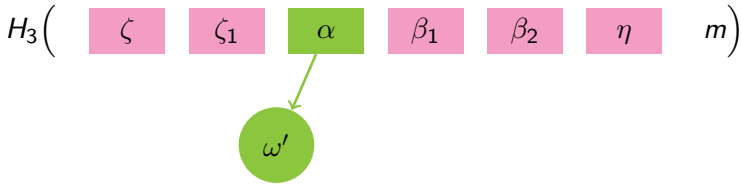
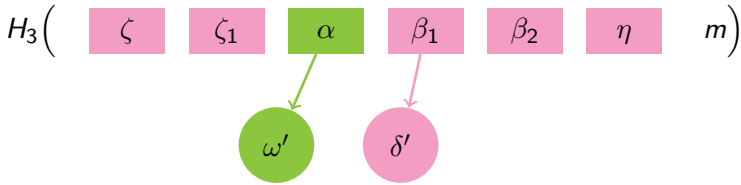
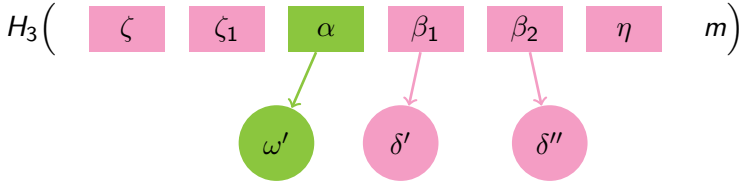


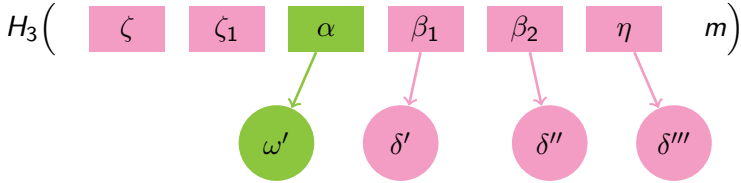
Preliminary Components



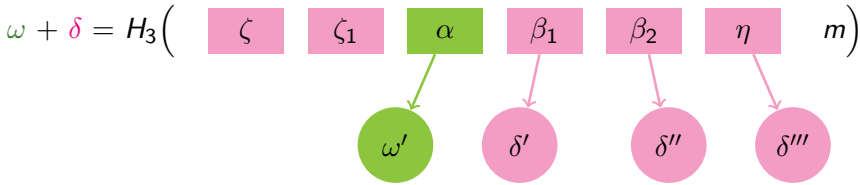
Preliminary Components



Preliminary Components



Preliminary Components



How can the preliminary components be the same?

- guess hash value

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- guess hash value
- solve some discrete log

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- guess hash value
- solve some discrete log
- behave honestly
  - match linking components
  - solve 1-ROS

Recap

- DLog + AGM + ROM: Abe's scheme **concurrently** secure
- OMDL + AGM + ROM: Schnorr's scheme *sequentially* secure
- for algebraic OMDL reduction: need at least as many OMDL queries as closed signing sessions

Open questions

- Security of Abe's scheme in ROM only
- Schnorr's scheme with low concurrency in AGM + ROM

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