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Lockable Obfuscation from Circularly Insecure Fully Homomorphic Encryption

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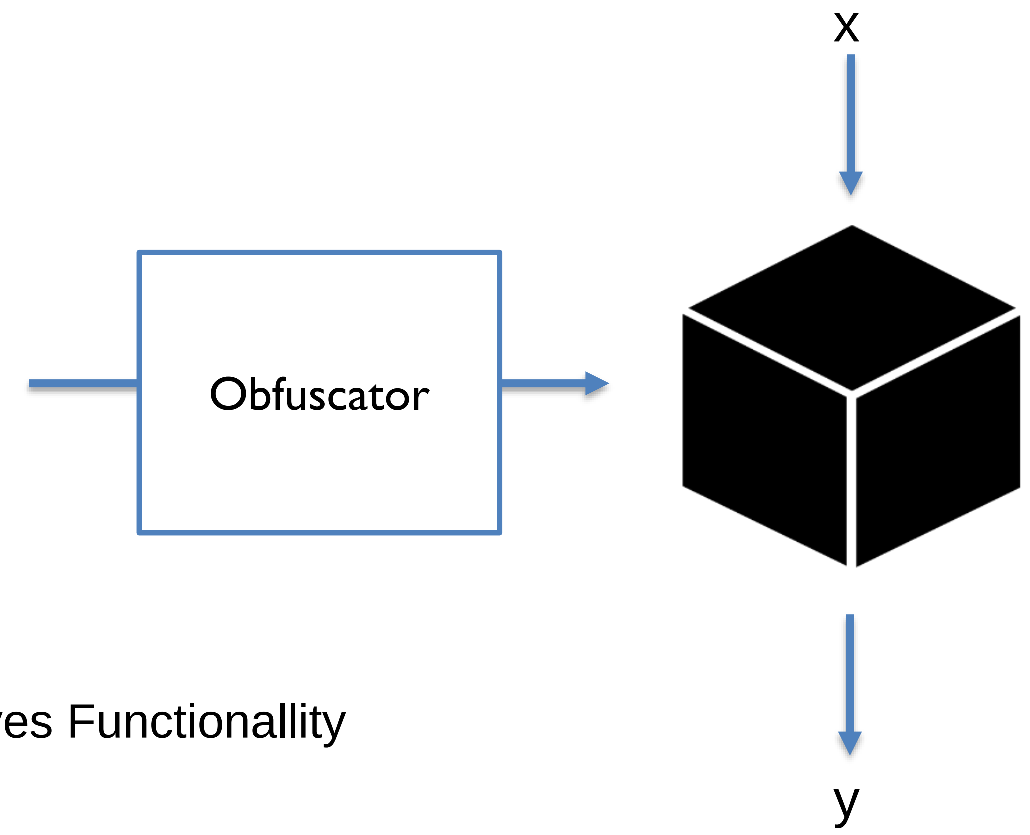
Obfuscation

```
int fun(int x){
    int y = (x + 5) % 11;
    return y;
}
```



- Preserves Functionallity

- Polynomial Slowdown

- Virtual Black-Box Secuity

What functions can we obfuscate from Standard Assumptions?

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Point Functions

$$\mathrm{PF}[\alpha](x) = \begin{cases} 1 & \text{if } \; x = \alpha \\ 0 & \text{otherwise} \end{cases}$$

Can97

CRV10

LPS04

GKPV10

DKL09

BS16

CD08

Wee05

KY18

Conjunctions

$$f(x_1, \ldots, x_n) = \neg x_3 \wedge x_5 \wedge \neg x_9 \wedge x_n$$

BR13

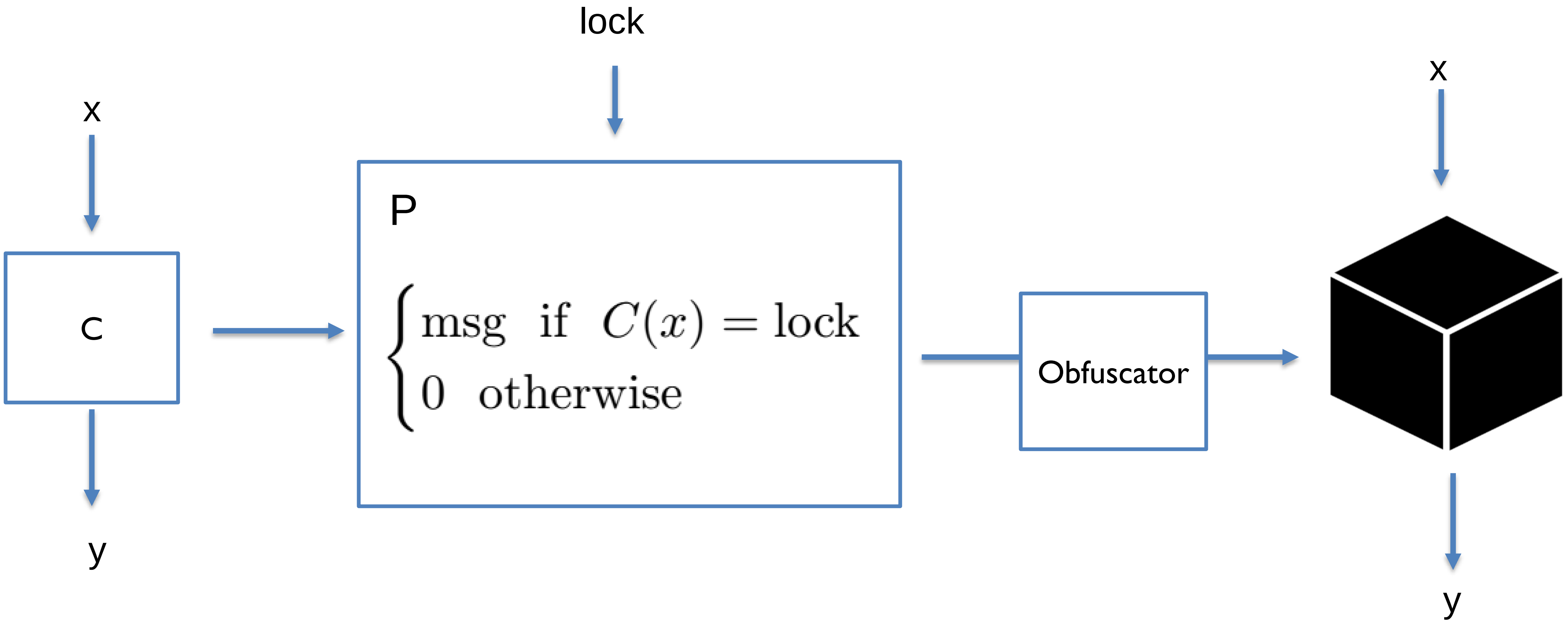
BKM+18

BVWW16

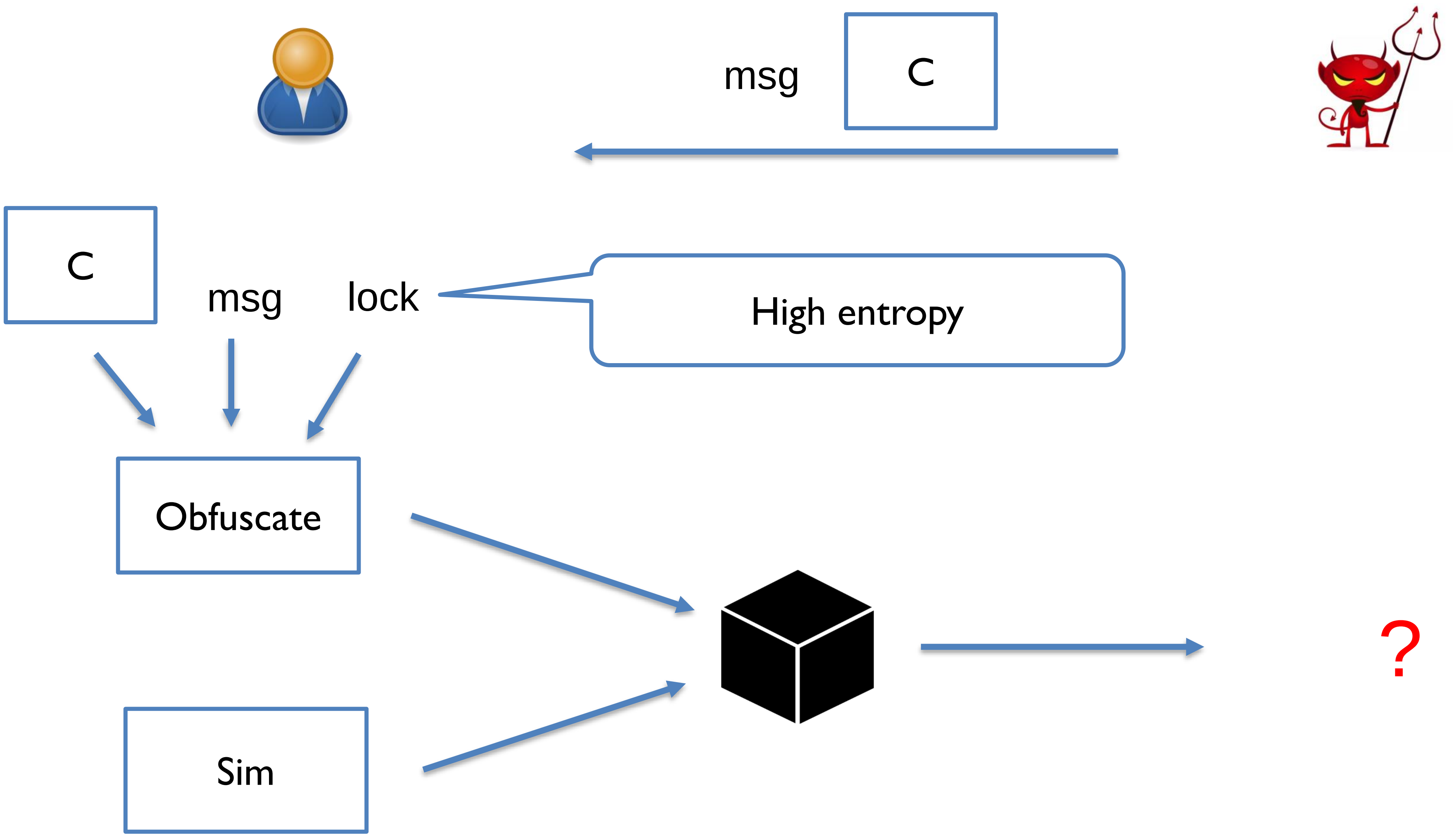
BW19

BLMZ19

Lockable Obfuscation: Obfuscating Compute-and-Compare Programs



Distributional Virtual Black-Box Security



Applications of Lockable Obfuscation

- Implications

Lockable Obfuscation ⟹ Point Obfuscation

Lockable Obfuscation ⟹ Conjunction Obfuscation

- Compilers

PKE + Lockable Obfuscation ⟹ Anonymous PKE

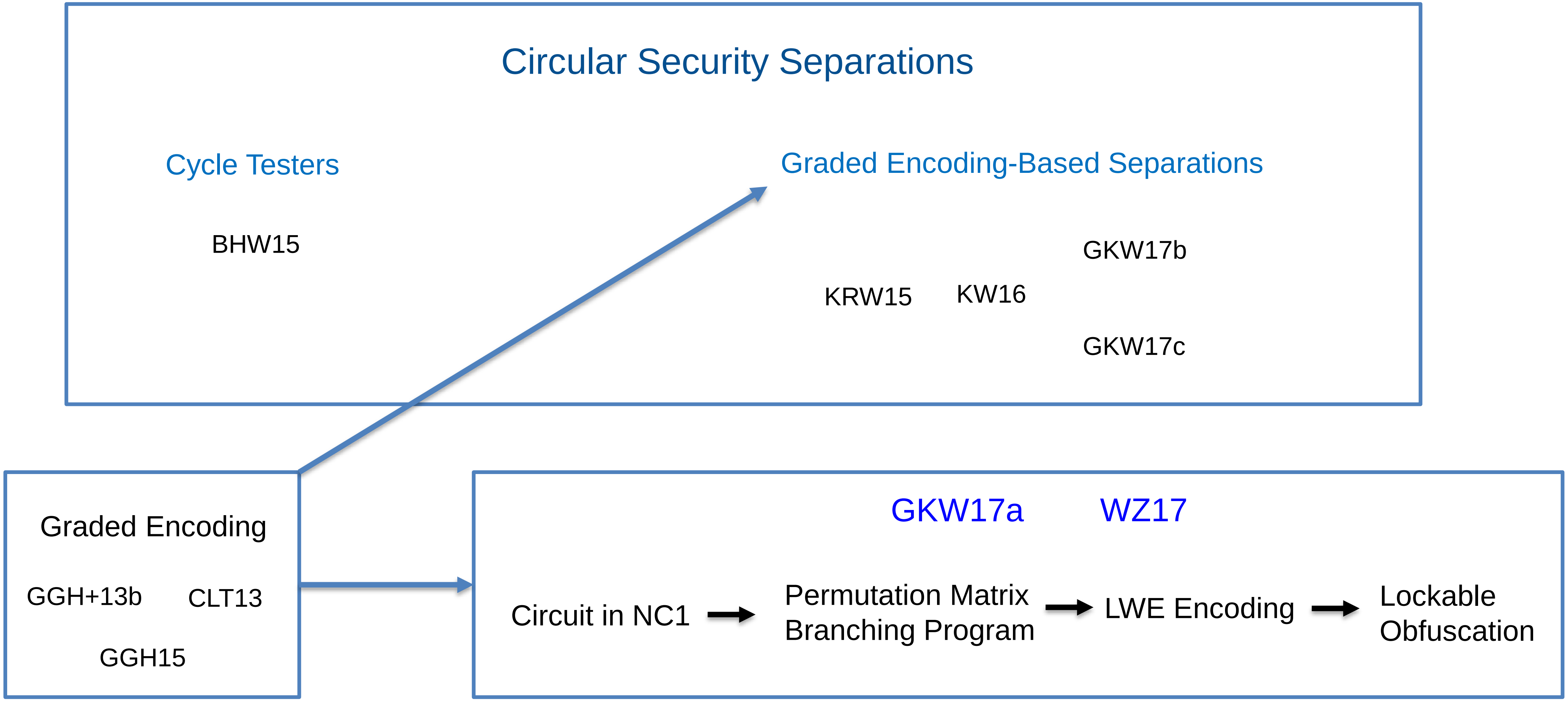
IBE + Lockable Obfuscation ⟹ Anonymous IBE

Predicate Encryption + Lockable Obfuscation ⟹ PE with One-Sided Privacy

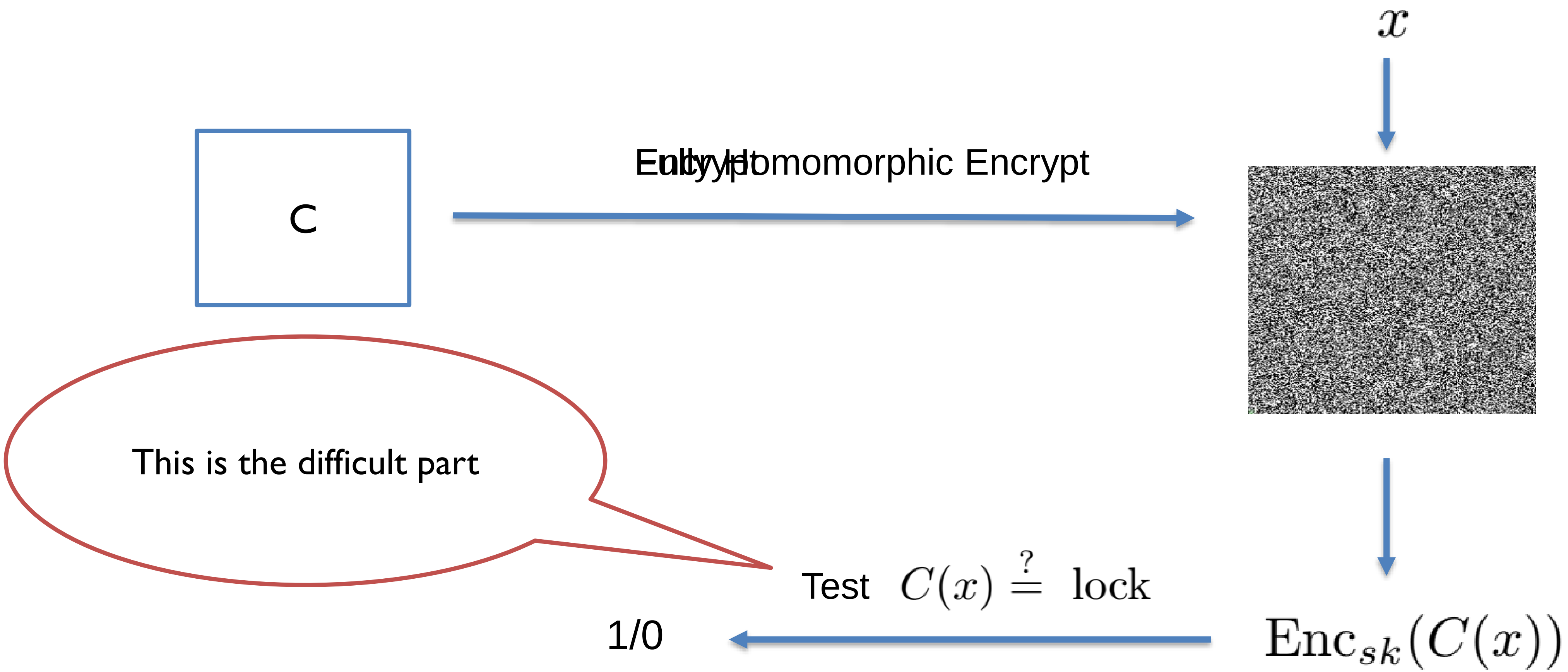
Broadcast Encryption + Lockable Obfuscation ⟹ Anonymous Broadcast Encryption

WE + Lockable Obfuscation ⟹ Indistinguishability Obfuscation for Rejecting Programs

Previous Construtions of Lockable Obfuscation



Intuitively: What is a Lockable Obfuscation?



Our base construction: Testing the Lock

Given $\mathrm{Enc}_{sk}(C(x))$ test $C(x) \stackrel{?}{=} \mathrm{lock}$

Idea: Use a fully homomorphic encryption scheme that is circularly insecure

(Equiped with a cycle tester)

Cycle Testers

$$\mathrm{Enc}_{sk_1}(sk_2), \ldots, \mathrm{Enc}_{sk_n}(sk_1) \not\approx \mathrm{Enc}_{sk_1}(0), \ldots, \mathrm{Enc}_{sk_n}(0)$$

$$\mathrm{Enc}_{sk}(sk) \not\approx \mathrm{Enc}_{sk}(0)$$

Our base construction: Putting Things Together

$$c \leftarrow \mathrm{Enc}_{sk}(C)$$

$$a \leftarrow \mathrm{Enc}_{\mathrm{lock}}(sk)$$

Evaluation

$$\mathrm{Eval}(U_x, c) \quad = \mathrm{Enc}_{sk}(C(x)) \quad = d$$

$$\mathrm{Eval}(\mathrm{Dec}(d, .), a) \quad = \mathrm{Enc}_{sk}(\mathrm{Dec}(C(x), \mathrm{Enc}_{\mathrm{lock}}(sk))) \quad = e$$

$$\text{if} \quad C(x) = \text{lock} \quad e = \mathrm{Enc}_{sk}(sk)$$

$$\text{otherwise} \quad e \neq \mathrm{Enc}_{sk}(sk) \quad \text{w.h.p.}$$

?

Cycle Tester

Security Proof

$$c \leftarrow \mathrm{Enc}_{sk}(C) \qquad a \leftarrow \mathrm{Enc}_{\mathrm{lock}}(sk)$$

↓ IND-CPA

$$c \leftarrow \mathrm{Enc}_{sk}(C) \qquad a \leftarrow \mathrm{Enc}_{\mathrm{lock}}(0)$$

↓ Pseudorandom Ciphertexts

$$c \leftarrow \mathrm{Enc}_{sk}(C) \qquad a \leftarrow \text{uniform random}$$

↓ IND-CPA

$$c \leftarrow \mathrm{Enc}_{sk}(0) \qquad a \leftarrow \text{uniform random}$$

In the paper, we have a version without this assumption

Extension: Distributional Security

$c \leftarrow \mathrm{Enc}_{sk}(C) \qquad a \leftarrow \mathrm{Enc}_{\mathrm{lock}}(sk)$

IND-CPA → Enc may be CPA-secure with respect to key from other distributions (not necessarily uniform)

$c \leftarrow \mathrm{Enc}_{sk}(C) \qquad a \leftarrow \mathrm{Enc}_{\mathrm{lock}}(0)$

Unpredictable Distribution:

$(x, \mathsf{aux}) \leftarrow_{\mathsf{D}} D_\lambda$

$\mathsf{aux} \rightarrow$  $\rightarrow x$

DKL09 DGK+10 CKVW10

α-Pseudo Entropy Distribution:

$(x, \mathsf{aux}) \leftarrow_{\mathsf{D}} D_\lambda \qquad \mathbf{H}_{\mathsf{HILL}}(x|\mathsf{aux}) \geq \alpha(\lambda)$

AGV09 GKPV10 BLSV18 DP08

ADN+10 NS09 Pie09 Zha16

DHLW10 BG10 AKPW13

Extention: Multi-bit messages

$$c \leftarrow \mathrm{Enc}_{sk}(C) \qquad a \leftarrow \mathrm{Enc}_{\mathrm{lock}}(sk) \qquad f_1 \leftarrow \mathrm{Enc}_{sk}(m_1), \ldots, f_n \leftarrow \mathrm{Enc}_{sk}(m_n)$$

Evaluation

$$\mathrm{Eval}(U_x, c) = \mathrm{Enc}_{sk}(C(x)) = d$$

$$\mathrm{Eval}(\mathrm{Dec}(d, .), a) = \mathrm{Enc}_{sk}(\mathrm{Dec}(C(x), \mathrm{Enc}_{\mathrm{lock}}(sk))) = e$$

Suppose $C(x) = \text{lock}$ and $e = \mathrm{Enc}_{sk}(sk)$

$$f_1 \cdot e = \mathrm{Enc}_{sk}(m_1 \cdot sk)$$

$m_1 = 0$ → $\mathrm{Enc}_{sk}(0)$

$m_1 = 1$ → $\mathrm{Enc}_{sk}(sk)$

?

Cycle Tester

Summary

- Generic Construction of Lockable Obfuscation from FHE with Cycle Testers
- In the paper: the construction for arbitrary cycle lenght

Implications

Lockable Obfuscation + (FH)E ⟹ (FH)E with a Cycle Tester

FHE + Cycle Tester ⟹ Lockable Obfuscation

Thank You

https://eprint.iacr.org/2021/1324