Lower Bounds for the Number of Decryption Updates in Registration-Based Encryption

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MAX PLANCK INSTITUTE FOR SECURITY AND PRIVACY

Public Key Encryption [DH76,RSA78,GM82]

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Alice

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$c_1 \leftarrow \mathrm{Enc}(pk_1,\ m_1)$

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Susan

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Too many public keys!

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Bob

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Identity Based Encryption [Shamir86, BF01]

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(MSK)

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$$c \leftarrow \mathrm{Enc}(\mathrm{PP},\ \text{“Alice”},\ m)$$

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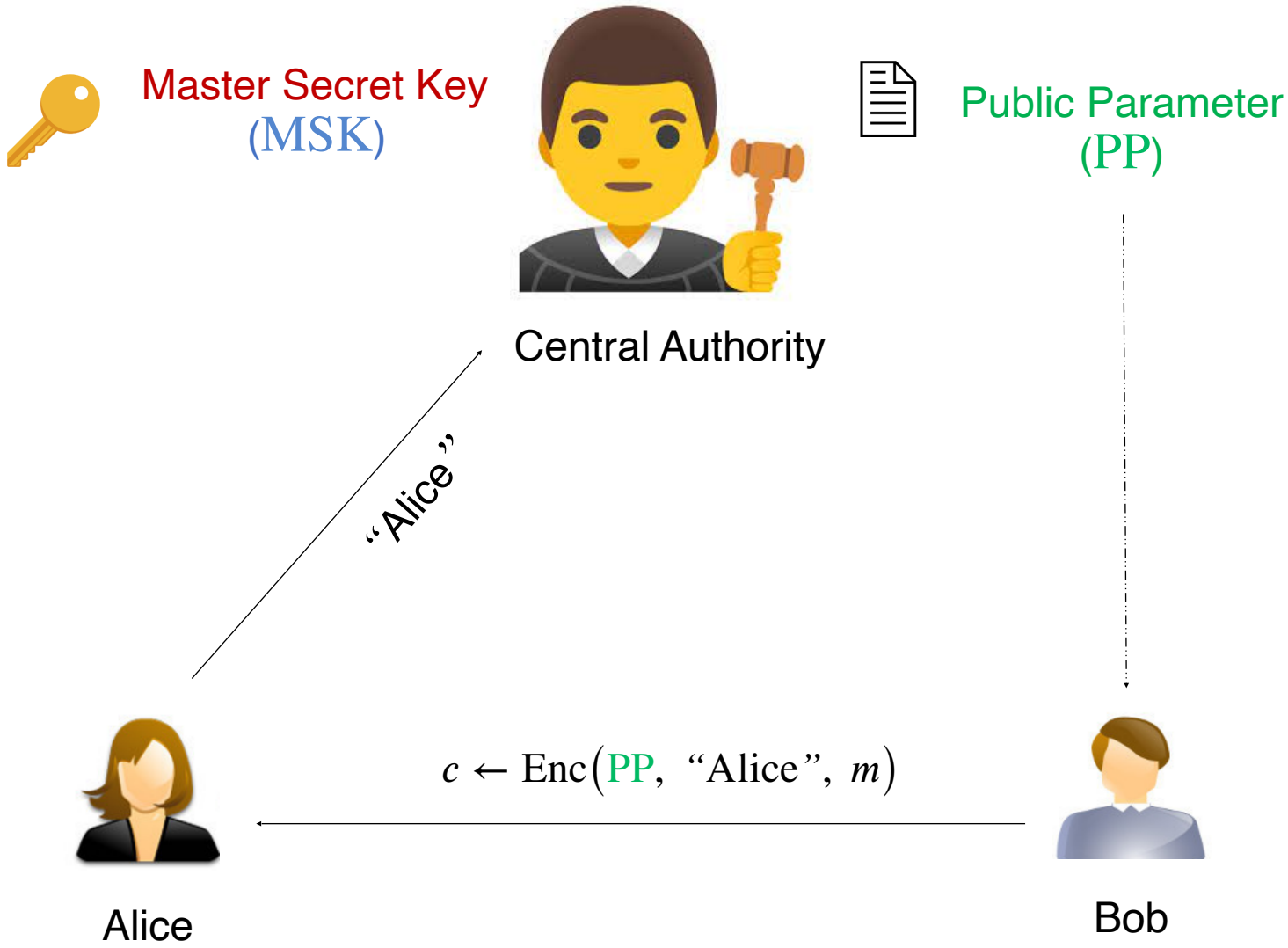

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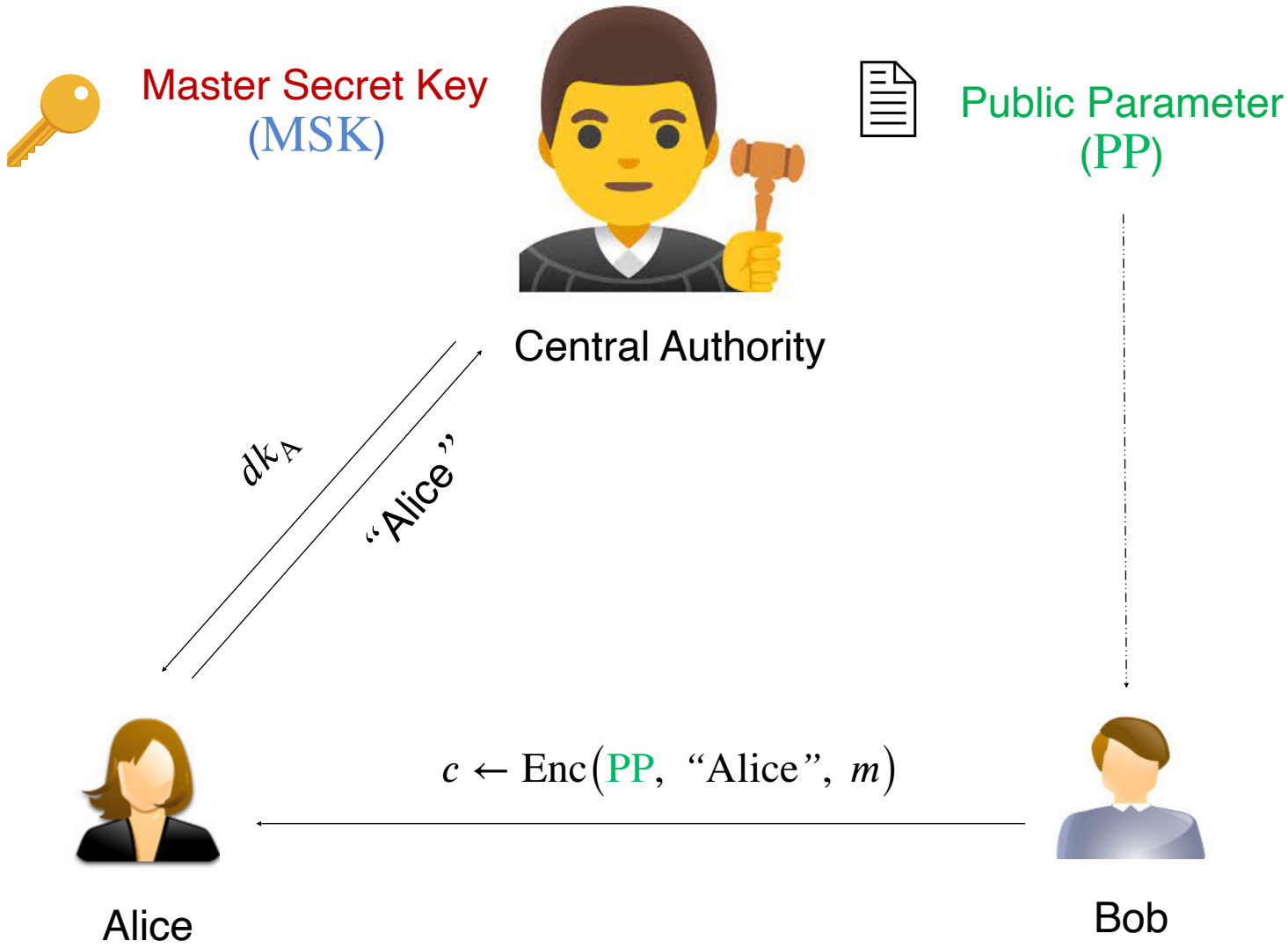

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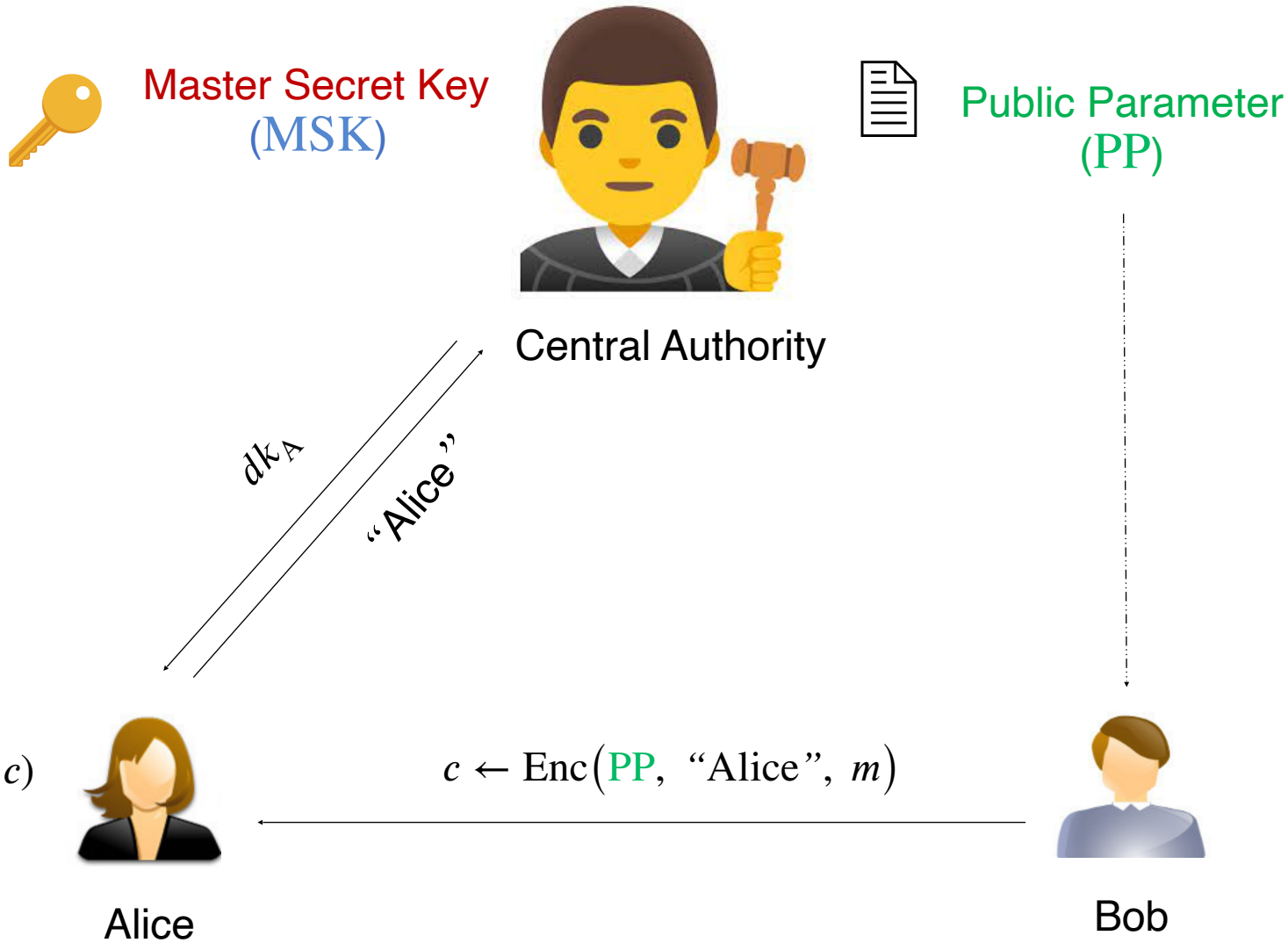

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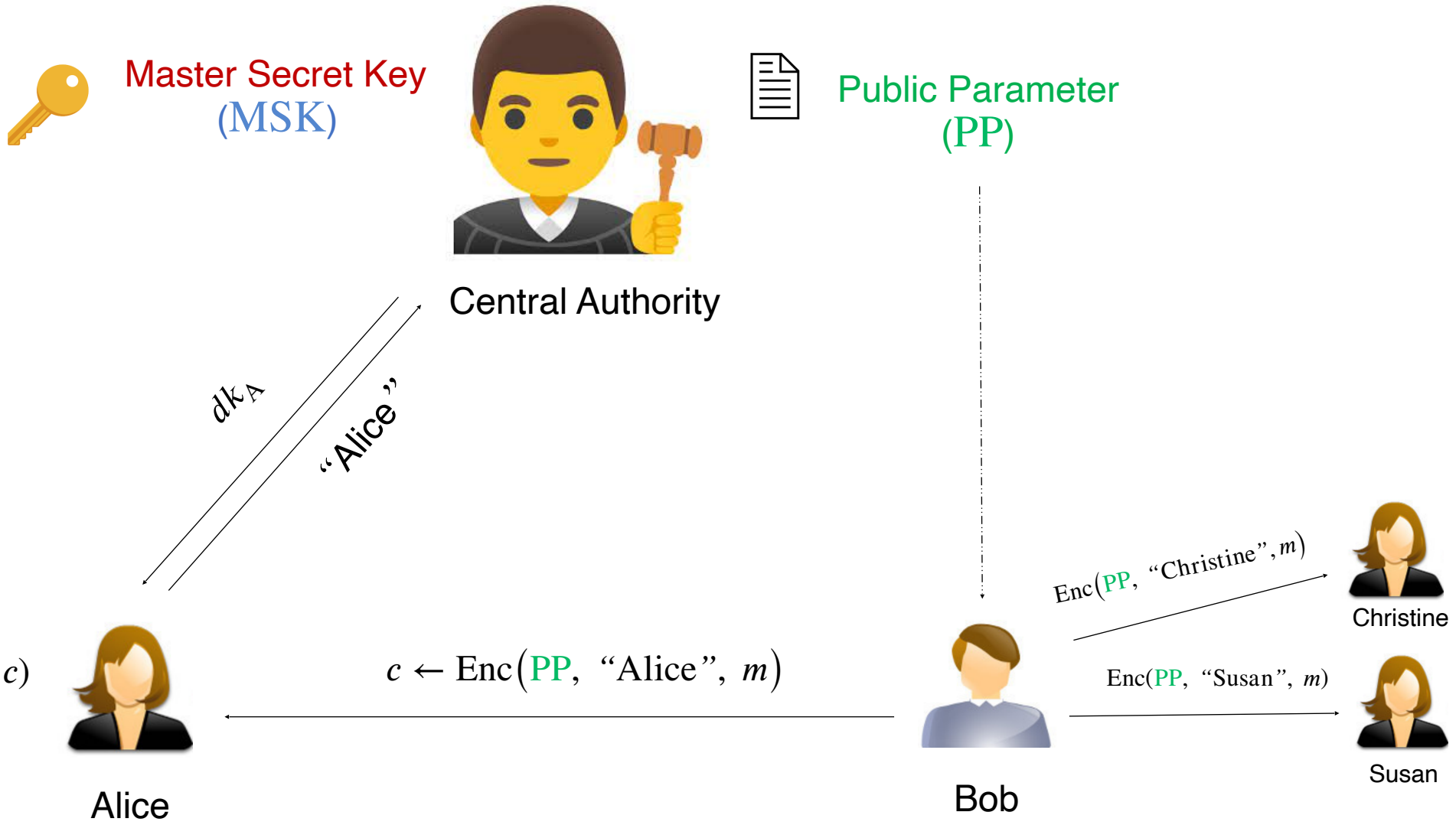

Identity Based Encryption [Shamir86, BF01]

Master Secret Key
(MSK)

Central Authority

Public Parameter
(PP)

One public key

dk_A

"Alice"

$m \leftarrow \mathrm{Dec}(dk_A, c)$

Alice

$c \leftarrow \mathrm{Enc}(\mathrm{PP}, \text{"Alice"}, m)$

Bob

$\mathrm{Enc}(\mathrm{PP}, \text{"Christine"}, m)$

Christine

$\mathrm{Enc}(\mathrm{PP}, \text{"Susan"}, m)$

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The Key Escrow Problem with IBE

Master Secret Key
(MSK)

Central Authority

Public Parameter
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Too much power!
It can decrypt everything.

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$m \leftarrow \mathrm{Dec}(dk_A, c)$

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Registration Based Encryption (RBE) [GHMR18]
Aiming for compact public parameter without key escrow

Identities generate their secret keys and register their public keys.

id_1

id_2

id_3

Key Curator (KC)

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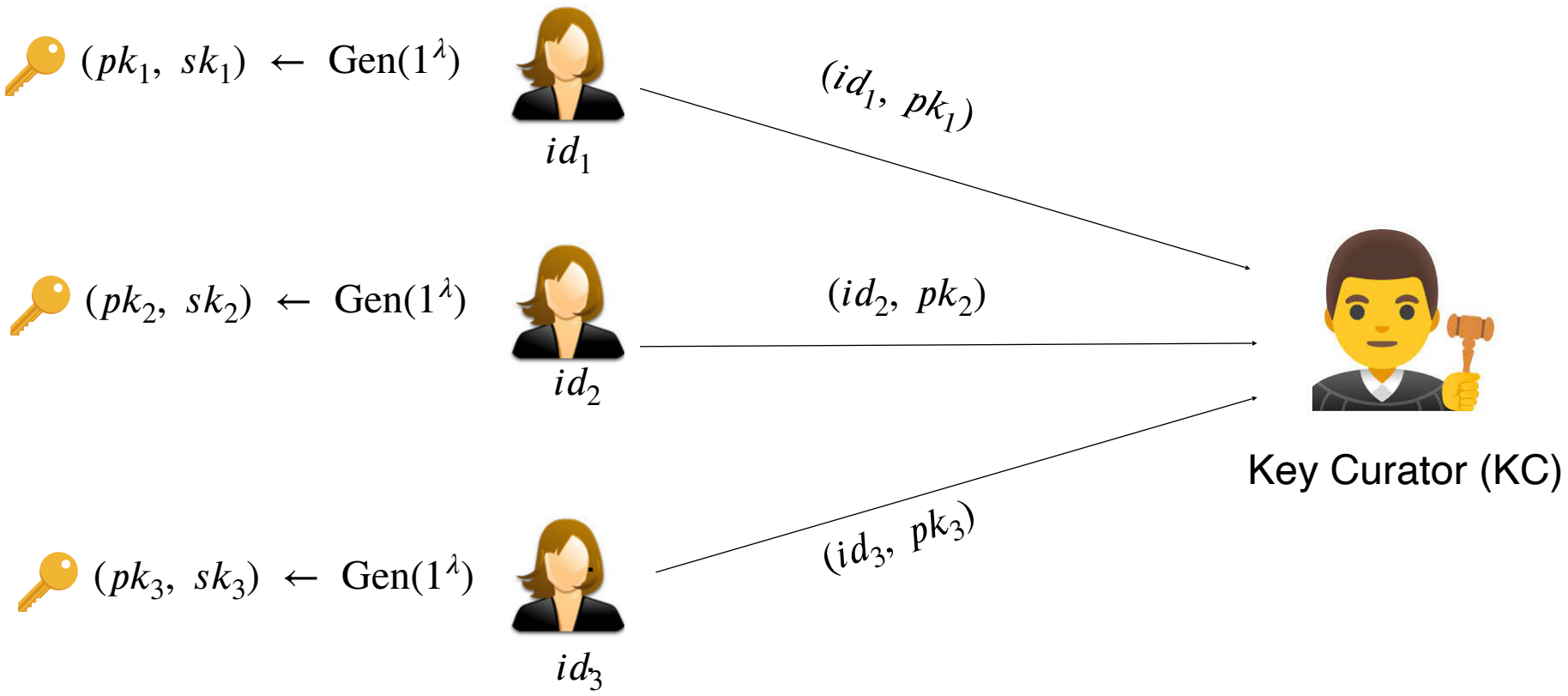

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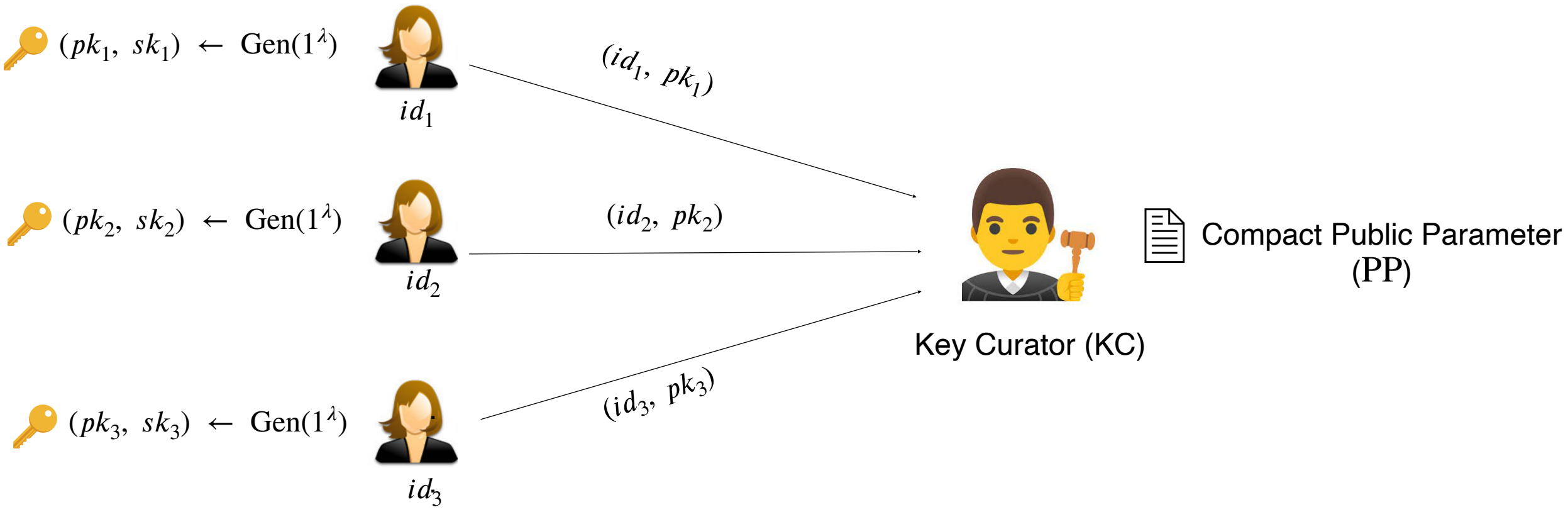

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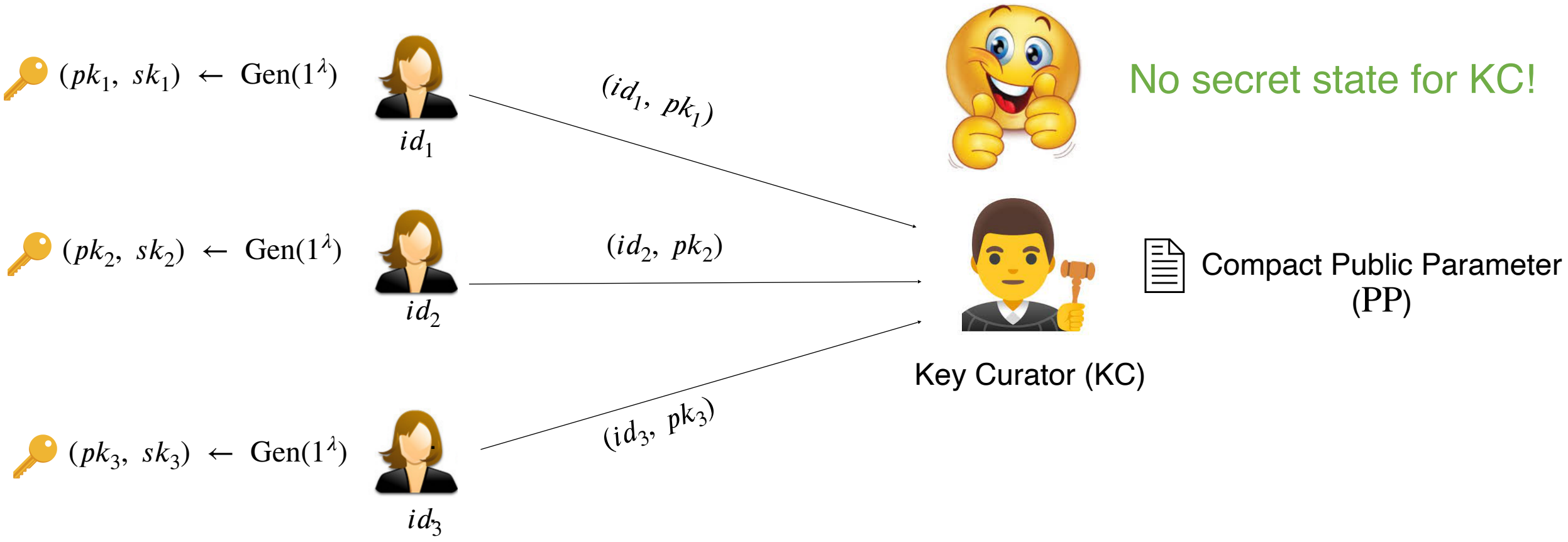

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More formally:

Key Curator

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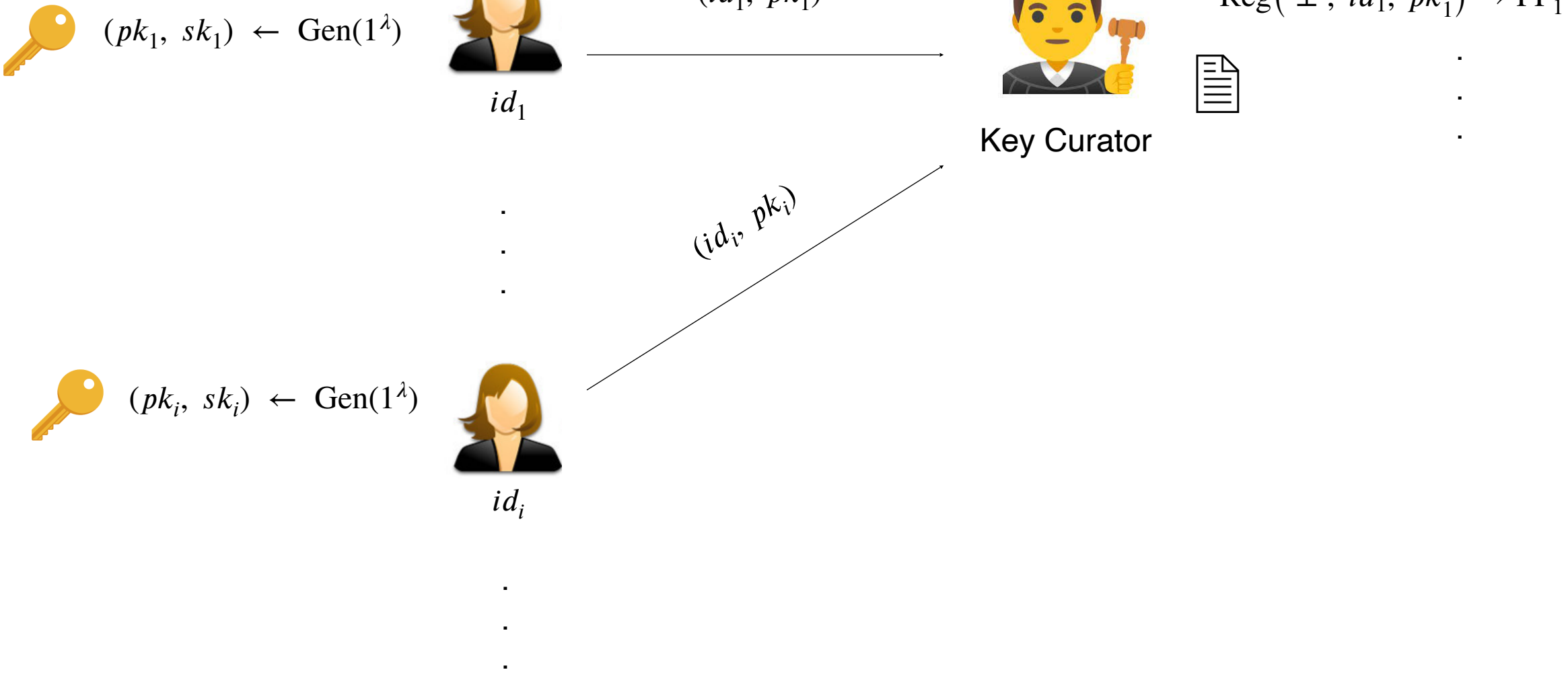

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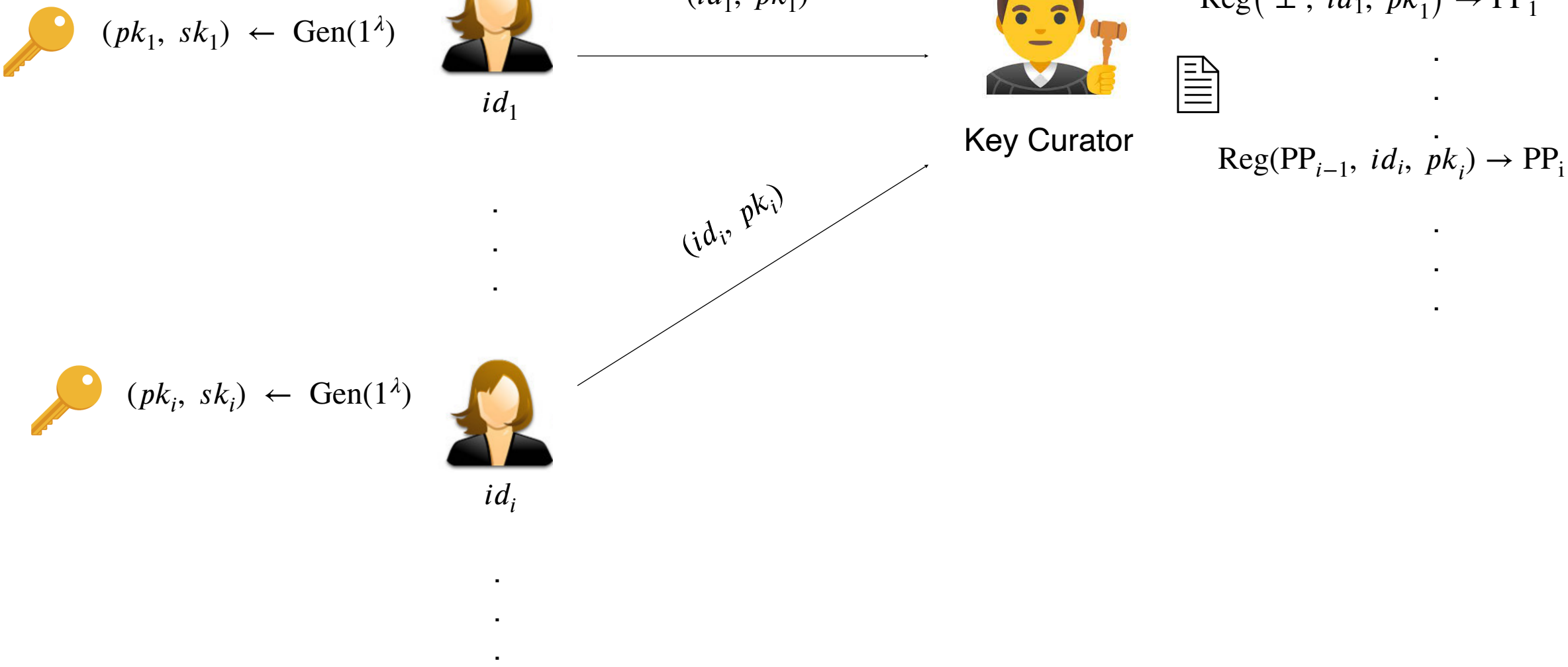

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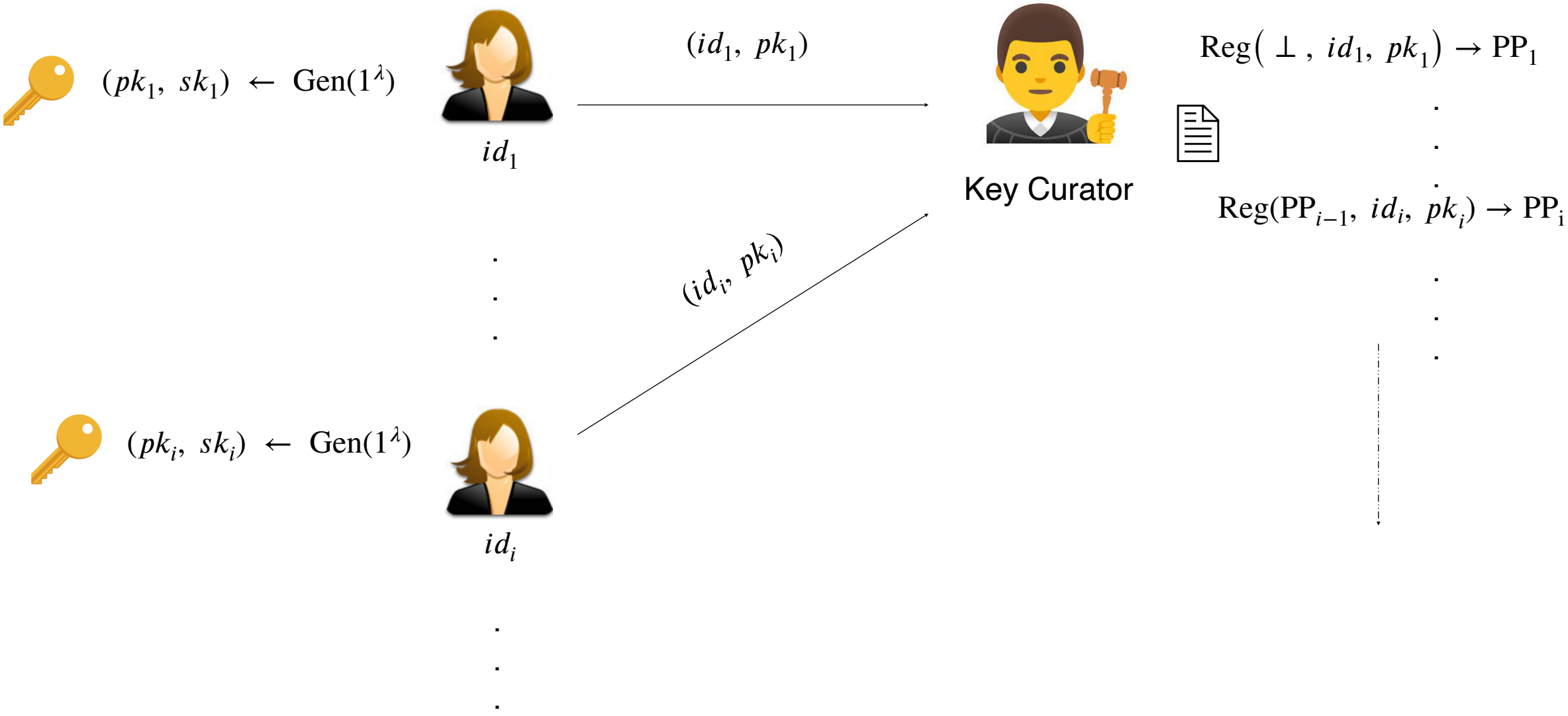

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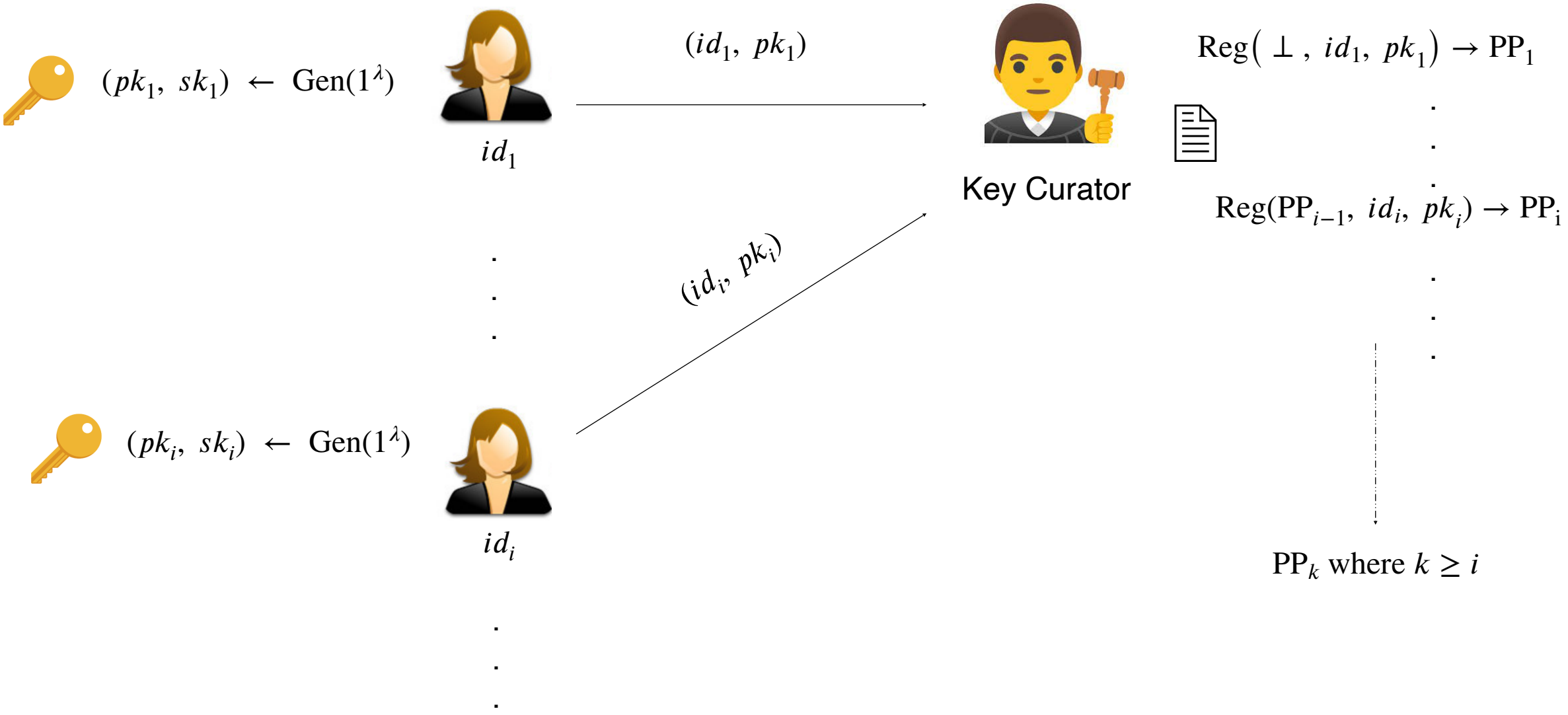

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PP_k where $k \geq i$

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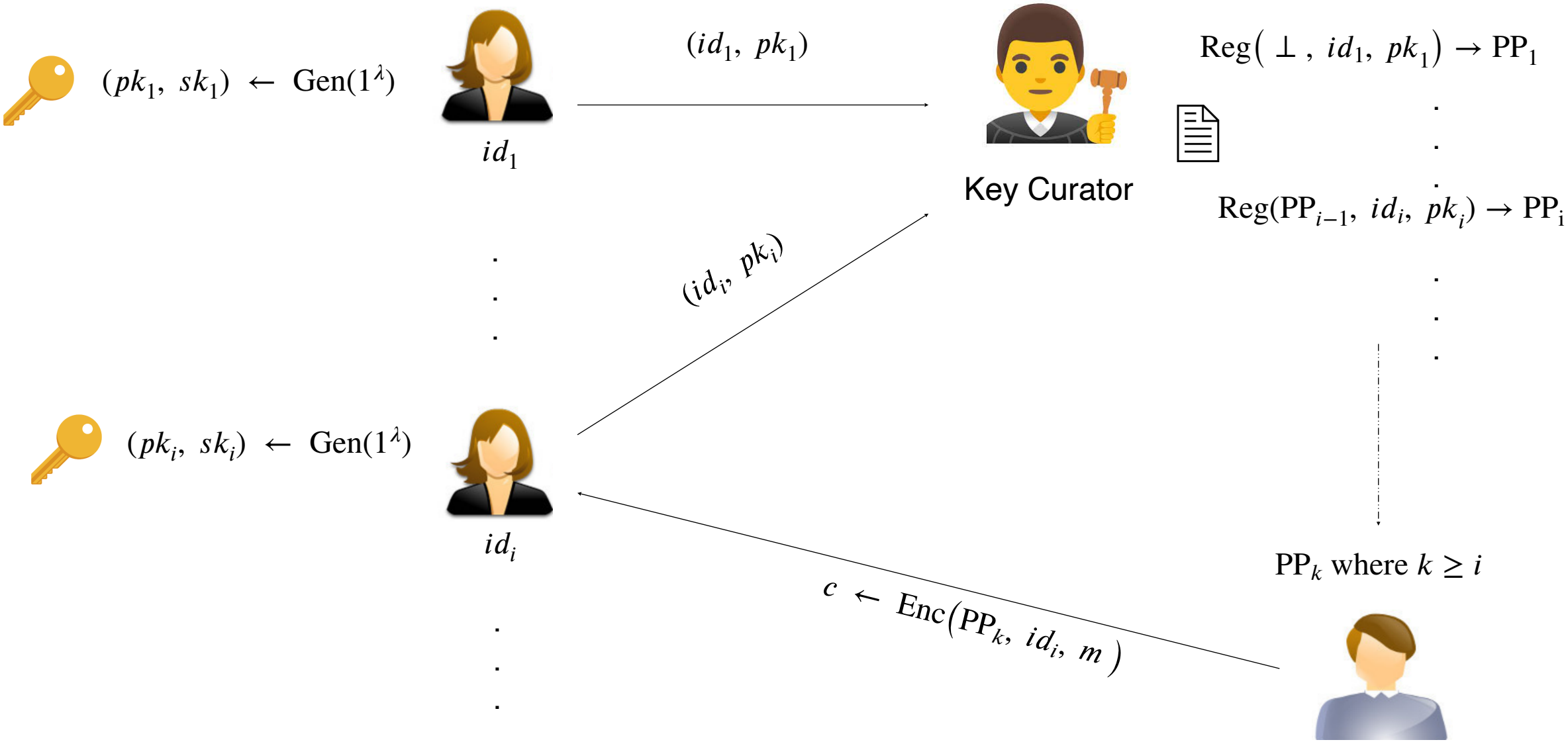

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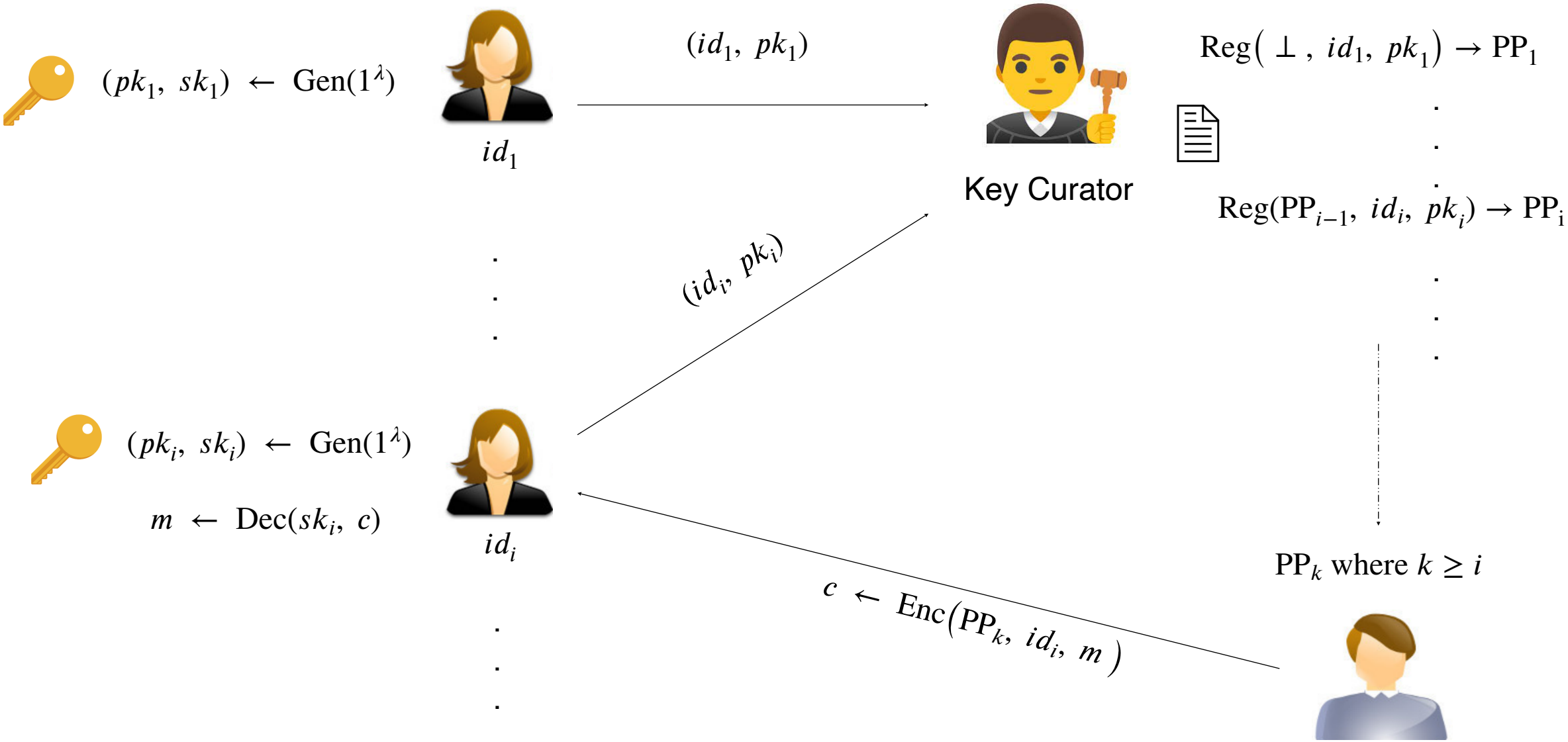

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RBE security : similar to IBE:

No colluding parties can decrypt others' messages.

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Let n := number of registered identities

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We need:

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Moreover:

- Updates are compact and have size $\leq \mathrm{polylog}(n)$
- Generating updates and registration can be done in time $\mathrm{polylog}(n)$

Main Question

Are $\approx \log n$ decryption updates necessary?

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All known constructions require $\Omega(\log n)$ updates.

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- Remark: Result holds even if some parties get many updates.
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Outline

- Introduction
- Our Results
- Proof Ideas

High Level Steps of the Proof

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(3) Use the **structure** to attack successfully, using **info theory**

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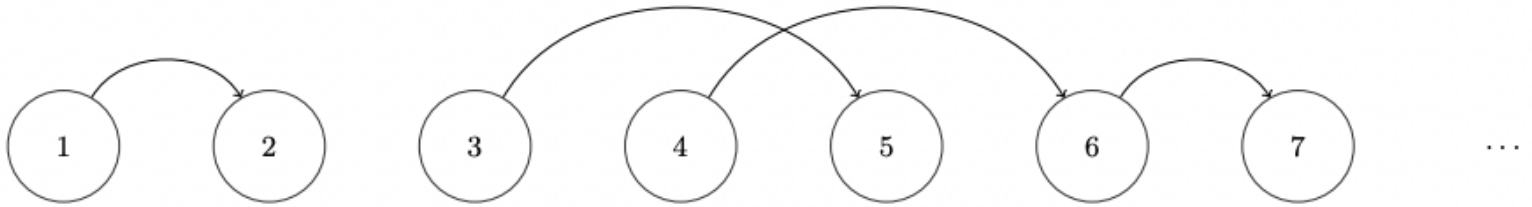

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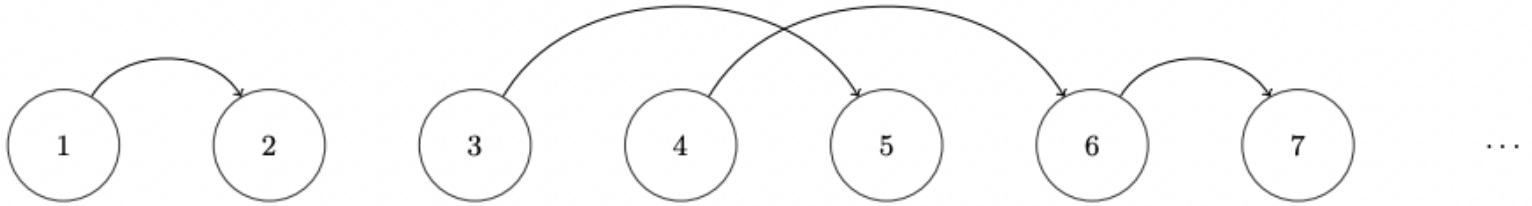

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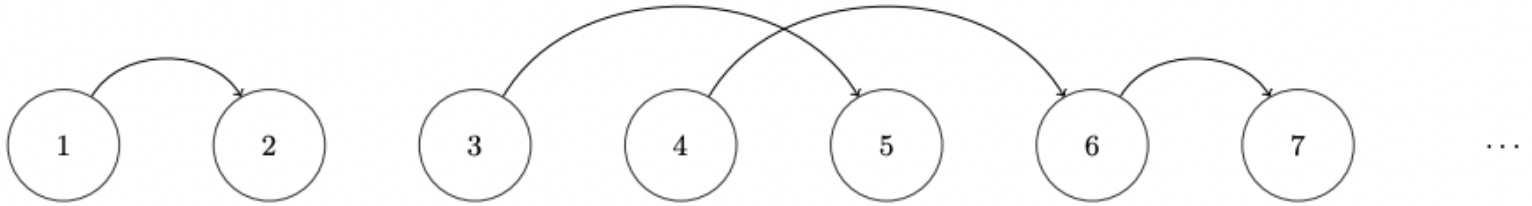


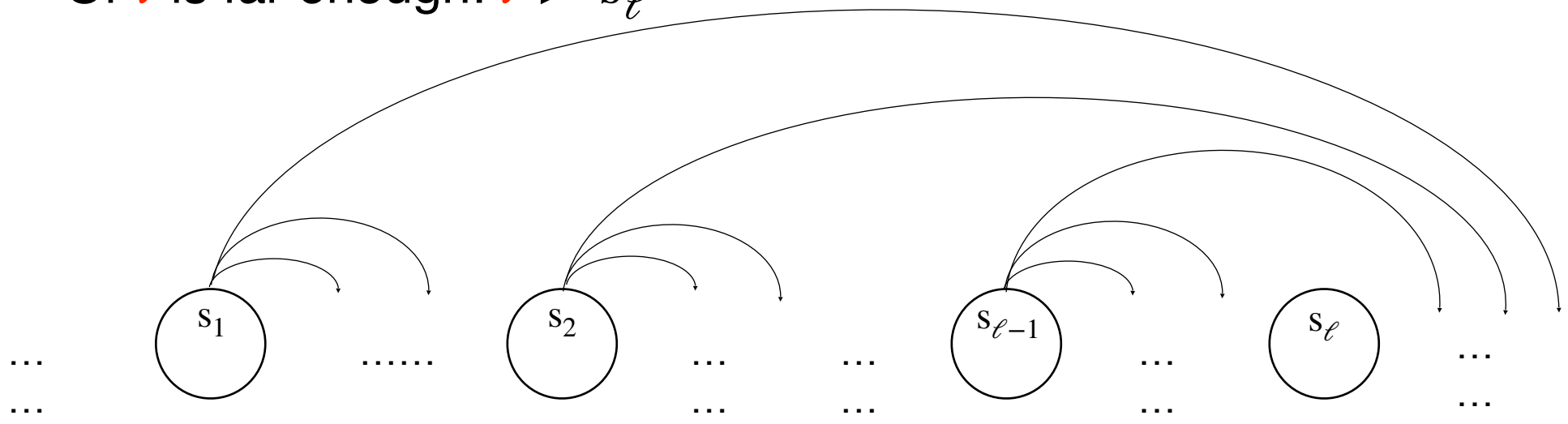
- Edge $(i \rightarrow j)$ exists iff user i needs an update after user j is registered.

By assumption, we know when users need update.

Step 2: Defining a Useful Structure in DAGs

- $\{s_1 < \ldots < s_\ell\}$ is a **skipping sequence** if

  for any outgoing edge (s_i, t)
  - Either t is close enough: $t < s_{i+1}$
  - Or t is far enough: $t > s_\ell$



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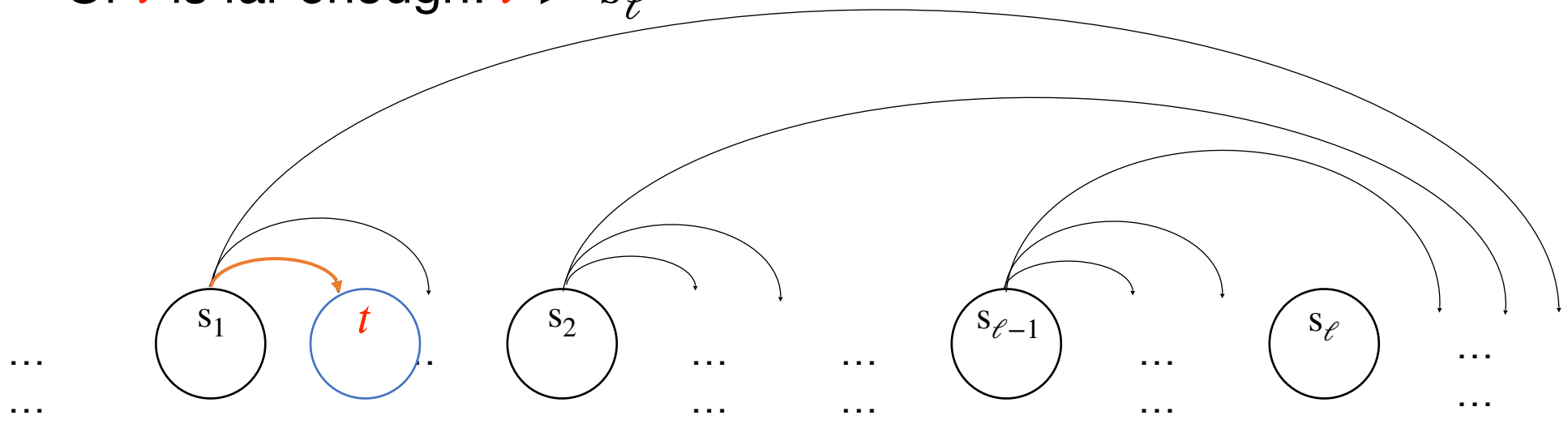

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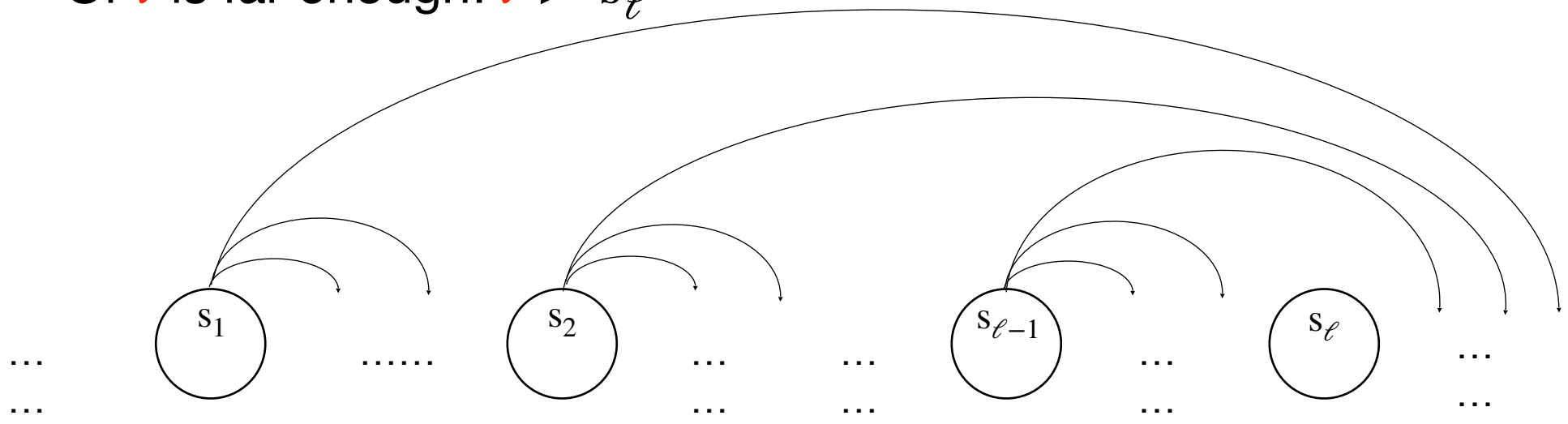

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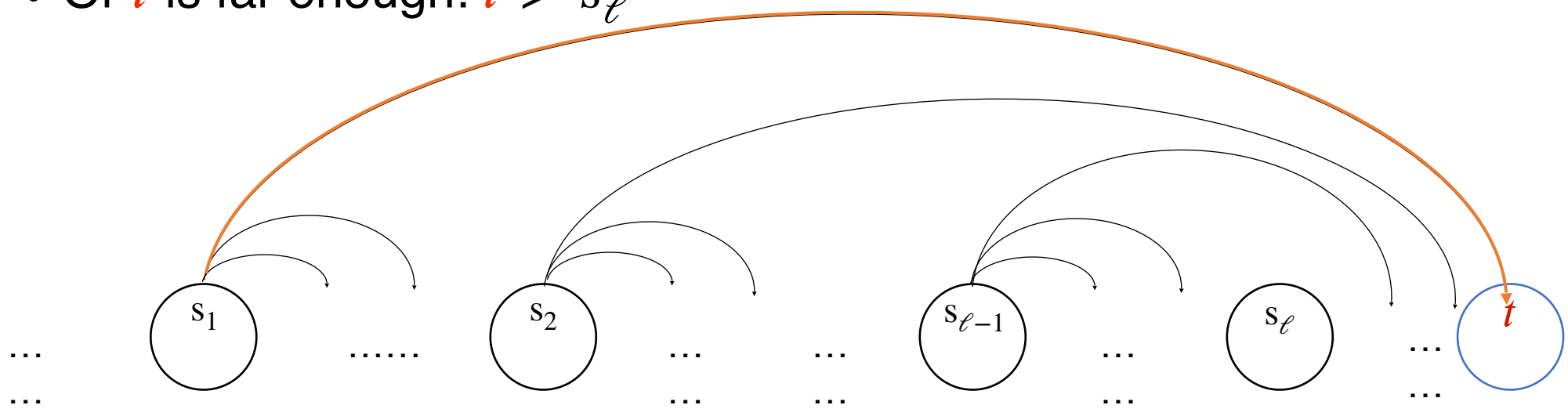

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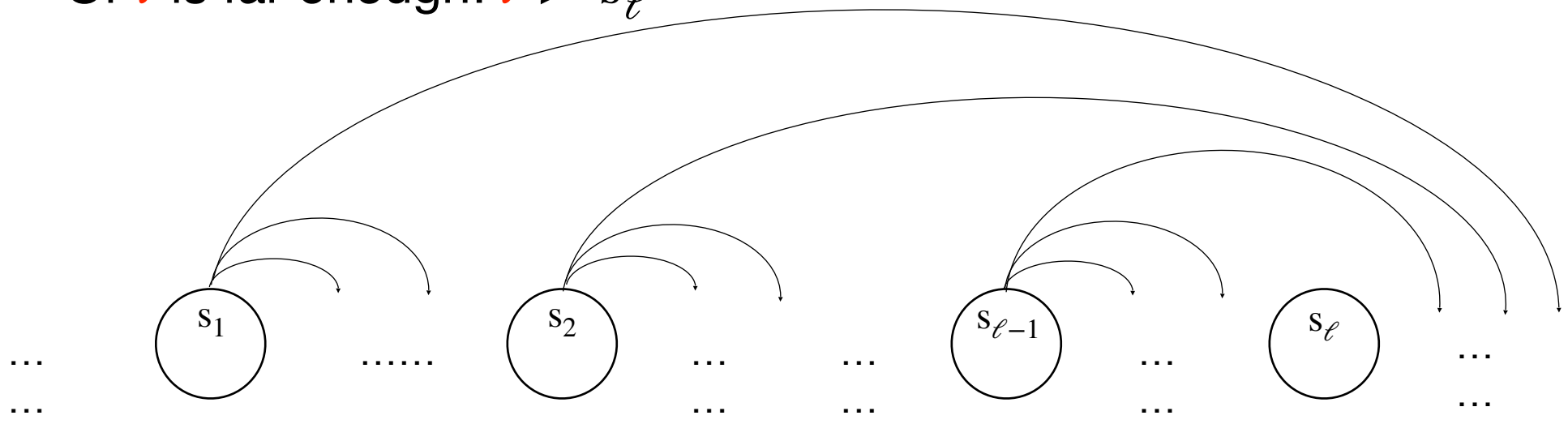

Step 2: Defining a Useful Structure in DAGs

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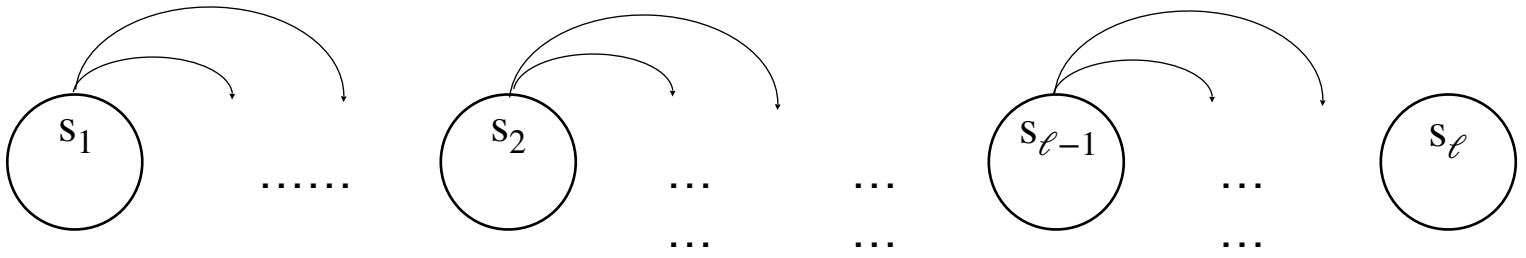

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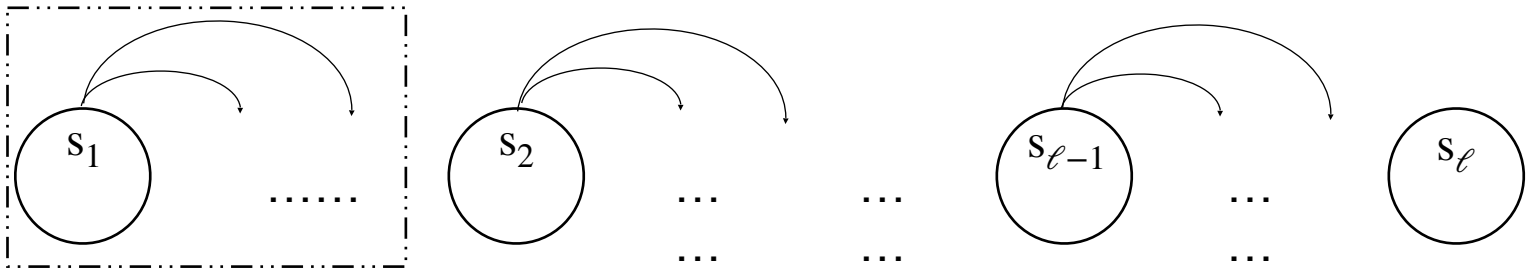

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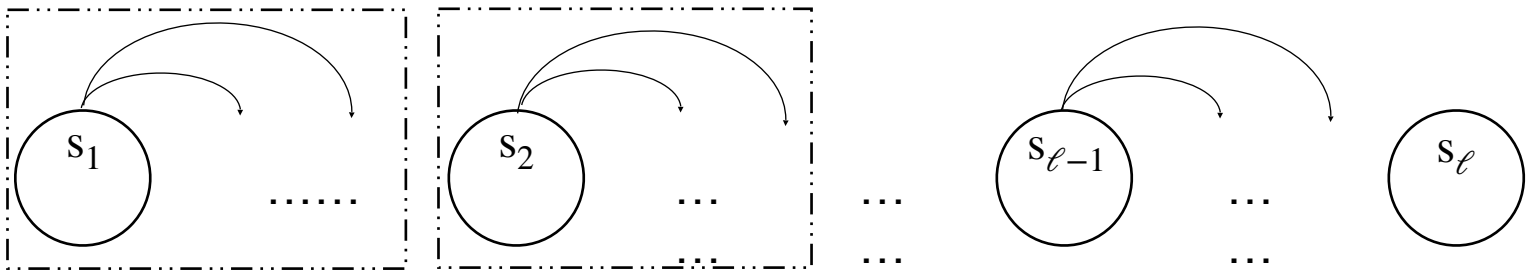

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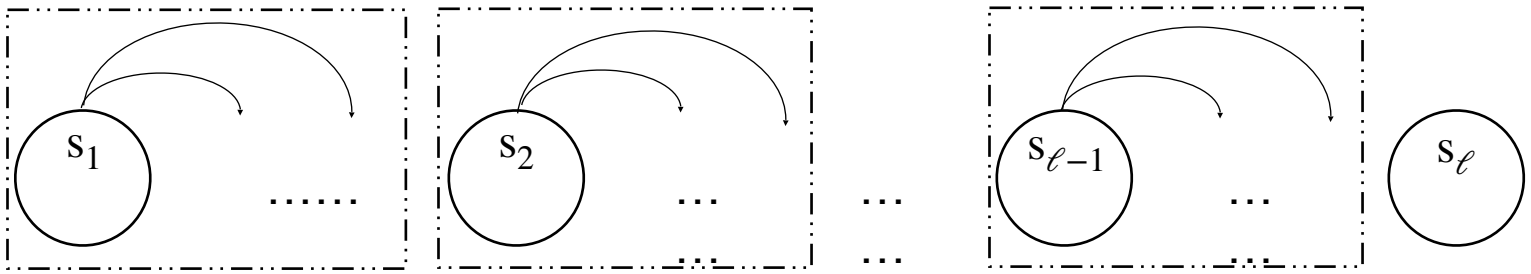

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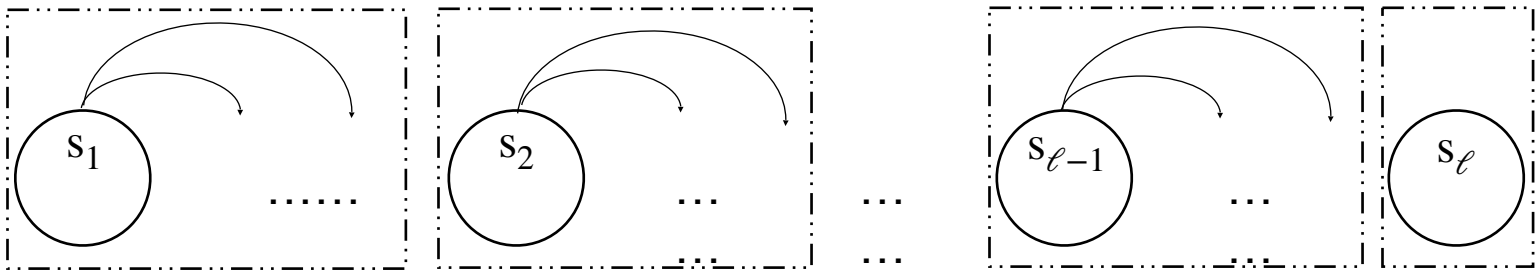

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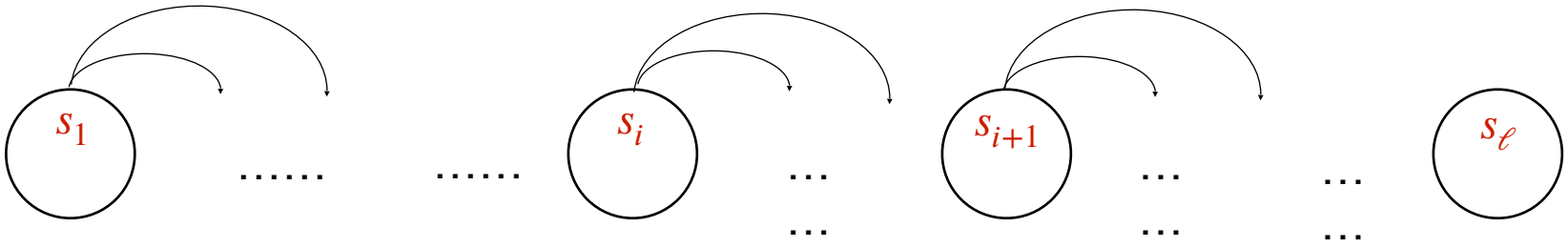
Proof: Induction on d.

Step 3: Using skipping sequence to attack



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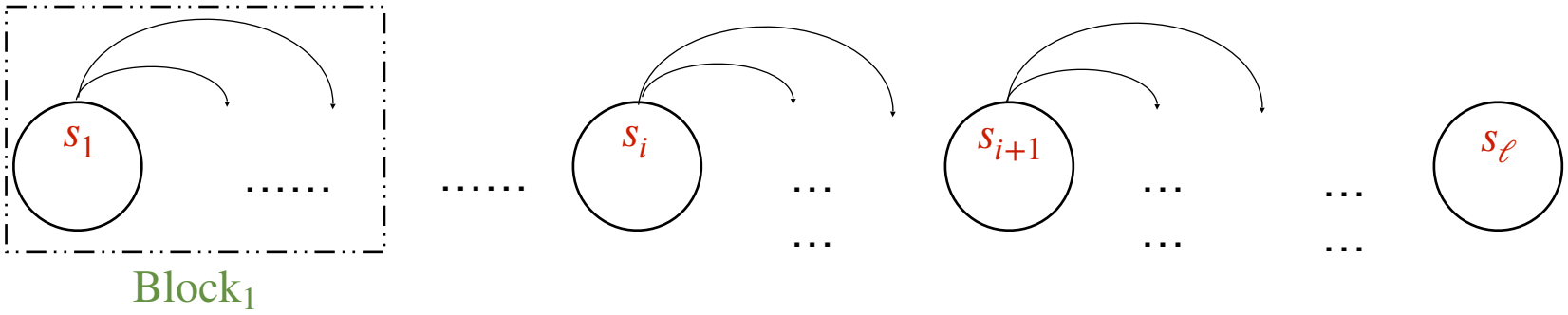

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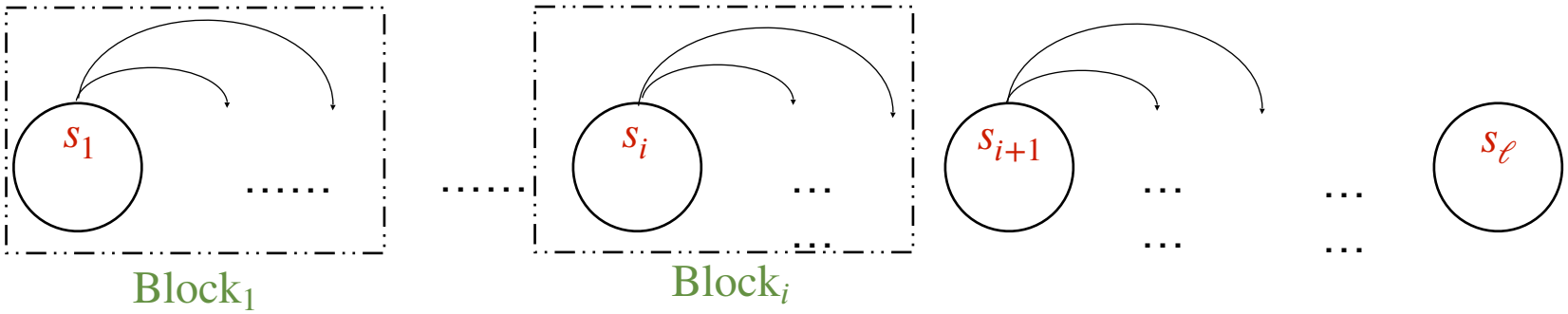

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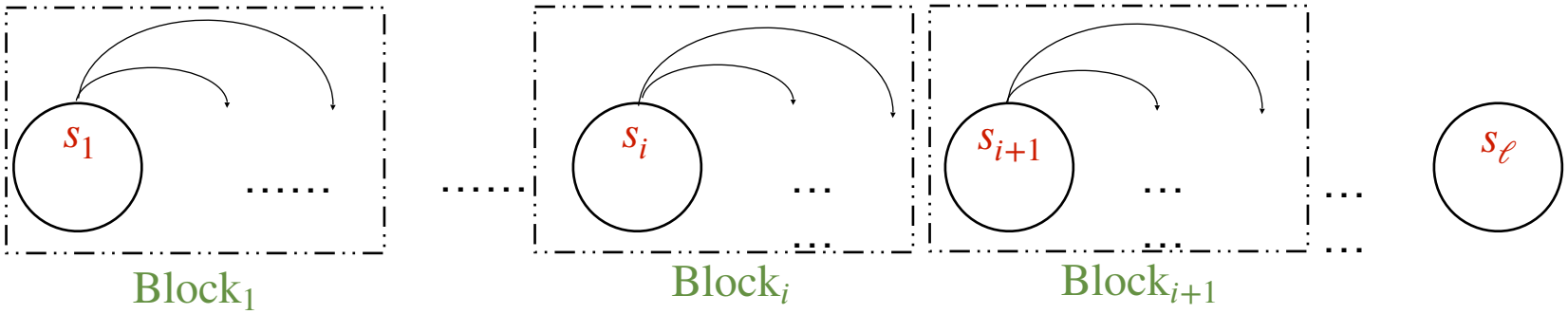

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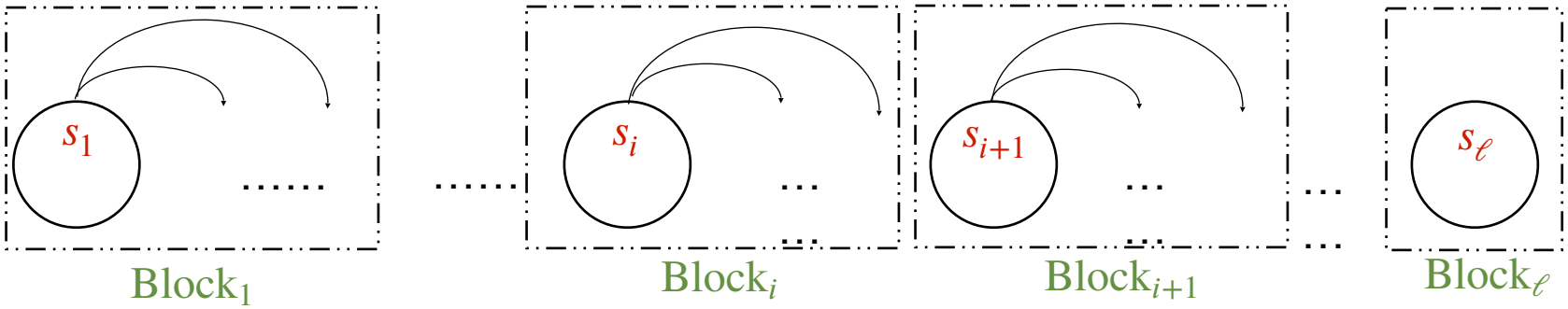

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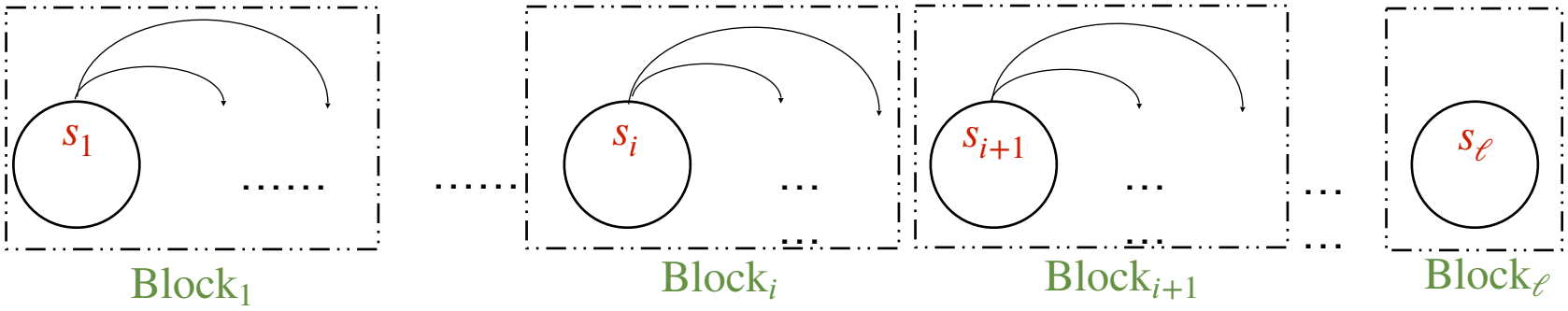

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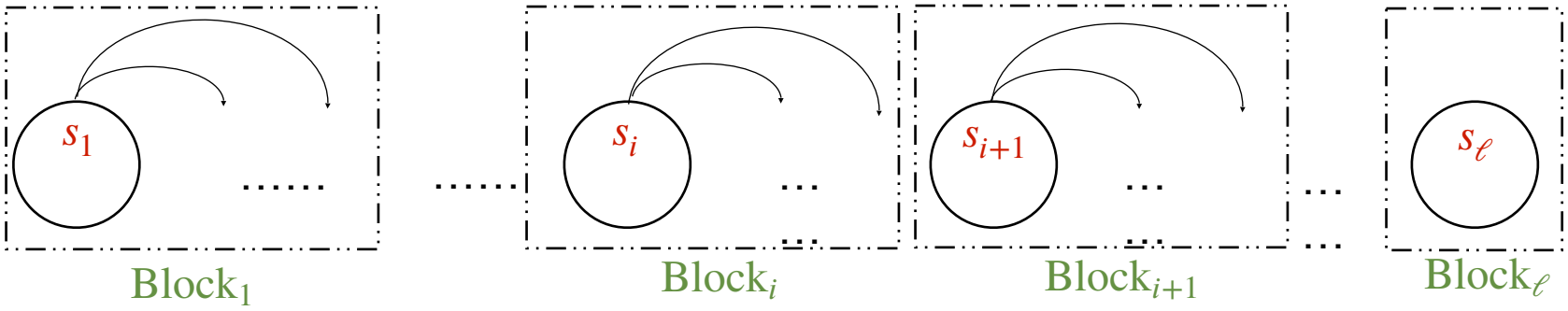

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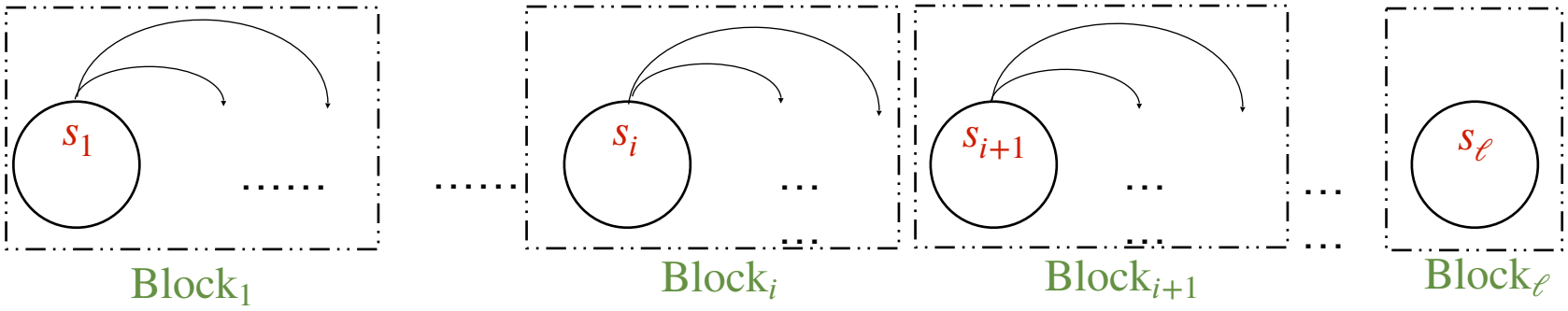

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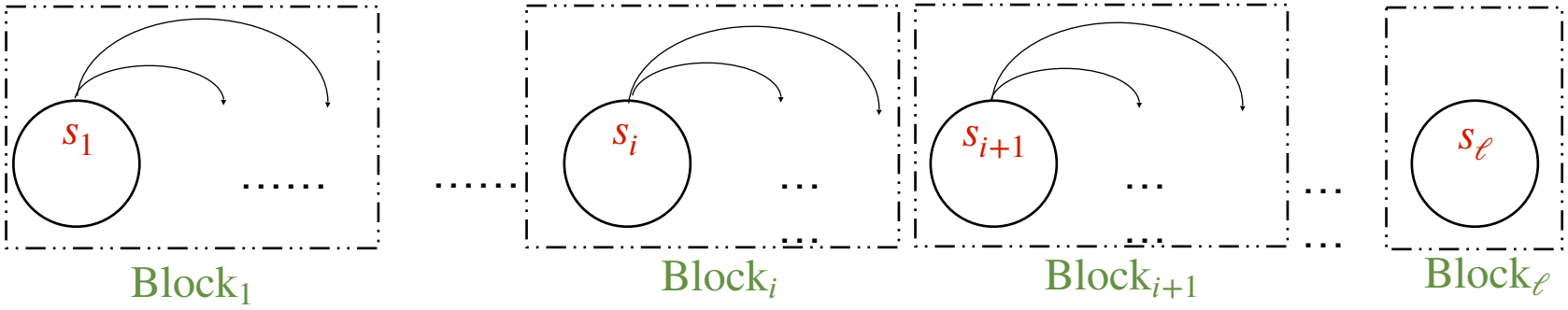

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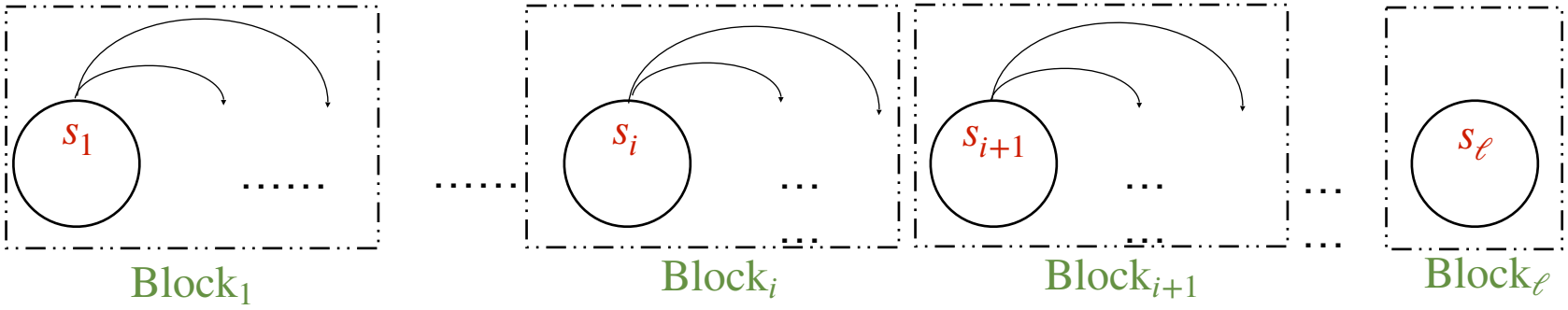

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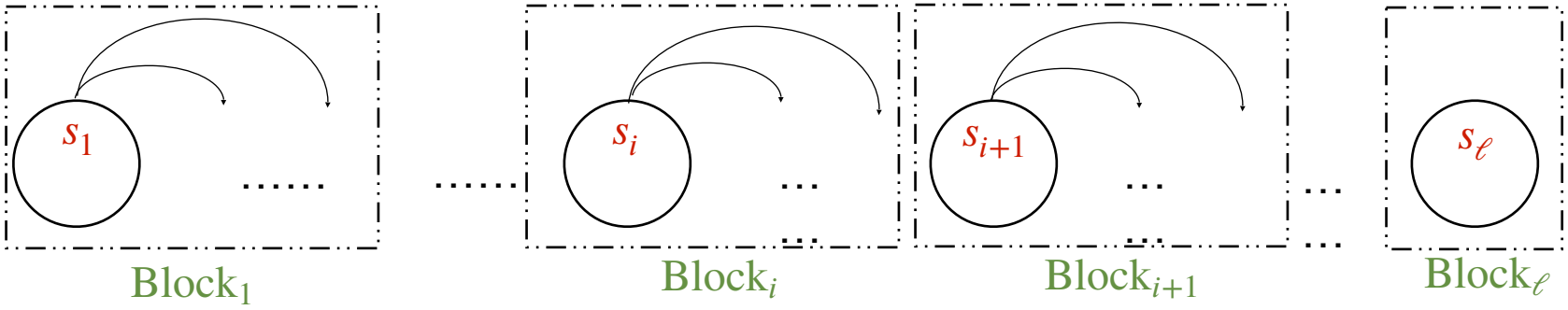

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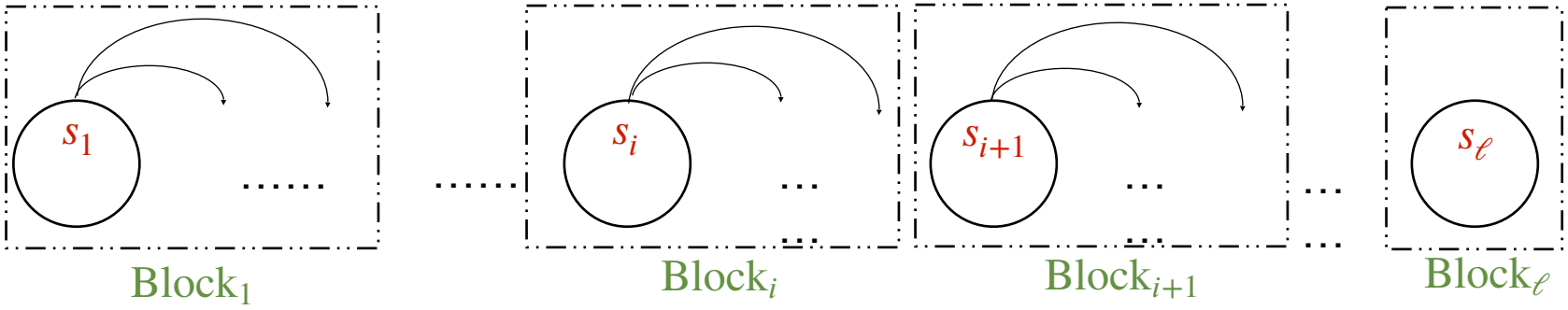

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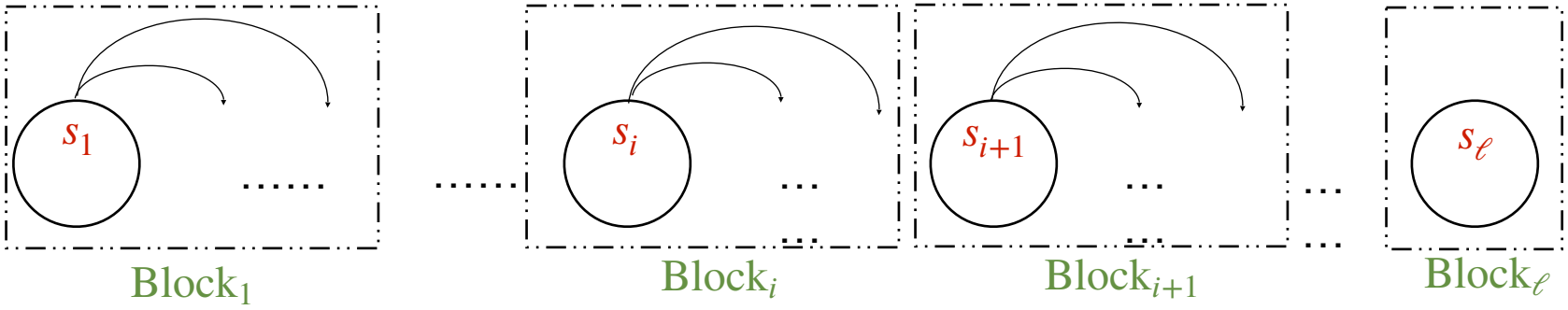

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- Main question: break this barrier using dynamic update times (that depend on the registered public keys) ?

Thanks!

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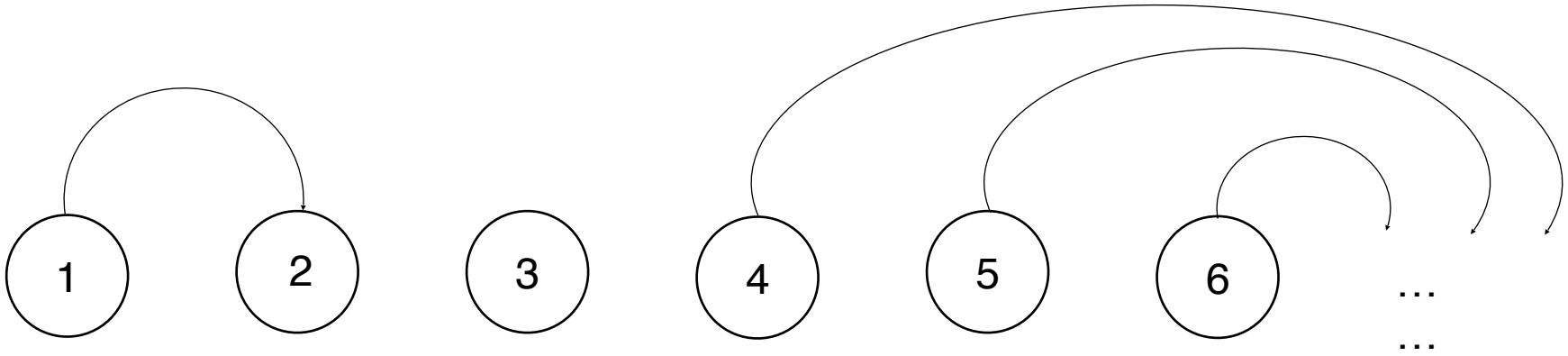

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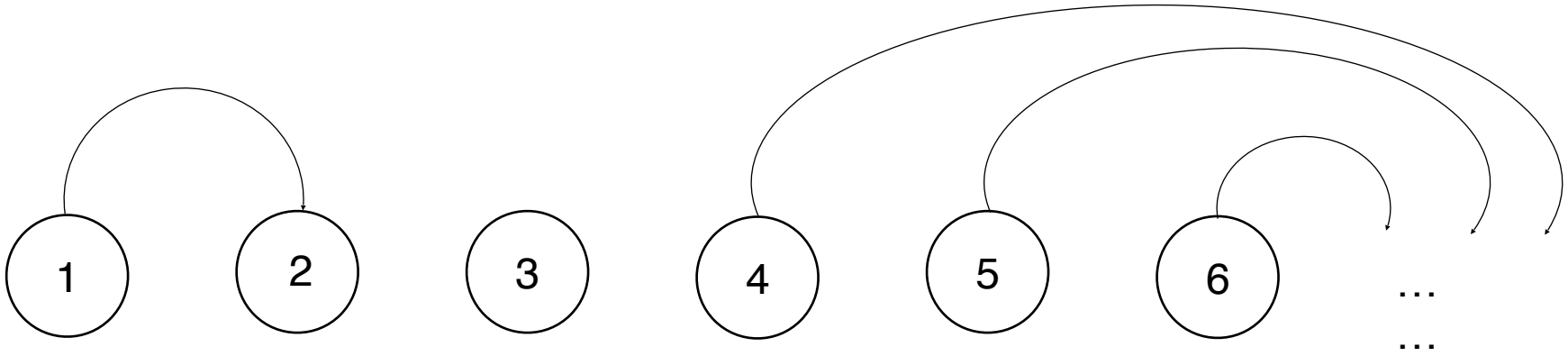

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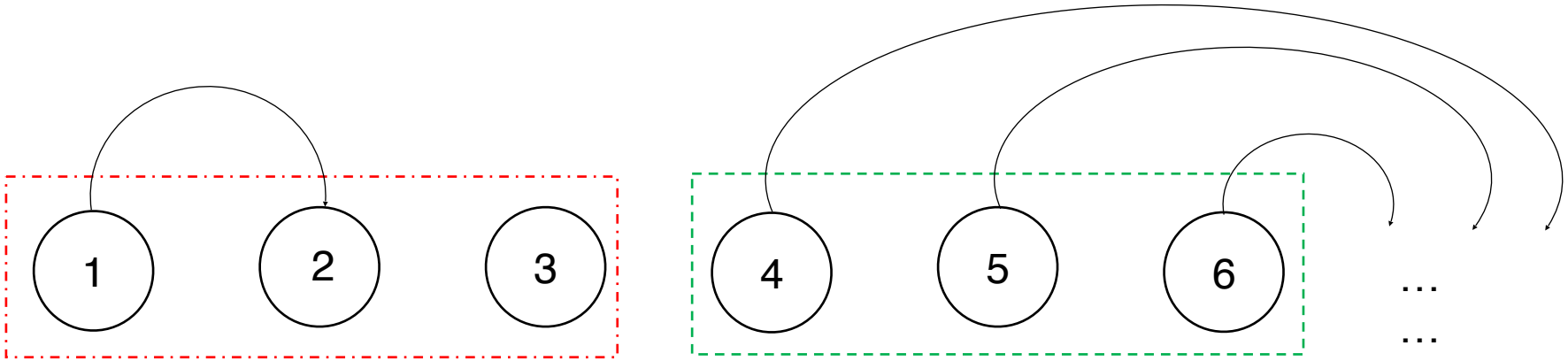

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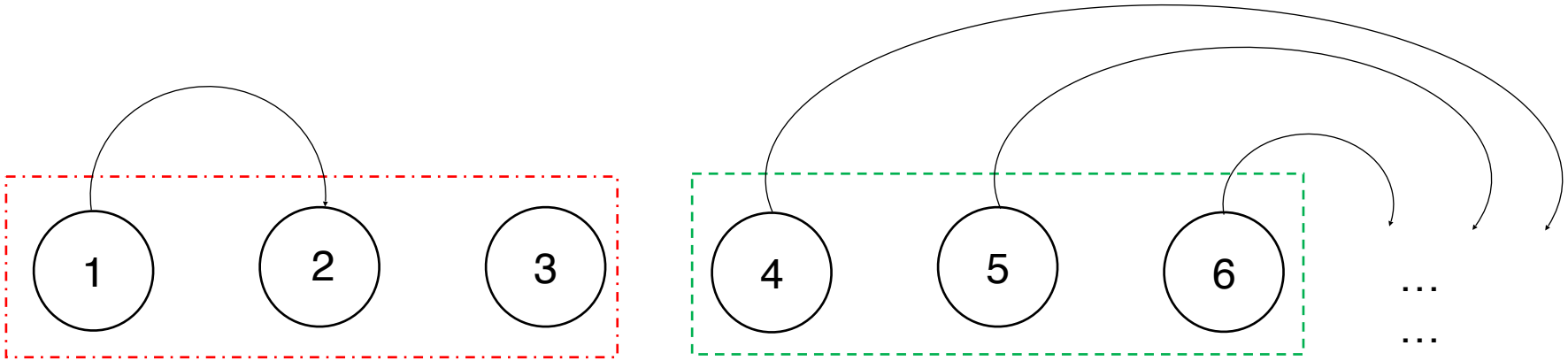

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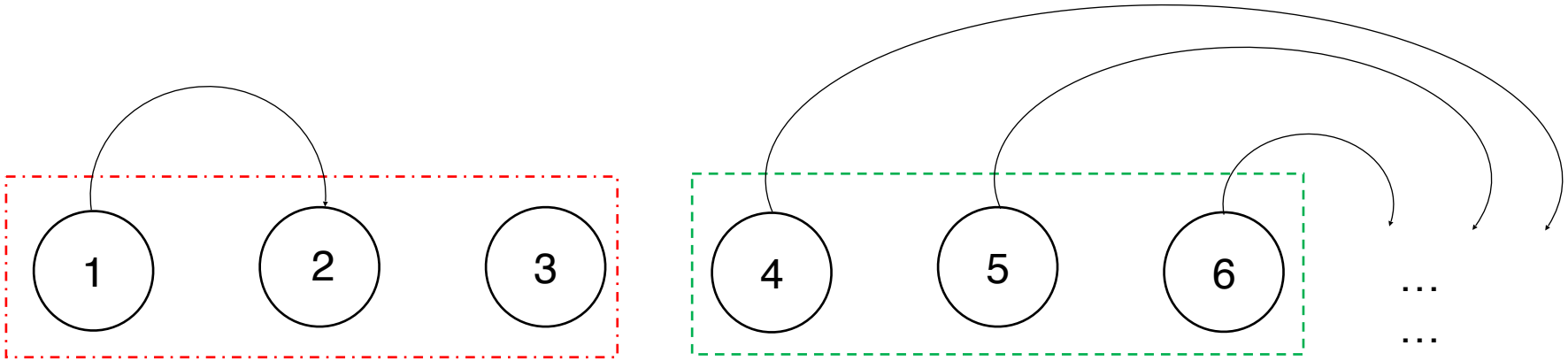

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