Universal Reductions

Reductions relative to stateful oracles

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November 10 2022

Joint work with Cody Freitag & Rafael Pass

Suppose we had a weak OWF **f**

f

$\forall$ A,
Pr[A inverts f] < 3/4

We want to build a strong OWF **f'**

f'

$\forall$ A,
Pr[A inverts f] = negl(.)

How do we prove the security of **f’**?

How do we prove the security of **f'**? By security reduction.

Suppose ∃ some **A** that inverts **f'** with 1/poly probability:

$$\mathbf{f'} \rightleftarrows \mathbf{A}$$

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Then ∃ **A'** that inverts **f** with probability >3/4:

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Contradiction! (with the weak one-wayness of f)

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Observe:
This proof is only useful “in the real world” if our model for attackers correctly captures the behavior of “real-life” adversaries!

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Extended Physical Church Turing Hypothesis:
All "real-life" attackers are captured by PPT (resp. QPT) Turing Machines

How do we prove the security of f’? By security reduction.

Suppose ∃ some PPT **A** that inverts f’ with 1/poly probability:

$$f' \rightleftarrows A$$

Then $\mathbf{R}^{A}$ inverts **f** with probability >3/4:

$$f \rightleftarrows R^{\boxed{A}}$$

Classically, we can write black-box reductions R^A:
R queries A many times

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say we want to invert **y** = f(x):

A(~, ~, **y**, ~, ~)
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Takeaway:
R^A utilizes many independent copies of **A**!

f **A**

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f **$R^{\boxed{A}}$**

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Suppose ∃ some PPT **A** th ability:

Takeaway:
$\mathbf{R}^{\mathbf{A}}$ utilizes many independent copies of **A**!

This is possible because we model **A** as an algorithm, which can be copied and run again.

Then $\mathbf{R}^{A}$ inverts **f** with prob

Classically, we can write black-box reductions $\mathbf{R}^{A}$:
R queries A many times

$$\mathbf{f} \rightleftarrows \mathbf{R}^{\boxed{\mathbf{A}}}$$

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What if we can’t run A many times?

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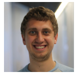
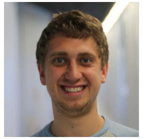
Maybe **A** is your **"next door neighbor"**  who happens to break **f'**:

f' $\rightleftarrows$ 

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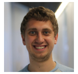
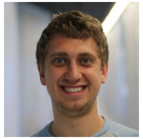
But you only have **"interactive access"** to  when trying to break **f**:

f $\rightleftarrows$ **R** $\rightleftarrows$ 

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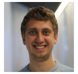
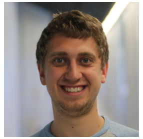
Maybe **A** is your **“next door neighbor”**  who happens to break **f’**:

f’ $\rightleftarrows$ 

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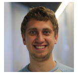
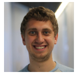
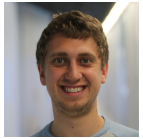
You have no clue how  **works.**

But you only have **“interactive access”** to  when trying to break **f:**

f $\rightleftarrows$ **R** $\rightleftarrows$ 

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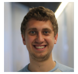
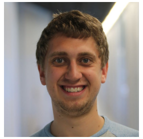
Maybe **A** is your **"next door neighbor"**  who happens to break **f'**:

$$f' \rightleftarrows$$ 

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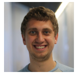
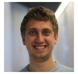
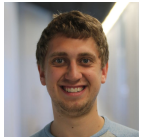
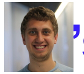
You have no clue how  works.

But you only have **"interactive access"** to  when trying to break **f:**

$$f \rightleftarrows R \rightleftarrows$$ 

You don't know 's code…

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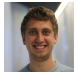
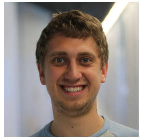
Maybe **A** is your **"next door neighbor"**  who happens to break **f'**:

$$f' \rightleftarrows$$ 

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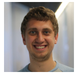
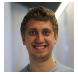
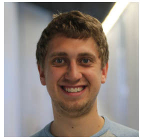
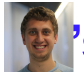
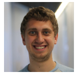
You have no clue how  works.

But you only have **"interactive access"** to  when trying to break **f**:

$$f \rightleftarrows R \rightleftarrows$$ 

You don't know 's code…

You can't "copy" 

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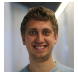
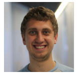
No “rewinding” 

Maybe **A** is your **“next door neighbor”**  who happens to break **f’**:

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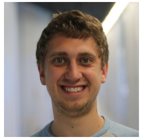

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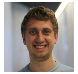
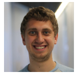
You have no clue how  **works.**

But you only have **“interactive access”** to  when trying to break **f:**

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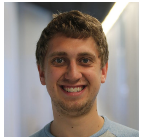
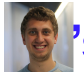
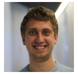


You don’t know  **’s code…**

You can’t “copy” 

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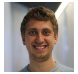
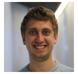
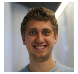
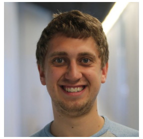
No “rewinding” 

Maybe A is your **“next door neighbor”**  who happens to break **f’**:

 **might have access to “cosmic resources” as far as you’re concerned**

⇄ 

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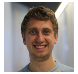
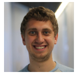
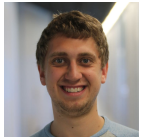
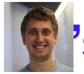
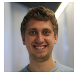
You have no clue how  **works.**

But you only have **“interactive access”** to  when trying to break **f**:

f ⇄ **R** ⇄ 

You don’t know  **’s code…**

You can’t “copy” 

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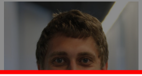

You have no clue how A rks.

Claim: we need to revisit classical proofs!

But you only have **"interactive access"** to when trying to break **f**:

f ⇄ **R** ⇄

You don't know 's code…

You can't "copy"

Looking forward, even just sending Cody multiple queries…

(~, ~, **y**, ~, ~)

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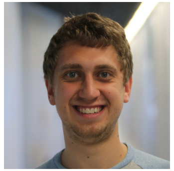

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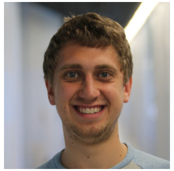

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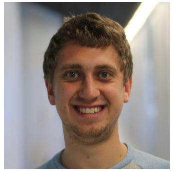

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A stateful adversary will remember that they've already answered a query.
"That's enough winning for today!"

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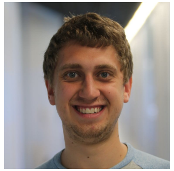

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Looking forward, we will assume that the adversary *wins "repeatedly" when given fresh challenges*. But even this is non-trivial to exploit.

“Stateful attackers” are already well motivated:

Quantum computers break existing proof techniques:

- No-cloning theorem: cannot copy quantum advice.
- Can’t be “rewound” when playing interactive security games

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Theoretically:

- We prefer a theory of cryptography that makes as few assumptions as possible!
- Can we get by without assuming that attackers are PPT (or QPT)?

This Talk:

We propose a reduction-based
theory of computational cryptography
with minimal assumptions on
the Nature of real-world attackers.

Next up: Defining Universal Reductions

After that: Feasibility and Impossibility Results

Defining Universal Reductions

A $\leftrightarrow$ Nat

"Attacker"
uniform **PPT**

"Nature"
nonuniform **any choice of runtime**
Stateful

"Augmented Adversary" (A, Nat)

A new model of attacker: “Augmented Adversaries”

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“Augmented Adversary” (A, Nat)

Some unknown power in the cosmos to which we only have “interactive access”

A new model of attacker: “Augmented Adversaries”

A classic attacker
that uses Nature
to break some scheme

A ⟷ **Nat**

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Augmented Security Game

$$\mathsf{C} \leftrightarrow \mathsf{A} \leftrightarrow \mathsf{Nat}$$

"Challenger"
uniform **PPT**
outputs "win"/"lose"

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***Observe*: the attacker can alter the state of Nature during the interaction. This is intentional and a key property of our definition.**

Note: all communication is classical (and **C**/**A** are PPT) because we want universal reductions to work in a PPT world!

Robust winning: “winning repeatedly”

Recall: We want adversaries that win “repeatedly” when given fresh challenges.

$$\mathbf{C} \leftrightarrow \mathbf{A} \leftrightarrow \mathbf{Nat}(\rho)$$

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Interaction prefix ρ:
a transcript of messages previously sent to Nat before the beginning of execution, including coins flipped by Nat.

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Definition: (A, Nat) has **robust advantage** a(.) for C, if ∀ *interaction prefixes* ρ, ∀ λ:
Pr[(**A**, **Nat**(ρ)) wins **C**] ≥ a(λ)

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Universal Reductions

∃ an **ε-universal reduction** from **C** to **C'** if ∀ PPT **A**, ∃ PPT **A'** s.t. ∀**Nat**:

Suppose (**A**, **Nat**) has **robust advantage a(·)** for **C**

$$\begin{tikzcd}
C \arrow[r, leftrightarrow, shift left=2] \arrow[r] & A \arrow[l, shift left=2] \arrow[r, leftrightarrow] & Nat
\end{tikzcd}$$

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C' \arrow[r, shift left, "f(x)"] & A' \arrow[l, shift left, "f^{-1}(f(x))"] \arrow[r, leftrightarrow] & Nat
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Universal Reductions

This is the central notion in our paper.

$\exists$ an **ε-universal reduction** from **C** to **C'** if $\forall$ PPT **A**, $\exists$ PPT **A'** s.t. $\forall$ **Nat**:

Suppose (**A**, **Nat**) has **robust advantage a(·)** for **C**

$$\begin{tikzcd} \mathbf{C} \arrow[r, leftrightarrow, shift left=3] \arrow[r, shift left=0] \arrow[r, leftarrow, shift right=3] & \mathbf{A} \arrow[r, leftrightarrow] & \mathbf{Nat} \end{tikzcd}$$

Then (**A'**, **Nat**) has **robust advantage ε(·,a(·))** for **C'**.

A' can now use the fact that (A, Nat) wins "on demand".

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Are Universal Reductions universal? Certainly, universal reductions imply reductions w.r.t. (nu)PPT, (nu)QPT.

Quick Comparisons

Relativized Reductions

- A *relativized reduction* gives attackers A^O access to some arbitrary oracle O
- O is modeled as a (perhaps uncomputable) function
- Universal reductions can be viewed as *relativized reductions* for *stateful, interactive oracles* O (in contrast to a *non-interactive*, *stateless* oracle).

Universal Composability [Canetti00]

- Universal reductions are *syntactically* similar to UC with unbounded environments
- *Semantically* very different: our notion is reduction-based & computational. (For instance, UC *security proofs* can rewind the environment [e.g. CLP10])

What can we do with universal reductions?

Warmup: a basic feasibility result

Thm 1. Classical 1-shot straight-line black-box reductions imply universal reductions.

> A straightforward argument, since a 1-shot reduction uses Nature once.

> **Corollaries**: Witness Indistinguishability/PRG Length Extension/PRFs/SKE/Commitments from PRGs

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What about problems that have classical reductions invoking the attacker multiple times?

Unfortunately, not all is possible...

<u>Thm 2</u> (*Impossibility of Hardness Amplification*):
There is no universal black-box reduction from the OWF security of $g^n(x_1 \ldots x_n) = (g(x_1), \ldots, g(x_n))$ to the OWF security of $g(x)$ that uses only black-box access to g, and that works for any function g.

<u>Thm 3</u> (*Impossibility of a Goldreich-Levin-Style Theorem*):
There is no universal black-box reduction from the security of the hardcore predicate $h(x,r) = \langle x,r \rangle$ w.r.t. $f(x, r) = (g(x), r)$ to the OWF security of g that uses only black-box access to g and that works for any function g.

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Let's go over the intuition of the proofs to understand universal reductions better.

Recall: classical Hardness Amplification

Let g be a weak OWF. Then $g^n = (g(x_1), \dots, g(x_n))$ is a strong OWF.

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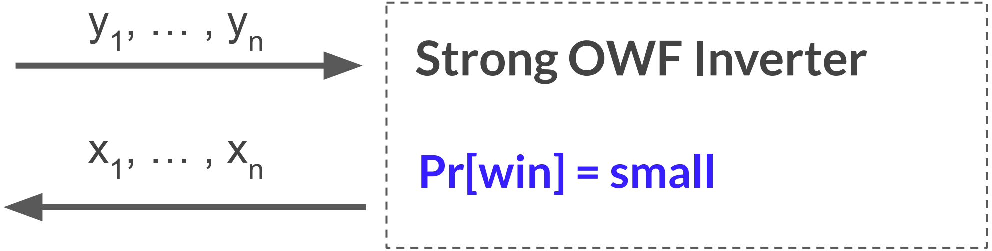

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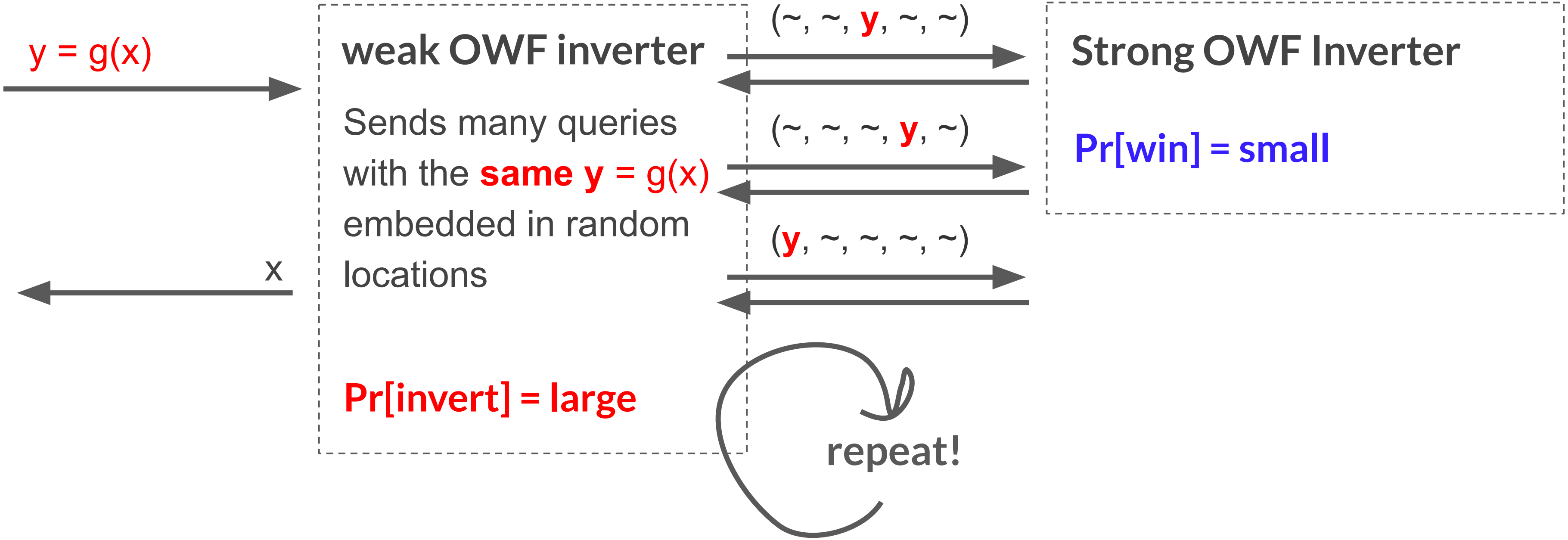

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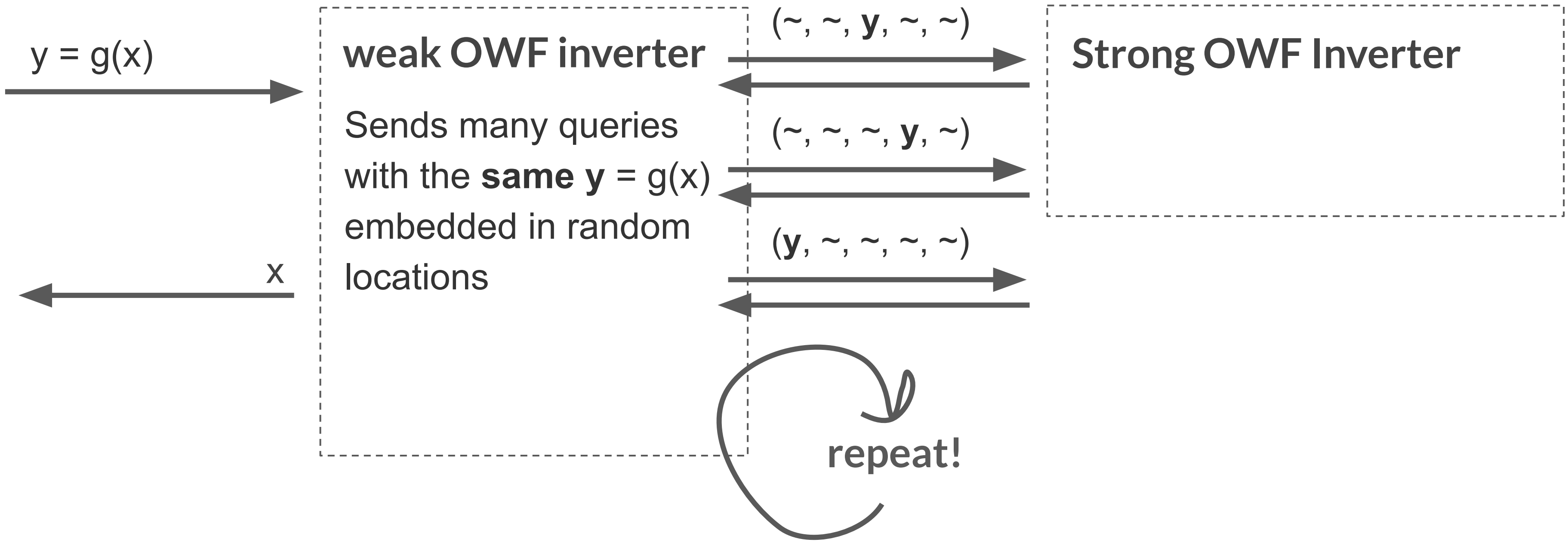

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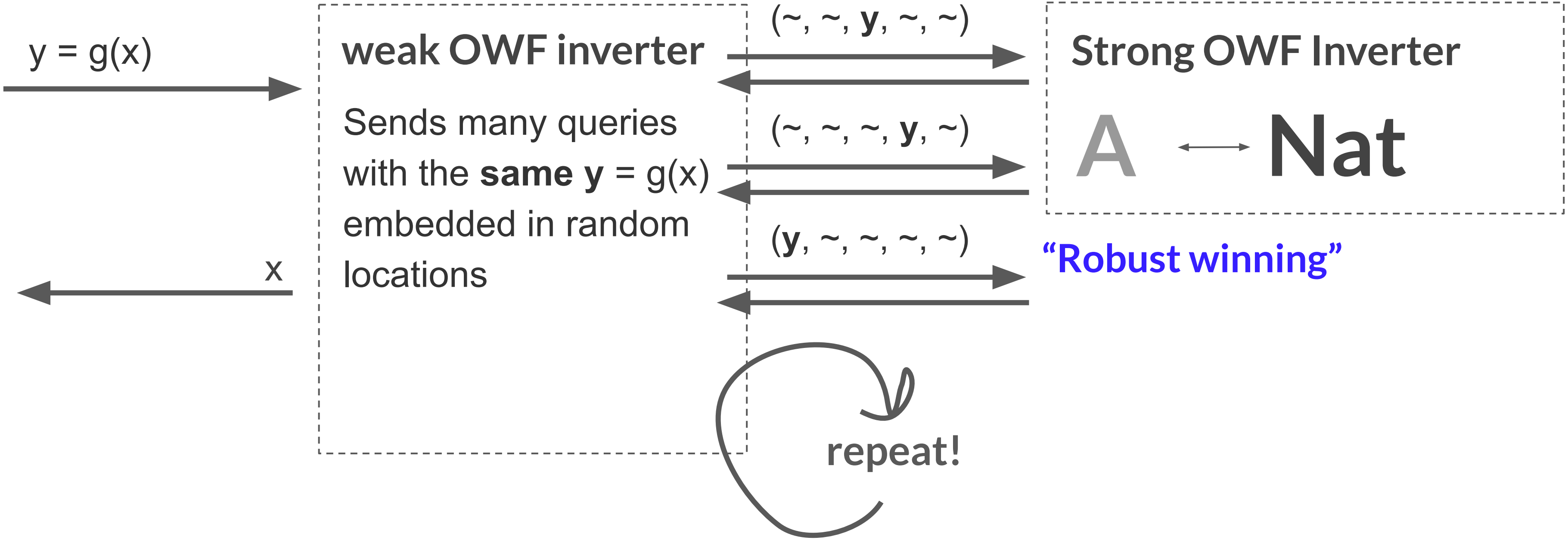

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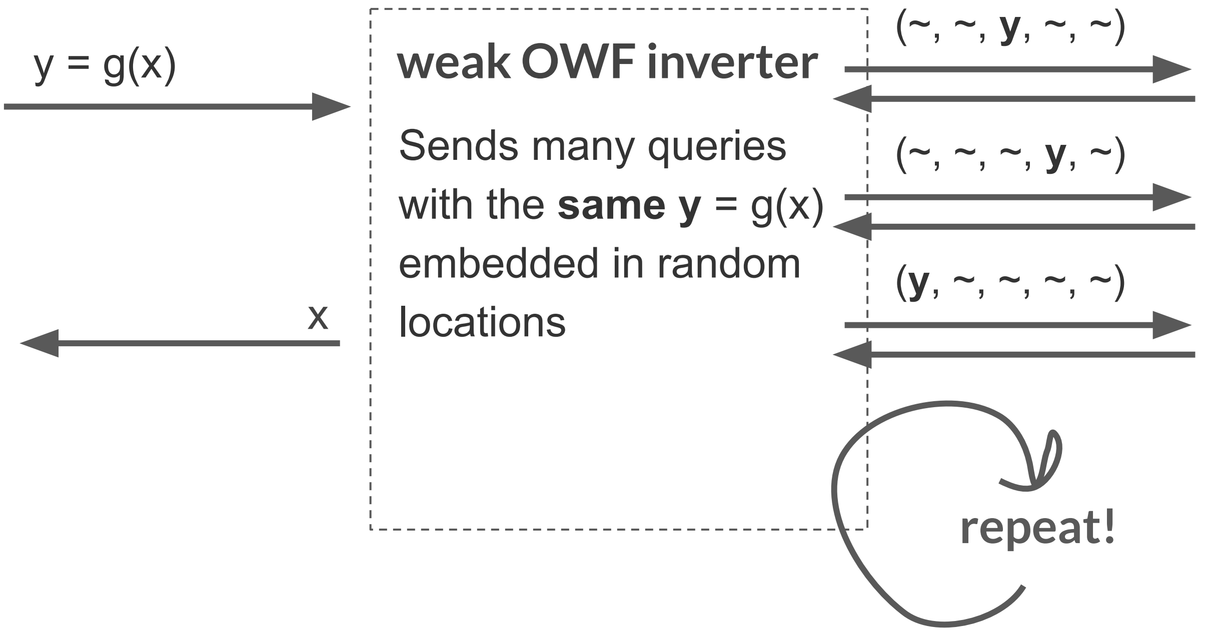

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- **A** forwards messages
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if it previously saw y, *it can ignore future correlated queries (e.g. that contain* ***y****.)*

$y = g(x)$

weak OWF inverter

Sends many queries with the **same y** = g(x) embedded in random locations

x

(~, ~, **y**, ~, ~)

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repeat!

Strong OWF Inverter

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Strong OWF Inverter

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repeat!

In the full proof, we show that any reduction – not just Yao's– that is black-box in g will have to send correlated queries to (**A**, **Nat**).
Thus, similarly broken!

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Indeed,
Hardness amplification is possible for specific one-way functions!

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Let f be a re-randomizable OWF. Then Yao's reduction is a universal reduction.

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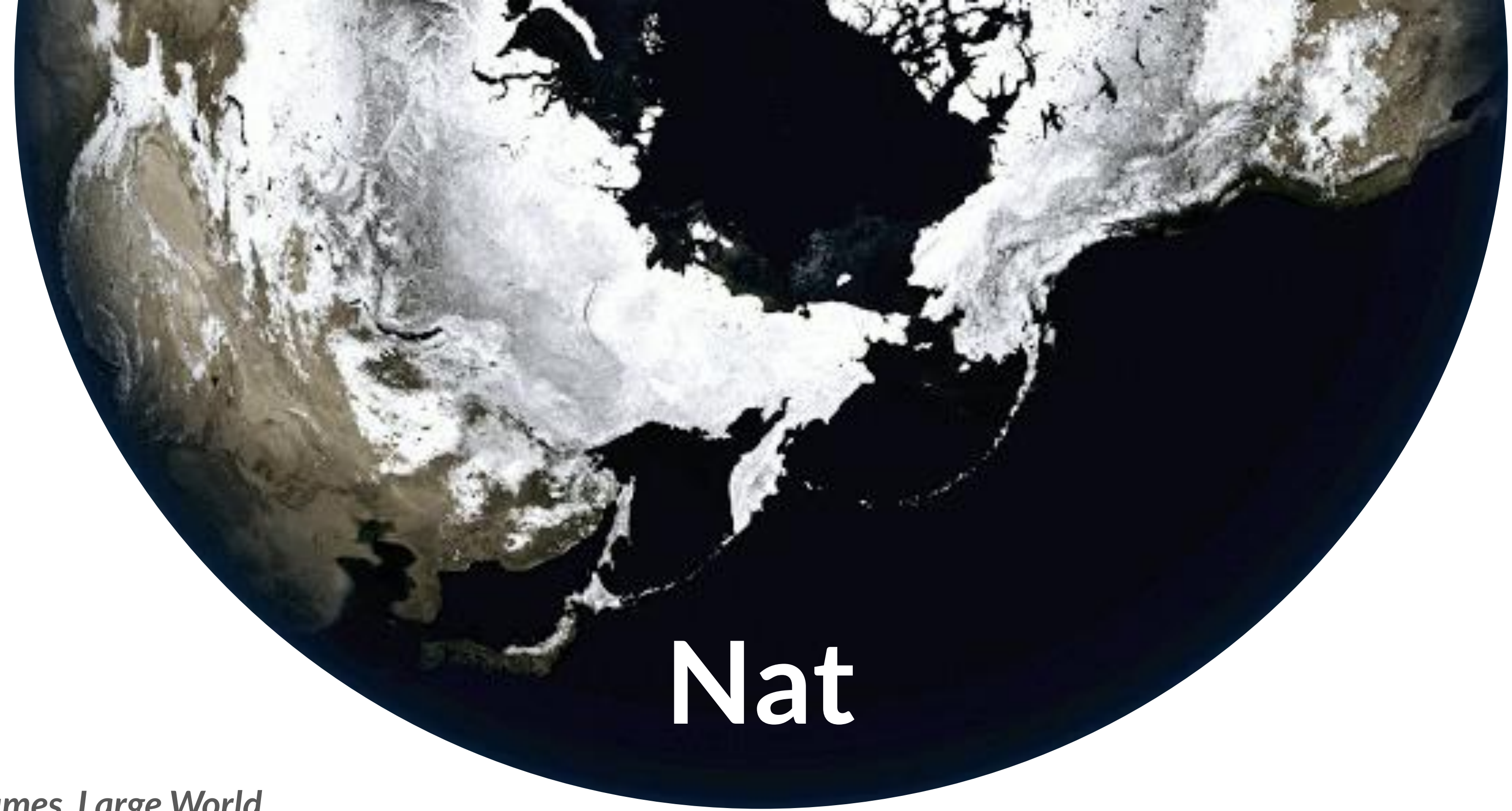
for now, let's try to climb a different mountain…

Briefly: Restricting Nature

Can we get non-trivial results by imposing constraints on Nature?



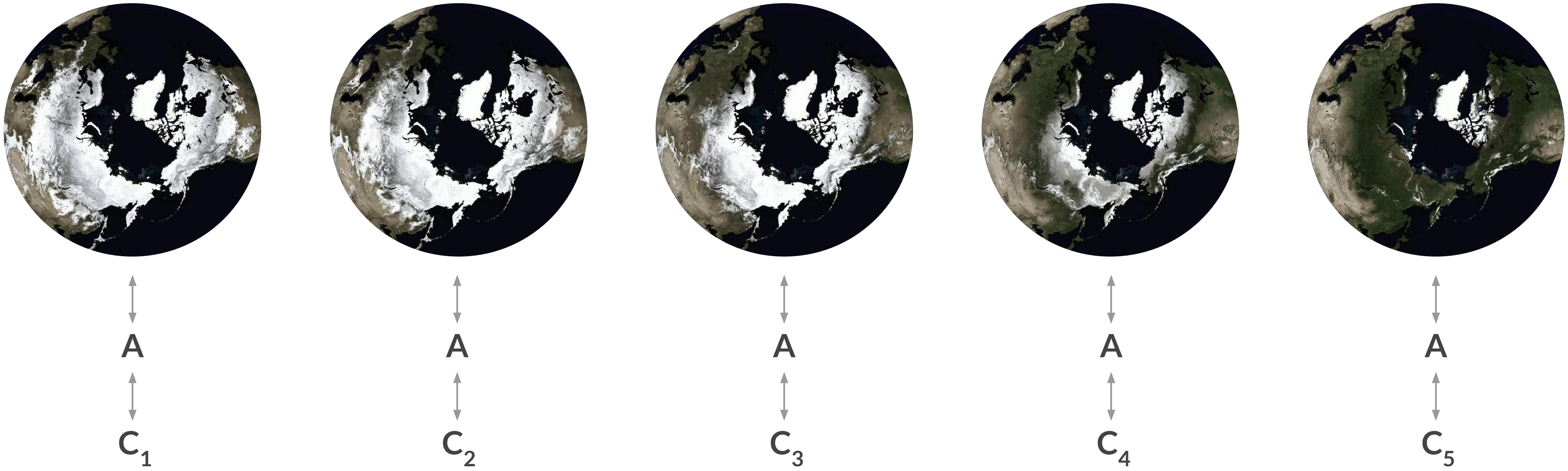
A

C

<u>***Small Games, Large World***</u>

It may be presumptuous to think that **C** or **A** can *influence* the future behavior of **Nat**.

What if **Nat** evolves over time (# of queries it has received)...



...but has a short term memory, and behaves independently of prior interactions?

Time-Evolving Windowed Natures

Theorem 5 (informal): Time-Evolving k-window Natures.
Suppose the behavior of "Nature" depends only on the number of messages it has seen, and the last k messages it has seen. Then classical non-adaptive straightline black-box reductions imply universal reductions w.r.t. this Nature.

We can think of (**A**, **Nat**) as a sequence of attackers $\mathbf{A_1}$ $\mathbf{A_2}$ $\mathbf{A_3}$ ...

How do we turn a "sequence of attackers" that must be queried in order into a single "restartable" adversary?

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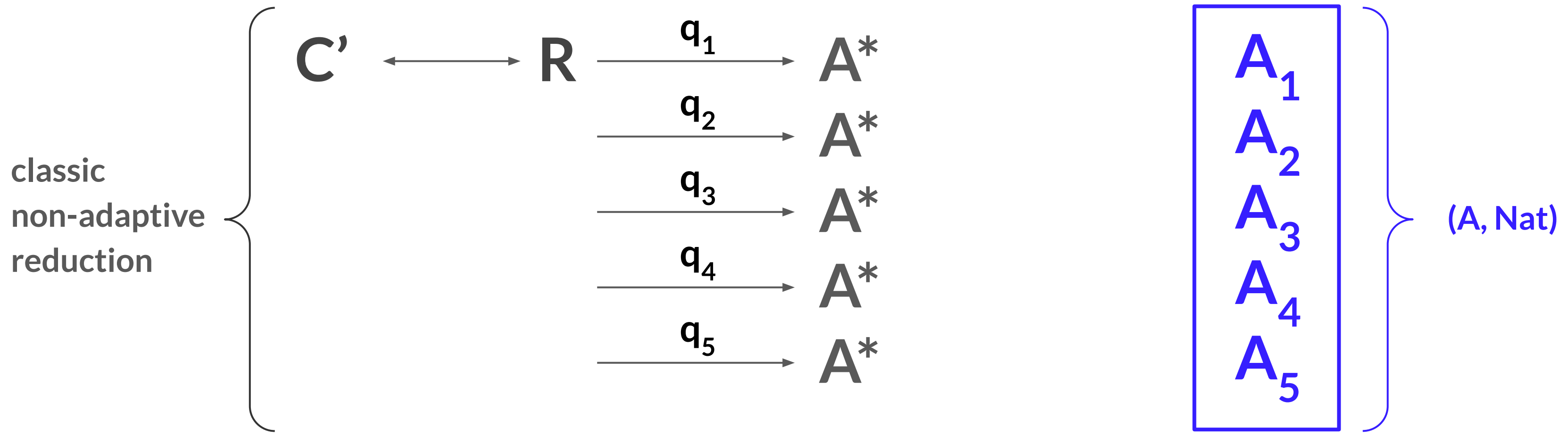

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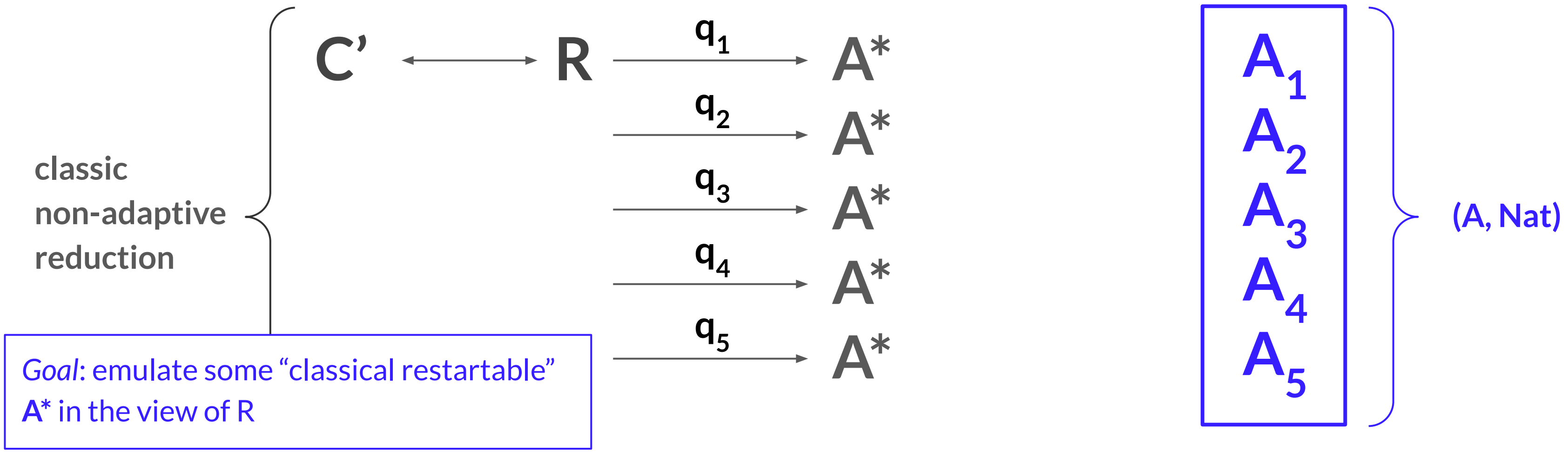

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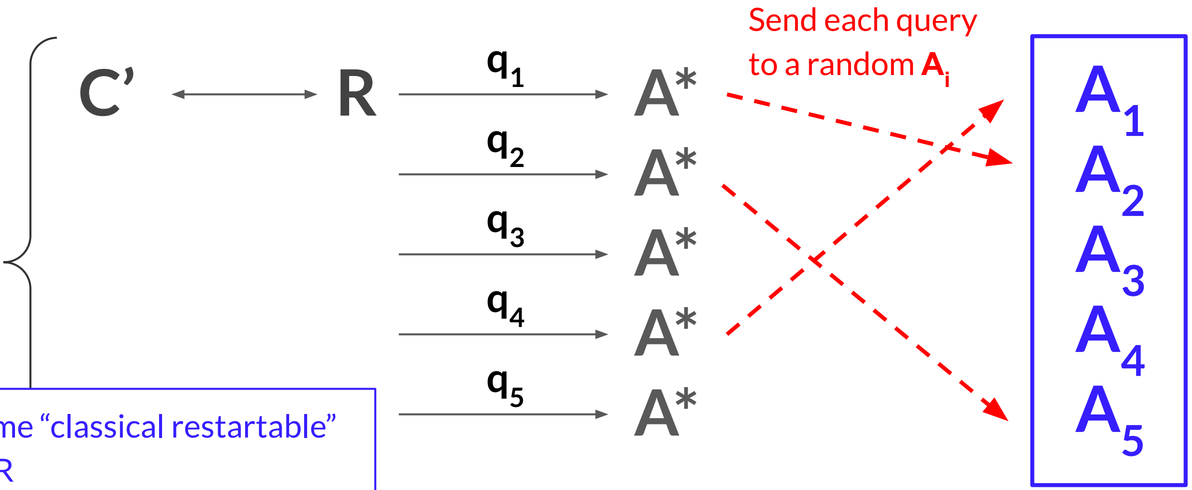
classic non-adaptive reduction



Nature is going to reply to queries out of order as it evolves $A_1, A_2, A_3 \ldots$

> Allowed since order of $q_1 \ldots q_5$ doesn't matter, by nonadaptivity.

Goal: emulate some "classical restartable" A* in the view of R

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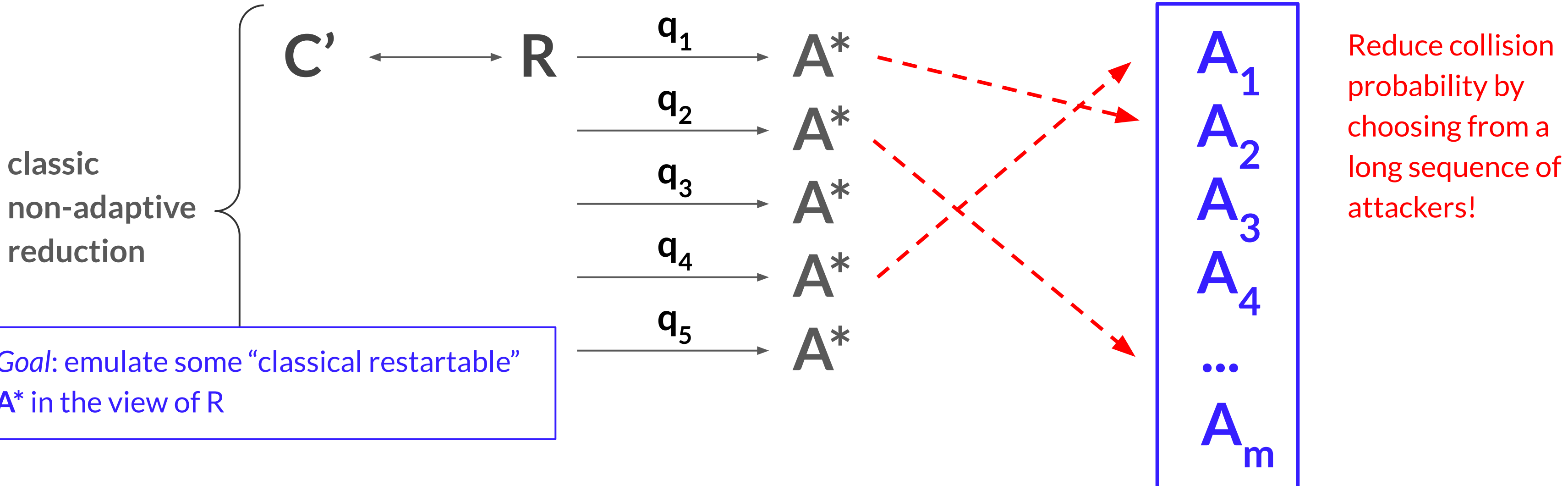

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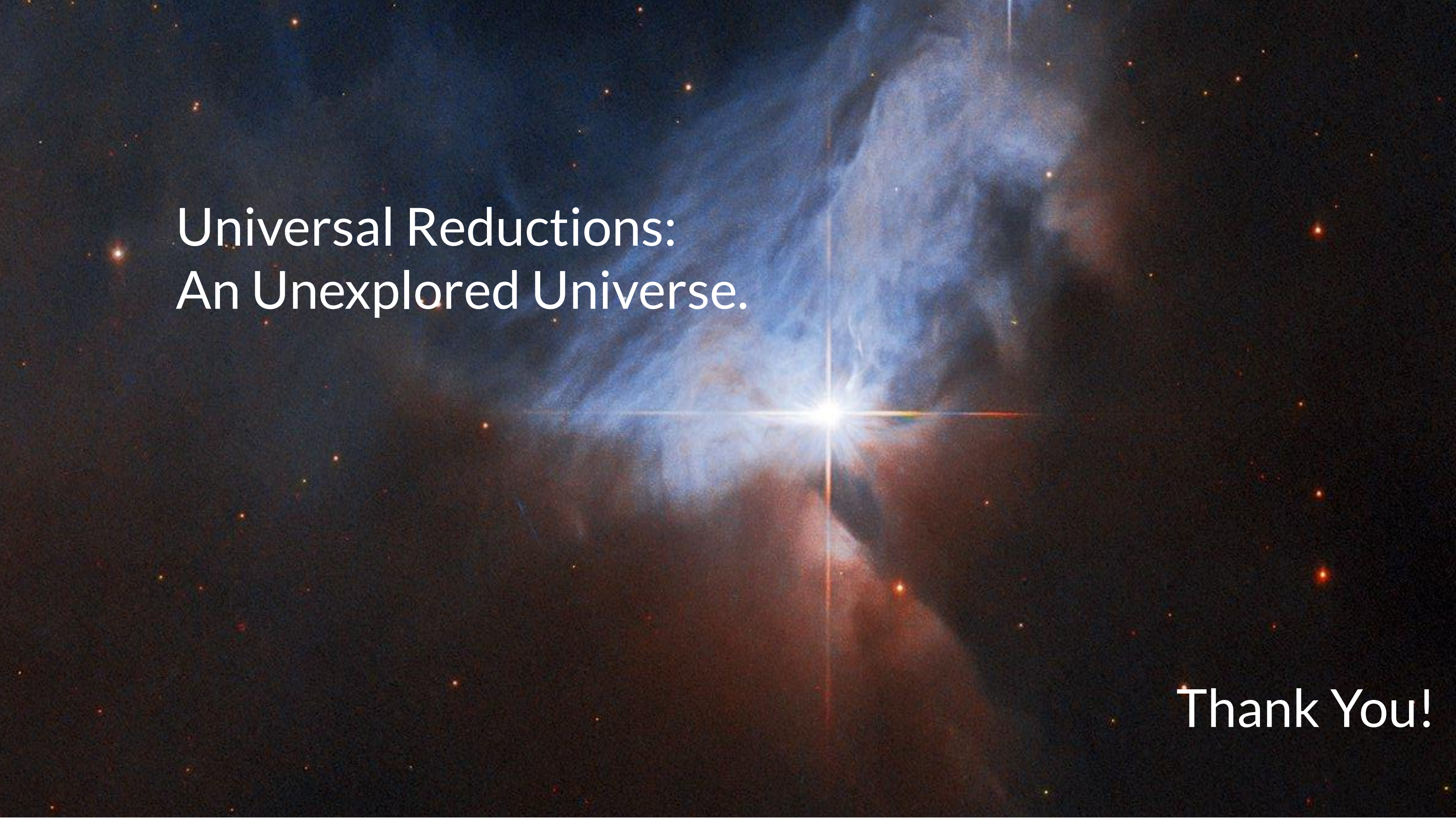
In conclusion: alot to unpack.

Takeaway: we can write meaningful security proofs w.r.t. stateful attackers!

Yet, at the same time, new techniques are clearly necessary.

- PRGs from OWFs?
- MPC?

We have hope for a “future-proof” notion of cryptography…


Universal Reductions:
An Unexplored Universe.
Thank You!