Fully Succinct and Updatable Batch Arguments for NP from Indistinguishability Obfuscation

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Batch Arguments for NP

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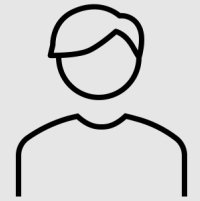
PROVER

VERIFIER

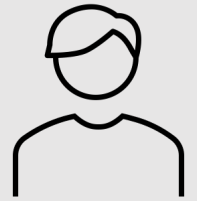


Batch Arguments for NP

PROVER

$(C, x_1, \ldots, x_t)$

VERIFIER



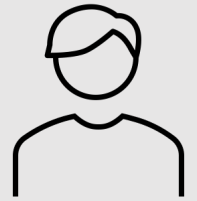
Batch Arguments for NP

$$\mathcal{L}_C = \{x \in \{0,1\}^n : \text{ exists } w \in \{0,1\}^m, C(x,w) = 1\}$$

PROVER

$$(C, x_1, \ldots, x_t)$$

VERIFIER



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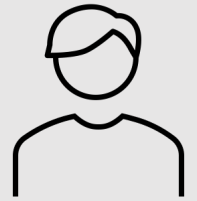

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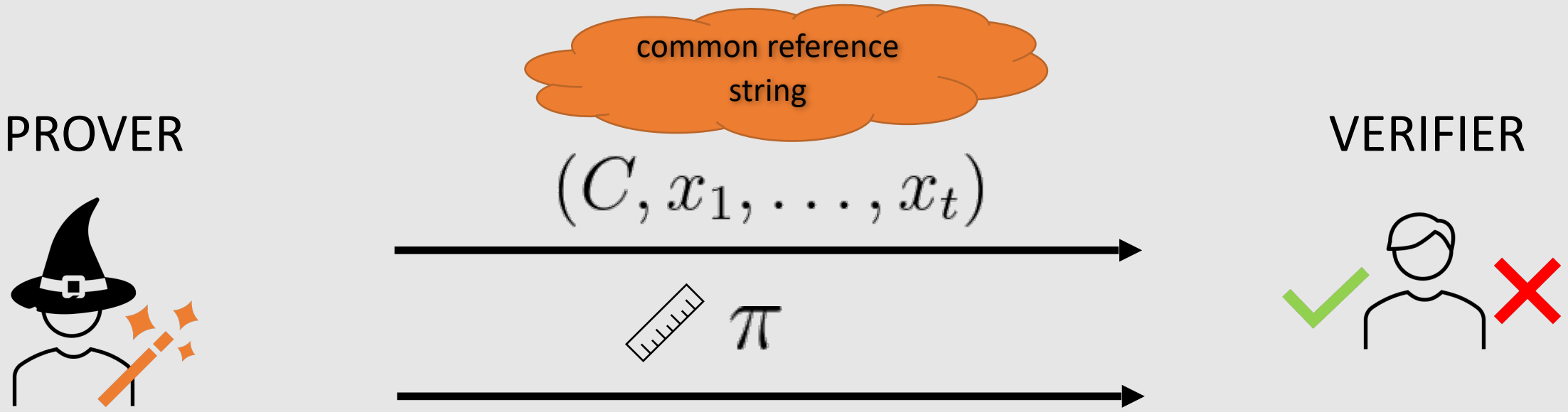

Batch Arguments for NP - BARGs

Proof π

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of instances

Circuit size

Proof π

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Proof π

$o(t)$

$\mathsf{poly}(\log t)$

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$\mathsf{poly}(|C|)$ $\quad$ $\mathsf{poly}(\log |C|)$

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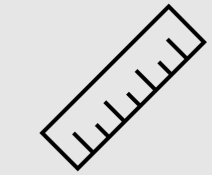
fully succinct $\quad$ $\mathsf{poly}(\lambda, \log t, \log |C|)$

Batch Arguments for NP - BARGs



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Completeness: *"Honest prover convinces verifier of true statements"*

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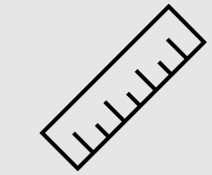

Proof π

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Circuit size

$\mathsf{poly}(|C|)$ $\quad$ $\mathsf{poly}(\log |C|)$

fully succinct $\quad$ $\boxed{\mathsf{poly}(\lambda, \log t, \log |C|)}$

Completeness: *"Honest prover convinces verifier of true statements"*

Soundness: *"No efficient prover can convince honest verifier of false statements"*

Prior Work (BARGs for NP)

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- Use SNARGs for NP - Proof independent of witness length

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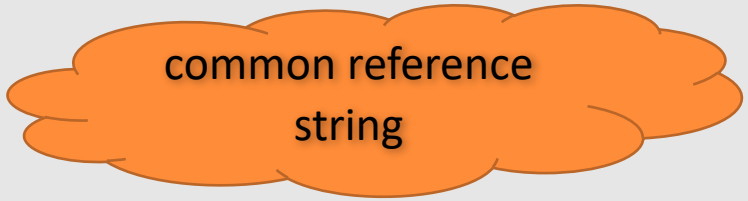
Sahai-Waters14



crs grows with t

supports apriori fixed instances

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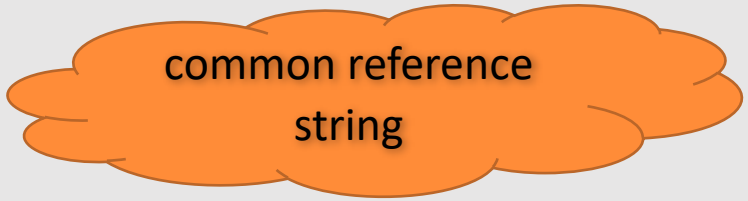

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Want to support arbitrary polynomial instances

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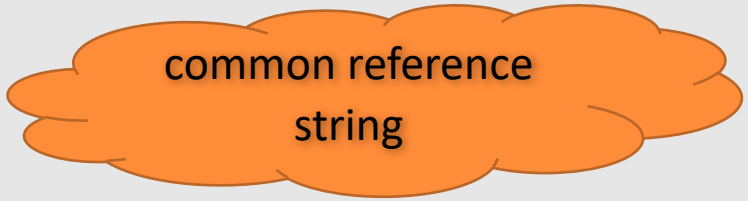
Sahai-Waters14



crs grows with t

supports apriori fixed instances

Want to support arbitrary polynomial instances

- CJJ21b,DGKV22 – from LWE, WW22 – from bilinear maps, CJJ21a + KLVW22 – DDH + QR

Not fully succinct

Our Results

BARGs for NP

Our Results

BARGs for NP

Support arbitrary polynomial instances

Our Results

BARGs for NP

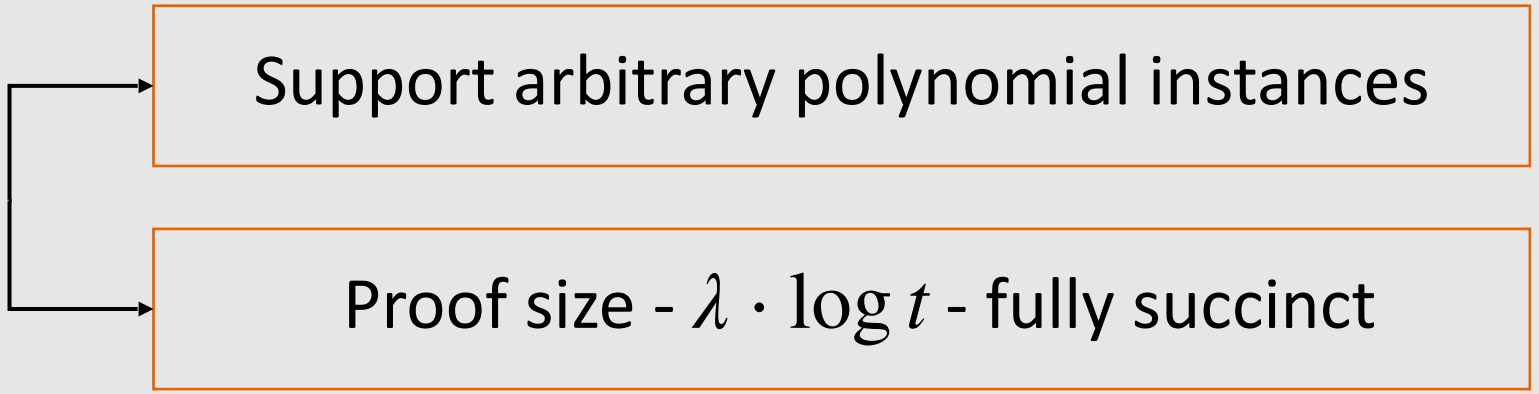
Support arbitrary polynomial instances

Proof size - $\lambda \cdot \log t$ - fully succinct

Our Results

BARGs for NP

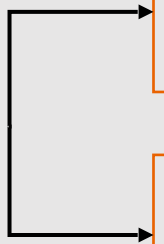
SW14+CJJ21b



Our Results

BARGs for NP

SW14+CJJ21b



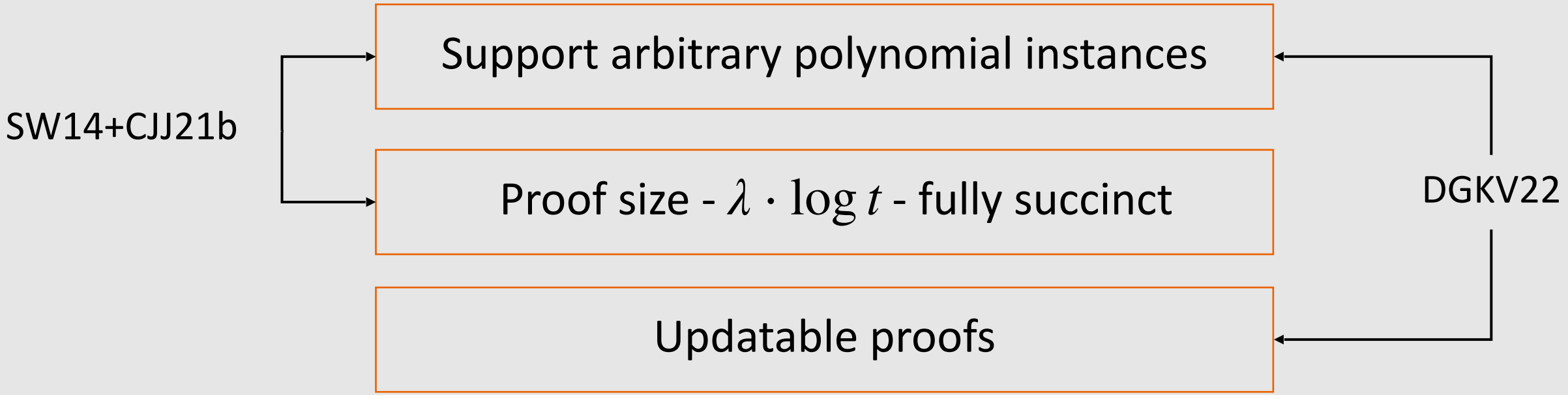
Support arbitrary polynomial instances

Proof size - $\lambda \cdot \log t$ - fully succinct

Updatable proofs

Our Results

BARGs for NP



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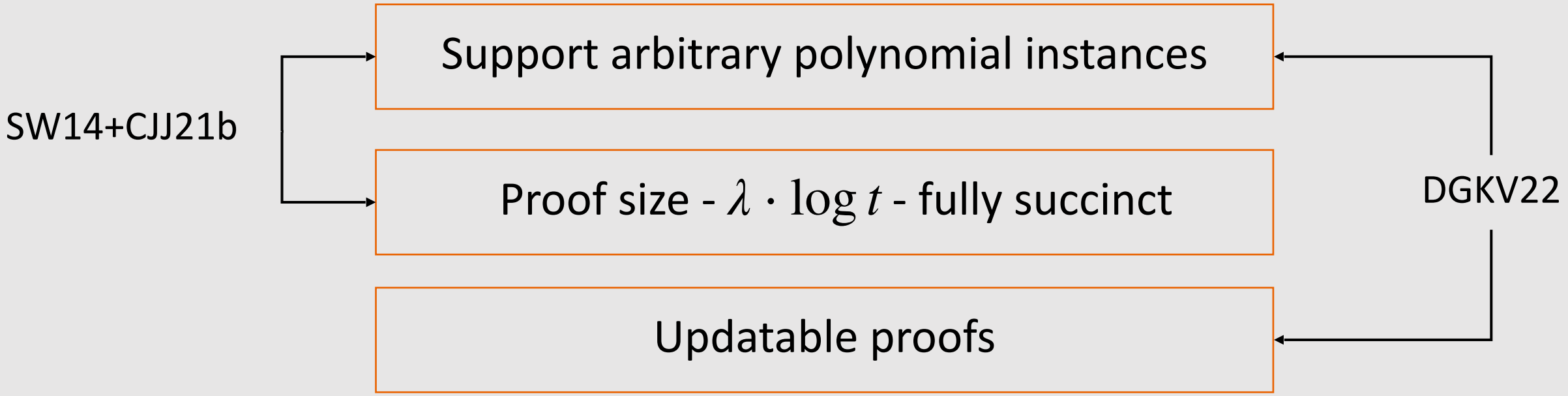

A more direct construction (similar to SW14) from iO+owf without Fiat-Shamir and correlation intractability

Updatable BARGs for NP

Updatable BARGs for NP

PROVER

common reference string

Updatable BARGs for NP

PROVER

common reference string

$(x_1, \ldots, x_t)$

C

π_1

Updatable BARGs for NP

PROVER

common reference string

C

$(x_1, \ldots, x_t)$ $(x_{t+1}, \ldots, x_{2t})$ poly $\cdots$ $(x_{(t'-1)\cdot t+1}, \ldots, x_{t'\cdot t})$

π_1 π_2 $\pi_{t'}$

Updatable BARGs for NP

PROVER

common reference string

poly

C

$(x_1, \ldots, x_t)$ $(x_{t+1}, \ldots, x_{2t})$ $\cdots$ $(x_{(t'-1)\cdot t+1}, \ldots, x_{t'\cdot t})$

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π

Updatable BARGs for NP

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π_1 π_2 $\pi_{t'}$

π

$\mathsf{poly}(\lambda, \log(t \cdot t'), \log|C|)$

Updatable BARGs for NP

Support arbitrary polynomial updates

PROVER

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$\mathsf{poly}(\lambda, \log(t \cdot t'), \log|C|)$

π

Ingredients

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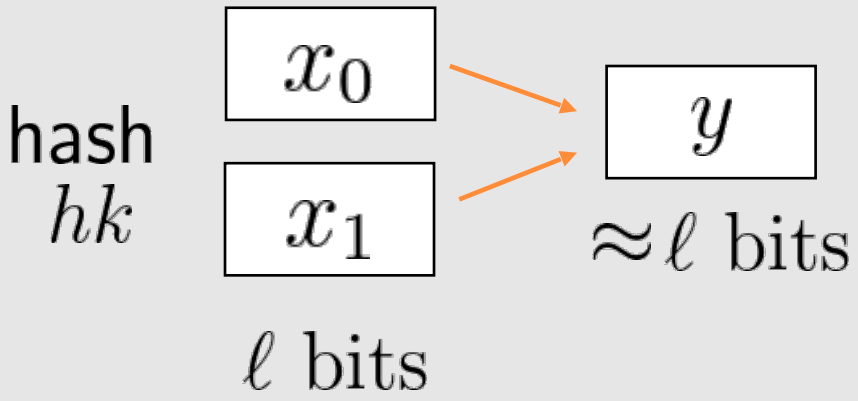
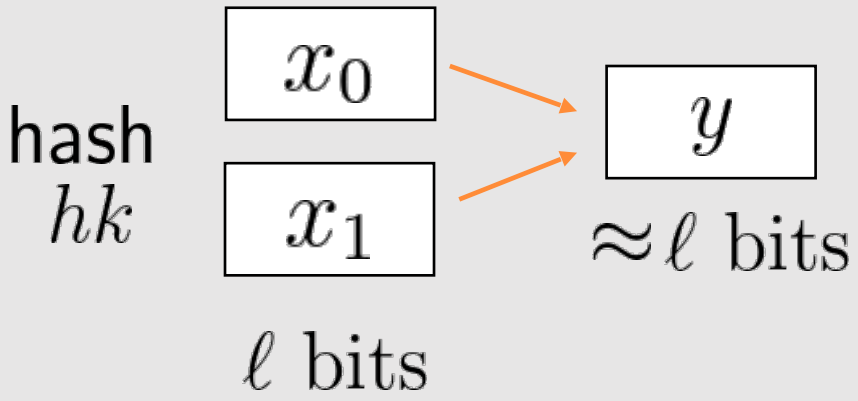
Two-to-one Hash function ★



Ingredients

★ **Two-to-one Hash function**



★ **Statistical Binding Property (positional accumulator)**

Ingredients

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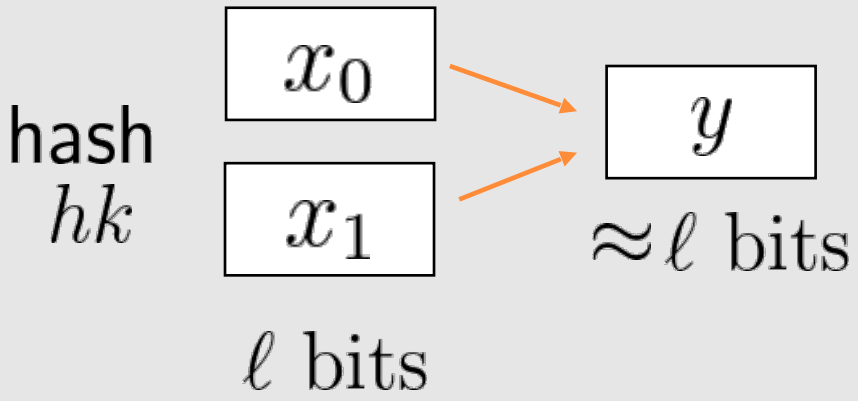

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Indistinguishability Obfuscation + **One-Way functions**

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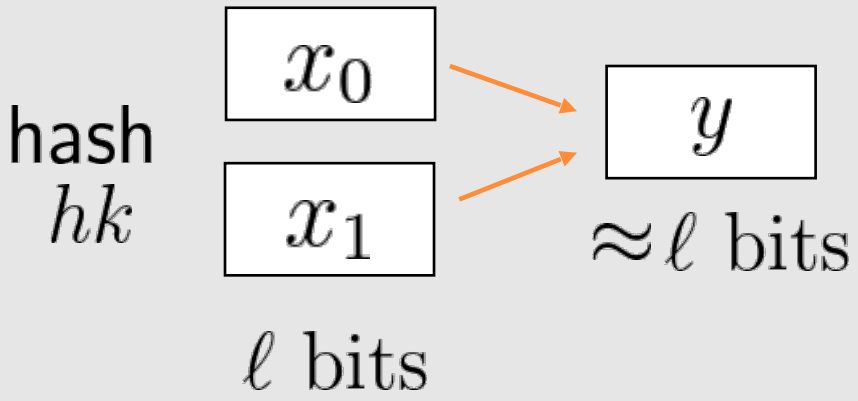

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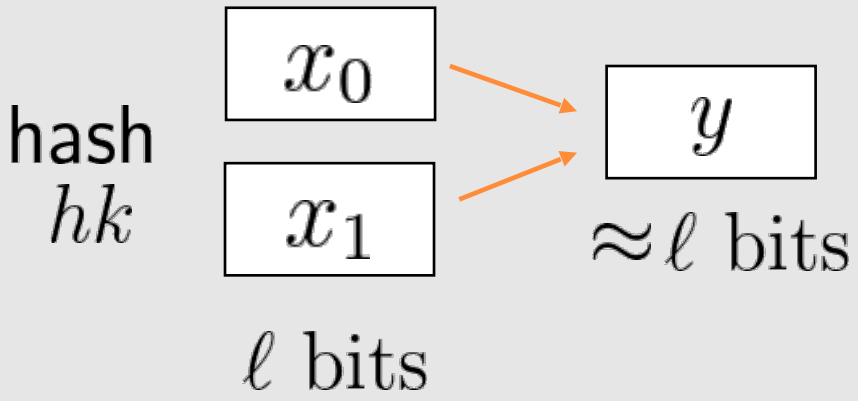

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crs

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If $C(x, w) = 1$,
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Verify

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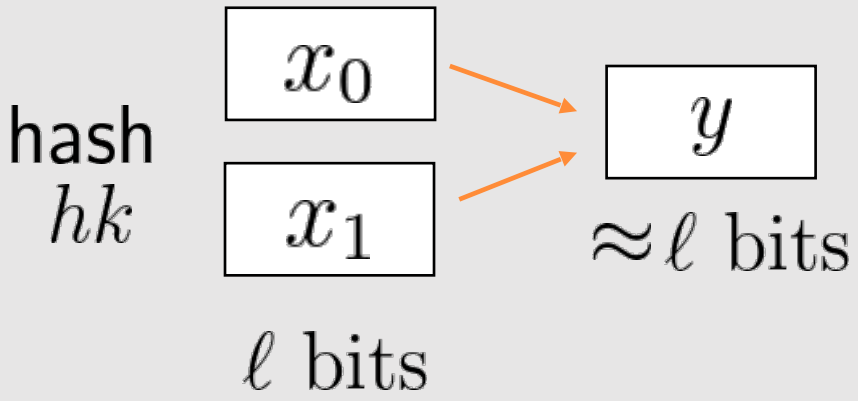

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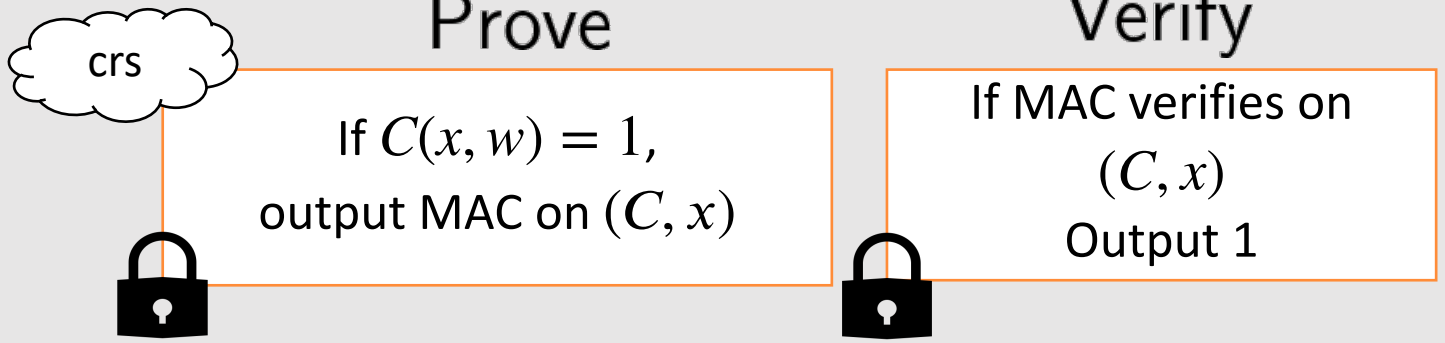

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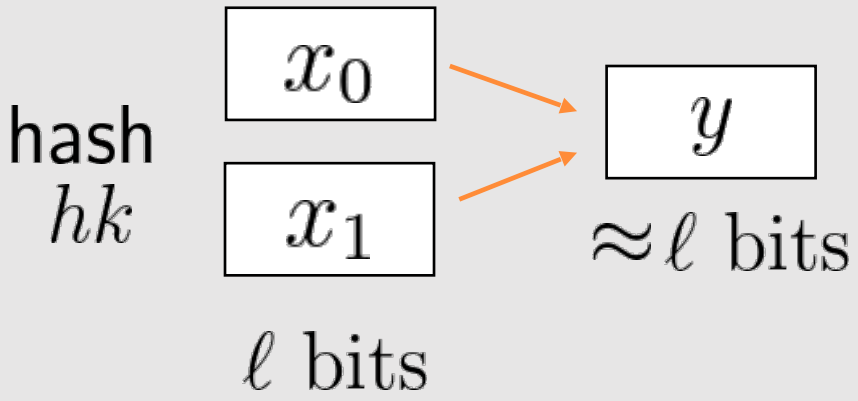

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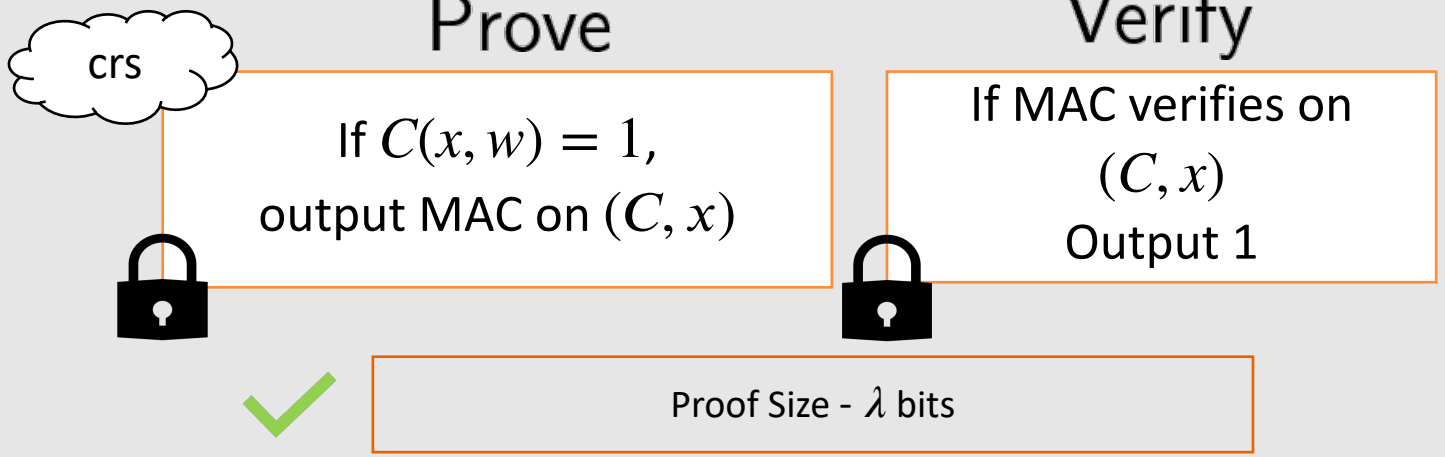

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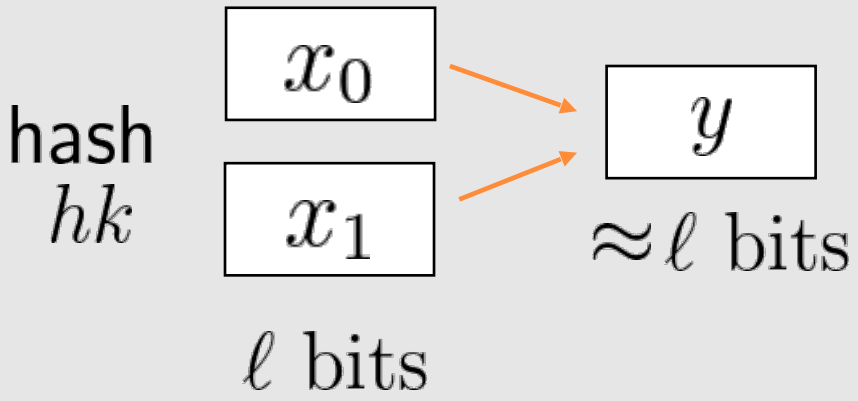

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✓ Proof Size - λ bits

✗ Support arbitrary polynomial instances

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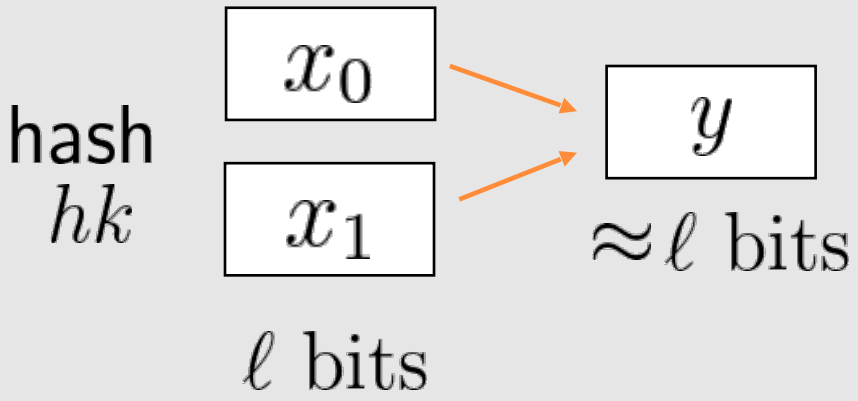

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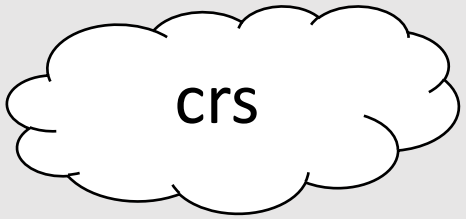
✓ Proof Size - λ bits

✗ Support arbitrary polynomial instances

✗ Can update proofs!

Two-to-one proof merger



hk

Prove

If $C(x_i, w_i) = 1$,
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Verify

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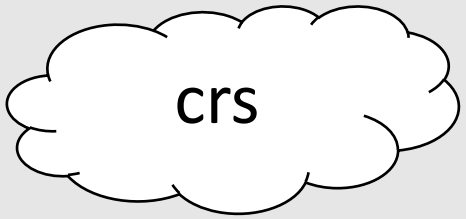
Two-to-one proof merger



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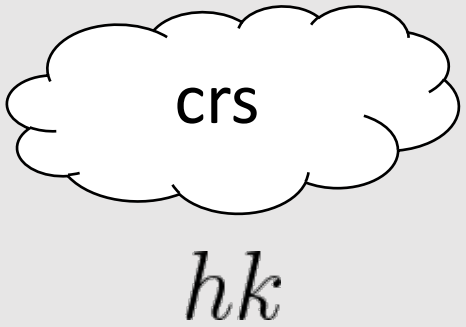


ProofMerge

1. Verify MAC $-\pi_{left}$ on $(C, h_{left}, left)$

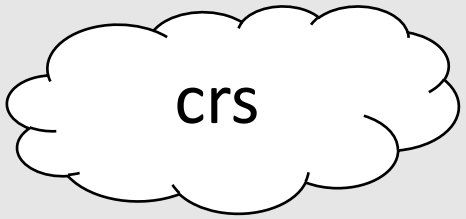
Prove

If $C(x_i, w_i) = 1$,
output MAC on (C, x, i)

Verify

If MAC verifies on $C(x_i, w_i)$,
Output 1

Two-to-one proof merger



hk

ProofMerge

1. Verify MAC $-\pi_{left}$ on $(C, h_{left}, left)$
2. Verify MAC $-\pi_{right}$ on $(C, h_{right}, right)$

$h_{left}\ \pi_{left}$ $h_{right}\ \pi_{right}$

Prove

If $C(x_i, w_i) = 1$,
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Verify

If MAC verifies on $C(x_i, w_i)$,
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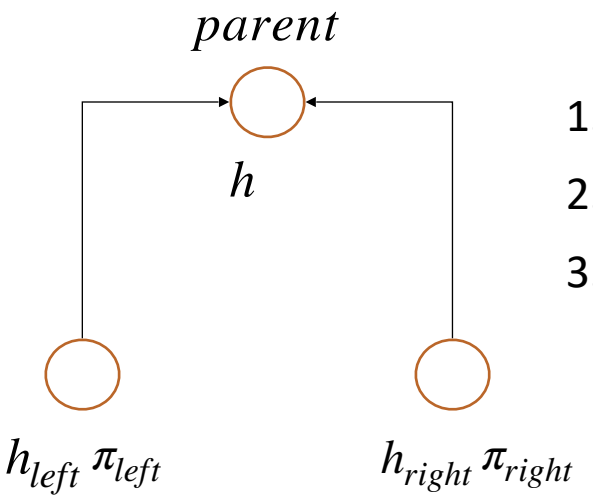
Two-to-one proof merger

crs

hk

ProofMerge



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3. Compute $h = hash(hk, h_{left}, h_{right})$

Prove

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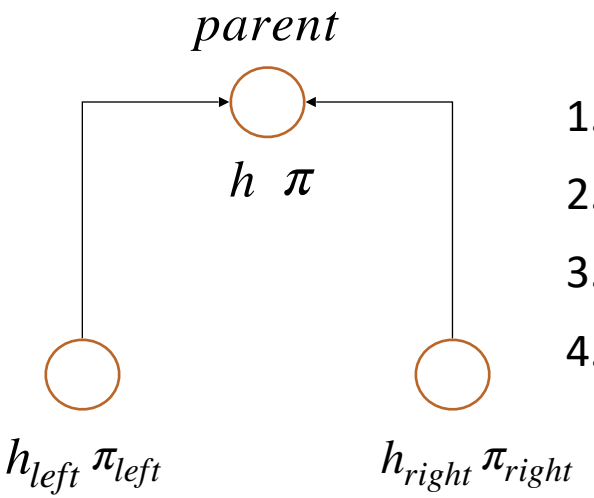

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3. Compute $h = hash(hk, h_{left}, h_{right})$
4. Output MAC π on $(C, h, parent)$

Prove

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Verify

If MAC verifies on $C(x_i, w_i)$,
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Two-to-one proof merger

crs

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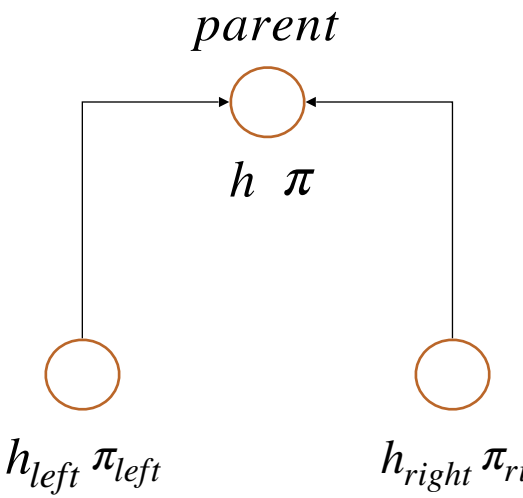
ProofMerge



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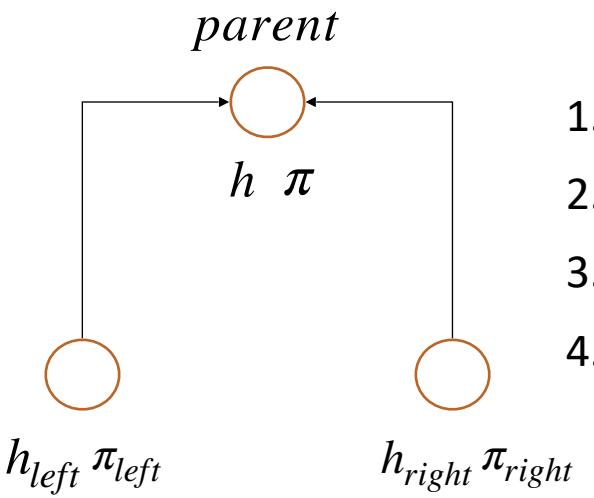
Proof Size λ $bits$

ProofMerge



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3. Compute $h = hash(hk, h_{left}, h_{right})$
4. Output MAC π on $(C, h, parent)$

Prove

If $C(x_i, w_i) = 1$,
output MAC on (C, x, i)

Verify

If MAC verifies on $C(x_i, w_i)$,
Output 1

Two-to-one proof merger

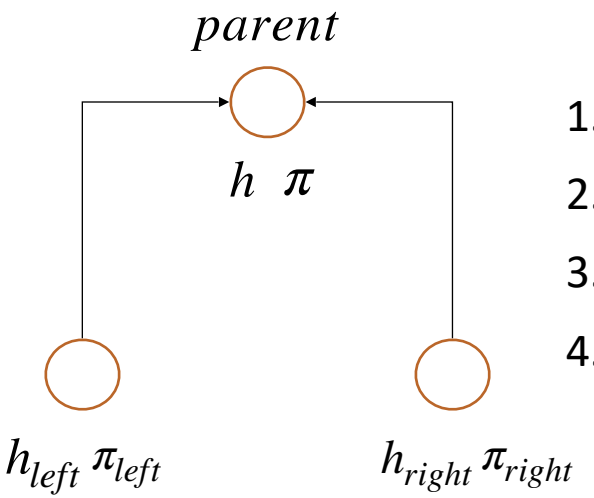
crs

hk

Proof Size λ $bits$

Support arbitrary polynomial instances

ProofMerge



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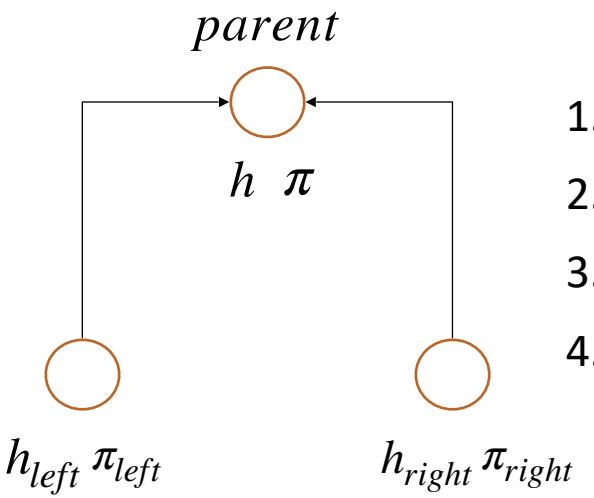

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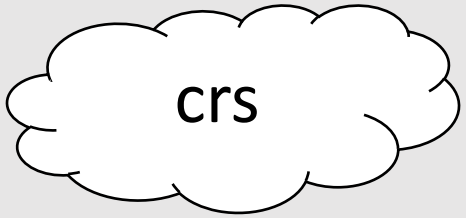
Updatable BARG



ProofMerge

Prove

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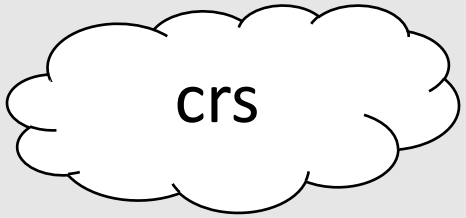
hk

Updatable BARG



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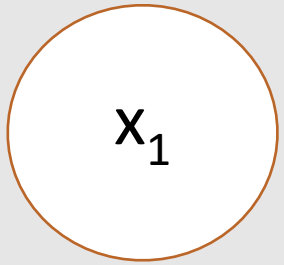
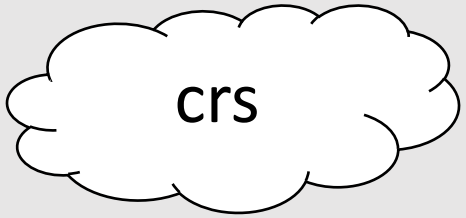


w_1

Updatable BARG

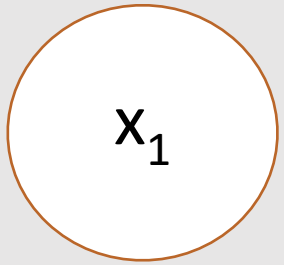


ProofMerge

Prove

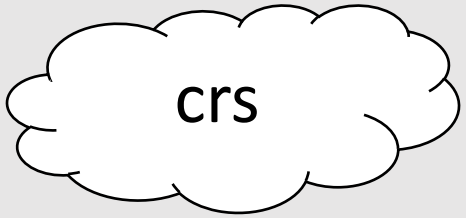
Verify

hk



$w_1\ \pi_1$

Updatable BARG



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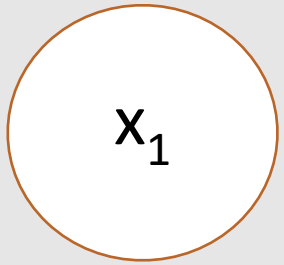
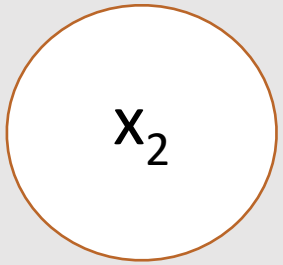
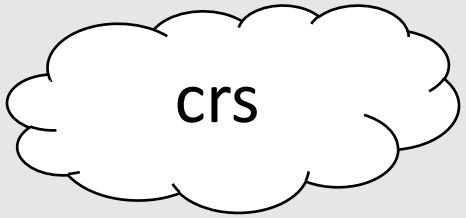
hk





$w_1\ \pi_1$ w_2

Updatable BARG

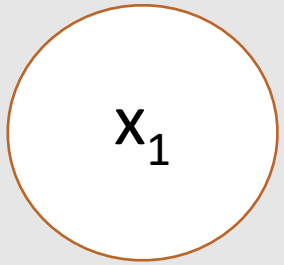


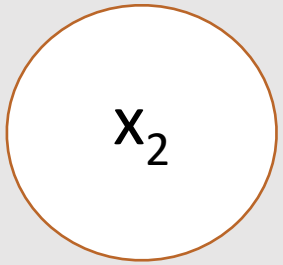
ProofMerge

Prove

Verify

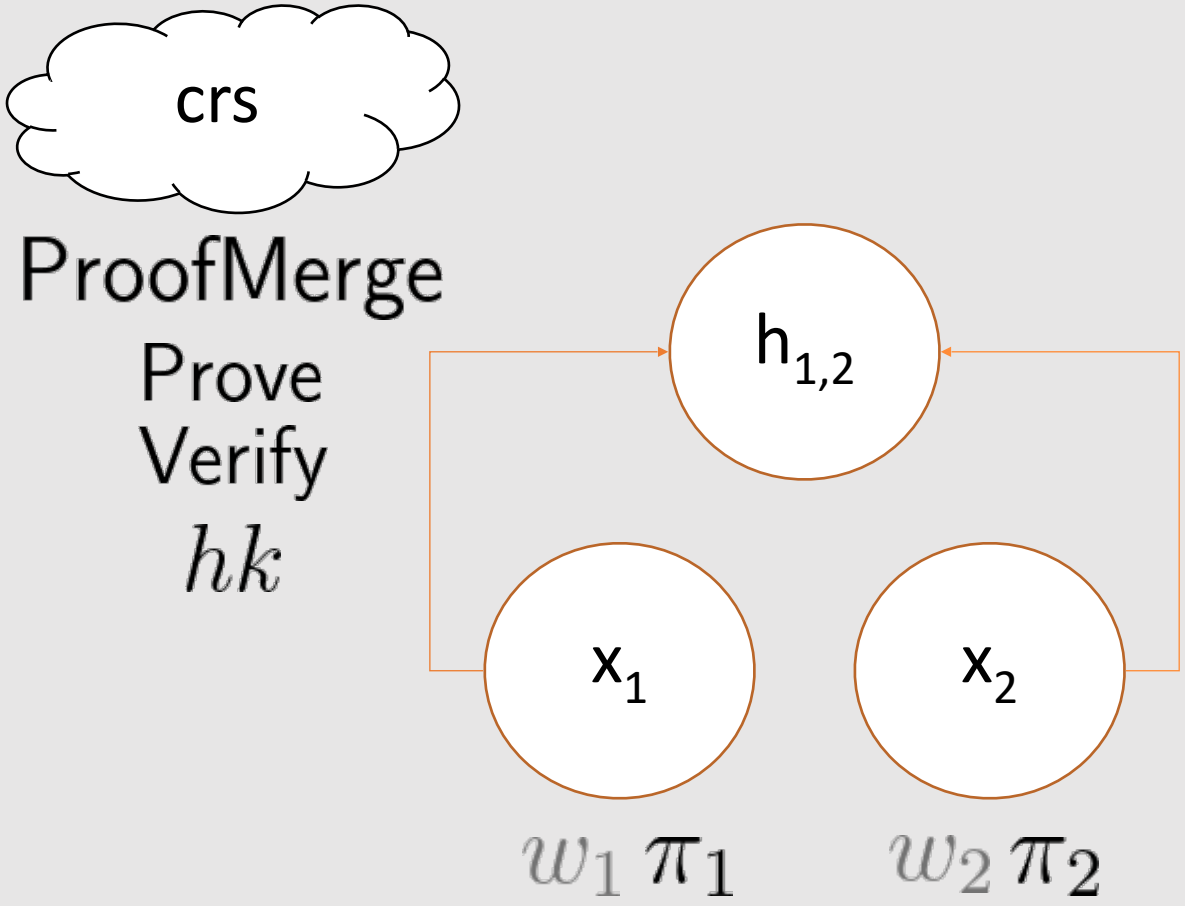
hk





$w_1\ \pi_1$ $w_2\ \pi_2$

Updatable BARG



Updatable BARG

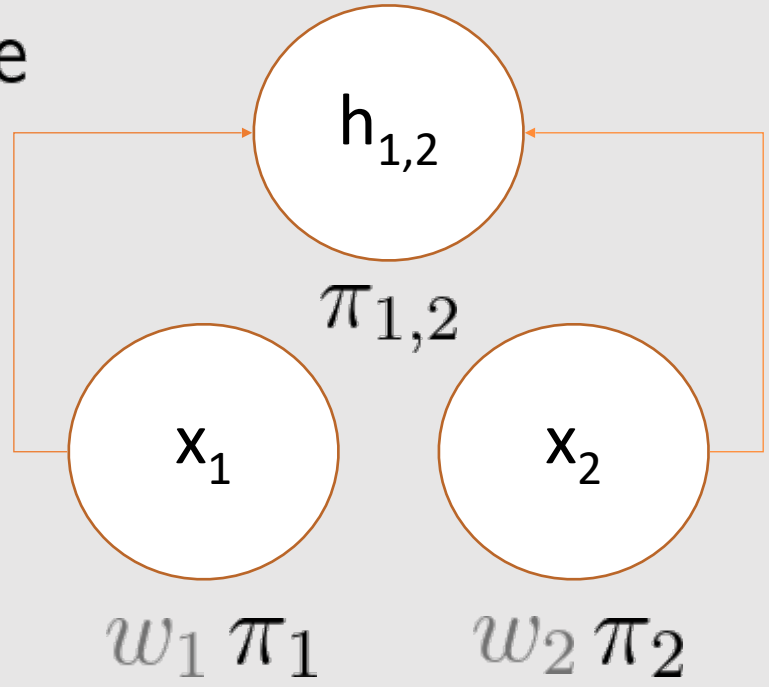
crs

ProofMerge

Prove

Verify

hk



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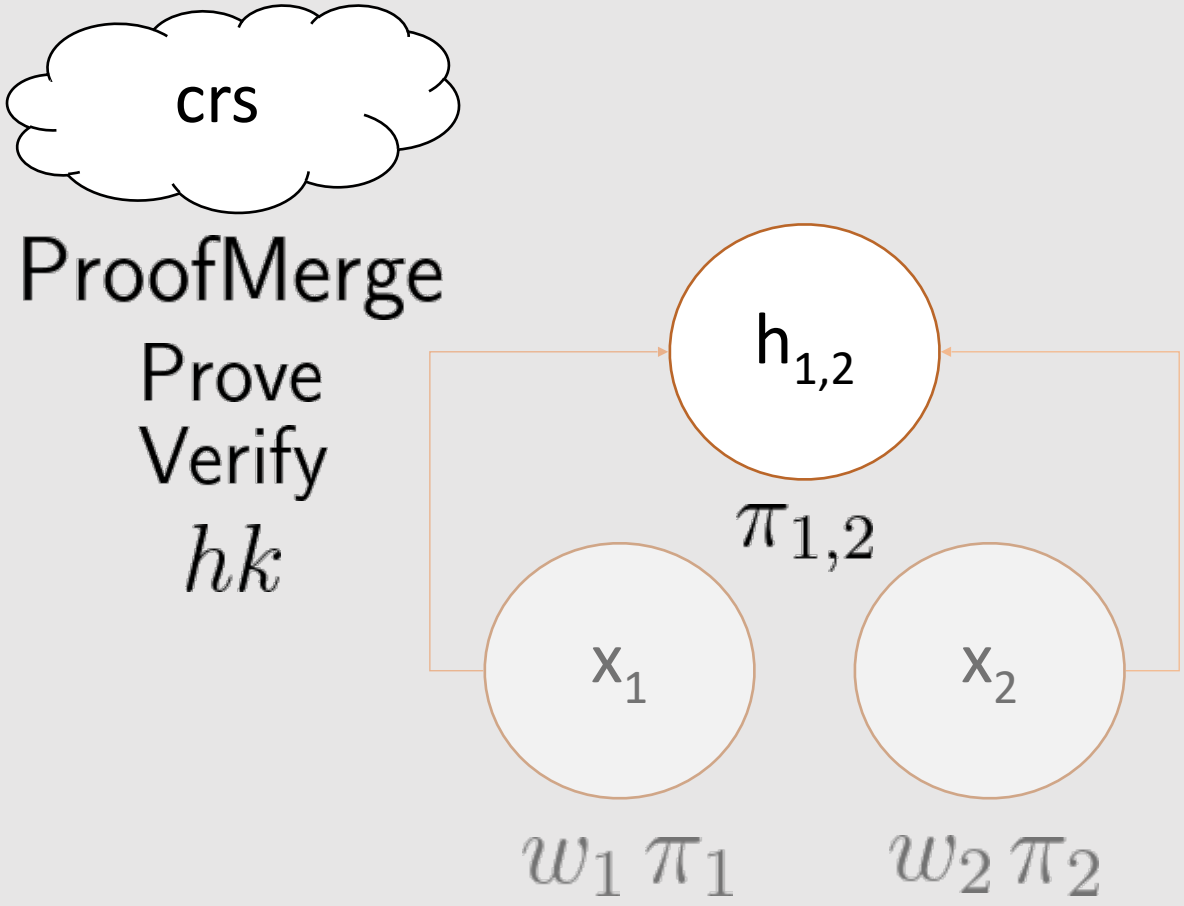

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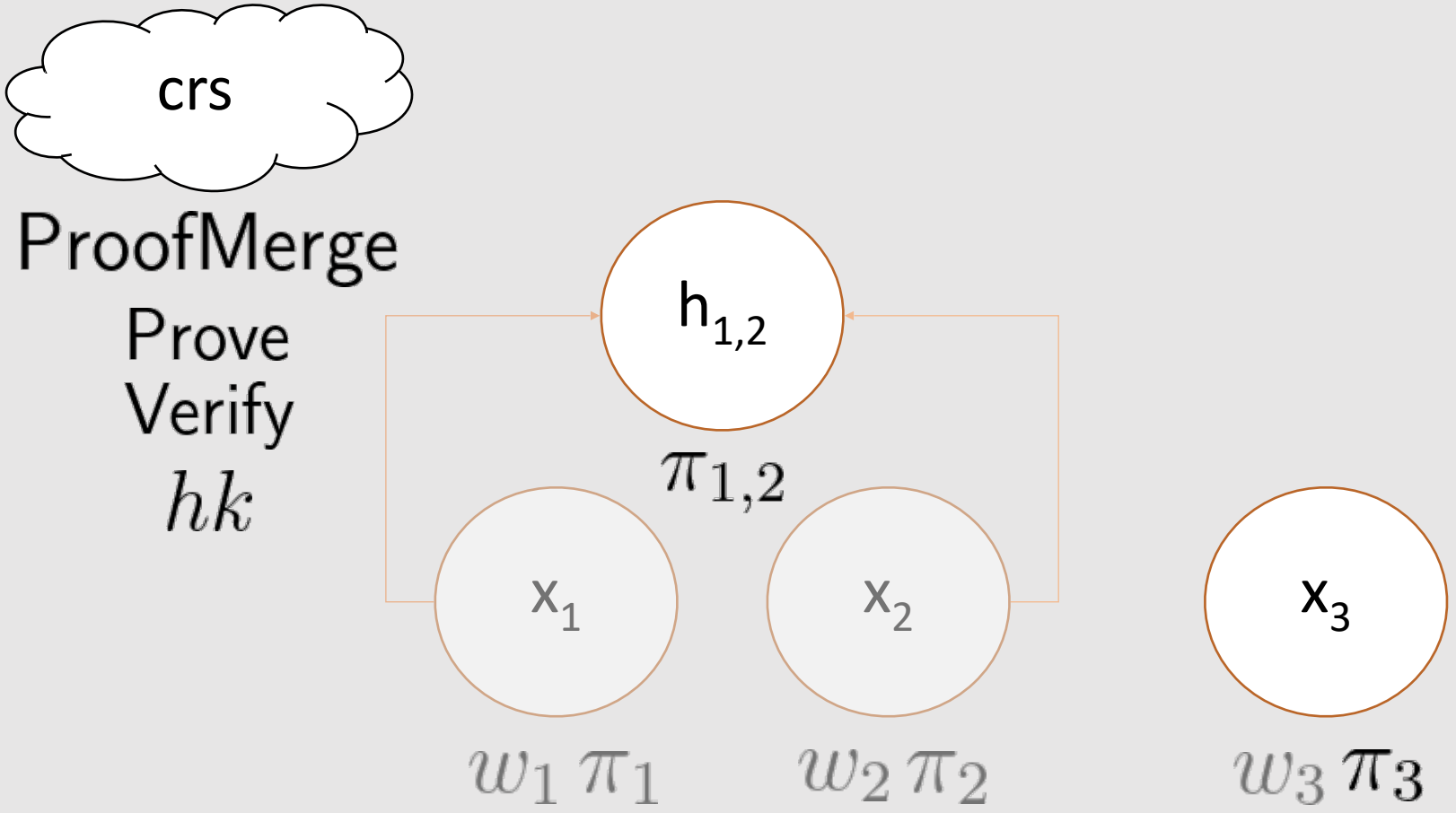

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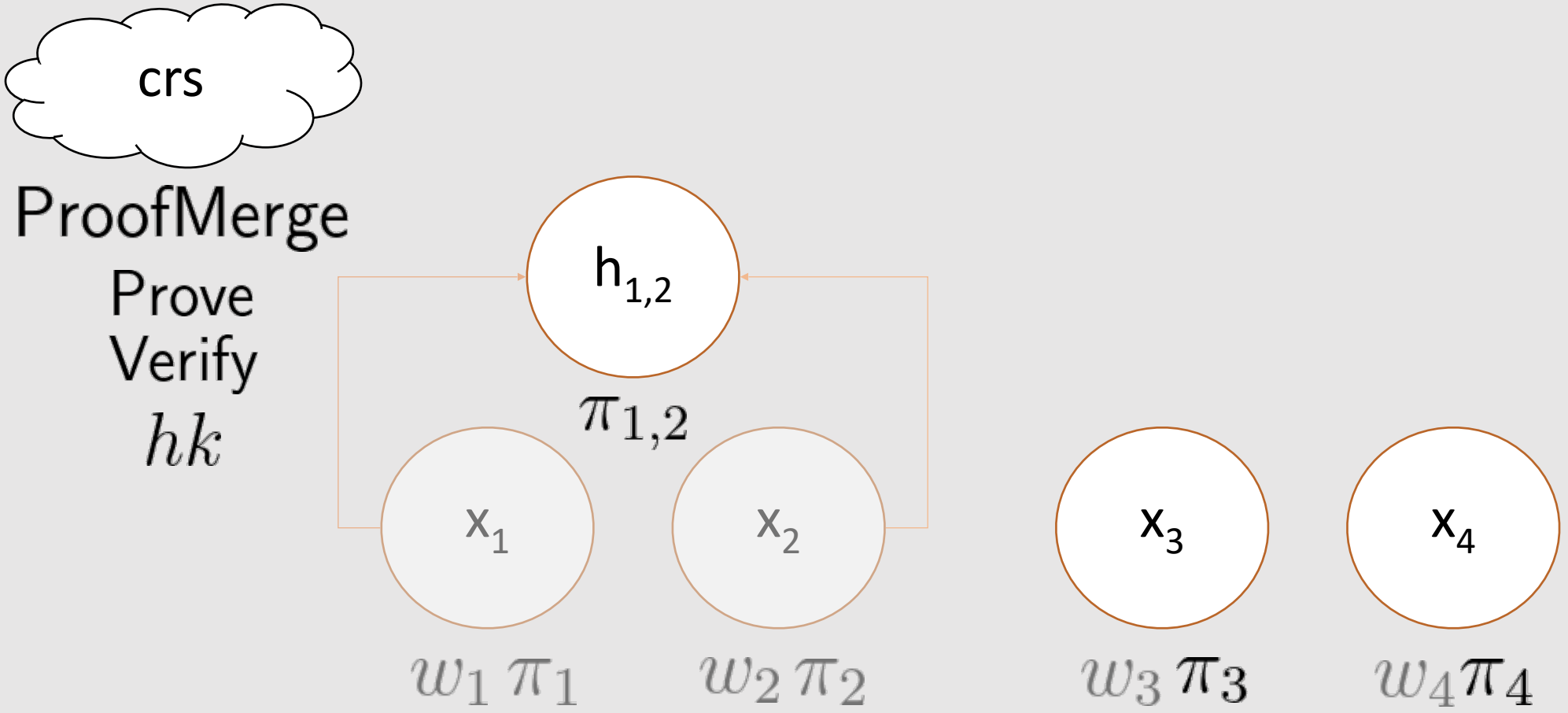

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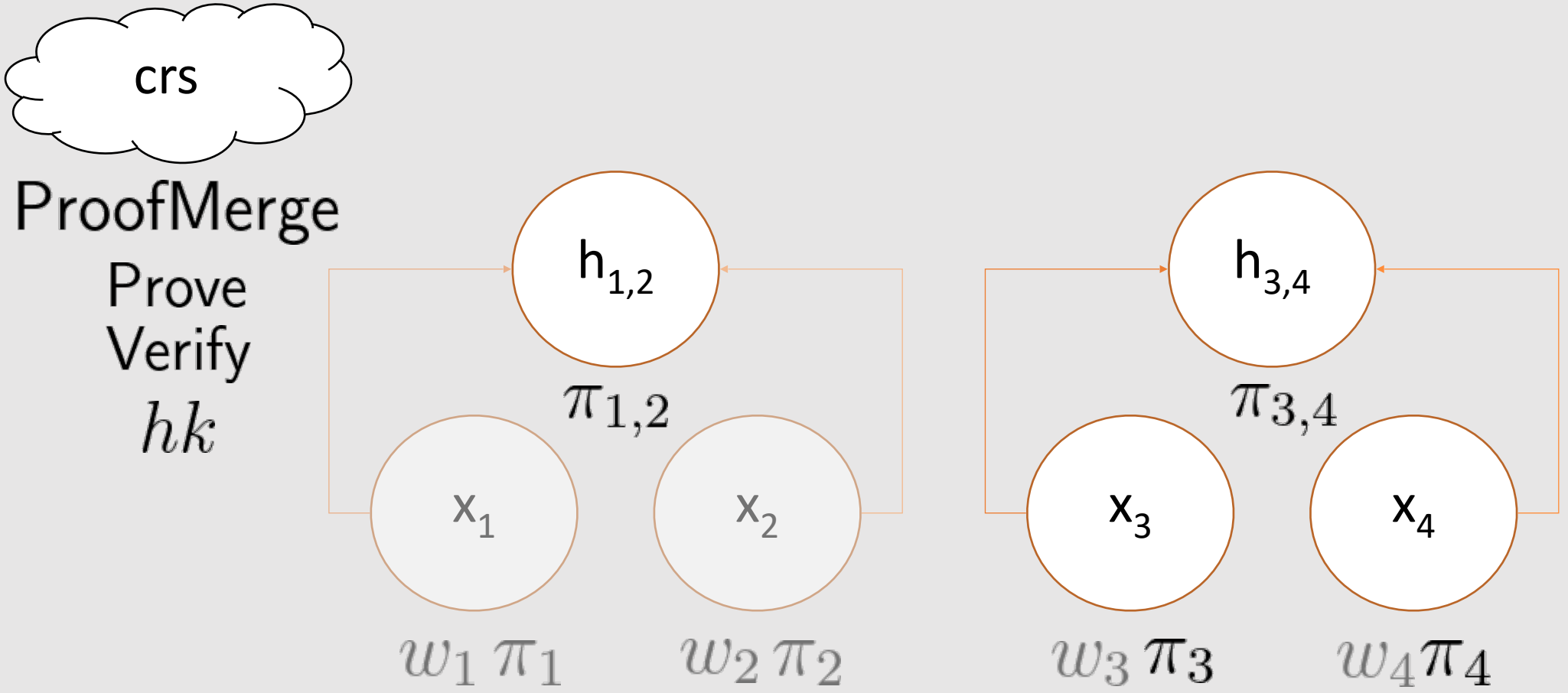

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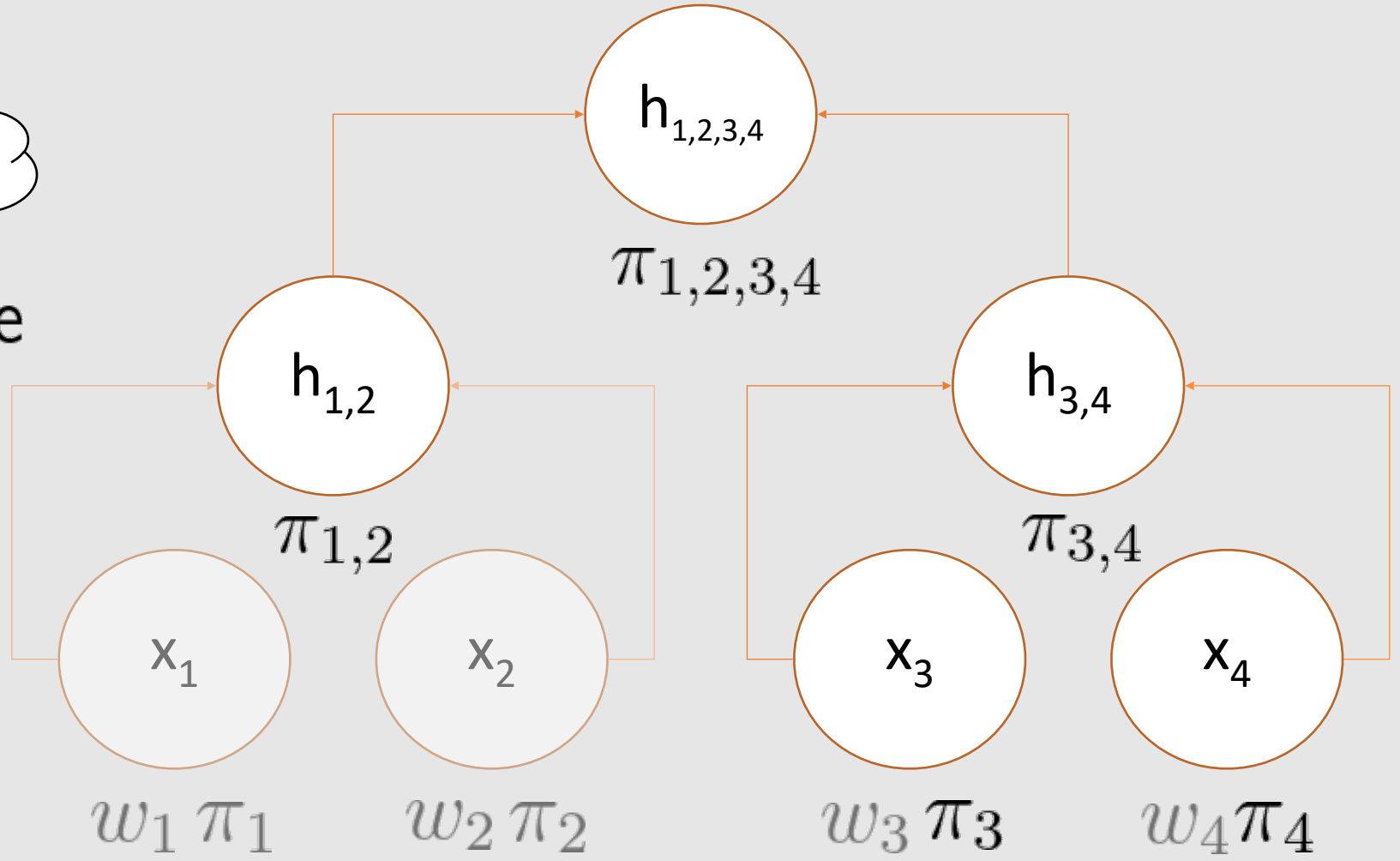

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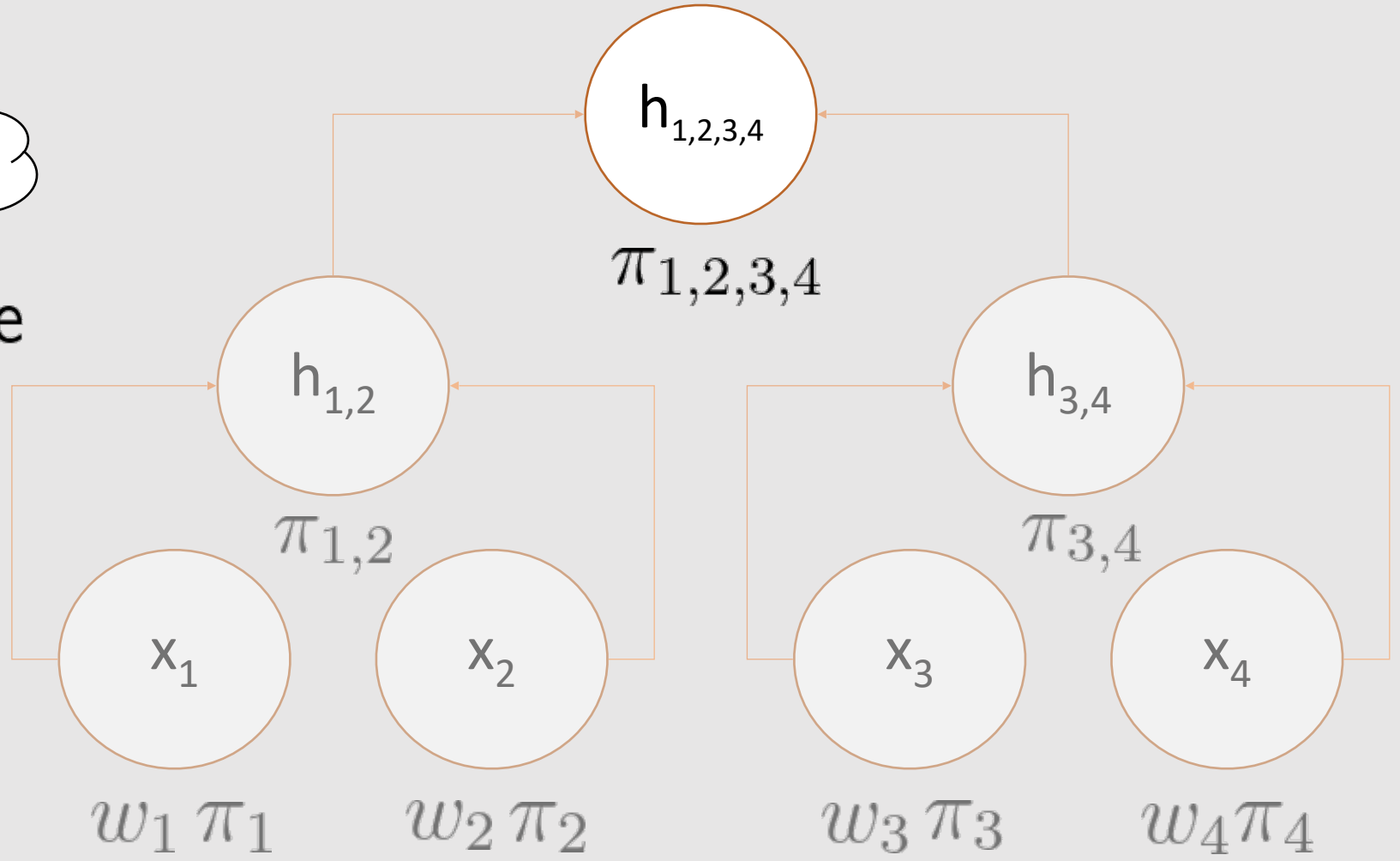

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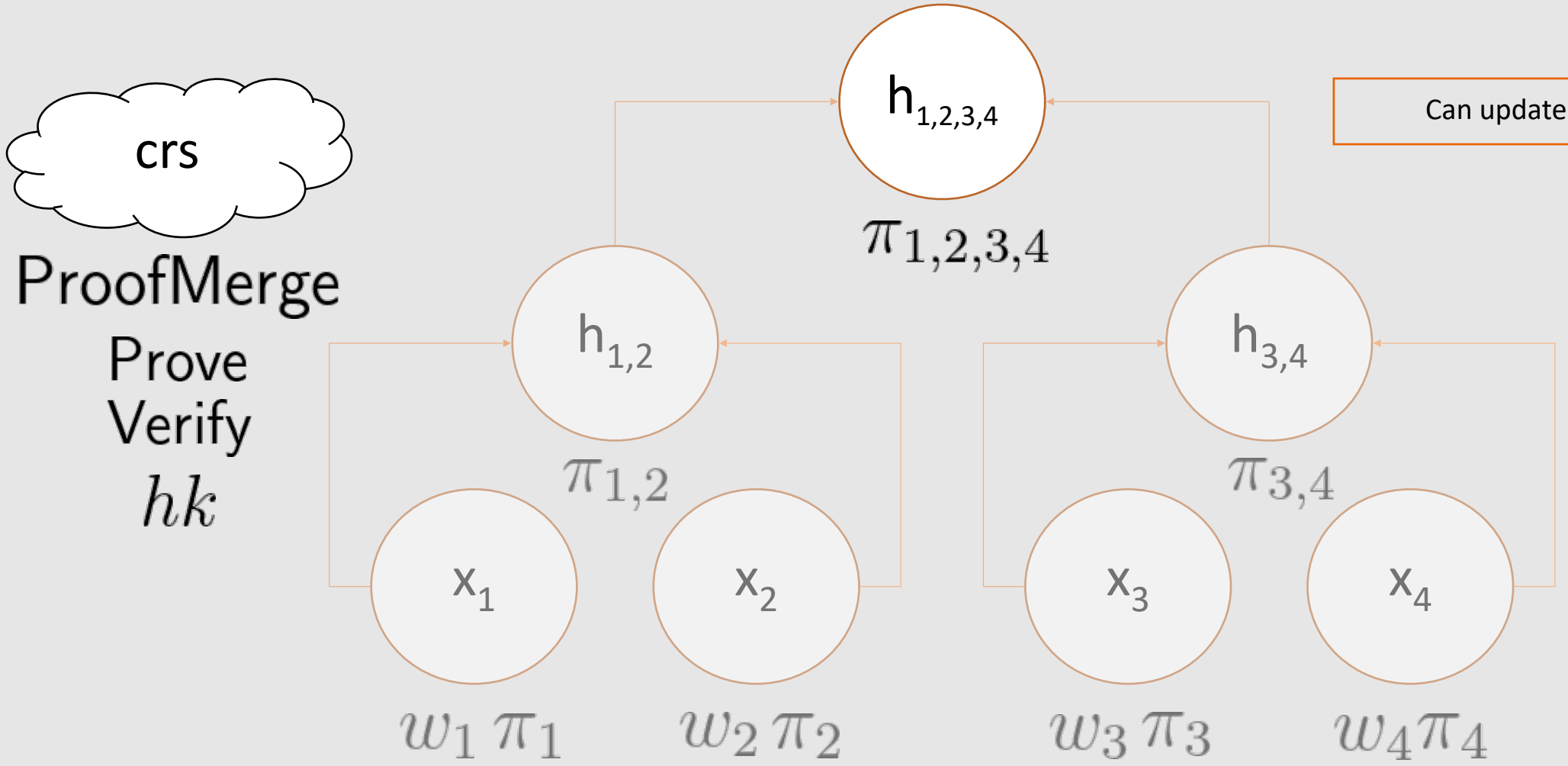


Non-Adaptive Soundness

Non-Adaptive Soundness

$\exists i, x_i$ is not circuit satisfiable

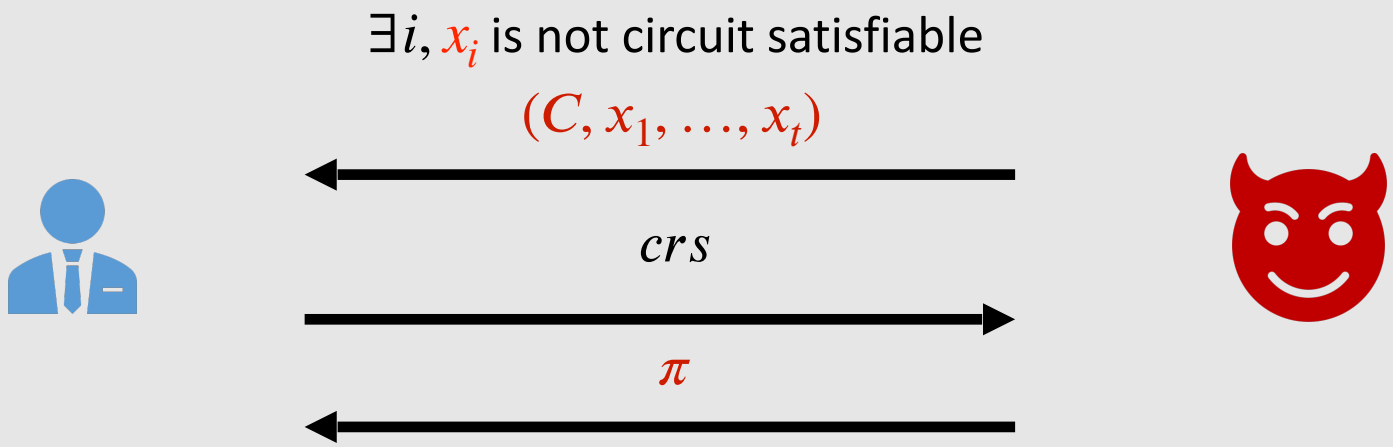
$(C, x_1, \ldots, x_t)$

crs

π

Outputs result of
Verify on proof π

Non-Adaptive Soundness



crs

Prove

If $C(x_i, w_i) = 1$,
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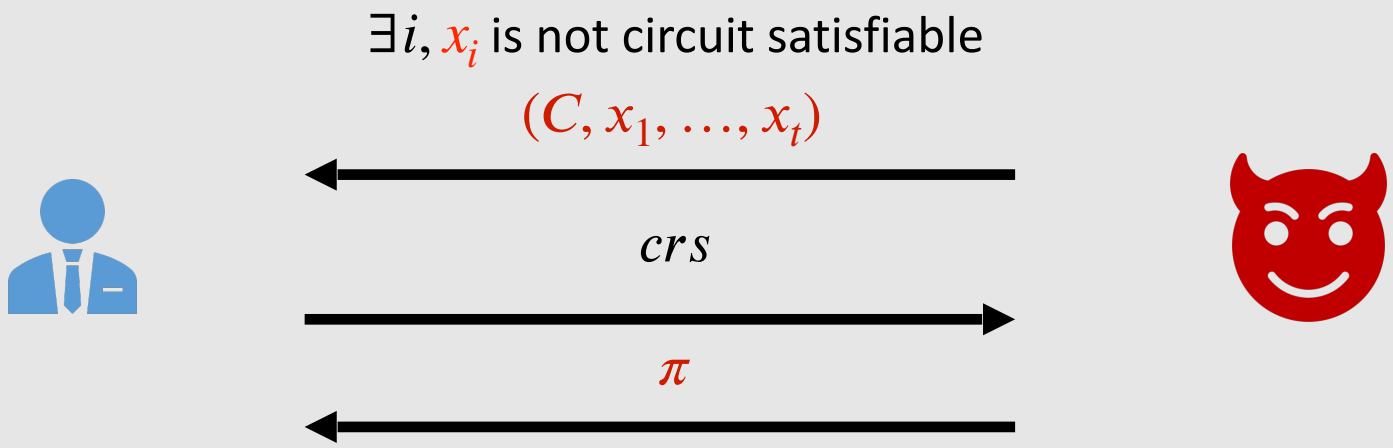
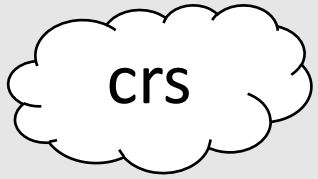
Prove

If $x = x_i$, then output $\perp$
If $C(x_i, w_i) = 1$,
output MAC on (C, x, i)

Non-Adaptive Soundness





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$\approx_{iO}$

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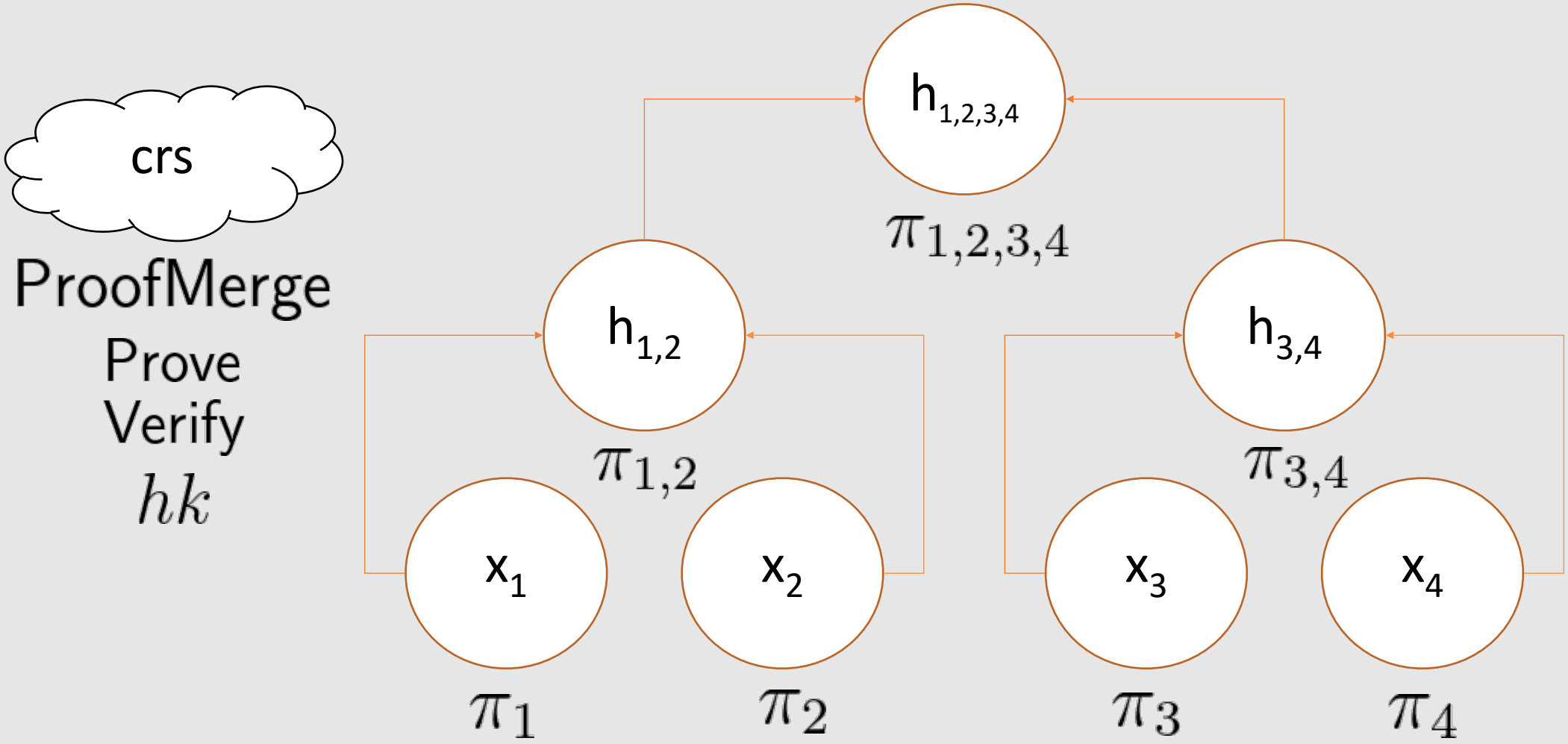
Non-Adaptive Soundness



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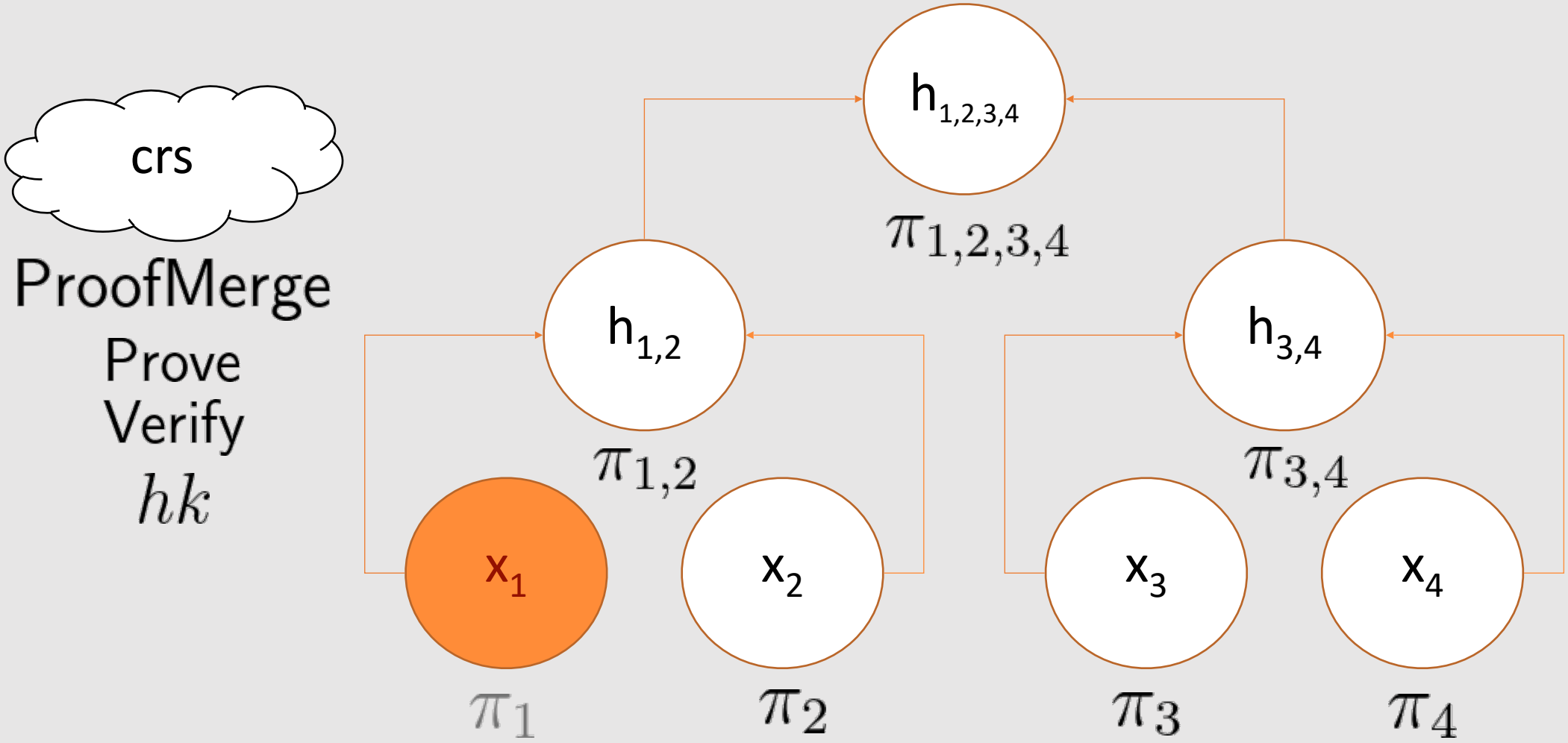

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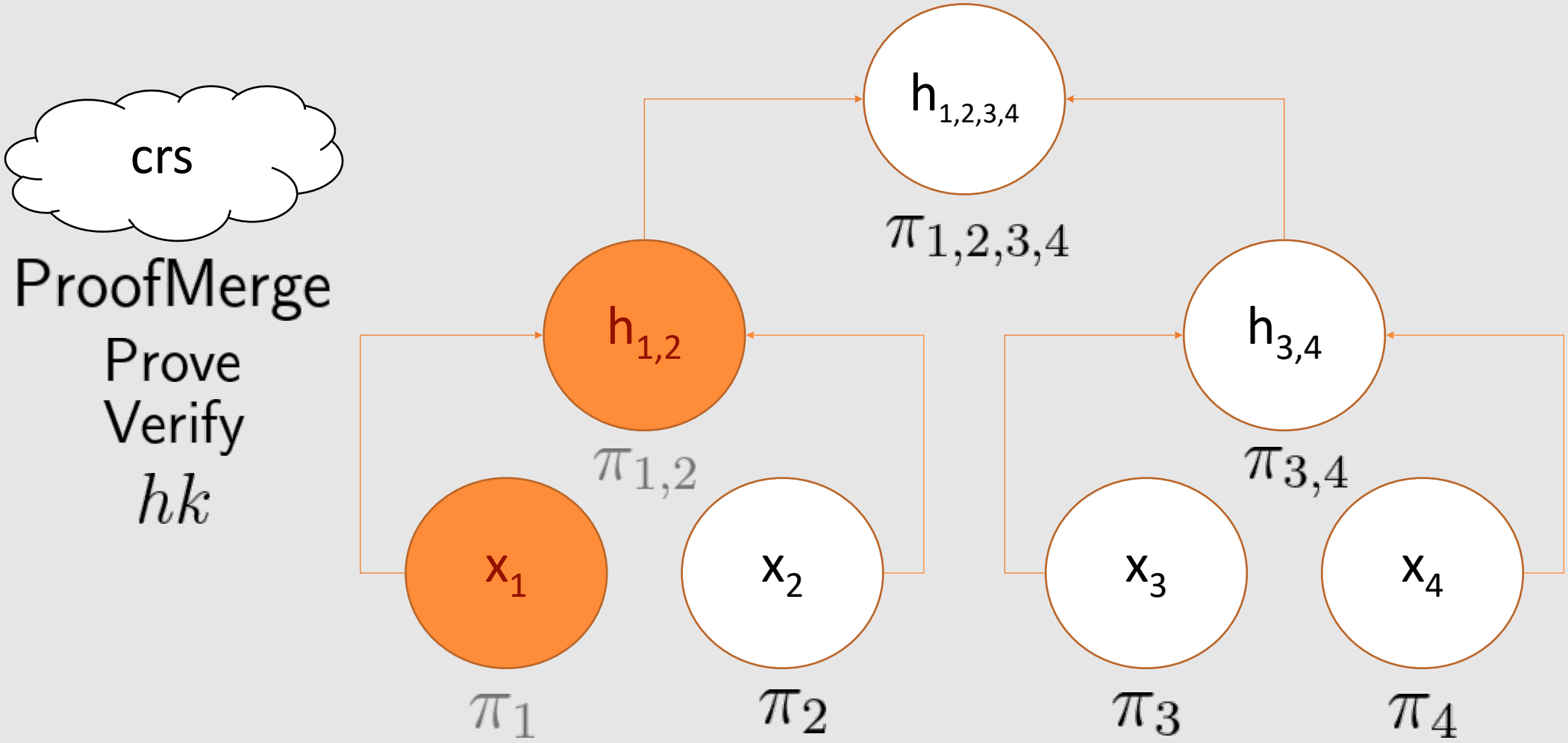

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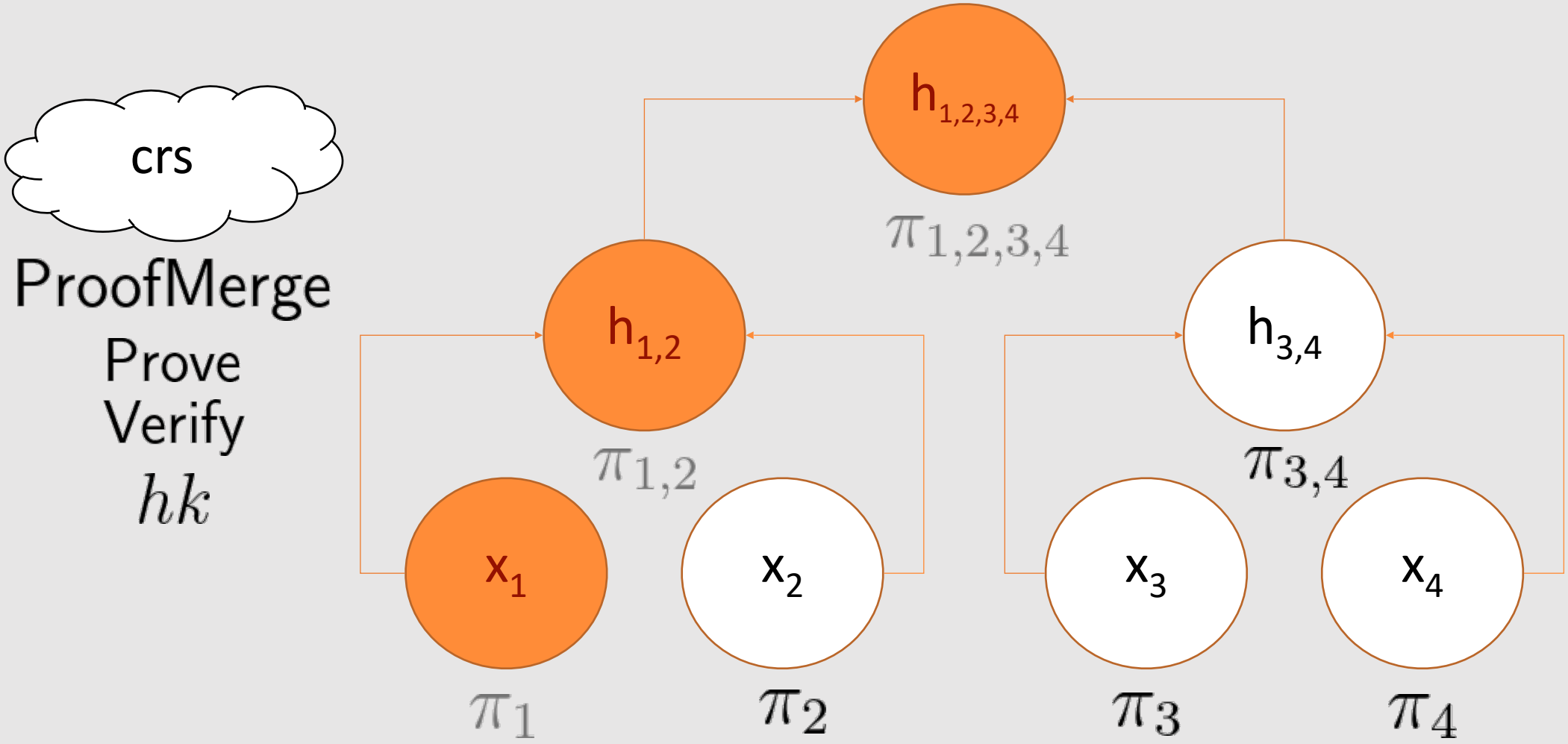

BARGs for NP from iO

Support arbitrary poly instances

Proof size - $\lambda \cdot \log t$ bits

Can update proofs!

Non-adaptive soundness

A direct construction from iO+owf

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A direct construction from iO+owf

Complexity leveraging
SSB hashing (LWE, DDH, DCR, φ -hiding)

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BARGs for NP from iO

Support arbitrary poly instances	Support arbitrary poly instances
Proof size - $\lambda \cdot \log t$ bits	Proof size – $\log t \cdot \text{poly}(\lambda, \lvert C \rvert)$ bits
Can update proofs!	Can update proofs!
Non-adaptive soundness	Adaptive soundness

A direct construction from iO+owf

Complexity leveraging
SSB hashing (LWE, DDH, DCR, φ -hiding)

BARGs for NP from iO

Support arbitrary poly instances	Support arbitrary poly instances
Proof size - $\lambda \cdot \log t$ bits	Proof size – $\log t \cdot \text{poly}(\lambda, \lvert C \rvert)$ bits
Can update proofs!	Can update proofs!
Non-adaptive soundness	Adaptive soundness
Zero Knowledge	Zero Knowledge
A direct construction from iO+owf	Complexity leveraging SSB hashing (LWE, DDH, DCR, φ -hiding)

Thanks!

Soon to be on eprint.iacr.org