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Secure Sampling with Sublinear Communication

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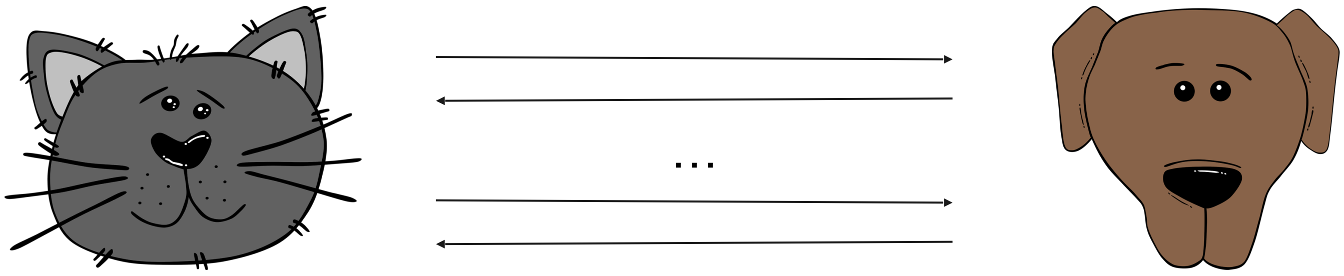
2-Party Sampling



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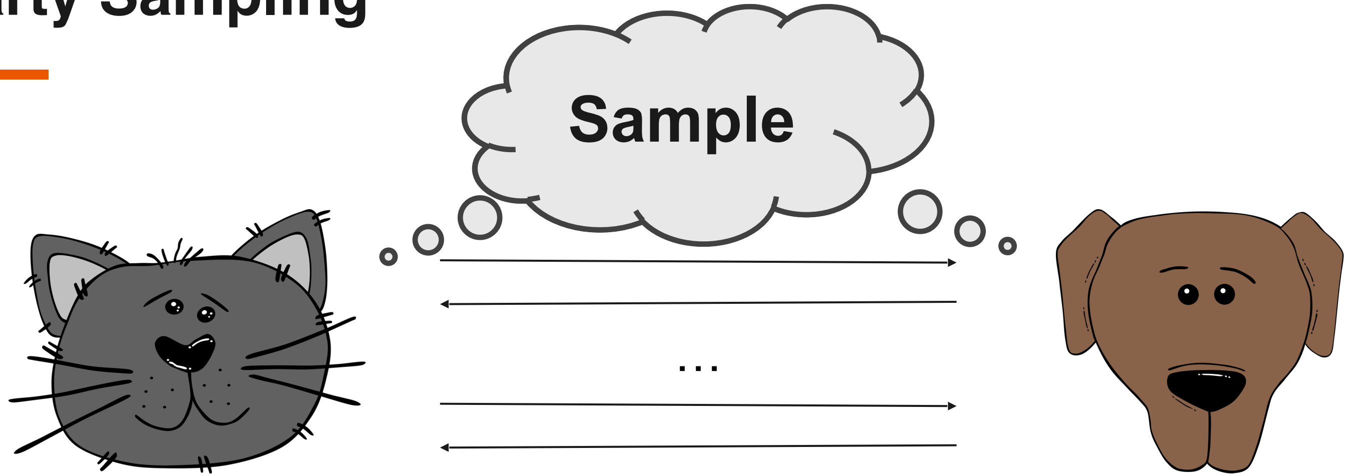

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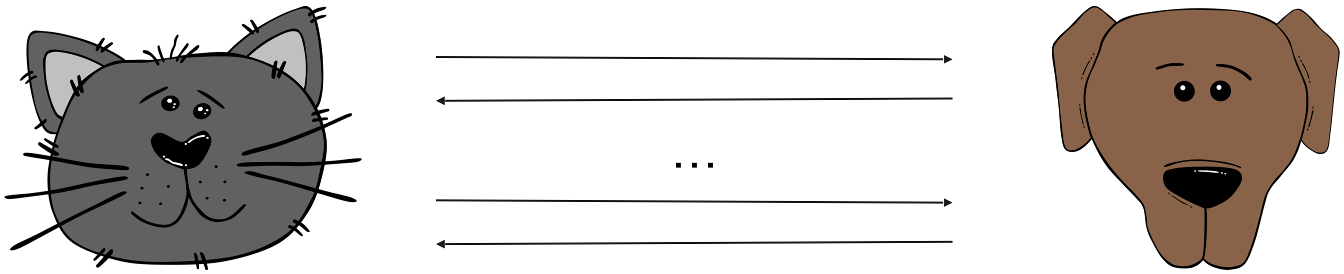
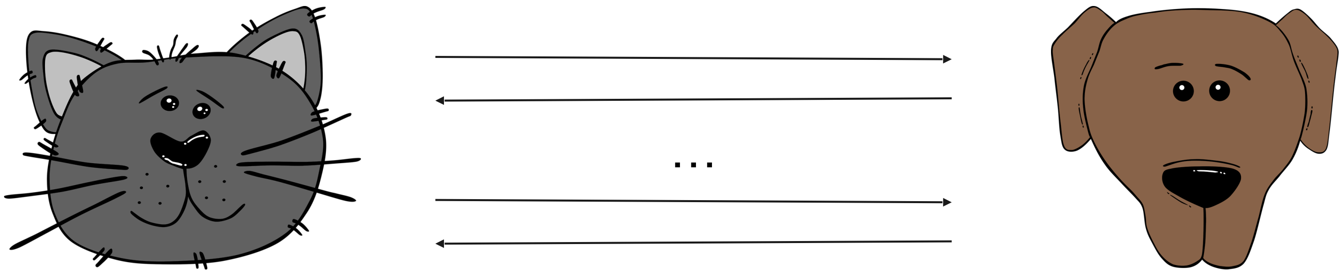


2-Party Sampling

$W_1 = (w_{1,1}, w_{1,2}, ..., w_{1,n})$ $W_2 = (w_{2,1}, w_{2,2}, ..., w_{2,n})$



Private sampling based on W_1 and W_2

- Output i with $\Pr[i] \propto f(w_{1,i}, w_{2,i})$
- L_1 sampling; L_2 sampling; Product sampling

Distribution is private

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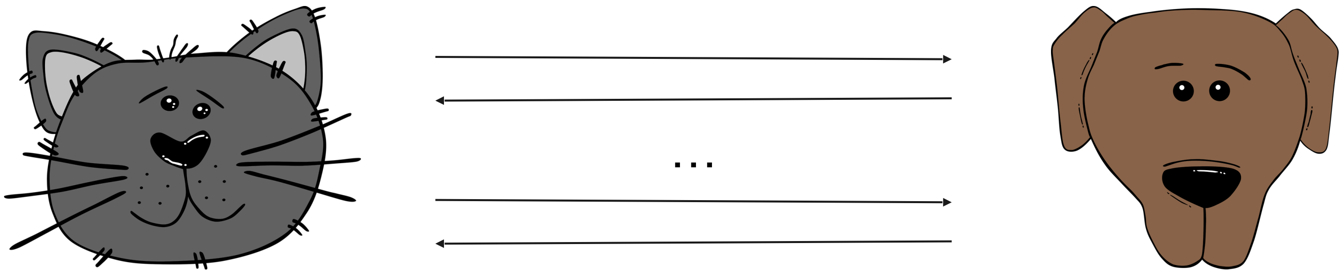

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Distribution is private

Question: Sample with sublinear communication?

Our results

- **Private sampling:**
  - L_1 sampling with sublinear communication
  - L_2 sampling with sublinear communication
- **Product sampling:**
  - Impossibility for arbitrary inputs
  - Sublinear communication with assumption on inputs

$$\langle \mathbf{W}_1, \mathbf{W}_2 \rangle \in \omega(\frac{\log n}{n})$$

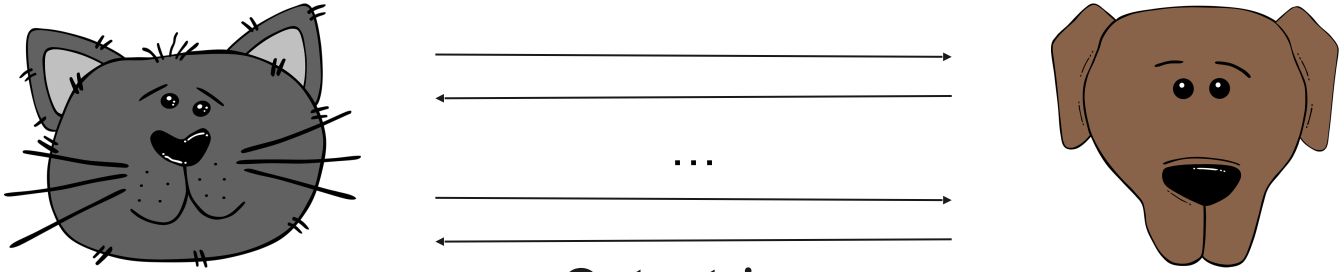
- **Exponential Mechanism**
  - 2-party Exponential Mechanism with sublinear communication

Outline

- Sublinear sampling
- Sampling from common distributions
  - L_1 sampling
  - L_2 sampling
  - Product sampling
- Exponential Mechanism

L_1 Sampling

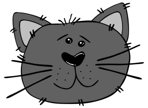
$$W_1 = (w_{1,1}, w_{1,2}, \ldots, w_{1,n}) \qquad W_2 = (w_{2,1}, w_{2,2}, \ldots, w_{2,n})$$

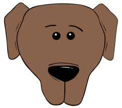


Output i

$$\Pr[i] = \frac{w_{1,i} + w_{2,i}}{\sum_j (w_{1,j} + w_{2,j})} = \frac{w_{1,i} + w_{2,i}}{\|W_1 + W_2\|_1}$$

L_1 Sampling

 $W_1 = (w_{1,1}, w_{1,2}, ..., w_{1,n})$

 $W_2 = (w_{2,1}, w_{2,2}, ..., w_{2,n})$

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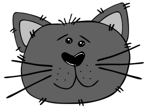

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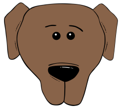

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- **L_1 is linear!**

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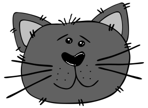

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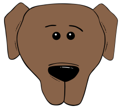

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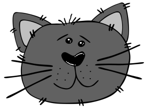

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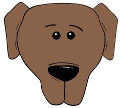

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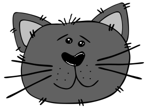
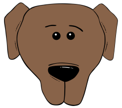
- **First attempt:**
  1. Party 1 samples i_1 locally.
  2. Party 2 samples i_2 locally.
  3. Flip a coin and output i_1 or i_2.

L_1 Sampling

 $W_1 = (w_{1,1}, w_{1,2}, \ldots, w_{1,n})$

 $W_2 = (w_{2,1}, w_{2,2}, \ldots, w_{2,n})$

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😄 **O(1) communication!**

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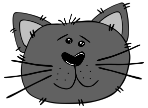

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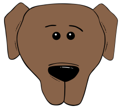

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😔 **This is not private!**

L_1 Sampling — Fix privacy issue

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  1. The parties obliviously sample and secret share i_1.

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  1. The parties obliviously sample and secret share i_1.
  2. The parties obliviously sample and secret share i_2.
  3. Reconstruct i_1 or i_2 based on a oblivious coin flip.

L$_1$ Sampling — Fi

Party 1 doesn't know i_1

Party 2 doesn't know i_2

- **Initial attempt**
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Outline

- Sublinear sampling
- Sampling from common distributions
    - L_1 sampling
    - L_2 sampling
    - Product sampling
- Exponential Mechanism

L_2 Sampling

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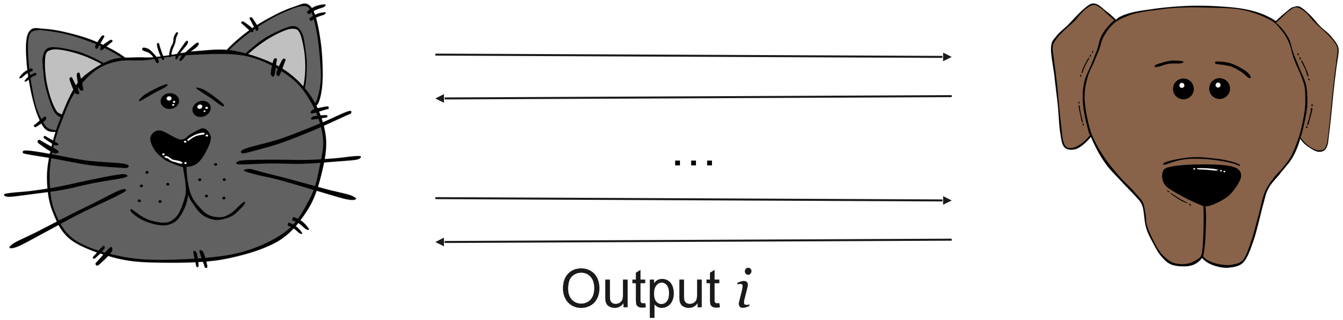

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Why is L_2 Sampling hard?

$$\Pr[i] = \frac{(w_{1,i} + w_{2,i})^2}{\|W_1 + W_2\|_2^2} = \frac{w_{1,i}^2 + w_{2,i}^2 + 2w_{1,i} \cdot w_{2,i}}{\|W_1\|_2^2 + \|W_2\|_2^2 + 2\sum_j (w_{1,j} \cdot w_{2,j})}$$

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- **Apply the linearity trick like L_1?**

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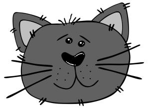
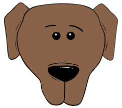
- **Apply the linearity trick like L_1?**
  - No longer linear
  - Cross terms. (Inner product)
  - Impossible to compute with sublinear communication

New trick

 $W_1 = (w_{1,1}, w_{1,2}, \ldots, w_{1,n})$

 $W_2 = (w_{2,1}, w_{2,2}, \ldots, w_{2,n})$

- **Corrective sampling**

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Goal: Output i with Pr[L_2 = i]

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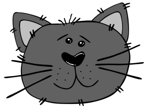
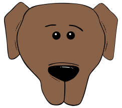
 $W_1 = (w_{1,1}, w_{1,2}, ..., w_{1,n})$

 $W_2 = (w_{2,1}, w_{2,2}, ..., w_{2,n})$

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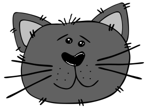
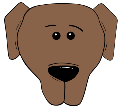
 $W_1 = (w_{1,1}, w_{1,2}, ..., w_{1,n})$

 $W_2 = (w_{2,1}, w_{2,2}, ..., w_{2,n})$

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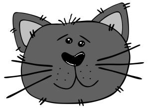
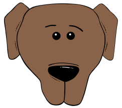
Goal: Output i with $\Pr[L_2 = i]$

  - Sample from a related distribution — "Drop the cross terms"
  - Correct it by "rejection sampling"
    - Output i with some probability p_i

New trick

 $W_1 = (w_{1,1}, w_{1,2}, \ldots, w_{1,n})$

 $W_2 = (w_{2,1}, w_{2,2}, \ldots, w_{2,n})$

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Avoid computing "Inner Product"

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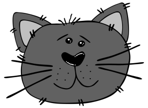
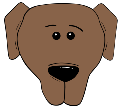
 $W_1 = (w_{1,1}, w_{1,2}, \dots, w_{1,n})$

 $W_2 = (w_{2,1}, w_{2,2}, \dots, w_{2,n})$

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Goal: Output i with Pr[$L_2 = i$]

- Sample from a related distribution — “Drop the cross terms”

Avoid computing “Inner Product”

- Correct it by “rejection sampling”
  - Output i with some probability p_i

Depends only on $w_{1,i}$ and $w_{2,i}$

Our protocol

Our protocol

- **The simple distribution A**

$$\Pr[A = i] = \frac{w_{1,i}^2 + w_{2,i}^2}{\|W_1\|_2^2 + \|W_2\|_2^2}$$

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Simply using our L1 sampling

$$V_1 = (w_{1,1}^2, w_{1,2}^2, \ldots, w_{1,n}^2)$$
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- **Acceptance probability p_i**

Goal for acceptance rate: $\dfrac{\Pr[L_2 = i]}{\Pr[A = i]}$

Computing p_i

Correctness

L_2 distribution

$$\Pr[L_2 = i] = \frac{(w_{1,i} + w_{2,i})^2}{\|W_1 + W_2\|_2^2}$$

Easy distribution A

$$\Pr[A = i] = \frac{w_{1,i}^2 + w_{2,i}^2}{\|W_1\|_2^2 + \|W_2\|_2^2}$$

Probability for i being accepted

$$\frac{\Pr[L_2 = i]}{\Pr[A = i]} = \frac{(w_{1,i} + w_{2,i})^2}{w_{1,i}^2 + w_{2,i}^2} \cdot \frac{\|W_1\|_2^2 + \|W_2\|_2^2}{\|W_1 + W_2\|_2^2}$$

Computing p_i

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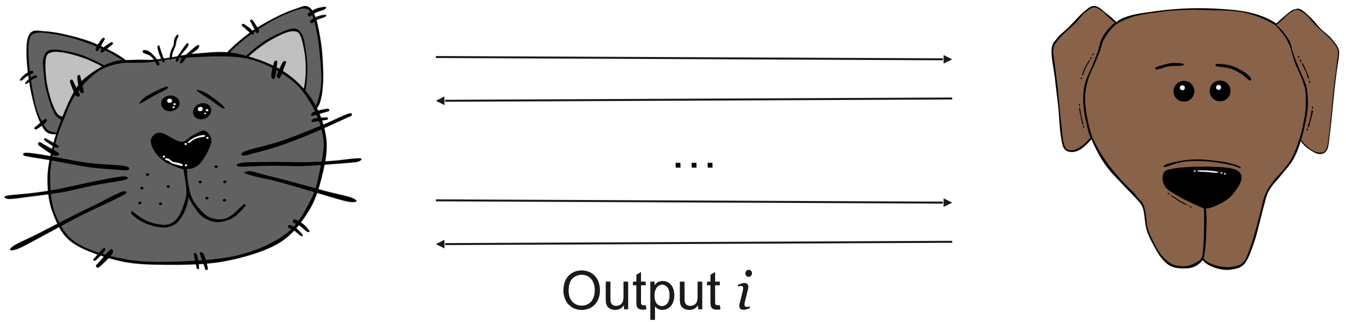
Expected constant round

Outline

- Sublinear sampling
- Sampling from common distributions
  - L_1 sampling
  - L_2 sampling
  - Product sampling
- Exponential Mechanism

Product Sampling

$$W_1 = (w_{1,1}, w_{1,2}, \ldots, w_{1,n}) \qquad W_2 = (w_{2,1}, w_{2,2}, \ldots, w_{2,n})$$



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2. The parties obliviously sample and secret share i_2.
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- **Impossible for sublinear communication**
  - Reduction from Set Disjointness Problem

Requires at least linear communication[R92]

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- Sublinear sampling
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Exponential Mechanism for Differential Privacy

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Product Sampling!

- **We have a problem — leaking inner product is not DP!**

✓ Solution: estimate Inner Product by DP version of Johnson-Lindenstrauss Transform[JL84, IM98]

Conclusion

- **Introduce a new problem of distributed sublinear sampling**
- **Achieve**
  - 2-party L_1, L_2, $L_p$sublinear sampling
  - Impossibility of sublinear product sampling in general
  - Sublinear product sampling protocol
    - for inputs w/ sufficient large inner product
  - 2-party exponential mechanism for differential privacy

Thank You!

http://eprint.iacr.org/2022/660

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