

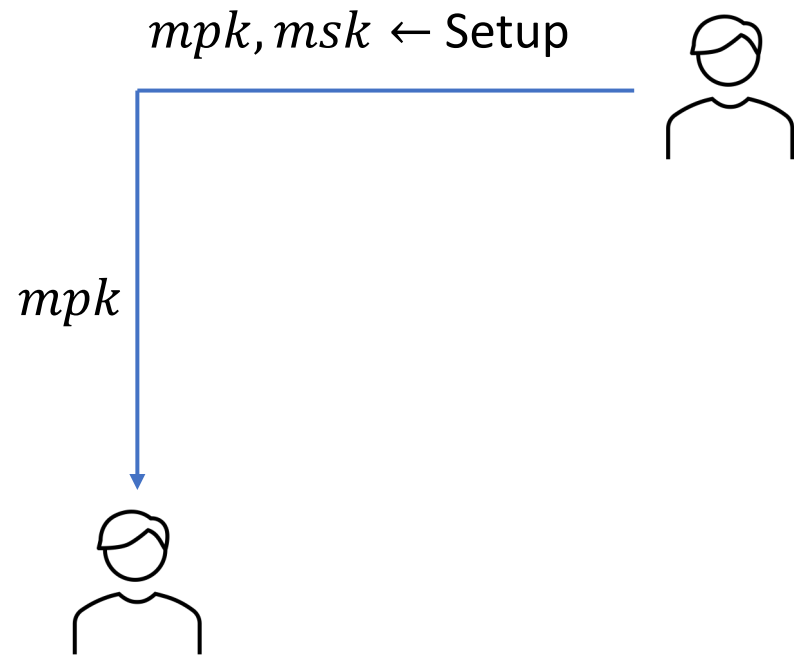
ABE for Circuits with Constant-Size Secret Keys and Adaptive Security

Hanjun Li, Huijia (Rachel) Lin, Ji Luo
University of Washington

Attribute Based Encryption (ABE) [SW05, GPSW06]

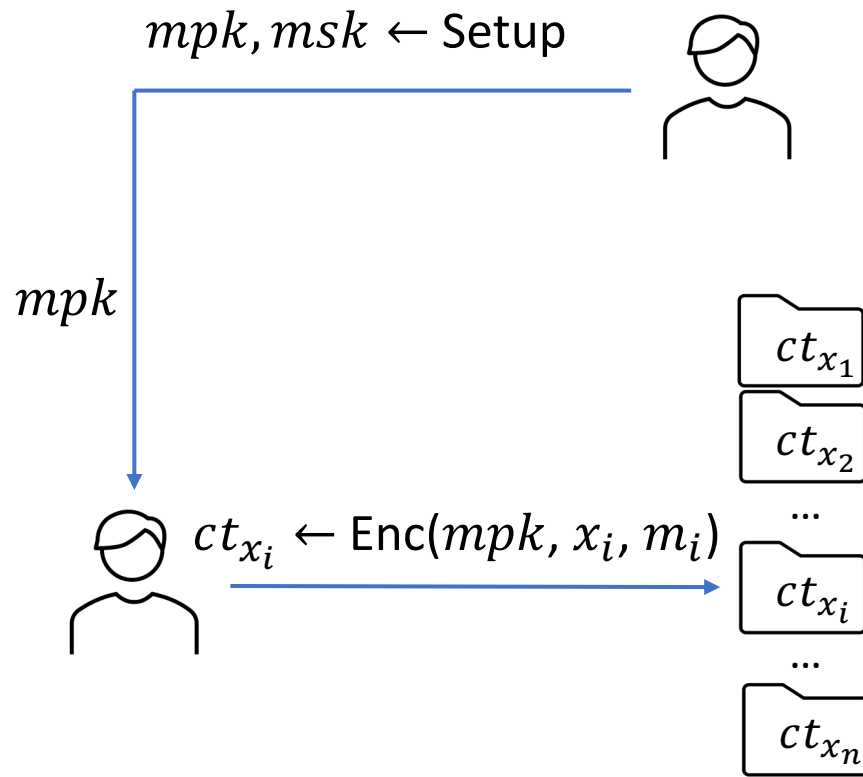
Key Policy (KP)
variant

Attribute Based Encryption (ABE) [SW05, GPSW06]



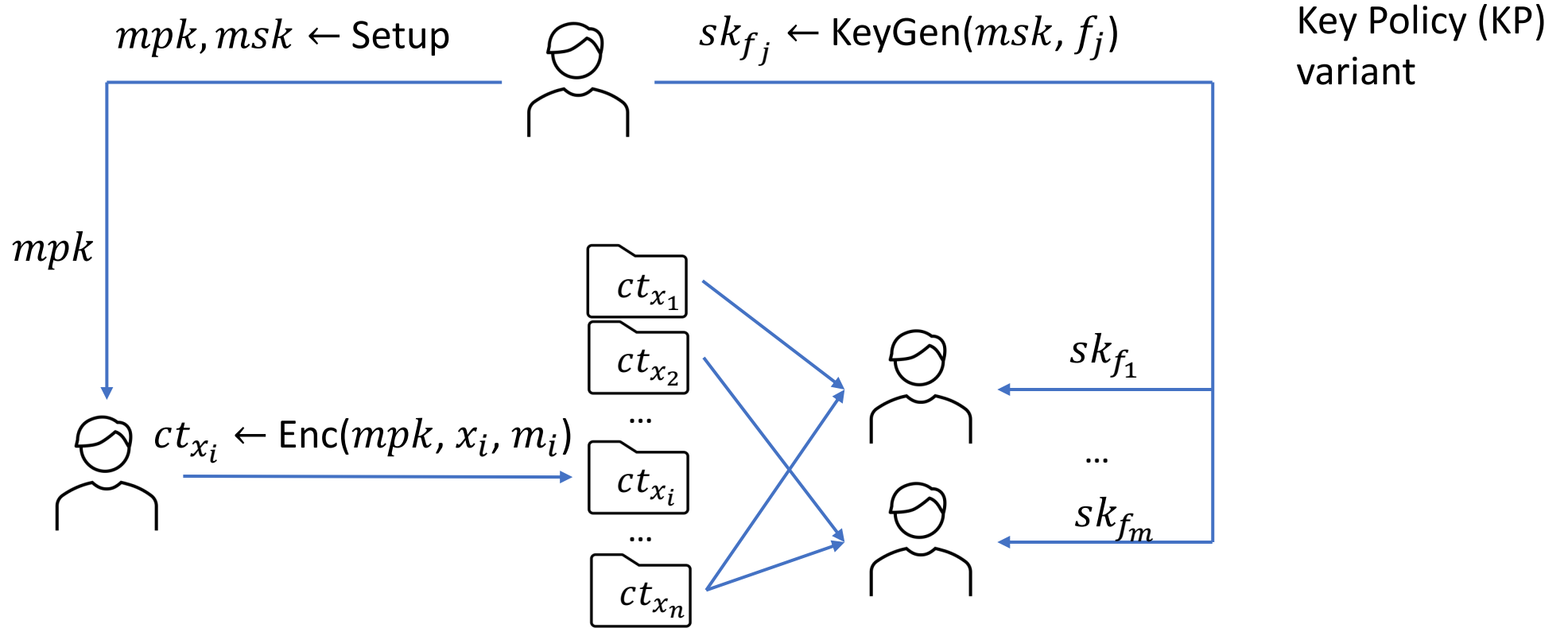
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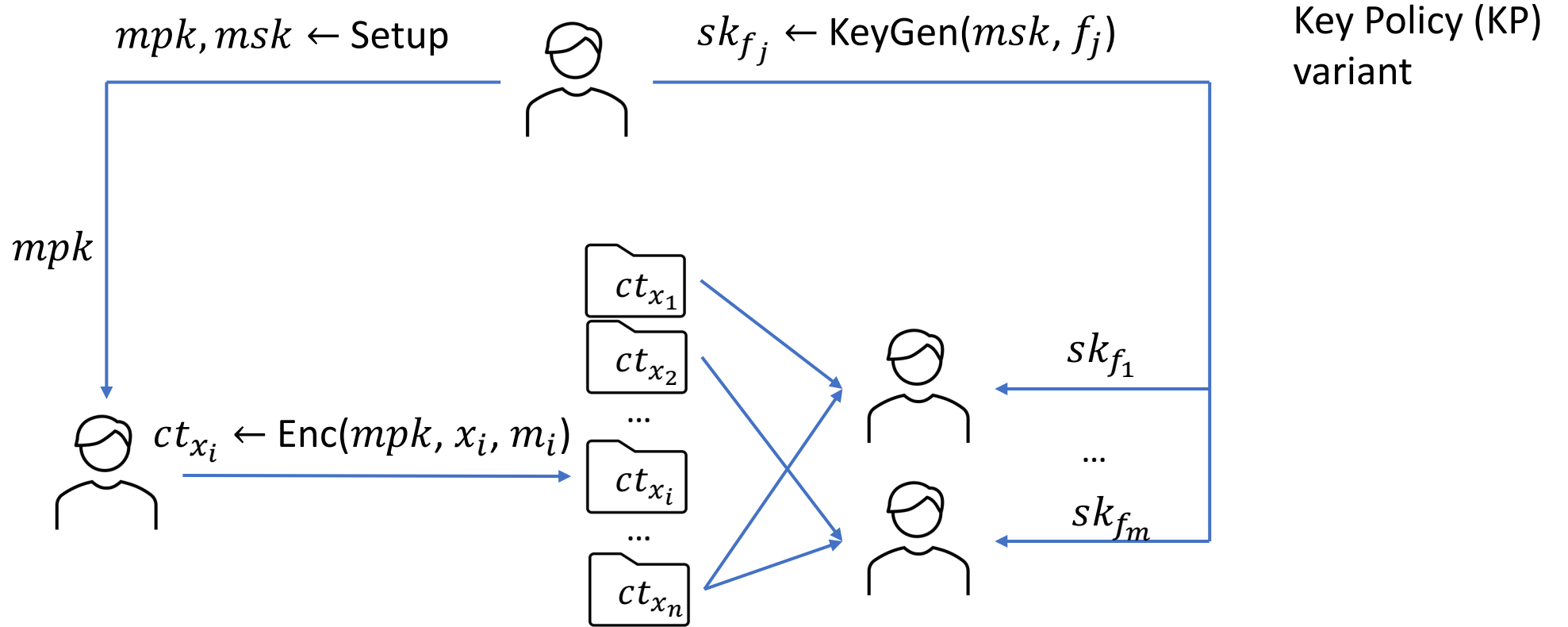


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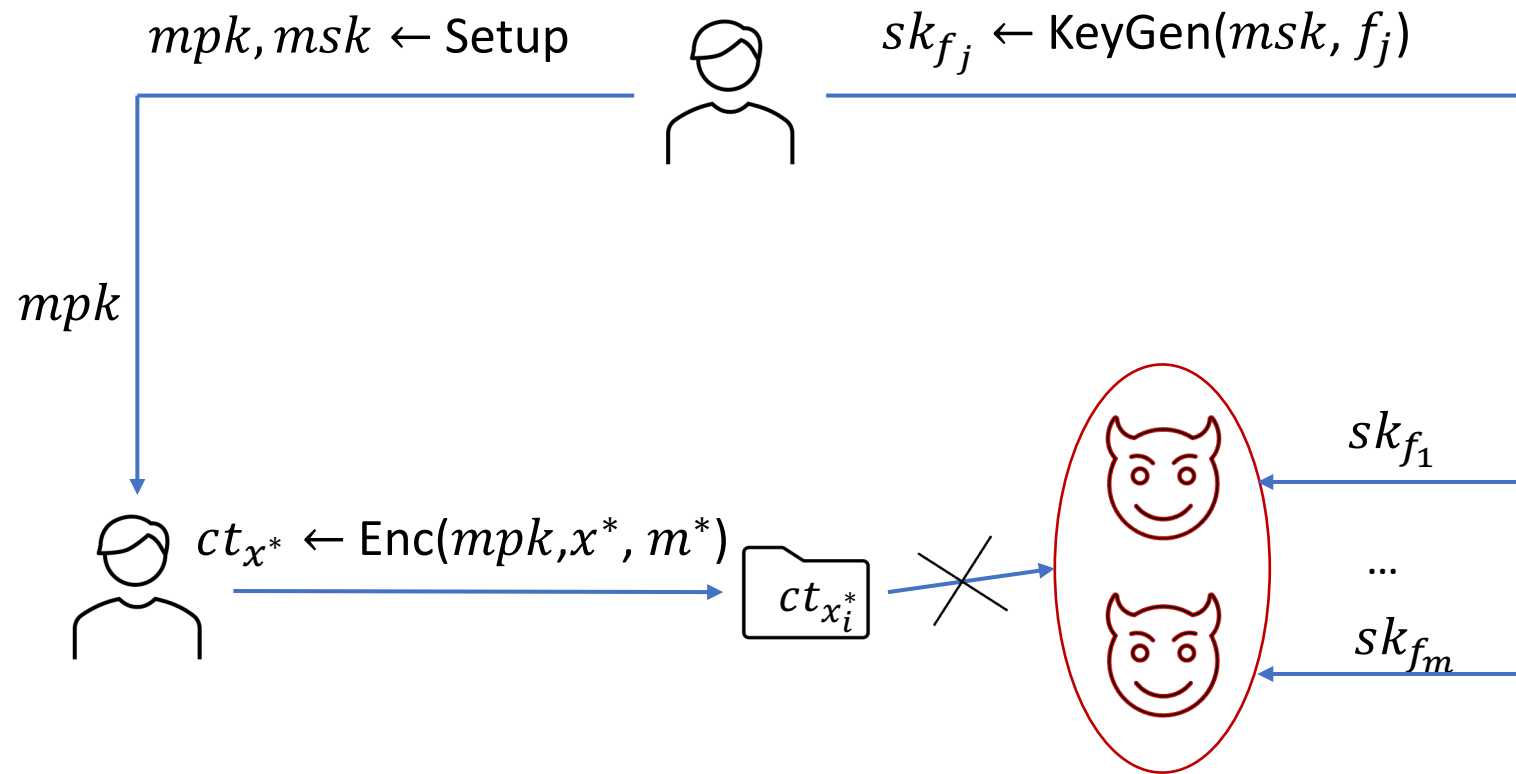


Attribute Based Encryption (ABE) [SW05, GPSW06]

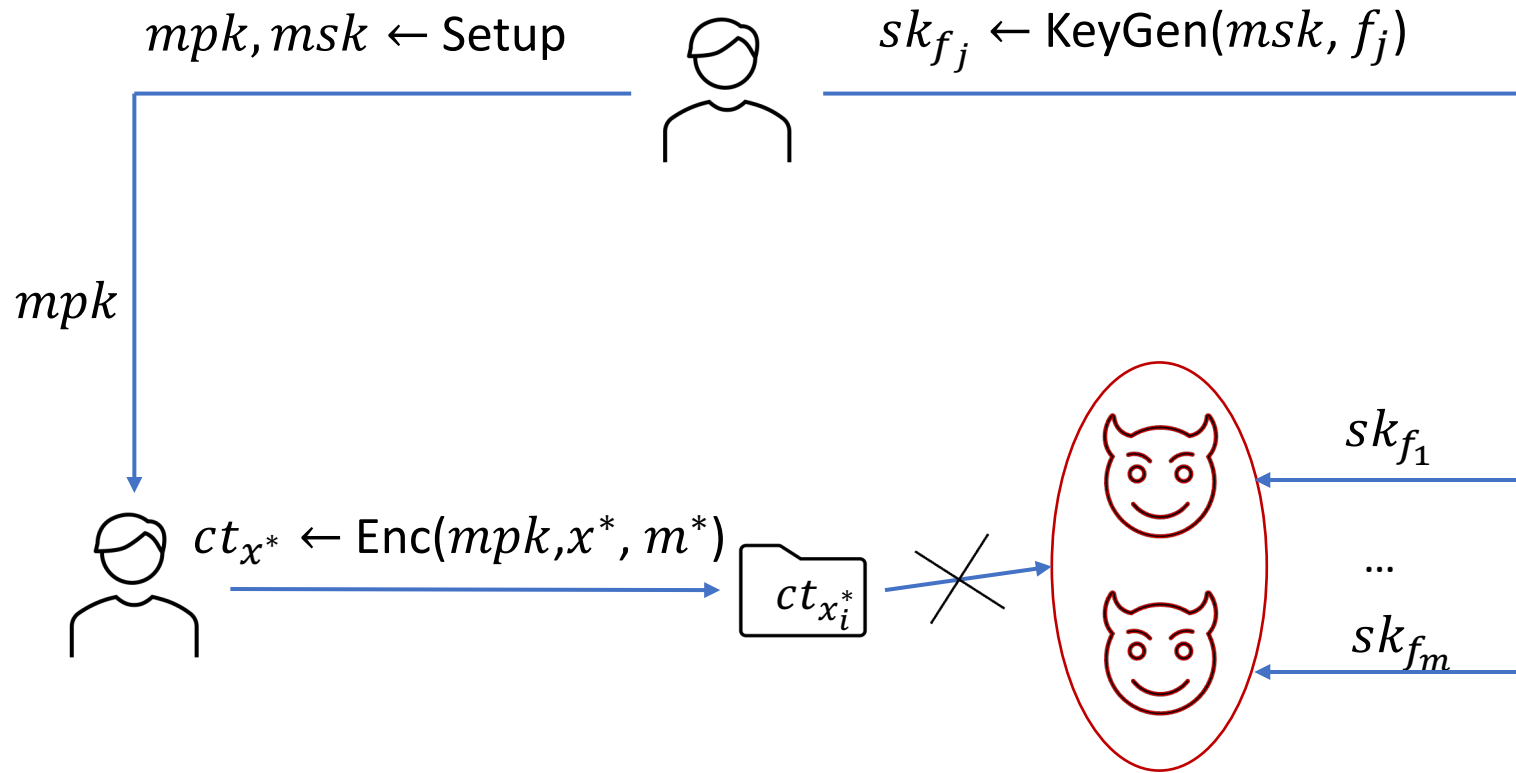


Correctness: $\text{Dec}(sk_{f_j}, ct_{x_i}) = m_i$
if $f_j(x_i) = 0$

Attribute Based Encryption (ABE) [SW05, GPSW06]

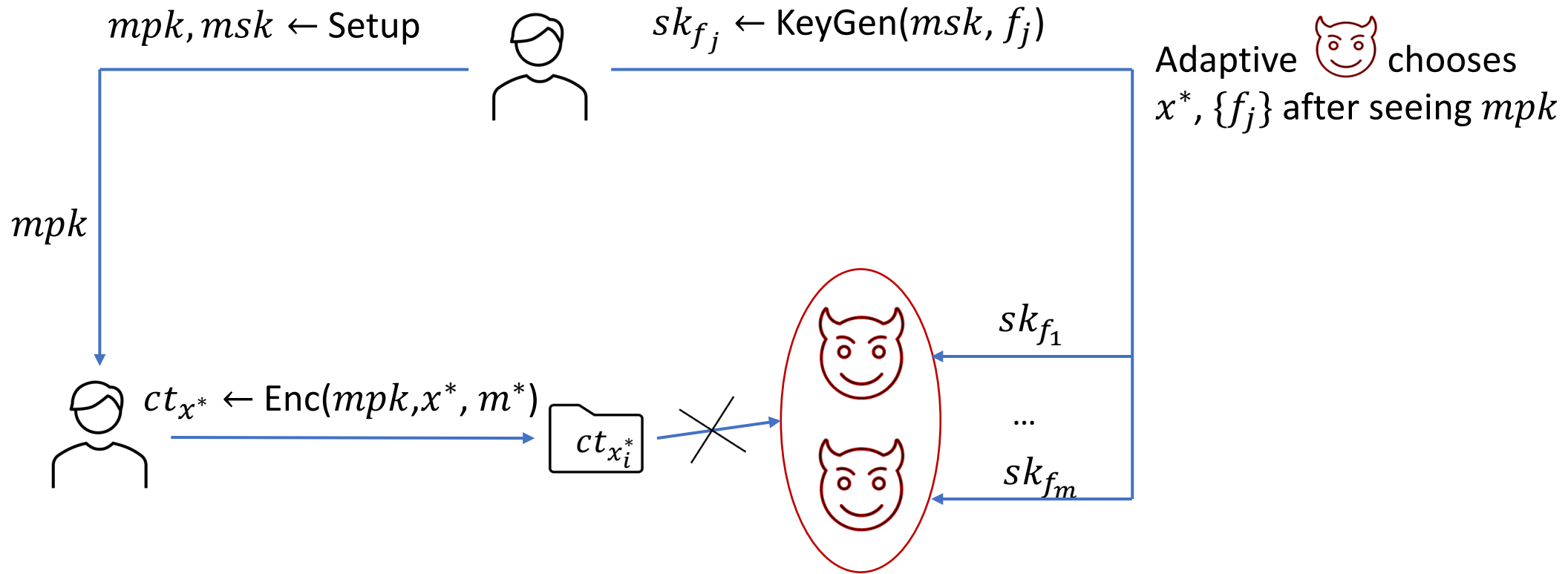


Attribute Based Encryption (ABE) [SW05, GPSW06]



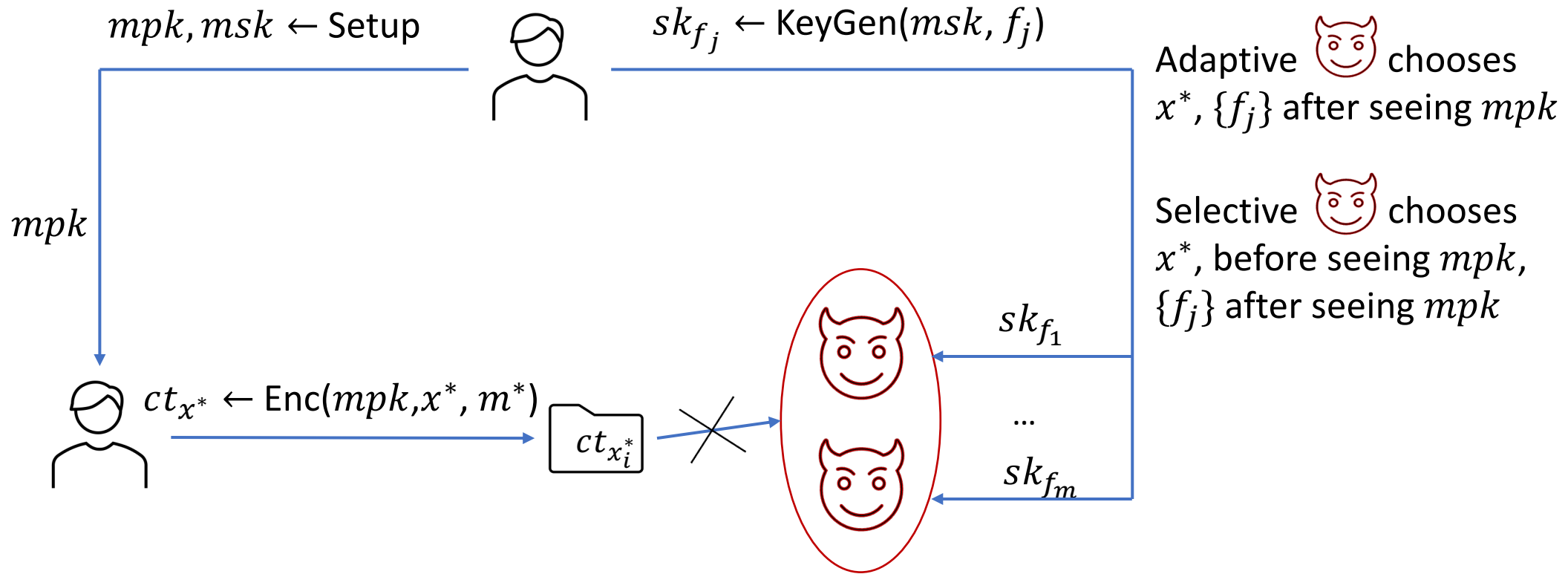
Security: m^* is hidden
if $\forall j, f_j(x^*) \neq 0$

Attribute Based Encryption (ABE) [SW05, GPSW06]




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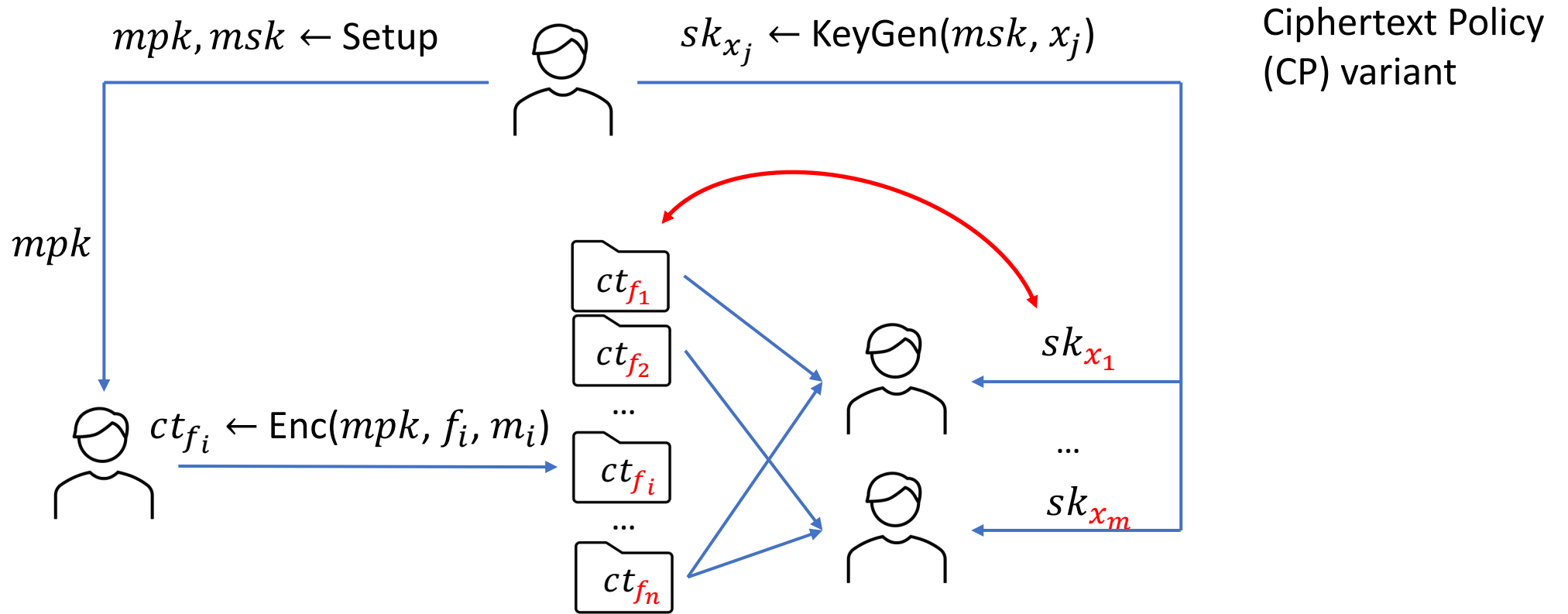


Adaptive  chooses $x^*, \{f_j\}$ after seeing mpk

Selective  chooses $x^*, \{f_j\}$ after seeing mpk

Security: m^* is hidden
if $\forall j, f_j(x^*) \neq 0$

Attribute Based Encryption (ABE) [SW05, GPSW06]

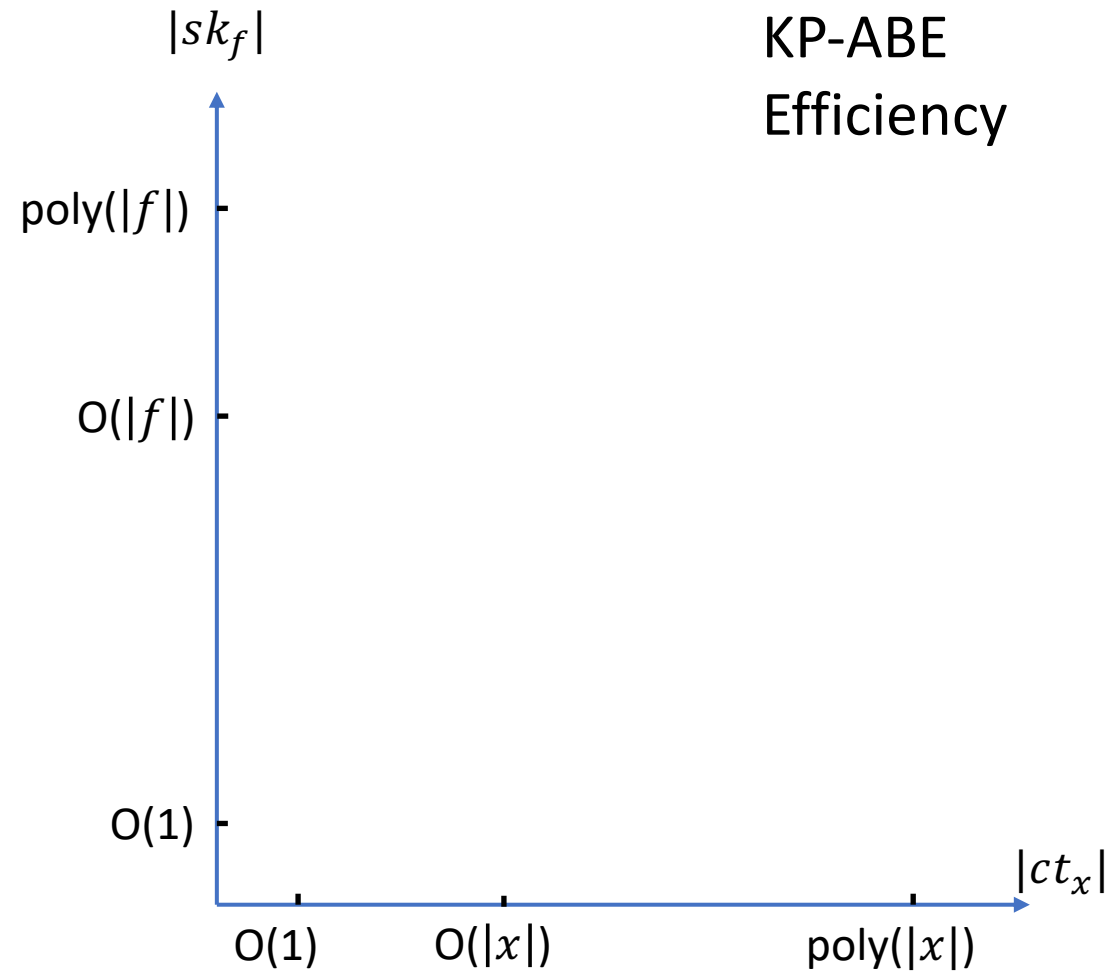


Correctness: $\text{Dec}(sk_{x_j}, ct_{f_i}) = m_i$
if $f_i(x_j) = 0$

Security: analogous

Efficiency of ABE: Size of $|ct|$, $|sk|$

compared to $|x|$, $|f|$, ignoring $\text{poly}(\lambda)$

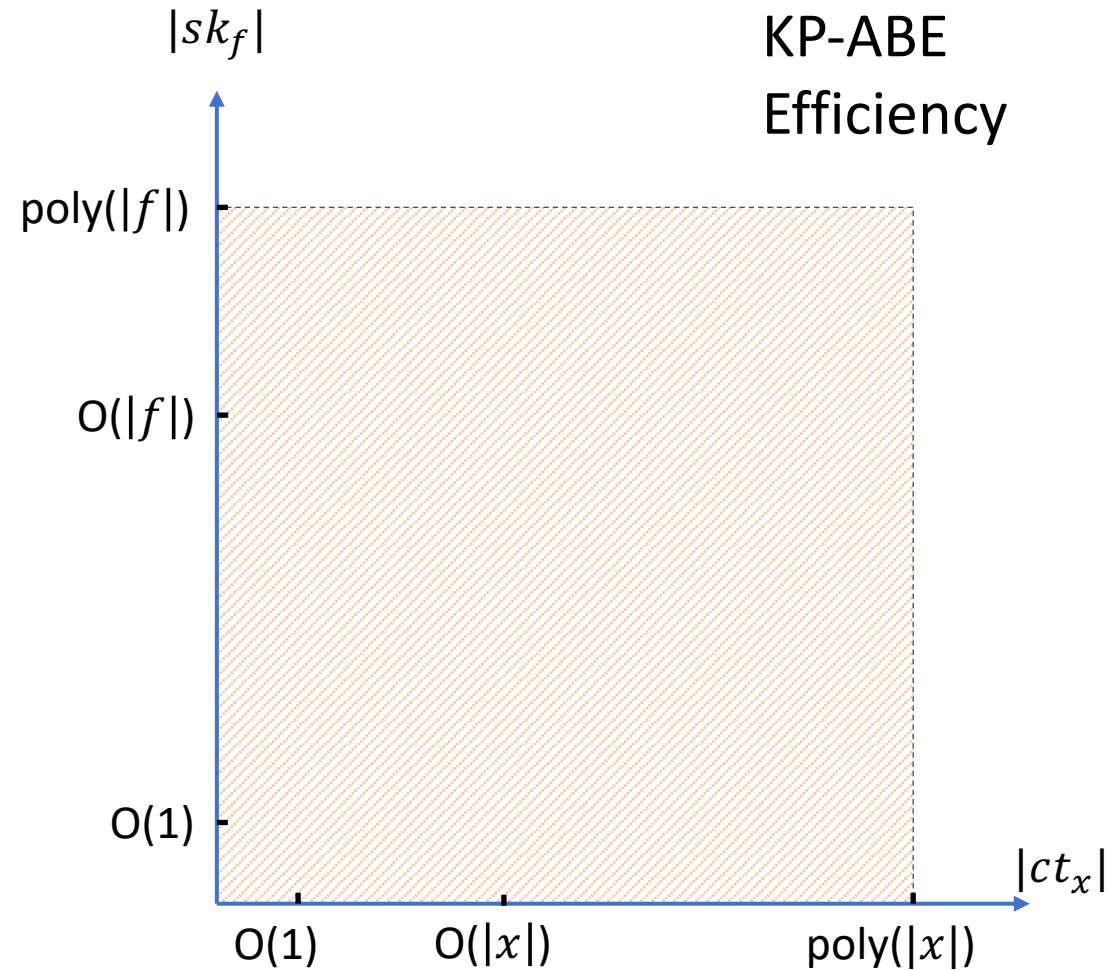


Efficiency of ABE: Size of $|ct|$, $|sk|$

compared to $|x|$, $|f|$, ignoring $\text{poly}(\lambda)$

- **Poly Efficiency:**

$$|ct_x| = \text{poly}(|x|), \quad |sk_f| = \text{poly}(|f|)$$



Efficiency of ABE: Size of $|ct|$, $|sk|$

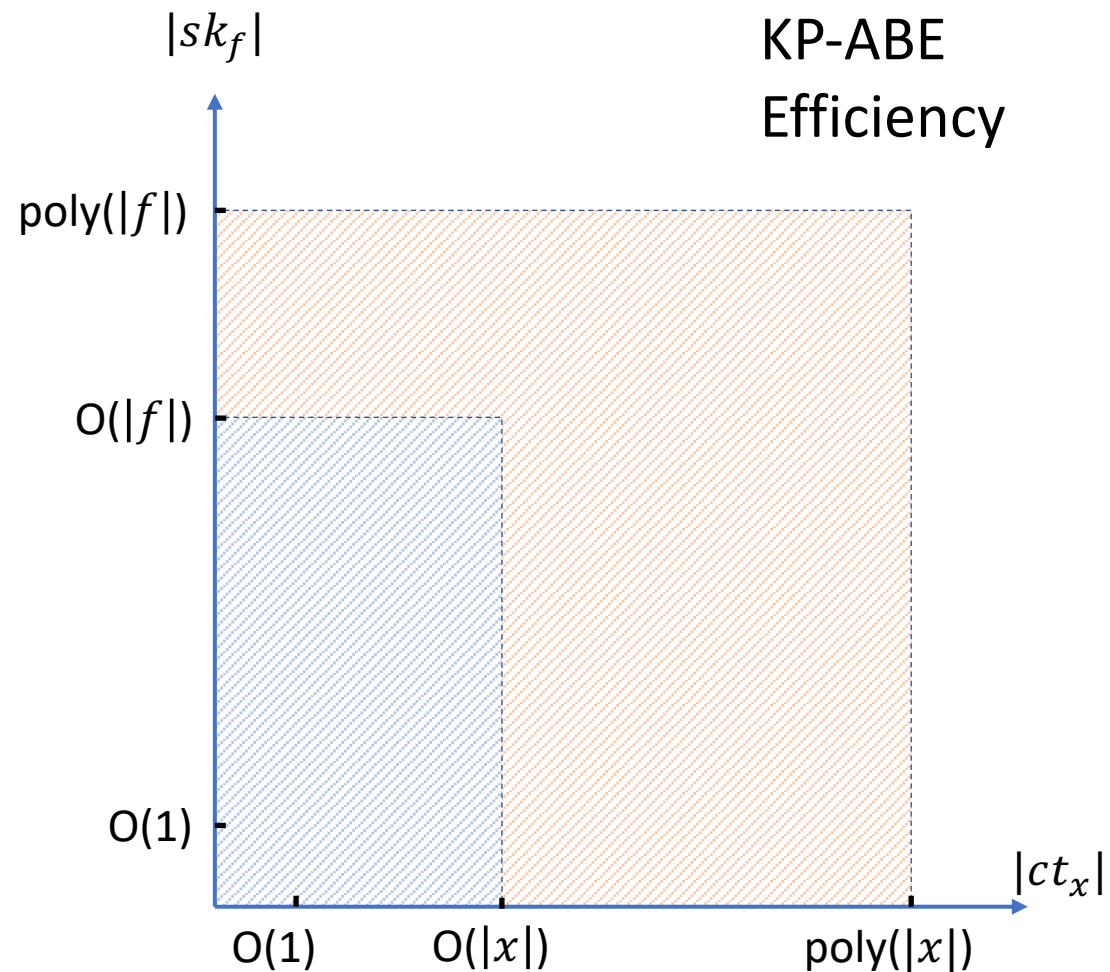
compared to $|x|$, $|f|$, ignoring $\text{poly}(\lambda)$

- **Poly Efficiency:**

$$|ct_x| = \text{poly}(|x|), |sk_f| = \text{poly}(|f|)$$

- **Compact:**

$$|ct_x| = O(|x|), |sk_f| = O(|f|)$$



Efficiency of ABE: Size of $|ct|$, $|sk|$

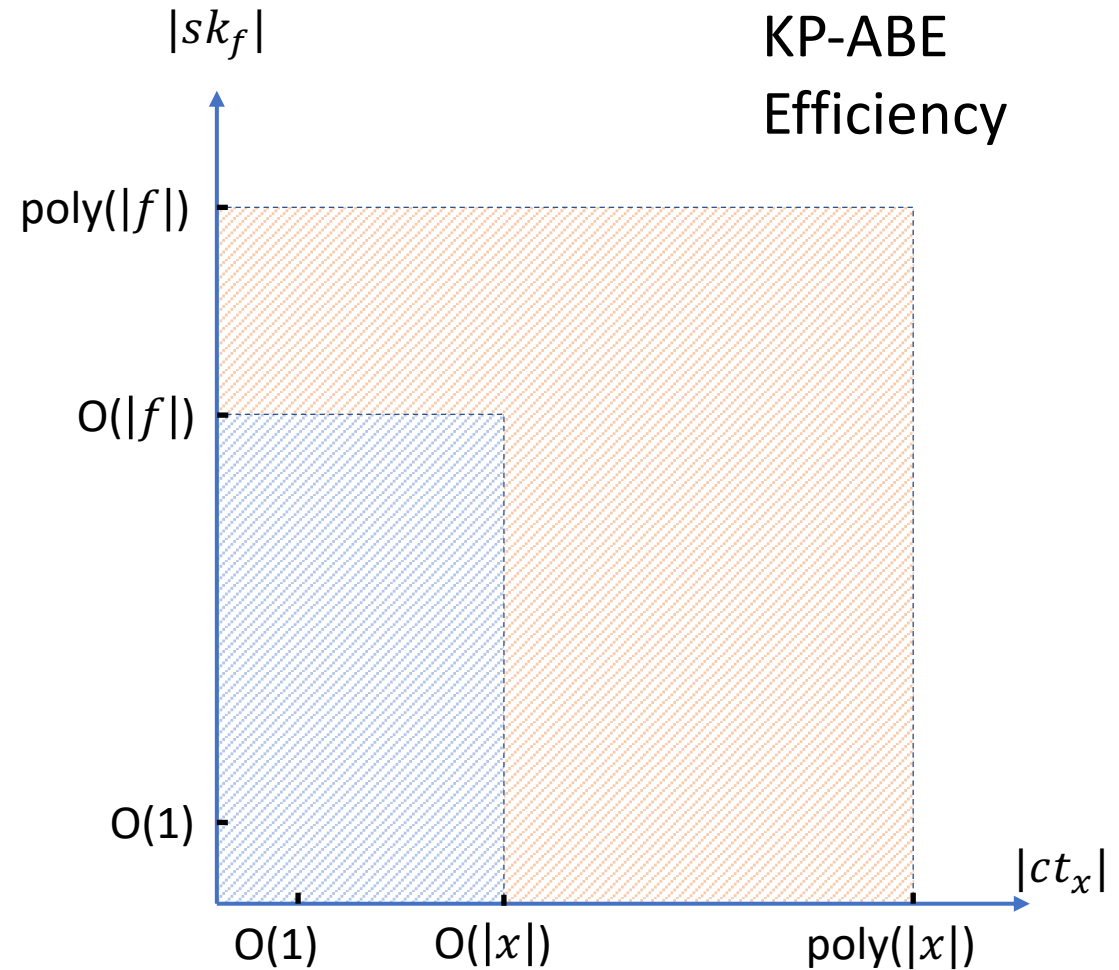
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- **Poly Efficiency:**

$$|ct_x| = \text{poly}(|x|), |sk_f| = \text{poly}(|f|)$$

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$$|ct_x| = O(|x|), |sk_f| = O(|f|)$$



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- **Poly Efficiency:**

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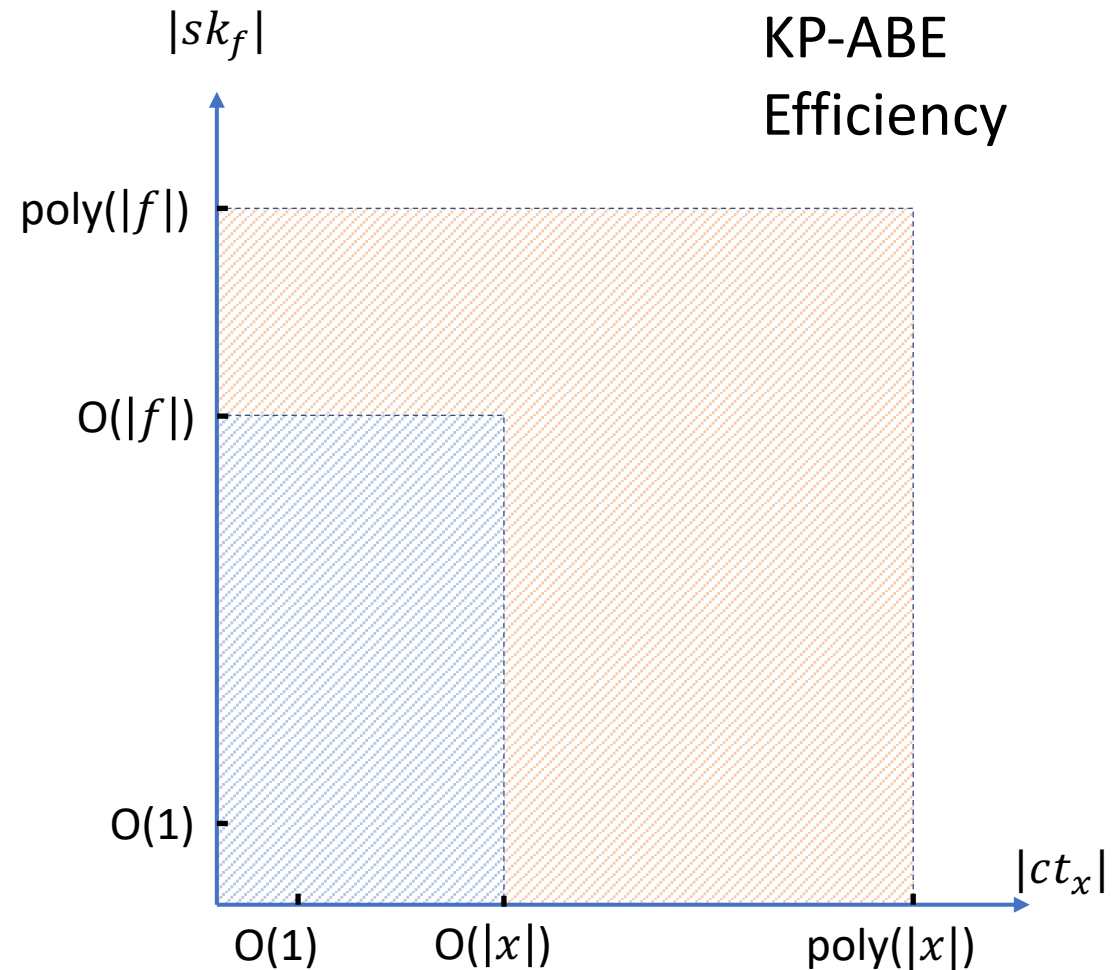
- **Compact: Best?**

$$|ct_x| = O(|x|), |sk_f| = O(|f|)$$

💡 No hiding x , f !

Transmit x , f in the clear, i.e. $\text{Dec}(x, f, sk_f, ct_x) = m$.

Possible to have $|ct_x| < |x|$, $|sk_f| < |f|$.



Efficiency of ABE: Size of $|ct|$, $|sk|$

compared to $|x|$, $|f|$, ignoring $\text{poly}(\lambda)$

- **Succinct:**

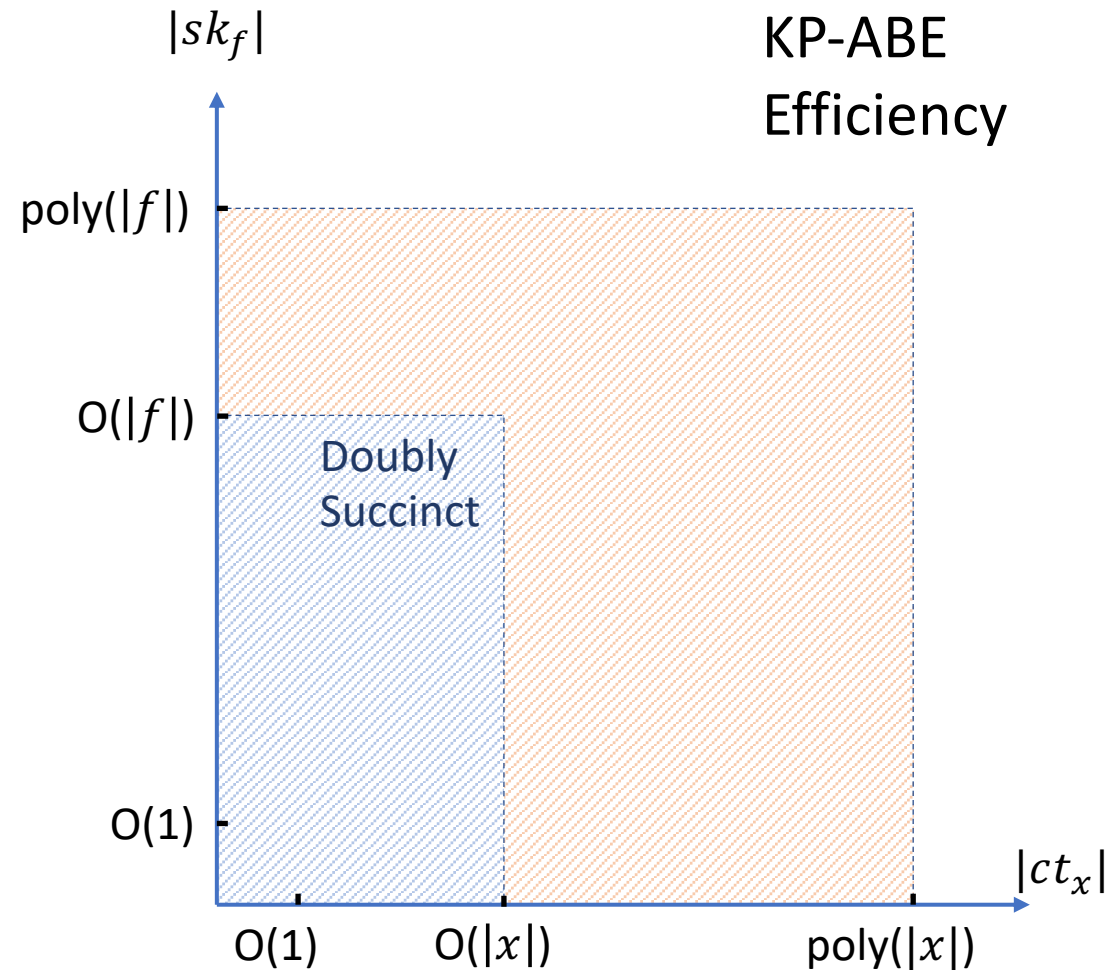
$$|ct_x| = o(|x|), |sk_f| = o(|f|)$$

(Doubly succinct if satisfying both.)

💡 No hiding x , f !

Transmit x , f in the clear, i.e. $\text{Dec}(x, f, sk_f, ct_x) = m$.

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Efficiency of ABE: Size of $|ct|$, $|sk|$

compared to $|x|$, $|f|$, ignoring $\text{poly}(\lambda)$

- **Succinct:**

$$|ct_x| = o(|x|), |sk_f| = o(|f|)$$

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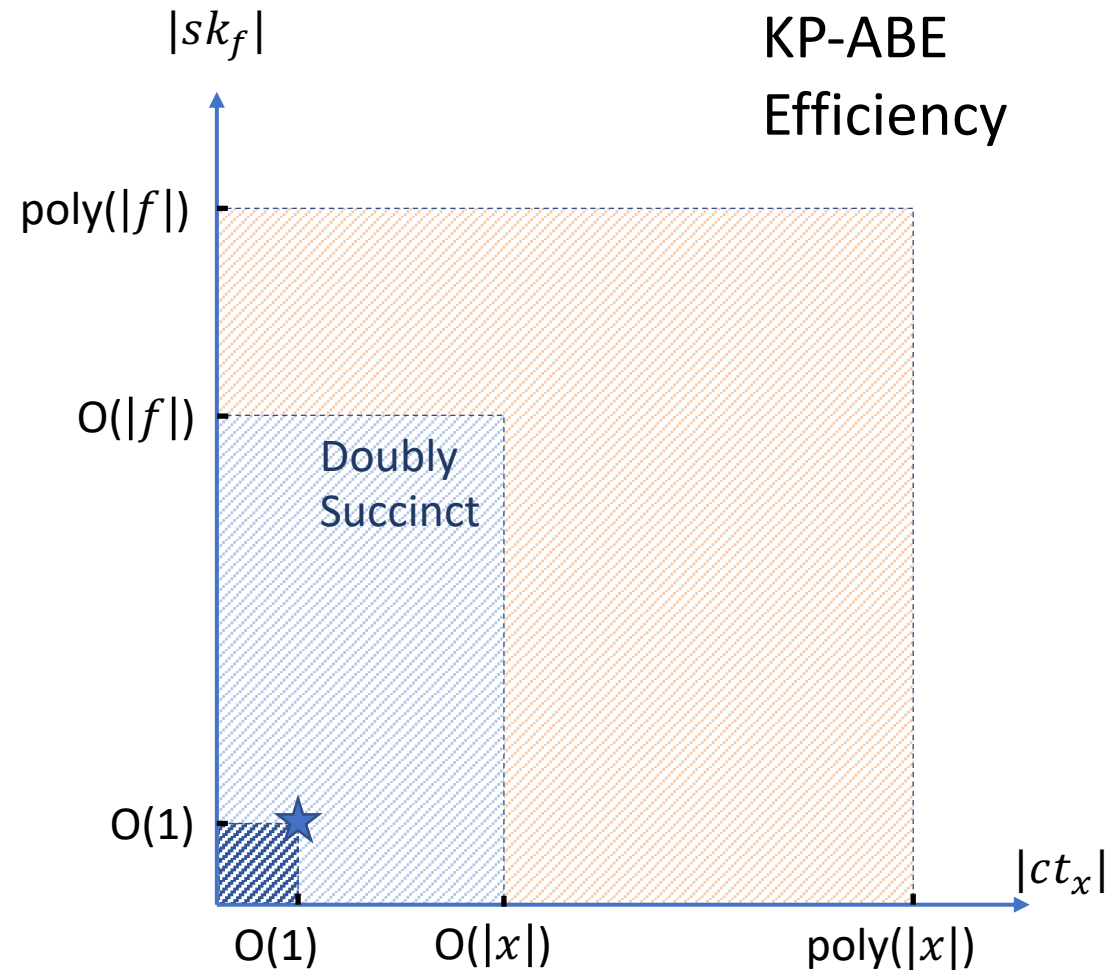
- **Constant: (Dream)**

$$|ct_x| = O(1), |sk_f| = O(1)$$

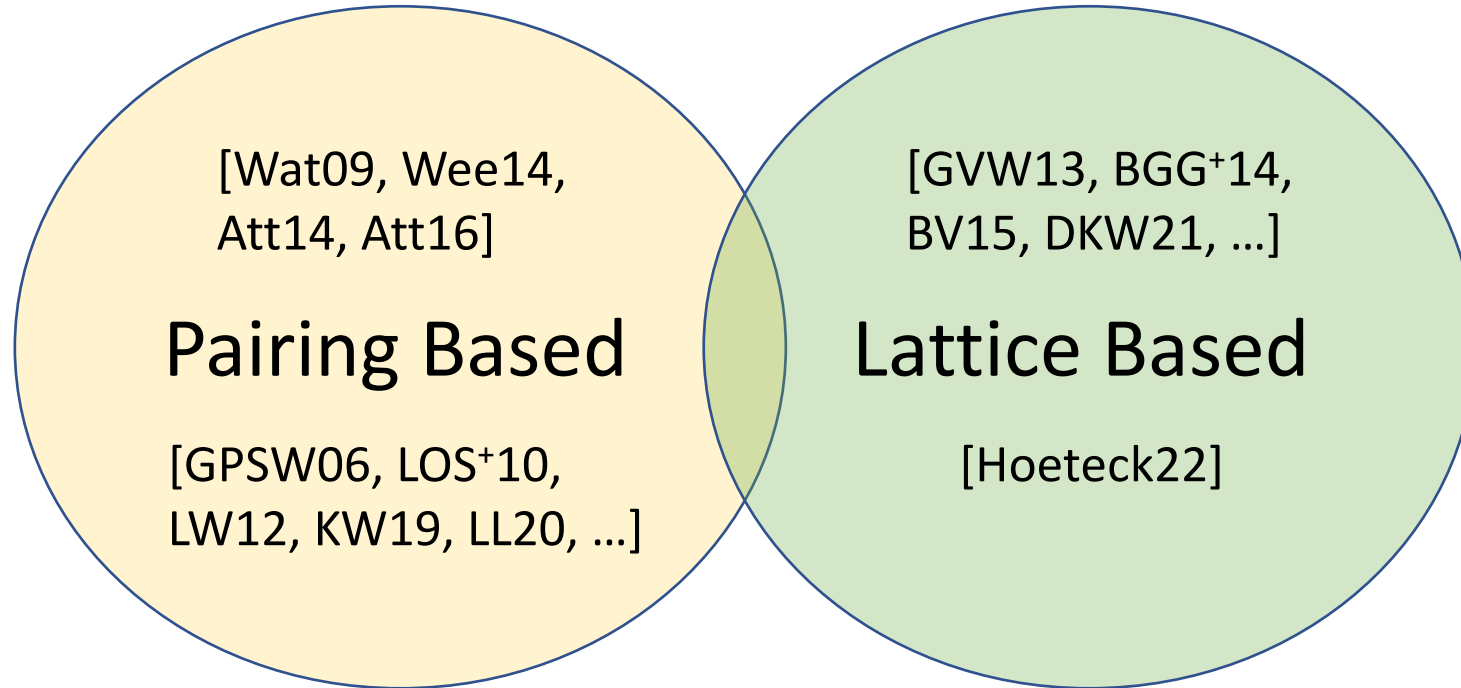
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Transmit x , f in the clear, i.e. $\text{Dec}(x, f, sk_f, ct_x) = m$.

Possible to have $|ct_x| < |x|$, $|sk_f| < |f|$.

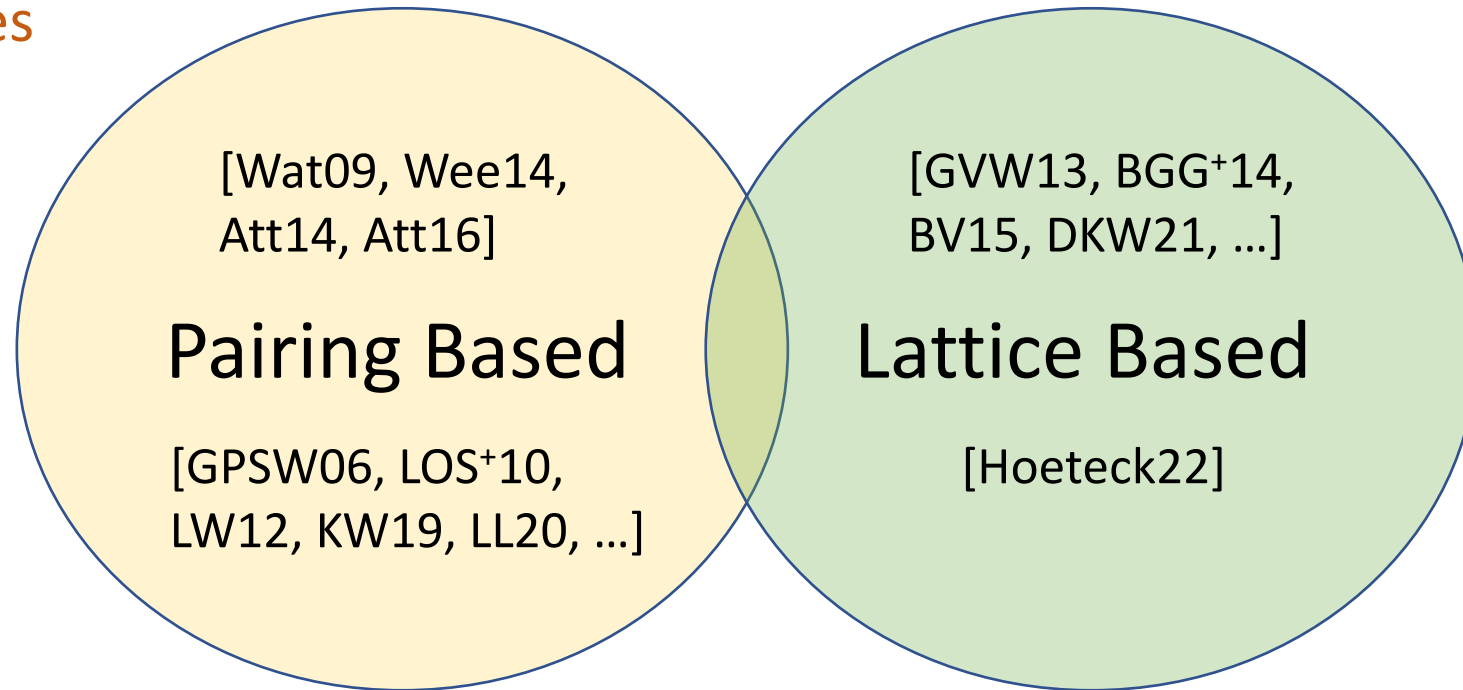


ABE Constructions



ABE Constructions

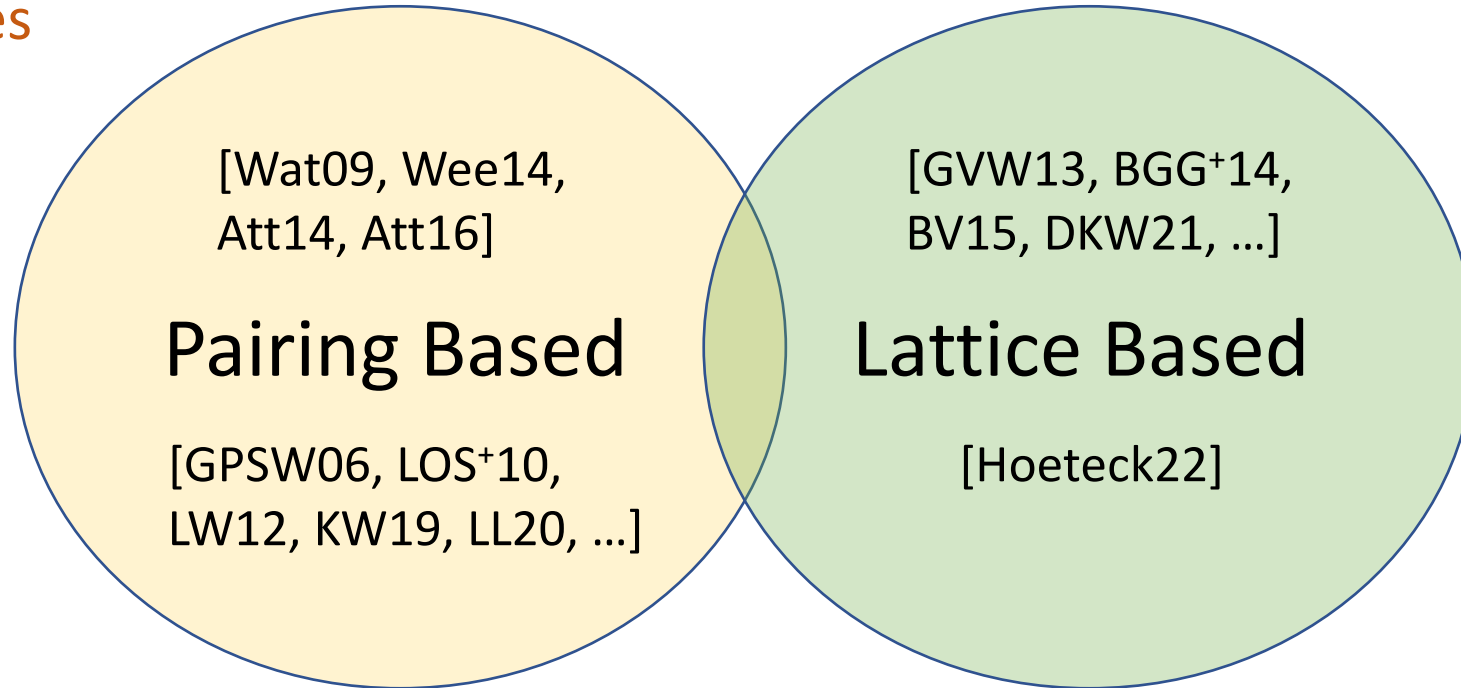
Low-depth policies



ABE Constructions

Low-depth policies

$$|sk| \cdot |ct| \geq |f| \cdot |x|$$

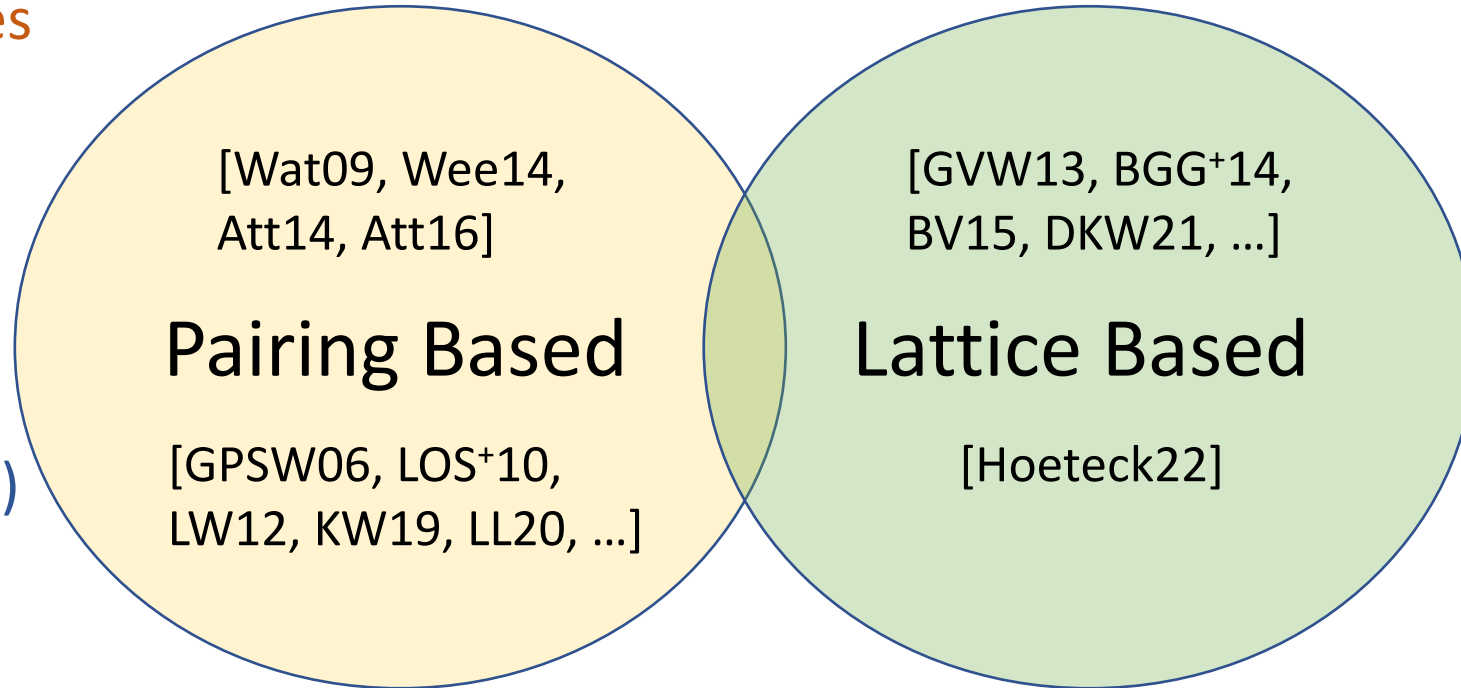


ABE Constructions

Low-depth policies

$$|sk| \cdot |ct| \geq |f| \cdot |x|$$

$$|sk| \text{ or } |ct| = O(1)$$



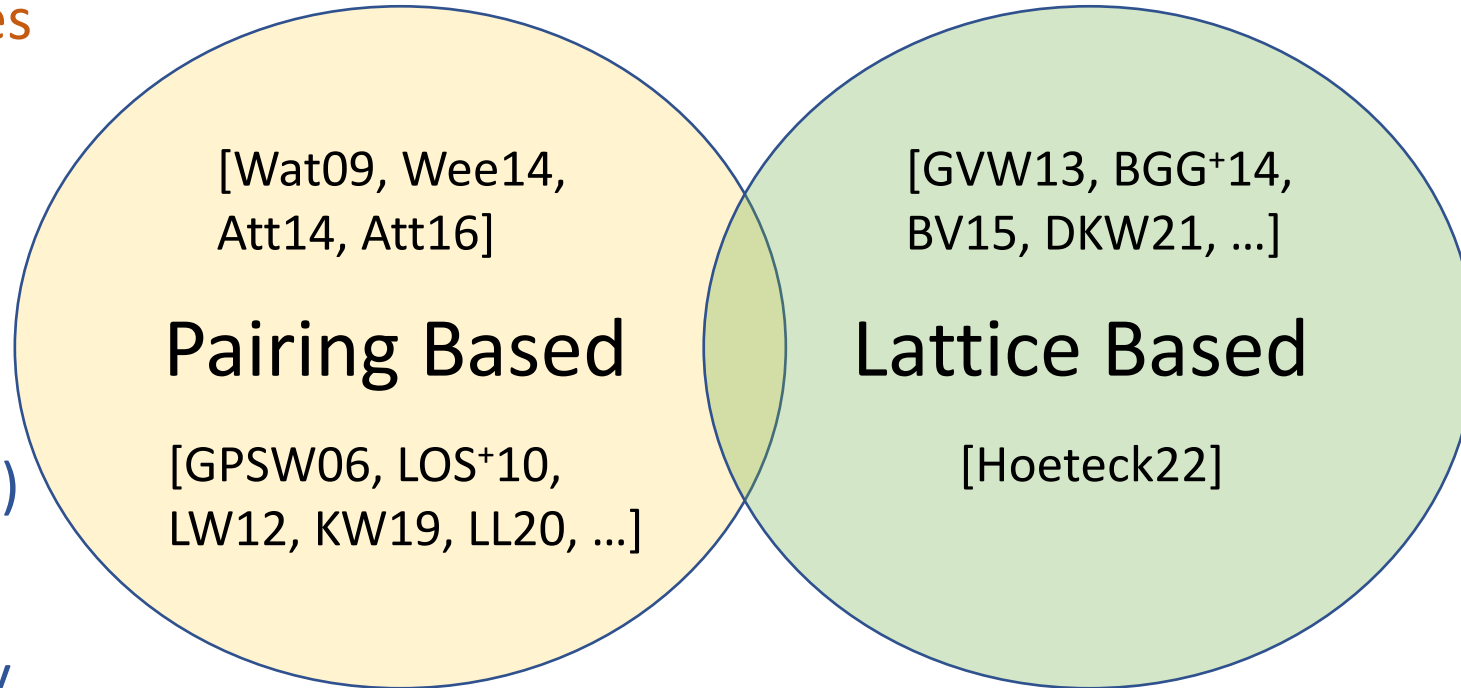
ABE Constructions

Low-depth policies

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Adaptive Security



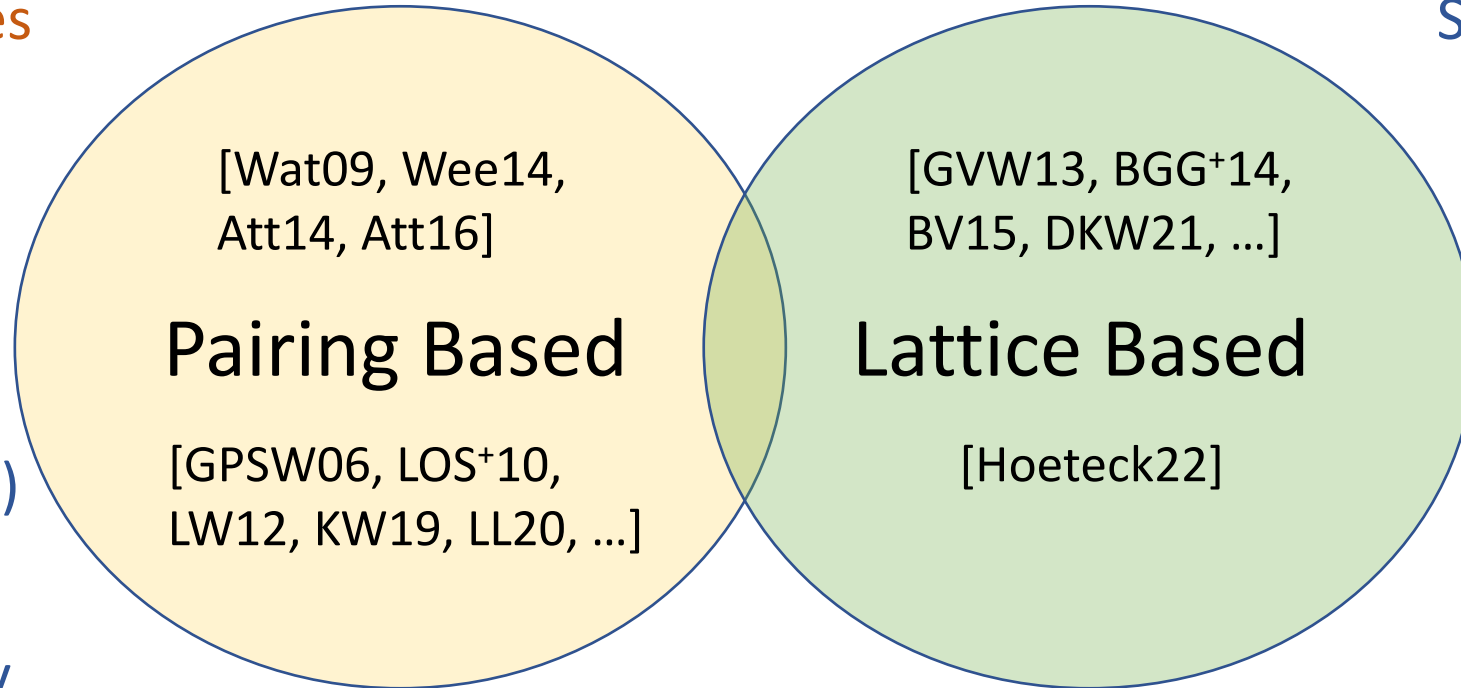
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Adaptive Security



Support Policies in P

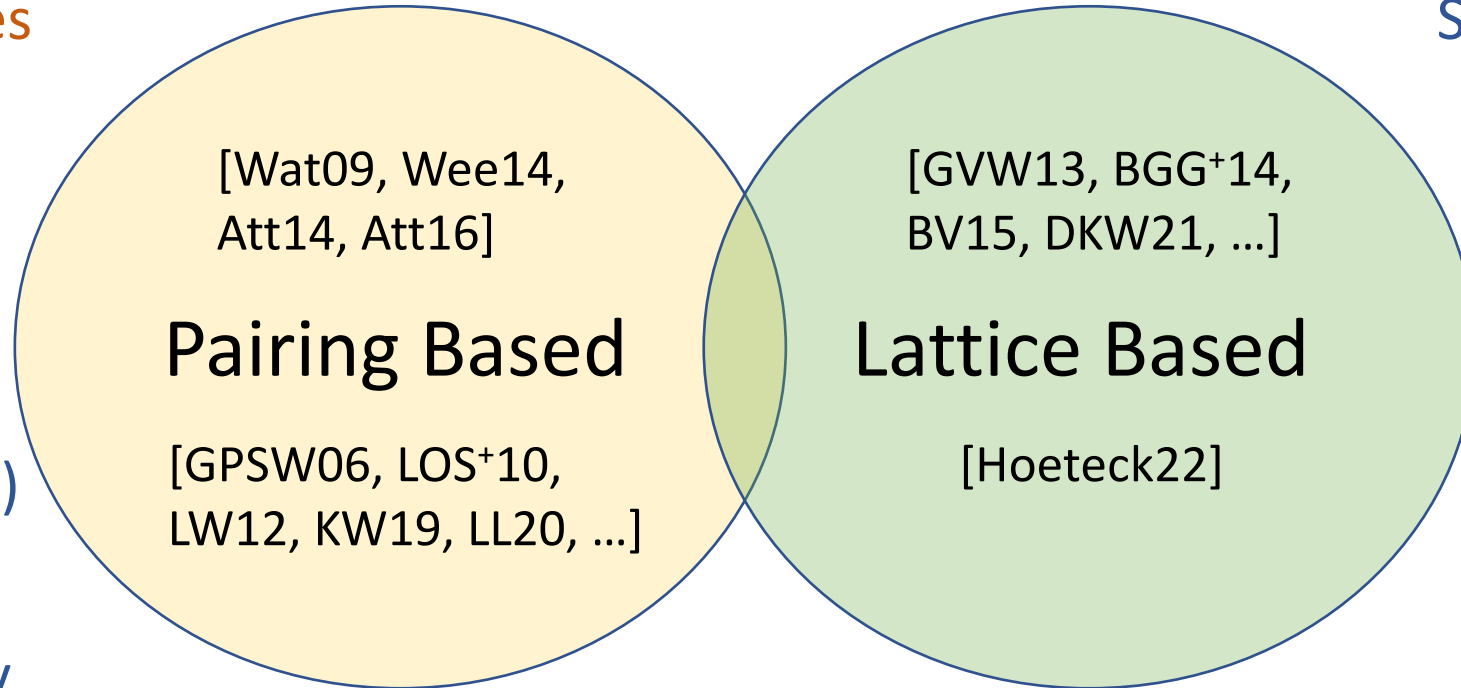
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Low-depth policies

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$$|sk| \text{ or } |ct| = O(1)$$

Adaptive Security



[Wat09, Wee14, Att14, Att16]

Pairing Based

[GPSW06, LOS⁺10, LW12, KW19, LL20, ...]

[GVW13, BGG⁺14, BV15, DKW21, ...]

Lattice Based

[Hoeteck22]

Support Policies in P

$$|sk|, |ct| = \text{poly}(d), |x| \text{poly}(d) \\ (d = \text{depth}(f))$$

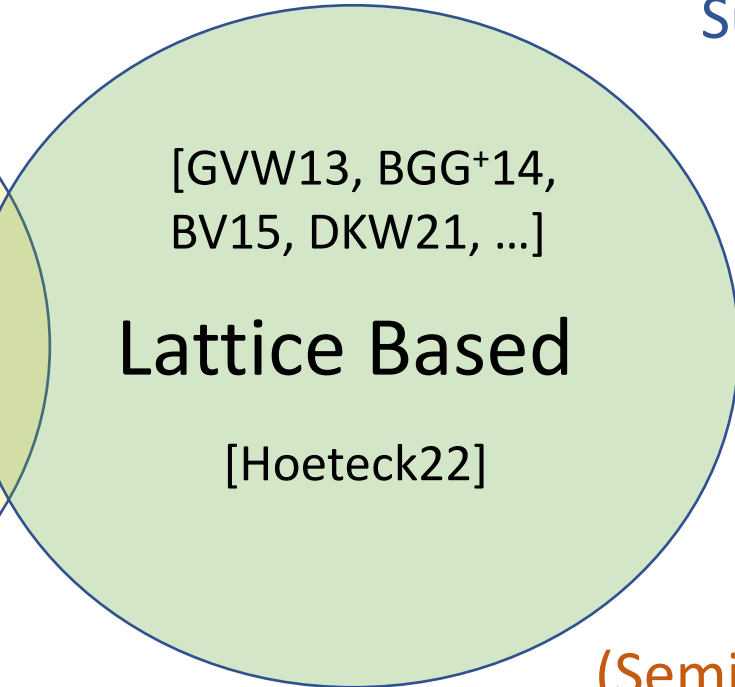
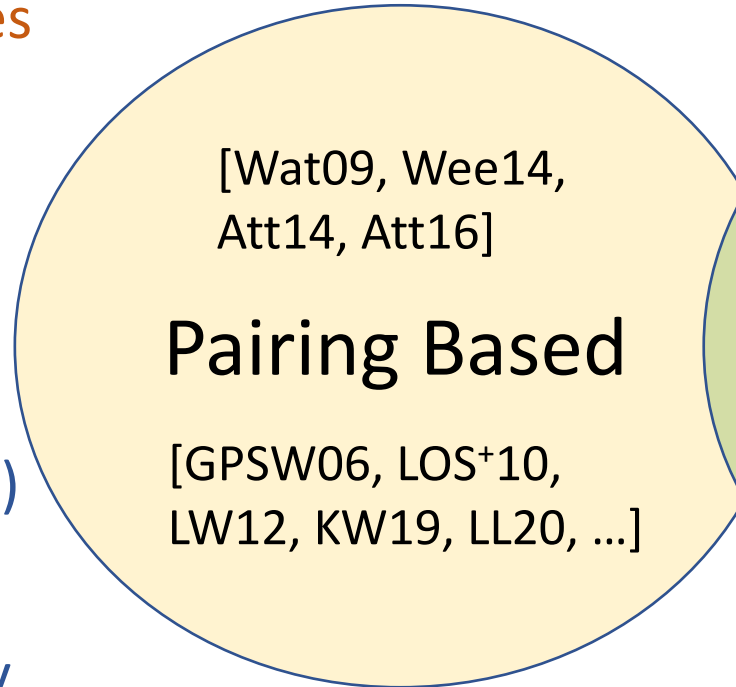
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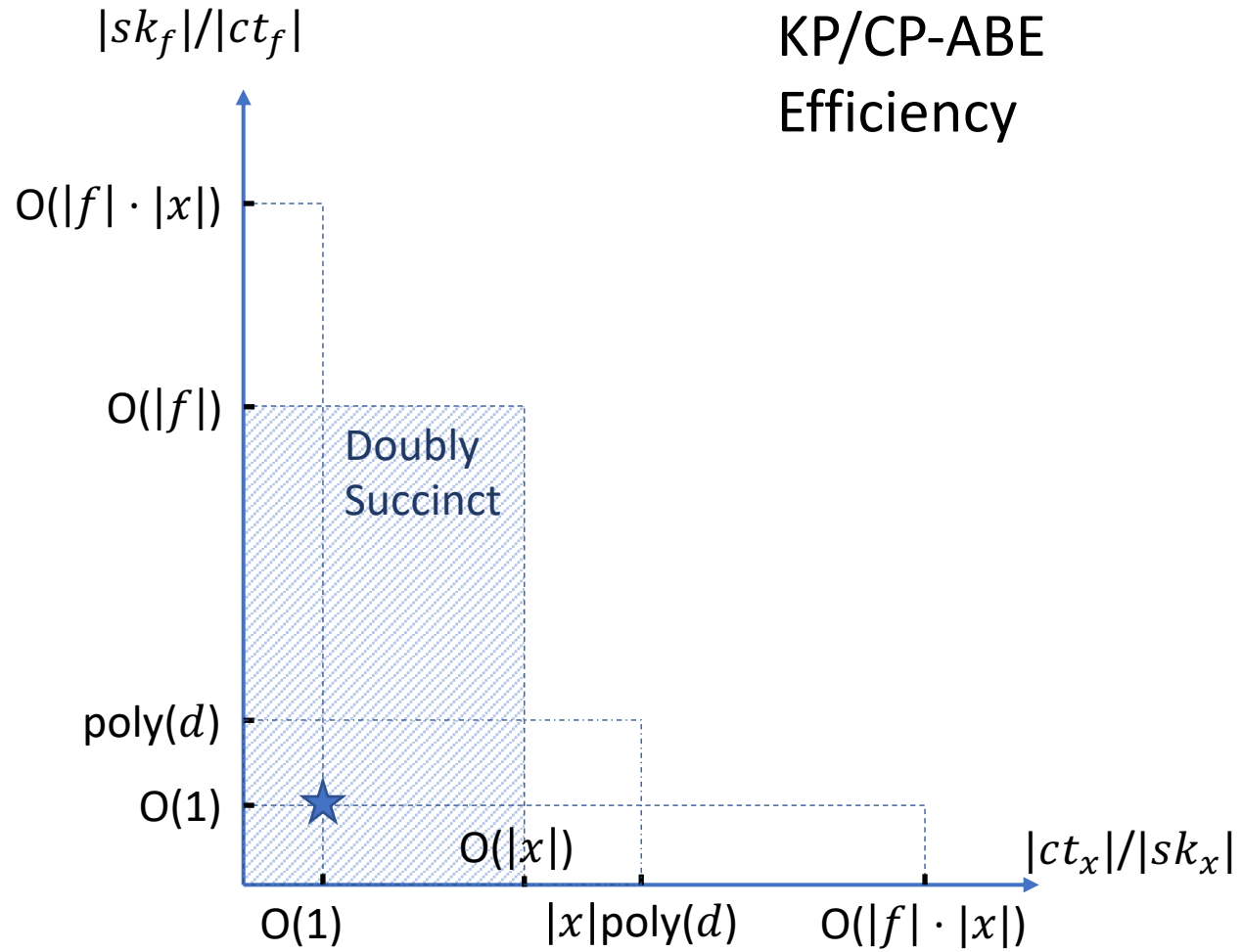


Support Policies in P

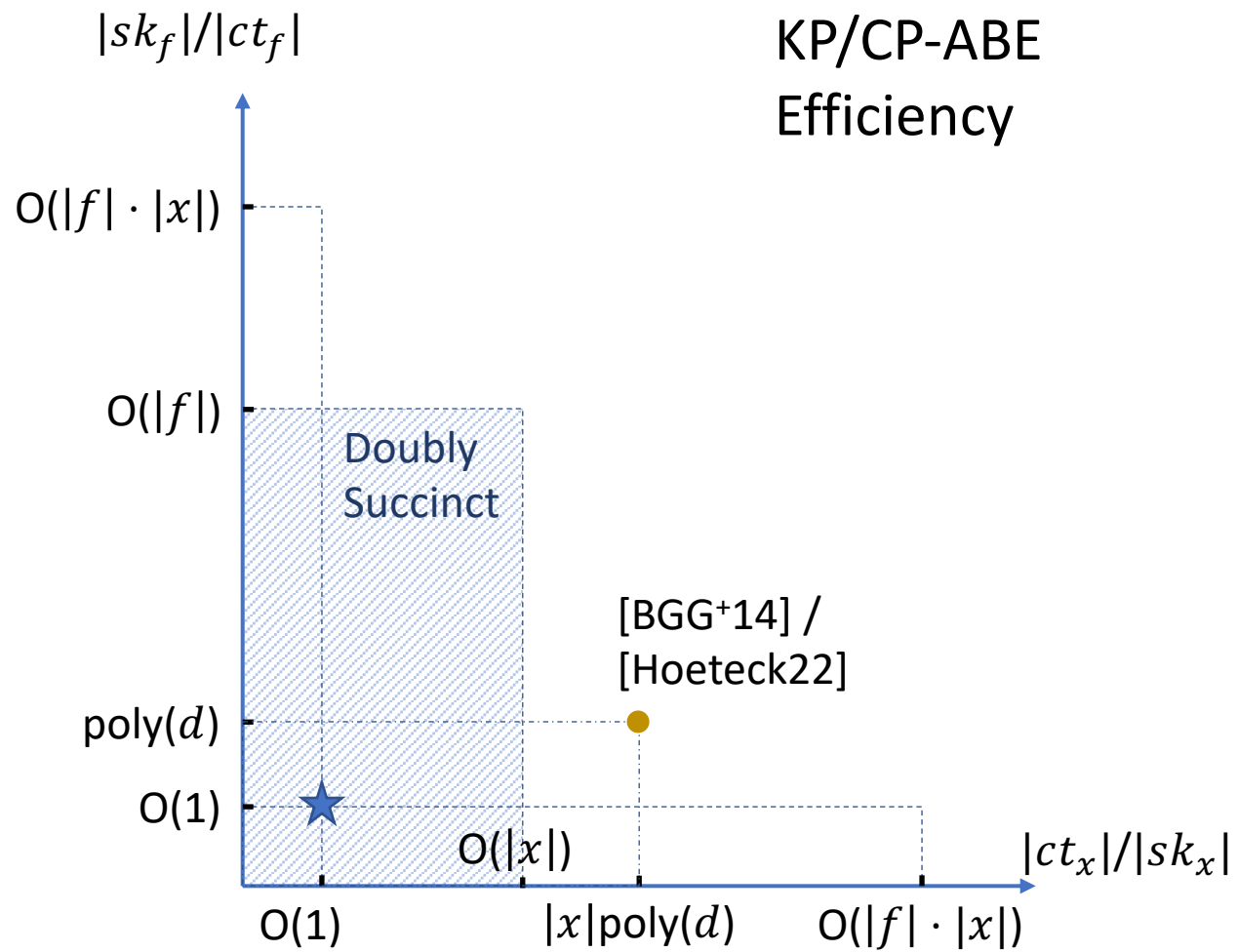
$$|sk|, |ct| = \text{poly}(d), |x| \text{poly}(d) \\ (d = \text{depth}(f))$$

(Semi) Selective Security
(fascinating barrier)

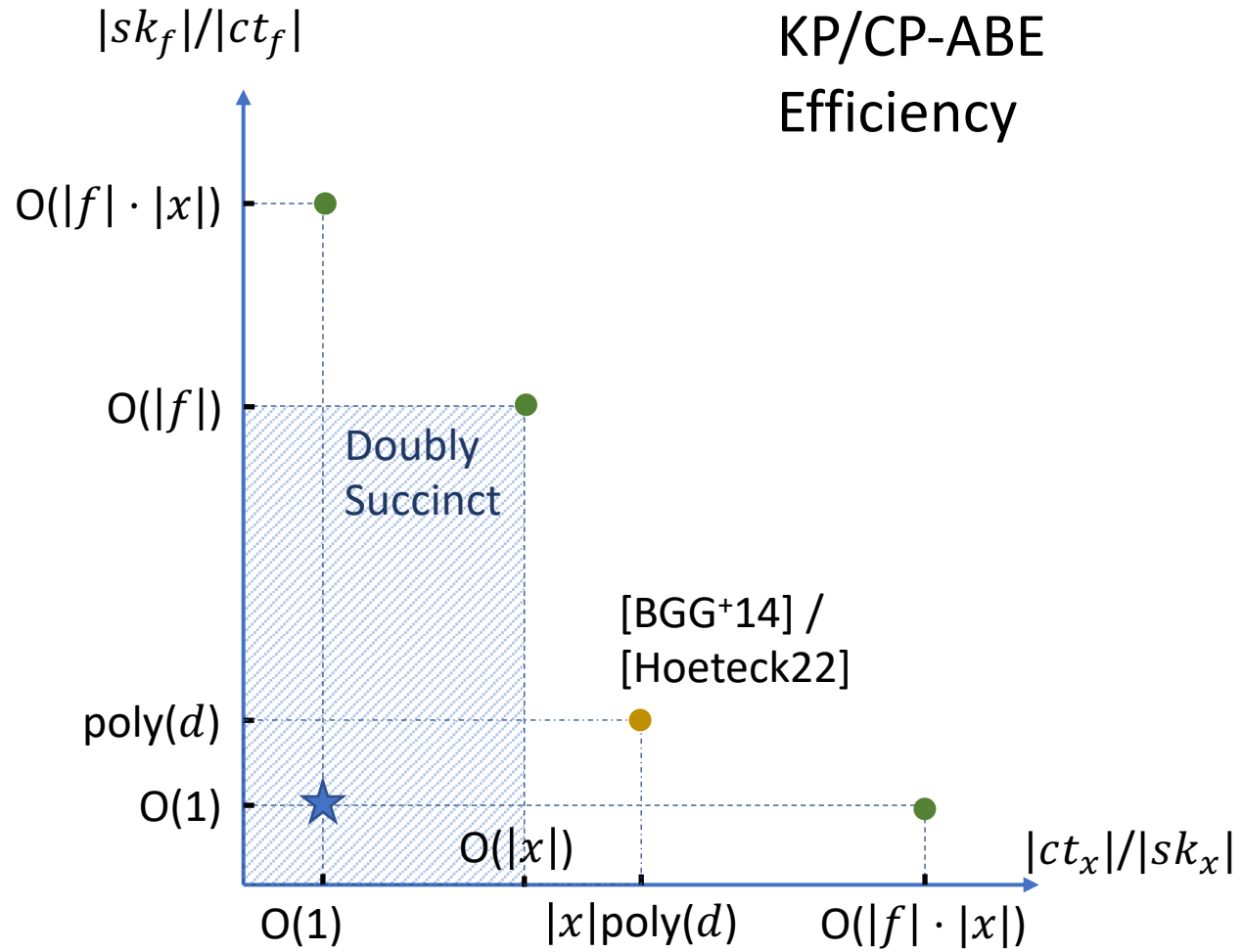
ABE Constructions



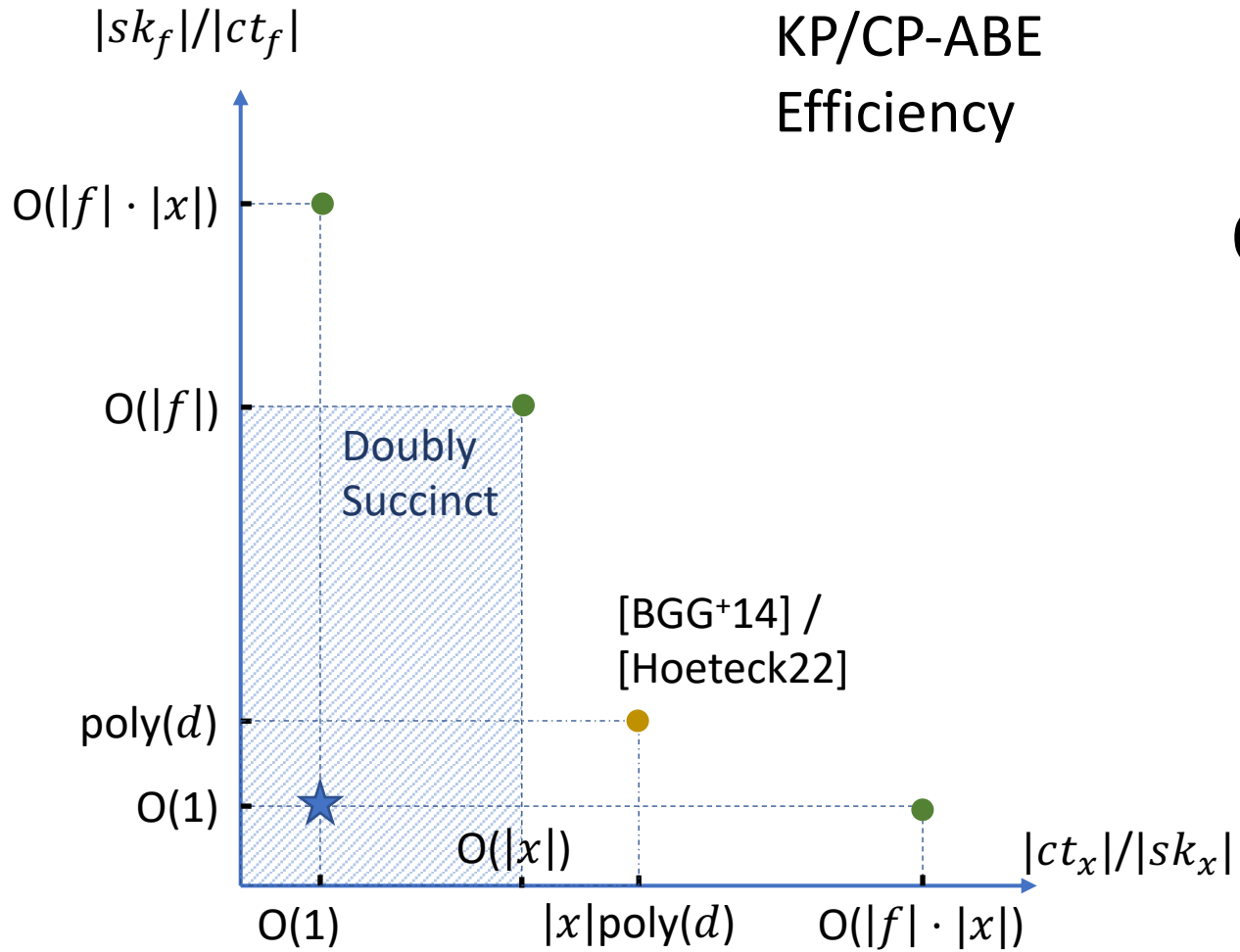
ABE Constructions



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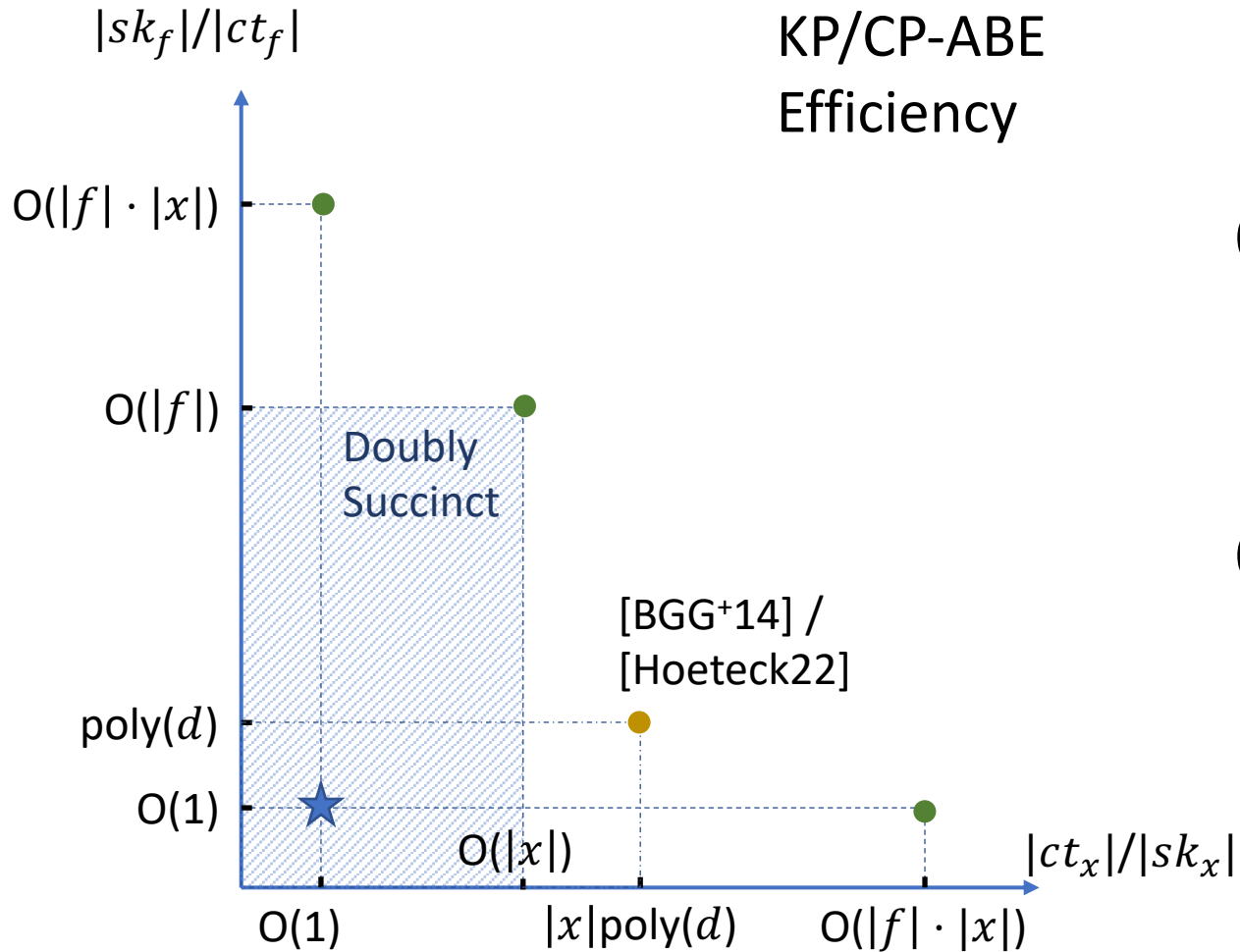


ABE Constructions



Q1: Closer to dream efficiency?

ABE Constructions



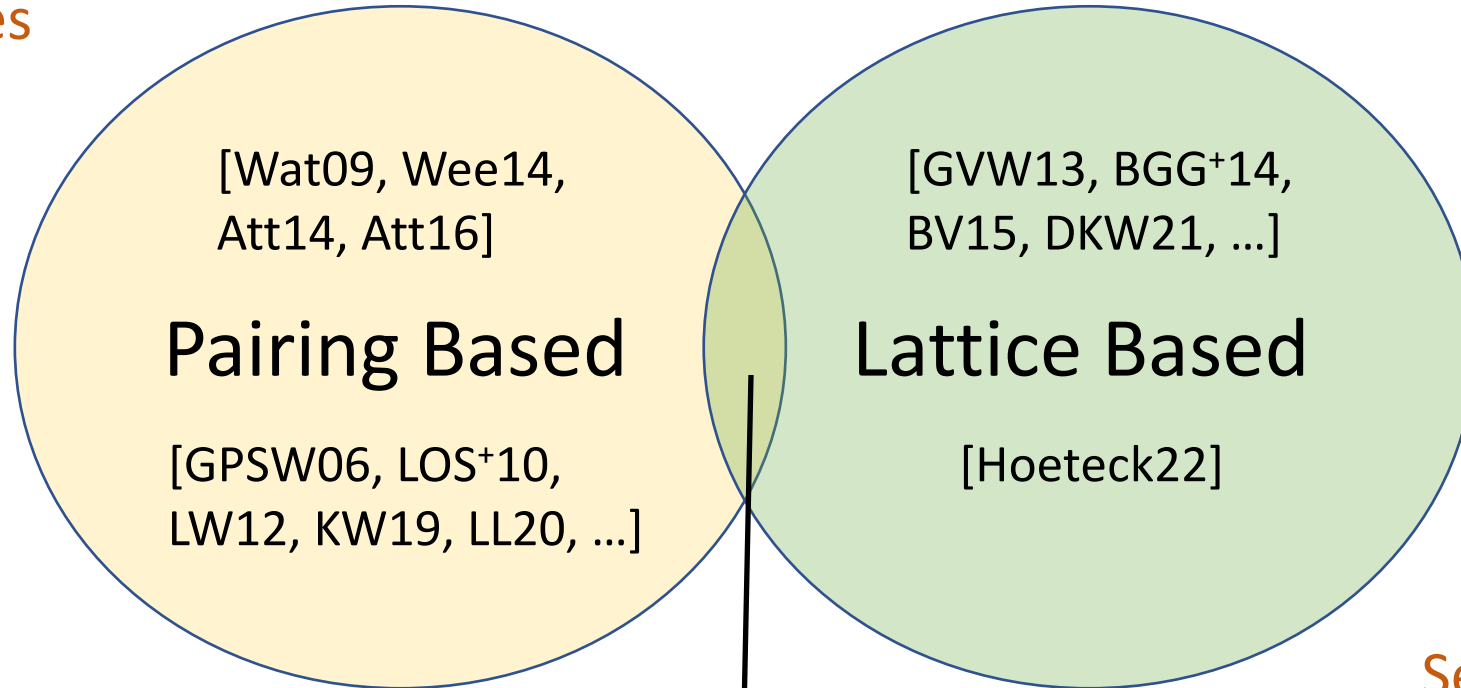
Q1: Closer to dream efficiency?

Q2: Can pairing + lattice help?

Recent Development

Low-depth policies

$$\begin{array}{l} |sk| \cdot |ct| \\ \not\approx |f| \cdot |x| \\ < \end{array}$$

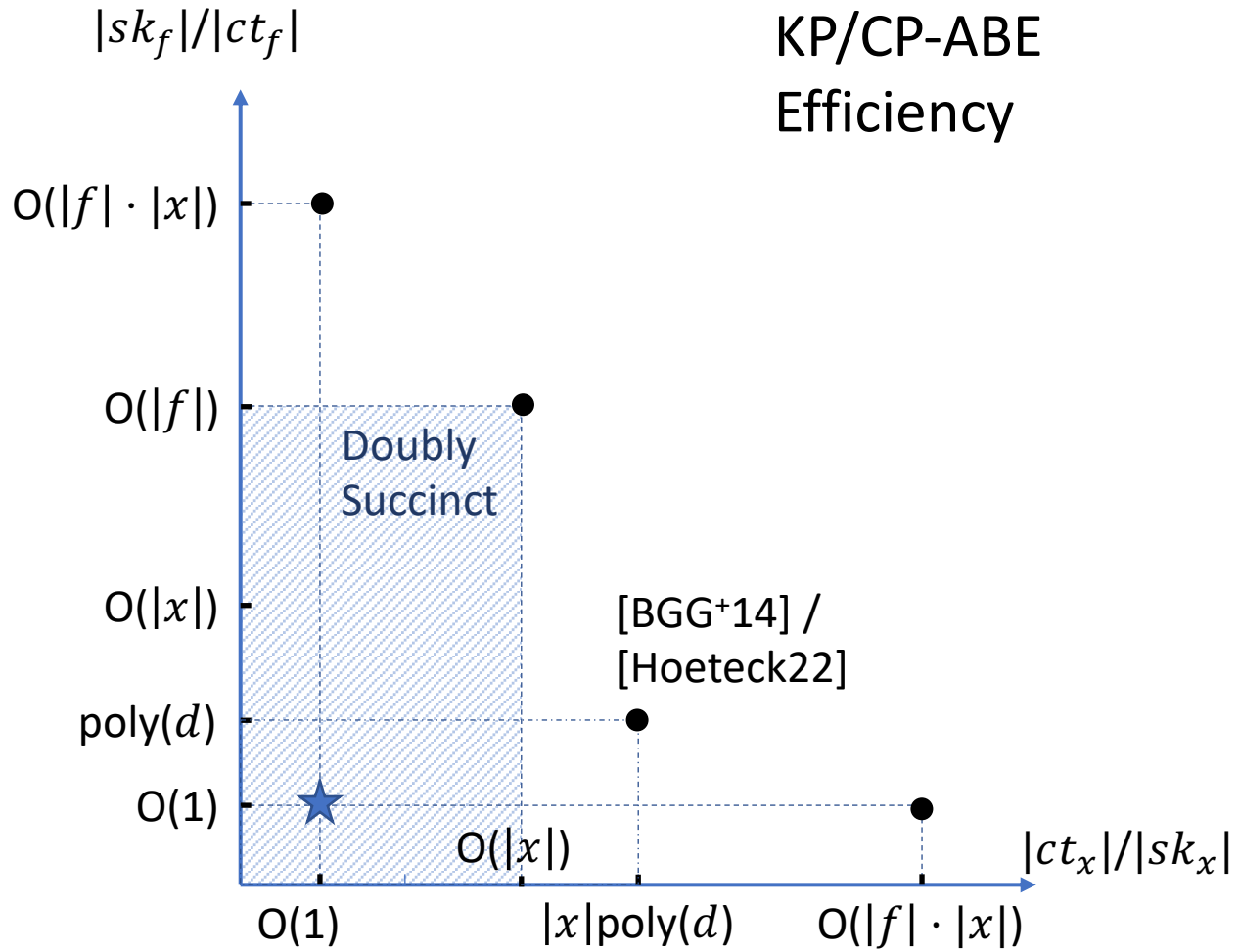


Selective Security
(fascinating barrier)

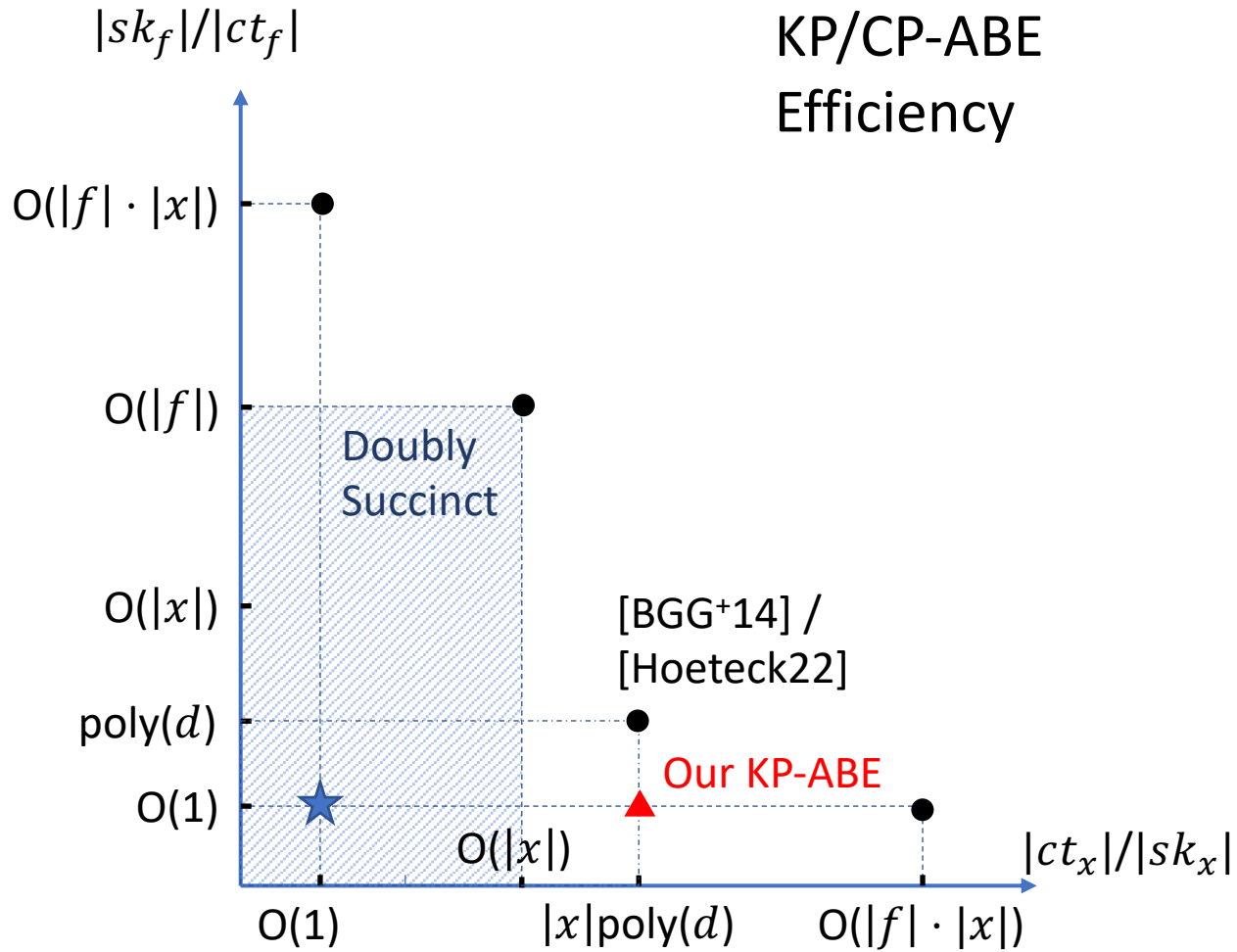
Succinct
 sk_f

- [AY, AWY20]: CP-ABE for NC1
- Lattice + pairing
 - $|sk_f| = O(|x|), |ct_x| = O(|x|)$
 - Selective security

Our Results



Our Results

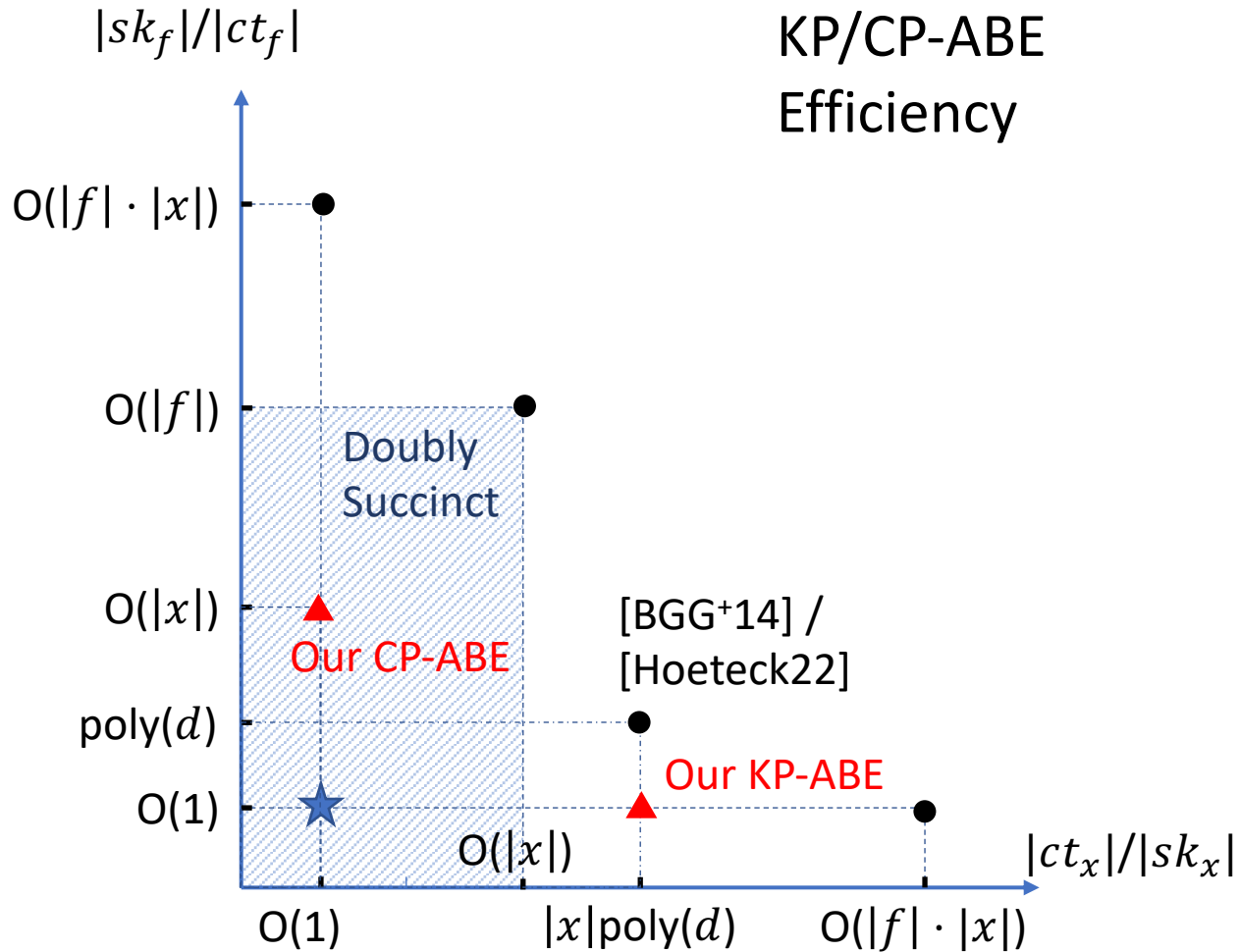


Thm1: Assuming LWE + GGM

\exists KP-ABE for P:

- $|sk_f| = O(1), |ct_x| = |x| \text{poly}(d),$
- Selective secure.

Our Results



Thm1: Assuming LWE + GGM

\exists KP-ABE for P:

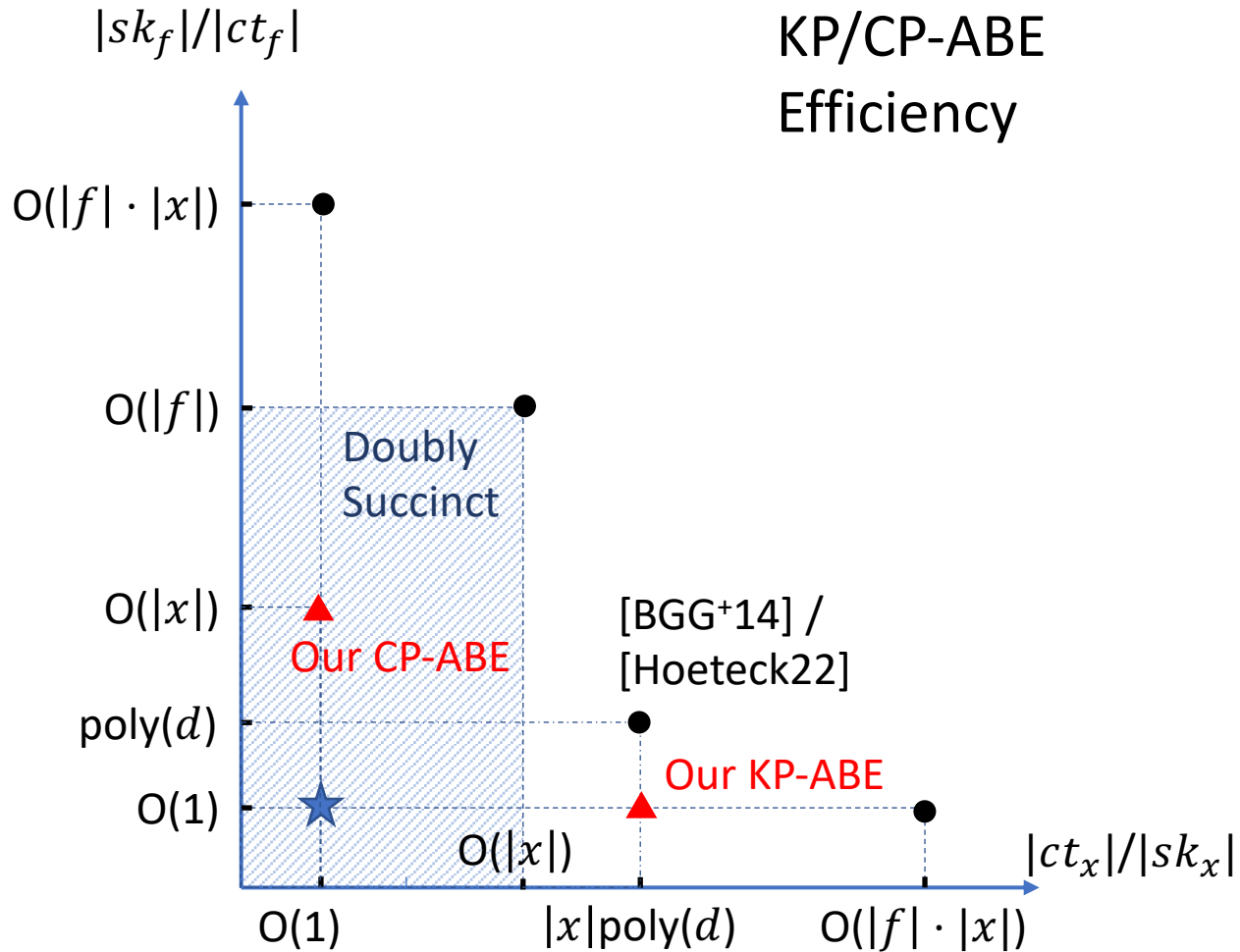
- $|sk_f| = O(1)$, $|ct_x| = |x|\text{poly}(d)$,
- Selective secure.

Thm2: Assuming LWE + GGM

\exists CP-ABE for NC1:

- $|sk_x| = O(1)$, $|ct_f| = |x|^2$,
- Selective secure.

Our Results



Adaptive LWE[QWW18]

Thm1: Assuming ~~LWE~~ + GGM

\exists KP-ABE for P:

- $|sk_f| = O(1)$, $|ct_x| = |x|poly(d)$,
- Adaptive secure.

Thm2: Assuming ~~LWE~~ + GGM

\exists CP-ABE for NC1:

- $|sk_x| = O(1)$, $|ct_f| = |x|^2$,
- Adaptive secure.

Technique In A Nutshell

Lattice based
Secure Function
Evaluation (SFE)
[QWW18]
($<$ 1-key ABE)

+ Pairing

—————→ Multi-key ABE

Technique In A Nutshell

KP-ABE for P [GVW13, BGG⁺14,..] uses lattice trapdoors

→ **Bottleneck: large keys, sel-security**

Lattice based

Secure Function

Evaluation (SFE)

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Multi-key ABE

Technique In A Nutshell

KP-ABE for P [GVW13, BGG⁺14,..] uses lattice trapdoors

→ **Bottleneck: large keys, sel-security**

Lattice based

Secure Function

Evaluation (SFE)

[QWW18]

(< 1-key ABE)

+ Pairing

Multi-key ABE



- Avoids trapdoors
- Adp-security from Adp-LWE

Technique In A Nutshell



Difficulty:

Lattice world:

Generates large noises
(e.g. HEval, noise flooding)

Pairing world:

Only allows small noises
(in the exponent)

The SFE Protocol [QWW18]

pp: A_1, \dots, A_n, \vec{u}

$f: \{0,1\}^n \rightarrow \{0,1\}$

$S(\vec{x}, m)$

$R(f)$ Goal: learn m
iff $f(\vec{x}) = 0$



The SFE Protocol [QWW18]

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[BGG+14]: $\text{EvalPP}(f, \{A_i\}_i) \rightarrow A_f$

The SFE Protocol [QWW18]

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A_f

$R(f)$

Goal: learn m
iff $f(\vec{x}) = 0$

$$ct_x \approx \{\vec{s}^\top (A_i + x_i G)\}_i$$

\vec{s} : LWE secret,

G : gadget matrix,

G^{-1} : public trapdoor

[BGG+14]: $\text{EvalPP}(f, \{A_i\}_i) \rightarrow A_f$

The SFE Protocol [QWW18]

pp: A_1, \dots, A_n, \vec{u}

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[BGG+14]: $\text{EvalPP}(f, \{A_i\}_i) \rightarrow A_f$

$\text{EvalCT}(f, \vec{x}, \{\vec{s}^\top (A_i + x_i G)\}_i) \rightarrow \vec{s}^\top (A_f + f(\vec{x})G)$

The SFE Protocol [QWW18]

pp: A_1, \dots, A_n, \vec{u}

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$\text{EvalCT}(f, \vec{x}, \{\vec{s}^\top (A_i + x_i G)\}_i) \rightarrow \vec{s}^\top (A_f + \cancel{f(\vec{x})} G)$

$f(\vec{x}) = 0$

The SFE Protocol [QWW18]

pp: A_1, \dots, A_n, \vec{u}

$S(\vec{x}, m)$

A_f

$R(f)$

Goal: learn m
iff $f(\vec{x}) = 0$

$$ct_x \approx \{\vec{s}^\top (A_i + x_i G)\}_i$$

$$\vec{s}^\top A_f G^{-1}(\vec{u})$$

[BGG+14]: $\text{EvalPP}(f, \{A_i\}_i) \rightarrow A_f$

$\text{EvalCT}(f, \vec{x}, \{\vec{s}^\top (A_i + x_i G)\}_i) \rightarrow \vec{s}^\top A_f$

$\times G^{-1}(\vec{u})$

The SFE Protocol [QWW18]

$$\text{pp: } A_1, \dots, A_n, \vec{u}$$

$$S(\vec{x}, m) \xleftarrow{A_f} R(f) \quad \text{Goal: learn } m \\ \text{iff } f(\vec{x}) = 0$$

$$ct_x \approx \{\vec{s}^\top (A_i + x_i G)\}_i \\ ct_m \approx \boxed{\vec{s}^\top A_f G^{-1}(\vec{u})} + m[q/2]$$

$$\boxed{\vec{s}^\top A_f G^{-1}(\vec{u})}$$

$$[\text{BGG+14}]: \text{EvalPP}(f, \{A_i\}_i) \rightarrow A_f$$

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The SFE Protocol [QWW18]

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$$ct_x \approx \{\vec{s}^\top (A_i + x_i G)\}_i$$

$$ct_m \approx \boxed{\vec{s}^\top A_f G^{-1}(\vec{u})} + m[q/2] \quad ct_m - \boxed{\vec{s}^\top A_f G^{-1}(\vec{u})} \approx m[q/2]$$

[BGG+14]: $\text{EvalPP}(f, \{A_i\}_i) \rightarrow A_f$

$\text{EvalCT}(f, \vec{x}, \{\vec{s}^\top (A_i + x_i G)\}_i) \rightarrow \vec{s}^\top A_f$

SFE to KP-ABE

$$\text{ABE.mpk} = \text{pp}: A_1, \dots, A_n, \vec{u}$$

$$S(\vec{x}, m) \xleftarrow{A_f} R(f) \quad \text{Goal: learn } m \text{ iff } f(\vec{x}) = 0$$

$$\text{ABE.ct}_x(m) = \begin{cases} ct_x \approx \{\vec{s}^\top (A_i + x_i G)\}_i \\ ct_m \approx \vec{s}^\top A_f G^{-1}(\vec{u}) + m[q/2] \end{cases}$$

$$ct_m - \boxed{\vec{s}^\top A_f G^{-1}(\vec{u})} \approx m[q/2]$$

[BGG+14]: $\text{EvalPP}(f, \{A_i\}_i) \rightarrow A_f,$

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SFE to KP-ABE

$$\text{ABE.mpk} = \text{pp}: A_1, \dots, A_n, \vec{u}$$

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$$ct_m - \boxed{\vec{s}^\top A_f G^{-1}(\vec{u})} \approx m[q/2]$$

- Dep. on f
- What's sk_f ?

[BGG+14]: $\text{EvalPP}(f, \{A_i\}_i) \rightarrow A_f$,

$\text{EvalCT}(f, \vec{x}, \{\vec{s}^\top (A_i + x_i G)\}_i) \rightarrow \vec{s}^\top A_f$

SFE to KP-ABE w/ IPFE

$$\text{ABE.mpk} = A_1, \dots, A_n, \vec{u}$$

$$\text{ABE.ct}_x(m): \left\{ ct_x \approx \{\vec{s}^\top (A_i + x_i G)\}_i \right.$$

$\text{ABE.sk}_f:$

Pairing based IPFE

$$\left. \begin{array}{l} \text{IPFE.ct}(\llbracket \vec{u} \rrbracket_1) \\ \text{IPFE.sk}(\llbracket \vec{v} \rrbracket_2) \end{array} \right\} \text{decrypt to } \llbracket \langle \vec{u}, \vec{v} \rangle \rrbracket_T$$

SFE to KP-ABE w/ IPFE

$$\text{ABE.mpk} = A_1, \dots, A_n, \vec{u}$$

$$\text{ABE.ct}_x(m): \begin{cases} ct_x \approx \{\vec{s}^\top (A_i + x_i G)\}_i \end{cases}$$

ABE.sk_f :

Pairing based IPFE

$$\left. \begin{array}{l} \text{IPFE.ct}(\llbracket \vec{u} \rrbracket_1) \\ \text{IPFE.sk}(\llbracket \vec{v} \rrbracket_2) \end{array} \right\} \text{decrypt to } \llbracket \langle \vec{u}, \vec{v} \rangle \rrbracket_T$$

\vec{u} is hidden. Only $\llbracket \vec{v} \rrbracket_2$ and $\llbracket \langle \vec{u}, \vec{v} \rangle \rrbracket_T$ are revealed.

SFE to KP-ABE w/ IPFE

$$\text{ABE.mpk} = A_1, \dots, A_n, \vec{u}$$

$$\begin{array}{l} \text{ABE.ct}_x(m): \\ \text{ABE.sk}_f: \end{array} \left\{ \begin{array}{l} ct_x \approx \{\vec{s}^\top (A_i + x_i G)\}_i \\ \end{array} \right\} \underbrace{\left[\vec{s}^\top A_f G^{-1}(\vec{u}) + m \lfloor q/2 \rfloor \right]_T}_{\llbracket ct_m \rrbracket_T}$$

Pairing based IPFE

$$\left. \begin{array}{l} \text{IPFE.ct}(\llbracket \vec{u} \rrbracket_1) \\ \text{IPFE.sk}(\llbracket \vec{v} \rrbracket_2) \end{array} \right\} \text{decrypt to } \llbracket \langle \vec{u}, \vec{v} \rangle \rrbracket_T$$

\vec{u} is hidden. Only $\llbracket \vec{v} \rrbracket_2$ and $\llbracket \langle \vec{u}, \vec{v} \rangle \rrbracket_T$ are revealed.

SFE to KP-ABE w/ IPFE

$$\text{ABE.mpk} = A_1, \dots, A_n, \vec{u}$$

$$\begin{aligned} \text{ABE.ct}_x(m) &: \left\{ \begin{array}{l} ct_x \approx \{\vec{s}^\top (A_i + x_i G)\}_i \\ \text{IPFE.ct}(\llbracket \vec{s}, m \rrbracket_1) \end{array} \right\} \\ \text{ABE.sk}_f &: \text{IPFE.sk}(\llbracket A_f G^{-1}(\vec{u}), [q/2] \rrbracket_2) \end{aligned} \quad \left. \vphantom{\begin{array}{l} \text{ABE.ct}_x(m) \\ \text{ABE.sk}_f \end{array}} \right\} \underbrace{\llbracket \vec{s}^\top A_f G^{-1}(\vec{u}) + m[q/2] \rrbracket_T}_{\llbracket ct_m \rrbracket_T}$$

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$$\left. \begin{array}{l} \text{IPFE.ct}(\llbracket \vec{u} \rrbracket_1) \\ \text{IPFE.sk}(\llbracket \vec{v} \rrbracket_2) \end{array} \right\} \text{decrypt to } \llbracket \langle \vec{u}, \vec{v} \rangle \rrbracket_T$$

\vec{u} is hidden. Only $\llbracket \vec{v} \rrbracket_2$ and $\llbracket \langle \vec{u}, \vec{v} \rangle \rrbracket_T$ are revealed.

KP-ABE w/ IPFE: Correctness

$$\text{ABE.mpk} = A_1, \dots, A_n, \vec{u}$$

$$\left. \begin{array}{l} \text{ABE.ct}_x(m): \begin{cases} ct_x \approx \{\vec{s}^\top (A_i + x_i G)\}_i \\ \text{IPFE.ct}(\llbracket \vec{s}, m \rrbracket_1) \end{cases} \\ \text{ABE.sk}_f: \text{IPFE.sk}(\llbracket A_f G^{-1}(\vec{u}), [q/2] \rrbracket_2) \end{array} \right\} \llbracket ct_m \rrbracket_T$$

ABE.Dec:

KP-ABE w/ IPFE: Correctness

$$\text{ABE.mpk} = A_1, \dots, A_n, \vec{u}$$

$$\text{ABE.ct}_x(m): \left\{ \begin{array}{l} ct_x \approx \{\vec{s}^\top (A_i + x_i G)\}_i \\ \text{IPFE.ct}(\llbracket \vec{s}, m \rrbracket_1) \end{array} \right\} \quad \left. \vphantom{\text{ABE.ct}_x(m)} \right\} \llbracket ct_m \rrbracket_T$$
$$\text{ABE.sk}_f: \text{IPFE.sk} \left(\llbracket A_f G^{-1}(\vec{u}), [q/2] \rrbracket_2 \right)$$

ABE.Dec:

$$\boxed{\vec{s}^\top A_f G^{-1}(\vec{u})}$$

KP-ABE w/ IPFE: Correctness

$$\text{ABE.mpk} = A_1, \dots, A_n, \vec{u}$$

$$\text{ABE.ct}_x(m): \left\{ \begin{array}{l} ct_x \approx \{\vec{s}^\top (A_i + x_i G)\}_i \\ \text{IPFE.ct}(\llbracket \vec{s}, m \rrbracket_1) \end{array} \right\} \text{ABE.sk}_f: \text{IPFE.sk}(\llbracket A_f G^{-1}(\vec{u}), [q/2] \rrbracket_2) \quad \left. \vphantom{\text{ABE.ct}_x(m)} \right\} \llbracket ct_m \rrbracket_T$$

ABE.Dec:

$$\llbracket ct_m \rrbracket_T - \boxed{\vec{s}^\top A_f G^{-1}(\vec{u})} \approx \llbracket m[q/2] \rrbracket_T$$

KP-ABE w/ IPFE: Correctness Problems

$$\text{ABE.mpk} = A_1, \dots, A_n, \vec{u}$$

$$\text{ABE.ct}_x(m): \left\{ \begin{array}{l} ct_x = \{\vec{s}^\top (A_i + x_i G)\}_i + e_i \\ \text{IPFE.ct}(\llbracket \vec{s}, m \rrbracket_1) \end{array} \right\} \text{ABE.sk}_f: \text{IPFE.sk}(\llbracket A_f G^{-1}(\vec{u}), [q/2] \rrbracket_2) \quad \left. \vphantom{\left\{ \right.} \right\} \llbracket ct_m \rrbracket_T$$

ABE.Dec:

KP-ABE w/ IPFE: Correctness Problems

$$\text{ABE.mpk} = A_1, \dots, A_n, \vec{u}$$

$$\text{ABE.ct}_x(m): \left\{ \begin{array}{l} ct_x = \{\vec{s}^\top (A_i + x_i G)\}_i + e_i \\ \text{IPFE.ct}(\llbracket \vec{s}, m \rrbracket_1) \end{array} \right. \left. \vphantom{\text{ABE.ct}_x(m)} \right\} \llbracket ct_m \rrbracket_T$$
$$\text{ABE.sk}_f: \text{IPFE.sk} \left(\llbracket A_f G^{-1}(\vec{u}), [q/2] \rrbracket_2 \right)$$

ABE.Dec:

$$\vec{s}^\top A_f G^{-1}(\vec{u}) + e_f$$

KP-ABE w/ IPFE: Correctness Problems

$$\text{ABE.mpk} = A_1, \dots, A_n, \vec{u}$$

$$\text{ABE.ct}_x(m): \left\{ \begin{array}{l} ct_x = \{\vec{s}^\top (A_i + x_i G)\}_i + e_i \\ \text{IPFE.ct}(\llbracket \vec{s}, m \rrbracket_1) \end{array} \right\} \text{ABE.sk}_f: \text{IPFE.sk}(\llbracket A_f G^{-1}(\vec{u}), [q/2] \rrbracket_2) \quad \left. \vphantom{\left\{ \right.} \right\} \llbracket ct_m \rrbracket_T$$

ABE.Dec:

$$\llbracket ct_m \rrbracket_T - \boxed{\vec{s}^\top A_f G^{-1}(\vec{u}) + e_f} = \llbracket m[q/2] + e_f \rrbracket_T$$

KP-ABE w/ IPFE: Correctness Problems

$$\text{ABE.mpk} = A_1, \dots, A_n, \vec{u}$$

$$\text{ABE.ct}_x(m): \left\{ \begin{array}{l} ct_x = \{\vec{s}^\top (A_i + x_i G)\}_i + e_i \\ \text{IPFE.ct}(\llbracket \vec{s}, m \rrbracket_1) \end{array} \right\} \text{ABE.sk}_f: \text{IPFE.sk}(\llbracket A_f G^{-1}(\vec{u}), [q/2] \rrbracket_2) \quad \left. \vphantom{\text{ABE.ct}_x(m)} \right\} \llbracket ct_m \rrbracket_T$$

ABE.Dec:

$$\llbracket ct_m \rrbracket_T - \boxed{\vec{s}^\top A_f G^{-1}(\vec{u}) + e_f} = \llbracket m[q/2] + e_f \rrbracket_T$$

Problem 1:

Huge noise! $2^{\text{poly}(d)}$

KP-ABE w/ IPFE: Correctness Problems

$$\text{ABE.mpk} = A_1, \dots, A_n, \vec{u}$$

Problem2:

$\text{poly}(q) = \text{poly}(d)$ size!

$$\text{ABE.ct}_x(m): \begin{cases} ct_x = \{\vec{s}^\top (A_i + x_i G)\}_i + e_i \\ \text{IPFE.ct}(\llbracket \vec{s}, m \rrbracket_1) \end{cases}$$

$\text{ABE.sk}_f: \text{IPFE.sk}(\llbracket A_f G^{-1}(\vec{u}), [q/2] \rrbracket_2)$

$\left. \begin{array}{l} \text{ABE.ct}_x(m) \\ \text{ABE.sk}_f \end{array} \right\} \llbracket ct_m \rrbracket_T$

ABE.Dec:

$$\llbracket ct_m \rrbracket_T - \boxed{\vec{s}^\top A_f G^{-1}(\vec{u}) + e_f} = \llbracket m[q/2] + e_f \rrbracket_T$$

Problem1:

Huge noise! $2^{\text{poly}(d)}$

KP-ABE w/ IPFE: Correctness by rounding

$$\text{ABE.mpk} = A_1, \dots, A_n, \vec{u}$$

$$\text{ABE.ct}_x(m): \left\{ \begin{array}{l} ct_x = \{\vec{s}^\top (A_i + x_i G)\}_i + e_i \\ \text{IPFE.ct}(\llbracket \vec{s}, m \rrbracket_1) \end{array} \right\} \text{ABE.sk}_f \quad \left. \vphantom{\left\{ \begin{array}{l} ct_x = \{\vec{s}^\top (A_i + x_i G)\}_i + e_i \\ \text{IPFE.ct}(\llbracket \vec{s}, m \rrbracket_1) \end{array} \right\}} \right\} \llbracket ct_m \rrbracket_T$$

ABE.Dec:

KP-ABE w/ IPFE: Correctness by rounding

$$\text{ABE.mpk} = A_1, \dots, A_n, \vec{u}$$

$$\text{ABE.ct}_x(m): \left\{ \begin{array}{l} ct_x = \{\vec{s}^\top (A_i + x_i G)\}_i + e_i \\ \text{IPFE.ct}(\llbracket \vec{s}, m \rrbracket_1) \end{array} \right\} \text{ABE.sk}_f: \left. \vphantom{\text{ABE.ct}_x(m)} \right\} \llbracket ct_m \rrbracket_T$$

ABE.Dec:

$$\boxed{\llbracket \vec{s}^\top A_f G^{-1}(\vec{u}) + e_f \rrbracket_p}$$

KP-ABE w/ IPFE: Correctness by rounding

$$\text{ABE.mpk} = A_1, \dots, A_n, \vec{u}$$

$$\text{ABE.ct}_x(m): \left\{ \begin{array}{l} ct_x = \{\vec{s}^\top (A_i + x_i G)\}_i + e_i \\ \text{IPFE.ct}(\llbracket \vec{s}, m \rrbracket_1) \end{array} \right\} \text{ABE.sk}_f: \left. \vphantom{\text{ABE.ct}_x(m)} \right\} \llbracket ct_m \rrbracket_T$$

ABE.Dec:

$$\begin{aligned} & \boxed{\llbracket \vec{s}^\top A_f G^{-1}(\vec{u}) + e_f \rrbracket_p} \\ & = \vec{s}^\top \llbracket A_f G^{-1}(\vec{u}) \rrbracket_p + \text{error} \end{aligned}$$

KP-ABE w/ IPFE: Correctness by rounding

$$\text{ABE.mpk} = A_1, \dots, A_n, \vec{u}$$

$$\text{ABE.ct}_x(m): \left\{ \begin{array}{l} ct_x = \{\vec{s}^\top (A_i + x_i G)\}_i + e_i \\ \text{IPFE.ct}(\llbracket \vec{s}, m \rrbracket_1) \end{array} \right\} \text{ABE.sk}_f: \text{IPFE.sk} \left(\llbracket [A_f G^{-1}(\vec{u})]_p, [p/2] \rrbracket_2 \right) \left. \vphantom{\text{ABE.ct}_x(m)} \right\} \llbracket ct_m \rrbracket_T$$

ABE.Dec:

$$\begin{aligned} \llbracket ct'_m \rrbracket_T - \boxed{\llbracket \vec{s}^\top A_f G^{-1}(\vec{u}) + e_f \rrbracket_p} &= \llbracket m[q/2] - \text{error} \rrbracket_T \\ &= \vec{s}^\top \llbracket A_f G^{-1}(\vec{u}) \rrbracket_p + \text{error} \end{aligned}$$


KP-ABE w/ IPFE: Correctness by rounding

$$\text{ABE.mpk} = A_1, \dots, A_n, \vec{u}$$

$$\text{ABE.ct}_x(m): \left\{ \begin{array}{l} ct_x = \{\vec{s}^\top (A_i + x_i G)\}_i + e_i \\ \text{IPFE.ct}(\llbracket \vec{s}, m \rrbracket_1) \end{array} \right\} \text{ABE.sk}_f: \text{IPFE.sk} \left(\llbracket [A_f G^{-1}(\vec{u})]_p, [p/2] \rrbracket_2 \right) \left. \vphantom{\text{ABE.ct}_x(m)} \right\} \llbracket ct_m \rrbracket_T$$

ABE.Dec:

$$\begin{aligned} \llbracket ct'_m \rrbracket_T - \boxed{\llbracket \vec{s}^\top A_f G^{-1}(\vec{u}) + e_f \rrbracket_p} &= \llbracket m[q/2] - \text{error} \rrbracket_T \\ &= \vec{s}^\top \llbracket A_f G^{-1}(\vec{u}) \rrbracket_p + \text{error} \end{aligned}$$

Small 

KP-ABE w/ IPFE: Correctness by rounding

$$\text{ABE.mpk} = A_1, \dots, A_n, \vec{u}$$

$$\text{ABE.ct}_x(m): \begin{cases} ct_x = \{\vec{s}^\top (A_i + x_i G)\}_i + e_i \\ \text{IPFE.ct}(\llbracket \vec{s}, m \rrbracket_1) \end{cases} \quad \text{poly}(p) = \text{poly}(\lambda) \text{ size!}$$
$$\text{ABE.sk}_f: \text{IPFE.sk} \left(\left(\llbracket A_f G^{-1}(\vec{u}) \rrbracket_p, \llbracket p/2 \rrbracket \right) \right)_2 \quad \left. \vphantom{\text{ABE.sk}_f} \right\} \llbracket ct_m \rrbracket_T$$

ABE.Dec:

$$\llbracket ct'_m \rrbracket_T - \llbracket \vec{s}^\top A_f G^{-1}(\vec{u}) + e_f \rrbracket_p = \llbracket m \llbracket q/2 \rrbracket - \text{error} \rrbracket_T$$
$$= \vec{s}^\top \llbracket A_f G^{-1}(\vec{u}) \rrbracket_p + \text{error}$$

Small

Other Challenges

- **1-key Security**
 - SFE relies on noise flooding

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Show ct_m entropic,
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- Show ct_m entropic,
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- Multi-key Security
 - Borrow ideas from [AY,AWY20]

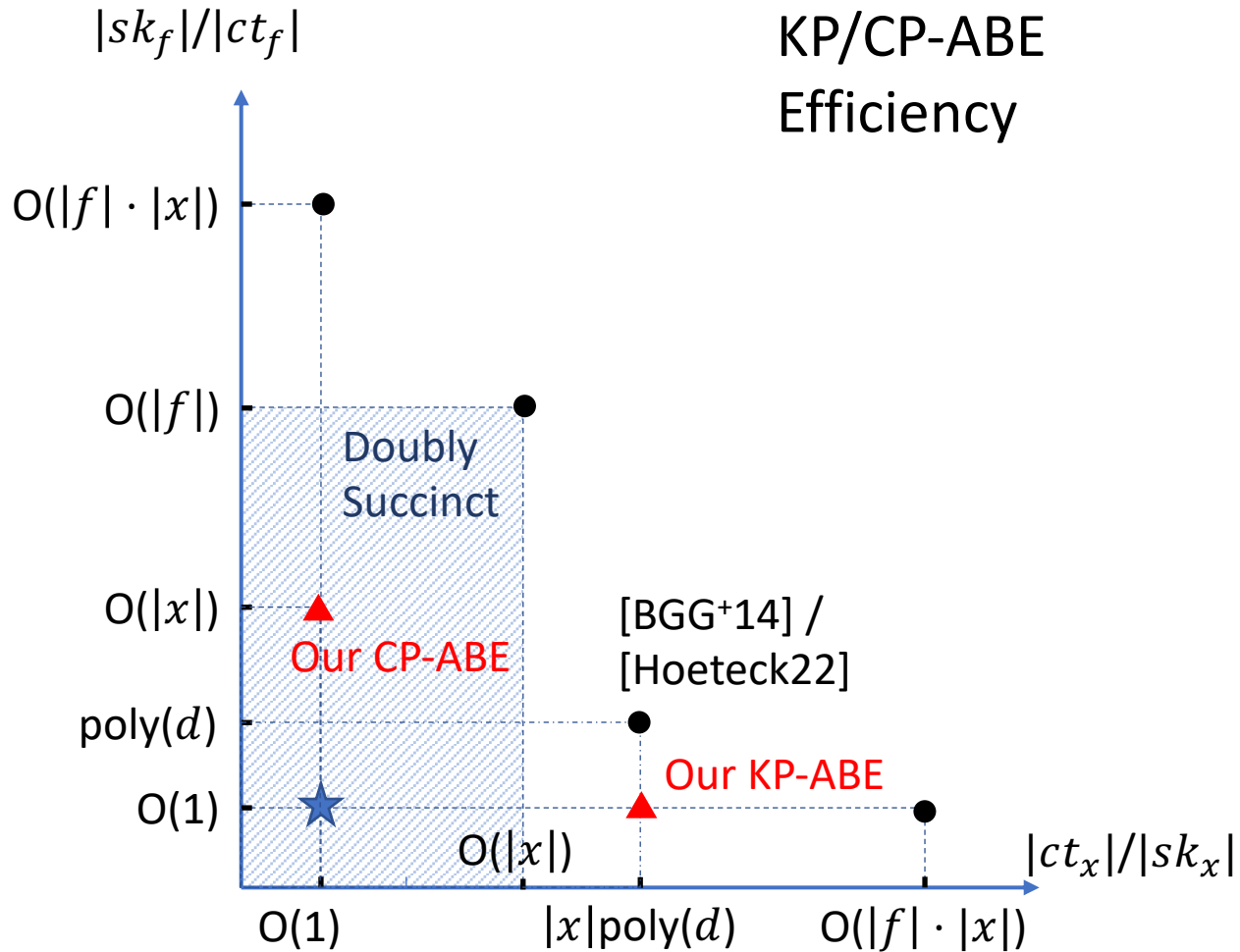
Other Challenges

- 1-key Security
 - SFE relies on noise flooding
 - Sol: Remove flooding noise Show ct_m entropic,
 \equiv random in GGM
- Multi-key Security
 - Borrow ideas from [AY,AWY20] Also relies on GGM

Thank You!

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Adaptive LWE



Thm1: Assuming ~~LWE~~ + GGM

\exists KP-ABE for P:

- $|sk_f| = O(1), |ct_x| = |x| \text{poly}(d),$
- Adaptive secure.

Thm2: Assuming ~~LWE~~ + GGM

\exists CP-ABE for NC1:

- $|sk_x| = O(1), |ct_f| = |x|^2,$
- Adaptive secure.