Complexity-Preserving Sublinear-Arguments from Symmetric Key Primitives

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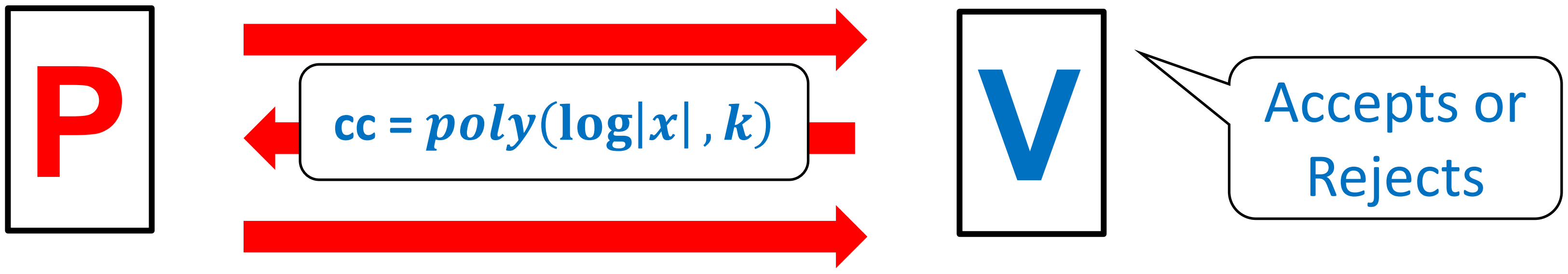
Succinct (ZK) Arguments [Kil99,Mic00]

Given a language L



Completeness: $\forall x \in L,\quad \langle P, V\rangle(x) = 1$ w.p. 1

Soundness: $\forall x \notin L, \forall$ **PPT** $P^*,\ \langle P^*, V\rangle(x) = 1$ w.p. $\leq\ negl(k)$

Zero-knowledge: $\forall x \in L, \forall$PPT $V^*,\quad \exists$ PPT S, s.t.

$$\{view_{V^*}\langle P, V^*\rangle(x)\} \approx \{S(x)\}$$

Main Question

Do there exist complexity-preserving public-coin succinct* arguments for NP from minimal assumptions?

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This Talk **YES***

* sublinear

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$\left.\begin{array}{l}\text{Time-preserving} \\ \text{Space-preserving}\end{array}\right\}$ Complexity-preserving

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Do there exist complexity-preserving public-coin succinct* arguments for NP from minimal assumptions?

This Talk **YES***

* sublinear

Taxonomy of Arguments

- Succinct* vs Non-succinct
- Interactive vs Non-interactive
- Trusted setup vs No setup (transparent)
- ZK vs (only) Integrity
- Public-Key Crypto vs (only) Symmetric-Key Crypto

Time-preserving
Space-preserving
$\Big\}$ Complexity-preserving

A SNARKy background

1992: Sublinear ZK for NP [Kil92]
2000: CSProofs [Mic00]
2007: IKO [IKO07], IVC [V07]
2008: IVC [Val08]
2010: Short-PB-NIZKA [Groth10], Preprocessing-Verifiable-Computation [GGP10]
2012: QSP [GGPR12], EfficientPCP [IMS12], Succinct-NIArgs-LIP [BCIOP12]
2013: Pinocchio [PGHR13], SNARKs-for-C [BCGTV13], ZK-vonNeumann [BCTV13]
2014: Geppetto [CFHKKNPZ14], CyclesOfCurves [BCTV14]
2015: IP4Muggles [GKR15], SNARKs-for-MapReduce [CTV15]
2016: ZKBoo [GMO16], BulletproofsPrequel [BCCGP16], Groth16 [Groth16], HybIntZK [CGM16]
2017: Ligero (AHIV17), ZKB++ and Picnic [CDGORRSZ17] (discuss alongside ZKBoo), Hyrax [WTsTW], zk-vSQL [ZGKPP17] (can add to existing vSQL section), Bulletproofs [BBBPWM17], SnarkySigs [GM17]
2018: Aurora [BCRSVW18], FRI [BBHR18], ZKStarks [BBHR18], Picnic2 [KKW18], vRAM [ZGKPP18], DIZK [WZCPS18], UpdatableNIZK [GKMM18], HybNIZK [AGM18]
2019: Fractal [COS19], Halo [BGH19], Plonk [GWC19], RedShift [KPV19], Spartan [Setty19], DeepFRI [BGKS19], LatticeZKPs [ESLL19], SubversionResistant [Bag19], Darks [BFS19], LatticeSnarkArithmetic [Nit19], ZKPSetMembership [BCFGD19]
2020: HaloInfinite [BDFG20], Quarks (Xiphos and Kopis) [SL20], Dory [Lee20],
Wolverine [WYKW20], Bulletproofs+ [CHJKS20], SPARKS [EFKP20], Plookup [GW20], SuperSonic [BFS20], CompressedSigma [AC20], LatticeZKviaOTC [LKS20], GeneralizedCompressedSigma [ACR20], PVZKfromBlockchain [SSV20], LinePointZK [DIO20], PublicCoinZKTime&Space [BHRRS20], Dory [Lee20], DoublyEfficientIP [ZLWZSXZ20], PqSnarks4Rsis-Rlwe [BCOS20], ZAPsAlgebraicLangs [CH20]
2021: Manta [CXZ21], Nova [KST21], Rinocchio [GNS21], Limbo [DGOT21], QuickSilver [YSWW21], Limbo [GOT21], IntRange [CKLR21], SubexpDDH [JJ21], Cerberus [LSTW21], ConstOverZKRamProgs [FKLOW21]
2022: NIZK Multiple Verifiers [YW22], Feta [BJOSS22], gOTzilla [BCGHM22], ZK UNSAT [LAHPTW22]
Credits: ZKProof.org

A SNARKy background

(Uniform) RAM program that takes time T and uses space S

- Succinct
- Public-coin or publicly verifiable
- Time-preserving
- Space-preserving
- “Black-box” in the underlying assumption

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[BC12] Designated Verifier Sublinear Arguments
[HR18] Non-interactive Sublinear Arguments

A SNARKy background

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[WTSTW18,XZZPS19,S20,SL20,KMP20,BCG20a,BCG20,…]
All require Prover to use space proportional to T!

A SNARKy background

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[Val08,BCCT13] Complexity preserving via recursive composition
[EFKP20] SNARKs for parallel RAM comp. using CRH and SNARKs
[BGH21,BCMS20,COS20] "Heuristic" assumptions

A SNARKy background

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[BBHR18,BFHVXZ20] *Folklore

A SNARKy background

(Uniform) RAM program that takes time T and uses space S

✓ • Succinct
✓ • Public-coin or publicly verifiable
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[BHRRS20] Based on hardness of discrete log
[BHRRS21] Based on hardness assumptions on hidden order groups

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(Uniform) RAM program that takes time T and uses space S

- ✓ Succinct
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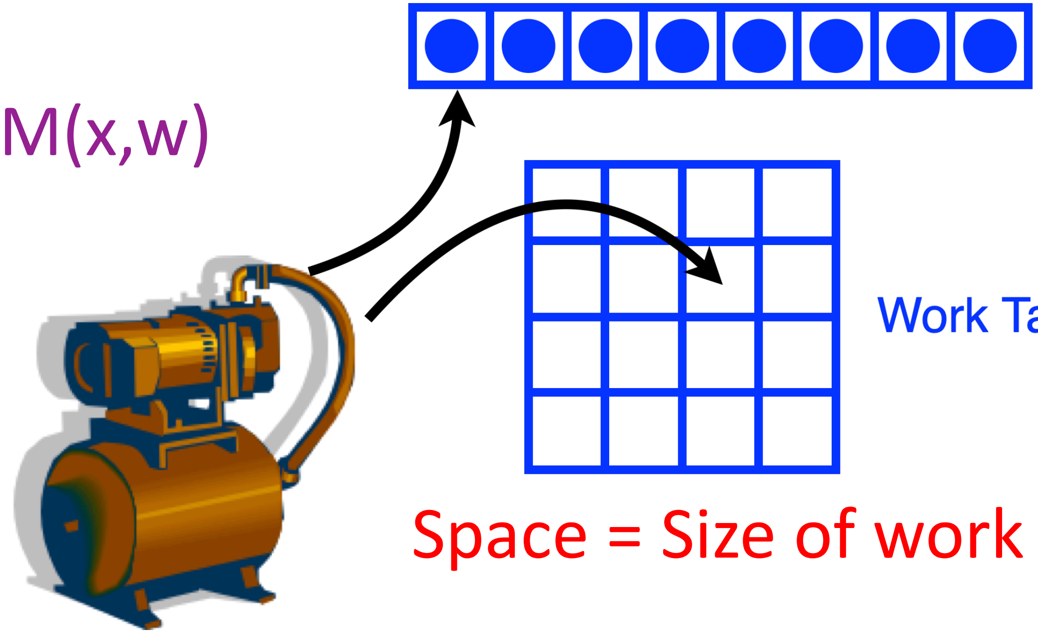
[BHRRS20] Based on hardness of discrete log
[BHRRS21] Based on hardness assumptions on hidden order groups

Main Question: Can we get based on symmetric cryptography?

RAM Model



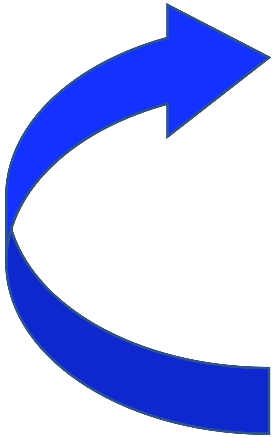
Input Tape - Linear Access

(Uniform) RAM program M(x,w)

- Runs in time T(|x|)
- Uses space S(|x|)
- It is of constant size

Work Tape - RAM Access

Space = Size of work tape



Modelled as a RAM

P has x, w on (two) input tapes with linear access.

V has x on input tape with linear access.

Main Result – Upper Bound

Theorem: Assume CRH exists. ∀ NP language verified by a uniform RAM machine running in **time** $\boldsymbol{T}$ and **space** $\boldsymbol{S}$ ∃ public-coin (ZK) arg. with soundness $\boldsymbol{negl(k)}$ such that:

	Time	Space
$\boxed{\mathbf{P}}$	$\boldsymbol{\tilde{O}(T)}$	$\boldsymbol{\tilde{O}(S)}$

where the CRH is used in a black-box way and $\boldsymbol{\tilde{O}(\quad)}$ ignores $\boldsymbol{poly(\log(T), k)}$

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$\boxed{\mathbf{P}}$	$\tilde{\boldsymbol{O}}(\boldsymbol{T})$	$\tilde{\boldsymbol{O}}(\boldsymbol{S})$

$\longrightarrow$ Complexity preserving

where the CRH is used in a black-box way and $\tilde{\boldsymbol{O}}(\quad)$ ignores $\boldsymbol{poly}(\mathbf{log}(\boldsymbol{T}), \boldsymbol{k})$

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	Time	Space
P	$\tilde{O}(T)$	$\tilde{O}(S)$
V	$\tilde{O}\left(\frac{T}{S} + S\right)$	$\tilde{O}(1)$

→ Complexity preserving

where the CRH is used in a black-box way and $\tilde{O}(\;)$ ignores $\boldsymbol{poly(\log(T), k)}$

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P	$\tilde{O}(T)$	$\tilde{O}(S)$	→ Complexity preserving
V	$\tilde{O}\left(\frac{T}{S}+S\right)$	$\tilde{O}(1)$	

Proof Length is $\tilde{O}\left(\frac{T}{S}\right)$

where the CRH is used in a black-box way and $\tilde{O}(\quad)$ ignores $\boldsymbol{poly}(\mathbf{log}(\boldsymbol{T}), \boldsymbol{k})$

Main Result – Upper Bound

Theorem: Assume CRH exists. ∀ NP language verified by a uniform RAM machine running in **time** $\boldsymbol{T}$ and **space** $\boldsymbol{S}$ ∃ public-coin (ZK) arg. with soundness $\boldsymbol{negl(k)}$ such that:

	Time	Space	
P	$\tilde{O}(T)$	$\tilde{O}(S)$	→ Complexity preserving
V	$\tilde{O}\left(\frac{T}{S}+S\right)$	$\tilde{O}(1)$	Sublinear

Proof Length is $\tilde{O}\left(\frac{T}{S}\right)$

where the CRH is used in a black-box way and $\tilde{O}(\ )$ ignores $\boldsymbol{poly(\log(T), k)}$

Main Result – Lower Bound*

Theorem: Assume CRH exists. ∀ NP language verified by a uniform RAM machine running in **time** $\boldsymbol{T}$ and **space** $\boldsymbol{S}$ ∃ public-coin (ZK) arg. with soundness $\boldsymbol{negl(k)}$ such that:

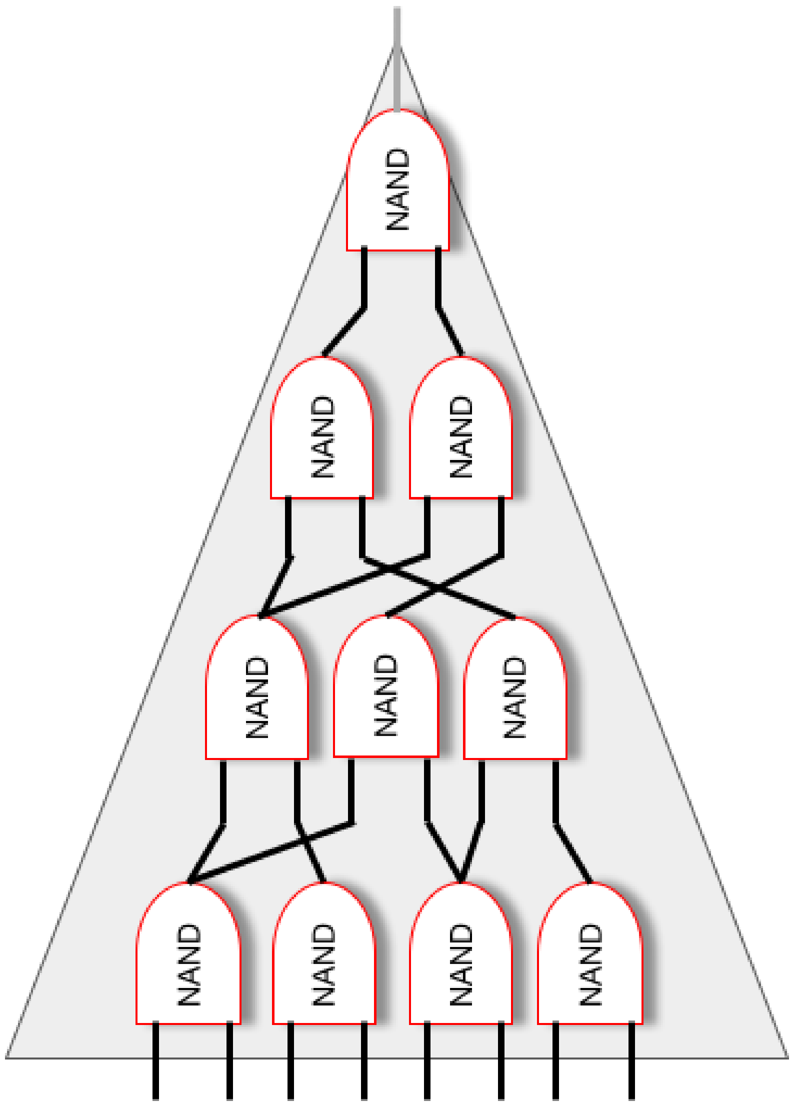
	Time	Space
P	$\tilde{O}(T)$	
V	$\tilde{O}\left(\frac{T}{S}+S\right)$	

Proof Length is $\tilde{O}\left(\frac{T}{S}\right)$

Our lower bound shows why it's hard to further improve the proof length.

where the CRH is used in a black-box way and $\tilde{O}(\quad)$ ignores $\boldsymbol{poly(\log(T), k)}$

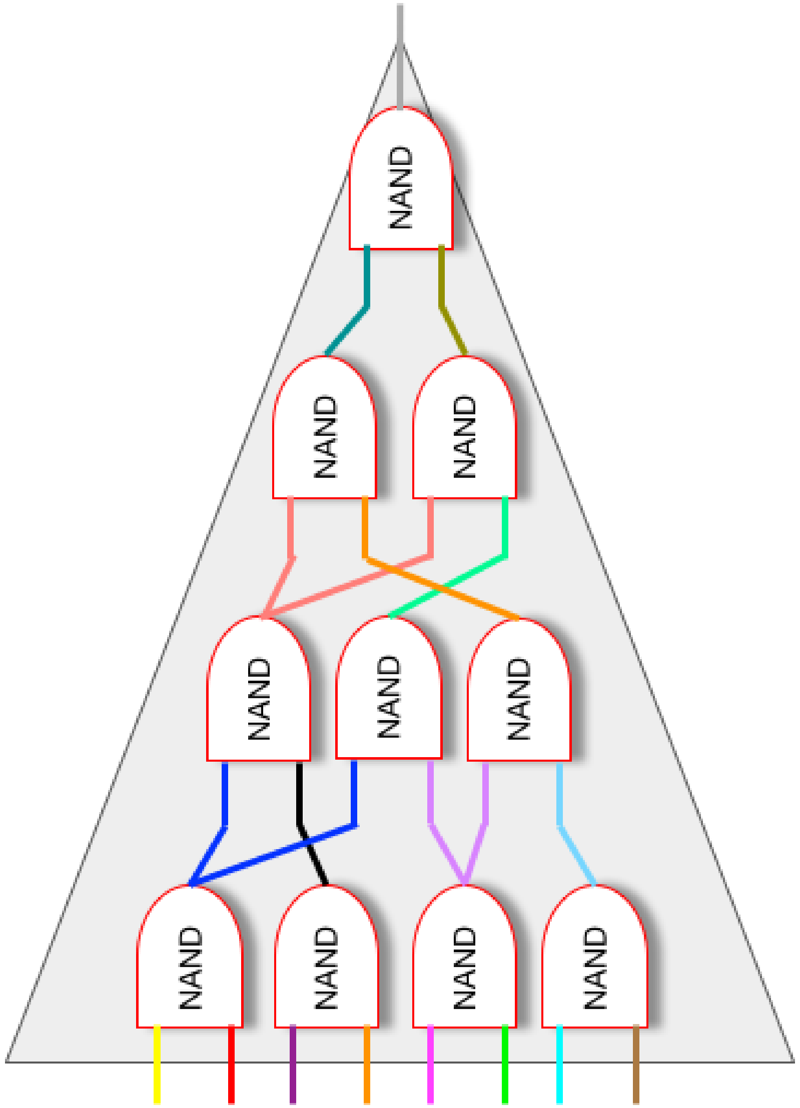
Ligero [AHIV17]
Proof Schematic



Given a circuit C

Prove that $\exists z$, such that $C(z) = 1$

Extended witness $\overline{w}$



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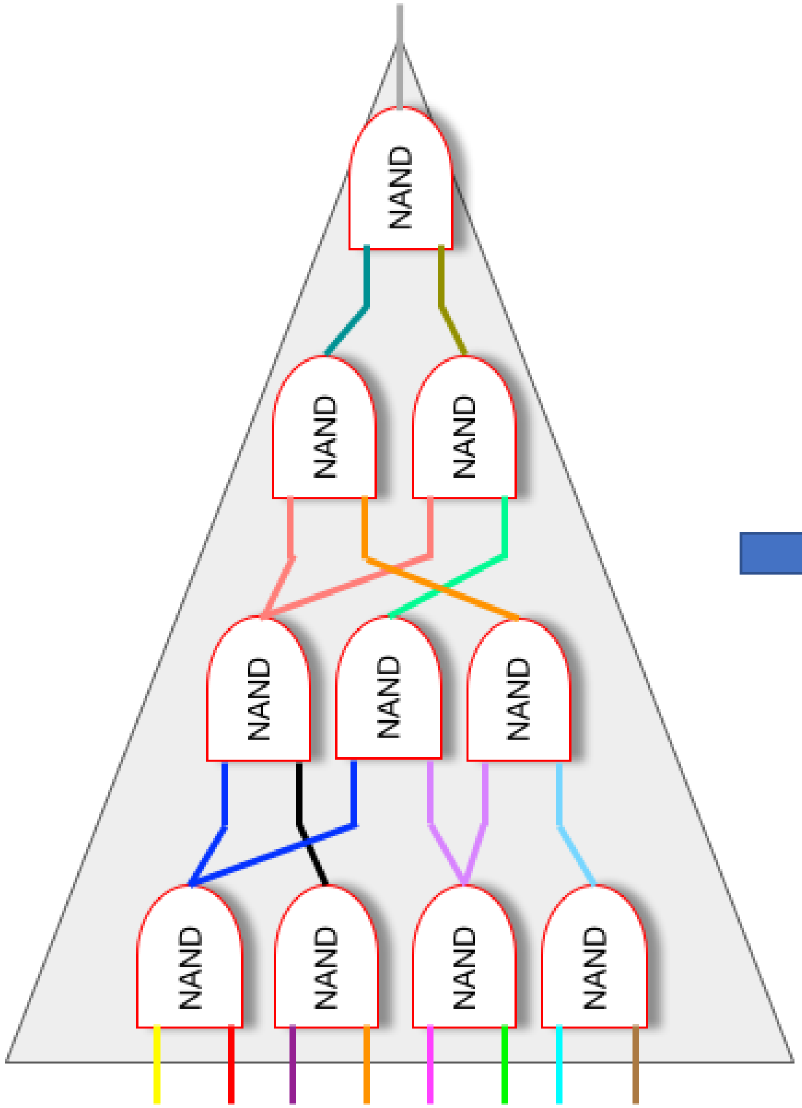
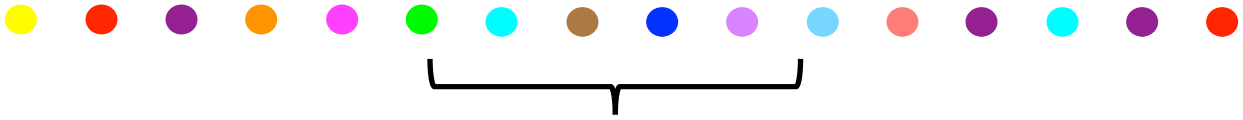
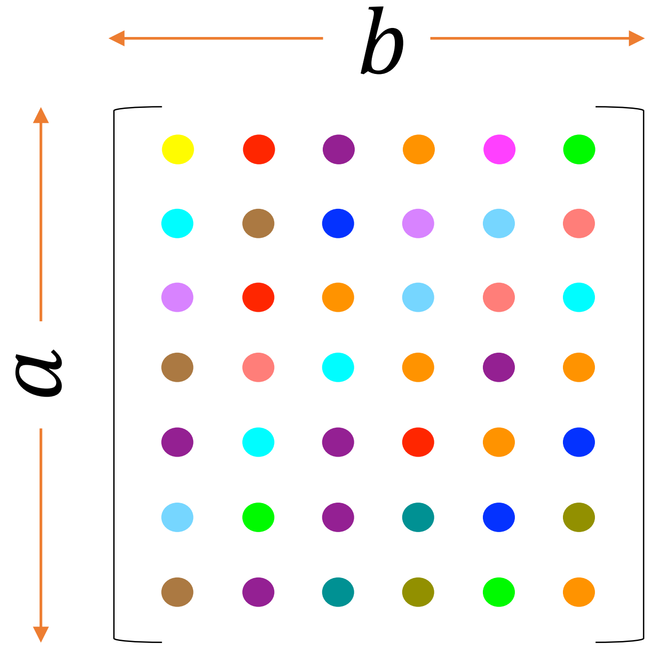


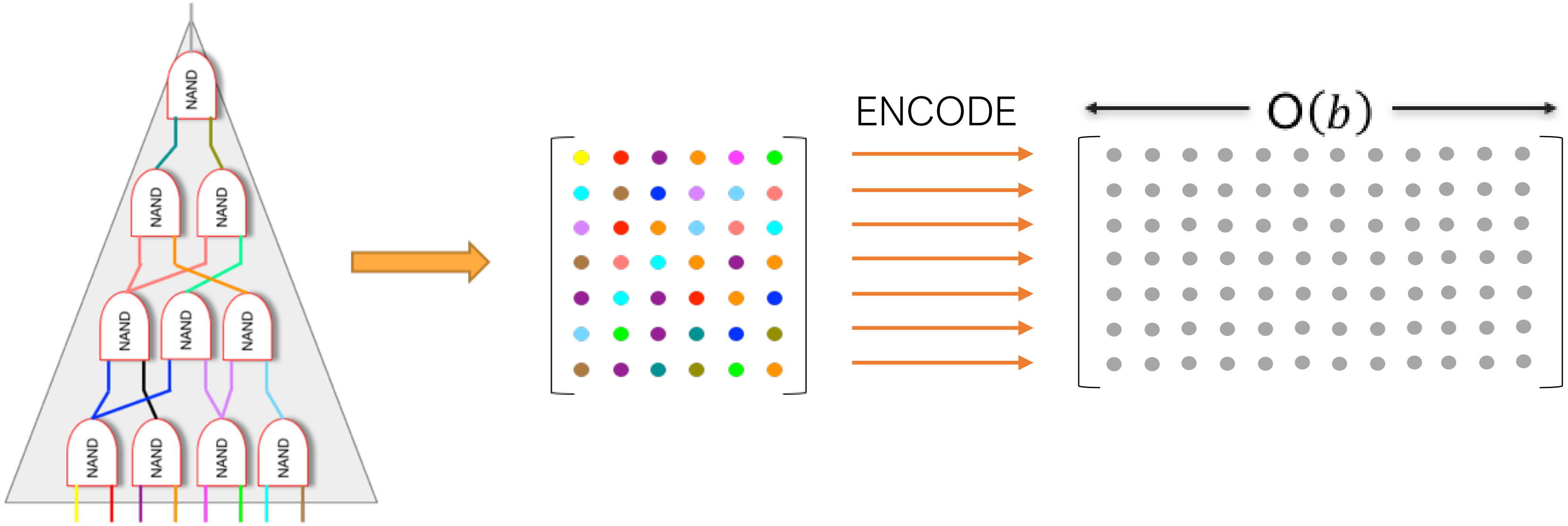


i^{th} chunk



i^{th} row $\overline{w}_i$
is i^{th} chunk of $\overline{w}$

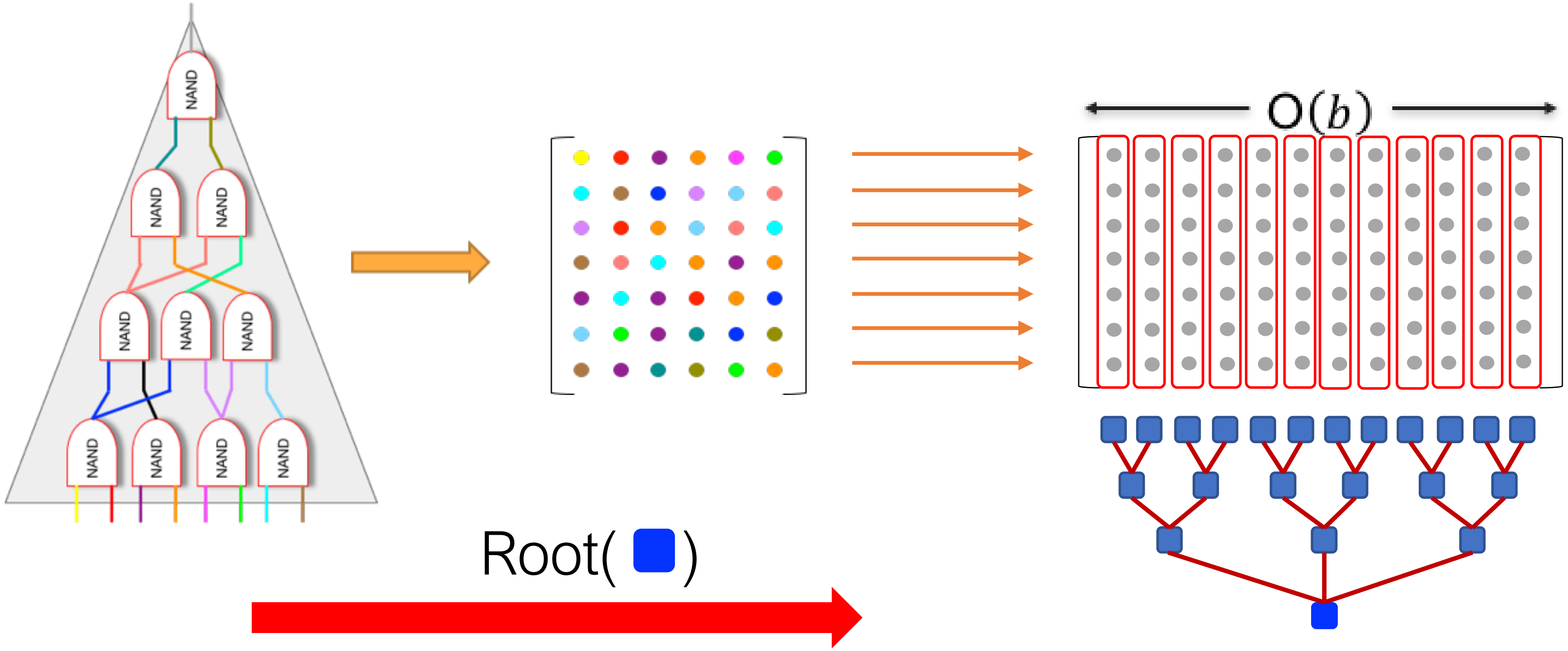
$$a \cdot b \geq X \cdot \#gates$$



P

V

Compute Merkle Root

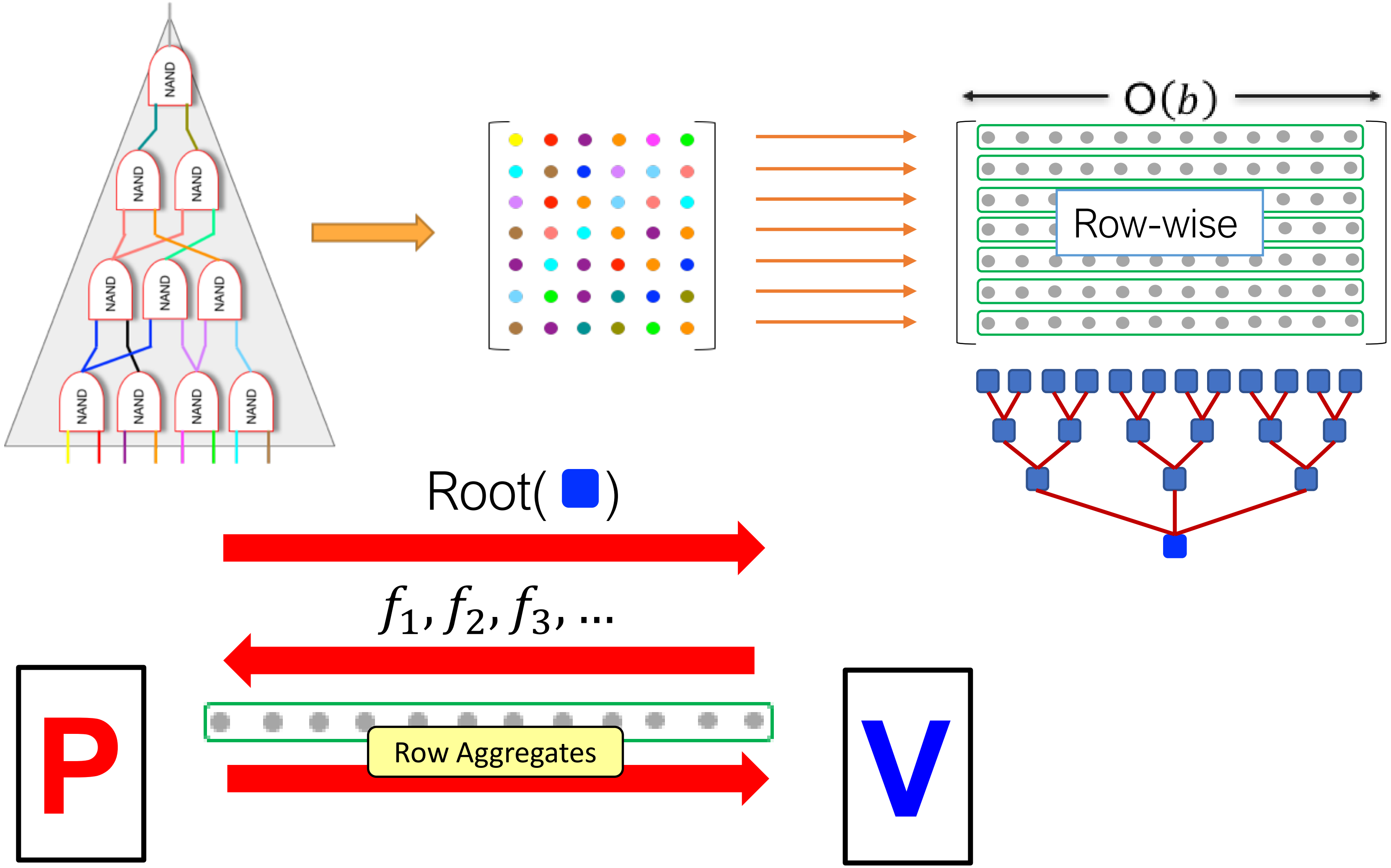


P

V

Compute Merkle Root

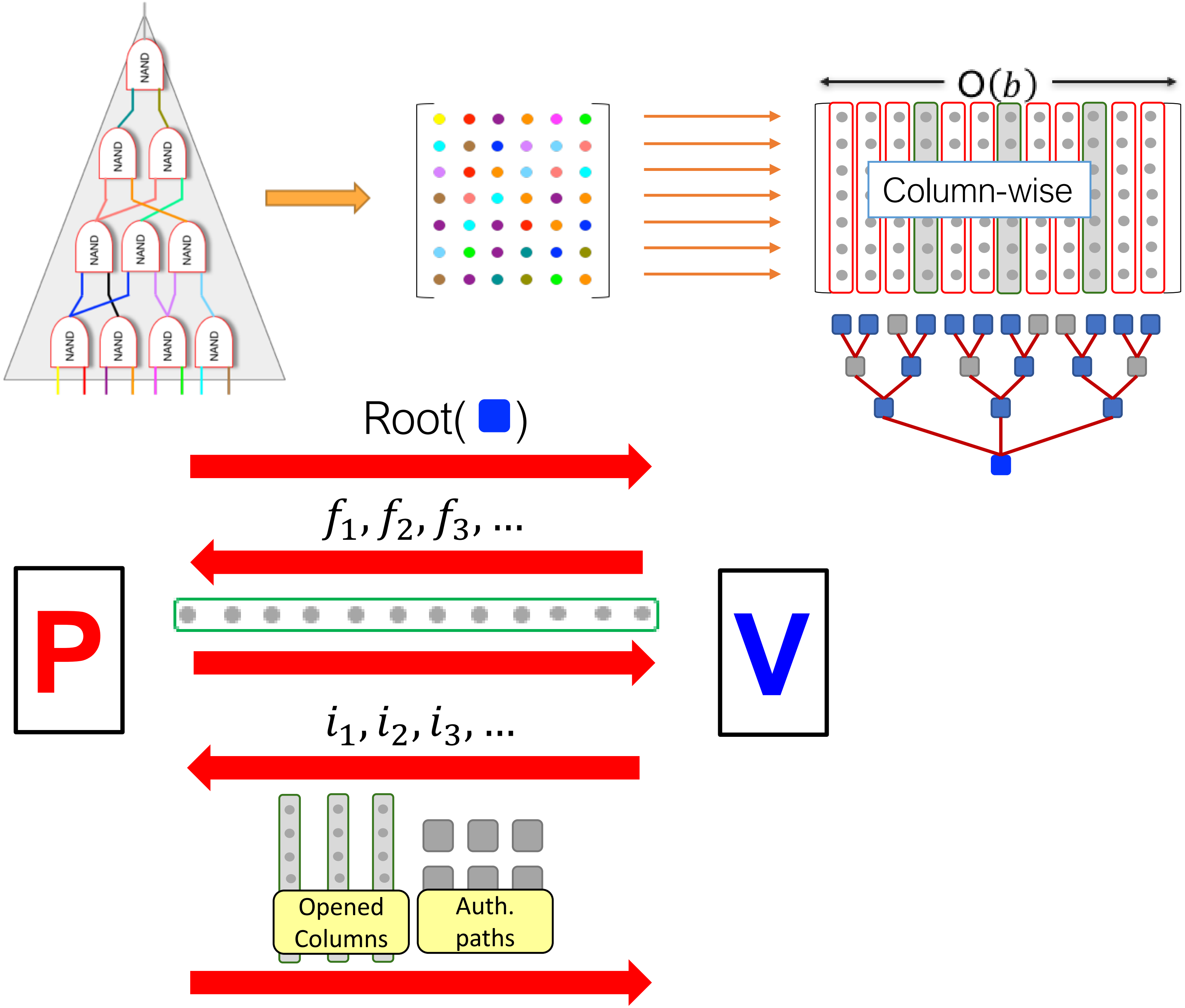
Compute Row Aggregates



Compute Merkle Root

Compute Row Aggregates

Decommit Columns



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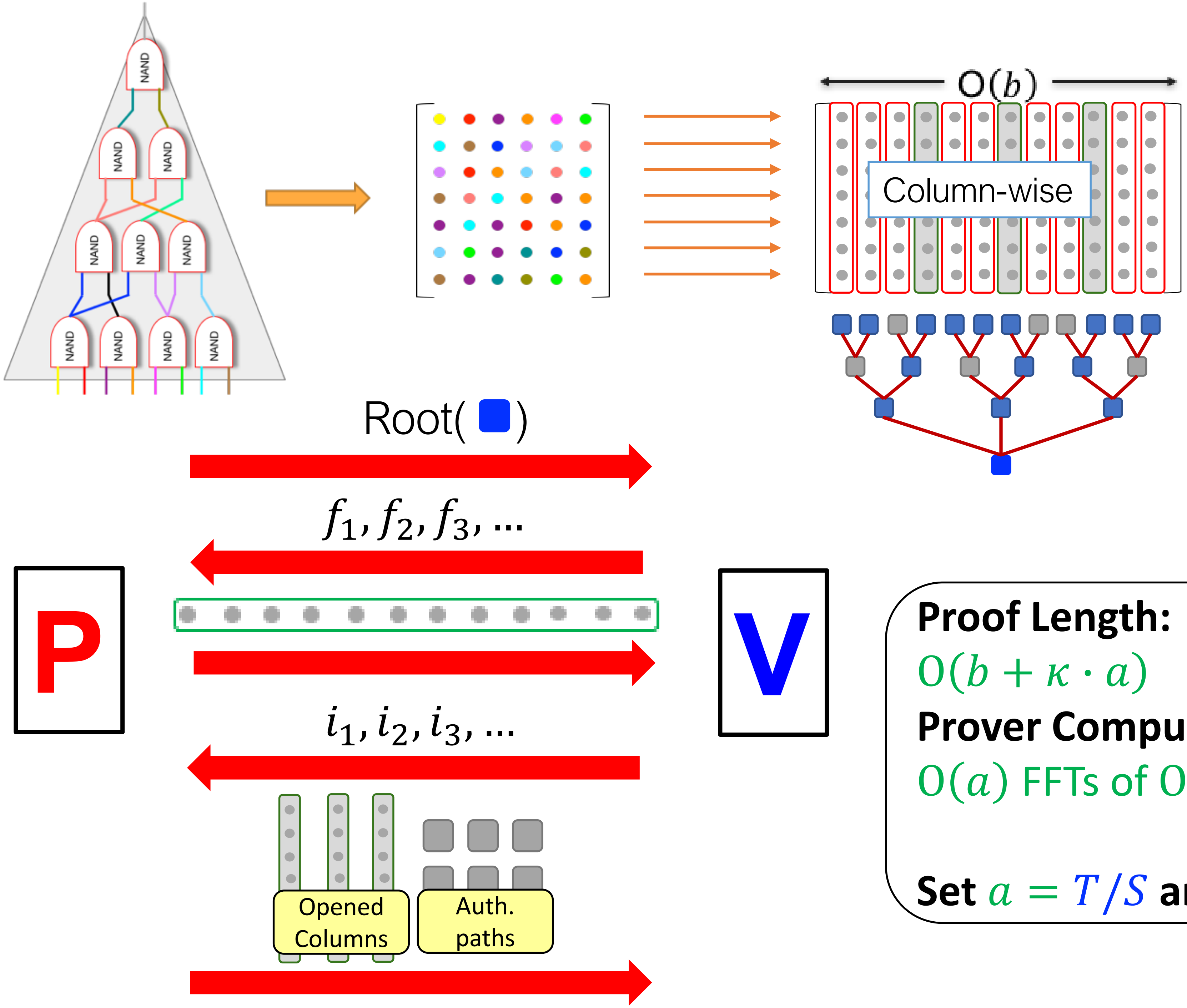


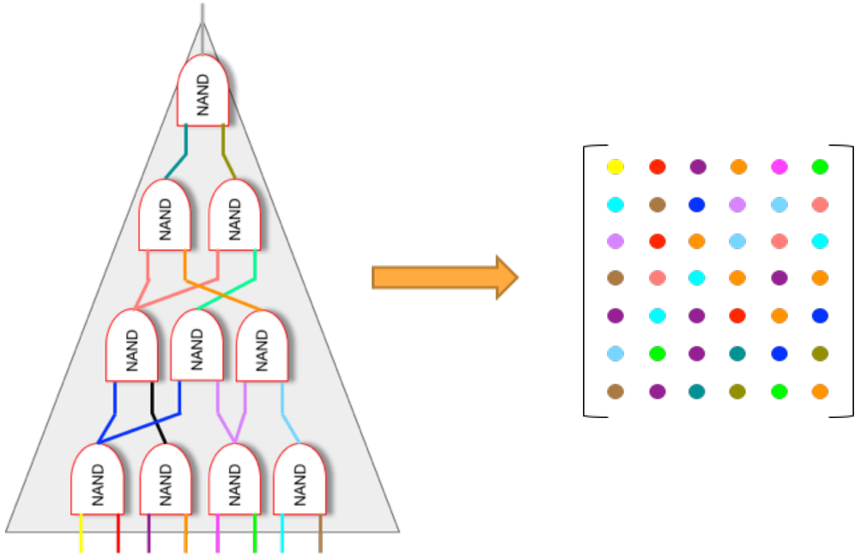
Proof Length:

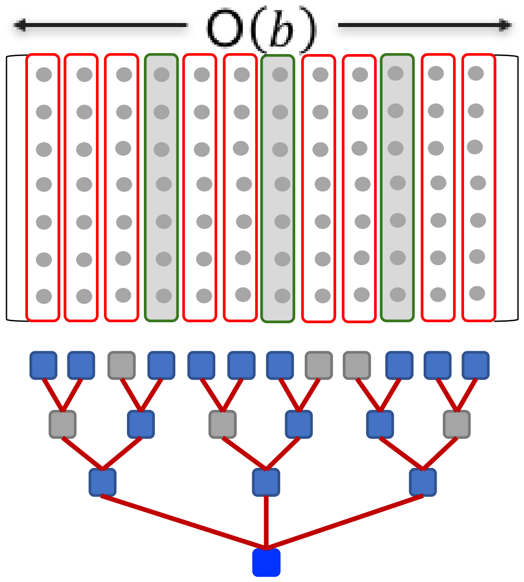
$O(b + \kappa \cdot a)$

Prover Computation:

$O(a)$ FFTs of $O(b)$

Set $a = T/S$ **and** $b = S$





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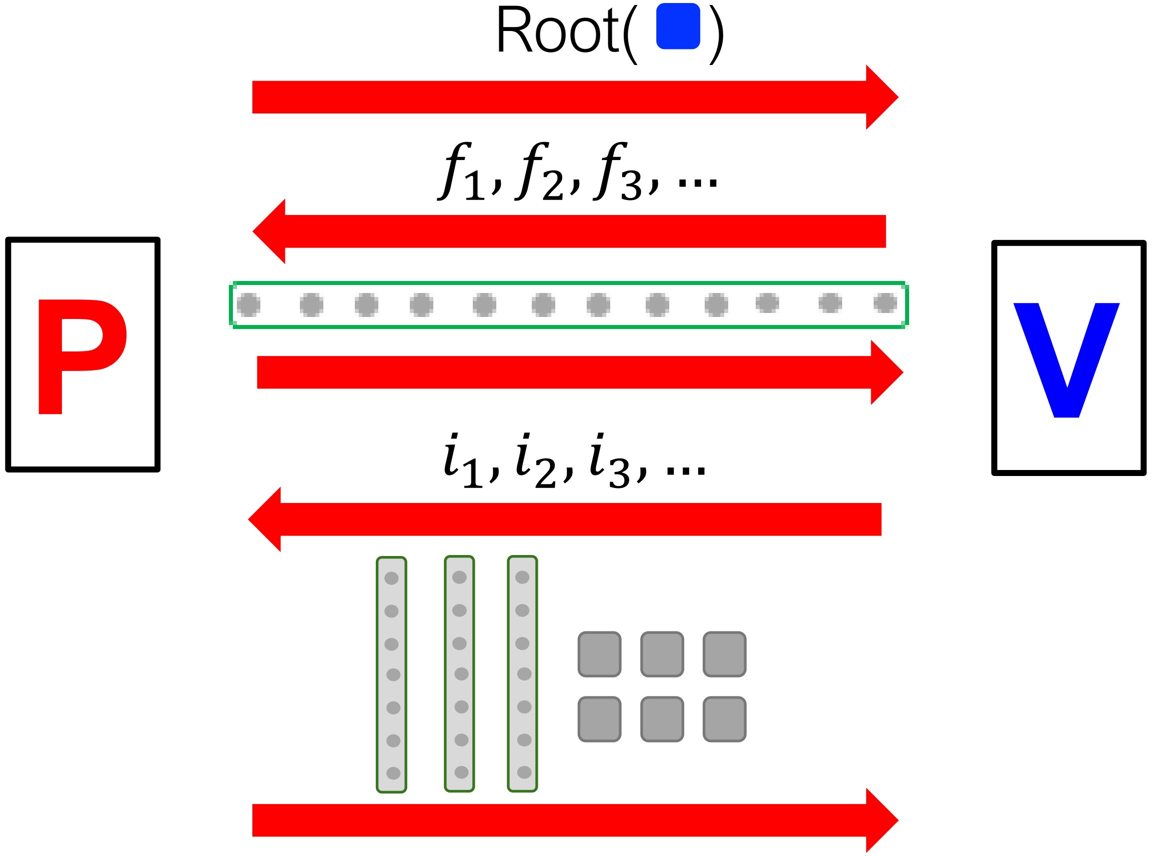

Proof Length:

$\tilde{O}\left(\frac{T}{S} + S\right)$

Prover Computation:

$\tilde{\boldsymbol{O}}(T)$

Set $a = T/S$ **and** $b = S$

What about space-efficiency?

Compute Merkle Root

Compute Row Aggregates

Decommit Columns

$O(S)$ space if the matrix can be computed row by row

What about space-efficiency?

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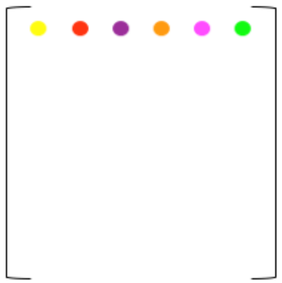
[BCGT13] RAM program => succinct circuits => Transcript generated in time T and space S

Computing Merkle Root Space Efficiently



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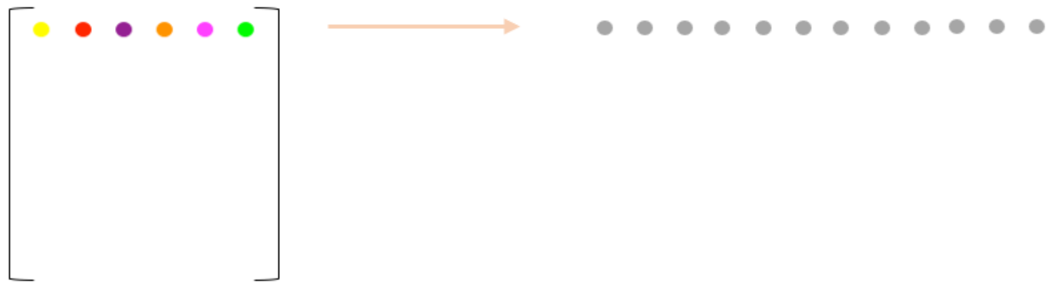

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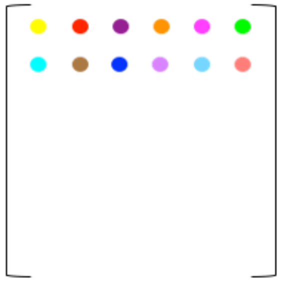

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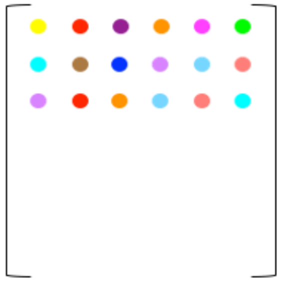

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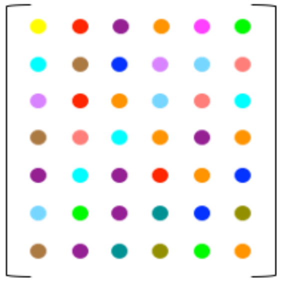

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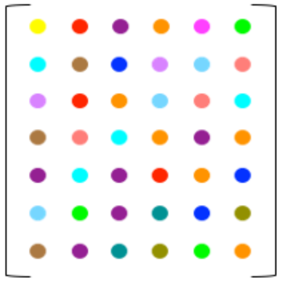

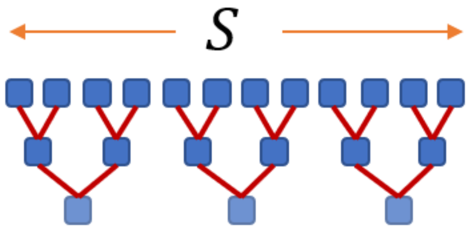

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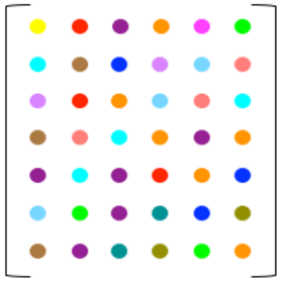
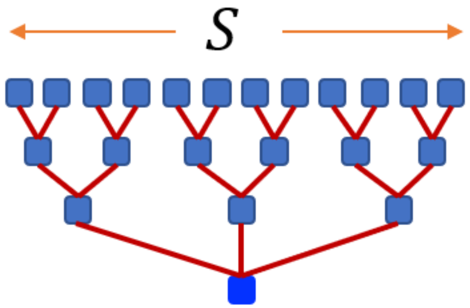




What about space-efficiency of Row Aggregates?

Compute Merkle Root

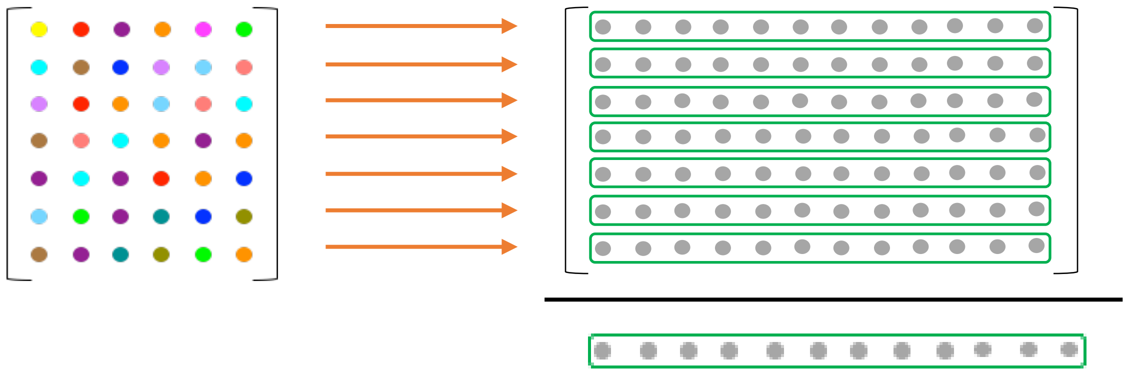
Compute Row Aggregates

Decommit Columns

Code test - the prover encoded each row correctly

Quadratic test - the prover computed multiplication gates correctly

Linear test - the prover computed “linear” gates correctly



Row Aggregate

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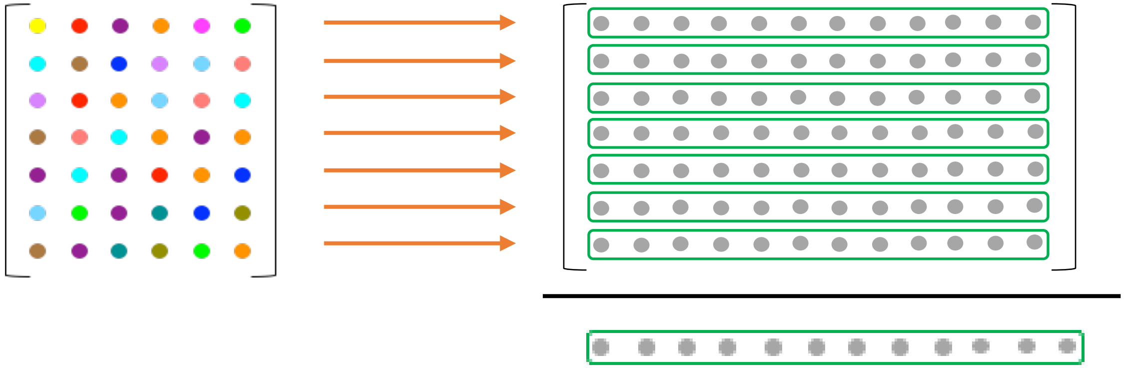
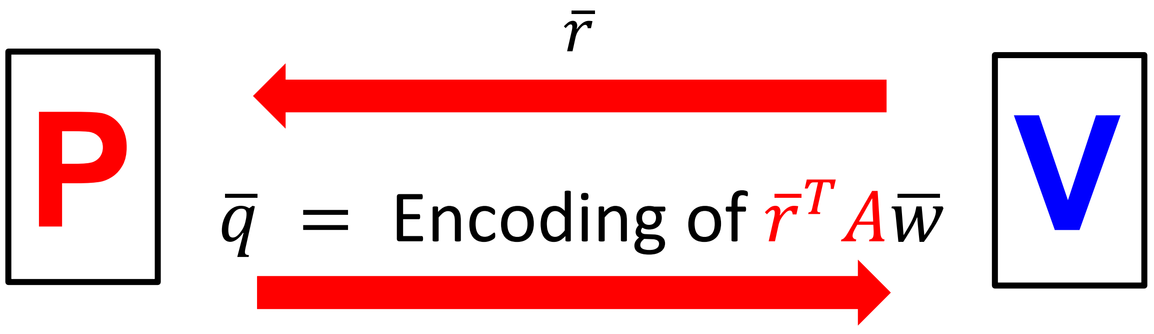


Row Aggregate

Linear gates were correct

Linear constraints can be expressed as $A\bar{w} = \bar{b}$



Check if $\bar{q}$ is a valid codeword encoding $\bar{r}^T \bar{b}$ and agrees on revealed columns

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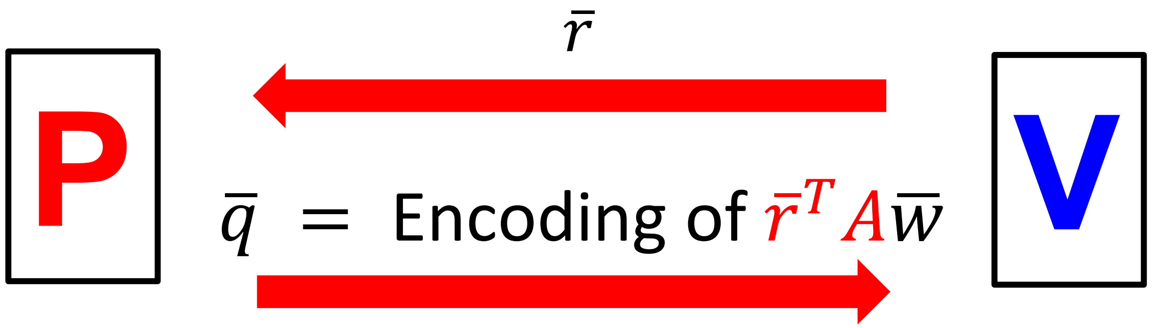
Linear constraints can be expressed as $A\bar{w} = \bar{b}$



Check if $\bar{q}$ is a valid codeword encoding $\bar{r}^T\bar{b}$ and agrees on revealed columns

Main Challenge: How to compute $\bar{r}^T A$ efficiently?

Linear gates were correct

Linear constraints can be expressed as $A\bar{w} = \bar{b}$

$\bar{r}$

P $\longleftarrow$ **V**

$\bar{q}$ = Encoding of $\bar{r}^T A\bar{w}$ $\longrightarrow$

Check if $\bar{q}$ is a valid codeword encoding $\bar{r}^T \bar{b}$ and agrees on revealed columns

Main Challenge: How to compute $\bar{r}^T A$ efficiently?

Size of r: $\tilde{O}(T)$

Size of A: $\tilde{O}(T) \times \tilde{O}(T)$

Linear gates were correct

Linear constraints can be expressed as $A\bar{w} = \bar{b}$

$\bar{r}$

P

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V

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Main Challenge: How to compute $\bar{r}^T A$ efficiently?

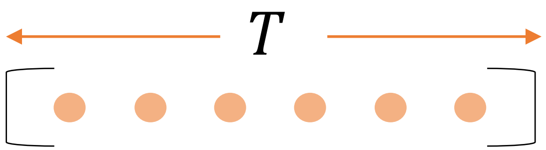
Size of r: $\tilde{O}(T)$

Size of A: $\tilde{O}(T) \times \tilde{O}(T)$

Naive Approach: $\tilde{O}(T^2)$ time & $\tilde{O}(T^2)$ space.

How to compute $\bar{r}^T A$ efficiently?

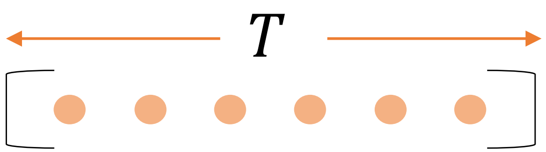
r



r requires $O(T)$ space!

How to compute $\bar{r}^T A$ efficiently?

r



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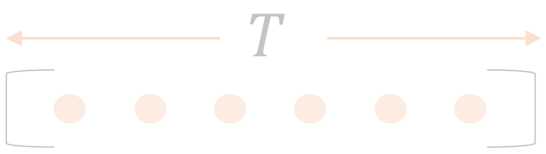
- Generate r from a seed
- Required space: $\tilde{O}(1)$

How to compute $\bar{r}^T A$ efficiently?

r



r requires $O(T)$ space!

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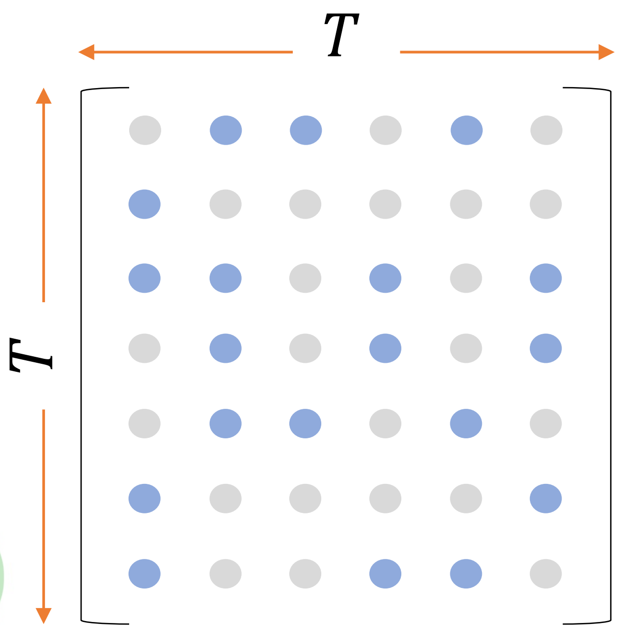
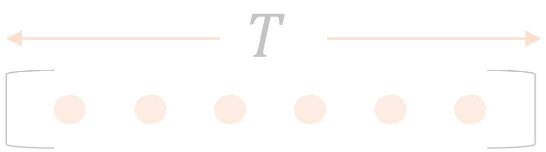
A



A is a large matrix!

How to compute $\bar{r}^T A$ efficiently?

r



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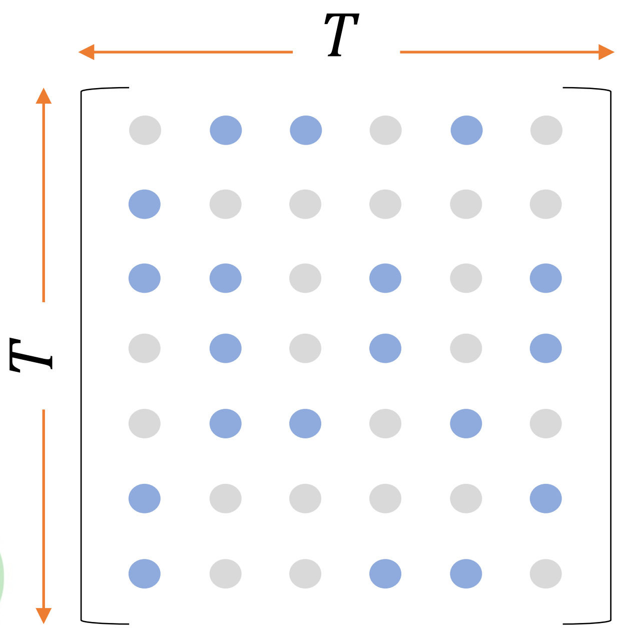
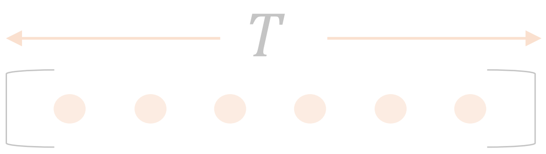


A is a large matrix!

- A is sparse and structured

How to compute $\bar{r}^T A$ efficiently?

r



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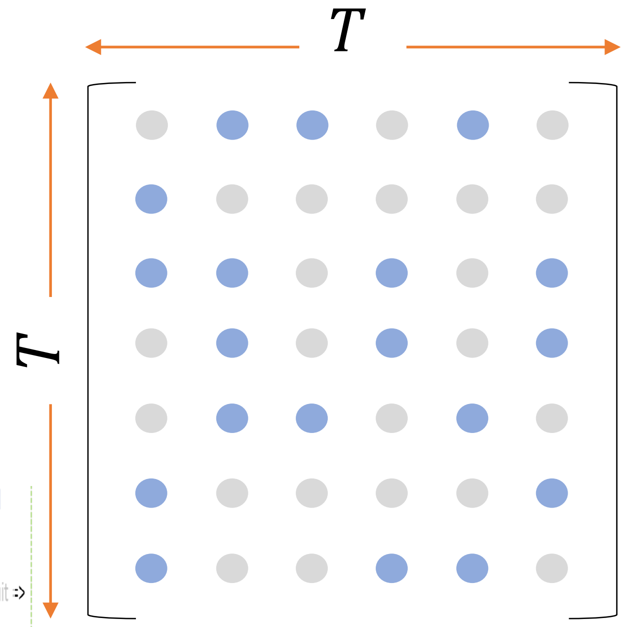

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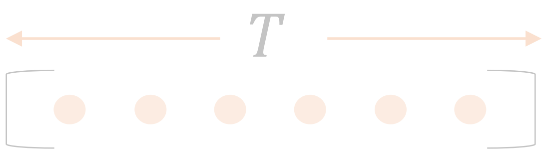

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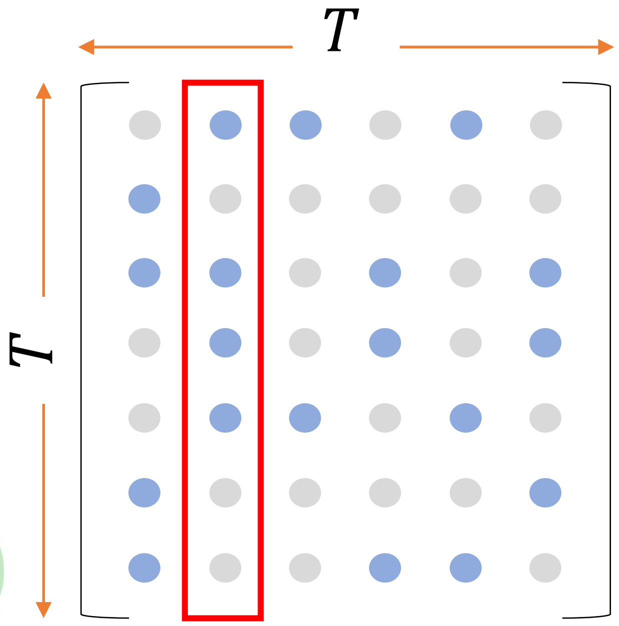

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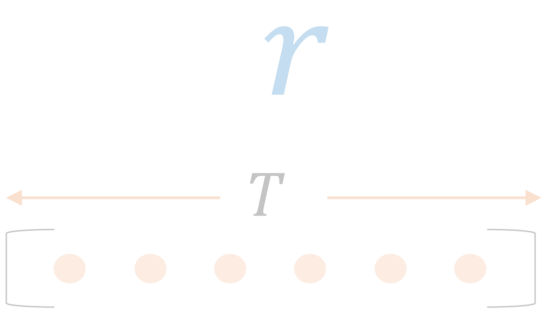

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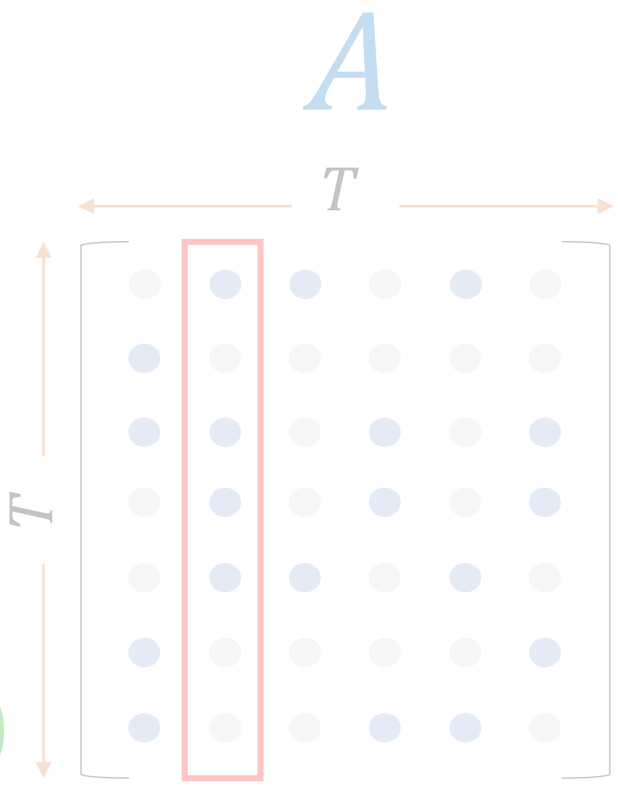

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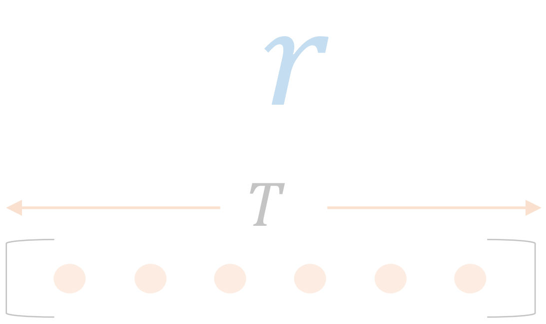

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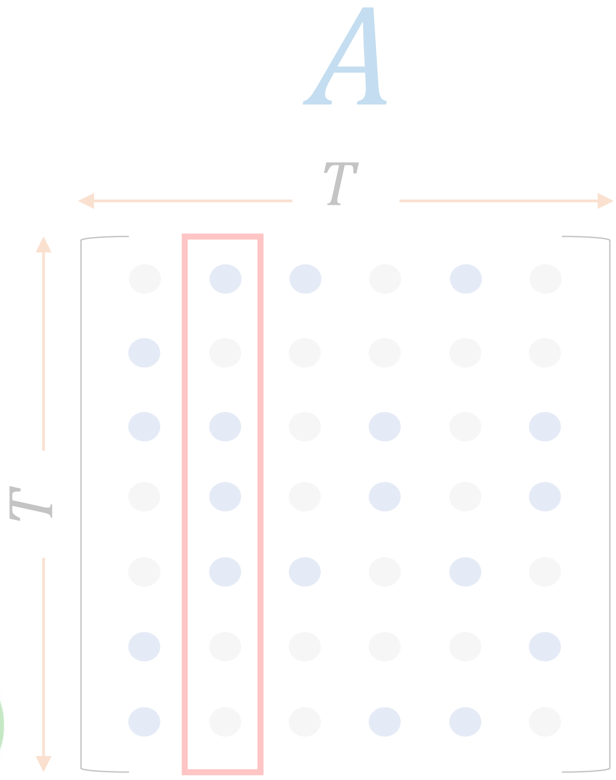

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P is complexity-preserving.

Main Result – Lower Bound*

Theorem: Assume CRH exists. ∀ NP language verified by a uniform RAM machine running in **time** $\boldsymbol{T}$ and **space** $\boldsymbol{S}$ ∃ public-coin (ZK) arg. with soundness $\boldsymbol{negl(k)}$ such that:

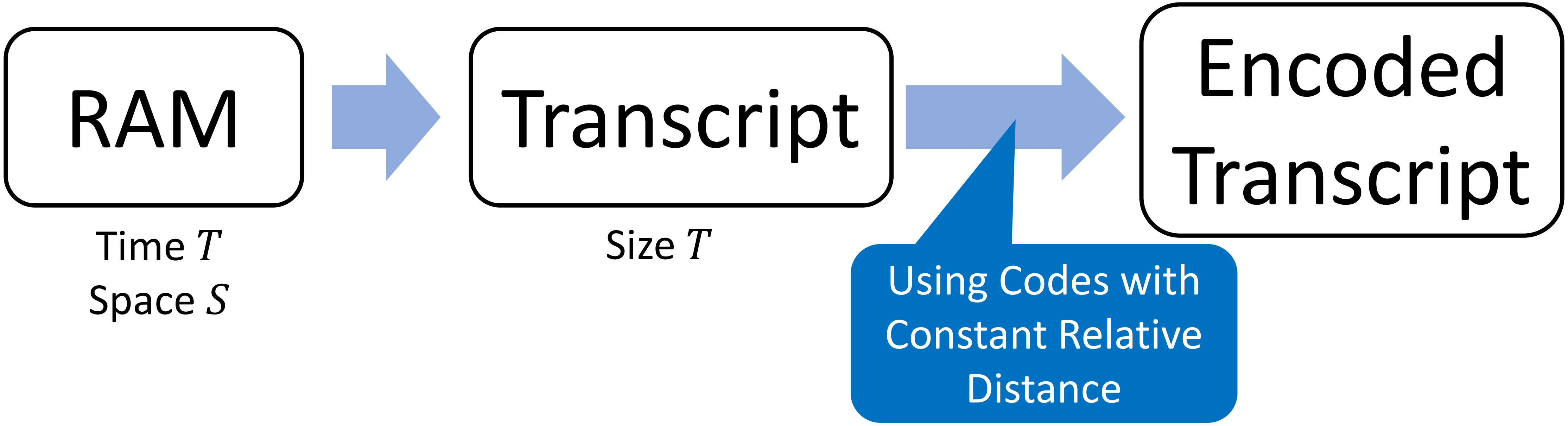
	Time	Space
P	$\tilde{O}(T)$	
V	$\tilde{O}\left(\frac{T}{S}+S\right)$	

Our lower bound shows why it's hard to further improve the proof length.

Proof Length is $\tilde{\boldsymbol{O}}\left(\frac{\boldsymbol{T}}{\boldsymbol{S}}\right)$

where the CRH is used in a black-box way and $\tilde{\boldsymbol{O}}(\quad)$ ignores $\boldsymbol{poly}(\mathbf{log}(\boldsymbol{T}), \boldsymbol{k})$

Current Techniques*: Most ZK-SNARKs rely on Codes…



Current Techniques*: Most ZK-SNARKs rely on Codes…

RAM → Transcript → Encoded Transcript

Time T
Space S

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Using Codes with Constant Relative Distance

"Lower bound" on the encoding algorithm

provides evidence

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"Hard" to reduce the proof length below $\tilde{O}\left(\frac{T}{S}\right)$

Lower Bound*

Theorem: Let C be $[n, m, \delta m]$ code over $\mathbb{F}$ then any r-pass Encode RAM algorithm requires space

$$S > \frac{\delta n}{r} \log|\mathbb{F}|$$

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[YRST02] Proved the lower bound for 1-pass algorithm

Setting $n = T$ and $|\mathbb{F}| = \log T$ we get that $\delta < \frac{S \cdot r}{T \cdot \log T}$

=> Minimum number of oracle queries is $\tilde{O}\left(\frac{1}{\delta}\right) \quad = \tilde{O}\left(\frac{T}{S}\right)$

Summary

- First construction of complexity-preserving sublinear (ZK) arguments from CRH in a black-box way

- Improving the communication will require new techniques – i.e., getting around constant-distance codes.

Thank You

Additional Slides

Warmup Lower Bound

Simplifying assumptions:

1. Only one pass over the message
2. No work tape
3. Codeword head is independent of the contents of the message

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