

TCC 2022

The Parallel Reversible Pebbling Game: Analyzing the Post-Quantum Security of iMHFs

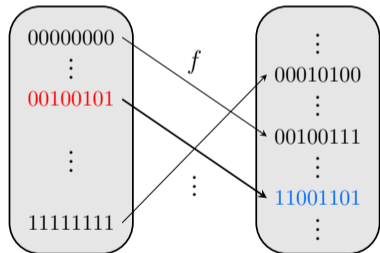
Jeremiah Blocki, Blake Holman, and Seunghoon Lee

November 7, 2022



Motivation: Quantum Pre-Image Attack

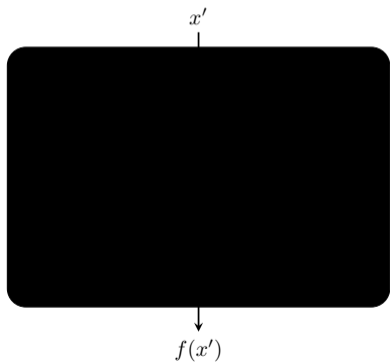
Problem. Given a function $f : \{0, 1\}^n \rightarrow \{0, 1\}^n$ and a target output y , find an input $x \in \{0, 1\}^n$ such that $y = f(x)$.



Find $x \in \{0, 1\}^8$ s.t. $f(x) = 11001101$

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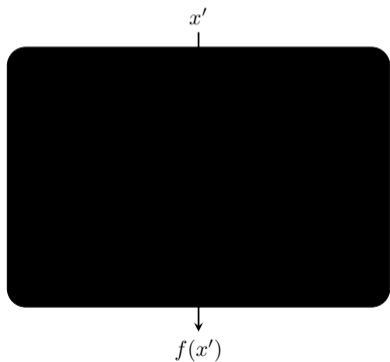
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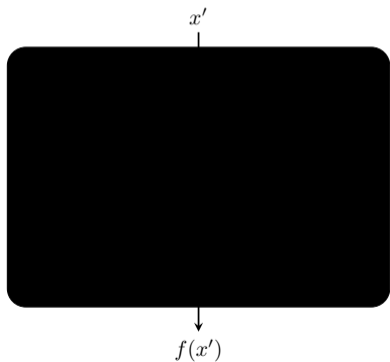


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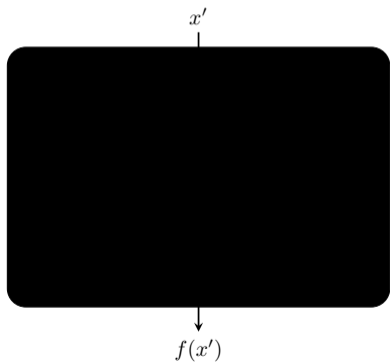


- **Classical Unstructured Search:** need $\Omega(2^n)$ black-box queries to the function f
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 - only requires $\mathcal{O}(2^{n/2})$ black-box queries to the function f
 - this is tight: any quantum algorithm using f as a black box must make $\Omega(2^{n/2})$ queries [BBBV97]

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- What is the full cost of a quantum pre-image attack?

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The full cost of a quantum pre-image attack is defined as the *space-time cost* (ST-cost), i.e.,

$$(\text{space}(C)) \times (\text{time}(C)),$$

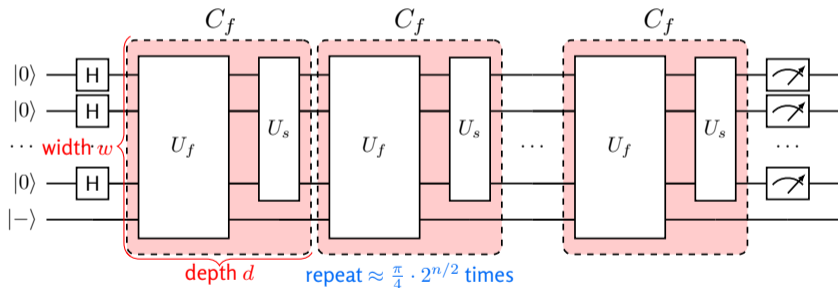
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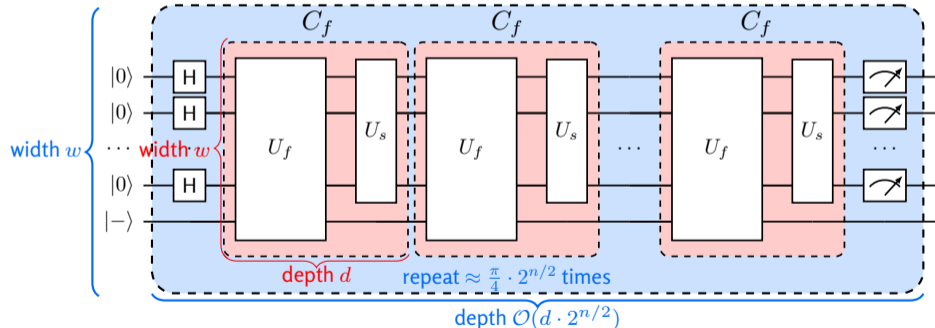
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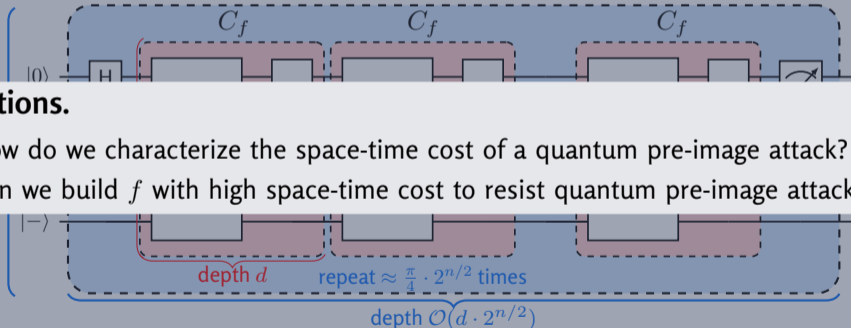
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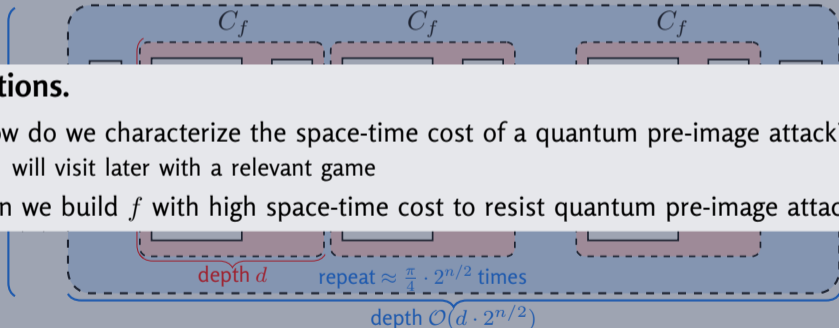
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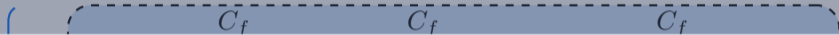
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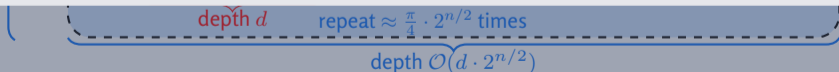
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 - Memory-Hard Functions!
 - Application: password hashing



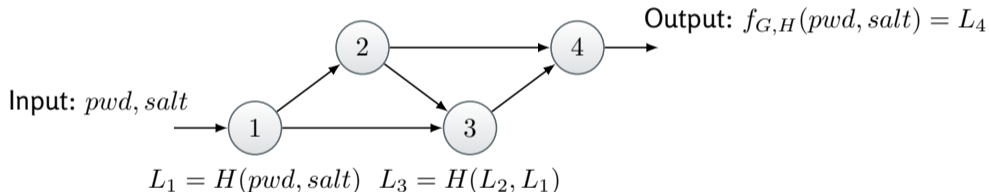
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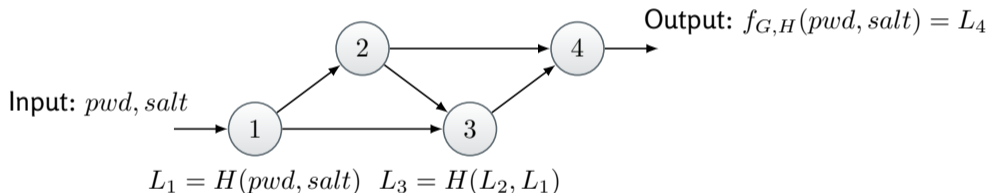
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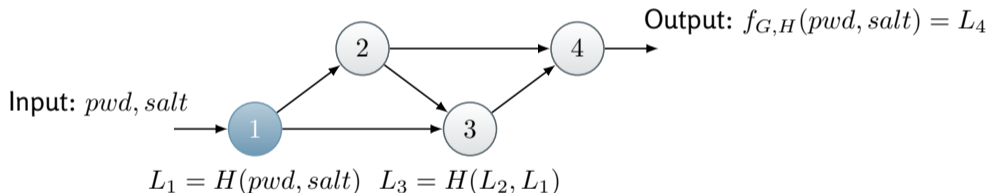


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 - Rule 1: should **start with no pebbles** on the graph and **end with target nodes**
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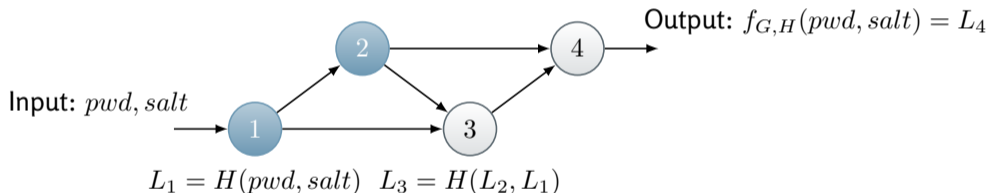


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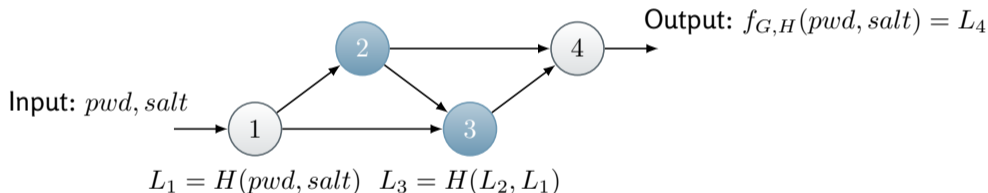


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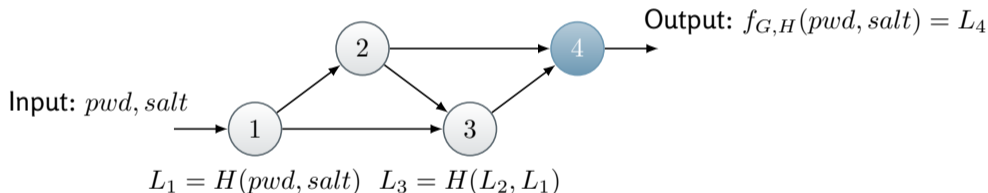


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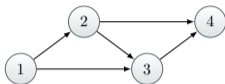
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Space-Time Complexity

In the Black Pebbling Game



A pebbling $P = (P_1 = \{1\}, P_2 = \{1, 2\}, P_3 = \{2, 3\}, P_4 = \{4\})$

Space-Time (ST) Complexity

- $ST(P) = (\text{time}) \times (\text{max space})$, and
 $ST(G) = \min_P ST(P)$
- For above example, we have

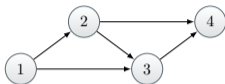
$$ST(P) = 4 \times 2 = 8$$

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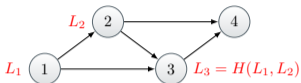
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No!

Why?

- Quantum circuits must be *reversible*
- $P_3 = \{2, 3\} \rightarrow P_4 = \{4\}$: not a *reversible* transition
- **Quantum Uncomputation** in the QROM:

$$|x, y\rangle \xrightarrow{H} |x, y \oplus H(x)\rangle$$

$$\begin{aligned} \therefore |(L_1, L_2), L_3\rangle &\xrightarrow{H} |(L_1, L_2), L_3 \oplus H(L_1, L_2)\rangle \\ &= |(L_1, L_2), 0^k\rangle \end{aligned}$$

- to remove a pebble from node 3 using uncomputation, we need have needed pebbles on nodes 1 and 2

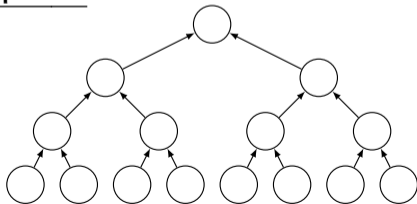
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- Prior work [Ben89, LV96, Krá01, MSR⁺19] introduced *sequential* reversible computation /reversible pebbling game
 - Added more constraints to capture reversible transitions by quantum uncomputation
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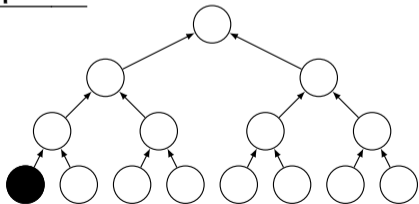
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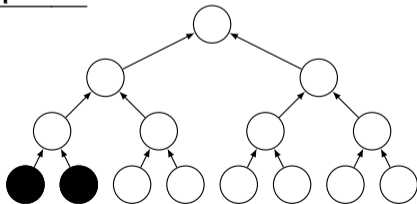
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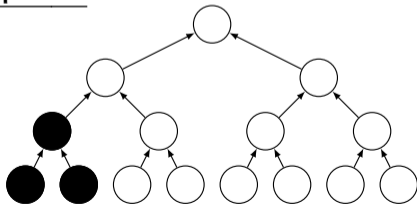
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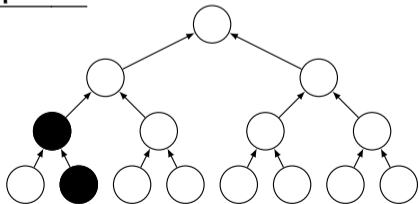
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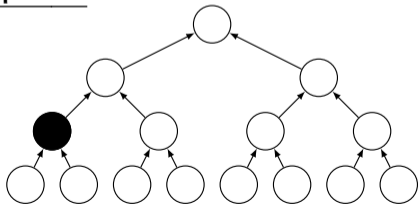
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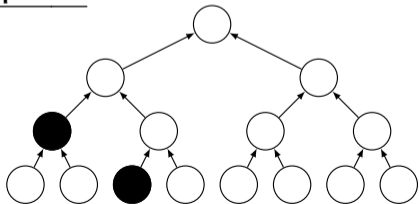
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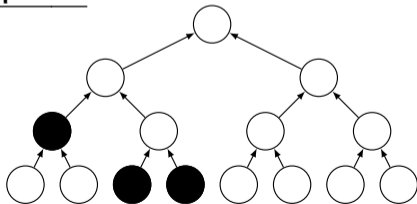
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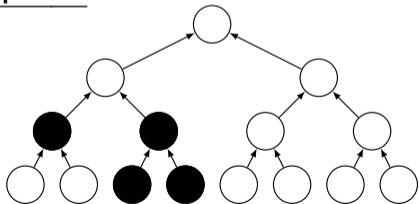
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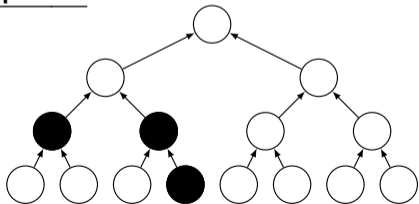
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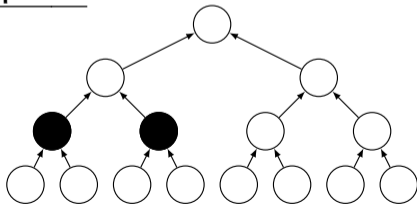
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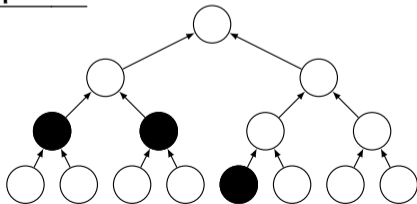
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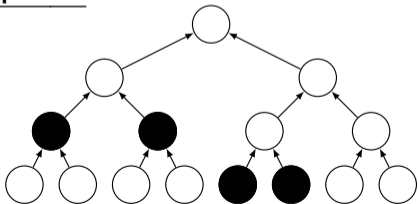
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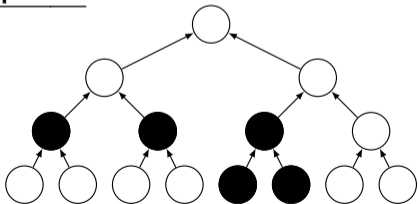
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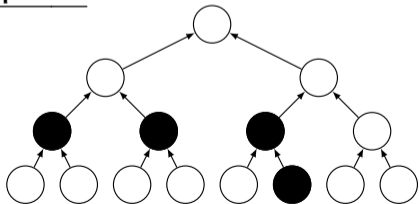
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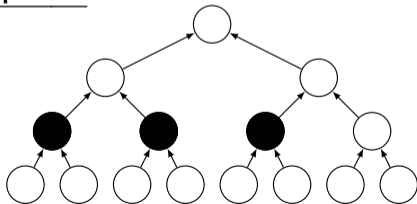
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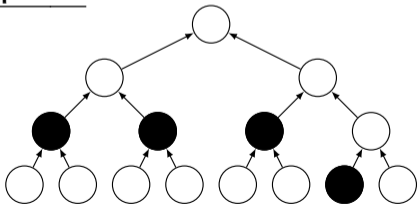
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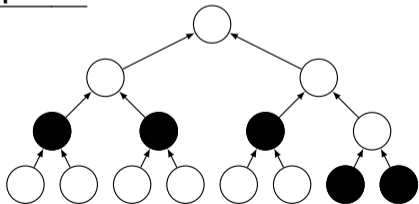
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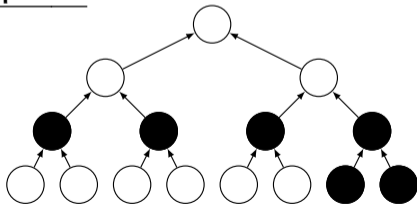
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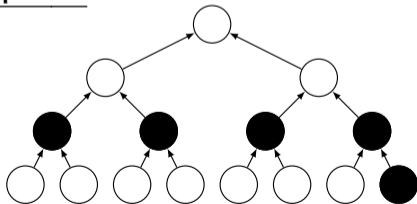
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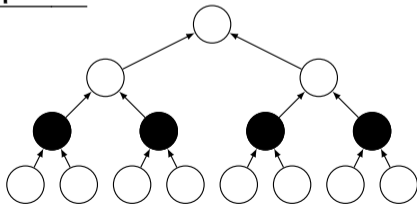
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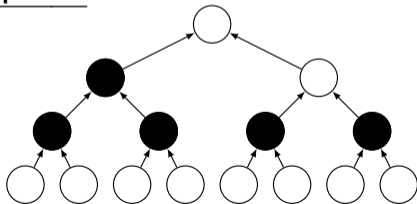
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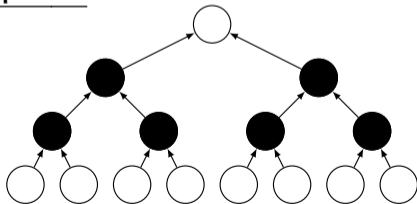
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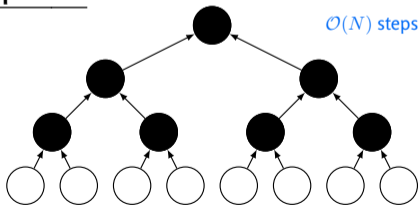
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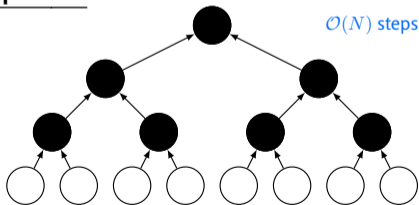
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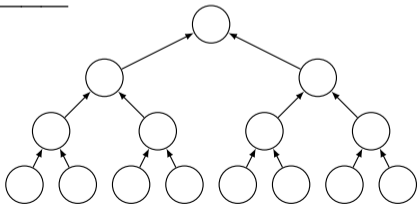
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$$\begin{aligned}\Rightarrow (\text{ST-Cost}) &= (\text{space}) \times (\text{time}) \\ &= 7 \times 23 = 161\end{aligned}$$

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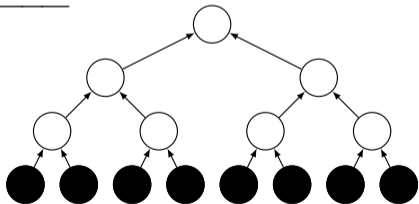
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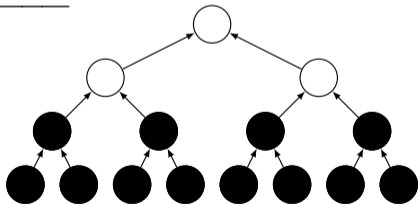
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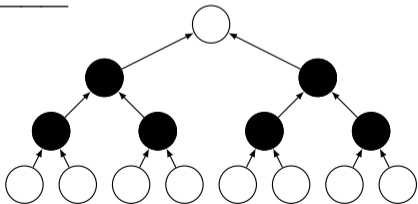
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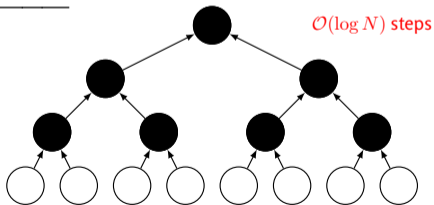
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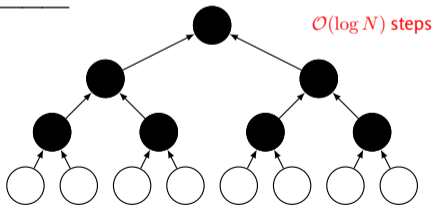
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Analyzing the Quantum Circuit:

$$\begin{aligned}\Rightarrow (\text{ST-Cost}) &= 12 \times 4 = 48 \\ \Rightarrow \text{the time cost can be decreased} \\ &\text{from } O(N) \text{ to } O(\log N)\end{aligned}$$

Key Research Question

Back to Our First Question

How to characterize the space-time cost of a quantum circuit $C_{f_{G,H}}$ for a data-independent Memory-Hard Function $f_{G,H}$?

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How to characterize the space-time cost of a quantum circuit $C_{f_{G,H}}$ for a data-independent Memory-Hard Function $f_{G,H}$?



Partial Answer: *The Parallel Reversible Pebbling Game*
& *Study some attacks against iMHFs in this pebbling model*

Definition: Parallel Reversible Pebbling Game

A *parallel reversible pebbling* $P = (P_0, \dots, P_t)$ is a sequence of pebbling configurations with the conditions (same as classical):

1. start with no pebbles (i.e., $P_0 = \emptyset$) and end with target nodes T (i.e., $T \subseteq P_t$)^(*),
 2. a new pebble can be added only if its parents were previously pebbled, and
- the following *additional* conditions:

Condition 3. (Quantum No-Deletion)

a pebble *can be deleted* only if **all of its parents were previously pebbled**

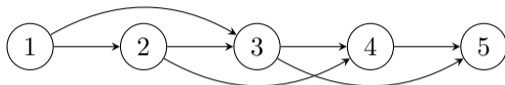
Condition 4. (Quantum Reversibility)

we must keep the pebble if **a pebble was required** to generate new pebbles (or delete pebbles)

(*) we can make this condition strict, i.e., $P_t = T$. See the paper for detail.

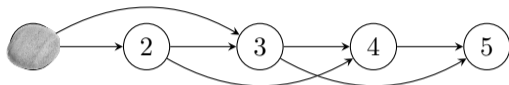
Example: A Parallel Pebbling

Classical vs. Reversible



Example: A Parallel Pebbling

Classical vs. Reversible



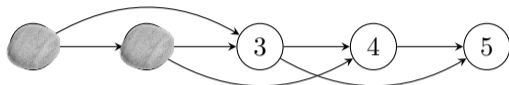
Classical

Round 1



Example: A Parallel Pebbling

Classical vs. Reversible

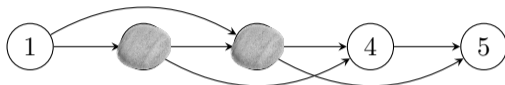


Classical

Round 1 ● ○ ○ ○ ○
Round 2 ● ● ○ ○ ○

Example: A Parallel Pebbling

Classical vs. Reversible

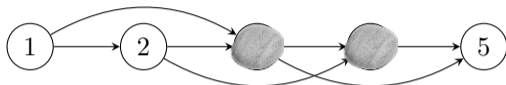


Classical

Round 1	●	○	○	○	○
Round 2	●	●	○	○	○
Round 3	○	●	●	○	○

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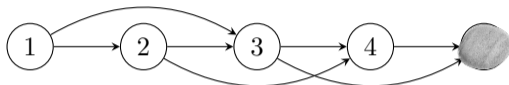


Classical

Round 1	●	○	○	○	○
Round 2	●	●	○	○	○
Round 3	○	●	●	○	○
Round 4	○	○	●	●	○

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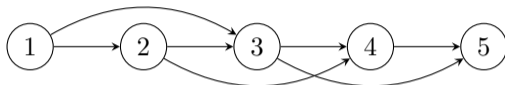


Classical

Round 1	●	○	○	○	○
Round 2	●	●	○	○	○
Round 3	○	●	●	○	○
Round 4	○	○	●	●	○
Round 5	○	○	○	○	●

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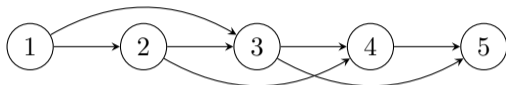
Classical

Round 1	●	○	○	○	○
Round 2	●	●	○	○	○
Round 3	○	●	●	○	○
Round 4	○	○	●	●	○
Round 5	○	○	○	○	●

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– cannot remove pebble since not all parents were pebbled

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Round 1	●	○	○	○	○
Round 2	●	●	○	○	○
Round 3	○	●	●	○	○
Round 4	○	○	●	●	○
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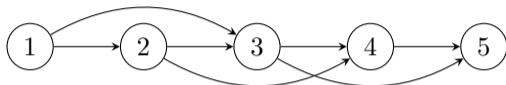
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Round 1	●	○	○	○	○						
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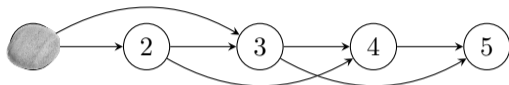
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Round 4	○	○	●	●	○	○	○	○	○	○		
Round 5	○	○	○	○	●	○	○	○	○	○		

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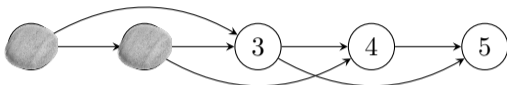
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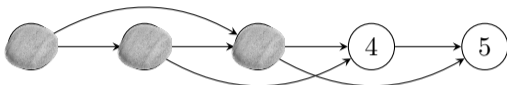
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Round 3	○	●	●	○	○	●	●	●	○	○	Round 3
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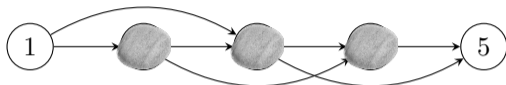
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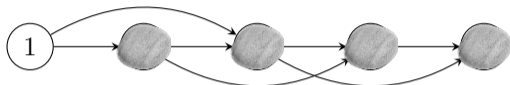
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Round 3	○	●	●	○	○	●	●	●	○	○	Round 3
Round 4	○	○	●	●	○	○	●	●	●	○	Round 4
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Reversible Pebbling Attack 1

Attack on a Line Graph

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 - can be characterized as a line graph

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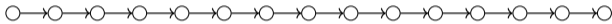
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Parallel Classical Pebbling

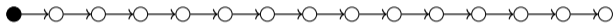


Reversible Pebbling Attack 1

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- **PBKDF2** [Kal00] and **BCRYPT** [PM99]: widely deployed hash functions
 - can be characterized as a line graph
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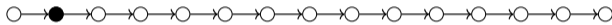


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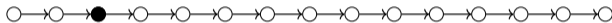


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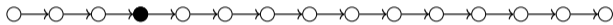


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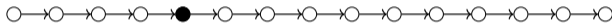


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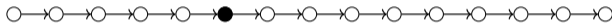


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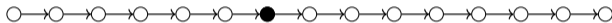


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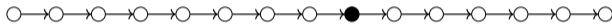


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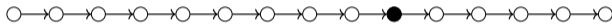


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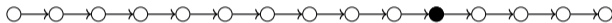


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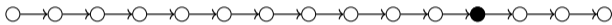


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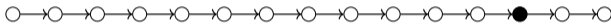


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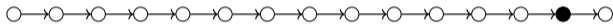


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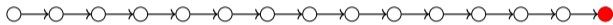


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(ST-Cost) = N

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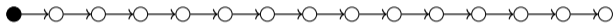


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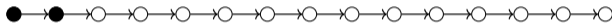


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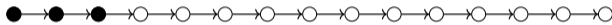


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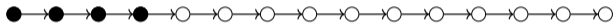


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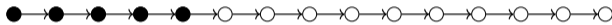


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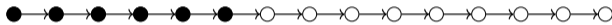


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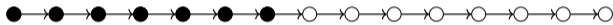


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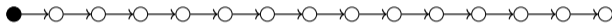


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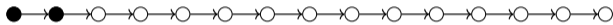


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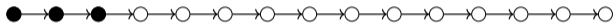


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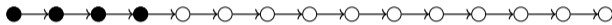


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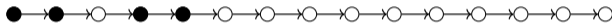


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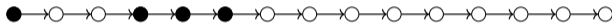


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||: parallel, \leftrightarrow : reversible

Our Result. For a line graph L_N with N nodes, we have $ST^{||, \leftrightarrow}(L_N) = \mathcal{O}\left(N^{1+\frac{2}{\sqrt{\log N}}}\right)$.

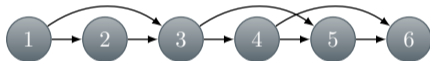
- We modified Li and Vitányi's (sequential) strategy [LV96]
- A similar (sequential) argument was implicitly assumed by Bennett [Ben89] but was not formalized as a reversible pebbling strategy

Reversible Pebbling Attack 2

Attack on Any (e, d) -Reducible DAGs

Definition. A DAG $G = (V, E)$ is (e, d) -*reducible* if there exists a *depth-reducing set* $S \subseteq V$ of size $|S| \leq e$ such that the longest path in $G - S$ has length $\leq d$.

Example. $(2, 2)$ -reducible graph



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- It becomes useful when $e \ll N$ and $d \ll \log N$ (which implies $ST^{\parallel, \leftrightarrow}(G) \ll \mathcal{O}(N^2)$)

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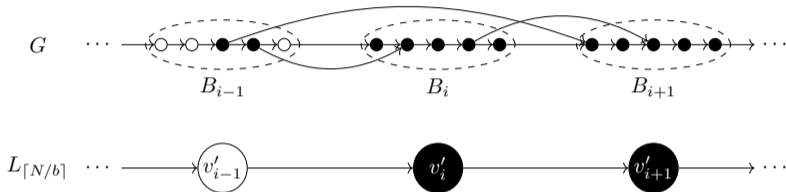


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- **Argon2i-A/B:** winner of the password hashing competition/standardized
- Using this result, we have $ST^{\parallel, \leftrightarrow}(\text{Argon2i-A}) = \mathcal{O}(N^2 \log \log N / \sqrt{\log N})$ and $ST^{\parallel, \leftrightarrow}(\text{Argon2i-B}) = \mathcal{O}(N^2 / \sqrt[3]{\log N})$

Reversible Pebbling Attack 3

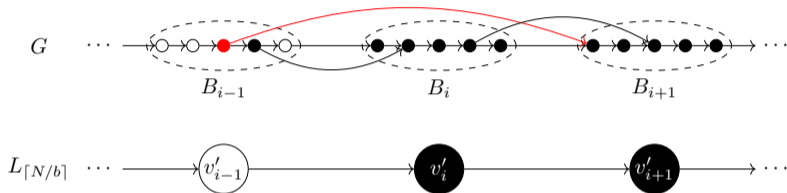
Using an Induced Line Graph



- Given a graph G , split into blocks of size b and create a line graph $L_{\lceil N/b \rceil}$ of size $\lceil N/b \rceil$
- **Transform** the reversible pebbling of $L_{\lceil N/b \rceil}$ to the original graph G

Reversible Pebbling Attack 3

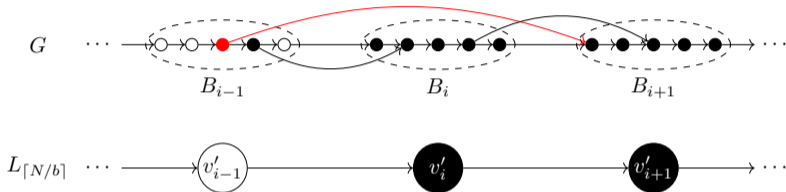
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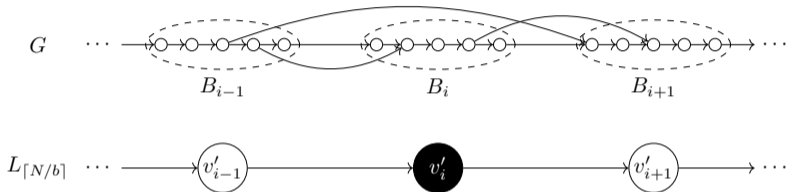
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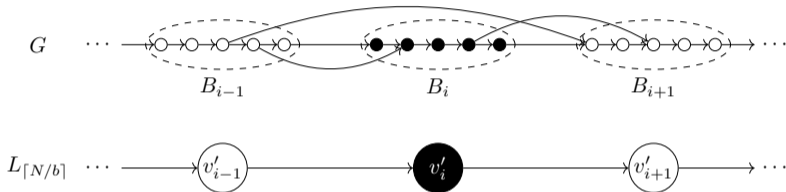
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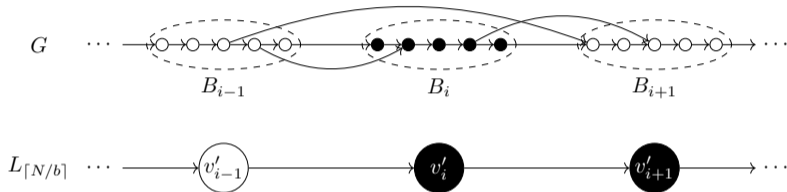
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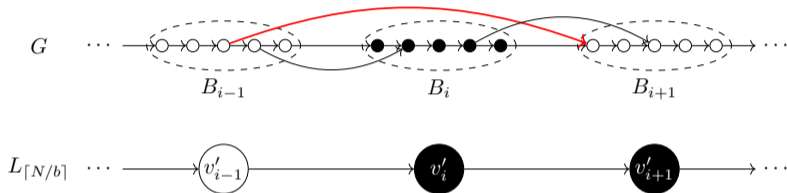
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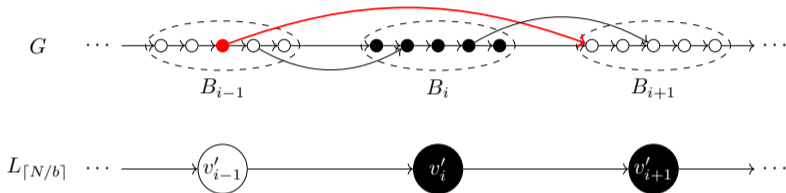
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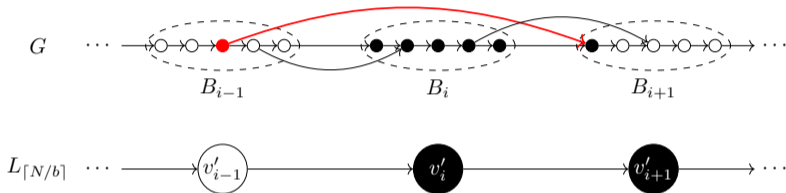
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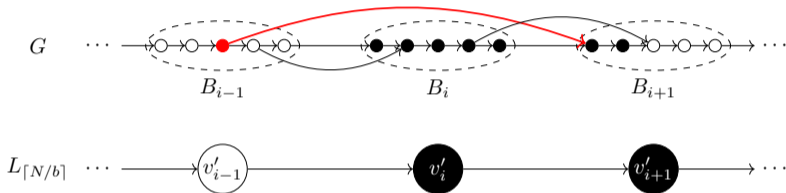
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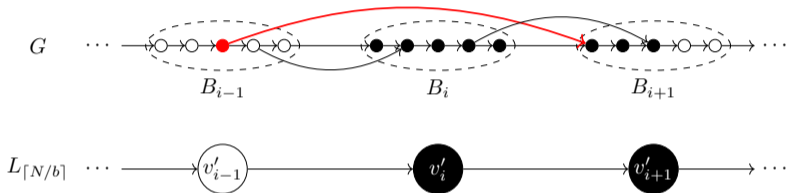
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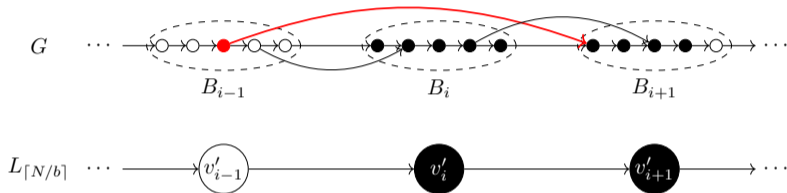
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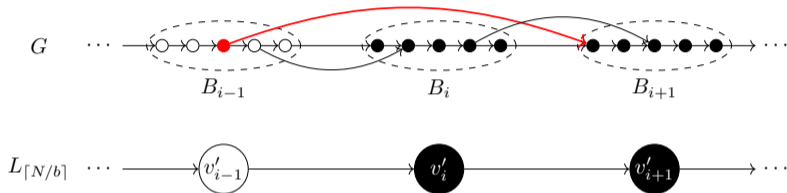
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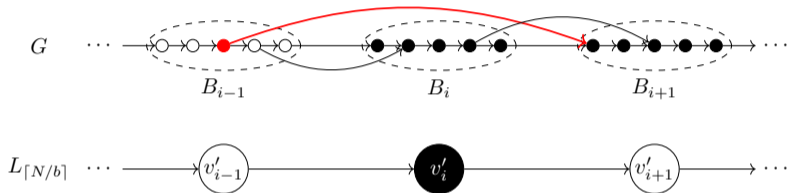
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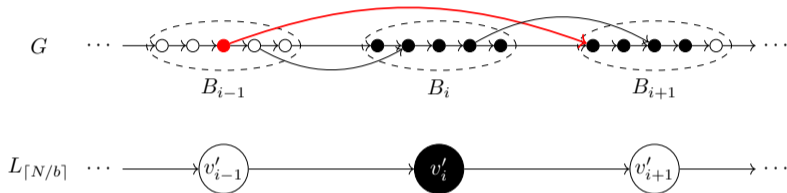
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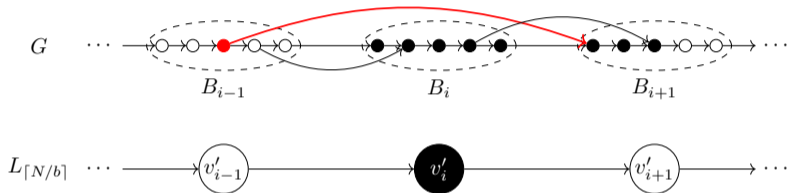
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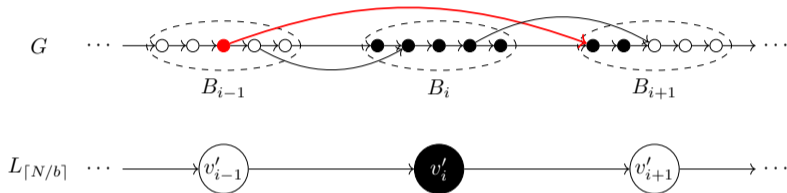
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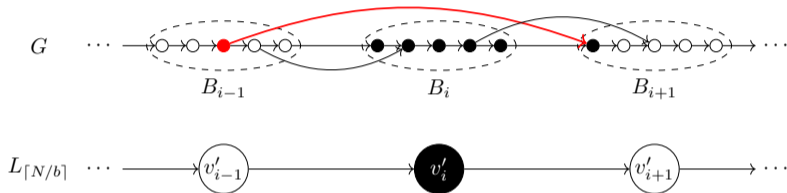
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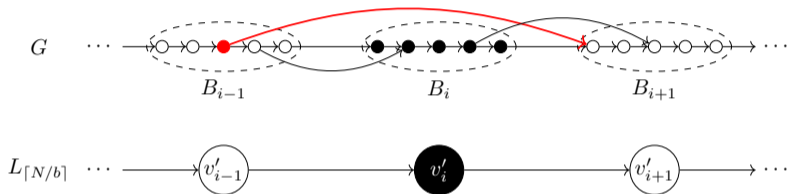
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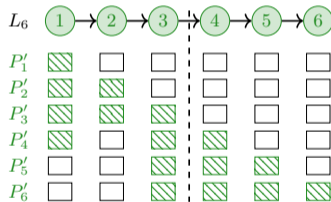
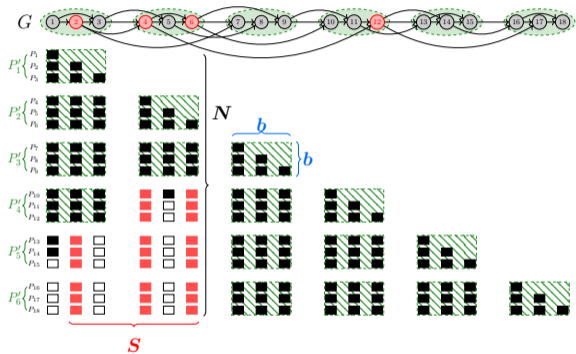
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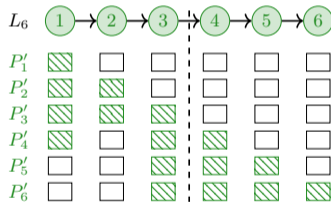
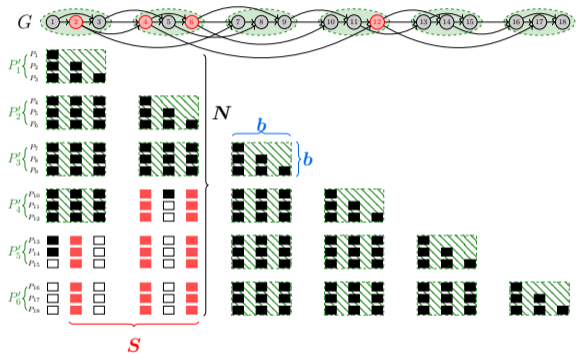
Example

Attack Using an Induced Line Graph



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Our Result.

$$ST^{\parallel, \leftrightarrow}(G) = \mathcal{O}\left(SN + b^2 \cdot ST^{\parallel, \leftrightarrow}(L_{\lceil N/b \rceil})\right), \text{ where } S = (\# \text{ skip nodes}) \text{ and } b > 0: \text{ block size}$$

iMHF Example: DRSample

Attack Using an Induced Line Graph

- **DRSample** [ABH17]: a practical iMHF candidate with stronger *classical* memory-hardness
- For DRSample, we showed that (whp) the number of *skip nodes* is at most

$$(\# \text{ skip nodes}) = \mathcal{O}\left(\frac{N \log \log N}{\log N}\right),$$

when we set the block size $b = \mathcal{O}(N/\log^2 N)$.

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- **Note.** DRSSample admits a more efficient reversible pebbling attack than Argon2i-A/B
cf.) $ST^{\parallel, \leftrightarrow}(\text{Argon2i-A}) = \mathcal{O}\left(\frac{N^2 \log \log N}{\sqrt{\log N}}\right)$ and $ST^{\parallel, \leftrightarrow}(\text{Argon2i-B}) = \mathcal{O}\left(\frac{N^2}{\sqrt[3]{\log N}}\right)$

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Our Result: We extend the (non-reversible) Alwen and Blocki's attack [AB16]

Theorem. If G is (e, d) -reducible with N nodes with indegree δ , then

$$\text{aST}^{\parallel, \leftrightarrow}(G) \leq \min_{g \geq d} \left\{ 2N \left(\frac{2Nd}{g} + e + (\delta + 1)g \right) + N + \frac{2Nd}{g} \right\}.$$

- **Corollary:** $\text{aST}^{\parallel, \leftrightarrow}(\text{Argon2-A}) = \mathcal{O}(N^{1.75} \log N)$ and $\text{aST}^{\parallel, \leftrightarrow}(\text{Argon2-B}) = \mathcal{O}(N^{1.8})$.

Conclusion

- We introduced **the parallel reversible pebbling game**, and
- We use this game to analyze the **reversible space-time complexity** of a line graph and **data-independent Memory-Hard Functions** such as Argon2i-A/B and DRSample
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Open Questions

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- Is there a DAG with constant indegree having (parallel) reversible ST-cost $\Omega(N^2)$?
 - Candidate: DRS+BRG [BHK⁺19], none of our attacks performed well against DRS+BRG
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References I

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