How to Obfuscate MPC Inputs

Theory of Cryptography Conference 2022

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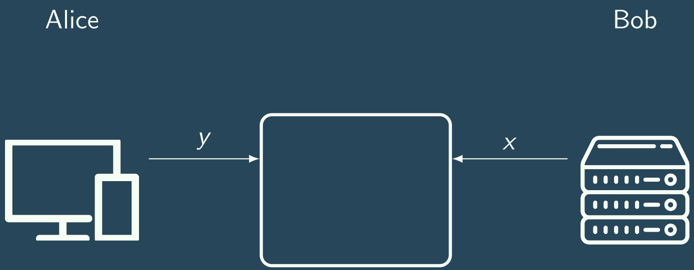
Introduction

- Bob wants to provide a service to Alice using his input x.



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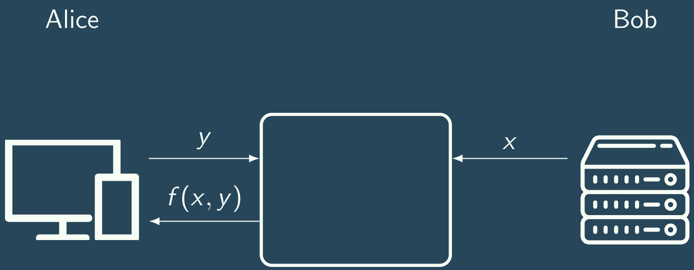
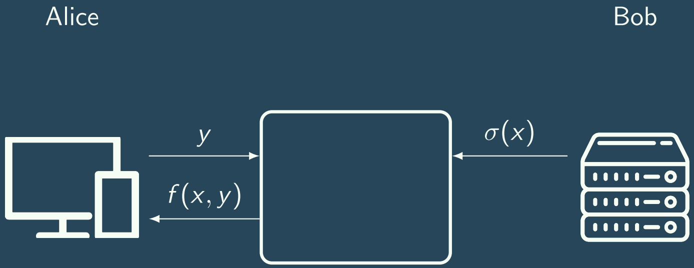
- But both Alice's and Bob's inputs contain private data.



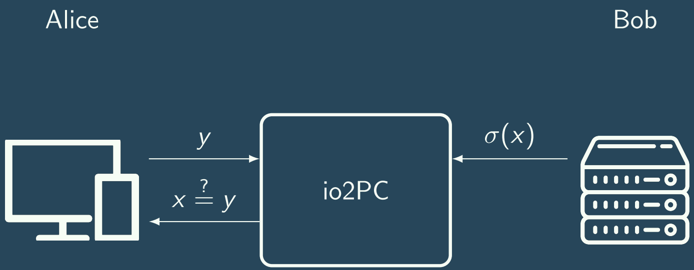
Introduction

- Bob wants to provide a service to Alice using his input x.
- But both Alice's and Bob's inputs contain private data.
- Bob is worried about compromise of his service leaking x.



io2PC- Point Functions

- Evaluating $x \stackrel{?}{=} y$ online is oblivious and interactive.



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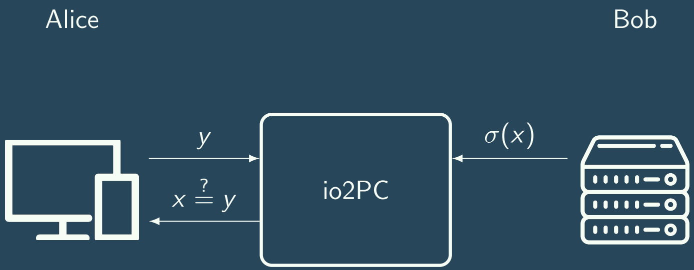

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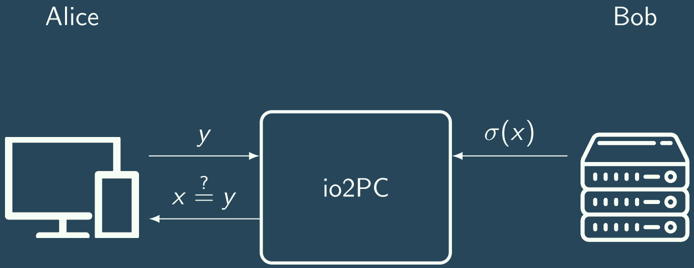
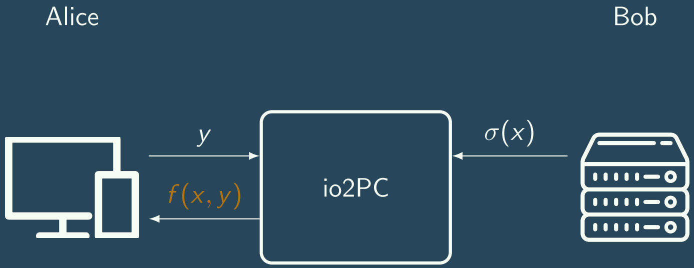


io2PC- General Case

- Evaluating $f(x, y)$ online is oblivious and interactive.
- Only an oracle $f(x, \cdot)$ is leaked on Bob's compromise.
- Offline evaluation of $f(x, y)$ must be done *post-compromise*.



io2PC

Theorem

There exists an UC-secure io2PC protocol for a function f, if the related class of functions $\mathcal{C}_f = \{f(x, \cdot) \mid x \in \{0,1\}^n\}$ has a VBB obfuscation in either the random oracle or generic group models.

io2PC

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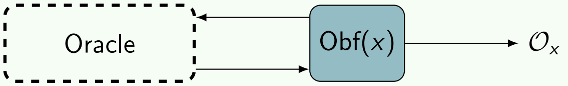
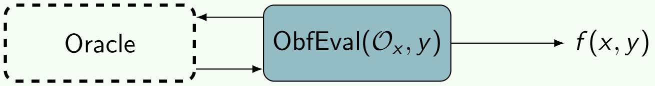
- We achieve this by replacing the corresponding non-interactive oracle queries with interactive protocols.

Virtual Black-Box Obfuscation

Obfuscation



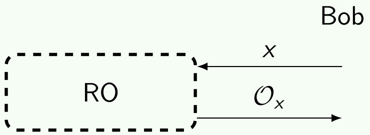
Evaluation



A VBB obfuscation $\mathcal{O}_x$ can be *simulated* with only oracle access to $f(x, \cdot)$.

Virtual Black-Box Obfuscation - Point Function

Obfuscation



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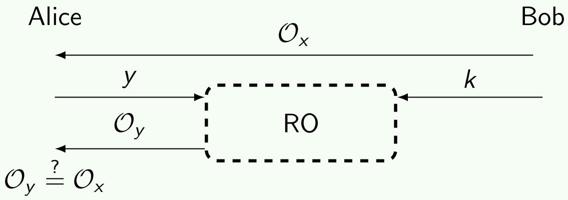
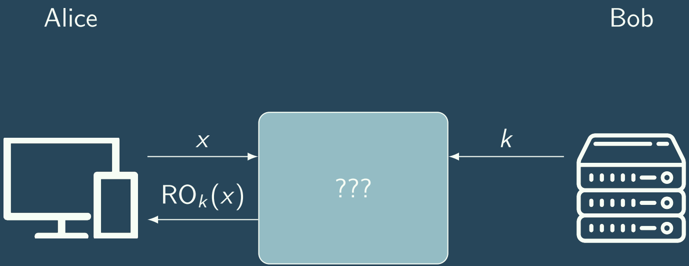


Interactive Random Oracles

What does an “interactive random oracle” look like?



Oblivious Psuedorandom Functions

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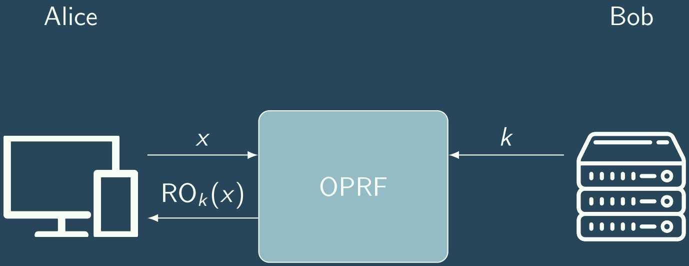
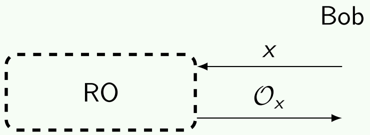


- JKKX16 provides an oblivious psuedorandom function (OPRF) achieving this property!

Input Obfuscation in the Random Oracle Model

Obfuscation



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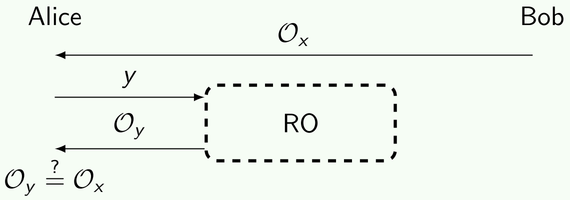

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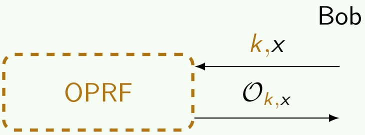

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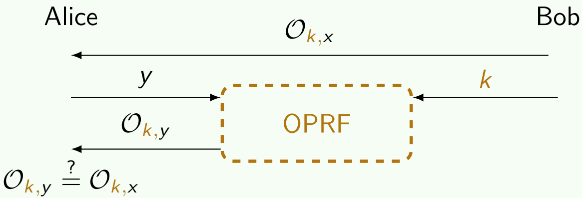

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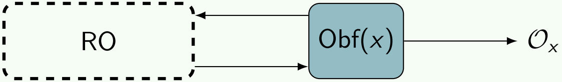

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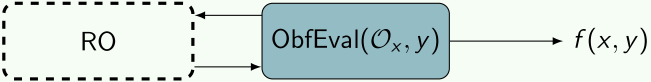

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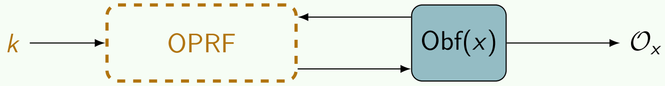

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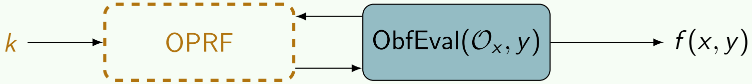

- Why is this not trivial in 2pc? — The idealized primitives are exponential in size!

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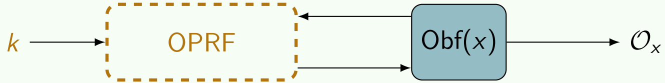

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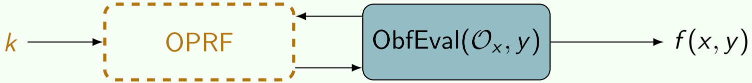

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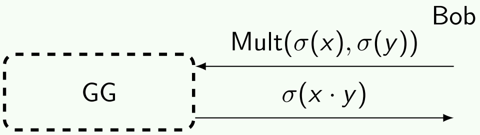
Can we construct interactive versions of other idealized primitives?

What about generic groups?

Generic Groups

For a uniform encoding $\sigma : \mathbb{Z}_p \to \{0,1\}^*$

Addition



Generic Groups

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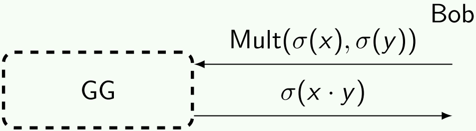
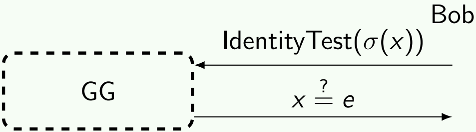
Multiplication



Identity Test



Interactive Generic Groups

- Given a publicly accessible GG $\mathcal{G} := (g, \cdot)$ and a "key" $(k \leftarrow \mathcal{K}, \hat{g} \leftarrow \mathcal{G})$.
- We construct an iGG where operations are interactive, oblivious, and require the key.
- Elements take the form $(F_k(m), \hat{g}^x \cdot g^m)$.

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- $\hat{g}^{x_1+x_2} \cdot g^{m_3}$ can be computed using the public group.
- $F_k(m_3)$ can be computed *interactively* in 2PC.

Personalized Generic Groups

- Given a publicly accessible GG $\mathcal{G} := (g, \cdot)$ and a "key" $(k \leftarrow \mathcal{K}, \hat{g} \leftarrow \mathcal{G})$.
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Identity Test

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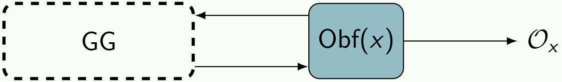
- $g_1 \stackrel{?}{=} g^m$ can be calculated using the public group.
- Alice *interactively* learns blindings ${g_1}^b$ and g^{bm} which she compares.

Input Obfuscation in the Generic Group Model

Obfuscation



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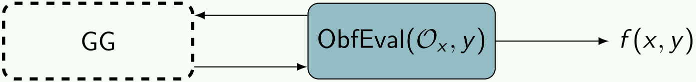

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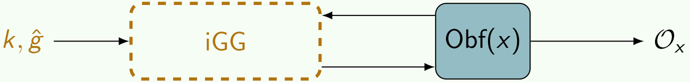

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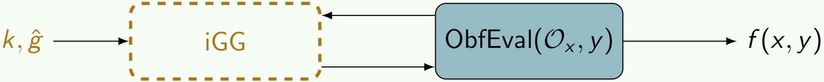

Conclusion

- We introduce the study of input obfuscation for secure two-party computation (io2PC).
- We provide a compiler from VBB in the GGM and ROM to io2PC.
- To construct the latter, we provide an oblivious, interactive GG analogous to an OPRF.
- We provide explicit io2PC protocols for point functions and hyperplane membership using our compiler.

Conclusion

- We introduce the study of input obfuscation for secure multi-party computation (io2PC).
- We provide a compiler from VBB in the GGM and ROM to UC-secure io2PC.
- To construct the latter, we provide an oblivious, interactive GG analogous to an OPRF.
- We prove that known VBB obfuscations of point functions and hyperplane membership are compatible.
- We conjecture that io2PC is possible for generic graded encodings and therefore all $\mathcal{P}$.