Pseudorandom (Function-Like) Quantum State Generators

New Definitions And Applications

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Henry Yuen (Columbia University)

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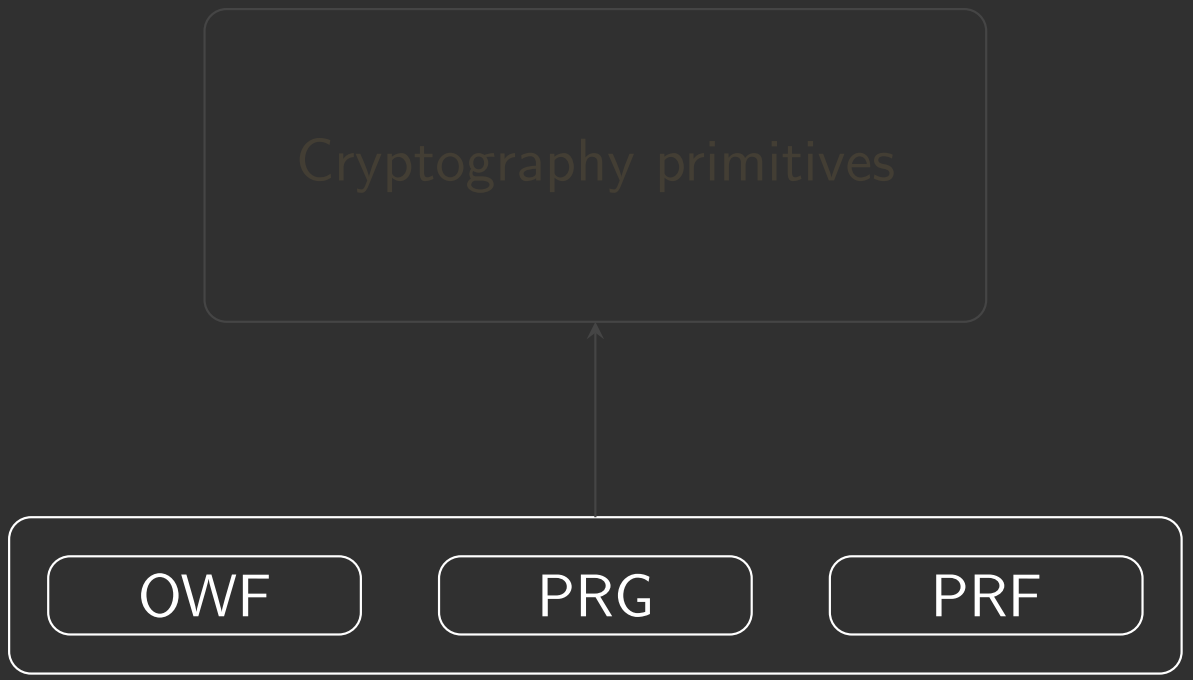
November 7, 2022

Pseudorandomness

Classical



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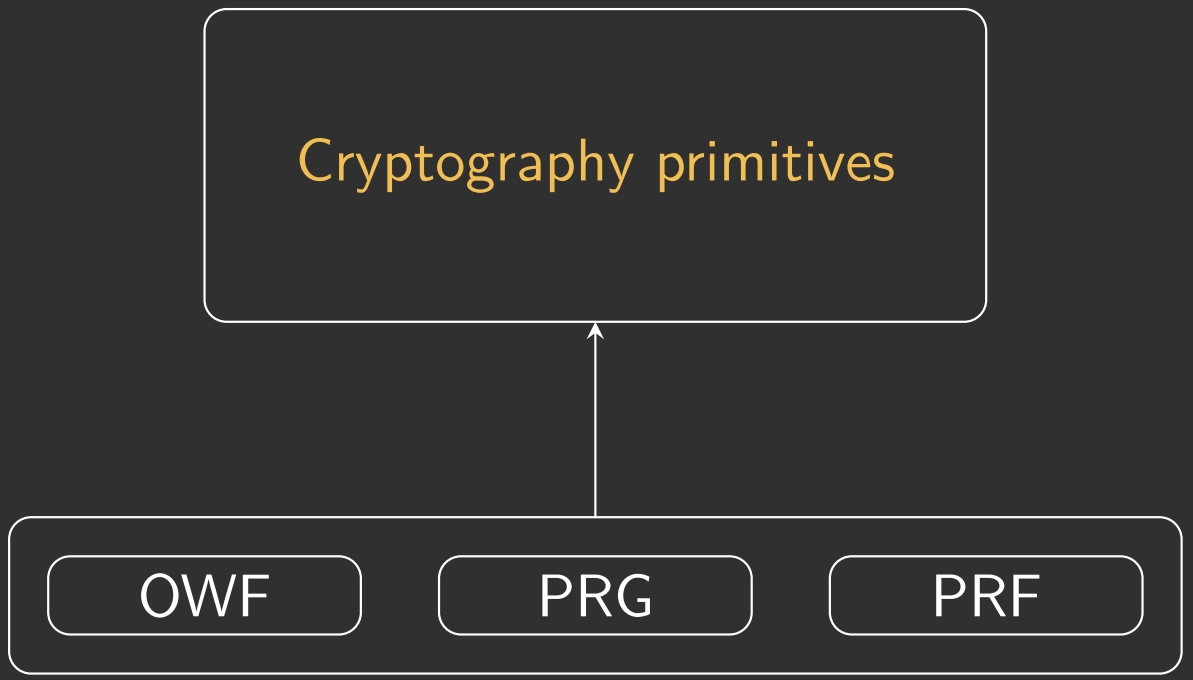

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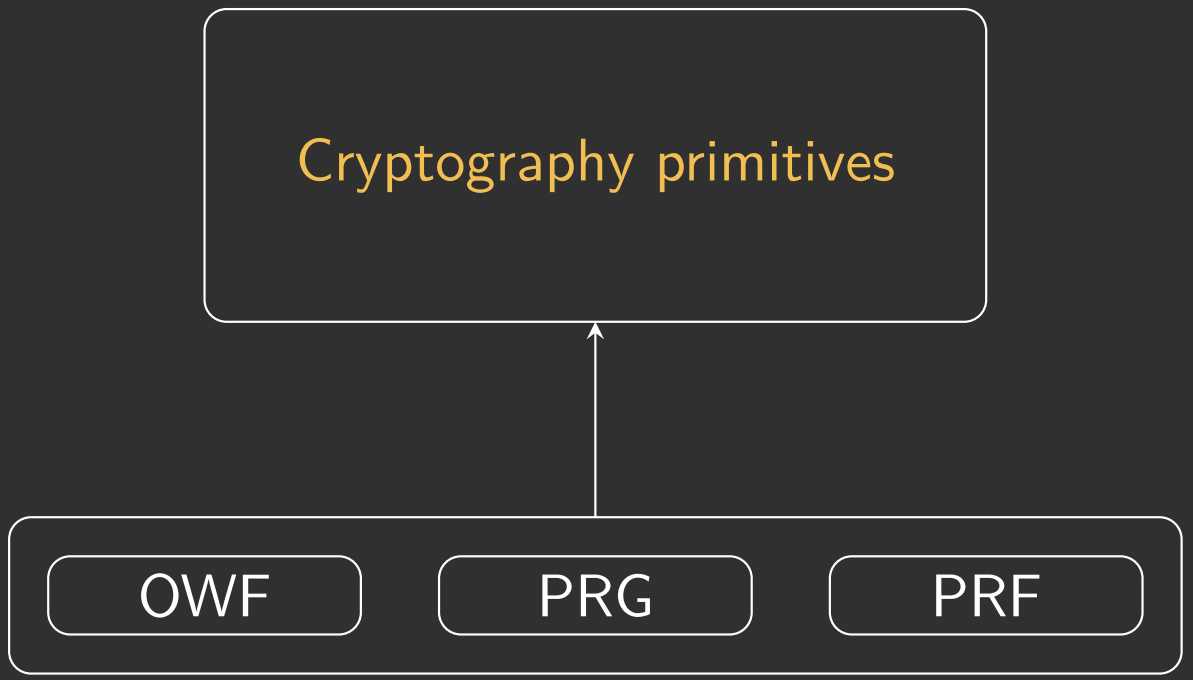

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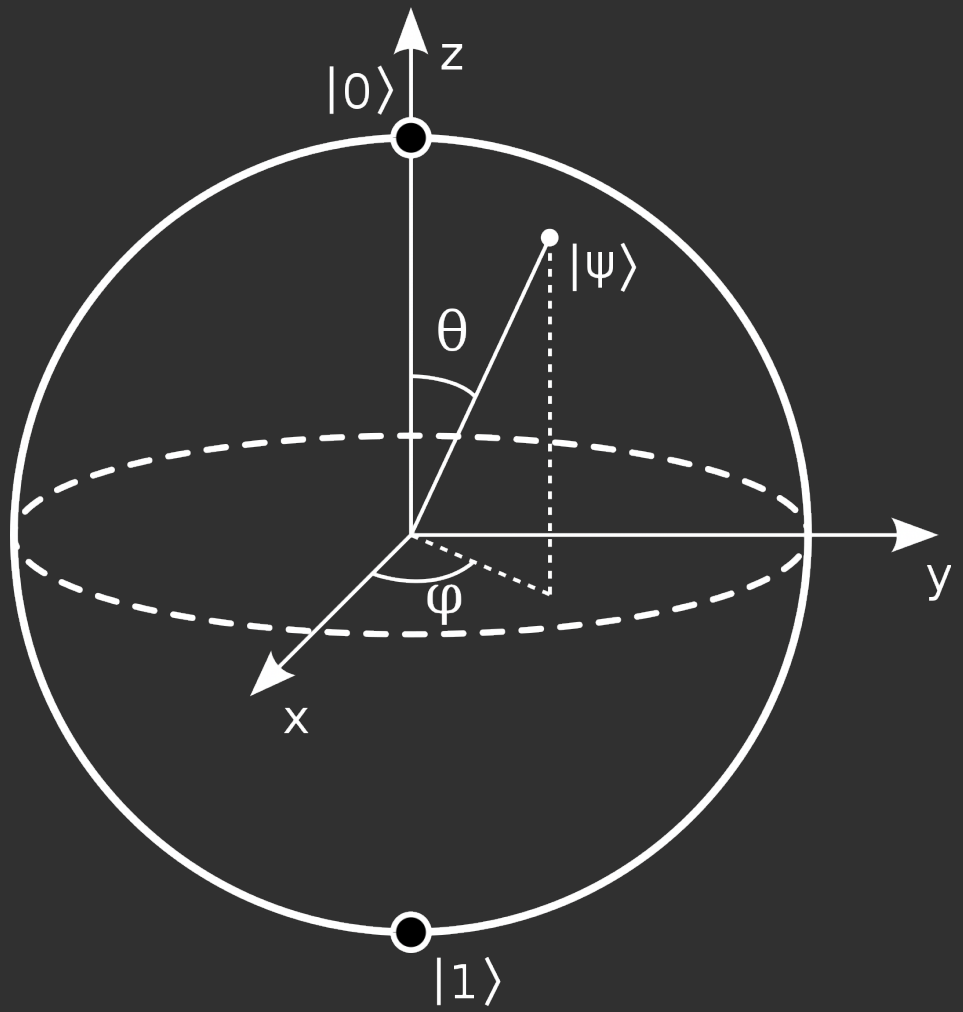
Haar-Random State



Haar random state looks like picking a random point on the bloch sphere.

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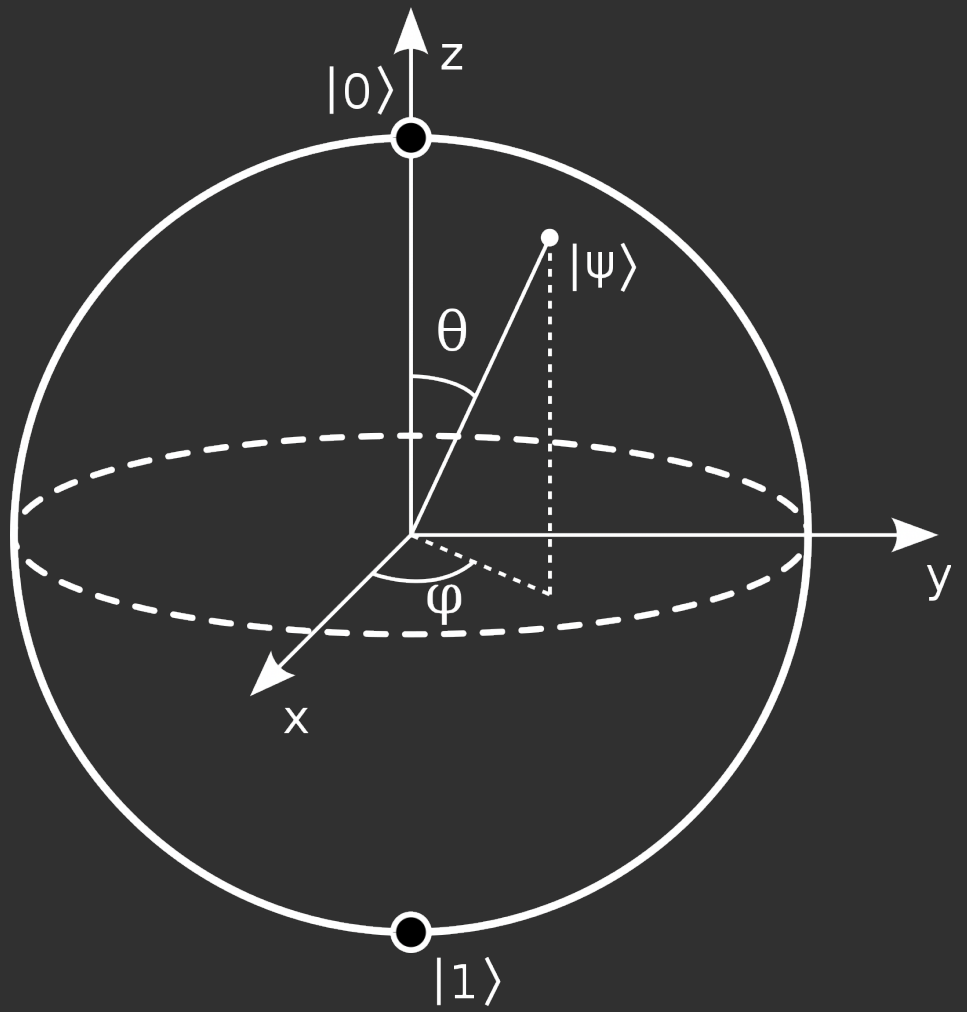

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Pseudorandom Quantum State (PRS)

Efficiently computable states that “look like” Haar random states.

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PRS generator (Poly-sized circuit)

Notation: n-PRS

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Can't copy quantum states!

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Constructions

PRS from one-way functions.

- Ji-Liu-Song'18
- Brakerski-Shmueli'19,20

PRF

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Binary phase PRS

PRS vs OWF

[Kretshmer'TQC'20]

There is an oracle with respect to which:

- OWF doesn't exist
- PRS exists

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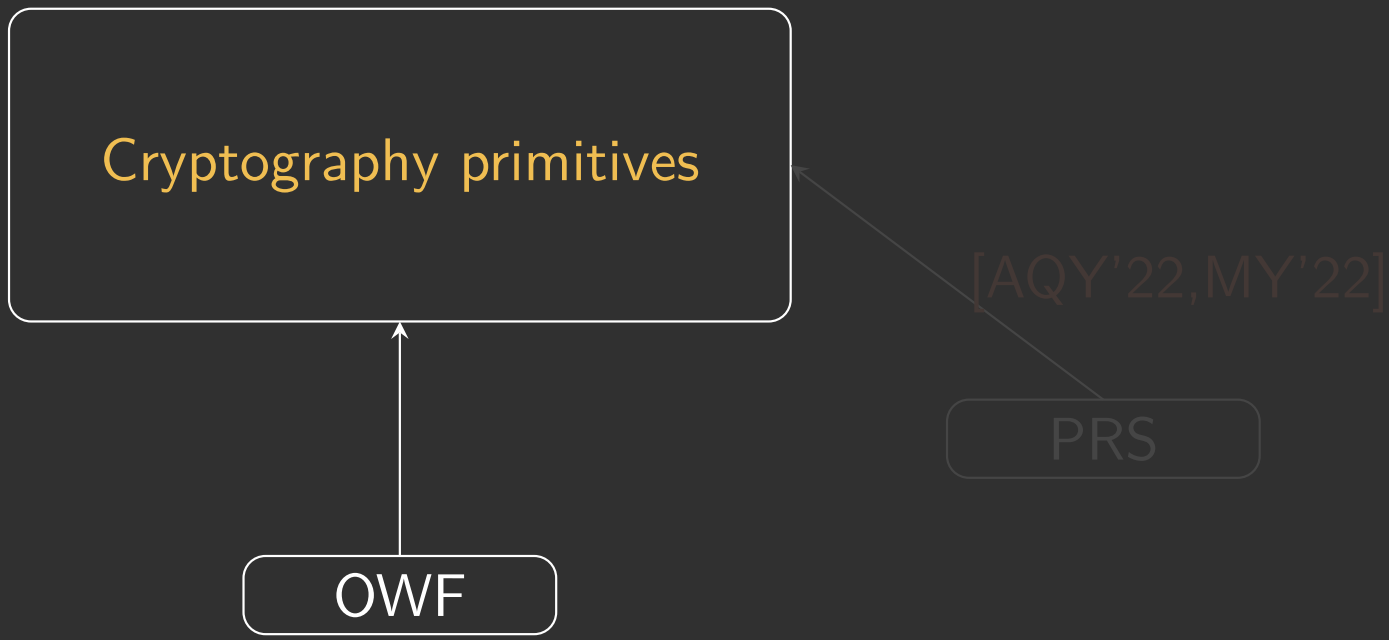

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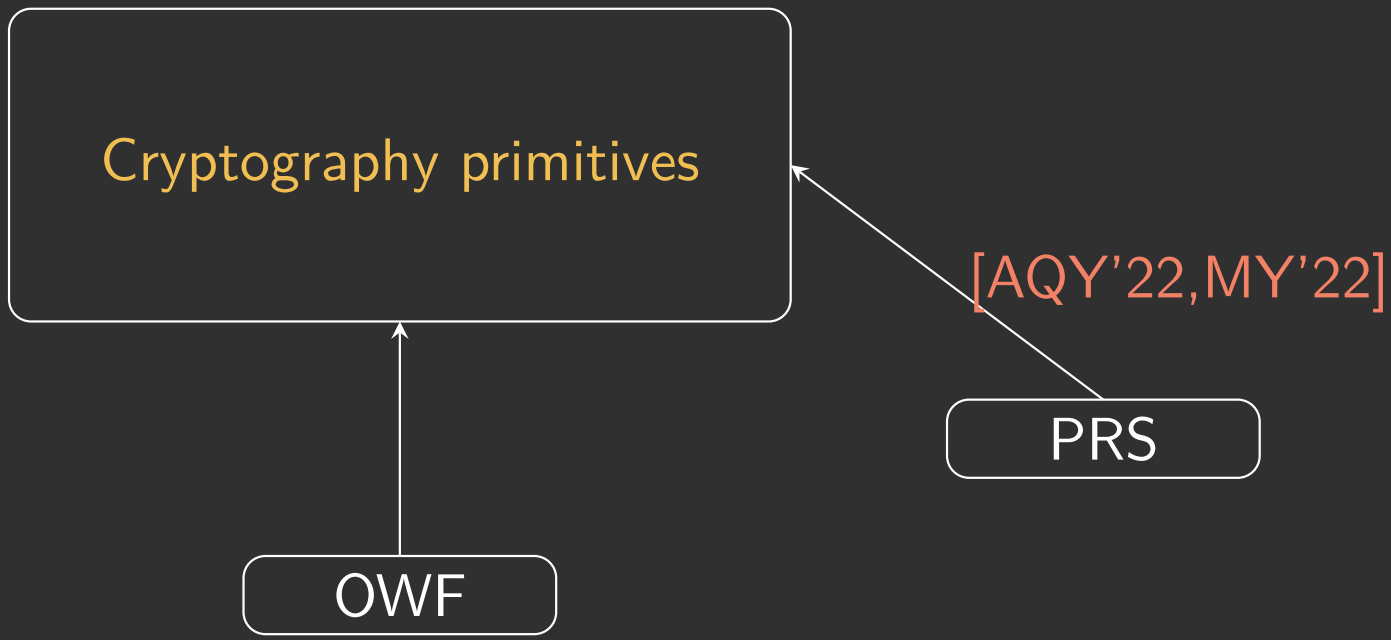

Psuedorandom Function-Like State (PRFS)

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1. Efficient generation:

Notation: (n, d)-$PRFS$

Pseudorandom Function-Like State (PRFS)

$\forall q(\lambda) = \mathsf{poly}(\lambda), x_1, \ldots, x_q,$

2. Selective Security:[AQY'22]

$$\{F(k,x_1)^{\otimes q}, \ldots, F(k,x_q)^{\otimes q}\} \approx_c \left\{|\psi_{x_1}\rangle^{\otimes q}, \ldots, |\psi_{x_q}\rangle^{\otimes q}\right\}$$

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Might not be sufficient for many applications.

2. Adaptive Security:

- Adversary can choose $x_1, \ldots, x_q$ adaptively.
- Quantum access: query $\sum_{x,y} \alpha_{x,y}|x,y\rangle$ and receive $\sum_{x,y} \alpha_{x,y}|x,y \oplus F(k,x)\rangle$.

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Pseudorandom Function-Like State (PRFS)

Q2. Does PRFS with these stronger notions exist?

Stronger security notion (Contribution)

- Construct Adaptive PRFS from post-quantum one-way functions.
- Construct Quantum Access Adaptive PRFS from post-quantum one-way functions.
- Simpler proof of Binary Phase PRS by getting a better representation of Haar random states
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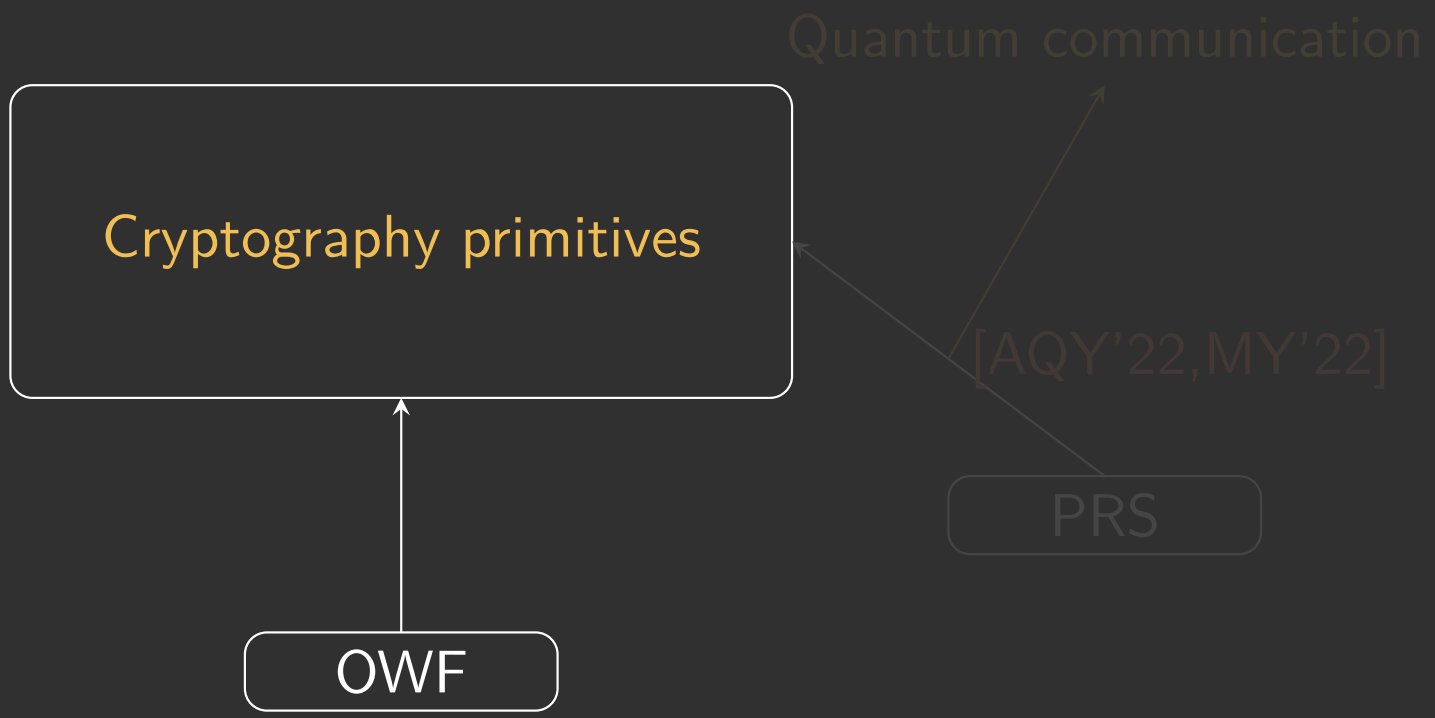
Applications to cryptography

Cryptography from PRS



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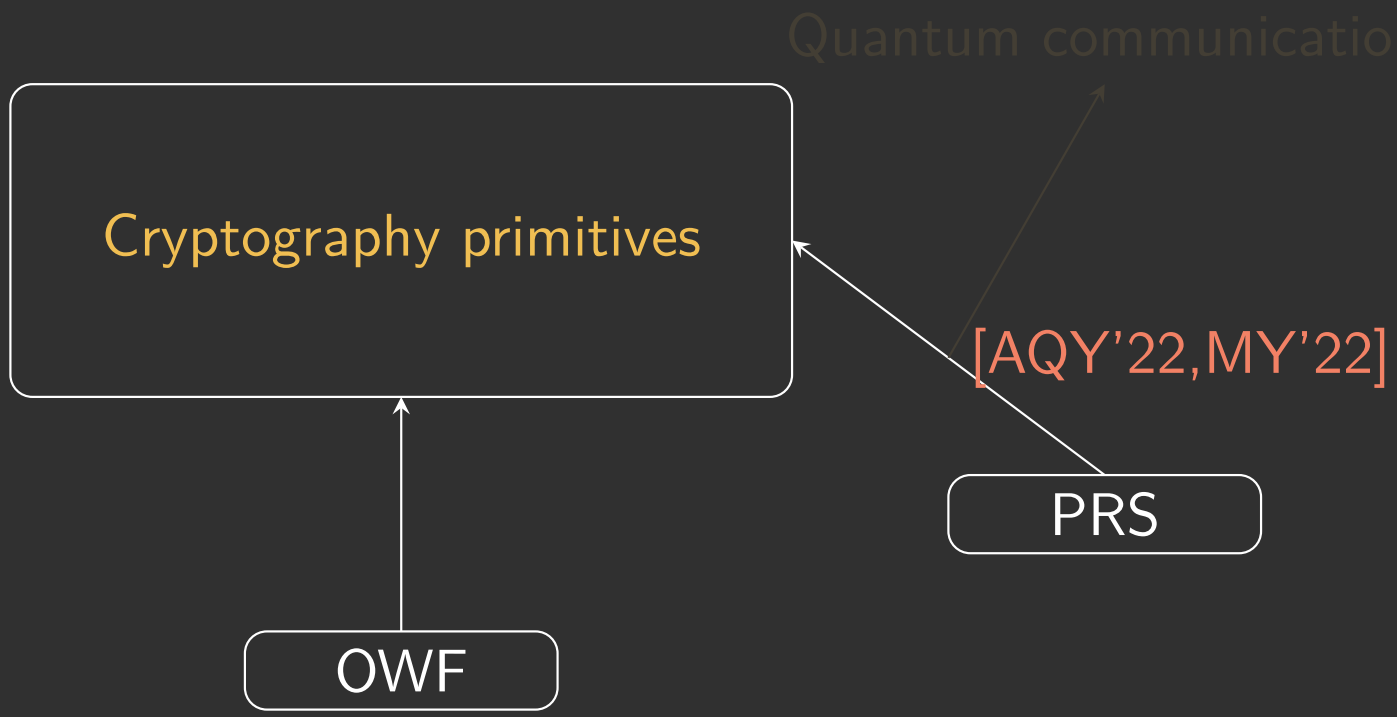

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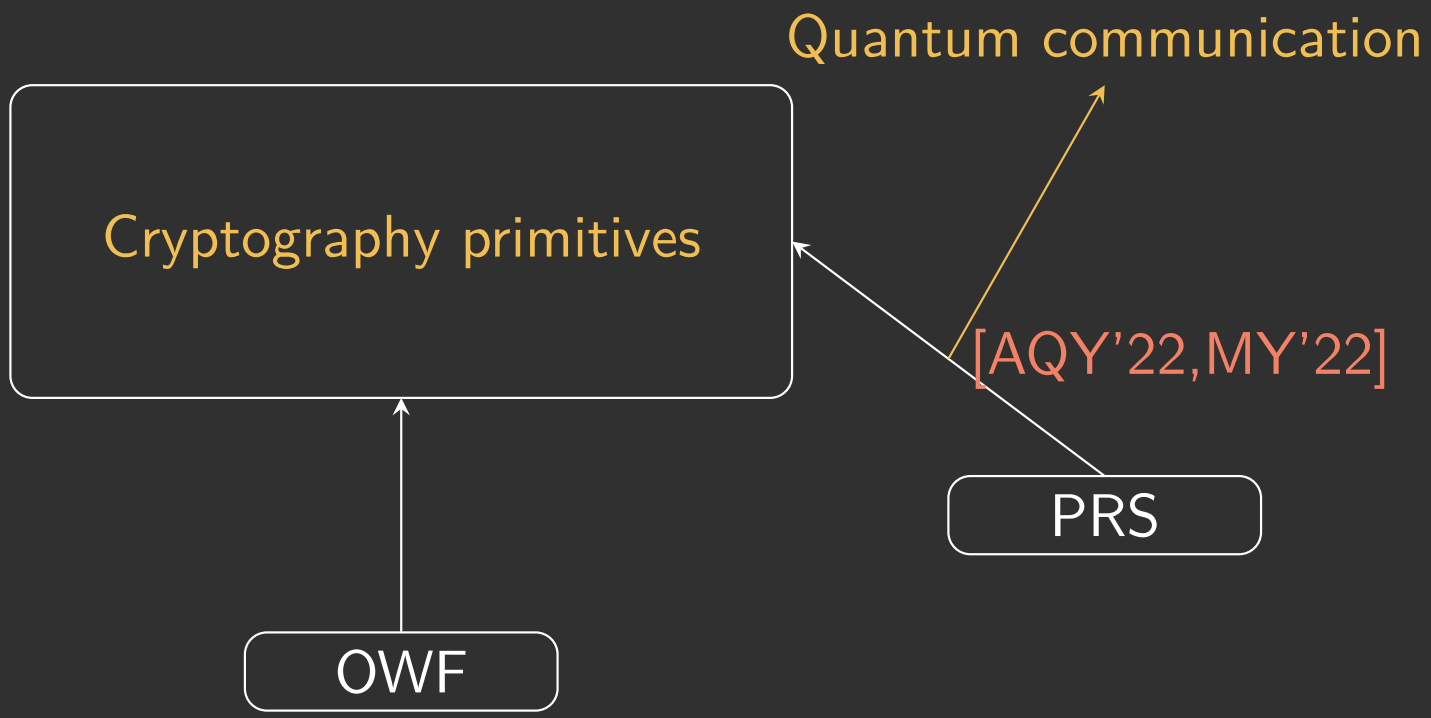

Applications to cryptography

Q3. Is quantum communication necessary in PRS-based primitives?

PRS-based crypto with classical communication (Contribution)

- PRS-based commitment scheme with computational binding and statistical hiding with only classical communication.
- PRS-based QOTP with only classical communication.

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Techniques
Need for computational assumption

Computational assumptions

Inefficient

G is a $\log(\lambda)$-PRS, then $\exists A_\lambda$,

$$\left|\Pr\left[A_\lambda\left(G(k)^{\otimes t}\right)=1\right]-\Pr\left[A_\lambda\left(|\psi\rangle^{\otimes t}\right)=1\right]\right|\geq\frac{1}{3}$$

Haar random

Computational assumptions

$G(k)$ is almost pure.

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Computational assumptions

Distinguish by projecting here

PRS

$$\dim\left(\text{Span}\left\{|\psi_k\rangle\langle\psi_k|^{\otimes t}\right\}\right) \leq 2^{\lambda}$$

Haar

$$\dim\left(\text{Span}\left\{|\psi\rangle\langle\psi|^{\otimes t}\right\}\right) = \binom{2^n + t - 1}{t}$$

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$$t = \lambda + 1$$

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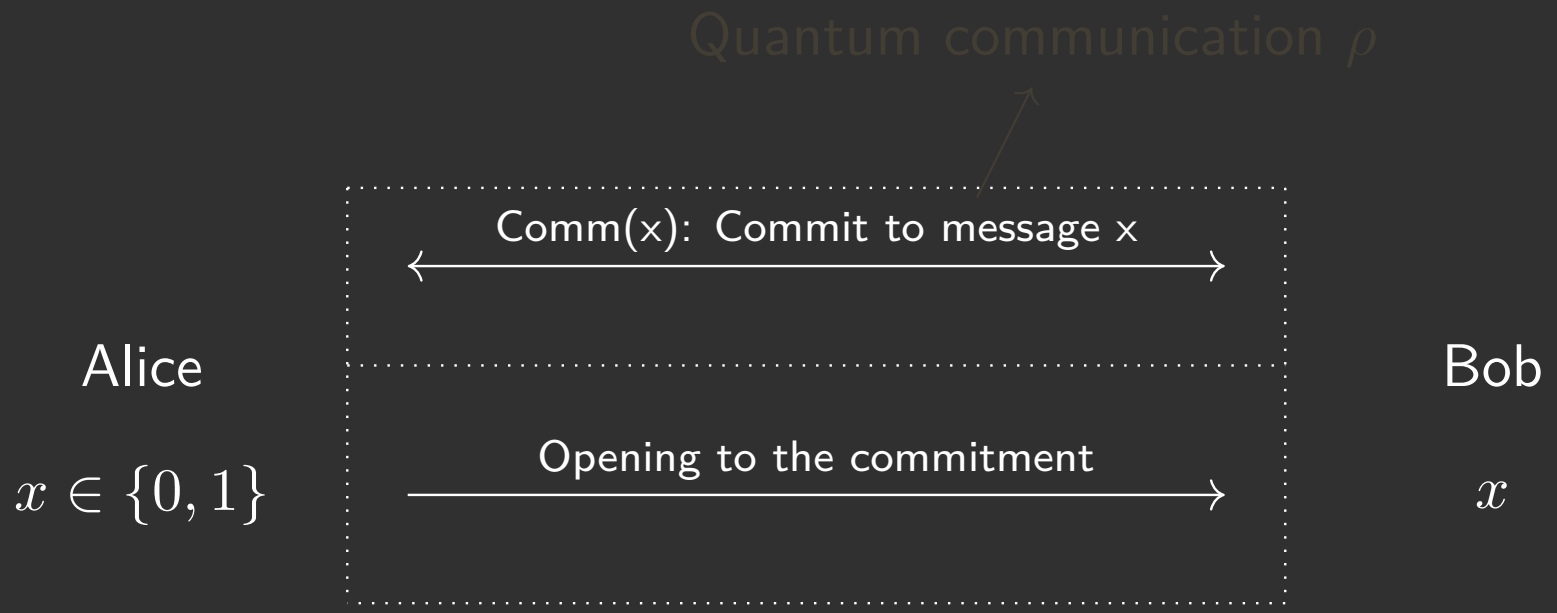
Techniques
Commitment scheme

Commitment scheme [AQY'22]



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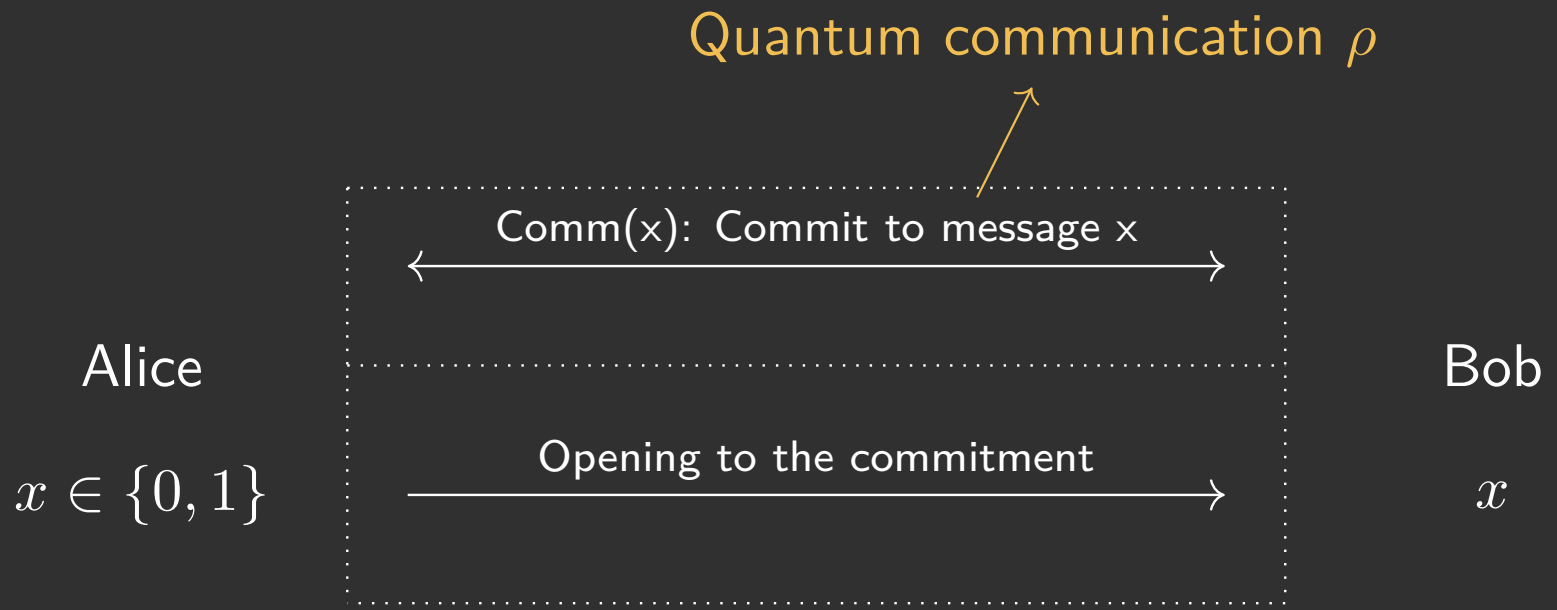

Tomography

Procedure to get a classical description of a quantum state:

$$\text{Tomography}(\rho^{\otimes \exp(n)}) = M \longrightarrow n = \log(\lambda)$$

such that

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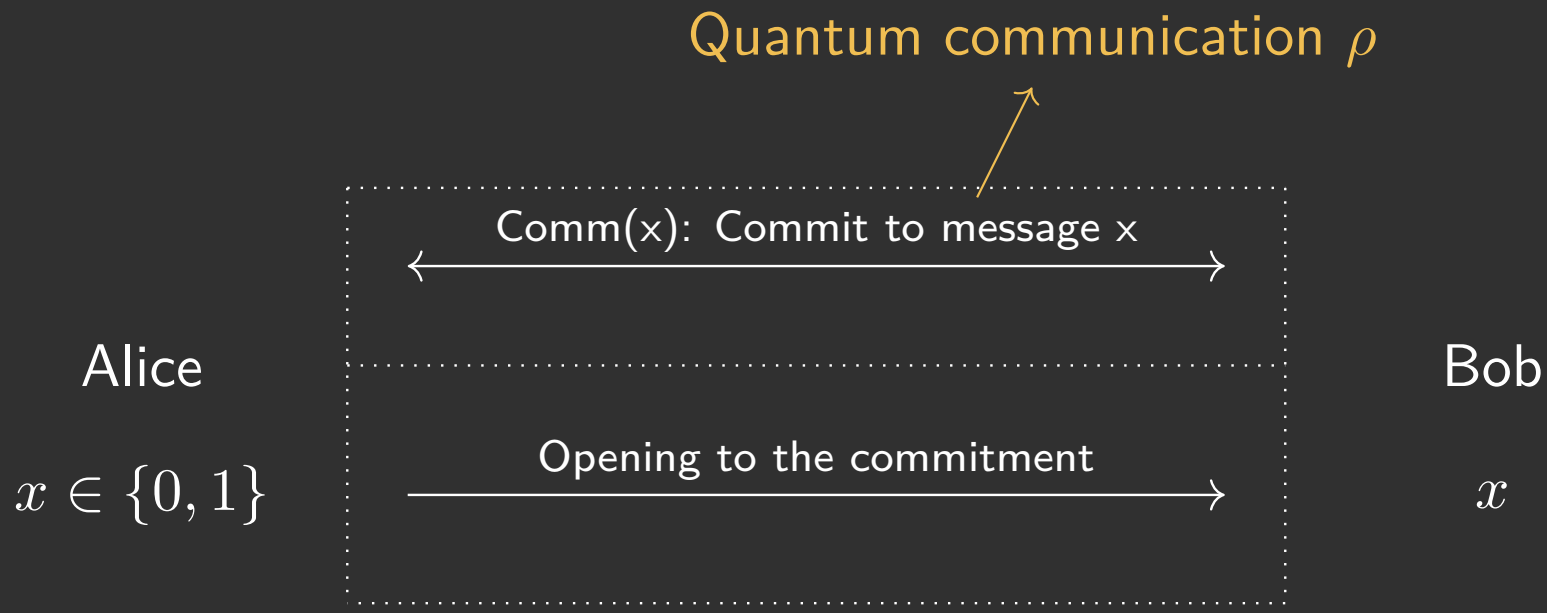

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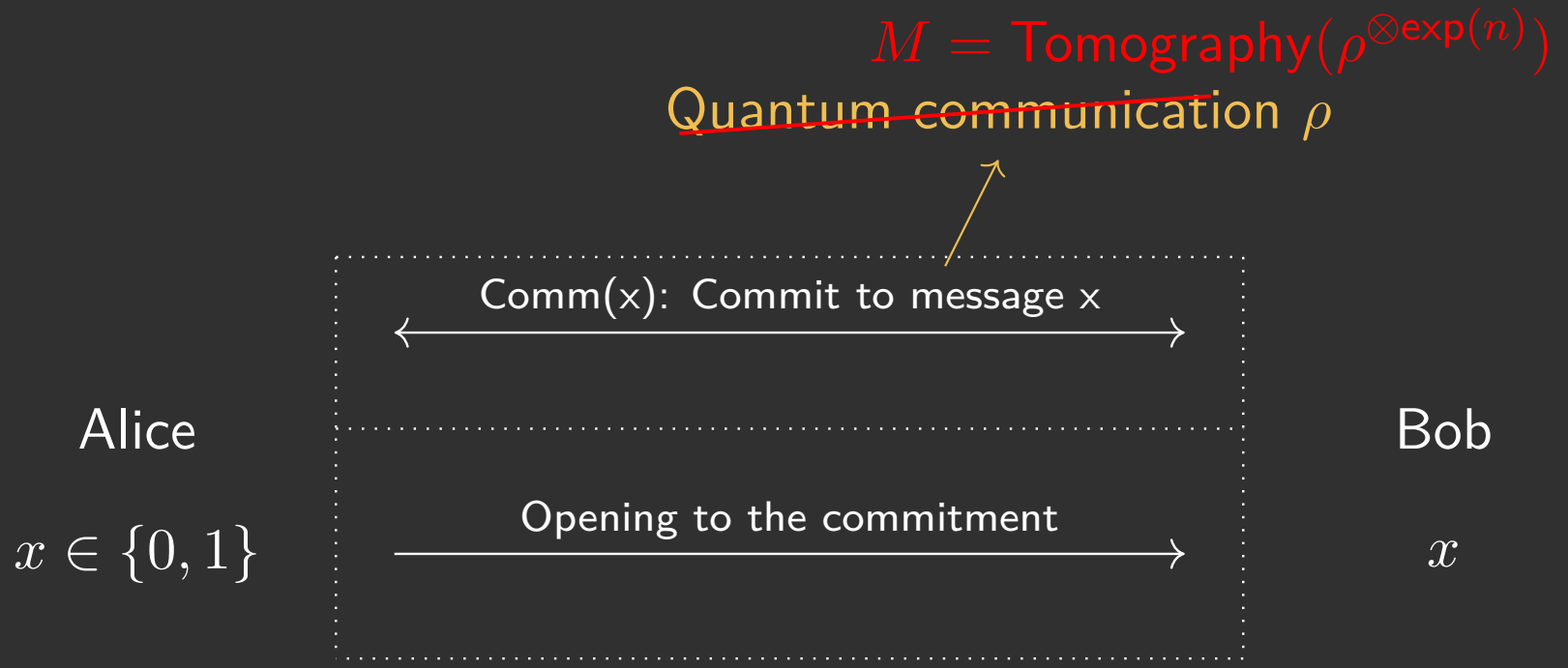

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Malicious Adversaries can cheat, hence we need a stronger notion!

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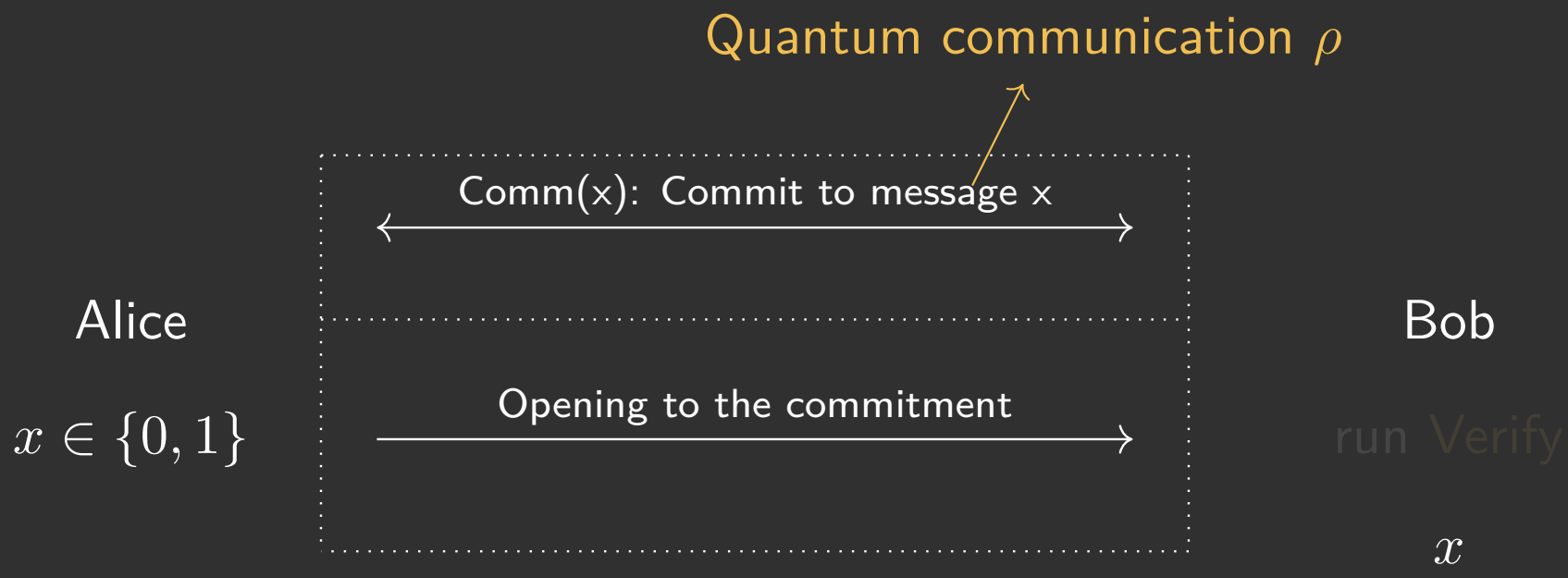

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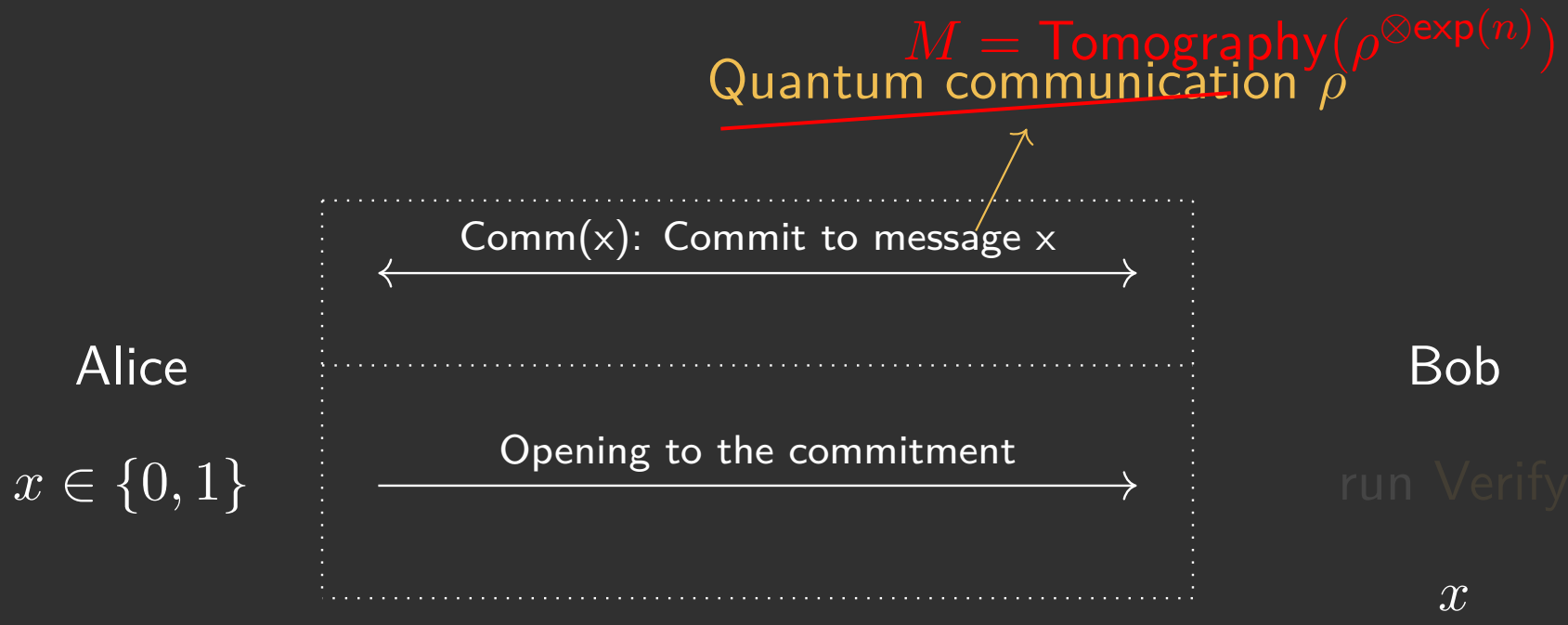

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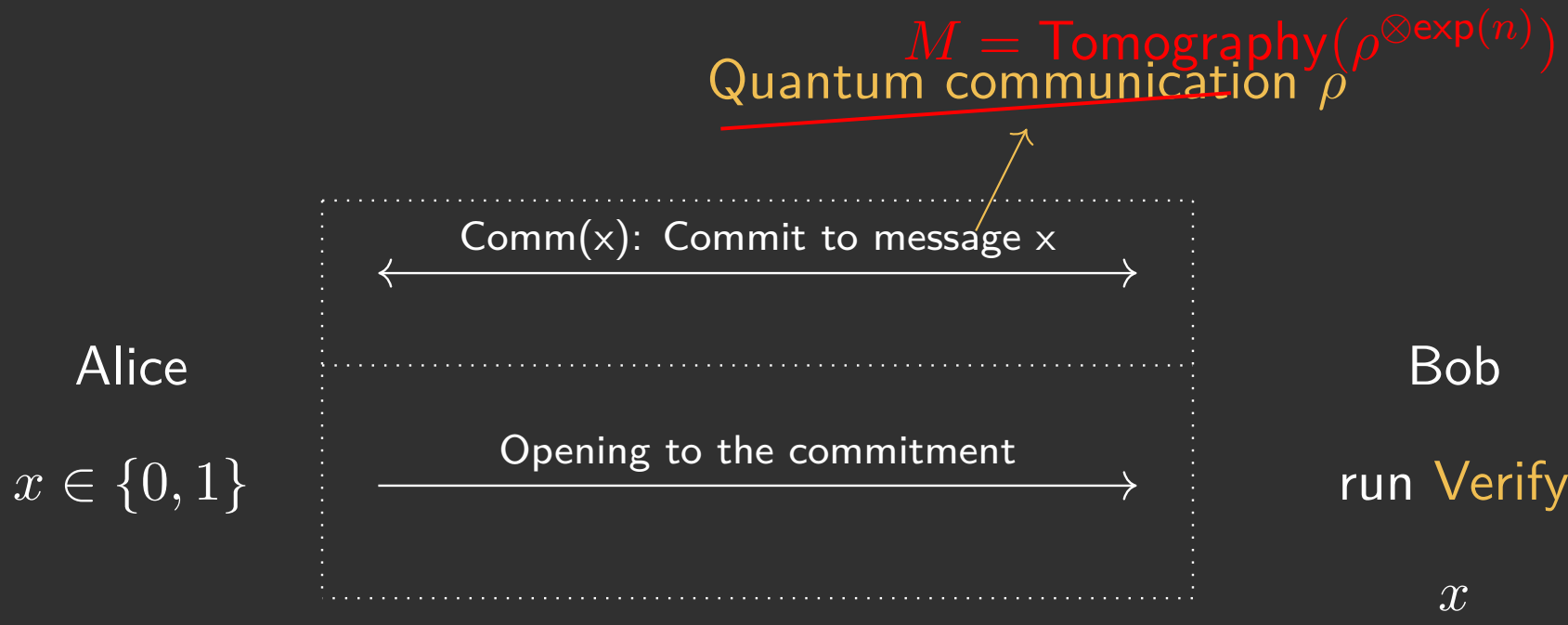

Summary

- PRS with $\omega(\log(\lambda))$-output length requires computational assumptions
- Simpler Proof of Binary Phase PRS
- New variants of PRS and their constructions
- Commitments (and other primitives) from PRFS with classical communication

Thank You