Collusion-Resistant Copy-Protection for Watermarkable Functionalities

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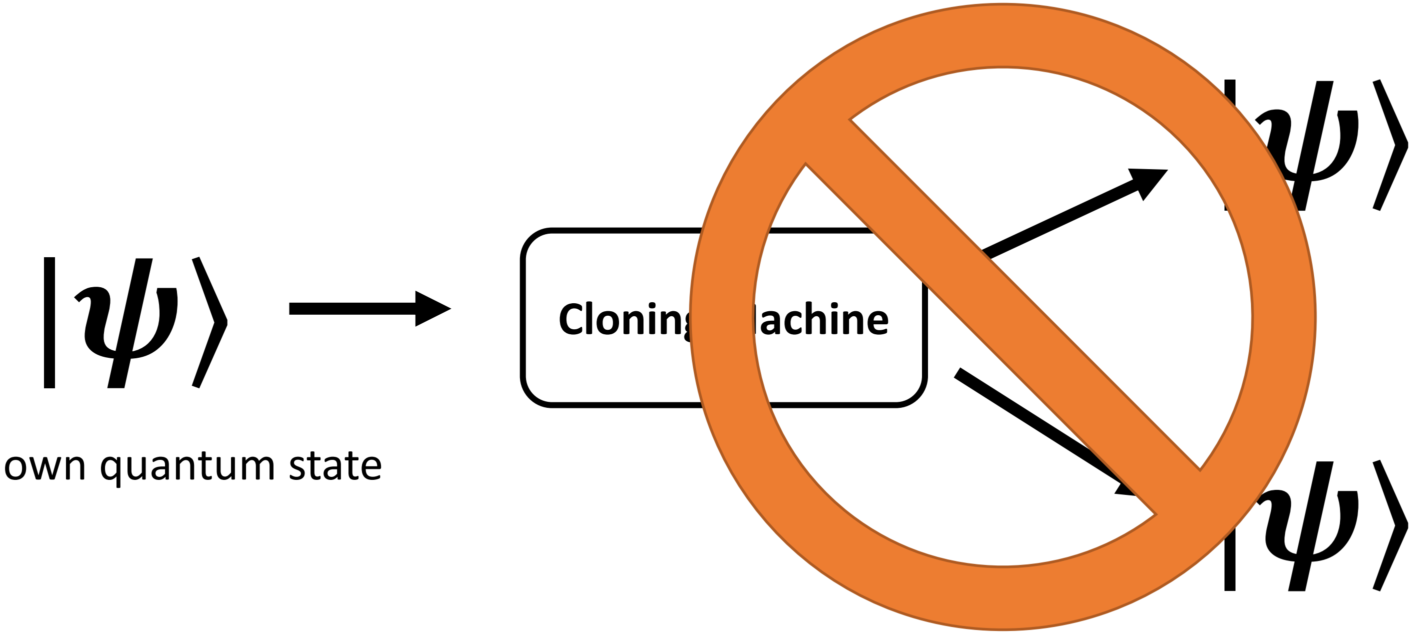
NTT Research

No-Cloning Theorem



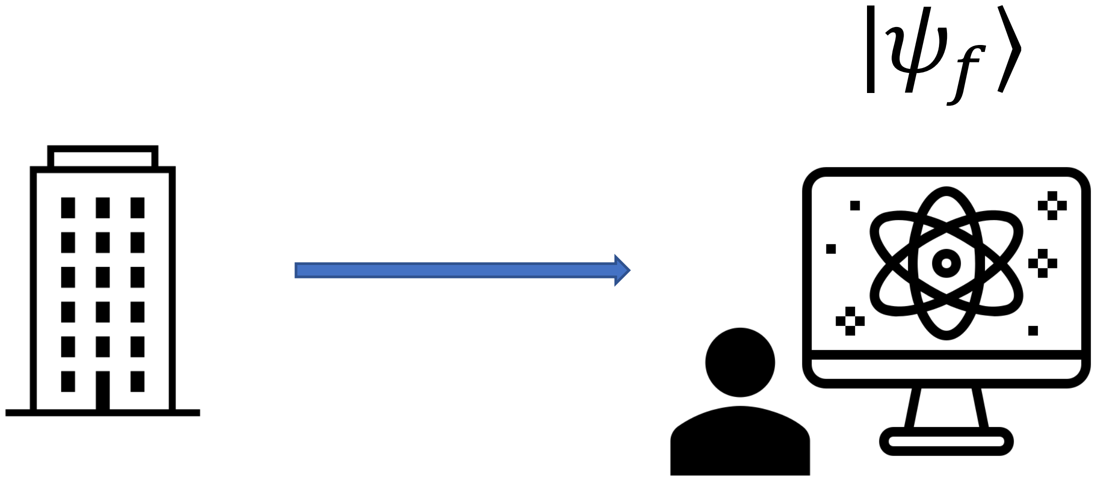
Classically Impossible Primitives

- QKD [BB’84]
- Quantum Money [Wiesner’69, AC’12, Zhandry’19…]
- Quantum Copy-Protection [Aaronson’09, CLLZ’21 …]
- Signature Token [BS’16, AGKZ’20, CLLZ’21]
- Unclonable Encryption, Decryption [Gottesman’02, BL’19, GZ’20, CLLZ’21]
- …

Quantum Copy Protection

Can we obtain unclonable states with ”functionalities”?

[Aaronson09]: Quantum Copy Protection for softwares

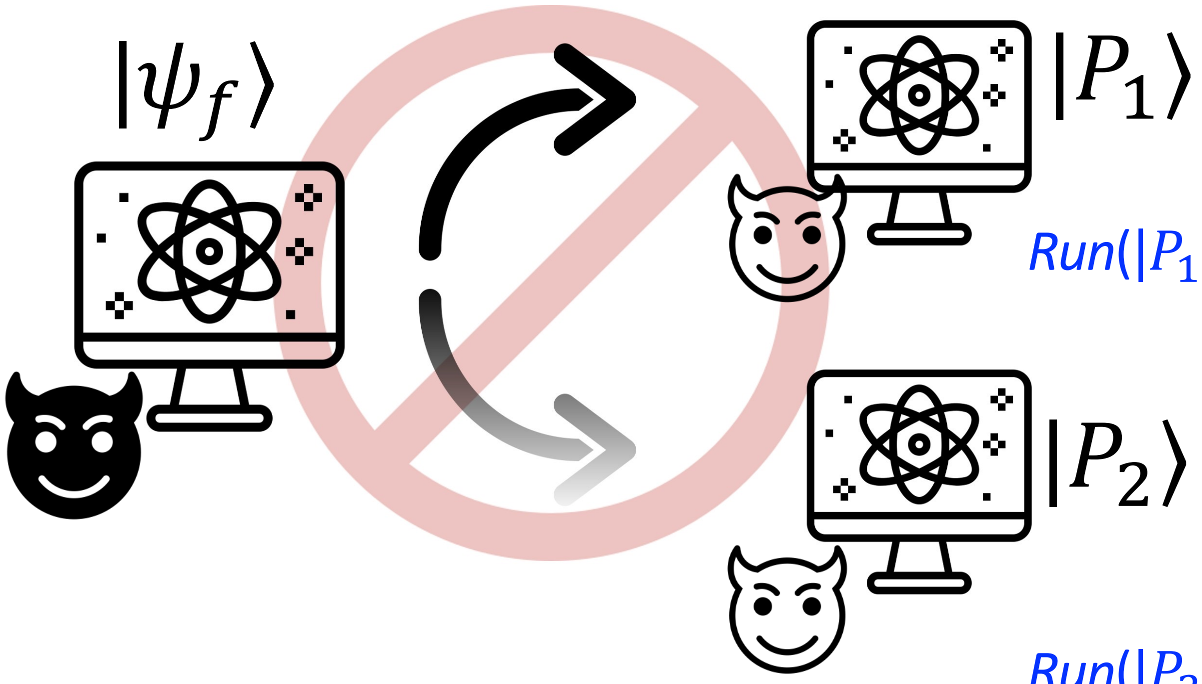


Classical: $f \xrightarrow{\text{copy−protect}} |\psi_f\rangle$

$\not\forall x,\ Run(|\psi_f\rangle\ ,x)\ =\ f(x)$

Quantum Copy Protection

[Aaronson09]: Quantum Copy Protection for softwares



Run$(|P_1\rangle, x) = f(x)$?

Run$(|P_2\rangle, x) = f(x)$?

Detour: Watermarking

- Watermarking:
  - cannot remove watermark without destroying functionality
- Copy-protection:
  - cannot clone without destroying functionality

- Intuitively, goal of adversary is to create illegal copies
  - Watermarking is the classical analogy of copy-protection

- Known watermarkable functionalities: decryption, PRF, signatures
  - [CHN+15, KW17, YAL+19, GKM+19, …]

Some Previous Results

	Unclonable Decryption	Copy-Protection PRF	Unclonable Signing Key
VBB/Oracle	VBB [GZ20, ALL+21]	VBB [ALL+21]	VBB [ALL+21]
Plain Model	iO + OWF [CLLZ21]	iO + OWF [CLLZ21]	Not known

[ALL+21]: Can all watermarkable functions be copy-protected?

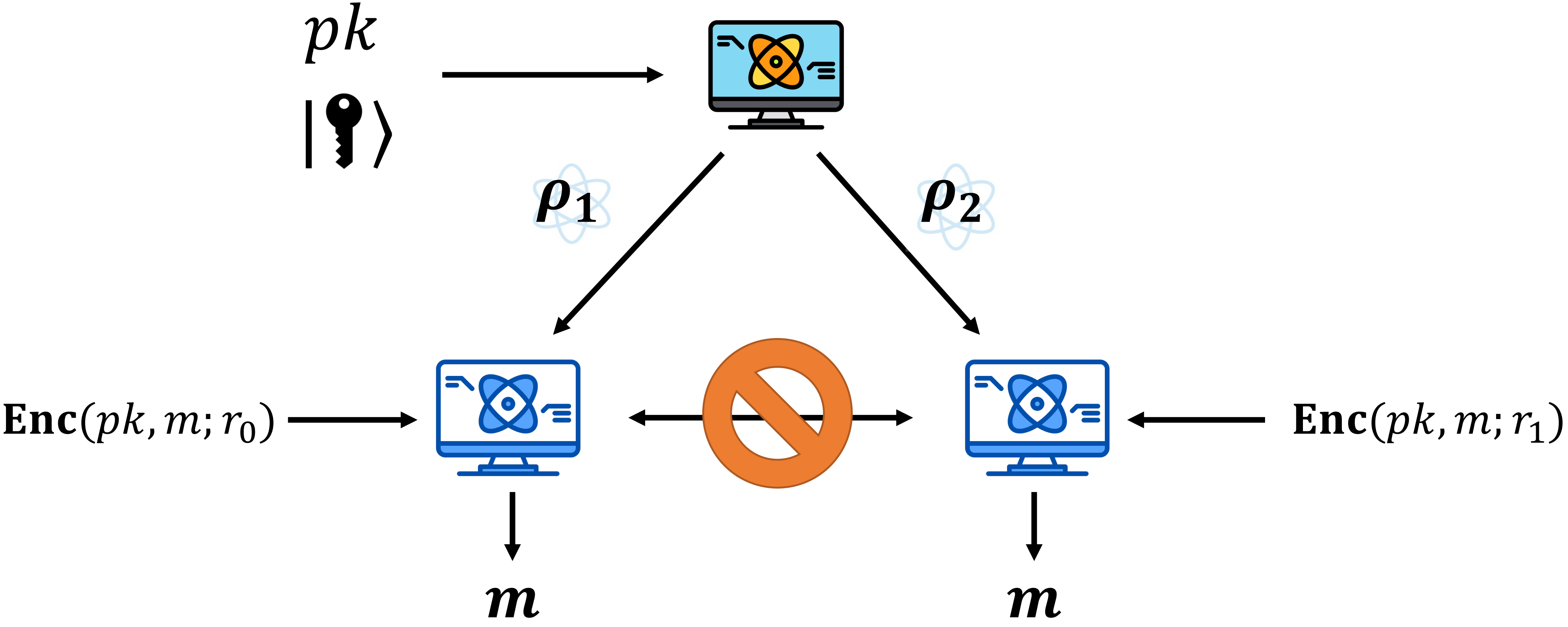
Unclonable Decryption (1->2) [CLLZ’21]

- **KeyGen**(λ): outputs pk, |🔑⟩

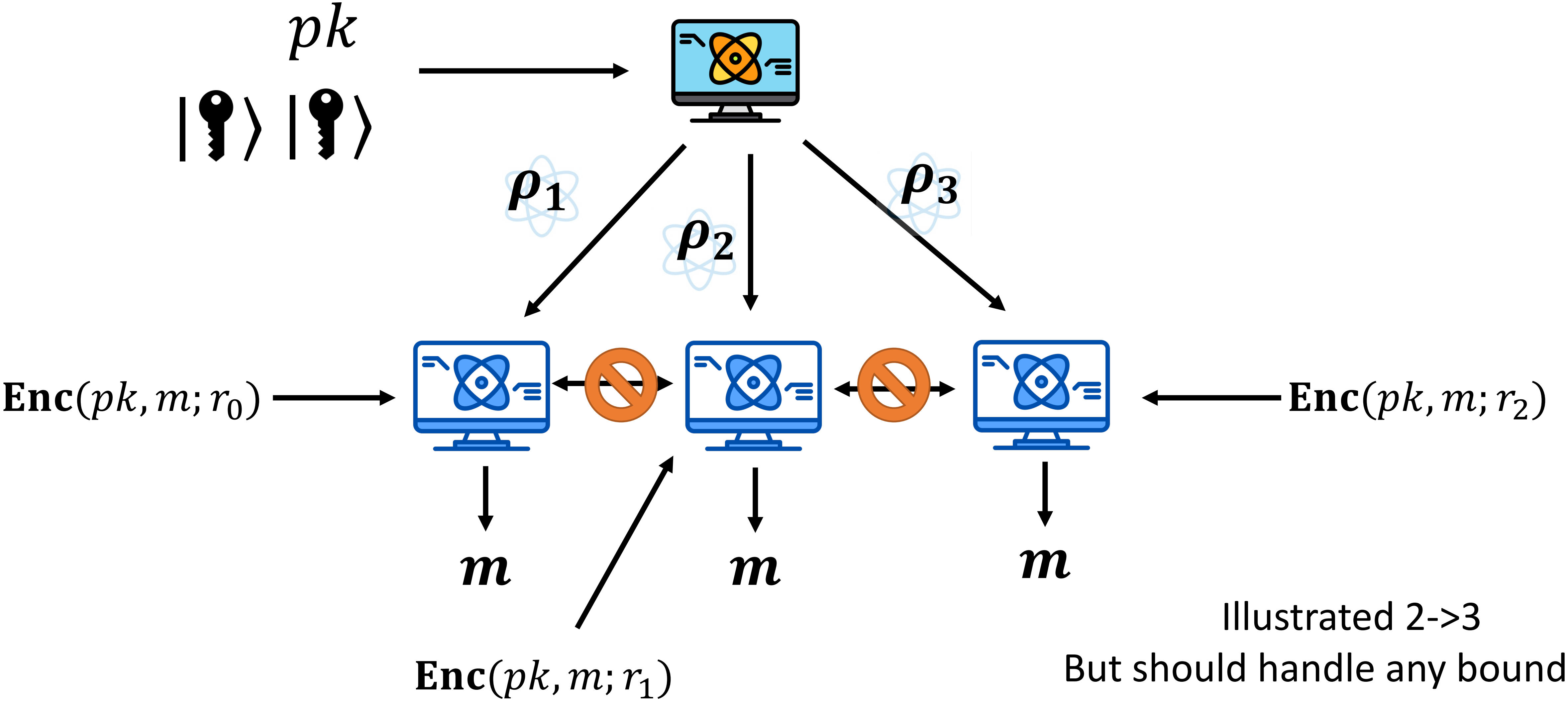
- **Enc**(pk, m): outputs c

- **Dec**(|🔑⟩, c): outputs m

Unclonable Decryption Key[GZ20,CLLZ21]



Collusion-Resistance (k->k+1)



pk

$|🔑\rangle\,|🔑\rangle$

$\boldsymbol{\rho_1}$

$\boldsymbol{\rho_2}$

$\boldsymbol{\rho_3}$

$\mathbf{Enc}(pk, m; r_0)$

$\mathbf{Enc}(pk, m; r_2)$

$\boldsymbol{m}$

$\boldsymbol{m}$

$\boldsymbol{m}$

$\mathbf{Enc}(pk, m; r_1)$

Illustrated 2->3

But should handle any bounded k

Previous Results

	Unclonable Decryption	Copy-Protection PRF	Unclonable Signing Key
VBB/Oracles	VBB [GZ20]	VBB [ALL+21]	VBB [ALL+21]
Plain Model	iO + OWF [CLLZ21]	iO + OWF [CLLZ21]	Not known

Not Collusion-Resistant!

Our Results: Collusion-Resistant CP

	Unclonable Decryption	**Copy-Protection PRF**	**Unclonable Signing Key**
Plain Model	[This Work] iO + OWF	[This Work] iO + OWF	[This Work] iO + OWF

All Made Collusion Resistant!

Unclonable Decryption (2->3) [This Work]

- **KeyGen**(λ): outputs pk pk $\lvert$🔑$\rangle$ $\lvert$🔑$\rangle$

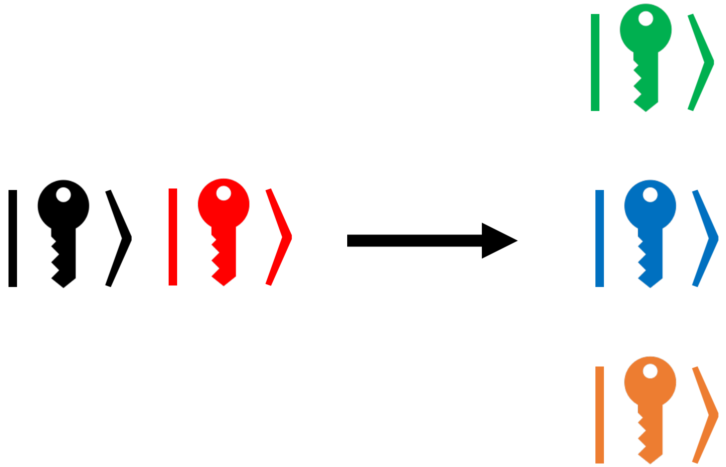
- **Enc**(pk, pk, m): outputs c=

  Enc(pk, m) **Enc**(pk, m)

- **Dec**($\lvert$🔑$\rangle$,$\lvert$🔑$\rangle$, c):
  - Use either key to decrypt any of the cipher $c = c_1\ c_2$

$c_1 \xrightarrow[\lvert 🔑 \rangle]{} m_1$ $c_2 \xrightarrow[\lvert 🔑 \rangle]{} m_2$

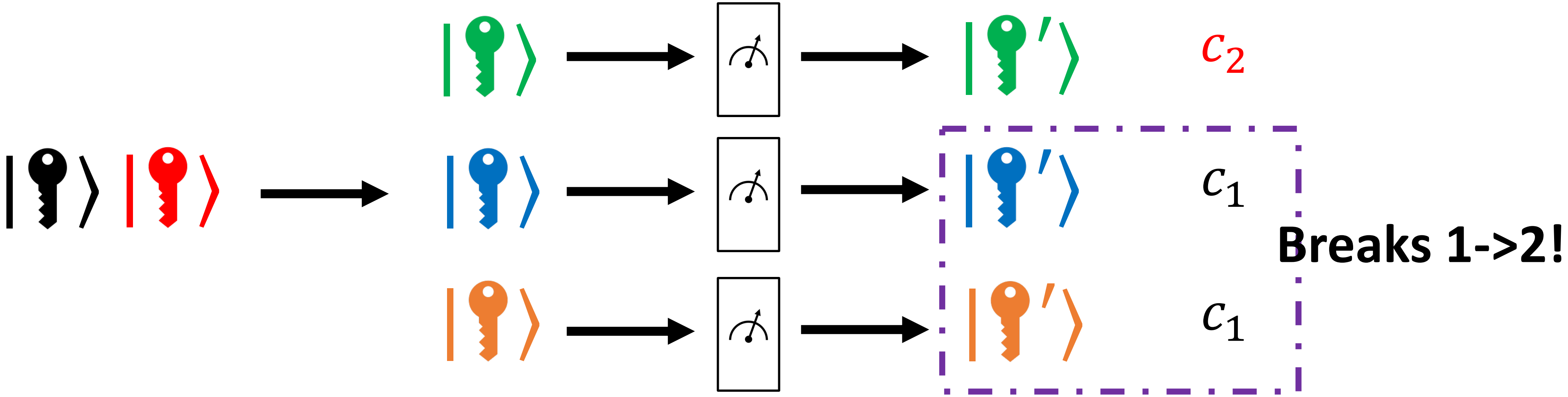
Reduction idea: reduce to 1-2 security



$c = c_1 \, {\color{red} c_2}$

- Produce three keys
- Each decrypts one of c_1 and ${\color{red} c_2}$
- (Classical) Pigeonhole principle?
  - **Not working!**

Reduction idea [This work]

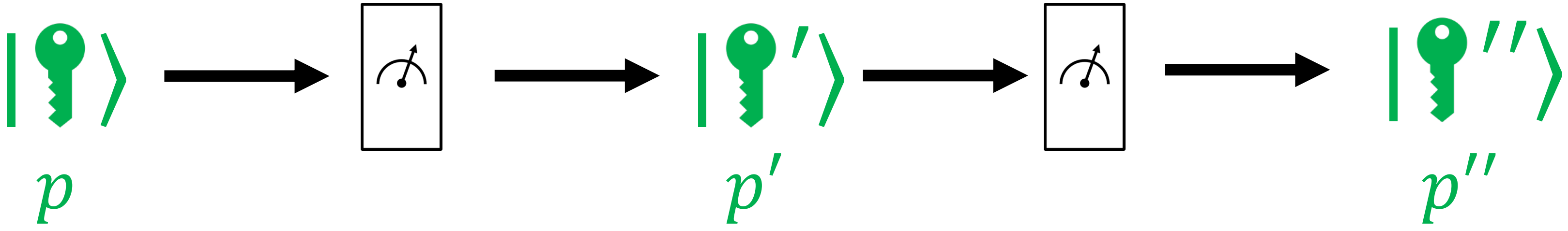


$c = c_1\ c_2$

Technique: Measure which ciphertext to decrypt

Technical Details: High-level

Pirate Decryotion keys:



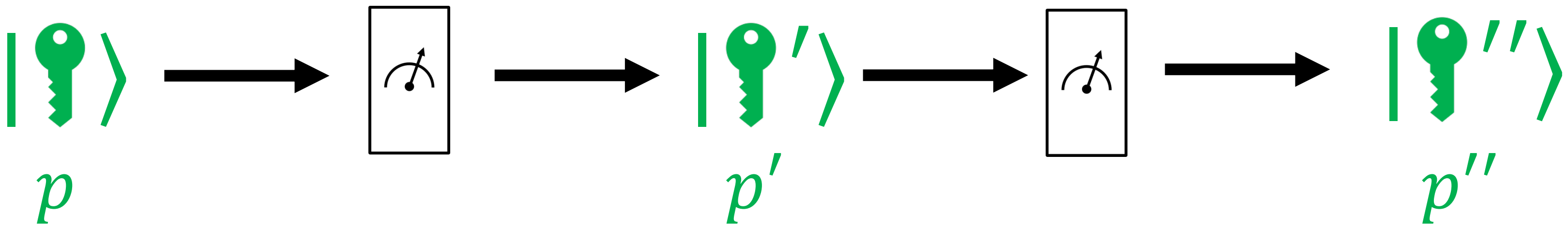
p p' p''

p: decryption probability over uniform $c = c_1\ c_2$

p': decryption probability over uniform $c = \perp\ c_2$

p'': decryption probability over uniform $c = \perp\ \perp$

Technical Details: High-level



p: decryption probability over uniform $c = c_1\ c_2$

p': decryption probability over uniform $c = \perp\ c_2$

If $p - p' \geq 1/\mathrm{poly}$, |🔑⟩ must decrypt c_1

More subtlety to handle!

Open Problems

- Unbounded Collusion-Resistant? i.e. KeyGen does not depend on number of keys issued
- Collusion-resistant copy protection for all unlearnable functions?

Thank you!